



Probing of strong interactions and hadron matter in hadron and heavy ion collisions

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Complex FAIR



- **HESR:** Storage ring for \bar{p} Injection of \bar{p} at 3.7 GeV/c
- Slow synchrotron (1.5-15 GeV/c) 8
- Luminosity up to L~ 2x10³² cm⁻²s⁻¹
- Beam cooling (stochastic & electron)

$$\sqrt{s} \approx 5.5 \text{ GeV}$$

Antiproton production

- Proton Linac 70 MeV
- Accelerate p in SIS18 / 100
- Produce p on Cu target
- Collection in CR, fast cooling
- Accumulation in RESR
- Storage and usage in HESR

Complex NICA

Collider basic parameters: *beams: from* p to Au; $L\sim 10^{27} \text{ cm}^{-2} \text{ c}^{-1} (\text{Au}), \sqrt{s_{NN}} = 4-11 \text{ GeV}; \sim 10^{32} \text{ cm}^{-2} \text{ c}^{-1} (\text{p}), \sqrt{s} = 12-26 \text{ GeV};$

BMON

Experiments with fixed targets with bld. 205 BM@h

NICA collider major parameters

Booster (600 MeV/u)

KRION-6T+HILac (3MeV/u)

PS and

LU-20 (5MeV/u)

NUCLOTRON

0.6-4.5 GeV/u

	Ring circumference, m	503.04				
	heavy ions					
1	β, m	0.35				
	energy range for Au⁷⁹⁺: √S _{NN} , GeV	4 - 11				
	r.m.s. ⊿p/p, 10 ⁻³	1.6				
į	peak Luminosity for Au⁷⁹⁺ , cm ⁻² s ⁻¹	1x10 ²⁷				
	polarized particles					
	max. energy for polarized p , Gev	27				
	peak Luminosity for p , cm ⁻² s ⁻¹	1x10 ³²				



MPD

Accelertor complex LHEP

SPD

Collider NICA

(1-4.5 GeV/u, C=503 m)

MultiPurpose Detector - MPD

existing

In preparation

MOTIVATION

To look for different charmonium-like states (charmonium and exotics) in *pp\bar* annihilation, *pp* and *pA* collisions to obtain complementary results to the ones from *e+e-* interactions and *B*-meson decays

Outline

- Physics case & motivation
- Conventional & exotic hadrons
- Review of recent experimental data
- Analysis & results
- Summary & perspectives

WHY WE CONCENTRATE ON PHYSICS WITH ANTIPROTONS, PROTONS AND HEAVY IONS



Expected masses of \overline{qq} -mesons, glueballs, hybrids and two-body production thresholds.

Motivation

- Predicted neutral charmonium states compared with found cc̄ states, & both neutral & charged exotic candidates
- Based on Olsen [arXiv:1511.01589]
- Added 4 new J/ψφ states



Charmonium-like states possess some well favored characteristics:

- is the simplest two-particle system consisting of quark & antiquark;
- is a compact bound system with small widths varying from several tens of keV to several tens of MeV compared to the light unflavored mesons and baryons
- charm quark c has a large mass $(1.27 \pm 0.07 \text{ GeV})$ compared to the masses of u, d & s (~ 0.1 GeV) quarks, that makes it plausible to attempt a description of the dynamical properties of charmonium-like system in terms of non-relativistic potential models and phenomenological models;
- quark motion velocities in charmonium-like systems are non-relativistic (the coupling constant, $\alpha_s \approx 0.3$ is not too large, and relativistic effects are manageable ($v^2/c^2 \approx 0.2$);
- the size of charmonium-like systems is of the order of less than 1 Fm $(R_{c\bar{c}} \sim \alpha_s \cdot m_q)$ so that one of the main doctrines of QCD asymptotic freedom is emerging;

Therefore:

- charmonium-like studies are promising for understanding the dynamics of quark interaction at small distances;
- charmonium-like spectroscopy represents itself a good testing ground for the theories of strong interactions:
 - QCD in both perturbative and nonperturbative regimes
 - QCD inspired potential models and phenomenological models



Coupling strength between two quarks as a function of their distance. For small distances $(\leq 10^{-16} \text{ m})$ the strengths α_s is ≈ 0.1 , allowing a theoretical description by perturbative QCD. For distances comparable to the size of the nucleon, the strength becomes so large (strong QCD) that quarks can not be further separated: they remain confined within the nucleon and another theoretical approaches must be developed and applicable. For charmonium (charmonium-like) states $\alpha_s \approx 0.3$ and $\langle v^2/c^2 \rangle \approx 0.2$.

The quark potential models have successfully described the charmonium spectrum, which generally assumes short-range coulomb interaction and long-range linear confining interaction plus spin dependent part coming from one gluon exchange. The zero-order potential is:

$$V_0^{(c\bar{c})}(r) = -\frac{4}{3}\frac{\alpha_s}{r} + br + \frac{32\pi\alpha_s}{9m_c^2}\tilde{\delta}_{\sigma}(r)\vec{S}_c\cdot\vec{S}_{\bar{c}},$$

where $\tilde{\delta}_{\sigma}(r) = (\sigma/\sqrt{\pi})^3 e^{-\sigma^2 r^2}$ defines a gaussian-smeared hyperfine interaction.

Solution of equation with $H_0 = p^2/2m_c + V_0^{(c\bar{c})}(r)$ gives zero order charmonium wavefunctions. **T. Barnes, S. Godfrey, E. Swangon, Phys. Rev. D* 72, 054026 (2005), hep-ph/0505002 & Ding G.J. et al., arXiV: 0708.3712 [hep-ph], 2008 The splitting between the multiplets is determined by taking the matrix element of the $V_{spin-dep}$ taken from one-gluon exchange Breit-Fermi-Hamiltonian between zero-order wave functions:

$$V_{\text{spin-dep}} = \frac{1}{m_c^2} \left[\left(\frac{2\alpha_s}{r^3} - \frac{b}{2r} \right) \vec{\mathbf{L}} \cdot \vec{\mathbf{S}} + \frac{4\alpha_s}{r^3} \mathbf{T} \right]$$

where α_s - coupling constant, *b* - string tension, σ - hyperfine interaction smear parameter.

Izmestev A. has shown * *Nucl. Phys., V.52, N.6 (1990) & *Nucl. Phys., V.53, N.5 (1991)* that in the case of curved coordinate space with radius *a* (confinement radius) and dimension *N* at the dominant time component of the gluonic potential the quark-antiquark potential defines via Gauss equations. If space of physical system is compact (sphere S³), the harmonic potential assures confinement: **Advances in Applied Clifford Algebras, V.8, N.2, p.235 - 270 (1998)*.

$$\Delta V_{N}(\vec{r}) = \operatorname{const} G_{N}^{-1/2}(r) \delta(\vec{r}), \qquad V_{N}(r) = V_{0} \int D(r) R^{1-N}(r) dr/r, \quad V_{0} = \operatorname{const} > 0.$$

$$R(r) = \sin(r/a), \quad D(r) = r/a, \qquad V_{3}(r) = -V_{0} \operatorname{ctg}(r/a) + B, \qquad V_{0} > 0, \quad B > 0.$$

When cotangent argument in V₃(r) is small: $r^2/a^2 \ll \pi^2$, we get: $ctg(r/a) \approx a/r - r/3a$, $V(r)|_{r \to 0} \sim 1/r$

where R(r), D(r) and $G_N(r)$ are scaling factor, gauging and determinant of metric tensor $G_{\mu\nu}(r)$.



