High-energy factorization beyond leading order

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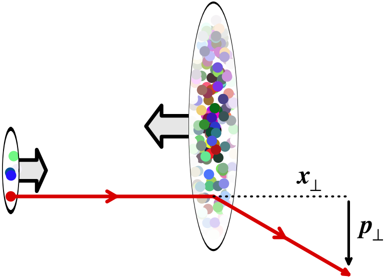
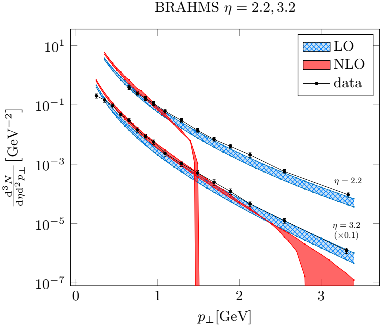
IPhT Saclay & CNRS

w/ A.H. Mueller and D.N. Triantafyllopoulos, arXiv:1608.05293



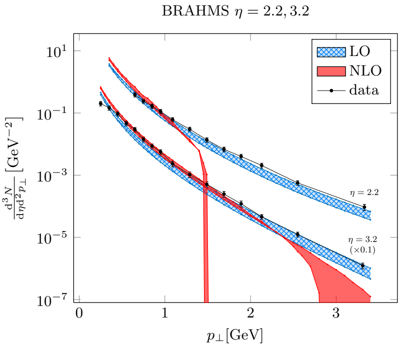


Introduction

- The CGC formalism is about to be promoted to NLO
  - NLO versions for the BK and B-JIMWLK equations
    (Balitsky and Chirilli, 2008, 2013; Kovner, Lublinsky, and Mulian, 2013)
  - NLO impact factor for particle production in pA collisions
    (Chirilli, Xiao, and Yuan, 2012; Mueller and Munier, 2012)
  - NLO impact factor for DIS
    (Balitsky and Chirilli, 2010-2013; Beuf, 2016-17)
- But the strict NLO approximations turn out to be problematic
- The NLO BK equation is unstable ... which calls for all-order "collinear" resummations *(cf. the previous talk by Dionysis T.)*
- Negative NLO cross-section for particle production in pA (and for DIS) *(Stasto, Xiao, and Zaslavsky, 2013)*

Particle production in pA collisions

- Good agreement at low $p_\perp$ ☺ ... but negative cross-section at larger $p_\perp$ ☹



- NLO calculation by CXY, 2012
- Numerics by Stasto, Xiao, and Zaslavsky, 2013
- The problem occurs for semi-hard momenta $p_\perp \sim Q_s$
- CGC is expected to apply there

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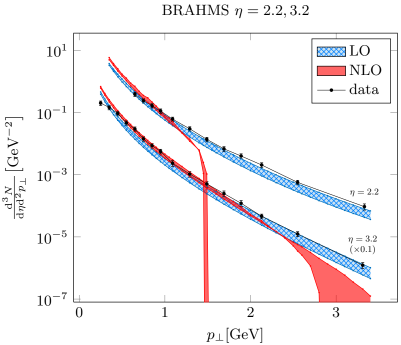

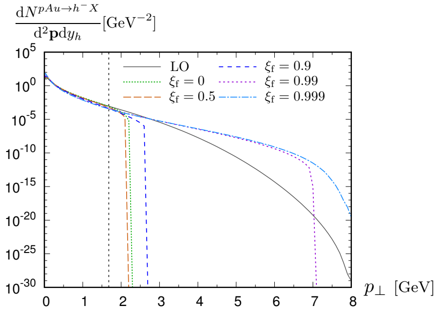

- Various proposals which alleviate the problem (pushed to higher $p_\perp$)
  - *Kang, Vitev, and Xing, arXiv:1403.5221*
  - *Altinoluk, Armesto, Beuf, Kovner, and Lublinsky, arXiv:1411.2869*
  - *Watanabe, Xiao, Yuan, Zaslavsky, arXiv:1505:05183*
  - *Ducloué, Lappi, and Zhu, arXiv:1604.00225*

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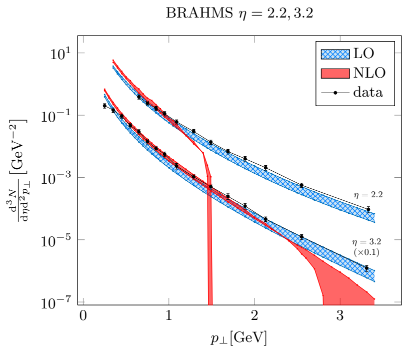
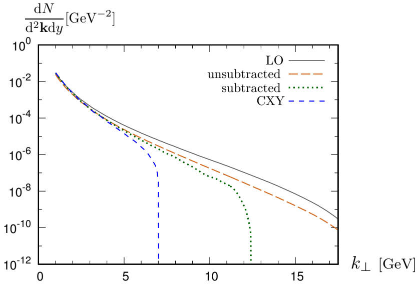




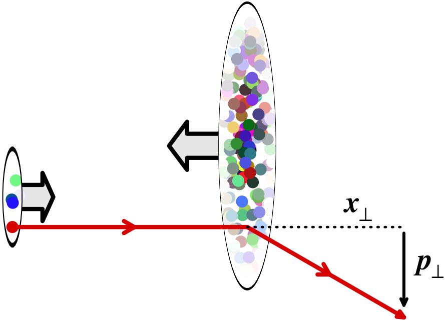
Ducloué, Lappi, and Zhu, arXiv:1703.04962

- An artefact of the 'k_T-factorization' commonly used at high energy
- New factorization scheme which avoids this problem

  (E.I., A. Mueller and D. Triantafyllopoulos, 2016)
- Positive cross-section (see also the next talk by Bertrand Ducloué)

Forward quark production in pA collisions

- A quark initially collinear with the proton acquires a transverse momentum $p_\perp$ via multiple scattering off the saturated gluons



$$\eta = -\ln\tan\frac{\theta}{2}$$

$$x_p \equiv \frac{p^+}{Q^+} = \frac{p_\perp}{\sqrt{s}}\,\mathrm{e}^{\eta}$$

$$X_g \equiv \frac{p^-}{P^-} = \frac{p_\perp}{\sqrt{s}}\,\mathrm{e}^{-\eta}$$

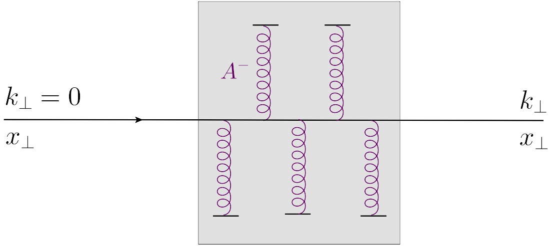
$X_g \ll x_p$ when $\eta > 0$

  - η : quark rapidity in the COM frame
  - x_p : longitudinal fraction of the quark in the proton
  - X_g : longitudinal fraction of the gluon in the target

