High-energy OPE: i) sub-eikonal spin corrections ii) Odderon and twist-3 light ray operators

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Outline: Sub-eikonal spin corrections

- Motivation
- Brief review Operator Product Expansion at high-energy
- Operator Product Expansion at high-energy with sub-eikonal corrections
  - Quark propagator with sub-eikonal corrections
- Leading Order Impact Factor for sub-eikonal spin correction
- Conclusions

Motivation

- Unpolarized DIS at low-x: dynamics is driven by gluon structure functions
  - gluon structure function grows as $(1/x)^{\lambda}$ with $\lambda > 1$.
- Polarized DIS at low-x: polarized gluon structure function grows as $(1/x)^{\lambda}$ with λ close to 0.
  - This implies that polarized quark and gluon structure functions are equally relevant.
- At Electron Ion Collider low-x spin TMDs and g_1 structure function are relevant
- Understand how the proton's spin arises from the intrinsic and orbital angular momenta of the constituent quarks and gluons.

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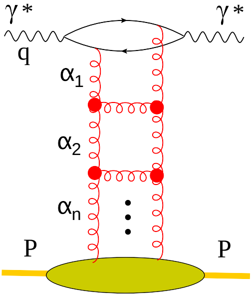
  - This implies that polarized quark and gluon structure functions are equally relevant.
- At Electron Ion Collider low-x spin TMDs and g_1 structure function are relevant
- Understand how the proton's spin arises from the intrinsic and orbital angular momenta of the constituent quarks and gluons.
- Compare with results obtained in the Leading Log approximation by Bartels-Ermolaev-Ryskin-(1995-1996) and recent work in Saturation formalism obtained by Kovchegov-Pytoniak-Sievert (20016-2017)

Unpolarized vs. Polarized DIS at low-x

- DGLAP: resums $\left(\alpha_s \ln \frac{Q^2}{\mu}\right)^n$ BFKL: resums $\left(\alpha_s \ln \frac{1}{x}\right)^n$
- overlap region resums $\left(\alpha_s \ln \frac{1}{x} \ln \frac{Q^2}{\mu}\right)^n$
- Scattering amplitude with fermion in t-channel in Regge limit we have $\left(\alpha_s \ln^2 \frac{1}{x}\right)^n$ contributions
  - such contribution not included in DGLAP asymptotic $x \to 0$
- Double Log of energy of quark distribution
  - unpolarized case: are not relevant since are suppressed by gluon distribution
  - polarized case: are relevant Bartels-Ermolaev-Ryskin-(1995-1996)

Semi-classical approach: Background field method

p_1^μ , p_2^μ light-cone vectors $\Rightarrow k^\mu = \alpha p_1^\mu + \beta p_2^\mu + k_\perp^\mu$
$\alpha_1 \gg \alpha_2 ... \gg \alpha_n$



Fields are ordered in their rapidities $\Rightarrow$

- large α gluons are treated as quantum fields
- low α fields are treated as classical fields

Boost of the fields

$$
\begin{aligned}
A_\bullet(x_\bullet, x_*, x_\perp) &\rightarrow \lambda A_\bullet(\lambda^{-1}x_\bullet, \lambda x_*, x_\perp)\\
A_*(x_\bullet, x_*, x_\perp) &\rightarrow \lambda^{-1} A_*(\lambda^{-1}x_\bullet, \lambda x_*, x_\perp)\\
A_\perp(x_\bullet, x_*, x_\perp) &\rightarrow A_\perp(\lambda^{-1}x_\bullet, \lambda x_*, x_\perp)
\end{aligned}
$$

λ is the boost parameter.

$$
\langle x|\frac{i}{\not{P}+i\epsilon}|y\rangle \;\rightarrow\; \langle x|\frac{i}{\not{p}+\alpha\frac{2}{s}\not{p}_2 A_\bullet + i\epsilon}|y\rangle
$$