The \overline{cc} system has been investigated in great detail first in e⁺e⁻-reactions, and afterwards on a restricted scale ($E_{\overline{p}} \leq 9$ GeV), but with high precision in \overline{pp} -annihilation (the experiments R704 at CERN and E760/E835 at Fermilab).

The number of unsolved questions related to charmonium has remained:

- singlet ${}^{1}D_{2}$ and triplet ${}^{3}D_{J}$ charmonium states are not determined yet;
- nothing is known about partial width of ${}^{1}D_{2}$ and ${}^{3}D_{J}$ charmonium states.
- higher laying singlet ${}^{1}S_{0}$, ${}^{1}P_{1}$ and triplet ${}^{3}S_{1}$, ${}^{3}P_{J}$ charmonium states are poorly investigated;
- only few partial widths of ³P_J-states are known (some of the measured decay widths don't fit theoretical schemes and additional experimental check or reconsideration of the corresponding theoretical models is needed, more data on different decay modes are desirable to clarify the situation);
 <u>AS RESULT</u>:
- little is known on charmonium states above the the $D\overline{D}$ threshold (*S*, *P*, *D*,....);
- many recently discovered states above $D\overline{D}$ threshold (*XYZ*-states) expect their verification and explanation (their interpretation now is far from being obvious).

IN GENERAL ONE CAN IDENTIFY FOUR MAIN CLASSES OF CHARMONIUM DECAYS:

- decays into particle-antiparticle or $D\overline{D}$ -pair: $\overline{cc} \to (\Psi, \eta_{c'}, \chi_{cJ'}...) \to \Sigma^0 \overline{\Sigma}^0, \ \Lambda \overline{\Lambda}, \ \Sigma^0 \overline{\Sigma}^0 \pi, \ \Lambda \overline{\Lambda} \pi;$
- decays into light hadrons: $\overline{cc} \to (\Psi, \eta_c, ...) \to \rho \pi; \overline{cc} \to \Psi \to \pi^+ \pi^-, \overline{cc} \to \Psi \to \omega \pi^0, \eta \pi^0, ...;$
- radiative decays: $\overline{cc} \rightarrow \gamma \eta_c, \gamma \chi_{cJ}, \gamma J/\Psi, \gamma \Psi', ...;$
- decays with J/Ψ , Ψ' and h_c in the final state: $\overline{cc} \to J/\Psi + X =>\overline{cc} \to J/\Psi \pi^+ \pi^-, \overline{cc} \to J/\Psi \pi^0 \pi^0;$ $\overline{cc} \to \Psi' + X =>\overline{cc} \to \Psi' \pi^+ \pi^-, \overline{cc} \to \Psi' \pi^0 \pi^0; \overline{cc} \to h_c + X =>\overline{cc} \to h_c \pi^+ \pi^-, \overline{cc} \to h_c \pi^0 \pi^0.$

non-standard hadrons

non-qq & non-qqq color-singlet combinations



Multiquark states have been discussed since the 1st page of the quark model

A SCHEMATIC MODEL OF BARYONS AND MESONS *

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If we assume that the strong interactions of baryons and mesons are correctly described in terms of the broken "eightfold way" 1-3), we are tempted to look for some fundamental explanation of the situation. A highly promised approach is the purely dynamical "bootstrap" model for all the strongly in teracting particles within which one may trypo rive isotopic spin and strangeness correctation and broken eightfold symmetry from sfl-consistency alone 4). Of course, with only a rong interactions, the orientation of the asymmetry in the unitary space cannot be specified; one hopes that in some way the selection of specific components of the Fspin by electromagnetism and the weak interactions determines the choice of isotopic spin and hypercharge directions.

Even if we consider the scattering amplitudes of strongly interacting particles on the mass shell only and treat the matrix elements of the weak, electromagnetic, and gravitational interactions by means ber $n_t - n_{\bar{t}}$ would be zero for all known baryons and mesons. The most interesting example of such a model is one in which the triplet has spin $\frac{1}{2}$ and z = 1, so bott the four particles d⁻, s⁻, u⁰ and b⁰ exhibit a parallel with the leptons.

A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" 6) q and the members of the anti-triplet as anti-quarks \bar{q} . Baryons can now be constructed from quarks by using the combinations (qqq), (qqqq \bar{q}), etc., while mesons are made out of (q \bar{q}). (qq $\bar{q}\bar{q}$), etc. It is assuming that the lowest baryon configuration (qqq) gives just the representations 1, 8, and 10 that have been observed, while the lowest meson configuration (q \bar{q}) similarly gives just 1 and 8.

Two different kinds of experiments to study exotics:

- production experiment $-\overline{ccg} \rightarrow X + M$, where $M = \pi$, η , ω ,... (conventional states plus states with exotic quantum numbers)
- formation experiment (annihilation process) $\overline{ccg} \rightarrow X \rightarrow M_1M_2$ (conventional states plus states with non-exotic quantum numbers)

 $\begin{array}{c|c} & \text{Gluon} \\ \hline (q\overline{q})_8 & 1^- \text{(TM)} & 1^+ \text{(TE)} \\ \hline {}^1S_0, 0^{-+} & 1^{++} & 1^{--} \\ \hline {}^3S_1, 1^{--} & 0^{+-} \leftarrow \text{exotic} & 0^{-+} \\ & 1^{+-} & 1^{-+} \leftarrow \text{exotic} \\ & 2^{+-} \leftarrow \text{exotic} & 2^{-+} \end{array}$

The low laying charmonium hybrid states:

Charmonium-like exotics (hybrids, tetraquarks) predominantly decay via electromagnetic and hadronic transitions and into the open charm final states:

• $\overline{ccg} \rightarrow (\Psi, \chi_{cJ})$ + light mesons $(\eta, \eta', \omega, \varphi)$ and $(\Psi, \chi_{cJ}) + \gamma$ - these modes supply small widths and significant branch fractions;

• $\overline{ccg} \rightarrow DD_J^*$. In this case S-wave (L = 0) + P-wave (L = 1) final states should dominate over decays to $D\overline{D}$ (are forbidden $\rightarrow CP$ violation) and partial width to should be very small.