- Gluons in the nucleus have a typical transverse momentum $k_\perp \sim Q_s(X_g)$

Multiple scattering

- Multiple scattering can be resummed in the eikonal approximation
  - fixed transverse coordinate & color precession

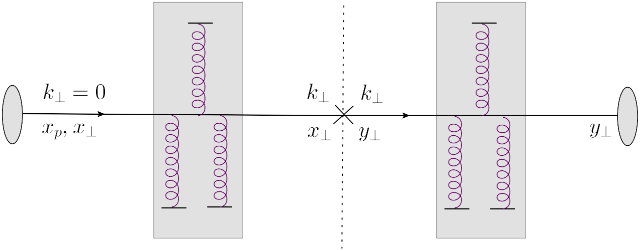


Amplitude: $$\mathcal{M}_{ij}(\boldsymbol{k}_\perp) \equiv \int \mathrm{d}^2\boldsymbol{x}_\perp\, \mathrm{e}^{-i\boldsymbol{x}_\perp\cdot\boldsymbol{k}_\perp}\, V_{ij}(\boldsymbol{x}_\perp)$$

Wilson line: $$V(\boldsymbol{x}_\perp) = \mathrm{P}\exp\left\{ ig\int \mathrm{d}x^+ A^-_a(x^+, \boldsymbol{x}_\perp)t^a \right\}$$

- A^-_a : color field representing small-x gluons in the nucleus

Multiple scattering



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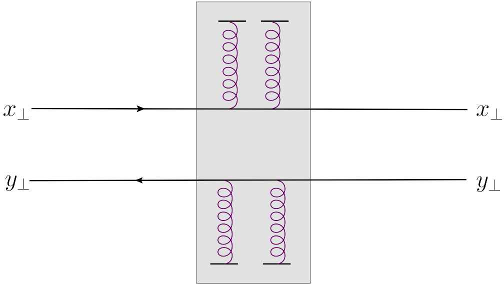
Cross-section: $$\frac{\mathrm{d}\sigma}{\mathrm{d}\eta\mathrm{d}^2\boldsymbol{k}_\perp} \simeq x_p q(x_p, Q^2) \, \frac{1}{N_c} \left\langle \sum_{ij} |\mathcal{M}_{ij}(\boldsymbol{k}_\perp)|^2 \right\rangle_{X_g}$$

- Average over the color fields A^- in the target (CGC)
- Two Wilson lines at different transverse coordinates, traced over color

Dipole picture

- Equivalently: the elastic S-matrix for a $q\bar{q}$ color dipole



$$S(\boldsymbol{x}, \boldsymbol{y}; X_g) \equiv \frac{1}{N_c} \left\langle \mathrm{tr}\left[V(\boldsymbol{x})V^\dagger(\boldsymbol{y})\right]\right\rangle_{X_g}$$

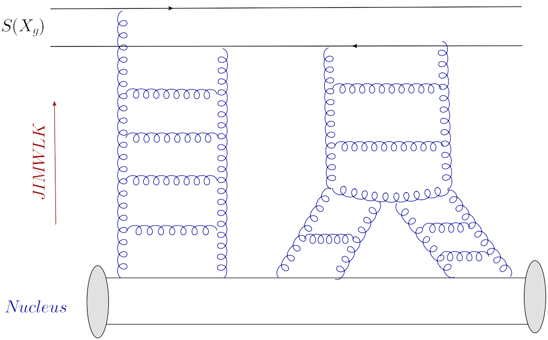
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\eta \mathrm{d}^2\boldsymbol{k}} \simeq x_p q(x_p) \int_{\boldsymbol{x},\boldsymbol{y}} \mathrm{e}^{-\mathrm{i}(\boldsymbol{x}-\boldsymbol{y})\cdot\boldsymbol{k}}\, S(\boldsymbol{x}, \boldsymbol{y}; X_g)$$

- F. transform $\mathcal{S}(\boldsymbol{k}, X_g)$: "unintegrated gluon distribution", or "dipole TMD"

The target average: CGC

$$\left\langle \mathrm{tr}\left(V_{\boldsymbol{x}} V_{\boldsymbol{y}}^\dagger\right)\right\rangle_{X_g} = \int [DA^-]\, \mathcal{W}_{X_g}[A^-]\, \mathrm{tr}\left(V_{\boldsymbol{x}} V_{\boldsymbol{y}}^\dagger\right)$$

- $\mathcal{W}_{X_g}[A^-]$: functional probability distribution for the target color fields
- High-energy evolution (JIMWLK) in presence of gluon saturation

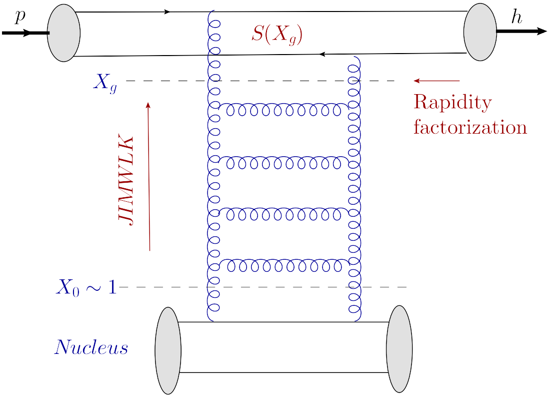


- Each emission brings in a factor $\bar{\alpha}_s \ln(1/X_g) \gtrsim \mathcal{O}(1)$

LO Hybrid Factorization

(Dumitru, Hayashigaki, and Jalilian-Marian, arXiv:hep-ph/0506308).

$$\frac{\mathrm{d}\sigma_h}{\mathrm{d}\eta\mathrm{d}^2\boldsymbol{p}} = \int \frac{\mathrm{d}z}{z^2}\, x_p q(x_p,\mu^2) \left[\int_{\boldsymbol{x},\boldsymbol{y}} \mathrm{e}^{-\mathrm{i}(\boldsymbol{x}-\boldsymbol{y})\cdot\boldsymbol{k}}\; S(\boldsymbol{x},\boldsymbol{y};X_g)\right] D_{h/q}(z,\mu^2)$$



