Exclusive diffractive processes in the CGC

Renaud Boussarie

Institute of Nuclear Physics
Polish Academy of Sciences

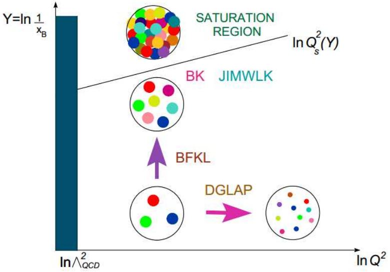
Probing QCD at the high energy frontier
Trento, May 2018

[RB, Grabovsky, Ivanov, Szymanowski, Wallon]
Phys.Rev.Lett 119 (2017) ; arXiv:1612.08026

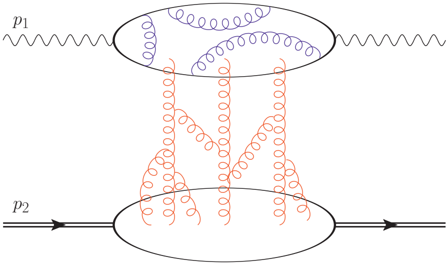
,

[RB, Grabovsky, Szymanowski, Wallon]
JHEP 1611 (2016) ; arXiv:1606.00419

NLO studies in the shockwave description of the Color Glass Condensate



Kinematics



$$p_1 = p^+ n_1 - \frac{Q^2}{2s} n_2$$

$$p_2 = \frac{m_t^2}{2p_2^-} n_1 + p_2^- n_2$$

$$p^+ \sim p_2^- \sim \sqrt{\frac{s}{2}}$$

Lightcone (Sudakov) vectors

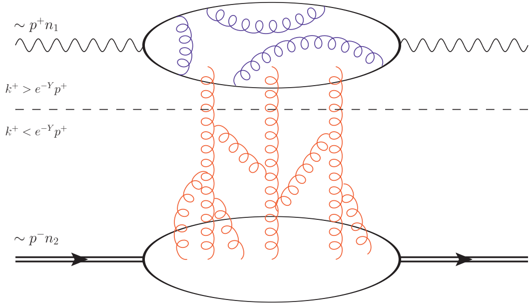
$$n_1 = \sqrt{\frac{1}{2}}(1, 0_\perp, 1), \quad n_2 = \sqrt{\frac{1}{2}}(1, 0_\perp, -1), \quad (n_1 \cdot n_2) = 1$$

Lightcone coordinates:

$$x = (x^0, x^1, x^2, x^3) \to (x^+, x^-, \vec{x})$$

$$x^+ = x_- = (x \cdot n_2) \quad x^- = x_+ = (x \cdot n_1)$$

Rapidity separation

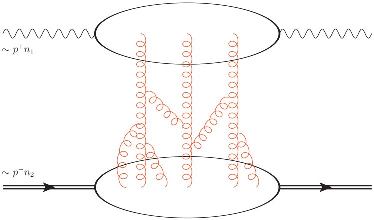


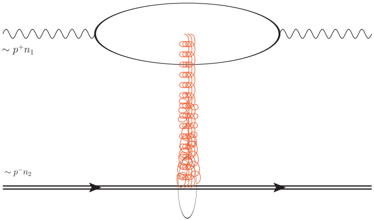
Let us split the gluonic field between "fast" and "slow" gluons

$$\begin{aligned}\mathcal{A}^{\mu a}(k^+, k^-, \vec{k}) &= A_\eta^{\mu a}(|k^+| > e^\eta p^+, k^-, \vec{k}) \\ &+ b_\eta^{\mu a}(|k^+| < e^\eta p^+, k^-, \vec{k})\end{aligned}$$

$$e^\eta = e^{-Y} \ll 1$$

Large longitudinal boost to the projectile frame





$$b^+(x^+, x^-, \vec{x}) \qquad \frac{1}{\Lambda} b^+(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x})$$

$$b^-(x^+, x^-, \vec{x}) \quad \longrightarrow \quad \Lambda b^-(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x})$$

$$b^k(x^+, x^-, \vec{x}) \qquad \Lambda \sim \sqrt{\frac{s}{m_t^2}} \qquad b^k(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x})$$

$$b^\mu(x) \to b^-(x)\, n_2^\mu = \delta(x^+)\, \mathbf{B}(\vec{x})\, n_2^\mu + O(\sqrt{\frac{m_t^2}{s}})$$

Shockwave approximation

Effective Feynman rules in the external shockwave field

Wilson lines :

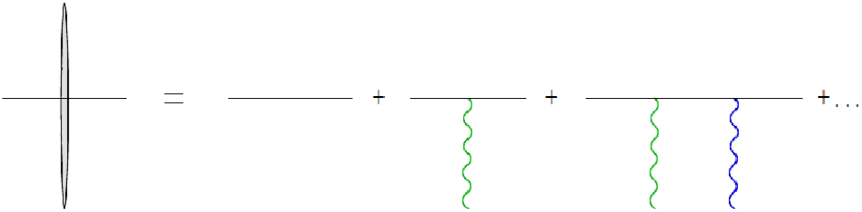
$$U_i^\eta = U_{\vec{z}_i}^\eta = P \exp\left[ig \int_{-\infty}^{+\infty} b_\eta^-(z_i^+, \vec{z}_i)\, dz_i^+ \right]$$

Fourier transform of a Wilson line

$$\tilde{U}^\eta(\vec{p}) = \int d^{D-2}\vec{z}\; e^{-i(\vec{p}\cdot\vec{z})}\, U_{\vec{z}}^\eta$$

$$U_i^\eta = 1 + ig \int_{-\infty}^{+\infty} b_\eta^-(z_i^+, \vec{z}_i) dz_i^+ + (ig)^2 \int_{-\infty}^{+\infty} b_\eta^-(z_i^+, \vec{z}_i) b_\eta^-(z_j^+, \vec{z}_j) \theta(z_{ji}^+) dz_i^+ dz_j^+$$

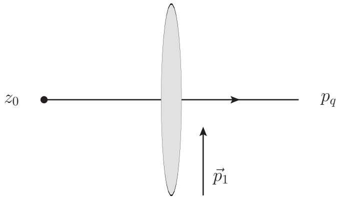
...



Quark line in the external field in momentum space

$$\bar{u}(p_q,\, z_0) = \int \frac{d^d \vec{p}_1}{(2\pi)^d} e^{i p_q^+ z_0^- + i z_0^+ \frac{(\vec{p}_q - \vec{p}_1)^2 - i0}{2 p_q^+} - i(\vec{p}_q - \vec{p}_1)\cdot \vec{z}_0}$$

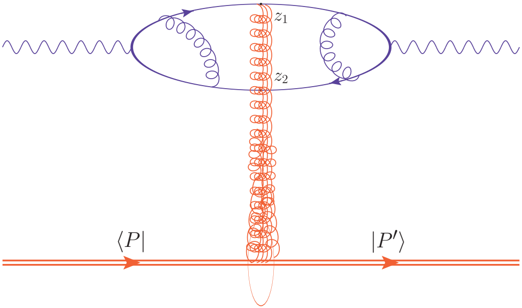
$$\times \bar{u}_{p_q} \gamma^+ \left[\tilde{U}_{\vec{p}_1} \theta(-z_0^+) + (2\pi)^d \delta(\vec{p}_1) \theta(z_0^+) \right] \frac{p_q^+ \gamma^- + \hat{p}_{q\perp} - \hat{p}_{1\perp}}{2 p_q^+}$$



Exchange in t-channel of an effective off-shell particle with 0 momentum along n_1

$$\bar{u}(p_q,\, z_0)\,|_{U=1} = e^{i(p_q \cdot z_0)} \bar{u}_{p_q} \left(1 - \frac{\hat{p}_q \gamma^+}{2 p_q^+}\right)$$

Factorized picture



Factorized amplitude

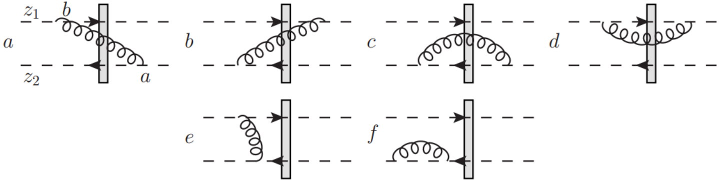
$$\mathcal{A}^\eta = \int d^{D-2}\vec{z}_1 d^{D-2}\vec{z}_2\, \Phi^\eta(\vec{z}_1, \vec{z}_2)\, \langle P'|[\mathrm{Tr}(U^\eta_{\vec{z}_1} U^{\eta\dagger}_{\vec{z}_2}) - N_c]|P\rangle$$

Dipole operator $\mathcal{U}^\eta_{ij} = \frac{1}{N_c}\mathrm{Tr}(U^\eta_{\vec{z}_i} U^{\eta\dagger}_{\vec{z}_j}) - 1$

Written similarly for any number of Wilson lines in any color representation!