The most interesting and promising decay channels of charmed hybrids have been, in particular, analyzed:

- $\overline{cc} \rightarrow \widetilde{\eta}_{c^{0,1,2}} (0^{-+}, 1^{-+}, 2^{-+}) \eta \rightarrow \chi_{c^{0,1,2}} (\eta, \pi\pi, \gamma; ...);$
- $\overline{cc} \rightarrow \widetilde{h}_{c0,1,2}(0^{+-}, 1^{+-}, 2^{+-}) \eta \rightarrow \chi_{c0,1,2}(\eta, \pi\pi, \gamma; ...); \qquad J^{PC} = 0^{--} \rightarrow \text{exotic!}$
- $\overline{cc} \rightarrow \widetilde{\Psi}(0 \leftarrow , \underline{l} \rightarrow , 2 \rightarrow) \rightarrow J/\Psi(\eta, \omega, \pi\pi, \gamma \dots);$
- $\overline{cc} \rightarrow \tilde{\eta}_{c0,1,2}, \quad h_{c0,1,2}, \quad \tilde{\chi}_{c1} \ (0^{-+}, 1^{-+}, 2^{-+}, 0^{+-}, 1^{+-}, 2^{+-}, 1^{++}) \ \eta \rightarrow D\overline{D}_{J}^{*}(\eta, \gamma).$

According the constituent quark model tetraquark states are classified in terms of the diquark and diantiquark spin S_{cq} , $S_{\overline{cq}}$, total spin of diquark-diantiquark system *S*, total angular momentum *J*, spacial parity *P* and charge conjugation *C*. The following states with definite quantum numbers J^{PC} are expected to exist:

- two states with J = 0 and positive *P*-parity $J^{PC} = 0^{++} i.e.$, $\begin{vmatrix} 0_{cq}, 0_{\overline{cq'}} \end{vmatrix}$; $S = 0, J = 0 \rangle$ and $\begin{vmatrix} 1_{cq}, 1_{\overline{cq'}} \end{vmatrix}$; $S = 0, J = 0 \rangle$;

- three states with J = 0 and negative *P*-parity *i.e.*, $|A\rangle = |1_{cq}, 0_{\overline{cq'}}; S = 1, J = 0\rangle; |B\rangle = |0_{cq}, 1_{\overline{cq'}}; S = 1, J = 0\rangle; |C\rangle = |1_{cq}, 1_{\overline{cq'}}; S = 1, J = 0\rangle$. State $|C\rangle$ is even under charge conjugation. Taking symmetric and antisymmetric combinations of states $|A\rangle$ and $|B\rangle$ we obtain a *C*-odd and *C*-even state respectively; therefore we have one state with $J^{PC} = 0^{--}$ *i.e.*, $|0^{--}\rangle = \frac{1}{\sqrt{2}}(|A\rangle + |B\rangle)$ and two states

with
$$\mathcal{J}^{PC} = 0^{-+} i.e., | 0^{-+} \rangle_1 = \frac{1}{\sqrt{2}} (|A\rangle - |B\rangle); | 0^{-+} \rangle_2 = | C \rangle.$$

- three states with J = 1 and positive *P*-parity *i.e.*, $|D\rangle = |1_{cq}, 0_{cq'}; S = 1, J = 1\rangle; |E\rangle = |0_{cq}, 1_{cq'}; S = 1, J = 1\rangle; S = 1, J = 1\rangle; |F\rangle = |1_{cq}, 1_{cq'}; S = 1, J = 1\rangle$. State $|F\rangle$ is odd under charge conjugation. Operating $|D\rangle$ and $|E\rangle$ in the same way as for states $|A\rangle$ and $|B\rangle$ we obtain one state with $J^{PC} = 1^{++}$ state *i.e.*, $|1^{++}\rangle = \frac{1}{\sqrt{2}}(|D\rangle + |E\rangle)$ and two states with $J^{PC} = 1^{+-}$ *i.e.*, $|1^{+-}\rangle_1 = \frac{1}{\sqrt{2}}(|D\rangle - |E\rangle); |1^{+-}\rangle_2 = |F\rangle$. - one state with J = 2 and positive *P*-parity $J^{PC} = 2^{++}$ *i.e.*, $|1_{cq}, 1_{cq'}; S = 1, J = 2\rangle$.

• $\overline{p}p \to X \to J/\Psi \rho \to J/\Psi \pi\pi; \quad \overline{p}p \to X \to J/\Psi \omega \to J/\Psi \pi\pi\pi, \quad \overline{p}p \to X \to \chi_{cJ} \pi \text{ (decays into } J/\Psi, \Psi', \chi_{cJ} \text{ and light mesons)};$

• $\overline{p}p \to X \to D\overline{D}^* \to D\overline{D} \gamma$; $\overline{p}p \to X \to D\overline{D}^* \to D\overline{D} \eta$ (decays into $D\overline{D}^*$ -pair).

