Saturation Fronts in Resummed High Energy Evolution

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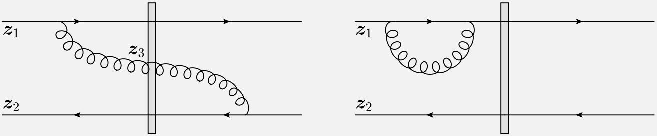
B. Ducloué, E. Iancu, A.H. Mueller, E. Iancu, DT, in preparation

Outline

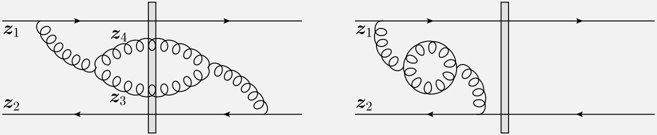
- Large transverse logarithms in NLO BK and instabilities
- Resummation (double logs) and stability
- Issues with resummed evolution in projectile rapidity (Y)
- Comparison of saturation fronts in Y and η
- Resummed evolution in target rapidity (η)

NLO BK Evolution

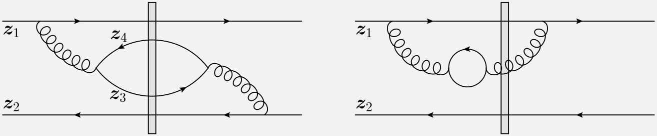
LO



NLO N_c



NLO N_f



Evolution of right moving projectile: modes with smaller longitudinal momentum k^+. Soft plus non-soft (in general).

NLO BK Evolution

Proj: (z_1, z_2), q^+, $q^- = 1/2z_{12}^2 q^+$ Targ: Q_0^2, q_0^-, $q_0^+ = Q_0^2/2q_0^-$

NLO evolution for $S_{12} \equiv S(z_1, z_2; Q_0^2; Y)$, with $Y = \ln q^+/q_0^+$:

$$\frac{\mathrm{d}S_{12}}{\mathrm{d}Y} = \frac{\bar{\alpha}_s}{2\pi}\int \frac{\mathrm{d}^2 z_3 z_{12}^2}{z_{13}^2 z_{32}^2}\Bigg[1 + \bar{\alpha}_s\Bigg(\underbrace{\bar{b}\ln z_{12}^2\mu^2 - \bar{b}\frac{z_{13}^2 - z_{23}^2}{z_{12}^2}\ln\frac{z_{13}^2}{z_{12}^2}}_{\text{RC: choose } \mu = 1/z_{ij\,\mathrm{min}}}$$

$$\underbrace{-\frac{1}{2}\ln\frac{z_{13}^2}{z_{12}^2}\ln\frac{z_{23}^2}{z_{12}^2}}_{\text{TO in Projectile}}\Bigg)\Bigg](S_{13}S_{32} - S_{12})$$

$$+\frac{\bar{\alpha}_s^2}{8\pi^2}\int \frac{\mathrm{d}^2 z_3 \mathrm{d}^2 z_4}{z_{34}^4}\Bigg(\underbrace{-2 + \frac{z_{13}^2 z_{24}^2 + z_{14}^2 z_{23}^2 - 4z_{12}^2 z_{34}^2}{z_{13}^2 z_{24}^2 - z_{14}^2 z_{23}^2}\ln\frac{z_{13}^2 z_{24}^2}{z_{14}^2 z_{23}^2}}_{\text{Proj \& Targ DGLAP} \rightsquigarrow -11/12}\Bigg)$$

$$\times(S_{13}S_{34}S_{42} - S_{13}S_{32})$$

$$+\mathcal{O}(N_f) + \mathcal{O}(1/N_c^2) + \text{regular terms}$$

Large Transverse Logarithms

Strongly ordered large perturbative dipoles (DLA)

$$z_{12} \ll z_{13} \simeq z_{23} \ll z_{14} \simeq z_{24} \simeq z_{34} \ll 1/Q_s$$

