The Initial Correlations of the Glasma Energy-Momentum Tensor

Pablo Guerrero Rodríguez [a]
with advisors:
Javier L. Albacete[a] and Cyrille Marquet[b]

[a] *CAFPE and Departamento de Física Teórica y del Cosmos, Universidad de Granada*
[b] *Centre de Physique Théorique at École Polytechnique, Palaiseau, France*

Probing QCD at the high energy frontier
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2. **Initial conditions: the Color-Glass Condensate**
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   - Initial conditions for the field in the forward light-cone.

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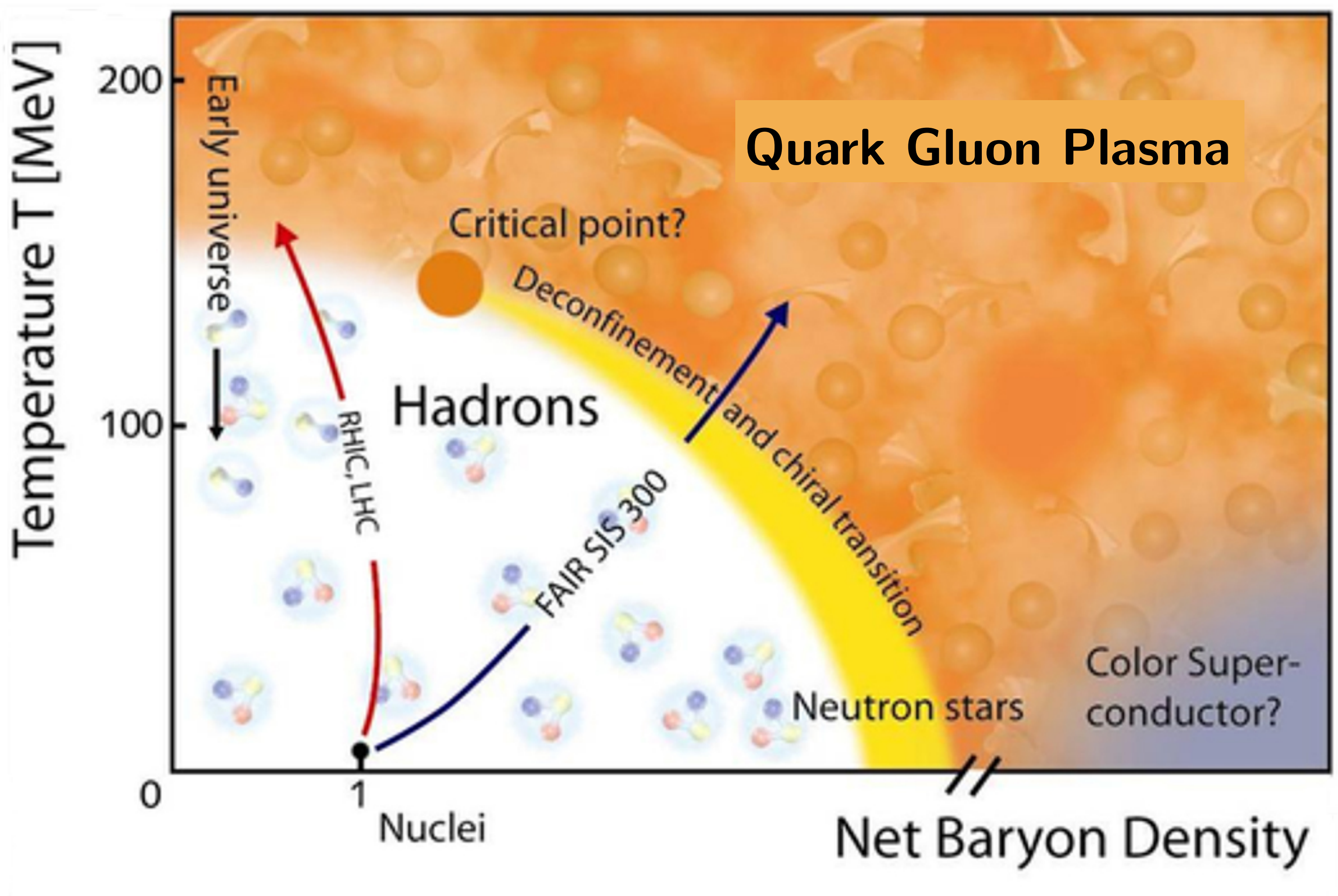
1. Introduction

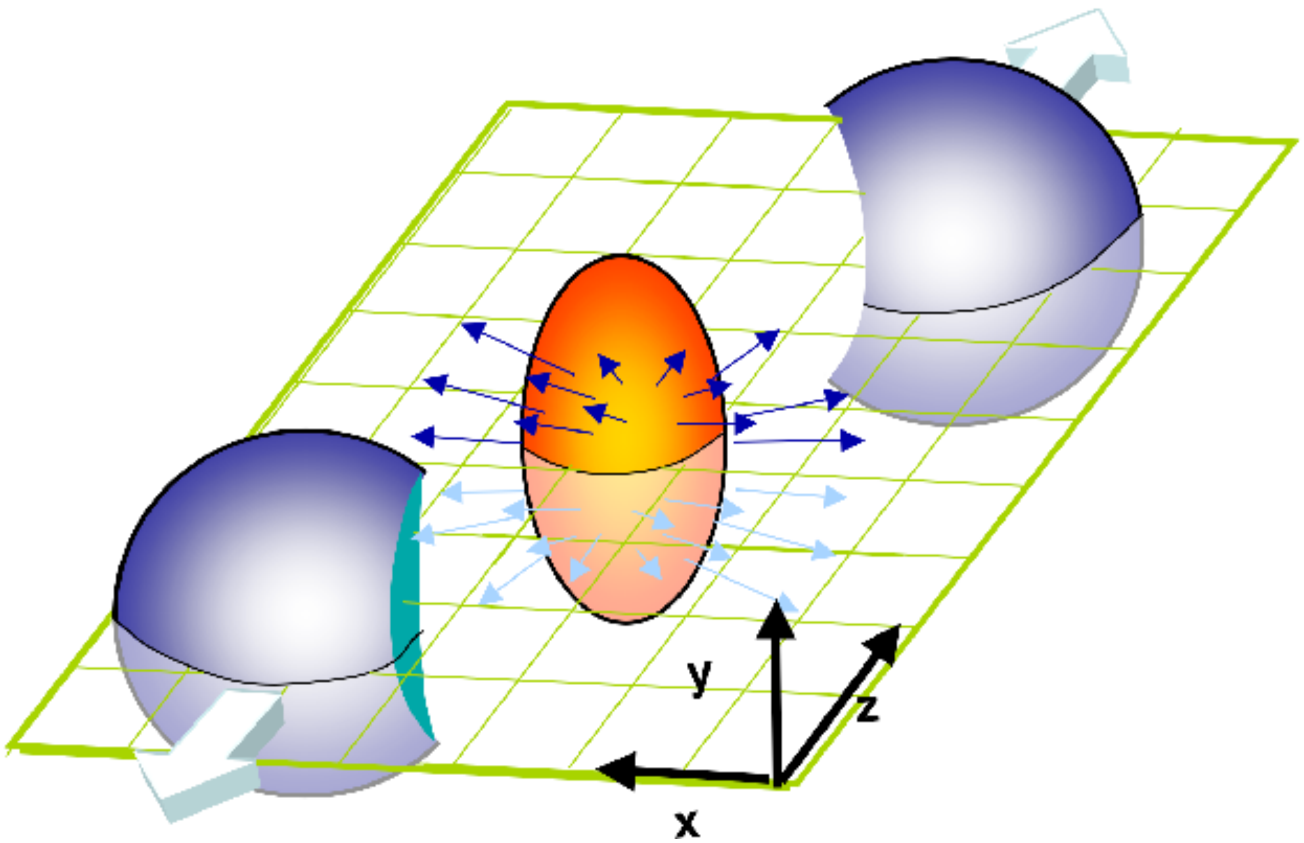
The QCD phase space

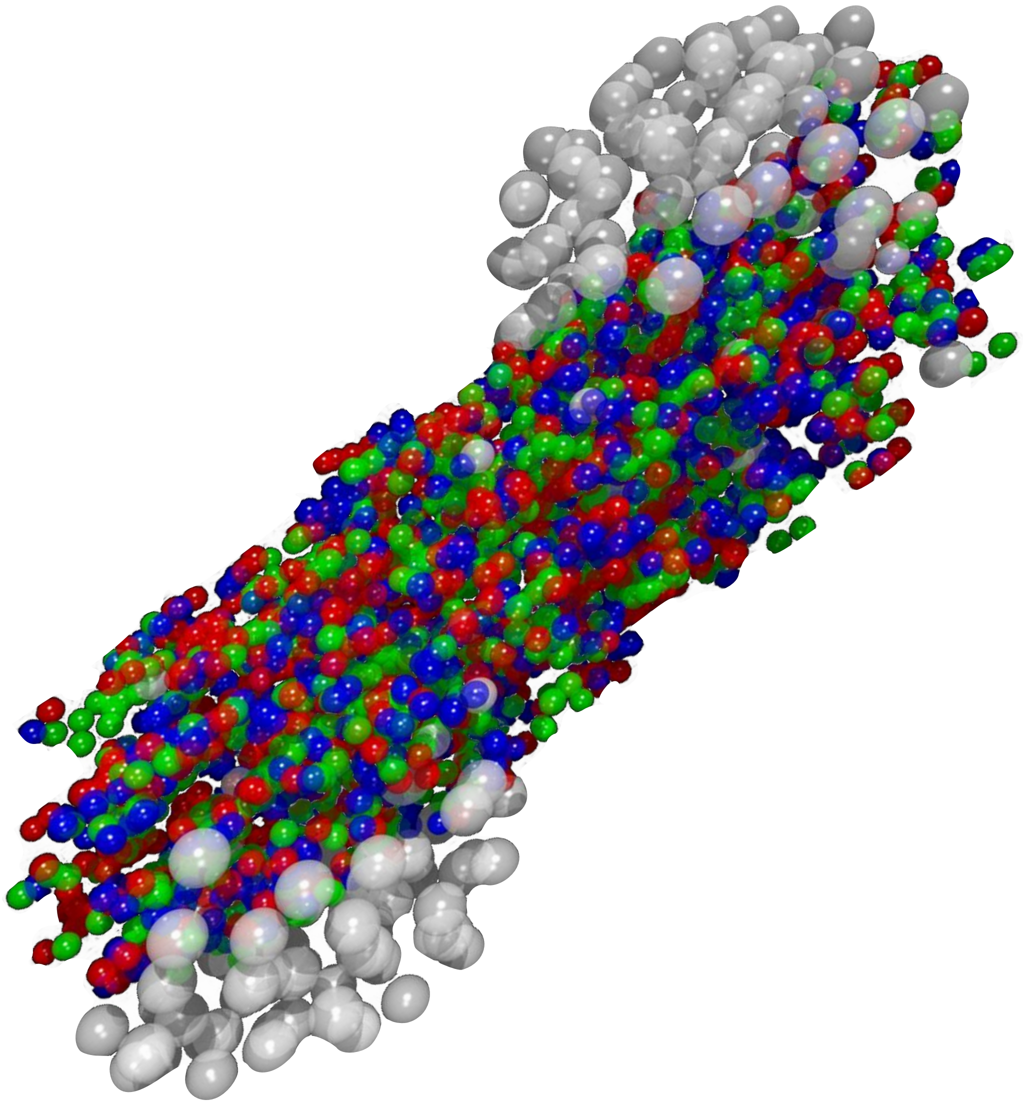


- QCD behaves differently depending on conditions of temperature and baryon density
- Low temperature and densities: **hadronic phase (confinement and spontaneously broken chiral symmetry)**
- Lattice simulations indicate a transition at high temperature to a **deconfined**, **chiral-symmetric phase:** The **QUARK-GLUON PLASMA**