Candidate exotic hadrons

	State	M (MeV)	Γ (MeV)	J^{PC}	Process (decay mode)	Experiment
	$\pi_1(1400)$	1354 ± 25	330 ± 25	1-+	$\pi^- p \rightarrow (\eta \pi^-) p$	MPS, Compass
Light quark sector	4				$par{p} o \pi^0(\pi^0\eta)$	Xtal Barrel
-0	X(1835)	$135.7^{+5.0}_{-3.2}0$	99 ± 50	0^{-+}	$J/\psi \rightarrow \gamma (p\bar{p})$	BESH, CLEOc, BESHI
	L				$J/\psi \to \gamma \left(\pi^+\pi^-\eta'\right)$	BESII, BESIII
	-X(3872)	3871.68 ± 0.17	< 1.2	1++	$B ightarrow K + (J/\psi\pi^+\pi^-)$	Belle, BaBar, LHCb
					$p\bar{p} \rightarrow (J/\psi \pi^+\pi^-) + \dots$	CDF, D0
					$B \rightarrow K + (J/\psi \pi^+ \pi^- \pi^0)$	Belle, BaBar
					$B \to K + (D^0 D^0 \pi^0)$	Belle , BaBar
					$B ightarrow K + (J/\psi\gamma)$	BaBar, Belle , LHCb
					$B ightarrow K + (\psi' \gamma)$	BaBar, Belle , LHCb
	V (004 T)	001= 1 1 0 =	00+10	0++	$pp \rightarrow (J/\psi \pi^+ \pi^-) + \dots$	LHCb, CMS
	X (3915)	3917.4 ± 2.7	28-10	0.1.1	$B \to K + (J/\psi \omega)$	Belle, BaBar
	(0.12)		0410	a++	$e^+e^- \rightarrow e^+e^- + (J/\psi\omega)$	Belle, BaBar
	$\chi_{c2}(2P)$	3927.2 ± 2.6	24 ± 6	2^{++}	$e'e \rightarrow e'e + (DD)$	Belle, BaBar
	X(3940)	3942_{-8}	37-17	0(?)	$e'e \rightarrow J/\psi + (D^*D)$	Belle
Charmonium like	(2(2000)	2042 + 01	E0-L11	1	$e'e \rightarrow J/\psi + ()$	Belle DaDan Dalla
Charmonium-like	G(3900) V(4008)	3943 ± 21	52±11	1	$e^+e^- \rightarrow \gamma + (DD)$	BaBar, Belle
	Y(4008) V(4140)	4008 49	220±97 82+300	1++	$e \cdot e \rightarrow \gamma + (J/\psi \pi \cdot \pi)$ $B \rightarrow K + (J/\psi \pi)$	CDF CMS LHCb
	X(4160)	4140.5 - 5.3 4156^{+29}	130+113	$0(2)^{-(?)+}$	$D \rightarrow K + (J/\psi \phi)$ $e^+e^- \rightarrow I/\psi + (D^*\bar{D})$	Belle
	Y(4260)	4263^{+8}	95+14	1	$e^+e^- \rightarrow \alpha + (1/\psi \pi^+\pi^-)$	BaBar CLEO Belle
	1 (4200)	4200_9	J0114	1	$e^+e^- \rightarrow (I/\psi\pi^+\pi^-)$	CLEO BESIII
					$e^+e^- \rightarrow (J/\psi \pi^0 \pi^0)$	CLEO, BESHI
	V(4274)	4273+19	56 ± 16	1++	$B \rightarrow K + (I/w \phi)$	CDF CMS LHCb
	X(4350)	$4350.6^{+4.6}$	$13.3^{+18.4}$	$0/2^{++}$	$e^+e^- \rightarrow e^+e^- (J/\psi \phi)$	Belle
	Y(4360)	4361 ± 13	74+18	1	$e^+e^- \rightarrow \gamma + (\psi'\pi^+\pi^-)$	BaBar, Belle
	X(4630)	$4634^{+,9}$	92^{+41}	1	$e^+e^- \rightarrow \gamma \left(\Lambda^+_e \Lambda^e\right)$	Belle
	Y(4660)	4664 ± 12	48 ± 15	1	$e^+e^- \rightarrow \gamma + (\psi'\pi^+\pi^-)$	Belle
	$Z_{c}^{+}(3900)$	3890 ± 3	33 ± 10	1+-	$Y(4260) \to \pi^- + (J/\psi \pi^+)$	BESIII, Belle
					$Y(4260) \to \pi^- + (D\bar{D}^*)^+$	BESIII
	$Z_{c}^{+}(4020)$	4024 ± 2	10 ± 3	$1(?)^{+(?)-}$	$Y(4260) \to \pi^- + (h_c \pi^+)$	BESIII
					$Y(4260) \to \pi^- + (D^*\bar{D}^*)^+$	BESIII
Charged	$Z_1^+(4050)$	4051^{+24}_{-43}	82^{+51}_{-55}	??+	$B \to K + (\chi_{c1} \pi^+)$	Belle, BaBar
charmonium-like	$Z^{+}(4200)$	4196^{+35}_{-32}	370^{+99}_{-149}	1+-	$B ightarrow K + (J/\psi \pi^+)$	Belle, LHCb
charmonium-like	$Z_2^+(4250)$	4248^{+185}_{-45}	177^{+321}_{-72}	??+	$B \to K + (\chi_{c1} \pi^+)$	Belle, BaBar
	$Z^+(4430)$	4477 ± 20	181 ± 31	1+-	$B \rightarrow K + (\psi' \pi^+)$	Belle, LHCb
Hidden charmed	L				$B ightarrow K + (J\psi \pi^+)$	Belle
poptaquarks	$P_{c}^{+}(4380)$	4380 ± 30	205 ± 88	$(3/2)^{-}$	$\Lambda_b^+ \to K + (J/\psi p)$	LHCb
pentaquarks	$P_{c}^{+}(4450)$	4449.8 ± 3.0	39 ± 20	$(5/2)^+$	$\Lambda_b^+ \to K + (J/\psi p)$	LHCb
	$Y_b(10890)$	$10888.4{\pm}3.0$	$30.7^{+8.9}_{-7.7}$	1	$e^+e^- ightarrow (\Upsilon(nS) \pi^+\pi^-)$	Belle
	$Z_b^+(10610)$) 10607.2 ± 2.0	18.4 ± 2.4	1+-	" $ Υ(5S)" → π^- + (Υ(nS)π^+), n = 1, 2, 3 $	Belle
h-quark sector					$"\Upsilon(5S)" \to \pi^- + (h_b(nP) \pi^+), \ n = 1, 2$	Belle
D-quark sector		10000 1 5			$"\Upsilon(5S)" \to \pi^- + (BB^*)^+, n = 1, 2$	Belle
	$Z_b^0(10610)$	10609 ± 6		1+-	"T(5S)" $\to \pi^{0} + (T(nS)\pi^{0}), n = 1, 2, 3$	Belle
	$Z_b^+(10650)$) 10652.2 ± 1.5	11.5 ± 2.2	1+-	" $T(5S)$ " $\to \pi^- + (T(nS)\pi^+), n = 1, 2, 3$	Belle
					$(16S)'' \to \pi^- + (h_b(nP)\pi^+), n = 1, 2$	Belle
					$(5S)^{\circ} \rightarrow \pi^{-} + (B^{\circ}B^{\circ})^{\circ}, n = 1, 2$	Belle



- Nature of these states? Isospin triplets?
- Different decay channels of the same states observed?
- Other decay modes?

The LHCb new resonances

In 2016 LHCb measured 4 new resonances with an amplitude analysis on $B^+ \rightarrow J/\Psi \phi K^+$ decay

- The X(4140) 1⁺⁺ state, Phys. Rev. Lett. 118, 022003 (2017), Phys. Rev. D 95, 012002 (2017)
 - 📃 Not seen by Belle, and BaBar
 - Seen by CDF and D0
 - **The** 1^{++} quantum numbers ruled out most of the multiquark models.
- The X(4274) 1⁺⁺, Phys. Rev. Lett. 118, 022003 (2017), Phys. Rev. D 95, 012002 (2017)
 - Seen by CDF and CMS and Belle with a higher mass.
- **The** $X(4500) 0^{++}$ and $X(4700) 0^{++}$, Phys. Rev. D 95, 012002 (2017)



X(1⁺⁺)



Contri-	sign.]	Fit results
bution	or Ref.	$M_0 { m MeV}$	$\Gamma_0 \mathrm{MeV}$	F.F. %
All $X(1^+)$		[]		$16\pm3 \ ^{+6}_{-2}$
X(4140)	8.4σ	$4146.5 \pm 4.5 {}^{+4.6}_{-2.8}$	$83\pm21^{+21}_{-14}$	$13\pm3.2^{+4.8}_{-2.0}$
ave.		4146.9 ± 2.3	17.8 ± 6.8	
X(4274)	6.0σ	$4273.3 \pm 8.3 \substack{+17.2 \\ -3.6}$	$56\pm11_{-11}^{+8}$	$7.1 \pm 2.5 {}^{+3.5}_{-2.4}$
CDF		$4274.4^{+8.4}_{-6.7} \pm 1.9$	$32^{+22}_{-15} \pm 8$	
\mathbf{CMS}		$4313.8 \pm 5.3 \pm 7.3$	$38 + 30 \pm 16$	