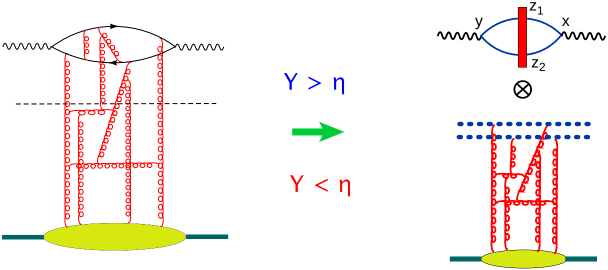
$[\hat{\alpha}, \hat{A}^{cl}_\mu] = 0$ with $\alpha = \sqrt{\frac{2}{s}}p^+$ and $\not{p}_2 \propto \gamma^+$

Eikonal interactions give a Wilson lines

$$U_z = [\infty p_1 + z_\perp, -\infty p_1 + z_\perp]$$

$$[x, y] = Pe^{ig\int_0^1 du(x-y)^\mu A_\mu(ux+(1-u)y)} \qquad p^\mu = \alpha p_1^\mu + \beta p_2^\mu + p_\perp^\mu$$

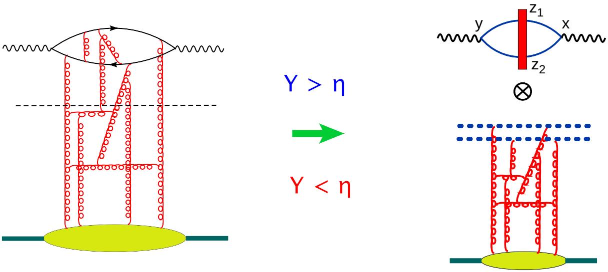
DIS amplitude is factorized in rapidity: η



$|B\rangle$ is the target state.

$$\langle B|T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\}|B\rangle = \int \frac{d^2z_1 d^2z_2}{z_{12}^4}\, I^{\rm LO}_{\mu\nu}(x,y;z_1,z_2)\langle B|{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}|B\rangle + \dots$$

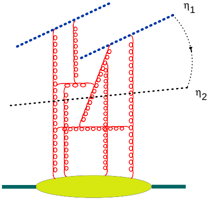
High-Energy Operator Product Expansion



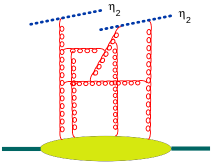
$$\langle B|T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\}|B\rangle = \int \frac{d^2z_1 d^2z_2}{z_{12}^4}\, I^{\rm LO}_{\mu\nu}(x,y;z_1,z_2)\langle B|\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}|B\rangle + \ldots$$

- If we use a model to evaluate $\langle B|\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}|B\rangle$ we can calculate the DIS cross-section.
- If we want to include energy dependence to the DIS cross section, we need to find the evolution of $\langle B|\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}|B\rangle$ with respect to the rapidity parameter η.

Evolution Equation



→ $\alpha_s(\eta_1 - \eta_2)K_{\text{evol}} \;\otimes$



- Separate fields in quantum and classical according to low and large rapidity.
  Formally we may write:

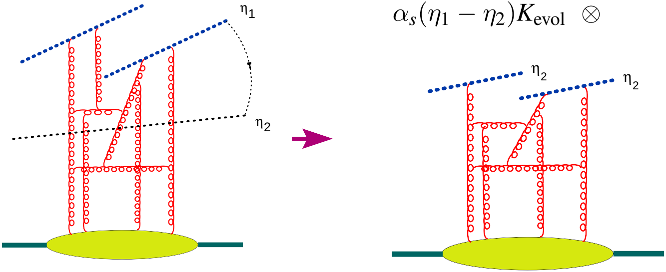
$$\langle B|\mathcal{O}^{\eta_1}|B\rangle \rightarrow \langle\mathcal{O}^{\eta_1}\rangle_A \rightarrow \langle\mathcal{O}'^{\eta_2} \otimes \mathcal{O}'^{\eta_1}\rangle_A$$

- Integrate over the quantum fields and get one-loop rapidity evolution of the operator $\mathcal{O}$

$$\langle\mathcal{O}^{\eta_1}\rangle_A = \alpha_s(\eta_1 - \eta_2)K_{\text{evol}} \otimes \langle\mathcal{O}'^{\eta_2}\rangle_A$$

- Where in principle $\mathcal{O}$ and $\mathcal{O}'$ may be different operators.