The QCD phase space

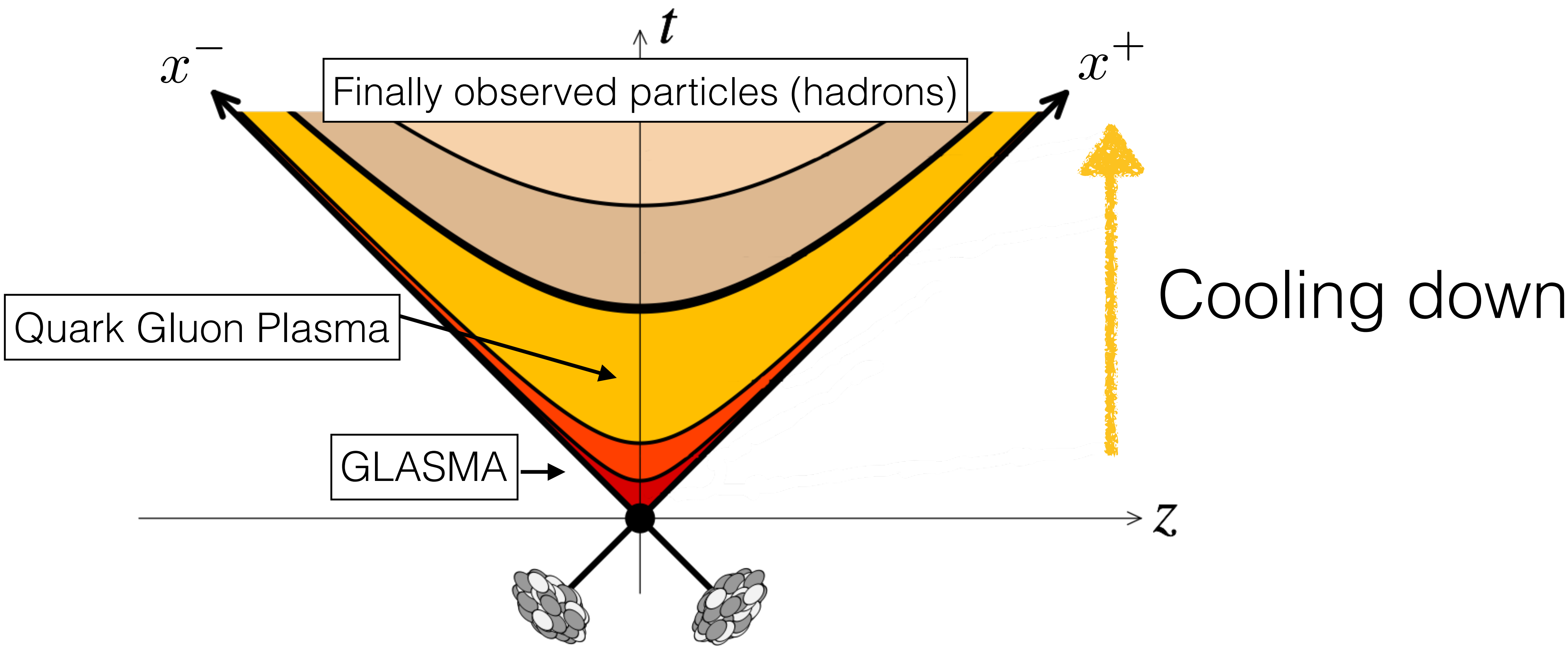
- This state of matter can be accessed in particle colliders through **Heavy Ion Collision** experiments





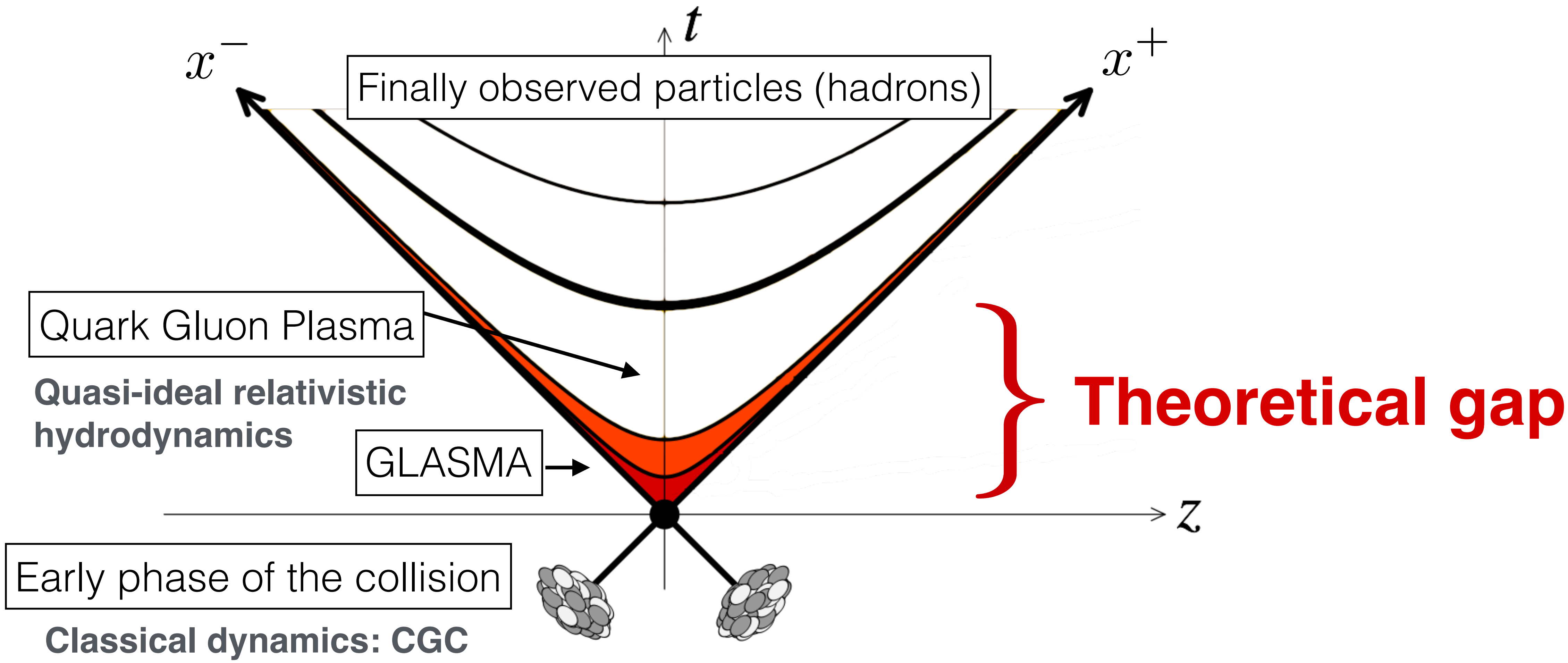
- Performed at Brookhaven National Laboratory's Relativistic Heavy Ion Collider (**RHIC**) and CERN's Large Hadron Collider (**ALICE** experiment)

Stages of a heavy ion collision



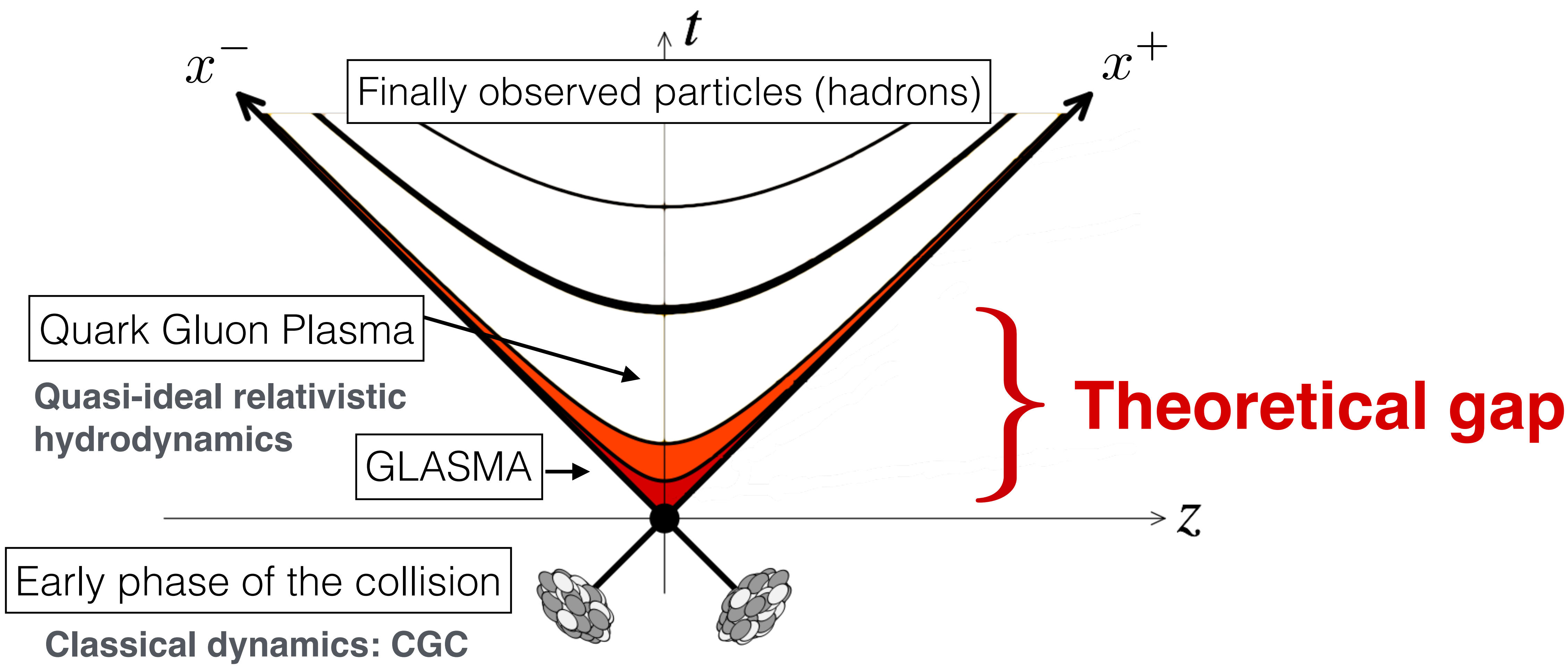
- After the collision, matter goes through different phases as it cools down
- In the last part, it reaches the hadronic phase, and this is how it appears in the detectors

Stages of a heavy ion collision



- There is a theoretical gap between the description of the early phase and the simulations of the expansion of the QGP

Stages of a heavy ion collision



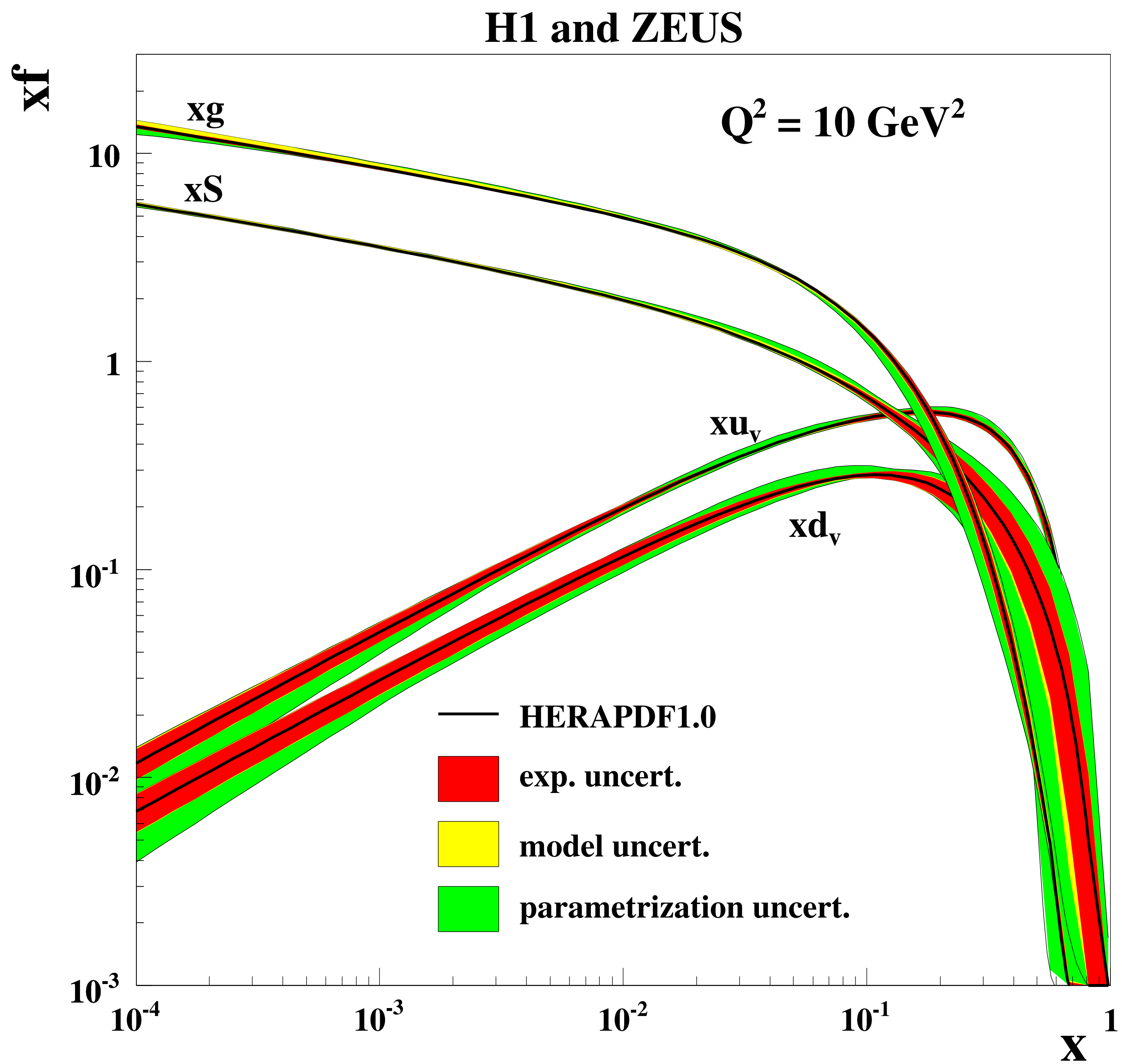
- There is a theoretical gap between the description of the early phase and the simulations of the expansion of the QGP
- Solid theoretical results are needed to mediate between both frameworks
- We provide an exact analytical calculation of the following object:

$$\mathrm{Cov}[T^{\mu\nu}](\tau = 0^+; x_\perp, y_\perp) \equiv \langle T_0^{\mu\nu}(x_\perp) T_0^{\sigma\rho}(y_\perp)\rangle - \langle T_0^{\mu\nu}(x_\perp)\rangle\langle T_0^{\sigma\rho}(y_\perp)\rangle$$

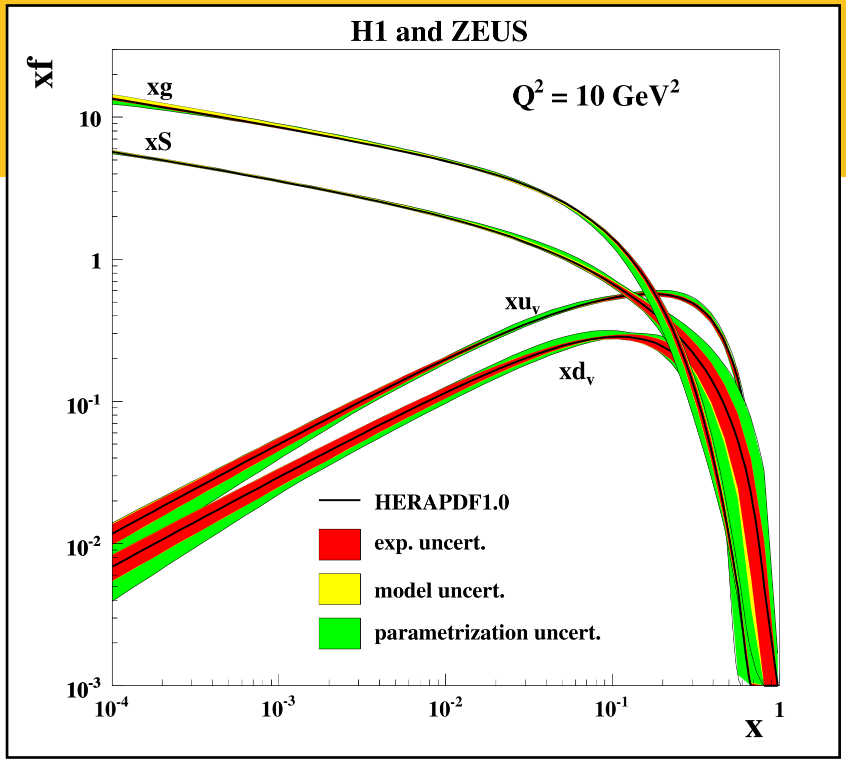
In the classical approximation (MV model)

2. Initial conditions: the Color-Glass Condensate

- At high energies (or equivalently, low x) the partonic content of protons and neutrons is **vastly dominated by a high density of gluons**

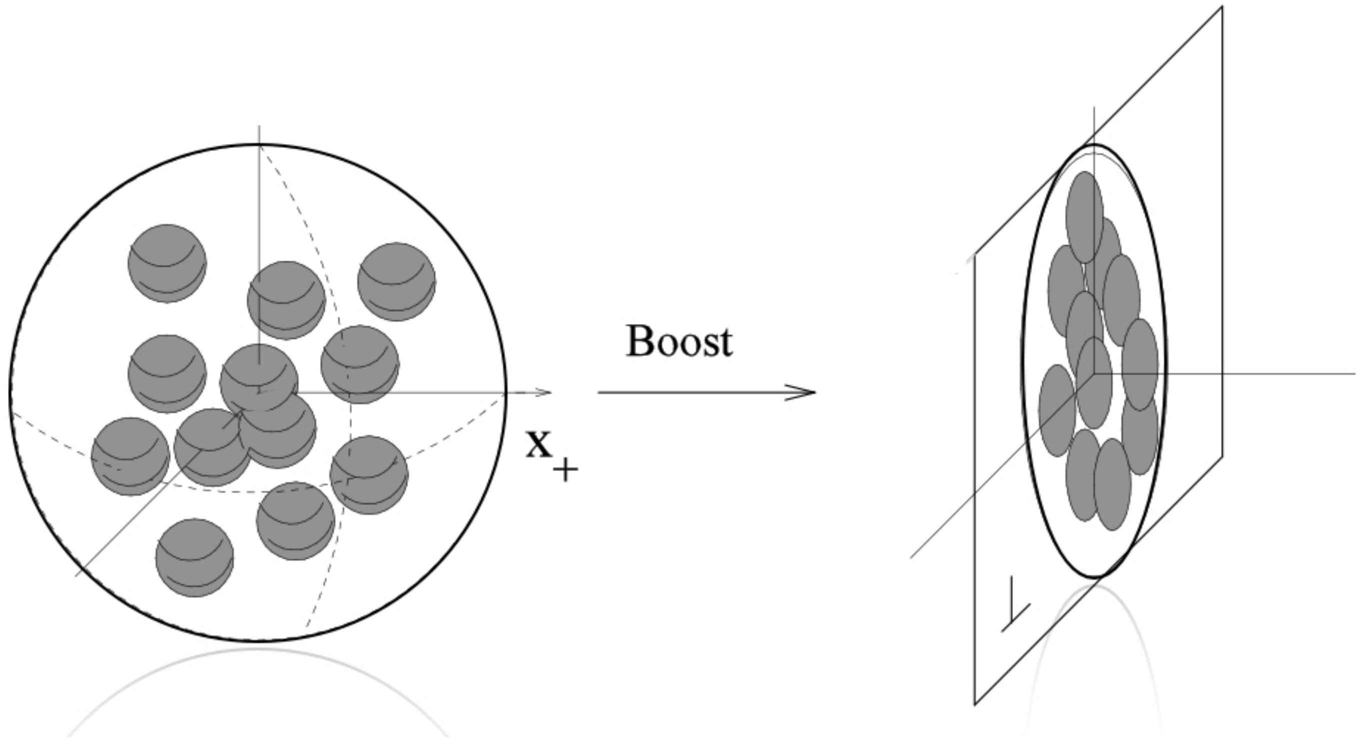


Highly Energetic Heavy Ion Collisions



- At high energies (or equivalently, low x) the partonic content of protons and neutrons is **vastly dominated by a high density of gluons**

- Relativistic kinematics: at high energies, the nuclei appear almost two-dimensional in the laboratory frame due to **Lorentz contraction**



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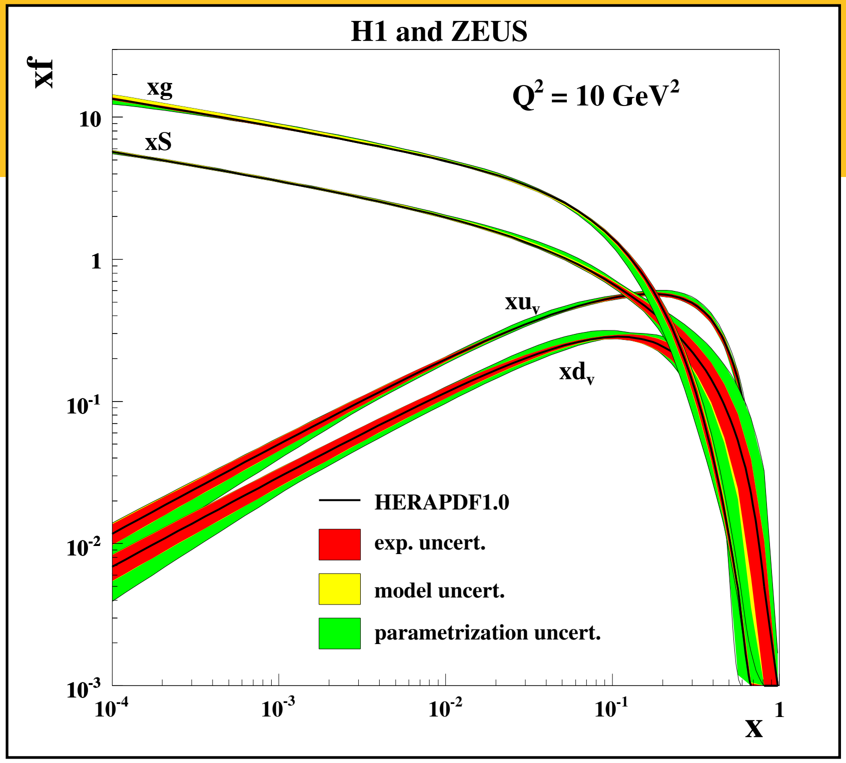