- Significant X(4140) 8.4σ,
 - mass consistent with the previous measurements, but the width substantially larger
 - $J^{PC}=1^{++}$ determined at 5.7σ including systematic errors
- Significant X(4274) 6.0σ,
 - Consistent with the unpublished CDF results. First significant claim for this structure.
 - $J^{PC}=1^{++}$ determined at 5.8 σ including systematic errors



 $X(0^{++})$



Contri-	sign.		Fit	results
bution		$M_0~{ m MeV}$	$\Gamma_0 \text{ MeV}$	F.F. %
All $X(0^+)$				$28\pm5^{+7}_{-7}$
$NR_{J/\psi \phi}$	6.4σ			$46\pm11^{+11}_{-21}$
X(4500)	6.1σ	$4506 \pm 11^{+12}_{-15}$	$92\pm21{}^{+21}_{-20}$	$6.6 \pm 2.4 {}^{+3.5}_{-2.3}$
X(4700)	5.6σ	$4704 \pm 10^{+14}_{-24}$	$120\pm31^{+42}_{-33}$	$12\pm 5^{+9}_{-5}$

- Significant structures at higher masses, best described by two new 0⁺⁺ resonances X(4500),X(4700):
 - Significances of 6.1σ , 5.6σ
 - $J^{PC}=0^{++}$ determined at 4.0 σ , 4.5 σ , respectively

NEW STATES WITH ZERO STRANGENESS from LHCb

- strangeness zero states charmonium (cssc) structures
- SU(3) symmetry suggests new X_s states near the thresholds:
 D D_s*, D_s D*, D_s*D_s* : observable in B decays?
 B -> X K: M_x < 4785 MeV



• No evidence in preliminary LQCD studies for (\overline{cssc}) tetraquark states.



6 observed states can fit* into charmonium table



* However, not easily: potential models need to be elaborated to describe new masses

What about others?

Panda meeting, ITEP, 19 May, 2015

P. Pakhlov

X(4700)

THE SPECTRUM OF TETRAQUARKS



M.Yu. Barabanov, S. L. Olsen, A.S. Vodopyanov, A.I. Zinchenko, Yad. Fiz., V.79, N.1, pp. 1-4 (2016) / Phys. At. Nucl., V.79, N.1, pp. 126 – 179 (2016)

What to look for

Does the Z(4433) exist??

Better to find charged X !

• Neutral partners of Z(4433)~X(1+,2S) should be close by few MeV and decaying to $\psi(2S) \pi/\eta$ or $\eta_c(2S) \rho/\omega$

What about X(1⁺⁻,1S)? Look for any charged state at ≈
 3880 MeV (decaying to ψπ or η_cρ)

Similarly one expects X(1++,2S) states. Look at M~4200-4300: X(1++,2S)->D^(*)D^(*)

Baryon-anti-baryon thresholds at hand (4572 MeV for 2M_{Λc} and 4379 MeV for M_{Λc}+M_{Σc}). X(2⁺⁺,2S) might be over bb-threshold.

(L.Maiani, A.D.Polosa, V.Riquer, 0708.3997

TETRAQUARK STATES

There are indrastions of Awatures in J/4 & of the kind [CS], tES], + tCS], tESJO - FROM LHCG.

SPECTRUM



PROBLEM: 4290 seems at the moment on 1++ !!

A.D. Polosa, "Bound states in QCD and beyond II", Germany, 20th - 23rd Feb, 2017

CALCULATION OF WIDTHS

The integral formalism (or in other words integral approach) is based on the possibility of appearance of the discrete quasi stationary states with finite width and positive values of energy in the barrier-type potential. This barrier is formed by the superposition of two type of potentials: short-range attractive potential $V_1(r)$ and long-distance repulsive potential $V_2(r)$.

Thus, the width of a quasi stationary state in the integral approach is defined by the following expression (integral formula):

$$\Gamma = 2\pi \left| \int_{0}^{\infty} \phi_{L}(r) V(r) F_{L}(r) r^{2} dr \right|^{2}$$

$$(r < R): \int_{0}^{R} |\phi_{L}(r)|^{2} dr = 1$$

where

where $F_L(r)$ – is the regular decision in the $V_2(r)$ potential, normalized on the energy delta-function; $\phi_{L(r)}$ – normalized wave function of the resonance state. This wave function transforms into irregular decision in the $V_2(r)$ potential far away from the internal turning point.

The integral can be estimated with the well known approximately methods: for example, the saddle-point technique or the other numerical method.





PHYSICS WITH pp & pA COLLISIONS:

• search for the bound states with gluonic degrees of freedom: glueballs and hybrids of the type gg, ggg, $\overline{Q}Qg$, Q^3g in mass range from 1.3 to 5.0 GeV. Especially pay attention at the states \overline{ssg} , \overline{ccg} in mass range from 1.8 – 5.0 GeV.

• charmonium-like states *cc*, *i.e.* $pp \rightarrow \overline{cc} pp$; $pp \rightarrow \overline{cq} cq' pp$ (q, q' = u, d, s)

• spectroscopy of heavy baryons with strangeness, charm and beauty:

 $\Omega_{c}^{0}, \Xi_{c}, \Xi_{c}^{\prime}, \Xi_{c}^{\prime}, \Xi_{c}^{+}, \Omega_{c}^{+}, \Sigma_{b}^{*}, \Omega_{b}^{-}, \Xi_{b}^{0}, \Xi_{b}^{-}.$ $pp \to \Lambda_{c}X; pp \to \Lambda_{c}pX; pp \to \Lambda_{c}pD_{s} \qquad pp \to \Lambda_{b}X, pp \to \Lambda_{b}pX; pp \to \Lambda_{b}pB_{s}$

• study of the hidden flavor component in nucleons and in light unflavored mesons such as η , η' , h, h', ω , φ , f, f'.

• search for exotic heavy quark resonances near the charm and bottom thresholds.

• *D*-meson spectroscopy and *D*-meson interactions: *D*-meson in pairs and rare *D*-meson decays to study the physics of electroweak processes to check the predictions of the Standard Model and the processes beyond it.