Evolution for the dipole operator

$$\mathcal{U}_{12}^{\eta+\delta\eta} - \mathcal{U}_{12}^{\eta}$$



B-JIMWLK hierarchy of equations

[Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner]

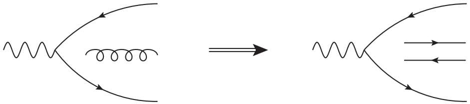
$$\frac{\partial \mathcal{U}_{12}^{\eta}}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2}\vec{z}_{23}^{\,2}} \left[\mathcal{U}_{13}^{\eta} + \mathcal{U}_{32}^{\eta} - \mathcal{U}_{12}^{\eta} + \mathcal{U}_{13}^{\eta}\mathcal{U}_{32}^{\eta}\right]$$

$$\frac{\partial \mathcal{U}_{13}^{\eta}\mathcal{U}_{32}^{\eta}}{\partial \eta} = \ldots$$

Evolves a dipole into a double dipole

The BK equation

Mean field approximation, or 't Hooft planar limit $N_c \to \infty$ in the dipole B-JIMWLK equation



[Mueller]

$\Rightarrow$ BK equation [Balitsky, 1995] [Kovchegov, 1999]

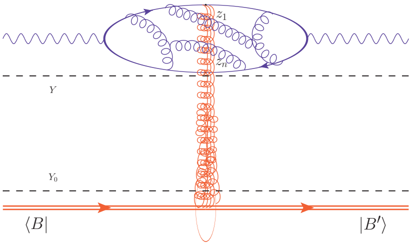
$$\frac{\partial \langle \mathcal{U}_{12}^{\eta} \rangle}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \, \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2} \vec{z}_{23}^{\,2}} \left[\langle \mathcal{U}_{13}^{\eta} \rangle + \langle \mathcal{U}_{32}^{\eta} \rangle - \langle \mathcal{U}_{12}^{\eta} \rangle + \langle \mathcal{U}_{13}^{\eta} \rangle \, \langle \mathcal{U}_{32}^{\eta} \rangle \right]$$

BFKL/BKP part   Triple pomeron vertex

Non-linear term : saturation

Practical use of the formalism

- Compute the upper impact factor using the effective Feynman rules
- Build non-perturbative models for the matrix elements of the Wilson line operators acting on the target states
- Solve the B-JIMWLK evolution for these matrix elements with such non-perturbative initial conditions at a typical target rapidity $\eta = Y_0$. May require adjustment. See E.Iancu's talk.
- Evaluate the solution at a typical projectile rapidity $\eta = Y$, or at the rapidity of the slowest gluon
- Convolute the solution and the impact factor



$$\mathcal{A} = \int d\vec{z}_1 ... d\vec{z}_n \; \Phi(\vec{z}_1, ..., \vec{z}_n)$$
$$\times \langle P' | U_{\vec{z}_1} ... U_{\vec{z}_n} | P \rangle$$

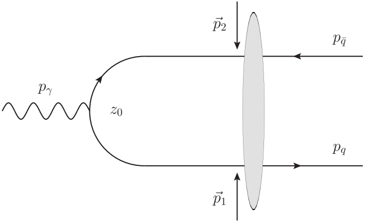
Exclusive diffraction allows one to probe the $b_\perp$-dependence of the non-perturbative scattering amplitude

First step: open parton production

- Regge-Gribov limit : $s \gg Q^2 \gg \Lambda_{QCD}$
- Otherwise completely general kinematics
- Shockwave (CGC) Wilson line approach: covariant description of the CGC
- Transverse dimensional regularization $d = 2 + 2\varepsilon$, longitudinal cutoff

$$|p_g^+| > \alpha p_\gamma^+$$

LO diagram

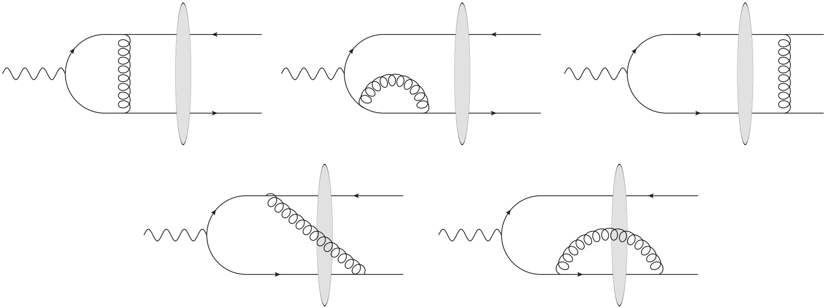


$$\mathcal{A} = \frac{\delta^{ik}}{\sqrt{N_c}} \int d^D z_0 [\bar{u}(p_q, z_0)]_{ij} (-ie_q) \hat{\varepsilon}_\gamma e^{-i(p_\gamma \cdot z_0)} [v(p_{\bar{q}}, z_0)]_{jk} \theta(-z_0^+)$$

$$= \delta(p_q^+ + p_{\bar{q}} - p_\gamma^+) \int d^d \vec{p}_1 d^d \vec{p}_2 \delta(\vec{p}_q + \vec{p}_{\bar{q}} - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2) \Phi_0(\vec{p}_1, \vec{p}_2) \times C_F \left\langle P' \middle| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \middle| P \right\rangle$$

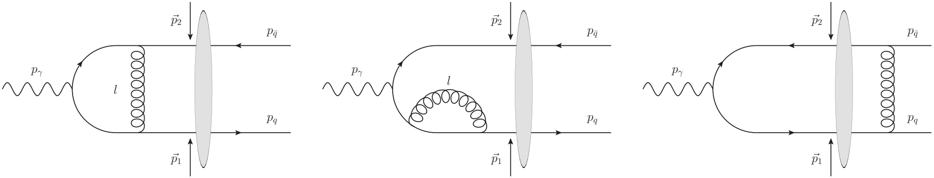
$$\tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) = \int d^d \vec{z}_1 d^d \vec{z}_2 \, e^{-i(\vec{p}_1 \cdot \vec{z}_1) - i(\vec{p}_2 \cdot \vec{z}_2)} [\tfrac{1}{N_c} \mathrm{Tr}(U^\alpha_{\vec{z}_1} U^{\alpha\dagger}_{\vec{z}_2}) - 1]$$

NLO open $q\bar{q}$ production



Diagrams contributing to the NLO correction

First kind of virtual corrections



Color factor

$$\frac{C_F}{\sqrt{N_c}} \tilde{\mathcal{U}}^{\alpha}(\vec{p}_1, \vec{p}_2)$$

Impact factor

$$\mathcal{A}_{NLO}^{(1)} \propto \delta(p_q^+ + p_{\bar{q}} - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 \delta(\vec{p}_q + \vec{p}_{\bar{q}} - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2)\, \Phi_{V1}(\vec{p}_1, \vec{p}_2)$$
$$\times C_F \left\langle P' \middle| \tilde{\mathcal{U}}^{\alpha}(\vec{p}_1, \vec{p}_2) \middle| P \right\rangle$$

Second kind of virtual corrections

Color factor

$$\frac{\delta^{ik}}{\sqrt{N_c}}(t^a U_1 t^b U_2^\dagger)_{ik}(U_3)^{ab}$$

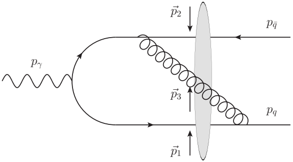
Action of the Wilson line in the adjoint representation

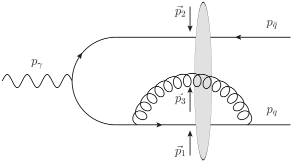
$$(U_3)^{ab} t^b = U_3 t^a U_3^\dagger \quad \Rightarrow \quad (U_3)^{ab} = 2\mathrm{Tr}(t^a U_3 t^b U_3^\dagger)$$

+ Fierz identity

$$C_F \mathcal{U}_{12} + \frac{1}{2}[\mathcal{U}_{13} + \mathcal{U}_{32} - \mathcal{U}_{12} + \mathcal{U}_{13}\mathcal{U}_{32}] \quad = C_F \mathcal{U}_{12} + \mathcal{W}_{123}$$

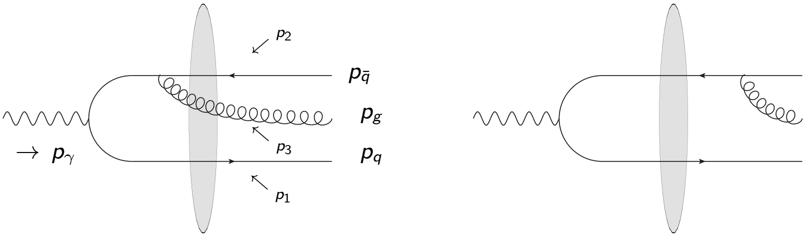
Second kind of virtual corrections





$$\mathcal{A}_{NLO}^{(2)} \propto \delta(p_q^+ + p_{\bar{q}} - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 d^d\vec{p}_3 \delta(\vec{p}_q + \vec{p}_{\bar{q}} - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2 - \vec{p}_3)$$
$$\times [\Phi'_{V1}(\vec{p}_1, \vec{p}_2) C_F \left\langle P' \right| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \left| P \right\rangle$$
$$+ \Phi_{V2}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \left\langle P' \right| \tilde{\mathcal{W}}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \left| P \right\rangle]$$