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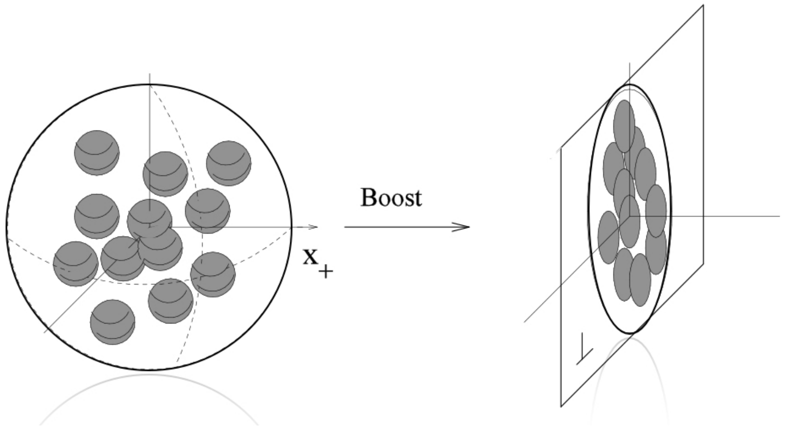
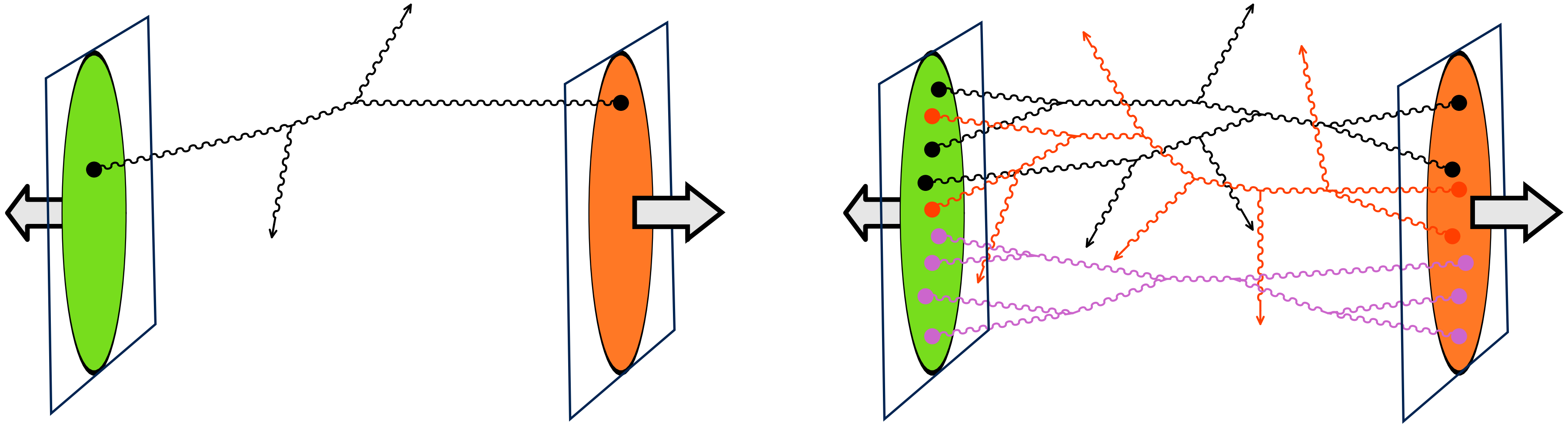
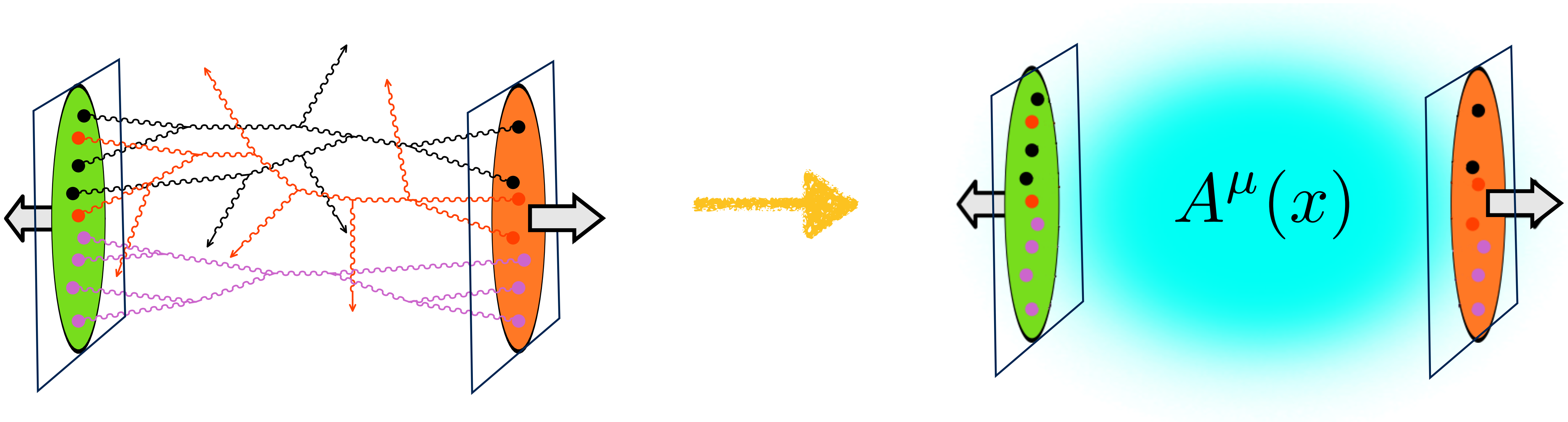


- QCD becomes **non-linear** and **non-perturbative**!



Color Glass Condensate: McLerran-Venugopalan model

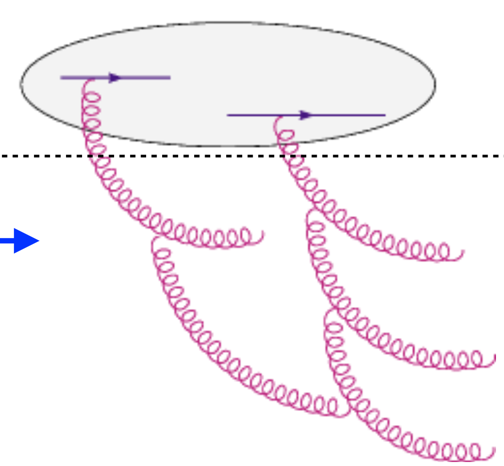
- We use an approximation of QCD for high gluon densities where we replace the gluons with a **classical field** generated by the valence quarks



- Dynamics of the field described by **Yang-Mills** classical equations:

$$[D_\mu, F^{\mu\nu}] = J^\nu \propto \rho(x)$$

$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - ig\,[A^\mu, A^\nu] \longrightarrow$



$\longleftarrow J^{\nu,a} = \delta^{\nu+}\rho^a(x^-, x_\perp)$

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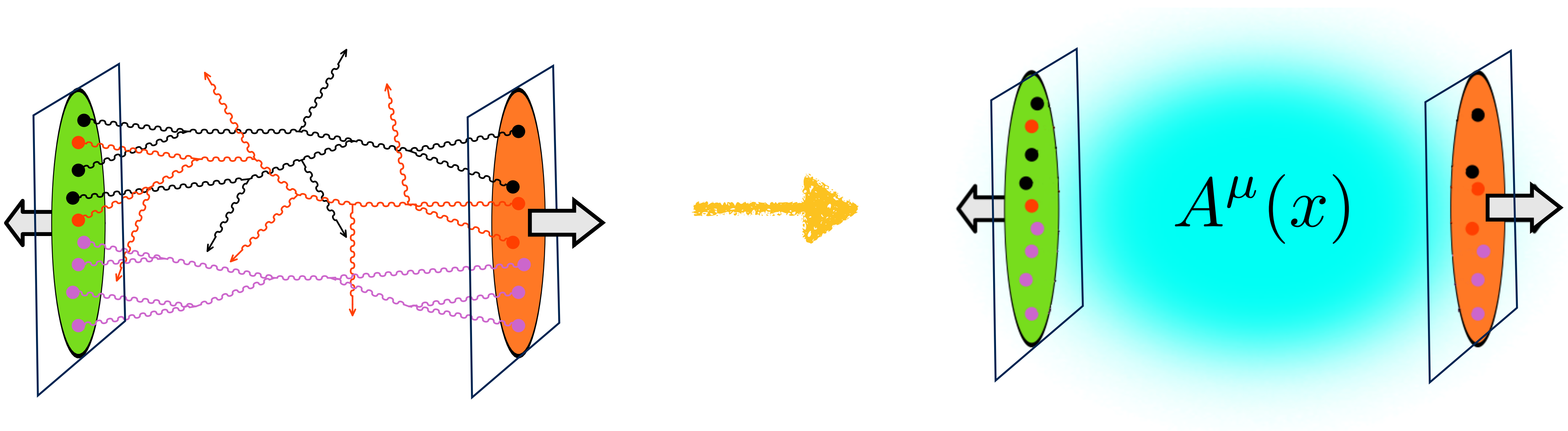

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- Calculation of observables: **average** over background classical fields

$$\langle \mathcal{O}[\rho] \rangle = \int [d\rho] \exp\left\{ - \int dx \, \mathrm{Tr}\left[\rho^2\right] \right\} \mathcal{O}[\rho]$$

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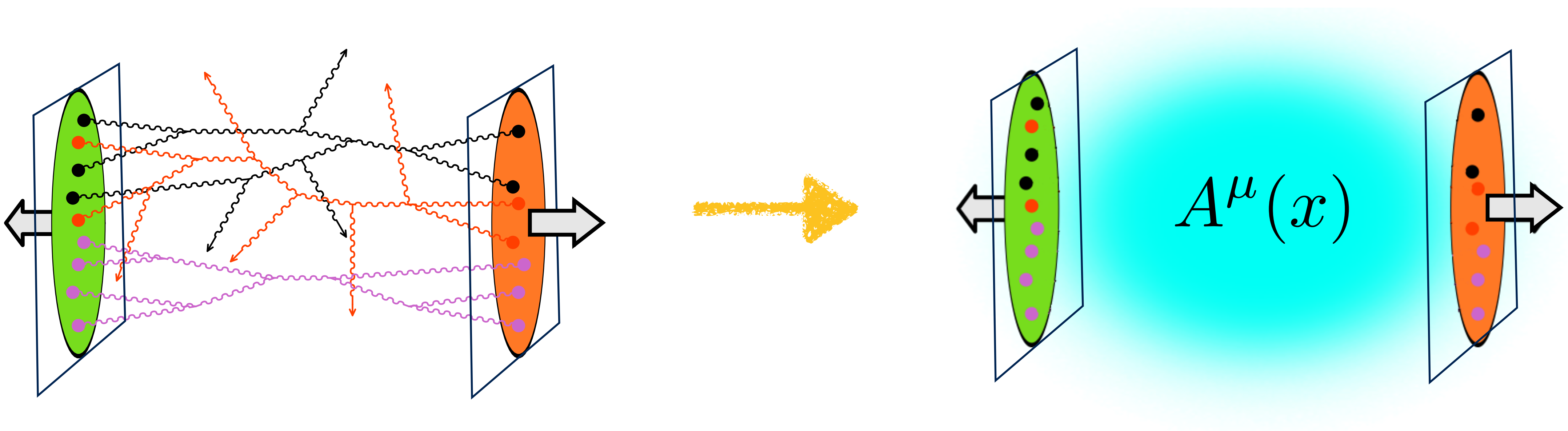

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$$[D_\mu, F^{\mu\nu}] = J^\nu \propto \rho(x)$$

- Calculation of observables: **average** over background classical fields

- Basic building block: **2-point correlator**

$$\langle \rho^a(x^-, x_\perp)\rho^b(y^-, y_\perp)\rangle = \mu^2(x^-)\delta^{ab}\delta(x^- - y^-)\delta^{(2)}(x_\perp - y_\perp)$$

Color Glass Condensate: McLerran-Venugopalan model

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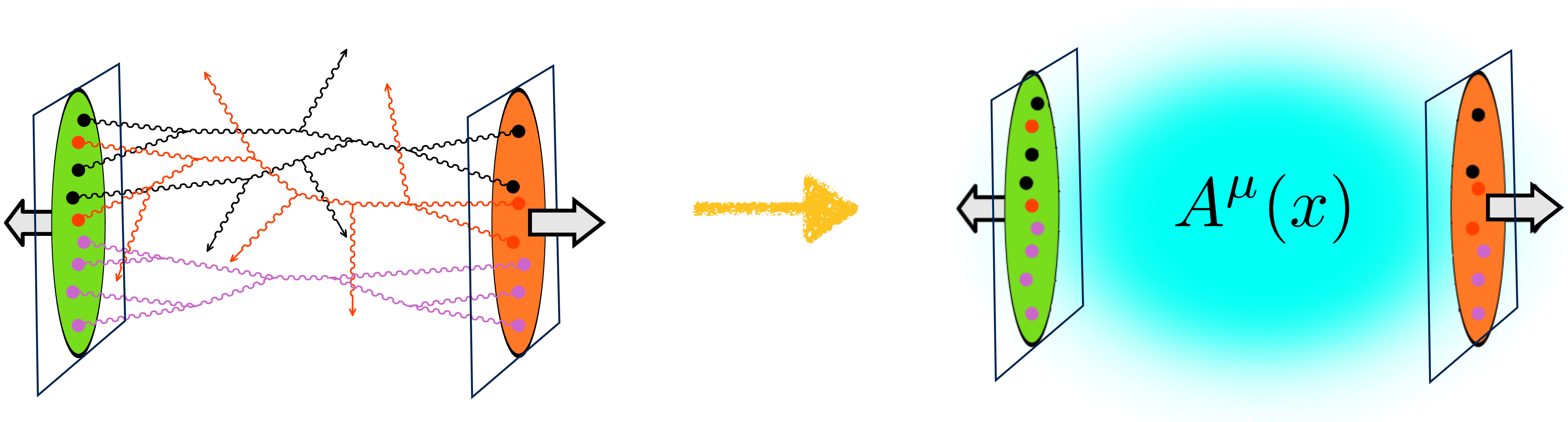