Running conditions

1. p+p at $\sqrt{s} = 25 \text{ GeV}$

2. Luminosity $L = 10^{29} \text{ cm}^{-2} \text{c}^{-1} - 10^{31} \text{ cm}^{-2} \text{c}^{-1}$

3. Running time 10 weeks: integrated luminosity $L_{int} = 604.8 \text{ nb}^{-1} - 60.48 \text{ pb}^{-1}$

Expectations for J/ψ

2. Statistics: $N_{J/\psi} = L_{int} \cdot \sigma_{J/\psi} \cdot Br_{J/\psi \to e^+e^-} \cdot Eff_{\Delta \eta = \pm 1.5} = 604.8 \cdot 108.7 \cdot 0.06 \cdot 0.8 = 3156$

Invariant mass: e⁻ + e⁺



Y(4260) state

1. X-section in Pythia6 for heavy flavours with default PDF and $Y(4260) \equiv \chi_{c2}(4260)$ is 81.3 nb

2. X-section for Y(4260) 9.1 nb

3. Y(4260) decay table as for $\psi(2S)$:

Br (Y4260 \rightarrow J/ $\psi \pi$ + π –) = 32.4% Br (Y4260 \rightarrow e+e- π + π –) = 1.9% \rightarrow X-section = 0.18 nb 1000 events for 10 weeks: L = 9.2 · 10²⁹ cm⁻²s⁻¹

Br (Y4260 \rightarrow J/ ψ K+K-) = 7.8% Br (Y4260 \rightarrow e+e- K+K-) = 0.5% \rightarrow X-section = 0.045 nb 1000 events for 10 weeks: L = 3.7 · 10³⁰ cm⁻²s⁻¹

Br $(Y4260 \rightarrow \chi_{c1} \gamma) = 8.7\%$ Br $(\chi_{c1} \rightarrow \gamma J/\psi) = 27.3\%$ Br $(Y4260 \rightarrow e+e-\gamma\gamma) = 0.14\% \rightarrow X$ -section = 0.013 nb 1000 events for 10 weeks: L = 1.3 $\cdot 10^{31}$ cm⁻²s⁻¹

X(3872) state

1. X-section in Pythia6 for heavy flavours with default PDF and $X(3872) \equiv \chi_{c2}(3872)$ is 92.9 nb

2. X-section for X(3872) 20.9 nb

3. X(3872) decay table as for $\psi(2S)$:

Br (X3872 \rightarrow J/ $\psi \pi$ + π –) = 32.4% Br (X3872 \rightarrow e+e- π + π –) = 1.9% \rightarrow X-section = 0.42 nb 1000 events for 10 weeks: L = 3.9 \cdot 10²⁹ cm⁻²s⁻¹



Is the "Y(4260)" produced on pp collisions?

-- possibility for NICA?? --



S/N problem the same as for the X(3872)

Probing the X(3872) meson structure with near-threshold pp and pA collisions at NICA

M.Yu. Barabanov¹, S.-K. Choi², S.L. Olsen^{3†}, A.S. Vodopyanov¹ and A.I. Zinchenko¹

- Joint Institute for Nuclear Research, Joliot-Curie 6 Dubna Moscow region Russia 141980
 (2) Department of Physics, Gyeongsang National University, Jinju 660-701, Korea
 - (3) Center for Underground Physics, Institute for Basic Science, Daejeon 34074, Korea

Pythia8 predictions for X(3872)

1. X-section of $\psi(3770)$ with m = 3.872 GeV at pp 12.5+6.5 GeV: 1.3 nb

- 2. X-section at pCu: 1.3 * A (=63) = 81.9 nb
- 3. Br (X(3872)→J/ $\psi \pi + \pi -$) = 5.00% Br (X(3872)→D⁺D⁻) = 40.45% Br (X(3872)→D⁰D*⁰bar) = 54.55% ⇒ D⁰D⁰bar π^{0} = 35.29%
- 4. Br $(D+->K-\pi+\pi+) = 9.2\%$, Br $(DO->K-\pi+) = 3.8\%$

5. $\sigma(pCu) * Br(J/\psi \pi + \pi -) * Br(e+e-) = 81.9 * 0.05 * 0.06 = 0.246 \text{ nb}$ $\sigma(pCu) * Br(D+D-) * Br(K\pi\pi)^2 = 81.9 * 0.4045 * 0.092 * 0.092 = 0.280 \text{ nb}$ $\sigma(pCu) * Br(D^0D^0bar\pi^0) * Br(K\pi)^2 = 81.9 * 0.3529 * 0.039 * 0.039$

 $\sigma(pCu) * Br(D^{\circ}D^{\circ}bar\pi^{\circ}) * Br(K\pi)^{2} = 81.9 * 0.3529 * 0.039 * 0.039 = 0.044 \text{ nb}$

0.280 nb => L = 5.9 x 10²⁹ (1000 events / 10 weeks)

Probably a mixture of DD ^{*} & a cc ^{*} core^{*}



Near-threshold prod. via pp & pA



Use NICA, a new pp/pA/AA collider at JINR (Dubna)?

 $\sqrt{s_{pN}} \simeq 8 \text{ GeV}$

Summary

- Many observed states remain puzzling and can not be explained for many years. This stimulates and motivates for new searches and ideas. New theoretical models are needed to obtain the nature of charmonium-like states.
- A combined approach based on quarkonium potential model and confinement model has been proposed and applied to study charmonium and exotics.
- The most promising decay channels of charmonium-like states have been analyzed.
- Different charmonium-like states are expected to exist in the framework of the combined approach.
- It is expected that charge / neutral tetraquarks with hidden charm must have neutral / charge partners with mass values which differ by few tens of MeV.
- Using the integral approach for the hadron resonance decay the widths of the expected states were calculated. They turn out to be relatively narrow of the order of several tens of MeV.
- Physics analysis for $\overline{p}p$ annihilation, pp and pA collisions is in progress nowadays.
- NICA & FAIR can provide important complimentary information and new discoveries.
 The necessity for further charmonium and exotics research has been demonstrated.

PERSECTIVES AND FUTURE PLANS

- *D*-meson spectroscopy:
- -CP-violation
- -Flavour mixing
- -Rare decays
- Baryon spectroscopy: -Strange baryons -Charmed baryons

in progress!



JOINT INSTITUTE FOR NUCLEAR RESEARCH

XXIV INTERNATIONAL BALDIN SEMINAR ON HIGH ENERGY PHYSICS PROBLEMS

RELATIVISTIC NUCLEAR PHYSICS & QUANTUM CHROMODYNAMICS

Dubna, Russia, September 17-22, 2018

The program is supposed to present reports from major experimental collaborations and summaries of major theoretical and experimental advances made in relativistic nuclear physics.

The Seminar programme will consist of review talks and contributions presenting original researches.

The working language of the Seminar is English.

http://relnp.jinr.ru/ishepp/



Seminar Topics

- Quantum chromodynamics at large distances
- Relativistic heavy ion collisions
- Hadron spectroscopy, multiquarks
- Cumulative processes
- Structure functions of hadrons and nuclei
- Multiparticle dynamics
- Polarization phenomena, spin physics
- Studies of exotic nuclei in relativistic beams
- Applied use of relativistic beams
- Accelerator facilities: status and perspectives
- New project NICA/MPD (Nuclotron-based Ion Collider fAcility/ Multippurposed Detector) at JINR
- Progress in experimental studies in the high energy centers JINR, CERN, BNL, JLAB, GSI, etc.
- Heavy flavours in hadron and heavy ion collisions (May be?)