LO open $q\bar{q}g$ production



$$\mathcal{A}_R^{(2)} \propto \delta(p_q^+ + p_{\bar{q}} + p_g^+ - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 d^d\vec{p}_3 \delta(\vec{p}_q + \vec{p}_{\bar{q}} + \vec{p}_g - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2 - \vec{p}_3)$$
$$\times [\Phi'_{R1}(\vec{p}_1, \vec{p}_2) C_F \left\langle P' \right| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \left| P \right\rangle$$
$$+ \Phi_{R2}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \left\langle P' \right| \tilde{\mathcal{W}}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \left| P \right\rangle]$$

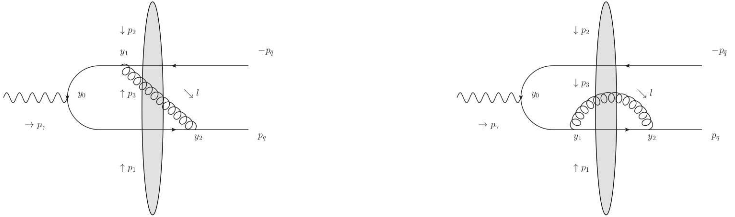
$$\mathcal{A}_R^{(1)} \propto \delta(p_q^+ + p_{\bar{q}} + p_g^+ - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 \delta(\vec{p}_q + \vec{p}_{\bar{q}} + \vec{p}_g - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2)$$
$$\times \Phi_{R1}(\vec{p}_1, \vec{p}_2) C_F \left\langle P' \right| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \left| P \right\rangle$$

Divergences

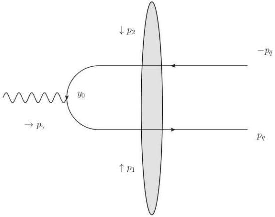
Divergences

- Rapidity divergence $p_g^+ \to 0$ $\Phi_{V2}\Phi_0^* + \Phi_0\Phi_{V2}^*$
- UV divergence $\vec{p}_g^{\,2} \to +\infty$ $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$
- Soft divergence $p_g \to 0$ $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$, $\Phi_{R1}\Phi_{R1}^*$
- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$ $\Phi_{R1}\Phi_{R1}^*$
- Soft and collinear divergence $p_g = \frac{p_g^+}{p_q^+} p_q$ or $\frac{p_g^+}{p_{\bar{q}}^+} p_{\bar{q}}$, $p_g^+ \to 0$ $\Phi_{R1}\Phi_{R1}^*$

Rapidity divergence



Double dipole virtual correction Φ_{V2}



B-JIMWLK evolution of the LO term : $\Phi_0 \otimes \mathcal{K}_{BK}$

Rapidity divergence

B-JIMWLK equation for the dipole operator

$$\frac{\partial \tilde{\mathcal{U}}_{12}^{\alpha}}{\partial \log \alpha} = 2\alpha_s N_c \mu^{2-d} \int \frac{d^d \vec{k}_1 d^d \vec{k}_2 d^d \vec{k}_3}{(2\pi)^{2d}} \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 - \vec{p}_1 - \vec{p}_2) \left(\tilde{\mathcal{U}}_{13}^{\alpha} \tilde{\mathcal{U}}_{32}^{\alpha} + \tilde{\mathcal{U}}_{13}^{\alpha} + \tilde{\mathcal{U}}_{32}^{\alpha} - \tilde{\mathcal{U}}_{12}^{\alpha} \right)$$

$$\times \left[2 \frac{(\vec{k}_1 - \vec{p}_1) \cdot (\vec{k}_2 - \vec{p}_2)}{(\vec{k}_1 - \vec{p}_1)^2 (\vec{k}_2 - \vec{p}_2)^2} + \frac{\pi^{\frac{d}{2}} \Gamma(1 - \frac{d}{2}) \Gamma^2(\frac{d}{2})}{\Gamma(d-1)} \left(\frac{\delta(\vec{k}_2 - \vec{p}_2)}{\left[(\vec{k}_1 - \vec{p}_1)^2\right]^{1-\frac{d}{2}}} + \frac{\delta(\vec{k}_1 - \vec{p}_1)}{\left[(\vec{k}_2 - \vec{p}_2)^2\right]^{1-\frac{d}{2}}} \right) \right]$$

η rapidity divide, which separates the upper and the lower impact factors

$$\Phi_0 \tilde{\mathcal{U}}_{12}^{\alpha} \to \Phi_0 \tilde{\mathcal{U}}_{12}^{\eta} + 2 \log \left(\frac{e^{\eta}}{\alpha} \right) \mathcal{K}_{BK} \Phi_0 \tilde{\mathcal{W}}_{123}$$

Rapidity divergence

Virtual contribution

$$(\Phi^{\mu}_{V2})_{div} \propto \Phi^{\mu}_{0} \left\{ 4 \ln\left(\frac{x\bar{x}}{\alpha^2}\right) \left[\frac{1}{\varepsilon} + \ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)\right] - \frac{6}{\varepsilon} \right\}$$

BK contribution

$$(\Phi^{\mu}_{BK})_{div} \propto \Phi^{\mu}_{0} \left\{ 4 \ln\left(\frac{\alpha^2}{e^{2\eta}}\right) \left[\frac{1}{\varepsilon} + \ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)\right] \right\}$$

Sum : the α dependence cancels

$$(\Phi'^{\mu}_{V2})_{div} \propto \Phi^{\mu}_{0} \left\{ 4 \ln\left(\frac{x\bar{x}}{e^{2\eta}}\right) \left[\frac{1}{\varepsilon} + \ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)\right] - \frac{6}{\varepsilon} \right\}$$

Rapidity divergence

Cancellation of the remaining $1/\epsilon$ divergence

Convolution

$$
\begin{aligned}
(\Phi'^{\mu}_{V2} \otimes \mathcal{W}) &= 2 \int d^d\vec{p}_1 d^d\vec{p}_2 d^d\vec{p}_3 \left\{ 4 \ln\left(\frac{x\bar{x}}{e^{2\eta}}\right) \left[\frac{1}{\varepsilon} + \ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)\right] - \frac{6}{\varepsilon} \right\} \\
&\times \quad \delta(\vec{p}_{q1} + \vec{p}_{\bar{q}2} - \vec{p}_3) \left[\tilde{\mathcal{U}}_{13} + \tilde{\mathcal{U}}_{32} - \tilde{\mathcal{U}}_{12} - \tilde{\mathcal{U}}_{13}\tilde{\mathcal{U}}_{32}\right] \Phi_0^{\mu}(\vec{p}_1, \vec{p}_2)
\end{aligned}
$$

Rq :

- $\Phi_0(\vec{p}_1, \vec{p}_2)$ only depends on one of the t-channel momenta.
- The double-dipole operators cancels when $\vec{z}_3 = \vec{z}_1$ or $\vec{z}_3 = \vec{z}_2$.

This permits one to show that the convolution cancels the remaining $\frac{1}{\varepsilon}$ divergence.

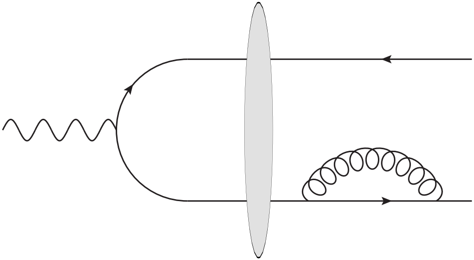
Then $\tilde{\mathcal{U}}^{\alpha}_{12}\Phi_0 + \Phi_{V2}$ is finite

Divergences

- Rapidity divergence
- UV divergence $\vec{p}_g^{\,2} \to +\infty$ — $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$
- Soft divergence $p_g \to 0$ — $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*, \Phi_{R1}\Phi_{R1}^*$
- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$ — $\Phi_{R1}\Phi_{R1}^*$
- Soft and collinear divergence $p_g = \frac{p_g^+}{p_q^+} p_q$ or $\frac{p_g^+}{p_{\bar{q}}^+} p_{\bar{q}}$, $p_g^+ \to 0$ — $\Phi_{R1}\Phi_{R1}^*$

UV divergence

Tadpole diagrams



Some null diagrams just contribute to turning UV divergences into IR divergences

$$\Phi = 0 \propto \left(\frac{1}{2\epsilon_{IR}} - \frac{1}{2\epsilon_{UV}}\right)$$

In the massless limit, renormalization of the external quark lines is absent in dimensional regularization.