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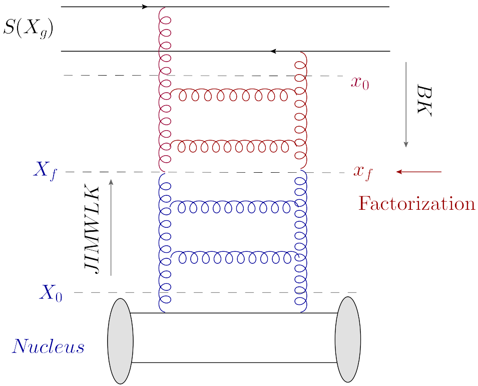
- Collinear factorization for the incoming proton/outgoing hadron
  - LO DGLAP evolution for quark distribution/ fragmentation
- High-energy (CGC) factorization for the quark-nucleus scattering
  - LO JIMWLK (BK) for target gluon distribution (dipole S-matrix)
- Natural, but non-trivial already at leading order
  - one needs to prove the factorization of the two types of evolution
- The dipole picture is preserved by the high-energy evolution up to NLO *(Kovchegov and Tuchin, 2002; Mueller and Munier, 2012)*

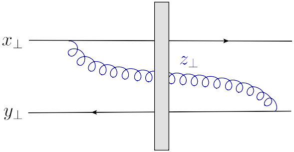
Shifting to projectile evolution

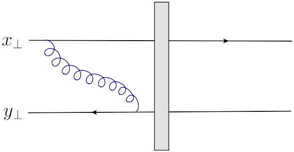
- The LO evolution can be shared between dilute projectile and dense target
  - JIMWLK evolution in $X = p^-/P^-$ from X_0 down to X_f
  - BK evolution in $x = p^+/Q^+$ from x_0 down to x_f
  - energy-momentum conservation implies $X_f = X_g/x_f$



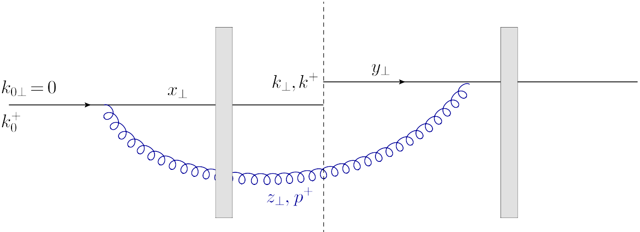
LO BK equation (1)

- The evolution of the dilute projectile (here, a dipole) is conceptually simpler





- Exchange graphs for the dipole:
  - the gluon crosses the shockwave ("N_c-terms"), or not ("C_F-terms")
- "Real gluon" correction to particle production: the gluon crosses the cut



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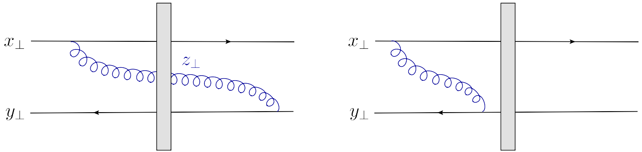
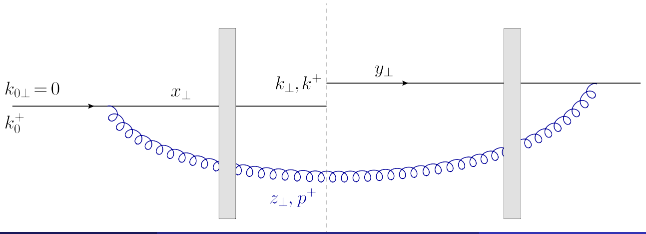
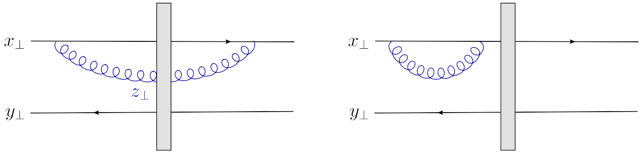


- Exchange graphs for the dipole:
  - the gluon crosses the shockwave ("N_c-terms"), or not ("C_F-terms")
- Effectively a C_F-term (gluon not measured): $\tilde{V}(\boldsymbol{z})\tilde{V}^\dagger(\boldsymbol{z}) = 1$

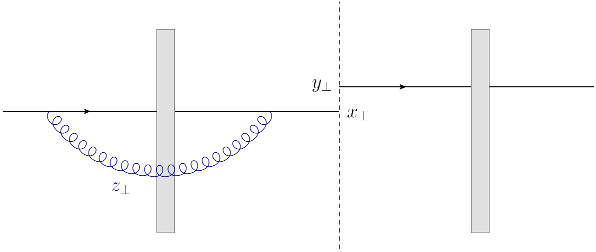


LO BK equation (2)

- The evolution of the dilute projectile (here, a dipole) is conceptually simpler



- Self-energy graphs for the dipole
  - the gluon crosses the shockwave ("N_c-terms"), or not ("C_F-terms")
- "Virtual" corrections to quark production: the gluon does not cross the cut



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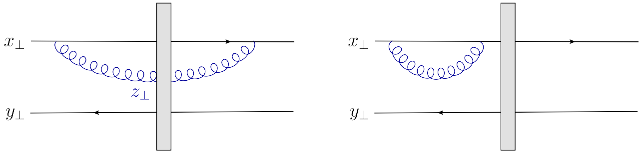

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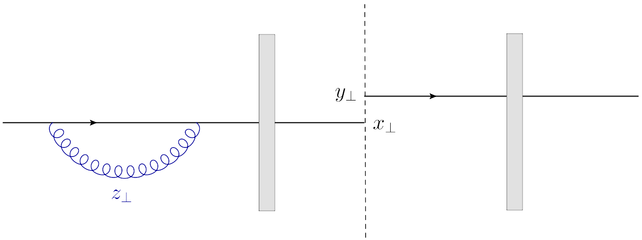
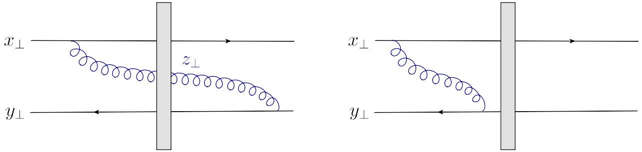


LO BK equation (3)

- Dipole evolution: the gluon crosses the shockwave, or not



- Evolution equation for the dipole S–matrix $S_{\boldsymbol{xy}}(Y)$ with $Y \equiv \ln(1/x)$

$$\frac{\partial S_{\boldsymbol{xy}}}{\partial Y} = \frac{\alpha_s N_c}{2\pi^2} \int \mathrm{d}^2\boldsymbol{z}\, \frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{y}-\boldsymbol{z})^2} \left[S_{\boldsymbol{xz}}S_{\boldsymbol{zy}} - S_{\boldsymbol{xy}}\right]$$