- Dynamics of the field described by **Yang-Mills** classical equations:

$$[D_\mu, F^{\mu\nu}] = J^\nu \propto \rho(x)$$

- Calculation of observables: **average** over background classical fields

- Basic building block: (**generalized**) **2-point correlator**

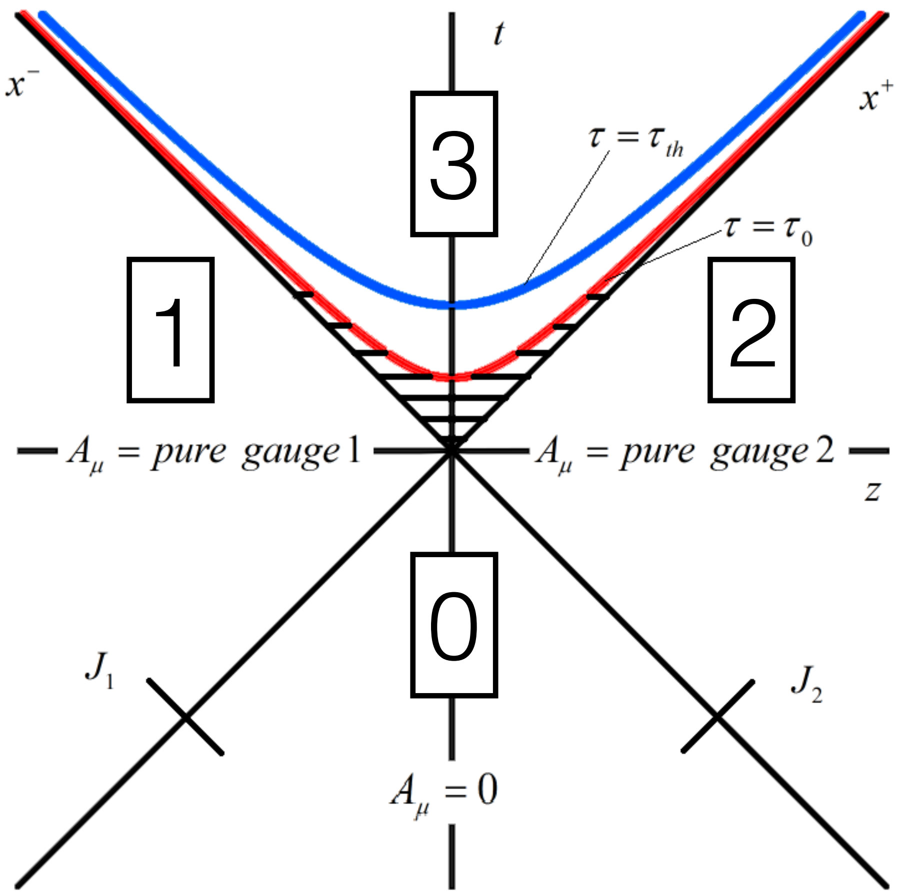
Non-Gaussianities

$$\langle \rho^a(x^-, x_\perp)\rho^b(y^-, y_\perp)\rangle = \mu^2(x^-)h(b_\perp)\delta^{ab}\delta(x^- - y^-)f(x_\perp - y_\perp)$$

Initial conditions for the field in the forward light-cone

Before the collision:

- In $\boxed{0}$ we take the fields as 0.
- As the fields in $\boxed{1}$ and $\boxed{2}$ are not in causal contact, they can be obtained independently.



Before the collision:

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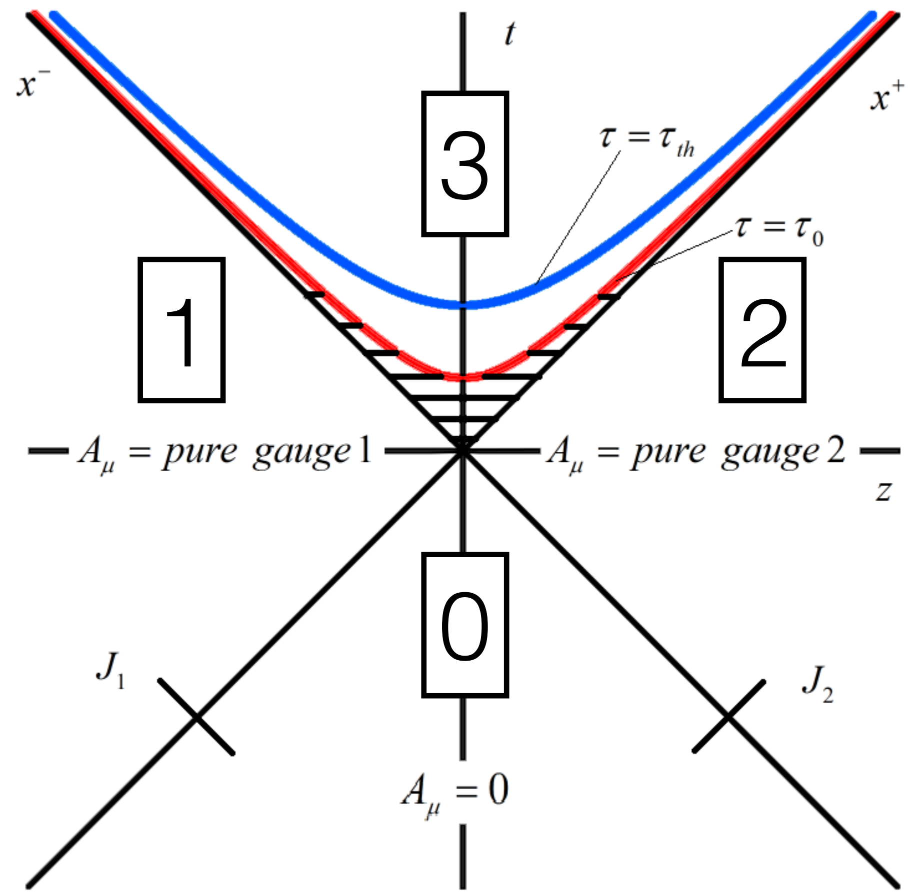
  We need to solve Yang-Mills equation with one source which, along with the covariant conservation of the current:



$$\frac{\partial}{\partial x^+} J^+ = ig\left[A^-, J^+\right]$$

Gives us: $A^{\pm}_{1,2} = 0$

$$A^i_{1,2} = \theta(x^\pm) \int_{-\infty}^{\infty} dz^\pm \frac{\partial^i \tilde{\rho}^a(z^\pm, x_\perp)}{\nabla^2_\perp} U^{ab}(z^\pm, x_\perp) t^b \equiv \theta(x^\pm) \alpha^i_{1,2}(x_\perp)$$

Where: $$U^{ab}(x^-, x_\perp) = \mathrm{P}^- \exp\left[ig \int_{-\infty}^{x^-} dz^- \int dz^2_\perp G(z_\perp - x_\perp) \rho^c(z^-, z_\perp) f^{abc}\right]$$

$\partial^2_\perp G(x_\perp - y_\perp) = \delta(x_\perp - y_\perp)$ ⟵ Green function of 2-D laplacian

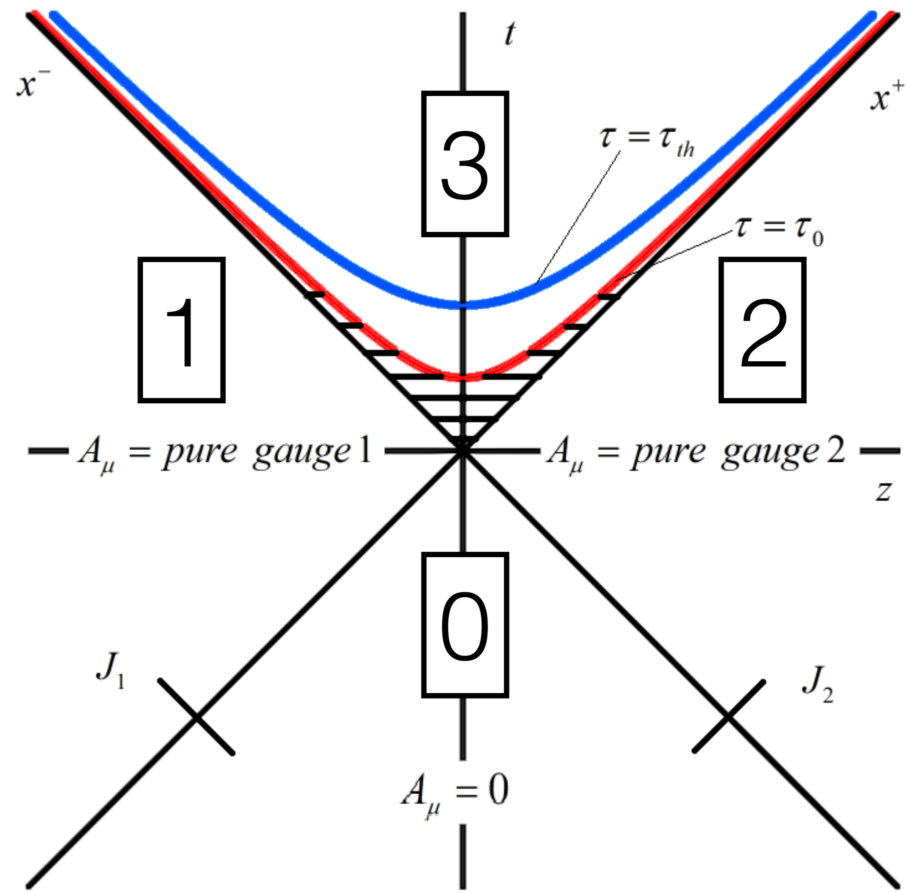
Initial conditions for the field in the forward light-cone

Before the collision:

- In $\boxed{0}$ we take the fields as 0.
- As the fields in $\boxed{1}$ and $\boxed{2}$ are not in causal contact, they can be obtained independently.

$$A_1^i = \theta(x^-)\alpha_1^i(x_\perp) \qquad A_2^i = \theta(x^+)\alpha_2^i(x_\perp)$$

$$A_1^\pm = 0 \qquad A_2^\pm = 0$$



'After' the collision: $\boxed{3}$

- Yang-Mills equations with two sources, $J^\nu = \delta^{\nu+}\rho_1(x^-, x_\perp) + \delta^{\nu-}\rho_2(x^+, x_\perp)$
- We calculate the gauge fields at $\tau = \sqrt{2x^+x^-} = 0^+$
- To do so we propose the following ansatz:

$$A^\pm = \pm x^\pm \alpha(\tau, x_\perp)$$
$$A^i = \alpha^i(\tau, x_\perp)$$

Initial conditions for the field in the forward light-cone

In the entire space-time: To relate our ansatz with the gluon fields of each nuclei:

$$
\begin{aligned}
A^+ &= \theta(x^+)\theta(x^-)x^+\alpha(\tau, x_\perp) \\
A^- &= -\theta(x^+)\theta(x^-)x^-\alpha(\tau, x_\perp) \\
A^i &= \theta(x^-)\theta(-x^+)\alpha_1^i(x_\perp) + \theta(x^+)\theta(-x^-)\alpha_2^i(x_\perp) + \theta(x^+)\theta(x^-)\alpha^i(\tau, x_\perp)
\end{aligned}
$$

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\end{aligned}
$$

- We impose that the Yang-Mills equations must be **regular**, which gives us the following initial conditions for the fields in the forward light-cone:

$$
\alpha^i(\tau = 0^+, x_\perp) = \alpha_1^i(x_\perp) + \alpha_2^i(x_\perp) \qquad \alpha(\tau = 0^+, x_\perp) = \frac{ig}{2}\left[\alpha_1^i(x_\perp), \alpha_2^i(x_\perp)\right]
$$

$$
\partial_\tau \alpha^i(\tau = 0^+, x_\perp) = 0 \qquad \partial_\tau \alpha(\tau = 0^+, x_\perp) = 0
$$

Initial conditions for the field in the forward light-cone

In the entire space-time: To relate our ansatz with the gluon fields of each nuclei:

$$A^+ = \theta(x^+)\theta(x^-)x^+\alpha(\tau, x_\perp)$$
$$A^- = -\theta(x^+)\theta(x^-)x^-\alpha(\tau, x_\perp)$$
$$A^i = \theta(x^-)\theta(-x^+)\alpha_1^i(x_\perp) + \theta(x^+)\theta(-x^-)\alpha_2^i(x_\perp) + \theta(x^+)\theta(x^-)\alpha^i(\tau, x_\perp)$$

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$$\alpha^i(\tau = 0^+, x_\perp) = \alpha_1^i(x_\perp) + \alpha_2^i(x_\perp) \qquad \alpha(\tau = 0^+, x_\perp) = \frac{ig}{2}\left[\alpha_1^i(x_\perp), \alpha_2^i(x_\perp)\right]$$
$$\partial_\tau \alpha^i(\tau = 0^+, x_\perp) = 0 \qquad \partial_\tau \alpha(\tau = 0^+, x_\perp) = 0$$

- We can obtain the early-time energy-momentum tensor $T^{\mu\nu}(\tau = 0^+)$ as:

$$\begin{aligned} T^{\mu\nu} &= \frac{1}{4} g^{\mu\nu} F^{\alpha\beta,a} F^a_{\alpha\beta} - F^{\mu\alpha,a} F^{\nu,a}_{\alpha} \\ &= -\frac{g^2}{2}(\delta^{ij}\delta^{kl} + \epsilon^{ij}\epsilon^{kl})\Big([\alpha_1^i, \alpha_2^j][\alpha_1^k, \alpha_2^l]\Big) \times \mathrm{diag}(1,1,1,-1) \\ &\equiv \epsilon_0 \times \mathrm{diag}(1,1,1,-1) \end{aligned}$$

Maximum pressure anisotropy in the longitudinal direction!