Divergences

- Rapidity divergence
- UV divergence
- Soft divergence $p_g \to 0$ $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$, $\Phi_{R1}\Phi_{R1}^*$
- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$ $\Phi_{R1}\Phi_{R1}^*$
- Soft and collinear divergence $p_g = \frac{p_g^+}{p_q^+} p_q$ or $\frac{p_g^+}{p_{\bar{q}}^+} p_{\bar{q}}$, $p_g^+ \to 0$ $\Phi_{R1}\Phi_{R1}^*$

Constructing a finite cross section

Exclusive diffractive production of a forward dijet

From partons to jets

[R.B.,A.V.Grabovsky,L.Szymanowski,S.Wallon]

JHEP 1611 (2016) 149

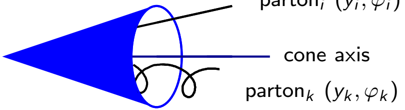
Soft and collinear divergence

Jet cone algorithm

We define a cone width for each pair of particles with momenta p_i and p_k, rapidity difference ΔY_{ik} and relative azimuthal angle $\Delta\varphi_{ik}$

$$(\Delta Y_{ik})^2 + (\Delta\varphi_{ik})^2 = R_{ik}^2$$

If $R_{ik}^2 < R^2$, then the two particles together define a single jet of momentum $p_i + p_k$.



Applying this in the small R^2 limit cancels our soft and collinear divergence.

Remaining divergence

Soft real emission

$$(\Phi_{R1}\Phi_{R1}^*)_{soft} \propto (\Phi_0\Phi_0^*) \int_{\text{outside the cones}} \left| \frac{p_q^\mu}{(p_q.p_g)} - \frac{p_{\bar{q}}^\mu}{(p_{\bar{q}}.p_g)} \right|^2 \frac{dp_g^+}{p_g^+} \frac{d^d p_g}{(2\pi)^d}$$

Collinear real emission

$$(\Phi_{R1}\Phi_{R1}^*)_{col} \propto (\Phi_0\Phi_0^*) \left(\mathcal{N}_q + \mathcal{N}_{\bar{q}}\right)$$

Where $\mathcal{N}$ is the number of jets in the quark or the antiquark

$$\mathcal{N}_k = \frac{(4\pi)^{\frac{d}{2}}}{\Gamma(2-\frac{d}{2})} \int_{\alpha p_\gamma^+}^{p_{jet}^+} \frac{dp_g^+ dp_k^+}{2p_g^+ 2p_k^+} \int_{\text{in cone k}} \frac{d^d \vec{p}_g d^d \vec{p}_k}{(2\pi)^d \mu^{d-2}} \frac{\mathrm{Tr}\left(\hat{p}_k \gamma^\mu \hat{p}_{jet} \gamma^\nu\right) d_{\mu\nu}(p_g)}{2p_{jet}^+ \left(p_k^- + p_g^- - p_{jet}^-\right)^2}$$

Those two contributions cancel exactly the virtual divergences

Cancellation of divergences

Total divergence

$$(d\sigma_1)_{div} = \alpha_s \frac{\Gamma(1-\varepsilon)}{(4\pi)^{1+\varepsilon}} \left(\frac{N_c^2-1}{2N_c}\right) \left(S_V + S_V^* + S_R + \mathcal{N}_{jet1} + \mathcal{N}_{jet2}\right) d\sigma_0$$

Virtual contribution

$$\begin{aligned} S_V &= \left[2\ln\left(\frac{x_j x_{\bar{j}}}{\alpha^2}\right) - 3\right]\left[\ln\left(\frac{x_j x_{\bar{j}}\mu^2}{(x_j \vec{p}_{\bar{j}} - x_{\bar{j}}\vec{p}_j)^2}\right) - \frac{1}{\epsilon}\right] \\ &+ 2i\pi\ln\left(\frac{x_j x_{\bar{j}}}{\alpha^2}\right) + \ln^2\left(\frac{x_j x_{\bar{j}}}{\alpha^2}\right) - \frac{\pi^2}{3} + 6 \end{aligned}$$

Real contribution

$$\begin{aligned} S_R + \mathcal{N}_{jet1} + \mathcal{N}_{jet2} &= 2\left[\ln\left(\frac{(x_{\bar{j}}\vec{p}_j - x_j\vec{p}_{\bar{j}})^4}{x_{\bar{j}}^2 x_j^2 R^4 \vec{p}_{\bar{j}}^{\,2}\vec{p}_j^{\,2}}\right)\ln\left(\frac{4E^2}{x_{\bar{j}}x_j(p_\gamma^+)^2}\right)\right. \\ &+ 2\ln\left(\frac{x_{\bar{j}}x_j}{\alpha^2}\right)\left(\frac{1}{\epsilon} - \ln\left(\frac{x_{\bar{j}}x_j\mu^2}{(x_{\bar{j}}\vec{p}_j - x_j\vec{p}_{\bar{j}})^2}\right)\right) - \ln^2\left(\frac{x_{\bar{j}}x_j}{\alpha^2}\right) \\ &+ \left.\frac{3}{2}\ln\left(\frac{16\mu^4}{R^4\vec{p}_j^{\,2}\vec{p}_{\bar{j}}^{\,2}}\right) - \ln\left(\frac{x_j}{x_{\bar{j}}}\right)\ln\left(\frac{x_j\vec{p}_{\bar{j}}^{\,2}}{x_{\bar{j}}\vec{p}_j^{\,2}}\right) - \frac{3}{\epsilon} - \frac{2\pi^2}{3} + 7\right] \end{aligned}$$

Cancellation of divergences

Total "divergence"

$$
\begin{aligned}
div &= S_V + S_V^* + S_R + \mathcal{N}_{jet1} + \mathcal{N}_{jet2} \\
&= 4\left[\frac{1}{2}\ln\left(\frac{(x_{\bar{j}}\vec{p}_j - x_j\vec{p}_{\bar{j}})^4}{x_{\bar{j}}^2 x_j^2 R^4 \vec{p}_{\bar{j}}^{\,2}\vec{p}_j^{\,2}}\right)\left(\ln\left(\frac{4E^2}{x_{\bar{j}}x_j(p_\gamma^+)^2}\right)+\frac{3}{2}\right)\right. \\
&+ \left.\ln(8) - \frac{1}{2}\ln\left(\frac{x_j}{x_{\bar{j}}}\right)\ln\left(\frac{x_j\vec{p}_{\bar{j}}^{\,2}}{x_{\bar{j}}\vec{p}_j^{\,2}}\right) + \frac{13-\pi^2}{2}\right]
\end{aligned}
$$

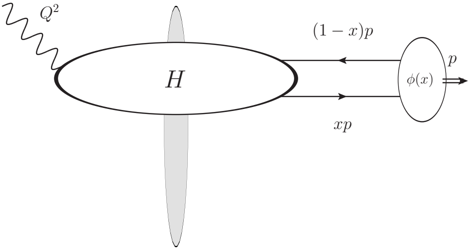
Our cross section is thus finite

Constructing a finite amplitude

Exclusive diffractive production of a light neutral vector meson

[RB, Grabovsky, Ivanov, Szymanowski, Wallon]
Phys.Rev.Lett. 119 (2017) ; arXiv:1612.08026

s-channel collinear factorization



Once the amplitude is factorized in terms of impact factors, we perform an additional twist expansion.

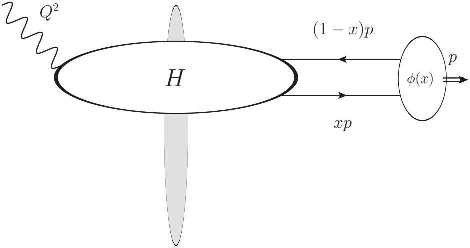
Two formalisms for collinear factorization:

- Light Cone Collinear Factorization (LCCF) [Anikin, Pire, Teryaev]
- Covariant Collinear Factorization (CCF) [Ball, Braun, Koike, Tanaka]

Straightforwardly equivalent at leading twist, shown to be equivalent for DVMP at twist 3 [Anikin, Ivanov, Pire, Szymanowski, Wallon]

s-channel collinear factorization

Twist 2: ρ_L production



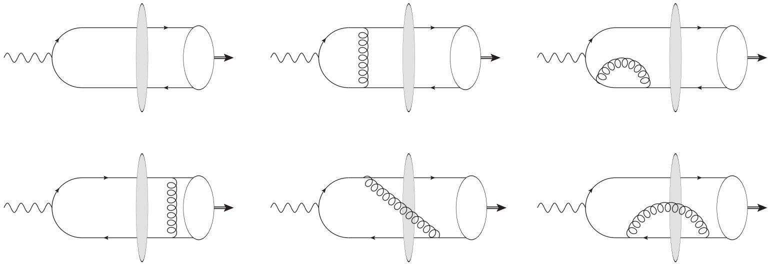
$$\int d^4z \mathcal{H}^{ij}_{\alpha\beta}(z) \left\langle \rho(p) \left| \bar{\psi}^i_\alpha(z) \psi^j_\beta(0) \right| 0 \right\rangle$$

$$\rightarrow \frac{1}{4N_c} \int d^4z \, \mathrm{tr}_{c,D} \left[\mathcal{H}(z) \Gamma^\lambda \right] \langle \rho(p) | \bar{\psi}(z) \Gamma_\lambda \psi(0) | 0 \rangle$$