- Large N_c: gluon emission $\approx$ dipole splitting and $C_F \simeq N_c/2$

- Non-linear extension of BFKL equation ensuring unitarity: $T \equiv 1 - S \leq 1$

LO BK equation (3)

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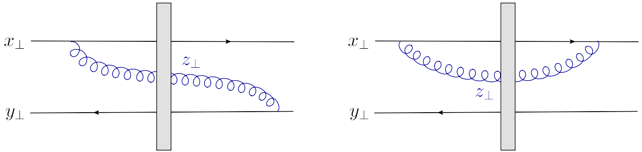

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- Dipole kernel: probability for the dipole to emit a soft gluon at $\boldsymbol{z}$

$$\frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{y}-\boldsymbol{z})^2} = \Bigg[\underbrace{2\,\frac{(x^i-z^i)(y^i-z^i)}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{x}-\boldsymbol{z})^2}}_{\text{real}} \underbrace{-\frac{1}{(\boldsymbol{x}-\boldsymbol{z})^2} - \frac{1}{(\boldsymbol{z}-\boldsymbol{y})^2}}_{\text{virtual}} \Bigg]$$

LO BK equation (3)

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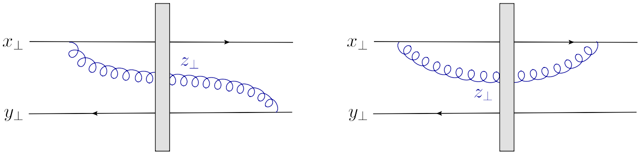

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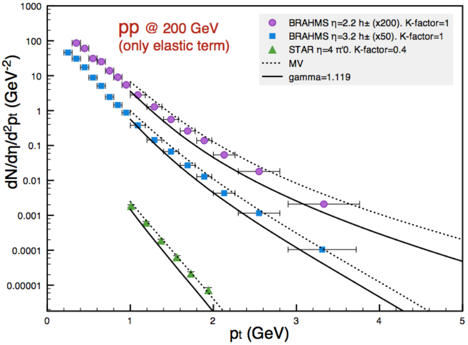
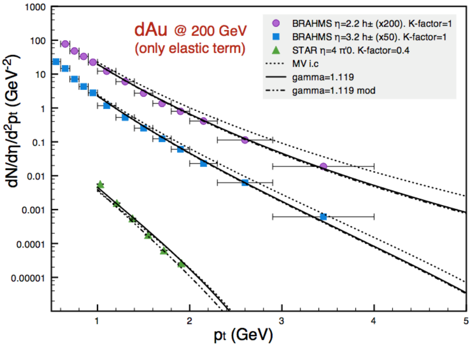
- The dominant large-$z_\perp$ behavior cancels between "real" and "virtual" graphs

LO phenomenology: rcBK

(Albacete, Dumitru, Fujii, Nara, arXiv:1209:2001)

- Fit parameters: initial condition for the rcBK equation + K-factors





$$\left.\frac{\mathrm{d}N_h}{\mathrm{d}\eta\,\mathrm{d}^2\boldsymbol{k}}\right|_{\rm LO} = K_h \int_{x_p}^{1} \frac{\mathrm{d}z}{z^2}\,\frac{x_p}{z} q\left(\frac{x_p}{z}\right) \mathcal{S}\left(\frac{\boldsymbol{k}}{z}, X_g\right) D_{h/q}(z)$$

- What about the (other) NLO corrections ?

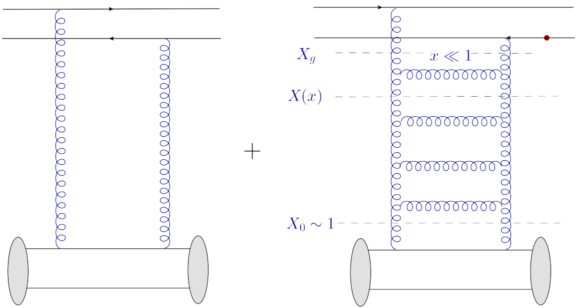
LO BK evolution in integral form

$$\left.\frac{\mathrm{d}N}{\mathrm{d}\eta\mathrm{d}^2\boldsymbol{k}}\right|_{\mathrm{LO}} = x_p q(x_p)\,\mathcal{S}(\boldsymbol{k},X_g)\,, \qquad \mathcal{S}(\boldsymbol{k},X_g) = \int \mathrm{d}^2\boldsymbol{r}\,\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}S(\boldsymbol{r},X_g)$$

- $S(\boldsymbol{r},X_g)$ is the solution to the LO BK equation and can be written as

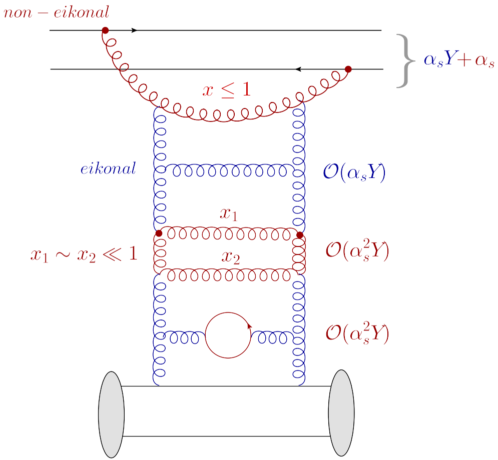
$$S_{\boldsymbol{xy}}(X_g) = S_{\boldsymbol{xy}}(X_0) + \bar{\alpha}_s \int_{X_g}^{1} \frac{\mathrm{d}x}{x} \int_{\boldsymbol{z}} \frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{y}-\boldsymbol{z})^2}\Big[S_{\boldsymbol{xz}}S_{\boldsymbol{zy}} - S_{\boldsymbol{xy}}\Big](X(x))$$

- The emission of the first gluon, with energy fraction x, is explicit.