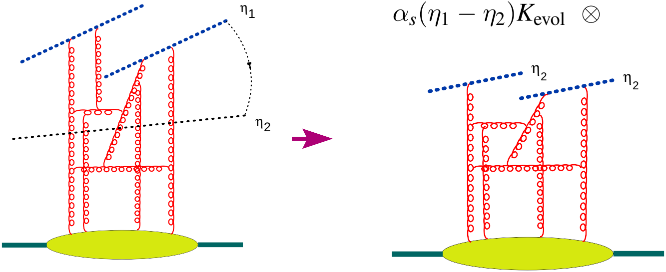
Non-linear evolution equation



$$\alpha_s(\eta_1 - \eta_2)K_{\text{evol}} \;\otimes$$

- Linear case $\quad \mathcal{O}^{\eta_1} = \alpha_s \Delta\eta \; K_{\text{evol}} \; \otimes \; \mathcal{O}^{\eta_2}$

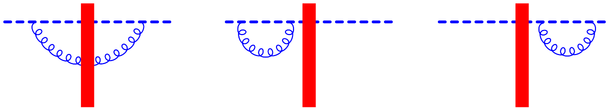
Non-linear evolution equation



$$\alpha_s(\eta_1 - \eta_2) K_{\text{evol}} \otimes$$

- Linear case $\mathcal{O}^{\eta_1} = \alpha_s \Delta\eta \; K_{\text{evol}} \; \otimes \; \mathcal{O}^{\eta_2}$
- Non-linear case $\mathcal{O}^{\eta_1} = \alpha_s \Delta\eta \; K_{\text{evol}} \; \otimes \; \{\mathcal{O}^{\eta_2}\mathcal{O}^{\eta_2}\}$

Non-linear evolution equation

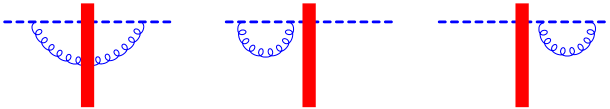


$$\langle\{U_x^{\eta_1}\}_{ij}\rangle_A = \frac{\alpha_s}{2\pi^2}\Delta\eta\int\frac{d^2z_\perp}{(x-z)_\perp^2}\left[\langle \mathrm{tr}\{U_x^{\eta_2}U_z^{\eta_2\dagger}\}\{U_z^{\eta_2}\}_{ij}\rangle_A - \langle\frac{1}{N_c}\{U_x^{\eta_2}\}_{ij}\rangle_A\right]$$

$\Delta\eta = \eta_1 - \eta_2$

$$\{U_x^{\dagger\eta_1}\}_{ij}, \quad \{U_x^{\eta_1}U_y^{\eta_1}\}_{ij}, \quad \{U_x^{\eta_1}U_y^{\dagger\eta_1}\}_{ij}, \quad \{U_x^{\dagger\eta_1}U_y^{\dagger\eta_1}\}_{ij}$$

Non-linear evolution equation



$$\langle\{U_x^{\eta_1}\}_{ij}\rangle_A = \frac{\alpha_s}{2\pi^2}\Delta\eta \int \frac{d^2 z_\perp}{(x-z)_\perp^2}\left[\langle \mathrm{tr}\{U_x^{\eta_2}U_z^{\eta_2\dagger}\}\{U_z^{\eta_2}\}_{ij}\rangle_A - \langle\frac{1}{N_c}\{U_x^{\eta_2}\}_{ij}\rangle_A\right]$$