3. Correlators of the energy-momentum tensor at $\tau = 0^+$

$\langle T^{\mu\nu}(x_\perp)\rangle$

- For the 1-point correlator of $T^{\mu\nu}$:

$$\langle\epsilon_0\rangle = -g^2(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})\left\langle \mathrm{Tr}\left\{[\alpha_1^i,\alpha_2^j][\alpha_1^k,\alpha_2^l]\right\}\right\rangle$$
$$= -g^2(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})\left\langle \alpha_1^{i,a}\alpha_2^{j,b}\alpha_1^{k,c}\alpha_2^{l,d}\right\rangle \mathrm{Tr}\left\{\left[t^a,t^b\right]\left[t^c,t^d\right]\right\}$$
$$= \frac{g^2}{2}(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})f^{abm}f^{cdm}\left\langle \alpha_1^{i,a}(x_\perp)\alpha_1^{k,c}(x_\perp)\right\rangle_1\left\langle \alpha_2^{j,b}(x_\perp)\alpha_2^{l,d}(x_\perp)\right\rangle_2$$

$\langle T^{\mu\nu}(x_\perp)\rangle$

- For the 1-point correlator of $T^{\mu\nu}$:

$$\begin{aligned}\langle\epsilon_0\rangle &= -g^2(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})\left\langle \mathrm{Tr}\left\{[\alpha_1^i,\alpha_2^j][\alpha_1^k,\alpha_2^l]\right\}\right\rangle \\ &= -g^2(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})\left\langle \alpha_1^{i,a}\alpha_2^{j,b}\alpha_1^{k,c}\alpha_2^{l,d}\right\rangle \mathrm{Tr}\left\{\left[t^a,t^b\right]\left[t^c,t^d\right]\right\} \\ &= \frac{g^2}{2}(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})f^{abm}f^{cdm}\left\langle \alpha_1^{i,a}(x_\perp)\alpha_1^{k,c}(x_\perp)\right\rangle_1\left\langle \alpha_2^{j,b}(x_\perp)\alpha_2^{l,d}(x_\perp)\right\rangle_2\end{aligned}$$

- We momentarily take two different transverse coordinates:

$$\langle\alpha^{i,a}(x_\perp)\alpha^{j,b}(y_\perp)\rangle = \int_{-\infty}^{\infty} dz^- dz^{-\prime}\left\langle \frac{\partial^i\tilde{\rho}^{a'}(z^-,x_\perp)}{\nabla^2}U^{a'a}(z^-,x_\perp)\frac{\partial^j\tilde{\rho}^{b'}(z^{-\prime},y_\perp)}{\nabla^2}U^{b'b}(z^{-\prime},y_\perp)\right\rangle$$

$\langle T^{\mu\nu}(x_\perp)\rangle$

- For the 1-point correlator of $T^{\mu\nu}$:

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&= -g^2(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})\left\langle \alpha_1^{i,a}\alpha_2^{j,b}\alpha_1^{k,c}\alpha_2^{l,d}\right\rangle \mathrm{Tr}\left\{\left[t^a,t^b\right]\left[t^c,t^d\right]\right\} \\
&= \frac{g^2}{2}(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})f^{abm}f^{cdm}\left\langle \alpha_1^{i,a}(x_\perp)\alpha_1^{k,c}(x_\perp)\right\rangle_1\left\langle \alpha_2^{j,b}(x_\perp)\alpha_2^{l,d}(x_\perp)\right\rangle_2
\end{aligned}
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- We momentarily take two different transverse coordinates:

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\left\langle\alpha^{i,a}(x_\perp)\alpha^{j,b}(y_\perp)\right\rangle = \int_{-\infty}^{\infty} dz^- dz^{-\prime}\left\langle \frac{\partial^i\tilde{\rho}^{a'}(z^-,x_\perp)}{\nabla_\perp^2}\frac{\partial^j\tilde{\rho}^{b'}(z^{-\prime}y_\perp)}{\nabla_\perp^2}\right\rangle\left\langle U^{a'a}(z^-,x_\perp)U^{b'b}(z^{-\prime},y_\perp)\right\rangle
$$

Luckily, Wilson lines and color source densities factorize

- For the 1-point correlator of $T^{\mu\nu}$:

$$\begin{aligned}\langle\epsilon_0\rangle &= -g^2(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})\left\langle \mathrm{Tr}\left\{[\alpha_1^i,\alpha_2^j][\alpha_1^k,\alpha_2^l]\right\}\right\rangle \\ &= -g^2(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})\left\langle \alpha_1^{i,a}\alpha_2^{j,b}\alpha_1^{k,c}\alpha_2^{l,d}\right\rangle \mathrm{Tr}\left\{\left[t^a,t^b\right]\left[t^c,t^d\right]\right\} \\ &= \frac{g^2}{2}(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})f^{abm}f^{cdm}\left\langle \alpha_1^{i,a}(x_\perp)\alpha_1^{k,c}(x_\perp)\right\rangle_1\left\langle \alpha_2^{j,b}(x_\perp)\alpha_2^{l,d}(x_\perp)\right\rangle_2\end{aligned}$$

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$$\delta^{a'b'}\mu^2(x^-)\delta(x^- - y^-)\partial_x^i\partial_y^j L(x_\perp - y_\perp)$$

Where:

$$L(x_\perp - y_\perp)=\int d^2z_\perp G(x_\perp - z_\perp)G(y_\perp - z_\perp).$$

$\langle T^{\mu\nu}(x_\perp)\rangle$

- For the 1-point correlator of $T^{\mu\nu}$:

$$\langle\epsilon_0\rangle = -g^2(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})\left\langle \mathrm{Tr}\left\{[\alpha_1^i,\alpha_2^j][\alpha_1^k,\alpha_2^l]\right\}\right\rangle$$

$$= -g^2(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})\left\langle \alpha_1^{i,a}\alpha_2^{j,b}\alpha_1^{k,c}\alpha_2^{l,d}\right\rangle \mathrm{Tr}\left\{\left[t^a,t^b\right]\left[t^c,t^d\right]\right\}$$

$$= \frac{g^2}{2}(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})f^{abm}f^{cdm}\left\langle \alpha_1^{i,a}(x_\perp)\alpha_1^{k,c}(x_\perp)\right\rangle_1\left\langle \alpha_2^{j,b}(x_\perp)\alpha_2^{l,d}(x_\perp)\right\rangle_2$$

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$$\frac{\delta^{ab}\delta^{a'b'}}{N}\exp\left[-g^2\frac{N}{2}\Gamma(x_\perp,y_\perp)\bar{\mu}^2(x^-)\right]$$

$\langle T^{\mu\nu}(x_\perp)\rangle$

- For the 1-point correlator of $T^{\mu\nu}$:

$$\begin{aligned}
\langle\epsilon_0\rangle &= -g^2(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})\Big\langle \mathrm{Tr}\left\{[\alpha_1^i,\alpha_2^j][\alpha_1^k,\alpha_2^l]\right\}\Big\rangle \\
&= -g^2(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})\left\langle \alpha_1^{i,a}\alpha_2^{j,b}\alpha_1^{k,c}\alpha_2^{l,d}\right\rangle \mathrm{Tr}\left\{\left[t^a,t^b\right]\left[t^c,t^d\right]\right\} \\
&= \frac{g^2}{2}(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})f^{abm}f^{cdm}\left\langle \alpha_1^{i,a}(x_\perp)\alpha_1^{k,c}(x_\perp)\right\rangle_1\left\langle \alpha_2^{j,b}(x_\perp)\alpha_2^{l,d}(x_\perp)\right\rangle_2 \\
&= \frac{g^2}{2}f^{abm}f^{cdm}(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})\delta^{ac}\delta^{ik}\delta^{bd}\delta^{jl}\bar\mu_1^2\bar\mu_2^2(\partial^2L(0_\perp))^2 \\
&= 4\,g^2C_A^2C_F\bar\mu_1^2\bar\mu_2^2(\partial^2L(0_\perp))^2
\end{aligned}$$

$\langle T^{\mu\nu}(x_\perp)\rangle$

- For the 1-point correlator of $T^{\mu\nu}$:

$$\begin{aligned}
\langle\epsilon_0\rangle &= -g^2(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})\left\langle \mathrm{Tr}\left\{[\alpha_1^i,\alpha_2^j][\alpha_1^k,\alpha_2^l]\right\}\right\rangle \\
&= -g^2(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})\left\langle \alpha_1^{i,a}\alpha_2^{j,b}\alpha_1^{k,c}\alpha_2^{l,d}\right\rangle \mathrm{Tr}\left\{\left[t^a,t^b\right]\left[t^c,t^d\right]\right\} \\
&= \frac{g^2}{2}(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})f^{abm}f^{cdm}\left\langle \alpha_1^{i,a}(x_\perp)\alpha_1^{k,c}(x_\perp)\right\rangle_1\left\langle \alpha_2^{j,b}(x_\perp)\alpha_2^{l,d}(x_\perp)\right\rangle_2 \\
&= \frac{g^2}{2}f^{abm}f^{cdm}(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})\delta^{ac}\delta^{ik}\delta^{bd}\delta^{jl}\bar{\mu}_1^2\bar{\mu}_2^2(\partial^2L(0_\perp))^2 \\
&= 4\,g^2C_A^2C_F\bar{\mu}_1^2\bar{\mu}_2^2(\partial^2L(0_\perp))^2 \\
&= \frac{4C_F}{g^2}Q_{s\,1}^2Q_{s\,2}^2
\end{aligned}$$

We absorb the logarithmic IR divergence from $\partial^2L(0_\perp)$ in the definition of a momentum scale: $Q_s^2=g^2C_A\bar{\mu}^2\partial^2L(0_\perp)$

$\langle T^{\mu\nu}(x_\perp)T^{\mu\nu}(y_\perp)\rangle$

- For the 2-point correlator of $T^{\mu\nu}$: prepare for trouble and make it double

$$\langle\epsilon(x_\perp)\epsilon(y_\perp)\rangle = \frac{g^4}{4}(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})(\delta^{i'j'}\delta^{k'l'}+\epsilon^{i'j'}\epsilon^{k'l'})f^{abn}f^{cdn}f^{a'b'm}f^{c'd'm}$$
$$\times\langle\alpha^{i\,a}_{1\,x}\alpha^{k\,c}_{1\,x}\alpha^{i'\,a'}_{1\,y}\alpha^{k'\,c'}_{1\,y}\rangle\langle\alpha^{j\,b}_{2\,x}\alpha^{l\,d}_{2\,x}\alpha^{j'\,b'}_{2\,y}\alpha^{l'\,d'}_{2\,y}\rangle$$

$\langle T^{\mu\nu}(x_\perp) T^{\mu\nu}(y_\perp)\rangle$

- For the 2-point correlator of $T^{\mu\nu}$: prepare for trouble and make it double

$$\langle \epsilon(x_\perp)\epsilon(y_\perp)\rangle = \frac{g^4}{4}(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})(\delta^{i'j'}\delta^{k'l'}+\epsilon^{i'j'}\epsilon^{k'l'})f^{abn}f^{cdn}f^{a'b'm}f^{c'd'm} \times \langle \alpha_{1\,x}^{i\,a}\alpha_{1\,x}^{k\,c}\alpha_{1\,y}^{i'a'}\alpha_{1\,y}^{k'c'}\rangle\langle \alpha_{2\,x}^{j\,b}\alpha_{2\,x}^{l\,d}\alpha_{2\,y}^{j'b'}\alpha_{2\,y}^{l'd'}\rangle$$