Charmonia measurements

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R. Vogt / Physics Reports 310 (1999) 197-260

Table 2

The ψ and ψ' cross sections in proton-induced interactions for $x_F > 0$ with the branching ratios to lepton pairs, $B(J/\psi \to \mu^+\mu^-) = (5.97 \pm 0.25)\%$ and $B(\psi' \to \mu^+\mu^-) = (0.77 \pm 0.17)\%$ [79], divided out. The ψ cross section into lepton pairs at y = 0, $B d\sigma(\psi)/dy|_{y=0}$ is also shown. All cross sections are per nucleon

Ref.	A^{a}	\sqrt{s} (GeV)	$\sigma(\psi)$ (nb)	$B d\sigma(\psi)/dy _{y=0}$ (nb)	$\sigma(\psi')$ (nb)	$\langle p_{\rm T}^2 \rangle$ (GeV ²)
[33]	Be	6.1	$0.1^{+0.10b}_{-0.05}$	~ 0.01		_
[34]	p	6.7	$0.31 \pm 0.09^{\circ}$	0.055 ± 0.02		0.62 ^{d,e}
[1]	Be	7.3	$1^{+1.0b}_{-0.5}$	~ 0.1		~ 0.62"
[35]	p	8.6	1.2 ± 0.6	0.2 ± 0.1^{r}		
[36]	Be	11.5	$11 \pm 3^{\circ}$	1.2 ± 0.4		$0.55 \pm 0.09^{\circ}$
[37]	Be	16.8		5.6 ± 1.5		~ 1°
[38]	Be	16.8	69 ± 23	7.2 ± 2.5^{s}		
[9]	p	16.8	47 ± 10^{h}			
[9]	p	19.4	61 ± 11	4.1 ± 0.3		1.23 ± 0.05
[39]	C	20.5	147 ± 7	14.3 ± 1.5^{s}	8.0 ± 4.5	
[40]	C	20.5	95 ± 13	9.5 ± 1.0	12 ± 7	1.25 ± 0.10
[18]	Li	23.8	162 ± 22		16 ± 6	-
[41]	p	24.3	71.8 + 9.3	6.2 ± 1.1		
[42]	Be	27.4	$110 \pm 27^{\circ}$	8.9 ± 2.2	15 ± 8	$0.91 \pm 0.29^{\circ}$
[43]	p	30	—	9.1 ± 2.5		-
[44]	p	30.6	-	6.6 ± 1.8		
[45]	Be	31.5	161 ± 35	8 ± 2		1.55 ± 0.11
[46]	p	52		7.5 ± 2.5		
[47]	p	52	$350 \pm 160^{\circ}$	12 ± 5		1.2 ± 0.3
[48]	p	52	_	12.8 ± 3.2		
[44]	p	52.7		11.0 ± 0.4	<u></u>	$1.92 \pm 0.15^{\circ}$
[43]	p	53		13.6 ± 3.1		
[44]	p	62.4		10.2 ± 0.7		$1.7 \pm 0.2^{\circ}$
[43]	p	63		14.8 ± 3.3		_

Charmonia measurements

NA38

NA51

Table 4: J/ψ and ψ' absolute cross sections, in the dimuon channel, for the measured p-A reactions. Systematic uncertainties, not included, amount to 7%.

	£	N ^y	$B^{\psi}_{\mu\mu}\sigma^{\psi}$	$B^{\phi'}_{\mu a} \sigma^{\phi'}$
	(nb^{-1})		(nb)	(nb)
C	2232.5	15014 ± 140	55.8 ± 0.6	1.06 ± 0.07
Al	136.4	1851 ± 48	112.1 ± 2.8	1.52 ± 0.39
Cu (2)	63.0	2083 ± 51	267.8 ± 6.3	4.66 ± 0.31
Cu (10.1)	518.4	16522 ± 140	263.5 ± 2.4	4.58 ± 0.29
W (1.5)	25.4	1896 ± 48	606.1 ± 14.8	9.63 ± 0.77
W (5.6)	136.7	11533 ± 118	692.6 ± 7.4	11.00 ± 0.87

Target	H2	D2
N_{ψ}	301236 ± 601	312204 ± 630
$N_{\psi'}$	5705 ± 127	6219 ± 131
N_{DY}	1910 ± 44	2120 ± 46
$B\sigma_{\psi} (nb)$	$5.50 \pm 0.01 \pm 0.36 (0.06)$	$11.32 \pm 0.03 \pm 0.75 (0.13)$
$B'\sigma_{\psi'}(nb)$	$0.086 \pm 0.002 \pm 0.006 (0.003)$	$0.188 \pm 0.004 \pm 0.015(0.006)$
$\sigma_{DY}~(pb)$	$25.3 \pm 0.6 \pm 1.8 (0.5)$	$55.0 \pm 1.2 \pm 3.9 (1.2)$
$B' \sigma_{\psi'} / B \sigma_{\psi} (\%)$	$1.60 \pm 0.04 \pm 0.02$	$1.72 \pm 0.04 \pm 0.025$
$B\sigma_\psi/\sigma_{DY}$	$54.7 \pm 1.0 \pm 1.3$	$53.8\pm1.0\pm0.5$

Table 3: Numbers of J/ψ , ψ' and Drell-Yan events in the mass range [4.3–8.0] GeV/ c^2 as well as the corresponding cross sections. B and B' are the branching ratios of the decay of J/ψ and ψ' resonances into two muons. Ratios of cross sections are also given. In the case of the ratio $B\sigma_{\psi}/\sigma_{DY}$, Drell-Yan pairs are taken in the mass range [2.9–4.5] GeV/ c^2 in order to allow the comparison with other data from the NA38 experiment. Finally, the numbers given in parenthesis correspond to the fraction of systematic error which has to be taken into account in the comparison of the two targets.

X(3872) decay channels



Models for the Y(3872)

$D^{0}-\overline{D}^{*0}$ molecule?

Lots of literature about this



Impossible to produce such an fragile extended object in prompt high energy hadron colliders at the rates reported by CDF & CMS

QCD diquark-diantiquark?



Predicts partner states (e.g., a nearby state with $u \rightarrow d$) that have yet be seen. * Advances in Applied Clifford Algebras, V.8, N.2, p.235 - 270 (1998).

SU(2) group and quantization of momentum.

The SO(4) symmetry of the Schrodinger- Pauli problem, generated by the angular momentum

$$\vec{M} = [\vec{r} \times \vec{p}]$$

and the normalized Laplace-Runge-Lenz vector

$$\vec{A} = (-2mH)^{-1/2} (\frac{\vec{r}}{r} + (2m\alpha)^{-1} (\vec{M} \times \vec{p} - \vec{p} \times \vec{M})),$$

can be used as follows:

1) the eigenvalue problem is solved directly on making use of properties of SU(2)-group:

$$H = \frac{1}{2(((\vec{M} \pm \vec{A}), \vec{\sigma}) + 2\hbar)^2} \to -mc^2 \frac{\alpha^2}{2(n+1)^2}, \ n = 0, 1, 2,$$

these eigenvalues (including degeneracy) are given by standard group theoretical arguments.

In this form \vec{A} and \vec{M} suffice to derive the bound states of the non-relativistic Coulomb problem.