$\Delta\eta = \eta_1 - \eta_2$

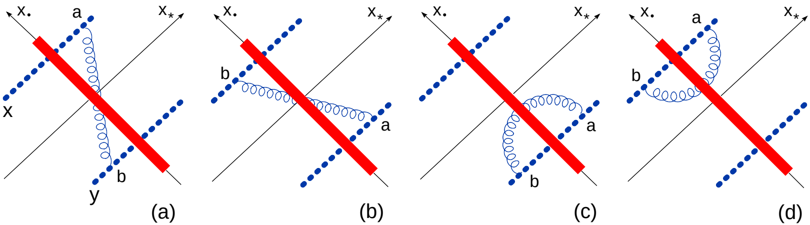
$$\{U_x^{\dagger\eta_1}\}_{ij}, \quad \{U_x^{\eta_1}U_y^{\eta_1}\}_{ij}, \quad \{U_x^{\eta_1}U_y^{\dagger\eta_1}\}_{ij}, \quad \{U_x^{\dagger\eta_1}U_y^{\dagger\eta_1}\}_{ij}$$

- Obtain a set of rules that allow one to get the LO evolution of any trace or product of traces of Wilson lines
- Hierarchy of evolution equation: B-JIMWLK equation.

Leading order evolution equation

$$\frac{d}{d\eta}\mathrm{tr}\{\hat{U}_x\hat{U}^\dagger_y\} = K_{\mathrm{LO}}\mathrm{tr}\{\hat{U}_x\hat{U}^\dagger_y\} + \ldots \quad \Rightarrow$$

$$\frac{d}{d\eta}\langle\mathrm{tr}\{\hat{U}_x\hat{U}^\dagger_y\}\rangle_{\mathrm{shockwave}} = \langle K_{\mathrm{LO}}\mathrm{tr}\{\hat{U}_x\hat{U}^\dagger_y\}\rangle_{\mathrm{shockwave}}$$



$x_\bullet = \sqrt{\frac{s}{2}}x^-$ $\qquad$ $x_* = \sqrt{\frac{s}{2}}x^+$

Non linear evolution equation: Balitsky-Kovchegov equation

$$\hat{\mathcal{U}}(x,y) \equiv 1 - \frac{1}{N_c}\mathrm{tr}\{\hat{U}(x_\perp)\hat{U}^\dagger(y_\perp)\}$$

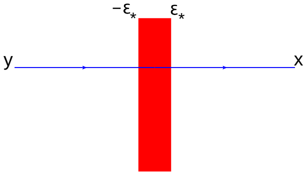
$$\frac{d}{d\eta}\hat{\mathcal{U}}(x,y) \;=\; \frac{\alpha_s N_c}{2\pi^2}\int \frac{d^2z\;(x-y)^2}{(x-z)^2(y-z)^2}\Big\{\hat{\mathcal{U}}(x,z)+\hat{\mathcal{U}}(z,y)-\hat{\mathcal{U}}(x,y)-\hat{\mathcal{U}}(x,z)\hat{\mathcal{U}}(z,y)\Big\}$$

- LLA for DIS in pQCD $\Rightarrow$ BFKL
  - (LLA: $\alpha_s \ll 1, \alpha_s\eta \sim 1$): describes proliferation of gluons.

- LLA for DIS in semi-classical-QCD $\Rightarrow$ BK eqn
  - background field method: describes recombination process.

- Note: if $x_\perp \;\to\; z_\perp$ or $y_\perp \;\to\; z_\perp$ divergences cancel out.