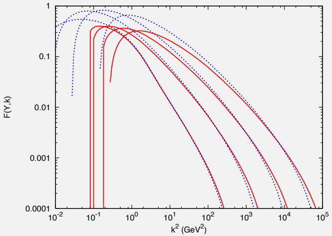
Linearize, large dipoles strong interaction, reals terms dominate

$$\frac{\mathrm{d}T_{12}}{\mathrm{d}Y} = \bar{\alpha}_s \int_{z_{12}^2}^{1/Q_s^2} \mathrm{d}z_{13}^2 \frac{z_{12}^2}{z_{13}^4} \left(1 - \frac{\bar{\alpha}_s}{2} \ln^2 \frac{z_{13}^2}{z_{12}^2} - \bar{\alpha}_s \frac{11}{12} \ln \frac{z_{13}^2}{z_{12}^2}\right) T_{13}$$

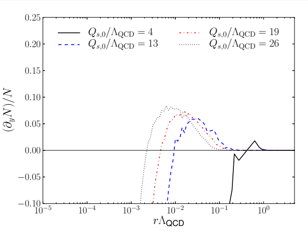
NLO$>$LO, unstable expansion in coupling

Even single iteration leads to negative solution

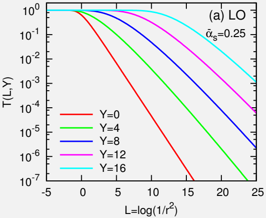
UNSTABLE NUMERICAL SOLUTIONS



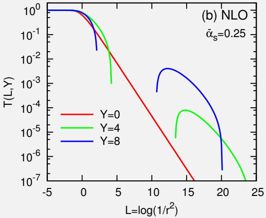
NLO BFKL + Sat Bound



NLO BK

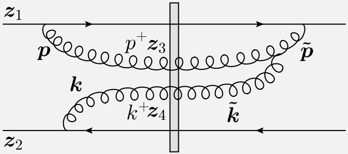


LO BK



LO BK + Double Log

Time Ordering and Resummation in DLA



Hard to soft projectile evolution: $\boldsymbol{k} \ll \boldsymbol{p}$ and $k^+ \ll p^+$

Time ordering non-trivial, requires: $\tau_k \sim k^+ z_4^2 \ll \tau_p \sim p^+ z_3^2$

Resum to all orders in a non-local equation

$$\frac{\mathrm{d}T_{12}(Y, z_{12}^2)}{\mathrm{d}Y} = \bar{\alpha}_s \int_{z_{12}^2}^{1/Q_s^2} \frac{\mathrm{d}z_{13}^2}{z_{13}^2} \frac{z_{12}^2}{z_{13}^2} \Theta\left(Y - \ln\frac{z_{13}^2}{z_{12}^2}\right) T\left(Y - \ln\frac{z_{13}^2}{z_{12}^2}, z_{13}^2\right)$$

Mathematically equivalent to local equation

$$\frac{\mathrm{d}T_{12}}{\mathrm{d}Y} = \bar{\alpha}_s \int_{z_{12}^2}^{1/Q_s^2} \frac{\mathrm{d}z_{13}^2}{z_{13}^2} \frac{z_{12}^2}{z_{13}^2} \frac{\mathrm{J}_1\left(2\sqrt{\bar{\alpha}_s \ln^2 \frac{z_{13}^2}{z_{12}^2}}\right)}{\sqrt{\bar{\alpha}_s \ln^2 \frac{z_{13}^2}{z_{12}^2}}} T_{13}$$

Resummation in BK

Match local resummed DLA equation to include BK physics

$$\frac{\mathrm{d}S_{12}}{\mathrm{d}Y} = \frac{\bar{\alpha}_s}{2\pi} \int \frac{\mathrm{d}^2 z_3 z_{12}^2}{z_{13}^2 z_{32}^2} \mathcal{K}_{\mathrm{DLA}} \left(\sqrt{\ln \frac{z_{13}^2}{z_{12}^2} \ln \frac{z_{23}^2}{z_{12}^2}} \right) (S_{13}S_{32} - S_{12})$$