NLO corrections to particle production

- $\mathcal{O}(\alpha_s)$ corrections to the evolution (i.e. to the dipole/BFKL kernel)
  - a pair of soft partons which are close in rapidity: $x_1 \sim x_2 \ll 1$
  - a contribution of $\mathcal{O}(\alpha_s^2 Y) \Rightarrow \mathcal{O}(\alpha_s)$ correction to the kernel



NLO corrections to particle production

- $\mathcal{O}(\alpha_s)$ corrections to the evolution *(cf. the talk by Dionysis T.)*
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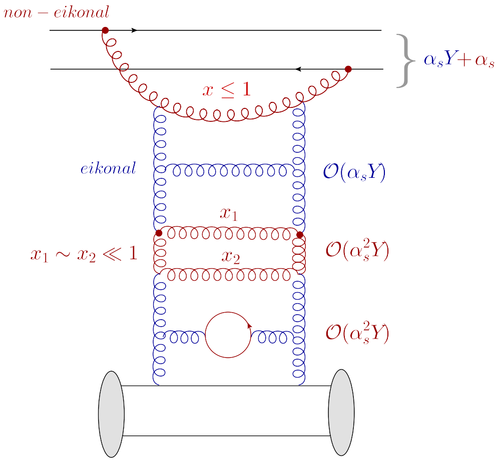
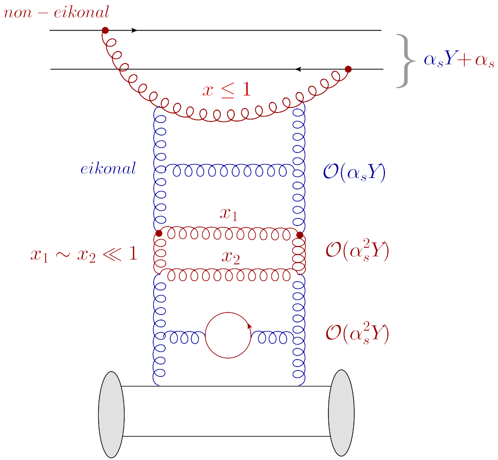
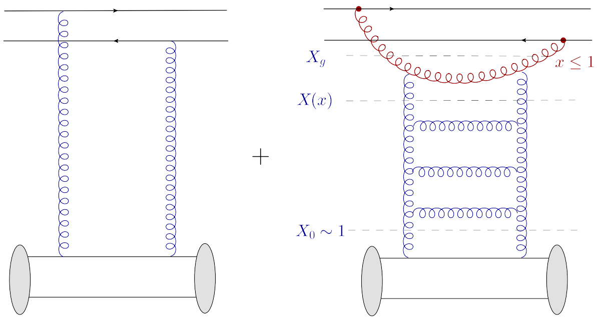


NLO corrections to particle production

- $\mathcal{O}(\alpha_s)$ corrections to the impact factor *(Chirilli, Xiao, and Yuan, 2012)*
  - the first emitted gluon is close in rapidity to the dipole: $x \sim \mathcal{O}(1)$
  - its emission must be computed with exact kinematics (beyond eikonal)



Just the impact factor

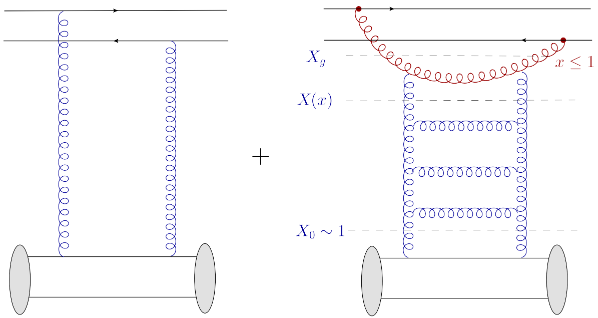


- The ensuing cross-section can be succinctly (but formally) written as

$$\frac{\mathrm{d}N}{\mathrm{d}\eta \mathrm{d}^2\boldsymbol{k}} = \mathcal{S}_0(\boldsymbol{k}) + \bar{\alpha}_s \int_{X_g}^{1} \frac{\mathrm{d}x}{x} \mathcal{K}(x)\, \mathcal{S}_{q\bar{q}g}\big(\boldsymbol{k}, X(x)\big)\,; \quad X(x) \simeq \frac{X_g}{x}$$

- $\mathcal{K}(x)$: kernel for emitting a gluon with exact kinematics *(CXY)*
- The evolution is evaluated at the floating scale $X(x)$: non-local in rapidity

Just the impact factor



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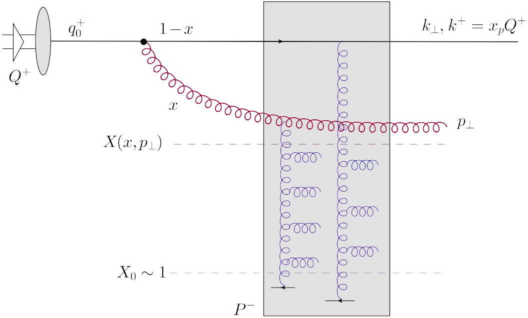
- This is in fact our proposal for the NLO factorization

  (E.I., A. Mueller and D. Triantafyllopoulos, arXiv:1608.05293)

- Almost obviously right (by construction) & positive definite

Physical picture & kinematics

- The quark q_0^+ from the proton radiates a recoil gluon with energy fraction x while scattering off the target



- LC energy conservation:

$$\frac{k_\perp^2}{2(1-x)q_0^+} + \frac{p_\perp^2}{2xq_0^+} = XP^-$$

$$\Longrightarrow \quad X = X(x, p_\perp)$$

- simplifies when $k_\perp \simeq p_\perp \gg Q_s$

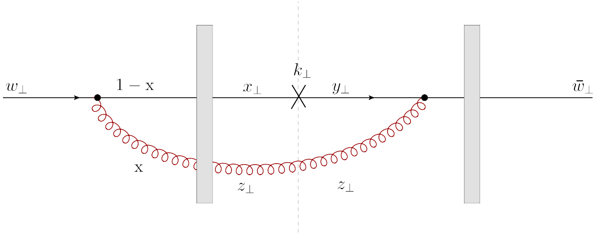
$$X(x) \simeq \frac{k_\perp^2}{xs} = \frac{X_g}{x}$$

- $X \leq 1 \Longrightarrow x \geq X_g$

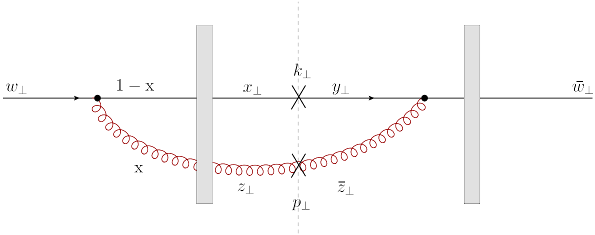
- The final quark carries a longitudinal momentum $k^+ = (1-x)q_0^+ = x_p Q^+$
- The original quark has a longitudinal fraction $q_0^+/Q^+ = x_p/(1-x)$