Singlet transition $\Rightarrow$ only virtual diagrams contribute.
Leading twist matrix element:

$$\left\langle \rho_L(p) \left| \bar{\psi}(z) \gamma^\lambda \psi(0) \right| 0 \right\rangle \quad \rightarrow \quad f_\rho m_\rho p^\lambda \int_0^1 dx \; e^{-ixp\cdot z} \varphi_\parallel(x)$$

Exclusive diffractive production of a light neutral vector meson



$$\mathcal{A}_0 = -\frac{e_V\, f_V\, \varepsilon_\beta}{N_c} \int_0^1 dx \varphi_\parallel (x) \int \frac{d^d \vec{p}_1}{(2\pi)^d} \frac{d^d \vec{p}_2}{(2\pi)^d}$$
$$\times \; (2\pi)^{d+1} \delta \left(p_V^+ - p_\gamma^+\right) \delta \left(\vec{p}_V - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2\right)$$
$$\times \; \Phi_0^\beta \left(x,\, \vec{p}_1,\, \vec{p}_2\right) \tilde{\mathcal{U}}_{12}^\eta .$$

Leading twist for a longitudinally polarized meson
Otherwise general kinematics, including transverse virtual photon (twist 3) contributions, and the photoproduction limit (for large $t-$channel momentum transfer)

ERBL evolution equation

Efremov, Radyushkin, Brodsky, Lepage evolution equation for a DA

Renormalization of the bilocal operator

$$\bar{\psi}(z)\gamma^{\mu}\psi(0)$$

$\Rightarrow$ Evolution equation for the distribution amplitude in the $\overline{MS}$ scheme

$$\frac{\partial\varphi(x,\mu_F^2)}{\partial\ln\mu_F^2} = \frac{\alpha_s C_F}{2\pi}\frac{\Gamma(1-\epsilon)}{(4\pi)^{\epsilon}}\left(\frac{\mu_F^2}{\mu^2}\right)^{\epsilon}\int_0^1 dz\varphi(z,\mu_F^2)\mathcal{K}(x,z),$$

$\mathcal{K}$ = ERBL kernel

ERBL evolution equation

Evolution equation for the distribution amplitude in the $\overline{MS}$ scheme

$$\frac{\partial\varphi(x,\mu_F^2)}{\partial\ln\mu_F^2} = \frac{\alpha_s C_F}{2\pi}\frac{\Gamma(1-\epsilon)}{(4\pi)^\epsilon}\left(\frac{\mu_F^2}{\mu^2}\right)^\epsilon\int_0^1 dz\varphi(z,\mu_F^2)\mathcal{K}(x,z),$$

where we parameterize the ERBL kernel for consistency as

$$\begin{aligned}\mathcal{K}(x,z) &= \frac{x}{z}\left[1+\frac{1}{z-x}\right]\theta(z-x-\alpha) \\ &+ \frac{1-x}{1-z}\left[1+\frac{1}{x-z}\right]\theta(x-z-\alpha) \\ &+ \left[\frac{3}{2}-\ln\left(\frac{x(1-x)}{\alpha^2}\right)\right]\delta(z-x).\end{aligned}$$

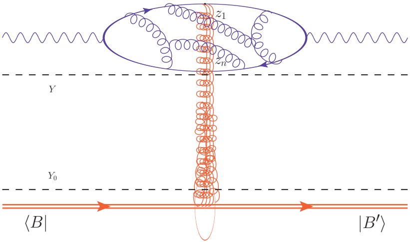
It is equivalent to the usual ERBL kernel

It provides the right counterterm to obtain a finite amplitude

Practical use of such results for phenomenology

Practical use of such results

- Compute the upper impact factor using the effective Feynman rules
- Build non-perturbative models for the matrix elements of the Wilson line operators acting on the target states
- Solve the B-JIMWLK evolution for these matrix elements with such non-perturbative initial conditions at a typical target rapidity $\eta = Y_0$
- Evaluate the solution at a typical projectile rapidity $\eta = Y$, or at the rapidity of the slowest gluon (cf. Bertrand's talk)
- Convolute the solution and the impact factor



$$\mathcal{A} = \int d\vec{z}_1 ... d\vec{z}_n \; \Phi(\vec{z}_1, ..., \vec{z}_n)$$
$$\times \langle P' | U_{\vec{z}_1} ... U_{\vec{z}_n} | P \rangle$$

Residual parameter dependence

Required parameters

- Renormalization scale μ_R
- Factorization scale μ_F if assumed that $\mu_F \neq \mu_R$
- Typical target rapidity Y_0
- Typical projectile rapidity Y

In the linear BFKL limit, the cross section only depends on $Y - Y_0$, so one only needs one arbirary parameter s_0 defined by

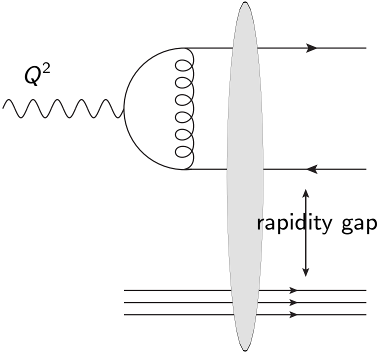
$$Y - Y_0 = \ln\left(\frac{s}{s_0}\right).$$

Modifying any of these parameter results in a higher order (NNLO) contribution

General amplitude

Very general result

- The hard scale can be Q^2, M_X^2 t
- The target can be either a proton or an ion, or another impact factor
- Finite results for $Q^2 = 0$ at large t
- One can study ultraperipheral collision in the limit $Q^2 \to 0$, at large t for meson production.
- Thus suited for HERA fitting, LHC UPC predictions, and perfectly suited for EIC studies



The general amplitude

Theoretical issues

Relation to lower density/lower energy formalisms

- How to get to the dilute BFKL limit at NLL?
- Relation to TMD factorization and the Wigner distributions
- Relation to GPDs, end-point singularities for the power-suppressed $\gamma_T \to \rho_L$ contribution?

Comparison with previous results:JIMWLK/BFKL equivalence

In the forward $t = 0$ limit and in the linear BFKL limit, the $\gamma_L \to \rho_L$ impact factor was computed at NLO [Ivanov, Kotsky, Papa].

JIMWLK convolution

$$\int d^d p_1 d^d p_2 \Phi_{CGC}(p_1, p_2)\tilde{\mathcal{U}}(p_1, p_2)$$

$\tilde{\mathcal{U}}(p_1, p_2)$ dipole scattering operator

BFKL convolution

$$\int d^d q_1 d^d q_2 \Phi_{BFKL}(q_1, q_2)\, R(q_1)R(q_2)$$

$R(q)$ Reggeon field

Defining the Reggeon field in the CGC as the logarithm of a Wilson line [Caron-Huot]

$$R^a(x) \equiv \frac{f^{abc}}{gC_A}\left(\ln U_x^{adj}\right)^{bc}$$

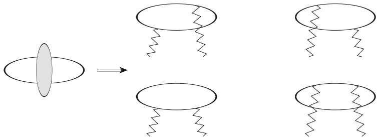
$$U_x = 1 + igt^a R^a(x) - \frac{g^2}{2} t^a t^b R^a(x) R^b(x) + O(g^3)$$

Such fields are Reggeized by the JIMWLK Hamiltonian, satisfy the BFKL equation and satisfy bootstrap equations.

JIMWLK/BFKL equivalence

Linear limit of diffractive CGC impact factors

$$\int d^2 p_1 d^2 p_2\, \varphi\left(p_1, p_2\right) \tilde{\mathcal{U}}\left(p_1, p_2\right)$$
$$= \frac{g^2}{4N_c} \int d^2 q_1 d^2 q_2 R^a\left(q_1\right) R^a\left(q_2\right) \left[2\varphi\left(q_1, q_2\right) - \varphi\left(q_1 + q_2, 0\right) - \varphi\left(0, q_1 + q_2\right)\right]$$



This matches our result to the leading order BFKL result.

At NLL accuracy, things are interestingly worse due to the ambiguity of distribution of radiative corrections between impact factors and kernels.

Equivalence with BFKL at NLL accuracy

Linear limit: usual k_t-factorization (BFKL framework)

s-channel discontinuity of $A + B \rightarrow A' + B'$ scattering amplitudes

$$\delta(p_{A'} + p_{B'} - p_A - p_B) Disc_s \mathcal{A}_{AB}^{A'B'} \propto \Phi(A', A) \otimes \mathcal{K} \otimes \Phi(B', B)$$

For any non-singular operator $\mathcal{O}$ this discontinuity is invariant under

$$\Phi(A', A) \rightarrow \Phi(A', A)\, \mathcal{O}, \quad \mathcal{K} \rightarrow \mathcal{O}^{-1} \mathcal{K} \mathcal{O}, \quad \Phi(B', B) \rightarrow \mathcal{O}^{-1} \Phi(B', B)$$

i.e. there is an ambiguity of distribution of corrections between the impact factors and the kernel. In the linear approximation of BK there exists an operator $\mathcal{O}$ such that

$$\Phi_{BK} \otimes \mathcal{K}_{BK} \otimes \Phi_{BK} = (\Phi_{BFKL} \otimes \mathcal{O}) \otimes (\mathcal{O}^{-1} \otimes \mathcal{K}_{BFKL} \otimes \mathcal{O}) \otimes (\mathcal{O}^{-1} \otimes \Phi_{BFKL})$$

The expression for $\mathcal{O}$ to make the kernels explicitly equivalent at NLO accuracy under such a change of variables is known [Fadin, Fiore, Grabovsky, Papa]
Comparing our NLL CGC impact factor with the NLL BFKL impact factor should confirm this expression.