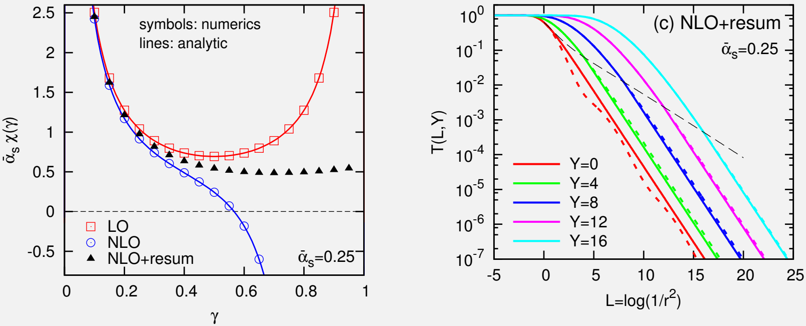
with

$$\mathcal{K}_{\mathrm{DLA}}(\rho) = \frac{\mathrm{J}_1\left(2\sqrt{\bar{\alpha}_s \rho^2}\right)}{\sqrt{\bar{\alpha}_s \rho^2}} = 1 - \frac{\bar{\alpha}_s \rho^2}{2} + \frac{(\bar{\alpha}_s \rho^2)^2}{12} + \cdots$$

Resums double logarithms to all orders (and nothing more)

LO BK + NLO double log when truncated to $\bar{\alpha}_s^2$

Numerical Solution



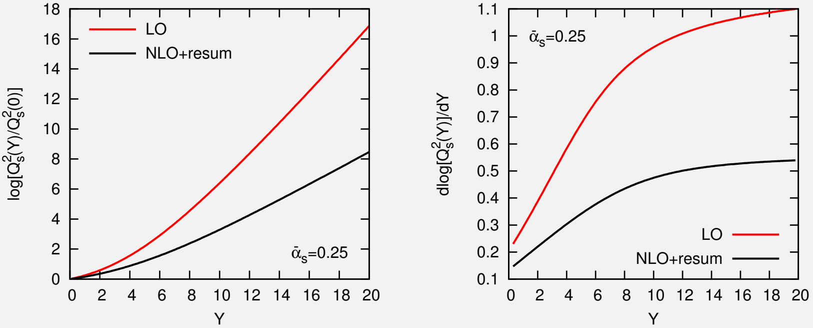
Linear on $z_{12}^{2\gamma} \equiv r^{2\gamma}$

Tail $\sim r^2$

$$\omega_{\rm LO} = \frac{\bar{\alpha}_s}{\gamma} + \frac{\bar{\alpha}_s}{1-\gamma} + \text{regular} \qquad \omega_{\rm NLO} = \frac{\bar{\alpha}_s}{\gamma} + \frac{\bar{\alpha}_s}{1-\gamma} - \frac{\bar{\alpha}_s^2}{(1-\gamma)^3} + \text{regular}$$

$$\omega = \frac{\bar{\alpha}_s}{\gamma} + \frac{1}{2}\left[-(1-\gamma) + \sqrt{(1-\gamma)^2 + 4\bar{\alpha}_s}\right] + \text{regular}$$

Numerical Solution



Considerable speed reduction, perhaps too much?

Target Rapidity η

Target rapidity determines the kinematics

$$\begin{aligned}\eta &\equiv \ln\frac{1}{x_{\rm Bj}} = \ln\frac{s}{Q^2} = \ln\frac{2q^+q_0^-}{2q^+q^-} = \ln\frac{q_0^-}{q^-} \\ &= \ln\frac{Q_0^2}{2q_0^+}\frac{2q^+}{Q^2} \to Y - \ln\frac{1}{r^2Q_0^2} \equiv Y - \rho\end{aligned}$$

NB:

Could have started from target DGLAP in η

Change variables from (η, ρ) to (Y, ρ)

Get large logarithms in the BK kernel

Issues/Comments with Evolution in Y

- Initial condition at $Y=0$ or boundary at $\eta=0 \Leftrightarrow Y=\rho$
- Erroneous use of MV or GBW type IC leads to unphysical pushed front
- $\gamma_s \sim 1$, where is BFKL dynamics? λ_s seems too small (compare to DT 03)
- For DIS express final result in terms of $\eta = Y - \rho$ and ρ
  Saturation: target property, need $Q_s^2(\eta)$
- Fronts in Y and η very different for relevant $\bar{\alpha}_s$ values

Starting the Evolution

- Physical initial condition given at $\eta = 0$
- Construct initial condition at unphysical value $Y = 0$