The NLO impact factor: "real" terms

- The same graphs as for one-step evolution, but with exact kinematics



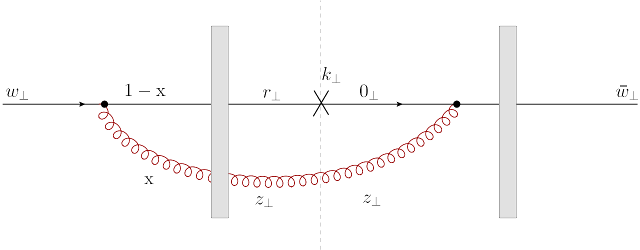
- Cyrille M. has computed quark-gluon production back in 2007



- "integrate out the gluon" $\int \mathrm{d}^2\boldsymbol{p}_\perp \implies \boldsymbol{z}_\perp = \bar{\boldsymbol{z}}_\perp$

"Real" N_c-terms

- The same graphs as for one-step evolution, but with exact kinematics

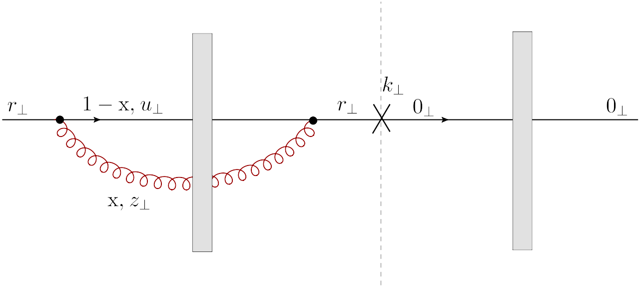


- N.B. slight change in notations: $\boldsymbol{x} \to \boldsymbol{r}$, $\boldsymbol{y} \to 0$ (translation invariance)
- $\boldsymbol{w} = x\boldsymbol{z} + (1-x)\boldsymbol{r}, \quad \bar{\boldsymbol{w}} = x\boldsymbol{z}$: center of energy of the final qg pair

$$\bar{\alpha}_s(k_\perp) \int_{\boldsymbol{r}} \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{r}} \int_{\boldsymbol{z}} \int_{X_g}^{1} \mathrm{d}x \, \frac{1+(1-x)^2}{2x} \, \frac{(\boldsymbol{z}-\boldsymbol{r})\cdot\boldsymbol{z}}{(\boldsymbol{z}-\boldsymbol{r})^2\boldsymbol{z}^2} \, S(\boldsymbol{r}-\boldsymbol{z})S(\boldsymbol{z}-\bar{\boldsymbol{w}})$$

- Same WW emission kernel as in the eikonal approx: gluon energy fraction x enters only via the coordinate $\bar{\boldsymbol{w}}$ of the parent quark

- There are corresponding “virtual” graphs (first computed by CXY, 2012)



$$-\bar{\alpha}_s(k_\perp)\int_{\boldsymbol{r}} \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\int_{\boldsymbol{z}}\int_{X_g}^{1}\mathrm{d}x\,\frac{1+(1-x)^2}{2x}\,\frac{1}{(\boldsymbol{z}-\boldsymbol{r})^2}\,S(\boldsymbol{u}-\boldsymbol{z})S(\boldsymbol{z})$$

- Here, $\boldsymbol{u}$ is defined by $\boldsymbol{r}\equiv x\boldsymbol{z}+(1-x)\boldsymbol{u}$

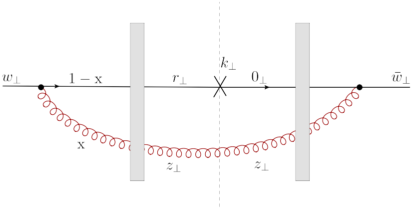
Putting "real" and "virtual" together

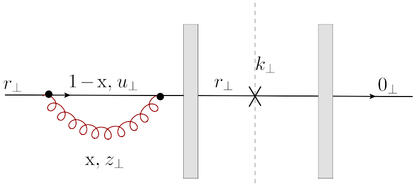
$$\bar{\alpha}_s(k_\perp)\int_{\boldsymbol{r}} \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{r}} \int_{\boldsymbol{z}}\int_{X_g}^{1}\mathrm{d}x\,\frac{1+(1-x)^2}{2x}$$
$$\left\{\frac{x_p}{1-x}q\Big(\frac{x_p}{1-x}\Big)\frac{(\boldsymbol{z}-\boldsymbol{r})\cdot\boldsymbol{z}}{(\boldsymbol{z}-\boldsymbol{r})^2\boldsymbol{z}^2}\,S(\boldsymbol{r}-\boldsymbol{z})S(\boldsymbol{z}-\bar{\boldsymbol{w}}) - \frac{x_p q(x_p)}{(\boldsymbol{z}-\boldsymbol{r})^2}\,S(\boldsymbol{u}-\boldsymbol{z})S(\boldsymbol{z})\right\}$$

- For generic $x < 1$, the "real" and "virtual" terms have
  - different coordinate arguments for the S-matrices
  - different weightings with the quark distribution
- For small $x \ll 1$: $\bar{\boldsymbol{w}} = x\boldsymbol{z} \to 0$ and $\boldsymbol{r} = x\boldsymbol{z} + (1-x)\boldsymbol{u} \implies \boldsymbol{u} \to \boldsymbol{r}$
  - for $x \ll 1$ one recovers the dipole kernel, hence LO BK, as expected
- For generic x, cancellations between "real" and "virtual" don't work anymore
- Complications for a running coupling: *see the talk by Bertrand Ducloué*

The C_F-terms: emergence of DGLAP

- The graphs in which the gluon is (effectively) non-interacting





$$-\frac{\alpha_s C_F}{2\pi}\int_{\boldsymbol{r}} \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{r}} \int_{\boldsymbol{z}}\int_{X_g}^{1}\mathrm{d}x\,\frac{1+(1-x)^2}{2x}$$

$$\left\{\frac{x_p}{1-x}q\left(\frac{x_p}{1-x}\right)\frac{(\boldsymbol{z}-\boldsymbol{r})\cdot\boldsymbol{z}}{(\boldsymbol{z}-\boldsymbol{r})^2\boldsymbol{z}^2}\,S(\boldsymbol{r})-\frac{x_p q(x_p)}{(\boldsymbol{z}-\boldsymbol{r})^2}\,S(\boldsymbol{r})\right\}$$