Shock-wave with finite width

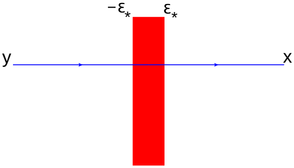


$x_* = \sqrt{\frac{s}{2}}x^+ \qquad x_\bullet = \sqrt{\frac{s}{2}}x^-$

$$A_\bullet(x_\bullet, x_*, x_\perp) \rightarrow \lambda A_\bullet(\lambda^{-1}x_\bullet, \lambda x_*, x_\perp)$$
$$A_*(x_\bullet, x_*, x_\perp) \rightarrow \lambda^{-1}A_*(\lambda^{-1}x_\bullet, \lambda x_*, x_\perp)$$
$$A_\perp(x_\bullet, x_*, x_\perp) \rightarrow A_\perp(\lambda^{-1}x_\bullet, \lambda x_*, x_\perp)$$

λ is the boost parameter

- $p^\mu = \alpha p_1^\mu + \beta p_2^\mu + p_\perp^\mu$
- small α gluons are classical fields   large α gluons are quantum fields.
- Longitudinal sized classical fields: $\epsilon_* = \frac{\alpha s}{l_\perp^2}$ with $l_\perp$ trans. mom. of classical fileds
- Distance traveled by quantum fields: $z_* = \frac{\alpha s}{k_\perp^2}$ with $k_\perp$ trans. mom. of classical fileds
- We are in the case $l_\perp \sim k_\perp$



$x_* = \sqrt{\frac{s}{2}}x^+ \qquad x_\bullet = \sqrt{\frac{s}{2}}x^-$

$$\begin{aligned}
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A_\perp(x_\bullet, x_*, x_\perp) &\rightarrow A_\perp(\lambda^{-1}x_\bullet, \lambda x_*, x_\perp)
\end{aligned}$$

λ is the boost parameter

sub-eikonal terms go like $\frac{1}{\lambda}$

$$\langle x|\frac{i}{\not{P} + i\epsilon}|y\rangle \rightarrow \langle x|\not{P}\frac{i}{p^2 + 2\alpha A_\bullet + ig\frac{2}{s}\not{p}_2\gamma^i F_{\bullet i} + \frac{1}{2}F_{ij}\sigma^{ij} + ... + i\epsilon}|y\rangle$$

- Note: $[\hat{\alpha}, \hat{A}^{cl}_\mu] = 0$ with $\alpha = \sqrt{\frac{2}{s}}p^+$ and $\not{p}_2 \propto \gamma^+$

$$e^{i\frac{\hat{p}_\perp^2}{\alpha s}z_*}\,\hat{A}_\bullet(z_*)\,e^{-i\frac{\hat{p}_\perp^2}{\alpha s}z_*} \simeq A_\bullet(z_*) - \frac{z_*}{\alpha s}\{p^i, F_{\bullet i}(z_*)\} - \frac{z_*^2}{2\alpha^2 s^2}\{p^j, \{p^i, D_j F_{\bullet i}(z_*)\}\} + ...$$

Quark propagator with sub-eikonal corrections

$x_* = \sqrt{\frac{s}{2}}x^+ \qquad x_\bullet = \sqrt{\frac{s}{2}}x^- \qquad \sqrt{\frac{2}{s}}p^+$