  Can do at level of DLA, e.g. for GBW physical IC

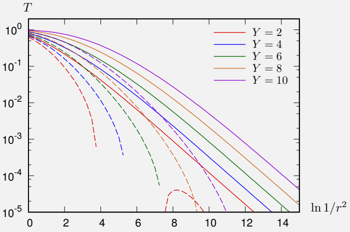
$$z_{12}^2 Q_0^2 \to z_{12}^2 Q_0^2 \mathrm{J}_0 \left(2\sqrt{\bar{\alpha}_s \ln^2 z_{12}^2 Q_0^2} \right) \quad \text{for} \quad z_{12}^2 Q_0^2 \ll 1$$

  Analytically continued backwards DLA evolution to

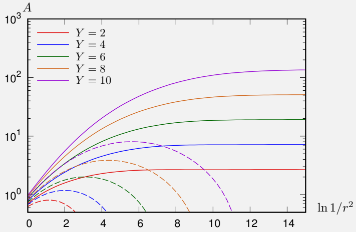
  $\eta = -\rho \Leftrightarrow Y = 0$

  Exponentiate to unitarize
- Evolution will not likely reproduce physical IC at $\eta = 0$

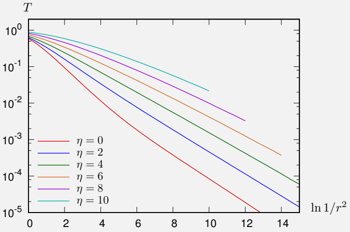
(A Little) Ugly Solution?

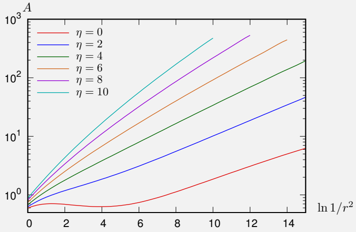


dashed: resummed J_0



$A = T/r^2$, dashed: resummed J_0

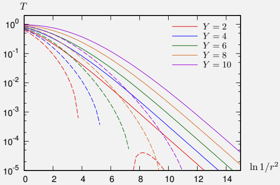




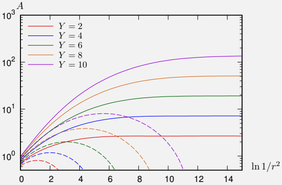
Difficult to trust solution, not able to solve BC problem

Try GBW IC at $Y = 0$?

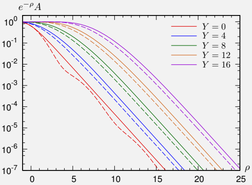
A mixture of GBW and resummed IC?



solid: GBW



$A = T/r^2$, solid: GBW



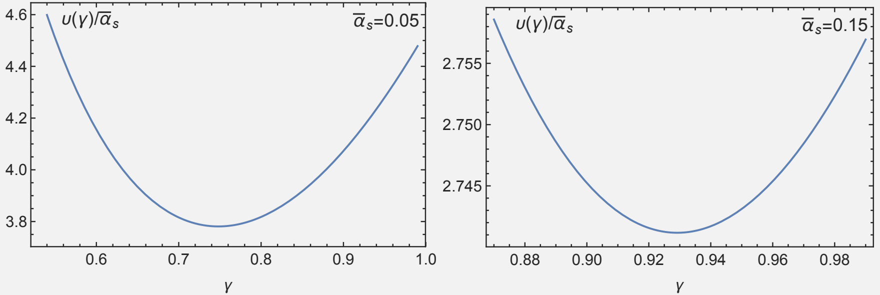
$[(1 + \mathrm{J}_0)/2]$GBW, GBW

Fronts look nice.

They are unphysical.

Why $\gamma_s = 1$?

Steepness of Initial Condition



Velocity function: $v(\gamma) = \omega(\gamma)/\gamma$

Saturation saddle point: $v'(\gamma_s) = 0$

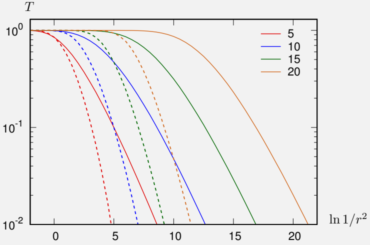
Front speed: $\lambda_s = \mathrm{d}\ln Q_s^2/\mathrm{d}Y = v(\gamma_s) + \mathcal{O}(1/Y)$