- The building block:

$$\langle \alpha^{i\,a}(x_\perp)\alpha^{k\,c}(x_\perp)\alpha^{i'a'}(y_\perp)\alpha^{k'c'}(y_\perp)\rangle = \int_{-\infty}^{\infty} dz^- dw^- dz^{-\prime} dw^{-\prime} \Big\langle \frac{\partial^i\tilde{\rho}^e(z^-,x_\perp)}{\nabla^2}U^{ea}(z^-,x_\perp) \frac{\partial^k\tilde{\rho}^f(w^-,x_\perp)}{\nabla^2}U^{fc}(w^-,x_\perp)\frac{\partial^{i'}\tilde{\rho}^{e'}(z^{-\prime},y_\perp)}{\nabla^2}U^{e'a'}(z^{-\prime},y_\perp)\frac{\partial^{k'}\tilde{\rho}^{f'}(w^{-\prime},y_\perp)}{\nabla^2}U^{f'c'}(w^{-\prime},y_\perp)\Big\rangle.$$

$\langle T^{\mu\nu}(x_\perp)T^{\mu\nu}(y_\perp)\rangle$

- For the 2-point correlator of $T^{\mu\nu}$: prepare for trouble and make it double

$$\langle\epsilon(x_\perp)\epsilon(y_\perp)\rangle = \frac{g^4}{4}(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})(\delta^{i'j'}\delta^{k'l'}+\epsilon^{i'j'}\epsilon^{k'l'})f^{abn}f^{cdn}f^{a'b'm}f^{c'd'm}$$
$$\times\langle\alpha^{i\,a}_{1\,x}\alpha^{k\,c}_{1\,x}\alpha^{i'a'}_{1\,y}\alpha^{k'c'}_{1\,y}\rangle\langle\alpha^{j\,b}_{2\,x}\alpha^{l\,d}_{2\,x}\alpha^{j'b'}_{2\,y}\alpha^{l'd'}_{2\,y}\rangle$$

- Technical difficulties:
  - The **expansion of the correlator** $\langle\alpha^{i\,a}(x_\perp)\alpha^{k\,c}(x_\perp)\alpha^{i'a'}(y_\perp)\alpha^{k'c'}(y_\perp)\rangle$ is far more difficult than that of $\langle\alpha^{i\,a}(x_\perp)\alpha^{k\,c}(y_\perp)\rangle$. Schematically:

$$\langle\alpha^{i\,a}(x_\perp)\alpha^{k\,c}(x_\perp)\alpha^{i'a'}(y_\perp)\alpha^{k'c'}(y_\perp)\rangle = \underbrace{\langle\rho^4\rangle\langle U^4\rangle}_{\text{3 terms}} + \underbrace{\langle\rho^2\rangle\langle\rho^2 U^4\rangle_c}_{\text{4 terms}}$$

(Wick's theorem)

Computed by application of the diagrammatic rules derived in:

F.m.c. Fillion-Gourdeau and S. Jeon: *Phys. Rev. C 79 (Feb, 2009) 025204.*

$\langle T^{\mu\nu}(x_\perp) T^{\mu\nu}(y_\perp)\rangle$

- For the 2-point correlator of $T^{\mu\nu}$: prepare for trouble and make it double

$$\langle \epsilon(x_\perp)\epsilon(y_\perp)\rangle = \frac{g^4}{4}(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})(\delta^{i'j'}\delta^{k'l'}+\epsilon^{i'j'}\epsilon^{k'l'})f^{abn}f^{cdn}f^{a'b'm}f^{c'd'm}$$
$$\times \langle \alpha_{1\,x}^{i\,a}\alpha_{1\,x}^{k\,c}\alpha_{1\,y}^{i'a'}\alpha_{1\,y}^{k'c'}\rangle\langle \alpha_{2\,x}^{j\,b}\alpha_{2\,x}^{l\,d}\alpha_{2\,y}^{j'b'}\alpha_{2\,y}^{l'd'}\rangle$$

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  - Instead of having to calculate the adjoint Wilson line dipole, we need the much more complex adjoint **Wilson line quadrupole**

$$\left\langle U^{ab}(z^-,x_\perp)U^{cd}(z^-,y_\perp)U^{ef}(z^-,x'_\perp)U^{gh}(z^-,y'_\perp)\right\rangle$$

Computed by application of the techniques outlined in:

A. Kovner and U. A. Wiedemann: *Phys. Rev. D 64 (Oct, 2001) 114002.*

$\langle T^{\mu\nu}(x_\perp)T^{\mu\nu}(y_\perp)\rangle$

- For the 2-point correlator of $T^{\mu\nu}$: prepare for trouble and make it double

$$\langle\epsilon(x_\perp)\epsilon(y_\perp)\rangle = \frac{g^4}{4}(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})(\delta^{i'j'}\delta^{k'l'}+\epsilon^{i'j'}\epsilon^{k'l'})f^{abn}f^{cdn}f^{a'b'm}f^{c'd'm}$$
$$\times\,\langle\alpha_{1\,x}^{i\,a}\alpha_{1\,x}^{k\,c}\alpha_{1\,y}^{i'a'}\alpha_{1\,y}^{k'c'}\rangle\langle\alpha_{2\,x}^{j\,b}\alpha_{2\,x}^{l\,d}\alpha_{2\,y}^{j'b'}\alpha_{2\,y}^{l'd'}\rangle$$

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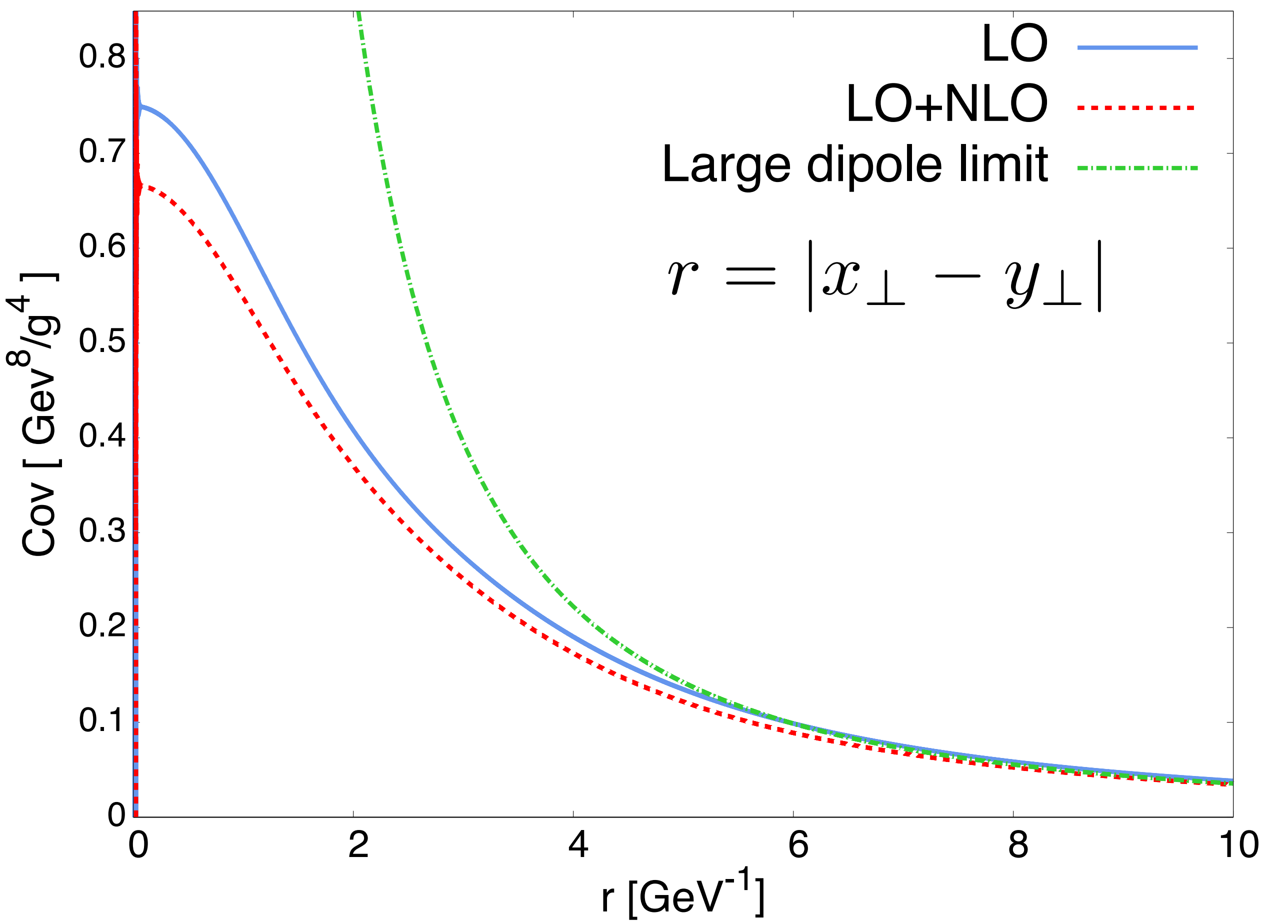
  - The **color structure** of this object is frustratingly complex. Even with all parts analytically calculated, the contraction of the color indices demands a computational treatment (via FeynCalc)

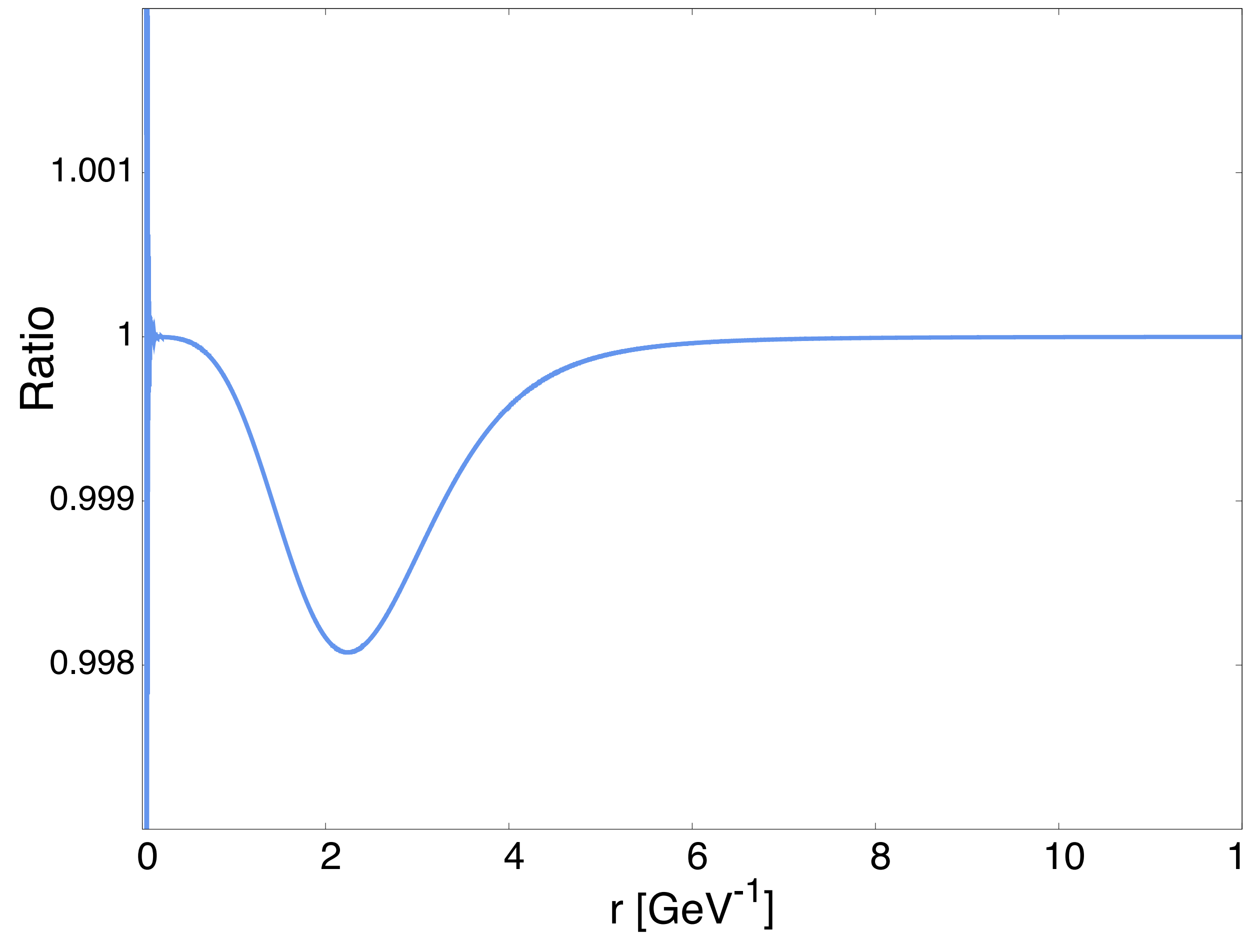
4. Preliminary results: Nc expansion

First orders of the Nc expansion: N_c^0 and N_c^{-2}

- Due to the lengthy result, we will only show the first orders of the Nc expansion.