Relation to (G)TMD factorization

The WW gluon GTMD

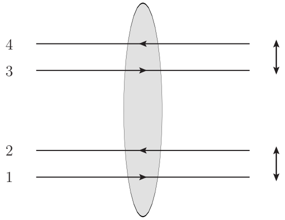
Exclusive dijet production and the Dipole Wigner distribution at small x

- Inclusive dijet production was used to relate CGC Wilson lines and TMD factorization at small x
  [Dominguez, Marquet, Xiao, Yuan]
  distinguishing two unpolarized gluon TMDs, the dipole TMD and the Weizsäcker-Williams TMD

- Exclusive processes give access to more information: exclusive diffractive dijet production can be shown to give access to the dipole GTMD, Fourier transform of the dipole Wigner distribution at small x.
  [Hatta,Xiao,Yuan] [Hagiwara,Hatta,Pasechnik,Tasevsky,Teryaev]

The Weizsäcker-Williams gluon GTMD at small x

In collaboration with Y. Hatta, B.-W. Xiao, F. Yuan



Wilson line derivatives

$$\partial_i U_x = ig \int_{-\infty}^{+\infty} dz^+ [-\infty, z^+]_x F^{i-}(z^+, x)[z^+, +\infty]_x$$

$$\int \prod_{k=1\ldots4} d^2\vec{z}_k e^{-i(\vec{p}_k\cdot\vec{z}_k)} \langle P'|\mathrm{Tr}(U_1 U_4^\dagger U_3 U_2^\dagger) - \frac{1}{N_c}\mathrm{Tr}(U_1 U_2^\dagger)\mathrm{Tr}(U_3 U_4^\dagger)|P\rangle$$

$$\simeq \int d^2\vec{b}_{1,2} d^2\vec{r}_{1,2}\, r_{1\perp}^i r_{2\perp}^j \langle P'|\mathrm{Tr}(\partial_i (U^\dagger U)_{\vec{b}_1} \partial_j (U^\dagger U)_{\vec{b}_2}|P\rangle e^{-i(\Delta_{\perp 1,2}\cdot b_{\perp 1,2}) - i(P_{\perp 1,2}\cdot r_{\perp 1,2})}$$

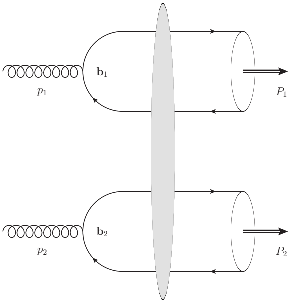
Hence the non-perturbative matrix element becomes

$$\mathcal{O}^{ij} \propto \int \frac{d\xi^+ d^2\vec{\xi}}{(2\pi)^3} e^{ixp^-\xi^+ - i(\vec{q}\cdot\vec{\xi})} \langle P'|F^{i-}(\xi)\mathcal{U}^{[+]\dagger} F^{j-}(0)\mathcal{U}^{[+]}|P\rangle$$

$\Rightarrow$ The Weizsäcker-Williams gluon GTMD at low-x can be probed in exclusive processes with two small dipoles at the amplitude level.

The Weizsäcker-Williams gluon GTMD at small x

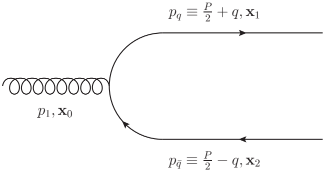
Example of such a process
Production of a forward pair of heavy quarkonia



At leading approximation, the WW GTMD encodes the exchange of a gluon pair in t channel. For connected diagrams to exist, we thus require the $gg \to M$ transitions to exist. Thus only C^+ quarkonia are allowed: η, χ_J mesons.

The Weizsäcker-Williams gluon GTMD at small x: quarkonium production

Non Relativistic QCD framework



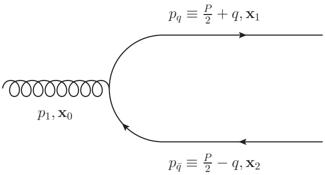
Expansion in the relative velocity v of the constituents of the quarkonia:

$$|\eta\rangle = O(1) \left| Q\bar{Q}[{}^1S_0^1] \right\rangle + O(v)$$

$$|\chi_J\rangle = O(1) \left| Q\bar{Q}[{}^3P_J^1] \right\rangle + O(v)$$

The Weizsäcker-Williams gluon GTMD at small x: quarkonium production

Non Relativistic QCD framework



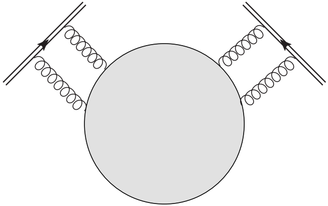
Hard part for η mesons

$$\left[\mathrm{Tr}\left(\frac{\hat{p}}{2} + \hat{q} + m\right)\gamma_5\left(\frac{\hat{p}}{2} - \hat{q} - m\right)\mathcal{A}_{Q\bar{Q}}\left(\frac{p}{2} + q, \frac{p}{2} - q\right)\right]_{q=0}$$

Hard part for χ_J mesons

$$\epsilon^{\mu}_{S_z}\epsilon^{\rho}_{L_z}\left[\frac{d}{dq^{\rho}}\mathrm{Tr}\left(\frac{\hat{p}}{2} + \hat{q} + m\right)\gamma_{\mu}\left(\frac{\hat{p}}{2} - \hat{q} - m\right)\mathcal{A}_{Q\bar{Q}}\left(\frac{p}{2} + q, \frac{p}{2} - q\right)\right]_{q=0}$$

The Weizsäcker-Williams gluon GTMD at small x: hybrid factorization



Forward production $\Rightarrow$ collinear factorization on the projectile side

Multiple scattering: Double PDF

$$\left\langle P \left| G^{+i'} G^{+i} G^{+j'} G^{+j} \right| P \right\rangle$$

Spin decomposition

$$H^{jj'} = \frac{1}{2}\delta^{jj'}\left(\Pi_g^{kk'} H^{kk'}\right) - \frac{1}{2} i\epsilon^{jj'}\left(\Pi_{\Delta g}^{kk'} H^{kk'}\right) + \tau^{jj',ll'}\left(\Pi_{\delta g}^{kk'}\right)^{ll'} H^{kk'},$$

thus 3×3 types of double PDFs: unpolarized ($\delta^{jj'}$), longitudinally polarized ($\epsilon^{jj'}$) and linearly polarized ($\tau^{jj',mm'}$).

The Weizsäcker-Williams gluon GTMD at small x: the GTMD

WW gluon GTMD

$$\mathcal{G}_{ij}\left(x,\boldsymbol{K},\boldsymbol{\Delta}\right) \equiv \int d^2\boldsymbol{b}_1 d^2\boldsymbol{b}_2 e^{-i\boldsymbol{\Delta}\cdot\frac{\boldsymbol{b}_1+\boldsymbol{b}_2}{2}-i\boldsymbol{K}\cdot(\boldsymbol{b}_1-\boldsymbol{b}_2)} \left\langle \mathrm{Tr}\left[\left(\partial_i U^\dagger_{\boldsymbol{b}_1}\right) U_{\boldsymbol{b}_1}\left(\partial_j U^\dagger_{\boldsymbol{b}_2}\right) U_{\boldsymbol{b}_2}\right]\right\rangle_x$$

The WW GTMD is real and $(i \leftrightarrow j)$ symmetric. We can decompose it as

$$\begin{aligned}\mathcal{G}_{ij}(x,\mathbf{P},\boldsymbol{\Delta}) \quad \equiv \quad & \frac{\delta_{ij}}{2}\mathrm{Tr}\mathcal{G} + \left(\frac{P_iP_j}{\mathbf{P}^2} - \frac{\delta_{ij}}{2}\right) f_1 + \frac{\boldsymbol{\Delta}^2}{4M^2}\left(\frac{\Delta_i\Delta_j}{\boldsymbol{\Delta}^2} - \frac{\delta_{ij}}{2}\right) f_2 \\ & - \frac{(\mathbf{P}\cdot\boldsymbol{\Delta})}{2M^2}\left(\frac{P_i\Delta_j}{M^2} + \frac{P_j\Delta_i}{M^2} - \frac{(\mathbf{P}\cdot\boldsymbol{\Delta})}{M^2}\delta_{ij}\right) f_3\end{aligned}$$