For $\bar{\alpha}_s < \bar{\alpha}_s^* \simeq 0.21$, $\gamma_s < \gamma_{\rm ic} = 1$: pulled front ✓

For $\bar{\alpha}_s > \bar{\alpha}_s^* \simeq 0.21$, $\gamma_s > \gamma_{\rm ic} = 1$: pushed front ✗

J_0 makes IC steep enough for all (relevant) values of $\bar{\alpha}_s$

Slope and Speed for the η-Front



Front in Y $\Leftrightarrow$ Front in η, physical

Scaling in Y: $-\ln T = \tilde{\gamma}_s(\rho - \tilde{\lambda}_s Y)$

Change variable $Y = \eta + \rho$

Scaling in η: $-\ln T = \gamma_s(\rho - \lambda_s \eta)$

(valid inside diffusion radius)

$$\boxed{\gamma_s = \tilde{\gamma}_s(1 - \tilde{\lambda}_s)} \quad \text{and} \quad \boxed{\lambda_s = \frac{\tilde{\lambda}_s}{1 - \tilde{\lambda}_s}}$$

$\tilde{\lambda}_s = \mathcal{O}(\bar{\alpha}_s)$, difference is NLO. In practice it is large.

Physical η-front less steep and faster

Constraints in Evolution: from Y to η

TO in hard to soft Y-evolution, local or non-local, forbids emission of large daughter dipoles $|\boldsymbol{x}-\boldsymbol{z}|, |\boldsymbol{y}-\boldsymbol{z}| \gtrsim r = |\boldsymbol{x}-\boldsymbol{y}|$

$$\boxed{\ln \frac{q^+}{k^+} > \ln \frac{r_>^2}{r^2}} \quad \text{with} \quad r_> = \max\{|\boldsymbol{x}-\boldsymbol{z}|, |\boldsymbol{y}-\boldsymbol{z}|\}$$

Make change of variables

$$q^+ = \frac{1}{2r^2 q^-} \quad , \quad k^+ = \frac{1}{2r_<^2 k^-}$$

Constraints in Evolution: from Y to η

TO contraints becomes

$$\ln\frac{k^-}{q^-} > \ln\frac{r_>^2}{r^2} - \ln\frac{r_<^2}{r^2} \Rightarrow \boxed{\ln\frac{k^-}{q^-} > \ln\frac{r_>^2}{r_<^2}}$$

TO in η-evolution forbids emission of small daughter dipoles (or equivalently forbids disparate daughter dipole sizes)

Is this relevant when scattering a small dipole of dense hadron?

Yes, BFKL diffusion $\rightsquigarrow$ not uni-directional evolution

Non-Local and Local Equation in η

- Non-local equation [NB: Real-virtual cancellation in UV]:

$$\frac{\mathrm{d}S_{\boldsymbol{xy}}(\eta)}{\mathrm{d}\eta} = \frac{\bar{\alpha}_s}{2\pi} \int \frac{\mathrm{d}^2\boldsymbol{z}\,(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}\,\Theta(\eta-\Delta_1)\Theta(\eta-\Delta_2) \Big[S_{\boldsymbol{xz}}(\eta-\Delta_1)S_{\boldsymbol{zy}}(\eta-\Delta_2) - S_{\boldsymbol{xy}}(\eta)\Big]$$

$$\Delta_1 = \ln r_>^2/|\boldsymbol{x}-\boldsymbol{z}|^2,\ \Delta_2 = \ln r_>^2/|\boldsymbol{z}-\boldsymbol{y}|^2$$

- Equivalent (to order of accuracy) local equation:

$$\boxed{\frac{\mathrm{d}S_{\boldsymbol{xy}}}{\mathrm{d}\eta} = \frac{\bar{\alpha}_s}{2\pi} \int \frac{\mathrm{d}^2\boldsymbol{z}\,(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}\,\mathcal{K}_{\mathrm{DLA}}\left(\ln\frac{(\boldsymbol{x}-\boldsymbol{z})^2}{(\boldsymbol{z}-\boldsymbol{y})^2}\right)(S_{\boldsymbol{xz}}S_{\boldsymbol{zy}} - S_{\boldsymbol{xy}})}$$