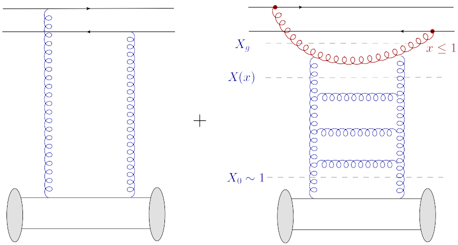
- Large $z \gg r$ and $\xi \equiv 1-x$: logarithmically IR-divergent integral over $\boldsymbol{z}$, recognized as one step in DGLAP (here, for the quark distribution)

$$-\frac{\alpha_s C_F}{2\pi}\,\mathcal{S}(\boldsymbol{k})\int_{x_p}^{1}\mathrm{d}\xi\left(\frac{1+\xi^2}{1-\xi}\right)_+\frac{x_p}{\xi}q\left(\frac{x_p}{\xi}\right)\int_r\frac{\mathrm{d}\boldsymbol{z}^2}{\boldsymbol{z}^2}\;\equiv x_p\Delta q(x_p,1/r^2)\mathcal{S}(\boldsymbol{k})$$

Factorization at NLO

- Return to our compact but formal expression for the cross-section at NLO:

$$\frac{\mathrm{d}N}{\mathrm{d}\eta \mathrm{d}^2\boldsymbol{k}} = \mathcal{S}_0(\boldsymbol{k}) + \bar{\alpha}_s \int_{X_g}^{1} \frac{\mathrm{d}x}{x} \mathcal{K}(x)\, \mathcal{S}_{q\bar{q}g}\big(\boldsymbol{k}, X(x)\big)\,; \quad X(x) \simeq \frac{X_g}{x}$$



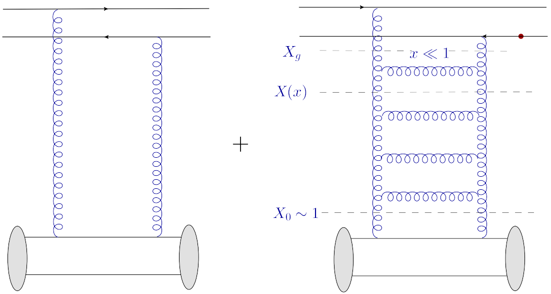
- Quark distribution and fragmentation functions are implicit
- $\mathcal{K}(x)$ includes the N_c-terms and the finite pieces of the C_F-terms
- The evolution is evaluated at the floating scale $X(x)$: non-local in rapidity
- Where is the origin of the negativity problem by CXY ?

k_T–factorization

- CXY insisted in writing a k_T–factorized expression for the cross-section
  - explicitly separate LO and NLO contributions to the impact factor
  - evaluate the NLO correction at the rapidity of the projectile: local in X
- This is indeed allowed to NLO accuracy ... at least formally
- ... But this is precisely the source of the problems !
  - separating LO and NLO contributions involves a high degree of fine-tuning
  - locality in X is merely an approximation, which becomes dangerous in this context: an over-subtraction

Recovering the LO result

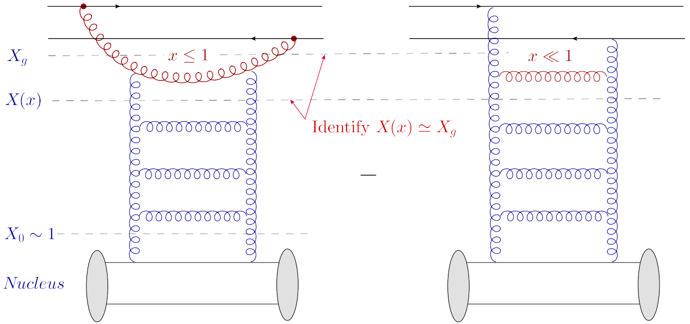
- The LO result is readily obtained as the limit $x \ll 1$ of our factorization: $x \ll 1 \implies \mathcal{K}(x) \to \mathcal{K}(0)$



$$\left.\frac{\mathrm{d}N}{\mathrm{d}\eta \mathrm{d}^2\boldsymbol{k}}\right|_{\mathrm{LO}} = \mathcal{S}_0(\boldsymbol{k}) + \bar{\alpha}_s \int_{X_g}^{1} \frac{\mathrm{d}x}{x}\, \mathcal{K}(0)\, \mathcal{S}_{q\bar{q}g}\big(\boldsymbol{k}, X(x)\big) = \mathcal{S}(\boldsymbol{k}, X_g)$$

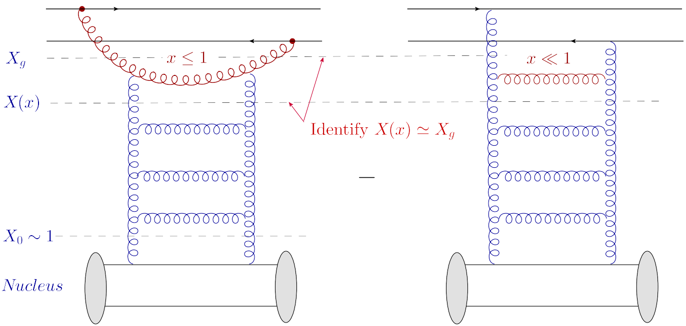
- This is just the BK equation in condensed notations

- One can of course subtract this LO piece from the NLO result



$$\frac{\mathrm{d}N}{\mathrm{d}\eta \mathrm{d}^2\boldsymbol{k}} = \mathcal{S}(\boldsymbol{k}, X_g) + \bar{\alpha}_s \int_{X_g}^{1} \frac{\mathrm{d}x}{x} \left[\mathcal{K}(x) - \mathcal{K}(0)\right] \mathcal{S}_{q\bar{q}g}\big(\boldsymbol{k}, X(x)\big)$$

- A mathematical identity ... but potentially tricky in practice !
  - one adds and subtracts a large, LO, contribution
- Not yet k_T-factorization: non-local in rapidity $(X(x))$