$$
\begin{aligned}
&\langle x|\frac{i}{\not{P}+i\epsilon}|y\rangle \\
&= \left[\int_0^{+\infty}\frac{d\!\!\!/\alpha}{2\alpha}\theta(x_*-y_*) - \int_{-\infty}^{0}\frac{d\!\!\!/\alpha}{2\alpha}\theta(y_*-x_*)\right] e^{-i\alpha(x_\bullet-y_\bullet)}\frac{1}{\alpha s}\langle x_\perp|\, e^{-i\frac{\hat{p}_\perp^2}{\alpha s}x_*}\Big\{\not{p}\not{p}_2[x_*,y_*]\not{p} \\
&+\frac{ig}{2\alpha}\int_{y_*}^{x_*} d\frac{2}{s}\omega_*\Big[\not{p}\not{p}_2\,[x_*,\omega_*]\frac{1}{2}F_{ij}(x_*)\sigma^{ij}[\omega_*,y_*]\not{p} + \not{p}\not{p}_2\{p^i,[x_*,\omega_*]\frac{2}{s}\omega_* F_{i\bullet}(\omega_*)[\omega_*,y_*]\}\not{p} \\
&+g\int_{\omega_*}^{x_*} d\frac{2}{s}\omega'_*\,\not{p}\not{p}_2\,\frac{2}{s}(\omega_*-\omega'_*)[x_*,\omega'_*]F^i{}_\bullet(\omega'_*)[\omega'_*,\omega_*]F_{i\bullet}[\omega_*,y_*]\,\not{p} \\
&+\not{p}\not{p}_2[x_*,\omega_*](i\not{D}_\perp\frac{1}{2}F_{ij}\sigma^{ij})[\omega_*,y_*] + \frac{2}{s}\omega_*\not{p}\not{p}_2\Big(ig\gamma_\perp^\rho[x_*,\omega_*]F_{i\bullet}(\omega_*)F_\rho{}^i(\omega_*)[\omega_*,y_*]\Big) \\
&+(\epsilon_{mij}\epsilon_{mlk}-\epsilon_{mjk}\epsilon_{mil})\frac{2}{s}\omega_*\not{p}\not{p}_2\Big(\{p_i,[x_*,\omega_*](iD_j\gamma_k F_{l\bullet})[\omega_*,y_*]\} \\
&\qquad -(iD_i[x_*,\omega_*])(iD_j\gamma_k F_{l\bullet})[\omega_*,y_*] - [x_*,\omega_*](iD_i\gamma_j F_{k\bullet})(iD_l[\omega_*,y_*])\Big)\Big]\Big\}e^{i\frac{\hat{p}_\perp^2}{\alpha s}y_*}|y_\perp\rangle
\end{aligned}
$$

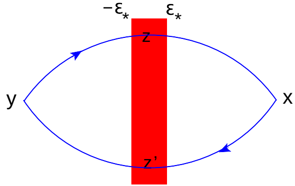
Quark propagator with sub-eikonal corrections

$x_* = \sqrt{\frac{s}{2}}x^+ \qquad x_\bullet = \sqrt{\frac{s}{2}}x^- \qquad \sqrt{\frac{2}{s}}p^+$

$$
\begin{aligned}
&\langle x|\frac{i}{\not{P}+i\epsilon}|y\rangle \\
&= \left[\int_0^{+\infty}\frac{\text{đ}\alpha}{2\alpha}\theta(x_*-y_*) - \int_{-\infty}^0\frac{\text{đ}\alpha}{2\alpha}\theta(y_*-x_*)\right] e^{-i\alpha(x_\bullet-y_\bullet)}\frac{1}{\alpha s}\langle x_\perp|\, e^{-i\frac{\hat{p}_\perp^2}{\alpha s}x_*}\Big\{\not{p}\not{p}_2[x_*,y_*]\not{p} \\
&+\frac{ig}{2\alpha}\int_{y_*}^{x_*}d\frac{2}{s}\omega_*\Big[\not{p}\,\not{p}_2\,[x_*,\omega_*]\frac{1}{2}F_{ij}(x_*)\sigma^{ij}[\omega_*,y_*]\not{p} + \not{p}\not{p}_2\{p^i,[x_*,\omega_*]\frac{2}{s}\omega_* F_{i\bullet}(\omega_*)[\omega_*,y_*]\}\not{p} \\
&+g\int_{\omega_*}^{x_*}d\frac{2}{s}\omega'_*\not{p}\not{p}_2\,\frac{2}{s}(\omega_*-\omega'_*)[x_*,\omega'_*]F^i{}_\bullet(\omega'_*)[\omega'_*,\omega_*]F_{i\bullet}[\omega_*,y_*]\,\not{p} \\
&+\not{p}\,\not{p}_2[x_*,\omega_*](i\not{D}_\perp\frac{1}{2}F_{ij}\sigma^{ij})[\omega_*,y_*] + \frac{2}{s}\omega_*\not{p}\,\not{p}_2\Big(ig\gamma_\perp^\rho[x_*,\omega_*]F_{i\bullet}(\omega_*)F_\rho{}^i(\omega_*)[\omega_*,y_*]\Big) \\
&+(\epsilon_{mij}\epsilon_{mlk}-\epsilon_{mjk}\epsilon_{mil})\frac{2}{s}\omega_*\not{p}\,\not{p}_2\Big(\{p_i,[x_*,\omega_*](iD_j\gamma_k F_{l\bullet})[\omega_*,y_*]\} \\
&\qquad -(iD_i[x_*,\omega_*])(iD_j\gamma_k F_{l\bullet})[\omega_*,y_*] - [x_*,\omega_*](iD_i\gamma_j F_{k\bullet})(iD_l[\omega_*,y_*])\Big)\Big]\Big\} e^{i\frac{\hat{p}_\perp^2}{\alpha s}y_*}|y_\perp\rangle
\end{aligned}
$$