The Weizsäcker-Williams gluon GTMD at small x: cross sections

Each quarkonium pair (η, χ_J), $(\chi_J, \chi_{J'})$ probes different double DPFs and different components of the WW GTMD. The pairs where one meson is a χ_1 are the nicest. In particular the (χ_1, χ_1) pair production cross section reads:

$$\frac{d\sigma(\chi_1, \chi_1)}{dY_1 dY_2 d^2\boldsymbol{K}\,|\boldsymbol{\Delta}|\,d\,|\boldsymbol{\Delta}|}$$

$$\simeq \frac{\alpha_s^2}{64 m_1^9 m_2^9} \frac{\boldsymbol{K}^4 \langle \mathcal{O}_{\chi_{c1}}(^3P_1)\rangle \langle \mathcal{O}_{\chi_{b1}}(^3P_1)\rangle}{(2\pi)^2 N_c^4 (N_c^2 - 1)^2} x_1 x_2 \mathcal{F}_{g,g}(x_1, x_2)$$

$$\times \left[\left(1 - \frac{\boldsymbol{\Delta}^2}{4\boldsymbol{K}^2}\right) \mathrm{Tr}\mathcal{G} + \left(1 + \frac{\boldsymbol{\Delta}^2}{2\boldsymbol{K}^2}\right) f_1 - \frac{\boldsymbol{\Delta}^2}{2\boldsymbol{K}^2}(f_2 - f_3)\right]^2$$

for $|\boldsymbol{\Delta}| \ll |\mathbf{K}|$, i.e. almost back-to-back quarkonium pairs.

Relation to GPDs, factorization breaking in DVMP

Relating shockwaves to GPDs at small x

[Hatta, Xiao, Yuan]

relations for CGC to GPD relations (for DVCS)

$$< \mathrm{Tr}(U_{b+r/2}U^{\dagger}_{b-r/2}) > \equiv \int d^2q_{\perp}d^2\Delta_{\perp}e^{-i(b_{\perp}\cdot\Delta_{\perp})-i(q_{\perp}\cdot r_{\perp})}F_x(q_{\perp},\Delta_{\perp}),$$

where the small-x Wigner distribution is decomposed as

$$F_x(q_{\perp},\Delta_{\perp}) = F_0(|q_{\perp}|,|\Delta_{\perp}|) + 2\cos\phi_{q,\Delta}F_{\epsilon}(|q_{\perp}|,|\Delta_{\perp}|)$$

Then one can show

$$\int d^2q_{\perp}q_{\perp}^2F_0(|q_{\perp}|,|\Delta_{\perp}|) = \frac{\alpha_s}{2N_c}x\,H_g(x=\xi,\Delta_{\perp})$$

$$\int d^2q_{\perp}q_{\perp}^2F_{\epsilon}(|q_{\perp}|,|\Delta_{\perp}|) = \frac{\alpha_s\Delta_{\perp}^2}{4N_cM_p^2}xE_{Tg}(x=\xi,\Delta_{\perp})$$

I am applying this method to DVMP, in view of studying factorization breaking mechanisms for the production of a transversely polarized light vector meson.

End point singularities and factorization

End point singularities?

Leading order impact factor for, respectively, $\gamma_L^* \to V_L$ and $\gamma_T^* \to V_L$ transitions:

$$\Phi_L^{(0)} = \frac{2x\bar{x}p_V^+ Q}{(\bar{x}\vec{p}_1 - x\vec{p}_2)^2 + x\bar{x}Q^2},$$

$$\Phi_T^{(0)} = -\frac{(x-\bar{x})p_V^+(\bar{x}\vec{p}_{1\perp} - x\vec{p}_{2\perp})\cdot\vec{\varepsilon}_{\gamma_T}}{(\bar{x}\vec{p}_1 - x\vec{p}_2)^2 + x\bar{x}Q^2}$$

No end point singularity, even for a transverse photon and even in the photoproduction limit and even at NLO.
With null transverse momenta in the t channel, one could encounter $x \in \{0, 1\}$ end point singularities as $\frac{1}{x\bar{x}Q^2}$ thus breaking collinear factorization.

Twist 3 production

Production of a transverse light vector meson

Twist 3 collinear factorization

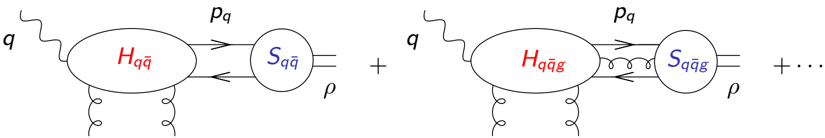
Non-forward and CGC extension of
[Anikin, Besse, Ivanov, Pire, Szymanowski, Wallon]

Twist 3 collinear factorization
Light Cone Collinear approach

- The impact factor can be written as

$$\Phi = \int d^4 p_q \cdots \mathrm{tr}[H(p_q \cdots) \quad S(p_q \cdots)]$$

hard part soft part



- At the 2-body level:

$$S_{q\bar{q}}(p_q) = \int d^4 z\, e^{-ip_q \cdot z} \langle \rho(p) | \psi(0)\, \bar{\psi}(z) | 0 \rangle,$$

- H and S are related by $\int d^4 p_q$ and by the summation over spinor indices

Collinear factorization

Light Cone Collinear approach: 2 steps of factorization (2-body case)

1 - Momentum factorization

- Define a Sudakov vector n such that $p \cdot n = 1$ and write $d^4 p_q = \int dx \, d^4 p_q \; \delta(x - p_q \cdot n)$.
- Taylor expansion of the hard part $H(p_q)$ along the collinear direction p:

$$
\begin{aligned}
& H(p_q) e^{-i p_q \cdot z} S(z) \\
&= H(xp) e^{-i p_q \cdot z} S(z) \quad + \quad \left. \frac{\partial H(p_q)}{\partial p_q^\mu} \right|_{p_q = xp} (p_q - x\, p)^\mu e^{-i p_q \cdot z} S(z) + \ldots
\end{aligned}
$$

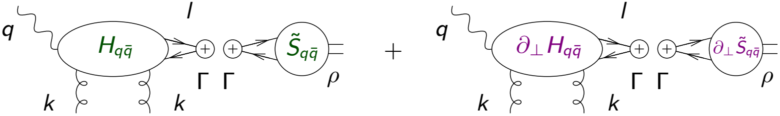
- $p_q^\mu \overset{IbP}{\longrightarrow}$ derivative of the soft term: $\int d^4 z \; e^{-i\ell \cdot z} \langle \rho(p) | \psi(0) \, i \, \overleftrightarrow{\partial_\mu} \, \bar{\psi}(z) | 0 \rangle$
- Standard derivative $\Rightarrow$ need for 3-body contributions to combine into covariant derivative.

Twist 3 collinear factorization

Light Cone Collinear approach: 2 steps of factorization (2-body case)

2 - Spinorial (and color) factorization

- Use Fierz decomposition of the Dirac (and color) matrices $\psi(0)\,\bar{\psi}(z)$ and $\psi(0)\, i\, \overleftrightarrow{\partial_\perp}\, \bar{\psi}(z)$:



- Φ has now the simple factorized form:

$$\Phi = \int dx \left\{ \mathrm{tr}\left[H_{q\bar{q}}(x\,p)\,\Gamma\right]\, S^{\Gamma}_{q\bar{q}}(x) + \mathrm{tr}\left[\partial_\perp H_{q\bar{q}}(x\,p)\,\Gamma\right]\, \partial_\perp S^{\Gamma}_{q\bar{q}}(x) \right\}$$

$\Gamma = \gamma^\mu$ and $\gamma^\mu\,\gamma^5$ matrices

$$S^{\Gamma}_{q\bar{q}}(x) = \int \frac{d\lambda}{2\pi}\, e^{-i\lambda x} \langle \rho(p)|\bar{\psi}(\lambda n)\,\Gamma\,\psi(0)|0\rangle$$

$$\partial_\perp S^{\Gamma}_{q\bar{q}}(x) = \int \frac{d\lambda}{2\pi}\, e^{-i\lambda x} \langle \rho(p)|\bar{\psi}(\lambda n)\,\Gamma\, i\, \overleftrightarrow{\partial_\perp}\, \psi(0)|0\rangle$$

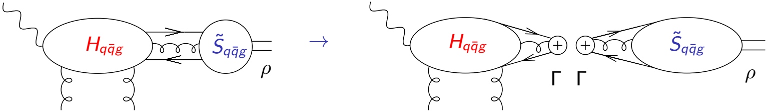
- choose axial gauge condition for gluons, *i.e.* $n \cdot A = 0 \Rightarrow$ no Wilson line

Twist 3 collinear factorization

Light Cone Collinear approach: 2 steps of factorization (3-body case)

Factorization of 3-body contributions

- 3-body contributions start at genuine twist 3
  $\Rightarrow$ no need for Taylor expansion
- Momentum factorization goes in the same way as for the 2-body case
- Spinorial (and color) factorization is similar:



Twist 3 Distribution Amplitudes

Required DAs for ρ_T production

- 2-body DAs

$$\left\langle \rho(p) \left| \bar{\psi}(z) \gamma_\mu \psi(0) \right| 0 \right\rangle \to m_\rho f_\rho \left[\varphi_1(x) \left(\varepsilon_\rho^* \cdot n \right) p_\mu + \varphi_3(x) \varepsilon_{\rho T \mu}^* \right]$$

$$\left\langle \rho(p) \left| \bar{\psi}(z) \gamma_5 \gamma_\mu \psi(0) \right| 0 \right\rangle \to m_\rho f_\rho i \varphi_A(x) \epsilon_{\mu\lambda\beta\delta} \varepsilon_{\rho T}^{\lambda *} p^\beta n^\delta$$

$$\left\langle \rho(p) \left| \bar{\psi}(z) \gamma_\mu i \overleftrightarrow{\partial}_\alpha \psi(0) \right| 0 \right\rangle \to m_\rho f_\rho \varphi_{1T}(x) p_\mu \varepsilon_{\rho T \alpha}^*$$

$$\left\langle \rho(p) \left| \bar{\psi}(z) \gamma_5 \gamma_\mu i \overleftrightarrow{\partial}_\alpha \psi(0) \right| 0 \right\rangle \to m_\rho f_\rho i \varphi_{AT}(x) p_\mu \epsilon_{\alpha\lambda\beta\delta} \varepsilon_{\rho T}^{*\lambda} p^\beta n^\delta$$

- 3-body DAs

$$\left\langle \rho(p) \left| \bar{\psi}(z_1) \gamma_\mu g A_\alpha(z_2) \psi(0) \right| 0 \right\rangle \to m_\rho f_3^V B(x_1, x_2) p_\mu \varepsilon_{\rho T \alpha}^*$$

$$\left\langle \rho(p) \left| \bar{\psi}(z_1) \gamma_5 \gamma_\mu g A_\alpha(z_2) \psi(0) \right| 0 \right\rangle \to m_\rho f_3^A i D(x_1, x_2) p_\mu \epsilon_{\alpha\lambda\beta\delta} \varepsilon_{\rho T}^{*\lambda} p^\beta n^\delta$$

Minimal set of DAs

7 required DAs

- Equations of motion: Dirac equation

$$\left\langle \left(i\hat{D}\psi_\alpha\right)(0)\,\bar{\psi}_\beta\,(z)\right\rangle = 0, \quad \left\langle \psi_\alpha\,(0)\left(i\hat{D}\bar{\psi}_\beta\right)(z)\right\rangle = 0$$

- Leads to two equations

$$x_1\varphi_3\,(x_1) + \bar{x}_1\varphi_A\,(x_1) + \varphi_{1T}\,(x_1) + \varphi_{AT}\,(x_1) + \int dx_2 \left[\zeta_3^V B\,(x_1, x_2) + \zeta_3^A D\,(x_1, x_2)\right] = 0$$

$$\bar{x}_1\varphi_3\,(x_1) - x_1\varphi_A\,(x_1) - \varphi_{1T}\,(x_1) + \varphi_{AT}\,(x_1) - \int dx_2 \left[\zeta_3^V B\,(x_2, x_1) - \zeta_3^A D\,(x_2, x_1)\right] = 0$$

7-2 required DAs

Minimal set of DAs

7-2 required DAs

- *n*-independence. n appeared in three constraints:
  - Lighcone direction of the separation z: $z = \lambda n$
  - Definition of the transverse polarization $\varepsilon_\rho \cdot n = 0$
  - Chosen gauge $n \cdot A = 0$
- Leads to 2 additional constraints for the DAs, plus the gauge invariance condition.

7-4 required DAs

$\varphi(x)$ ← 2-body twist 2 correlator
$B(x_1, x_2)$ ← 3-body genuine twist 3 vector correlator
$D(x_1, x_2)$ ← 3-body genuine twist 3 axial correlator

The 3-body contributions were shown to be suppressed for DVMP [Anikin,Besse,Ivanov,Pire,Szymanowski,Wallon] Thus we can restrict ourselves to 2-body contributions: Wandzura-Wilczek approximation.

Twist 3 collinear factorization

Covariant Collinear approach

- No arbitrary Sudakov vector, use of gauge invariant correlators
- Perform a twist expansion directly at the operator level
- 2-body correlators

$$\left\langle \rho\left(p\right)\left|\bar{\psi}\left(z\right)\left[z,0\right]\gamma^{\mu}\psi\left(0\right)\right|0\right\rangle \rightarrow f_{\rho}m_{\rho}\left[-ip^{\mu}\left(\varepsilon_{\rho}^{*}\cdot z\right)h\left(x\right)+\varepsilon_{\rho}^{\mu *}g_{\perp}^{(v)}\left(x\right)\right]$$

$$\left\langle \rho\left(p\right)\left|\bar{\psi}\left(z\right)\left[z,0\right]\gamma_{5}\gamma_{\mu}\psi\left(0\right)\right|0\right\rangle \rightarrow \frac{1}{4}f_{\rho}m_{\rho}\epsilon_{\mu\alpha\beta\delta}\varepsilon_{\rho\perp}^{\alpha}p^{\beta}z^{\delta}g_{\perp}^{(a)}\left(x\right)$$

- 3-body correlators

$$\left\langle \rho\left(p\right)\left|\bar{\psi}\left(z\right)\left[z,tz\right]\gamma_{\alpha}gG_{\mu\nu}\left(tz\right)\left[tz,0\right]\psi\left(0\right)\right|0\right\rangle$$
$$\rightarrow -im_{\rho}f_{3\rho}^{V}p_{\alpha}\left(p_{\mu}\varepsilon_{\rho\perp\nu}^{*}-p_{\nu}\varepsilon_{\rho\perp\mu}^{*}\right)V\left(x_{1},x_{2}\right)$$

$$\left\langle \rho\left(p\right)\left|\bar{\psi}\left(z\right)\left[z,tz\right]\gamma_{\alpha}\gamma_{5}g\tilde{G}_{\mu\nu}\left(tz\right)\left[tz,0\right]\psi\left(0\right)\right|0\right\rangle$$
$$\rightarrow -m_{\rho}f_{3\rho}^{A}p_{\alpha}\left(p_{\mu}\varepsilon_{\rho\perp\nu}^{*}-p_{\nu}\varepsilon_{\rho\perp\mu}^{*}\right)A\left(x_{1},x_{2}\right)$$

- Equations of motions $\Rightarrow$ only 3 DAs are required
- One-to-one correspondence to LCCF DAs is known

LO result, NLO prospects

Leading order, twist 3, Wandzura-Wilczek [preliminary] result:

$$
-\frac{e_\rho m_\rho f_\rho}{4N_c}\delta\left(p_\gamma^+ - p^+\right)\int d^d\vec{b}\, d^d\vec{r}\int_0^1 dx\, e^{i\left(\vec{p}_\gamma\cdot\vec{b}\right)}\mathrm{Tr}\left(U_{\vec{b}+\bar{x}\vec{r}}U^\dagger_{\vec{b}-x\vec{r}}\right)
$$

$$
\times\left\{g_\perp^{(a)}(x)\,\vec{\varepsilon}_\rho^{\,*}\cdot\left(p_\gamma^+\vec{\varepsilon}_\gamma - \varepsilon_\gamma^+\vec{p}_\gamma\right)\mu_x\left|\vec{r}\right| K_1\left(\mu_x\left|\vec{r}\right|\right)\right.
$$

$$
+\left[4\left(x-\bar{x}\right)h\left(x\right) - g_\perp^{(a)}\left(x\right)\right]\frac{\left(\vec{\varepsilon}_\rho^{\,*}\cdot\vec{r}\right)\left(p_\gamma^+\vec{\varepsilon}_\gamma - \varepsilon_\gamma^+\vec{p}_\gamma\right)\cdot\vec{r}}{\vec{r}^{\,2}}\mu_x\left|\vec{r}\right| K_1\left(\mu_x\left|\vec{r}\right|\right)
$$

$$
\left. + \; 8ih\left(x\right)\varepsilon_\gamma^+\left(\vec{\varepsilon}_\rho^{\,*}\cdot\vec{r}\right)\mu_x^2 K_0\left(\mu_x\left|\vec{r}\right|\right)\right\}
$$

Matches the previous results [Anikin, Besse, Ivanov, Pire, Szymanowski, Wallon] in the forward $t = 0$ limit using the CCF/LCCF correspondence. No end-point singularity.

To get the twist 3 amplitude at NLO:

- Use all twist-3 Fierz projections in our NLO open production result
- Take the derivative of the result
- Add counterterms from ERBL evolution of the DAs

Conclusion

- We provided the full computation of the impact factor for the exclusive diffractive production of a dijet and of a light neutral vector meson with NLO accuracy in the shockwave approach
- In the non-linear limit, our result will provide the consistency check of the JIMWLK/BFKL correspondence
- The computation can be adapted for twist 3 NLO production, removing factorization breaking end-point singularities
- Exclusive diffractive processes are perfectly suited for precision saturation physics and gluon tomography with $b_\perp$ dependence at the EIC