- Both are initial value problems at $\eta = 0$

Characteristic function

Truncation: NLO BFKL triple pole with $s_0 = Q^2$ ✓

$$\omega_{\mathrm{NLO}} = \frac{\bar{\alpha}_s}{\gamma} + \frac{\bar{\alpha}_s}{1-\gamma} - \frac{\bar{\alpha}_s^2}{\gamma^3} + \mathrm{regular}$$

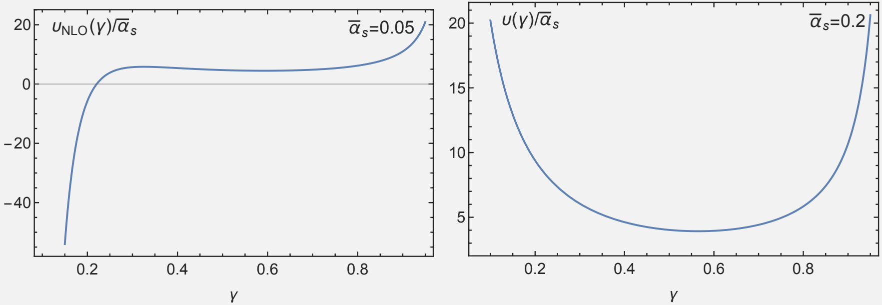
Resummed: Finite at $\gamma = 0$

$$\omega = \frac{\bar{\alpha}_s}{1-\gamma} + \underbrace{\frac{1}{2}\left[-\gamma + \sqrt{\gamma^2 + 4\bar{\alpha}_s}\right]}_{\omega_{\mathrm{DLA}}} + \mathrm{regular}$$

More precisely, matched to LO BFKL

$$\omega = \omega_{\mathrm{DLA}} + \bar{\alpha}_s\left(\chi_0 - \frac{1}{\gamma}\right) + \bar{\alpha}_s^2\left(\frac{4\zeta_3 - \chi_0^3 + 3\chi_0\chi_0' - \chi_0''}{6} + \frac{1}{\gamma^3}\right)$$

Velocity Function and Saddle Point



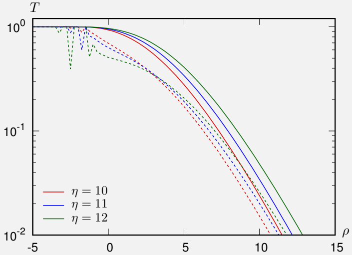
No real saddle point when truncating $\rightsquigarrow$ expect oscillations

Well defined saddle point with resummation

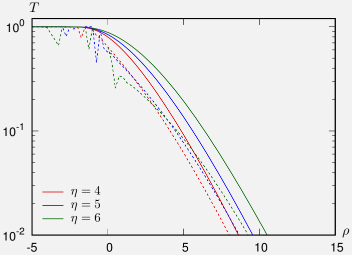
$\omega_{\mathbb{P}} = 1/2 < \gamma_s < \gamma_0 = 0.628$: pulled front with MV-GBW IC

$\delta\lambda_s/\lambda_0 \sim -\#\bar{\alpha}_s$ with $\# \simeq 1$

Numerical Solution



dashed: NLO, solid: res, $\bar{\alpha}_s = 0.2$



dashed: NLO, solid: res, $\bar{\alpha}_s = 0.3$

Oscillations milder than in Y-evolution. But still there.

Start to appear where expected: need large r (but smaller than $1/Q_s$) to allow for logarithmic integration in $z \ll r$

Resummation: forbids such log contribution $\rightsquigarrow$ well-defined answer

Other Corrections

- Resum single logarithms for projectile and target DGLAP
- Running coupling
- Include remaining regular NLO corrections, estimate: $\delta\omega = \pm\bar{\alpha}_s^2$, find $\delta\lambda_s/\lambda_0 \sim \pm\#\bar{\alpha}_s$ with $\# \simeq 0.35$

Still, what done here is a valid step

- Resummed unstable correction to render it $\mathcal{O}(\bar{\alpha}_s)$
- Definite sign (negative)
- Numerically larger than regular NLO corrections

Conclusions

- Very difficult (and not necessary?) to construct front in projectile rapidity Y
- Physical front is in terms of target rapidity η
- Front in η is easy to obtain (LO plus resummed)
- Front in η is faster and less steep