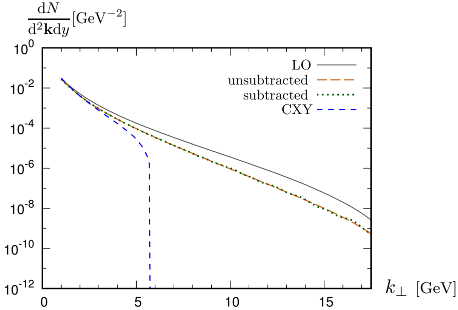


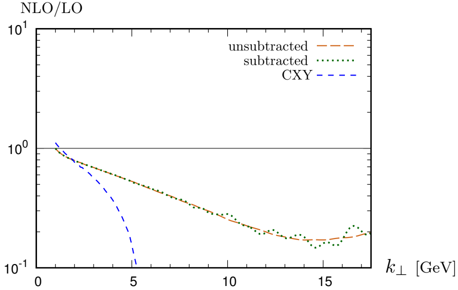
$$\frac{\mathrm{d}N}{\mathrm{d}\eta \mathrm{d}^2\boldsymbol{k}} = \mathcal{S}(\boldsymbol{k}, X_g) + \bar{\alpha}_s \int_0^1 \frac{\mathrm{d}x}{x} \left[\mathcal{K}(x) - \mathcal{K}(0)\right] \mathcal{S}_{q\bar{q}g}\big(\boldsymbol{k}, X_g\big)$$

- To NLO accuracy, one can perform additional approximations:
  - replace $\mathcal{S}\big(X(x)\big) \simeq \mathcal{S}(X_g)$ (since integral dominated by $x \sim 1$)
  - set $X_g \to 0$ in the lower limit ('plus prescription')
- Local in rapidity : k_T-factorization as presented by CXY

Numerical results: Fixed coupling $\alpha_s = 0.2$

Ducloué, Lappi, Zhu, arXiv:1703.04962 (see the talk by Bertrand Ducloué)

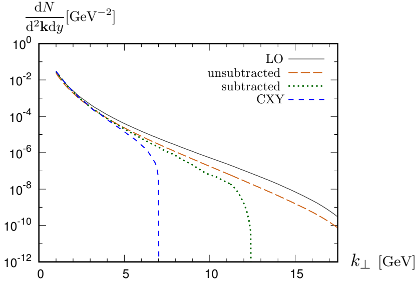


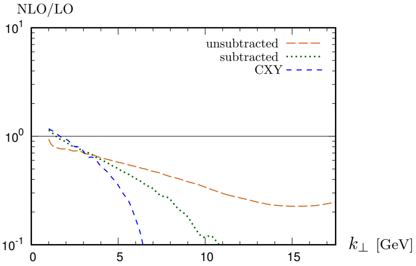


- Large NLO correction: $\gtrsim 50\,\%$ for $k_\perp \geq 5$ GeV
- The same results with and without subtracting the LO result
  - small oscillations in "subtracted" due to numerical errors
- $k_\perp$-factorization (CXY) rapidly becomes negative : over-subtraction

Numerical results: Running coupling

Ducloué, Lappi, Zhu, arXiv:1703.04962 (see the talk by Bertrand Ducloué)





- The running of the coupling renders the problem even more subtle:
  - already the "subtracted" result becomes negative
  - the "CXY" curve becomes negative even faster
- Mismatch between the running coupling prescriptions used ...
  - in coordinate space (for solving the BK equation)
  - ... and in momentum space (for computing the NLO impact factor)

Conclusions

- The hybrid factorization for pA collisions formally holds to NLO

  .... but this is rather subtle !

- On the nuclear side, one cannot 'automatically' apply k_T-factorization
  - one cannot enforce locality in rapidity in the NLO impact factor
  - with running coupling, the separation between LO and NLO contributions becomes dangerous

- But this is actually not needed: factorization is more general
  - no explicit separation between LO and NLO
  - non-local in rapidity

- Sensible physical results: positive cross-section, but smaller than at LO

- A similar strategy is necessary when computing DIS at NLO (dipole picture)

  Ducloué, Hänninen, Lappi, and Zhu, arXiv:1708.07328

- See the subsequent talk by Bertrand Ducloué !

Adding a running coupling

- The NLO impact factor is generally computed in momentum space
  - natural to use a running coupling $\bar{\alpha}_s(k_\perp^2)$ (at least for $k_\perp^2 \gtrsim Q_s^2$)

$$\frac{\mathrm{d}N}{\mathrm{d}\eta \mathrm{d}^2\boldsymbol{k}} = \mathcal{S}_0(\boldsymbol{k}) + \bar{\alpha}_s(k_\perp^2) \int_{X_g}^1 \frac{\mathrm{d}x}{x}\, \mathcal{K}(x)\, \mathcal{S}_{q\bar{q}g}\big(\boldsymbol{k}, X(x)\big)$$

  - more generally: $\bar{\alpha}_s(k_{\max}^2)$

- Dipole S-matrix is computed by solving rcBK in coordinate space

$$S_{\boldsymbol{xy}}(X_g) = S_{\boldsymbol{xy}}(X_0) + \int_{X_g}^1 \frac{\mathrm{d}x}{x} \int_{\boldsymbol{z}} \bar{\alpha}_s(r_{\min}^2) \frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{y}-\boldsymbol{z})^2} \Big[S_{\boldsymbol{xz}} S_{\boldsymbol{zy}} - S_{\boldsymbol{xy}}\Big]$$

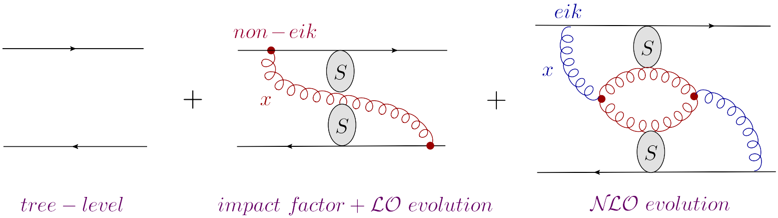
  - $r_{\min} \equiv \min\big\{|\boldsymbol{x}-\boldsymbol{y}|, |\boldsymbol{x}-\boldsymbol{z}|, |\boldsymbol{y}-\boldsymbol{z}|\big\}$

- Running coupling and Fourier transform do not "commute" with each other

Completing the NLO evolution

(E.I., A. Mueller and D. Triantafyllopoulos, arXiv:1608.05293)

- Recall: the NLO BK evolution also involves 2-loop graphs



$$\frac{\mathrm{d}N}{\mathrm{d}\eta \mathrm{d}^2\boldsymbol{k}} = \mathcal{S}_0 + \bar{\alpha}_s \int_{X_g}^{1} \frac{\mathrm{d}x}{x} \mathcal{K}(x)\, \mathcal{S}\big(X(x)\big) + \bar{\alpha}_s^2 \int_{X_g}^{1} \frac{\mathrm{d}x}{x} \mathcal{K}_2(0)\, \mathcal{S}\big(X(x)\big)$$

- $\mathcal{K}_2(0)$: NLO correction to the BK kernel with collinear improvement

  (Balitsky and Chirilli, 2008; Iancu et al, 2015)