* Advances in Applied Clifford Algebras, V.8, N.2, p.235 - 270 (1998).

In order to demonstrate this let us form two kinds of generators of the generators of SO(4) group

$$\vec{M} = [\vec{r} \times \vec{p}], \ \vec{N} = r_4 \vec{p} - \vec{r} p_4,$$

by taking the following linear combinations

$$\vec{\mathcal{M}}_{\pm} = \frac{1}{2} (\vec{M} \pm \vec{N}).$$

The SU(2) group generate the action on three-dimension sphere S^3 . This action consists of the translation with whirling around the direction of translation.

The operator \vec{N} on S^3 can be written as $\vec{N} = R\vec{p} + \vec{r}(\vec{r}\vec{p})/R$, where R is the radius of the sphere. Hamilton operator

$$H = \frac{2}{mR^2} \{\hbar + (\vec{\mathcal{M}}_{\pm}, \vec{\sigma})\} \{\hbar + (\vec{\mathcal{M}}_{\pm}, \vec{\sigma})\}$$

The spectrum of H:

$$H\Psi_n = \frac{\hbar^2}{2mR^2}(n+1)^2 \ \Psi_n, \quad n = 0, 1, 2, \dots .$$

The discreteness of the energy spectrum is a consequence of the compactness of the group SU(2), the space of which is the space of the solutions. When $R \to \infty$, the Hamiltonian tends to the Hamiltonian of Pauli equation. In this case

$$\mathcal{P}_{\pm} = (\vec{M} \pm \vec{N})/R \to \pm \vec{p},$$

The Dirac equation

$$2mH = \frac{(\vec{\sigma}\vec{\mathcal{M}} + 2\hbar)(\vec{\sigma}\vec{\mathcal{M}} + 2\hbar)}{R} \rightarrow$$
$$\rightarrow Det \left(\begin{array}{c} \frac{H_D}{c} - mc & \vec{\sigma}\vec{\mathcal{M}}_{\pm} + 2\hbar \\ \sigma\vec{\mathcal{M}}_{\pm} + 2\hbar & \frac{H_D}{c} + mc \end{array} \right) = 0 \rightarrow$$
$$\rightarrow \frac{H_D}{c} \Psi_{\pm} = (\vec{\alpha}\frac{\vec{\mathcal{M}}_{\pm}}{R} + \beta mc + \gamma_5 \frac{2\hbar}{R}) \Psi_{\pm}, \quad \gamma_5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}.$$
(2.5)

The spectrum of this Hamiltonian is given by

$$\mathcal{E} = c \sqrt{m^2 c^2 + \frac{\hbar^2 (n+1)^2}{R^2}}, \ n = 0, 1, 2, \dots$$

From a physical point of view, it is clear that we deal with quantization of the momentum:

$$\mathcal{E} = c\sqrt{m^2c^2 + \mathcal{P}^2}, \quad \mathcal{P} = \frac{\hbar^2(n+1)^2}{R^2}, \quad n = 0, 1, 2, \dots$$

Let us define the set of generators of SO(4) group $\longrightarrow \vec{M} = [\vec{r} \times \vec{p}]; \vec{N} = r_{_4}\vec{p} - \vec{r}p_{_4}$ where \vec{r} and \vec{p} are coordinate and momentum operators, \vec{M} is angular momentum operator. Dilatation operator \overline{N} defined on the sphere S³ has the form $\longrightarrow \overline{N} = R\overline{p} + \overline{r}(\overline{r}, \overline{p})/R$ The linear combinations of these orthonormal operators $\rightarrow \vec{\mu}_{+} = \left(\vec{M} \pm \vec{N}\right)$

contribute two set of generators of the SU(2) group. Thus the SU(2) group generates the action on a three-dimensional sphere S^3 . This action consists of the translation with whirling around the direction of translation. We get a Hamiltonian:

$$H = \frac{1}{2mR^2} \{2\hbar + (\vec{\mu}_{\pm}, \vec{\sigma})\} \{2\hbar + (\vec{\mu}_{\pm}, \vec{\sigma})\} \text{ where } \vec{\sigma} \text{ - spin operator, } m \text{ - mass of the top.}$$

 $\vec{\mu}_{\perp} / R = \left(\vec{M} + \vec{N}\right) / R \rightarrow \pm \vec{p}$ When radius of the sphere: $R \rightarrow \infty$

the Hamiltonian tends to the Pauli operator for the free particle motion:

$$H = \frac{1}{2mR^{2}} \{ 2\hbar + (\vec{\mu}_{\pm}, \vec{\sigma}) \} \{ 2\hbar + (\vec{\mu}_{\pm}, \vec{\sigma}) \} \rightarrow \frac{1}{2m} (\vec{p}, \vec{\sigma})^{2}$$

The spectrum is:

 $H\Psi_{n} = \frac{\hbar^{2}}{2mR^{2}}(n+1)^{2}\Psi_{n}, n = 0, 1, 2...$

The wave function: $\Psi_n = |LSJM_J\rangle$

$\Rightarrow \quad \vec{J}^2 = \left((\vec{\mu}_{\perp} + \vec{\sigma})/2 \right)^2 \quad \text{of the top.}$ was taken as eigenfunction of total momentum

* Advances in Applied Clifford Algebras, V.8, N.2, p.235-254 (1998) & V.8, N.2, p.255-270 (1998).

In the framework of this approach in the relativistic case the Hamiltonian of a decaying resonance is defined with the equation ($R \rightarrow a + b$ is a binary decay channel):

$$H = \sqrt{m_a^2 + \frac{1}{R^2} ((\vec{\mu}_{\pm}, \vec{\sigma}) + 2\hbar)^2} + \sqrt{m_b^2 + \frac{1}{R^2} ((\vec{\mu}_{\pm}, \vec{\sigma}) + 2\hbar)^2}$$

were m_a and m_b are the masses of resonance decay products (particles *a* and *b*). The spectrum of the Hamiltonian is:

$$E = \sqrt{m_a^2 + \frac{\hbar^2 (n+1)^2}{R^2}} + \sqrt{m_b^2 + \frac{\hbar^2 (n+1)^2}{R^2}}, \ n = 0, 1, 2...$$

Finally, the formula for resonance mass spectrum can be written in the following form (we used the system in which $\hbar = c = 1$):

$$E = M_{th} = \sqrt{m_a^2 + P_n^2} + \sqrt{m_b^2 + P_n^2} = \sqrt{m_a^2 + (nP_0)^2} + \sqrt{m_b^2 + (nP_0)^2} = \sqrt{m_a^2 + \left[\frac{n}{R_0}\right]^2} + \sqrt{m_b^2 + \left[\frac{n}{R_0}\right]^2}$$

where P_0 – is the basic momentum. The momentum of relative motion of decay products P_n (particles *a* and *b* in the center-of-mass system of decaying resonance) is quantized relatively $P_{0.}$. R_0 is the parameter with dimension of the length conjugated to P_0 .