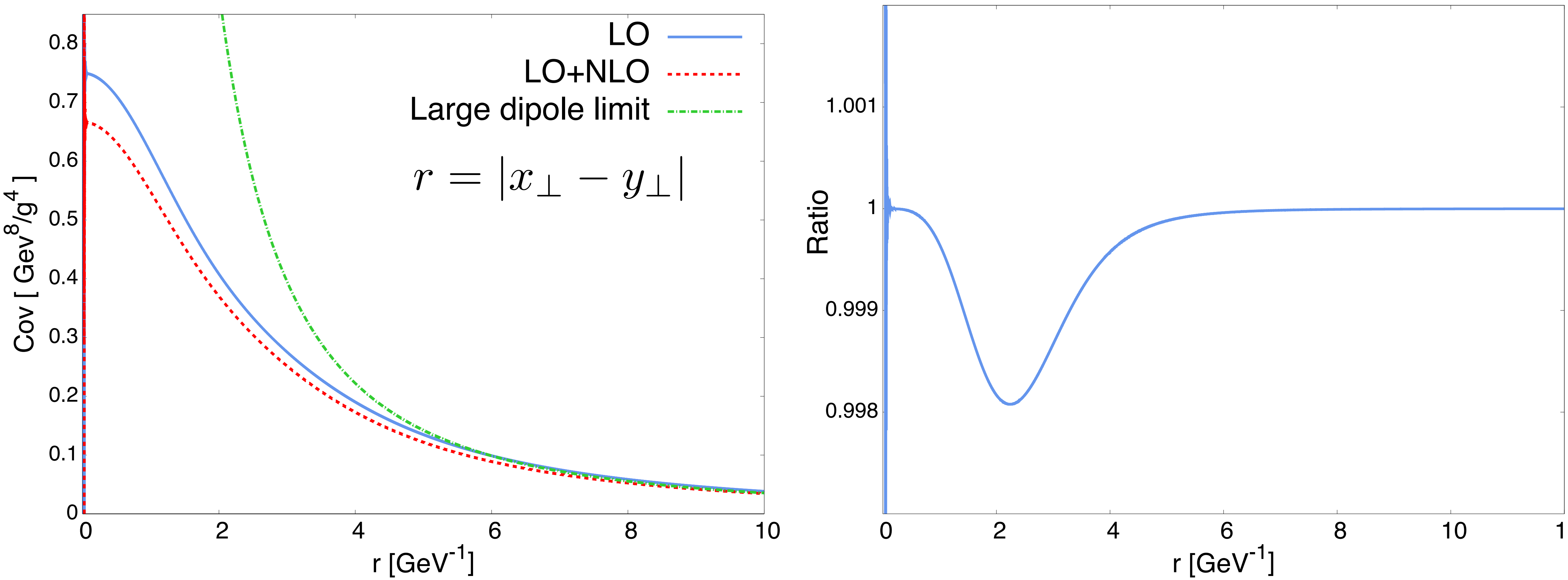
Sum of the first two orders of the N-expansion of the energy density covariance for N=3 in the classical MV model.



Ratio between the full result and the sum of the first two orders of the N-expansion, which turns out to be a very good approximation.

First orders of the Nc expansion

- Due to the lengthy result, we show the first orders of the Nc expansion.



- A remarkable aspect about our result is its highly non-trivial behavior in the limit of large correlation distances: a **power-law decay**.

$$\lim_{r\to\infty} \mathrm{Cov}[\epsilon_{\mathrm{MV}}](0^+; x_\perp, y_\perp) = \frac{2\left(N_c^2-1\right) Q_{s1}^2 Q_{s2}^2 \left(Q_{s1}^2+Q_{s2}^2\right)}{g^4 N_c^2 r^2}$$

Conclusions

- We have performed an exact analytical calculation of the covariance of the energy momentum tensor of the **Glasma** at $\tau = 0^+$, in the framework of the **MV model**.

- We expect to be able to **generalize this framework** by introducing an impact parameter dependence and relaxing some of the original assumptions, which could potentially open the door to **phenomenological applications**.

- The following steps are computing the **time evolution** of our result towards thermalization time, where it can serve as input for **hydro QGP simulations**.

Conclusions

- We have performed an exact analytical calculation of the covariance of the energy momentum tensor of the **Glasma** at $\tau = 0^+$, in the framework of the **MV model**.
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- The following steps are computing the **time evolution** of our result towards thermalization time, where it can serve as input for **hydro QGP simulations**.

Thanks for your attention

Back-up: Expressions of two first orders of Nc expansion

- Leading order:

$$\left[\mathrm{Cov}[\epsilon_{\mathrm{MV}}](0^+;x_\perp,y_\perp)\right]_{N_c^0} = \frac{1}{4g^4\,r^8}e^{-\frac{r^2}{2}\left(Q_{s1}^2+Q_{s2}^2\right)}\left(128+128\left(e^{\frac{Q_{s1}^2r^2}{2}}+e^{\frac{Q_{s2}^2r^2}{2}}\right)\right.$$
$$-\left[256e^{\frac{Q_{s1}^2r^2}{4}}+16e^{\frac{r^2}{4}\left(2Q_{s1}^2+Q_{s2}^2\right)}\left(Q_{s2}^4r^4+8Q_{s2}^2r^2-2Q_{s1}^4r^4+48\right)\right]-[1\leftrightarrow 2]$$
$$-e^{\frac{r^2}{4}\left(Q_{s1}^2+Q_{s2}^2\right)}\left[Q_{s1}^4Q_{s2}^4r^8+4Q_{s1}^2Q_{s2}^2r^6\left(Q_{s1}^2+Q_{s2}^2\right)\right.$$
$$\left.+128r^2\left(Q_{s1}^2+Q_{s2}^2\right)+16r^4\left(Q_{s1}^2+Q_{s2}^2\right)^2+1024\right]$$
$$\left.+8e^{\frac{r^2}{2}\left(Q_{s1}^2+Q_{s2}^2\right)}\left[-4r^4\left(Q_{s1}^4+Q_{s2}^4\right)+Q_{s1}^2Q_{s2}^2r^6\left(Q_{s1}^2+Q_{s2}^2\right)+80\right]\right)$$

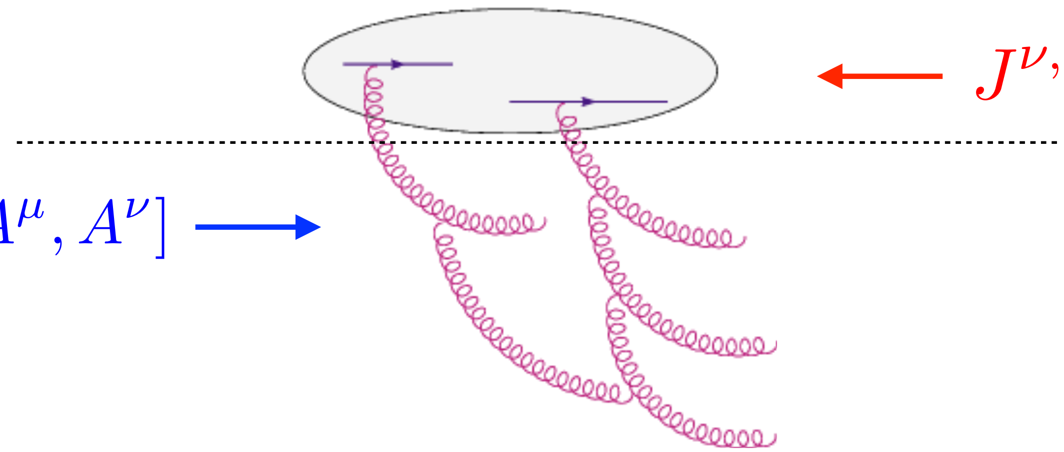
- First correction:

$$\left[\mathrm{Cov}[\epsilon_{\mathrm{MV}}](0^+;x_\perp,y_\perp)\right]_{N_c^{-2}} = \frac{1}{4N_c^2g^4\,r^8}e^{-\frac{r^2}{2}\left(Q_{s1}^2+Q_{s2}^2\right)}\left(16\left(Q_{s1}^2r^2+Q_{s2}^2r^2+8\right)^2\right.$$
$$+\left[16Q_{s1}^2r^2(8+Q_{s1}^2r^2)e^{\frac{Q_{s2}^2r^2}{2}}-32(8+Q_{s1}^2r^2)(4+Q_{s1}^2r^2)e^{\frac{Q_{s2}^2r^2}{4}}\right]+[1\leftrightarrow 2]$$
$$+\left[16\,r^2e^{\frac{1}{4}r^2\left(Q_{s1}^2+2Q_{s2}^2\right)}\left(r^2\left(Q_{s1}^4-2Q_{s2}^4\right)+8\left(Q_{s1}^2+2Q_{s2}^2\right)\right)\right]+[1\leftrightarrow 2]$$
$$-8e^{\frac{1}{2}r^2\left(Q_{s1}^2+Q_{s2}^2\right)}r^2\left(Q_{s1}^2+Q_{s2}^2\right)\left(Q_{s1}^2Q_{s2}^2r^4-4r^2\left(Q_{s1}^2+Q_{s2}^2\right)+32\right)$$
$$+e^{\frac{1}{4}r^2\left(Q_{s1}^2+Q_{s2}^2\right)}\left[Q_{s1}^4Q_{s2}^4r^8+4Q_{s1}^2Q_{s2}^2r^6\left(Q_{s1}^2+Q_{s2}^2\right)+128r^2\left(Q_{s1}^2+Q_{s2}^2\right)\right.$$
$$\left.\left.+16r^4\left(Q_{s1}^2+Q_{s2}^2\right)^2-1024\right]\right)$$

Back-up: More about the Color Glass Condensate

- Separation of ‘slow’ and ‘fast’ degrees of freedom

$$J^{\nu,a} = \delta^{\nu +}\rho^a(x^-, x_\perp)$$



$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - ig\,[A^\mu, A^\nu]$$

- Dynamic relation given by solution to classical Yang-Mills equations:

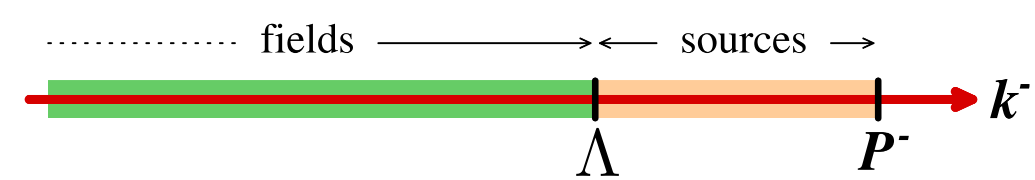
$$[D_\mu,\, F^{\mu\nu,a}] = J^{\nu,a}$$

- Calculation of observable quantities: average over color sources

$$\langle \mathcal{O} \rangle = \int [D\rho]\, W_\Lambda[\rho]\mathcal{O}[\rho]$$

- Scale (in)dependence: JIMWLK equations

$$\frac{\partial W_\Lambda}{\partial \log \Lambda} = \mathcal{H} W_\Lambda$$



- McLerran-Venugopalan model: W_Λ is a Gaussian distribution

$$\langle \rho^a(x^-, x_\perp)\rho^b(y^-, y_\perp)\rangle = \mu^2(x^-)\delta^{ab}\delta^2(x_\perp - y_\perp)\delta(x^- - y^-)$$