- Leading-eikonal term
- Sub-eikonal terms

Quark propagator with sub-eikonal corrections



Let $|B\rangle$ be proton or nuclear target

$$\langle B|J^\mu(x)J^\nu(y)|B\rangle \quad \rightarrow \quad \langle J^\mu(x)J^\nu(y)\rangle_A = \mathrm{Tr}\{\gamma^\mu\langle x|\frac{i}{\not{P}+i\epsilon}|y\rangle\,\gamma^\nu\langle y|\frac{i}{\not{P}+i\epsilon}|y\rangle\}$$

OPE in Wilson lines with sub-eikonal corrections

$$\langle P,S|\bar\psi(x)\gamma^\mu\psi(x)\bar\psi(y)\gamma^\nu\psi(y)|P,S\rangle$$

$$\stackrel{x_*>0>y_*}{=}\int d^2z_1 d^2z_2\,\frac{\mathrm{tr}\{\not{X}_1\not{p}_2\not{Y}_1\,\gamma^\nu\not{Y}_2\not{p}_2\not{X}_2\gamma^\mu\}}{4\pi^6\,x_*^4y_*^4\,\mathcal{Z}_1^3\mathcal{Z}_2^3}\,\mathrm{tr}\{[x_*,y_*]_{z_1}[y_*,z_*]_{z_2}\}$$

$$-g\int d^2z_1 d^2z_2\,\frac{\mathrm{tr}\{\not{X}_1\not{p}_2\not{Y}_1\gamma^\nu\not{Y}_2\not{p}_2\sigma^{\rho\sigma}\not{X}_2\gamma^\mu\}}{64\pi^6\,x_*^4y_*^4\,\mathcal{Z}_1^3\mathcal{Z}_2^2}$$

$$\times\int_{y_*}^{x_*}d\omega_*\Big[\mathrm{tr}\{[x_*,y_*]_{z_1}[y_*,\omega_*]_{z_2}F^{\perp}_{\rho\sigma}[\omega_*,x_*]_{z_2}\}+\mathrm{tr}\{[x_*,\omega_*]_{z_2}F^{\perp}_{\rho\sigma}[\omega_*,y_*]_{z_2}[y_*,x_*]_{z_1}\}\Big]$$

$$\mathcal{Z}_i\equiv\frac{(x-z_i)_\perp^2}{x_*}-\frac{(y-z_i)_\perp^2}{y_*}-\frac{4}{s}(x_\bullet-y_\bullet)$$

$$X_i\equiv x-z_i\quad\text{and}\quad Y_i\equiv y-z_i$$

$x_*=\sqrt{\frac{s}{2}}x^+$ and $x_\bullet=\sqrt{\frac{s}{2}}x^-$

OPE in Wilson lines with sub-eikonal corrections

$$\langle P,S|\bar\psi(x)\gamma^\mu\psi(x)\bar\psi(y)\gamma^\nu\psi(y)|P,S\rangle$$

$$\stackrel{x_*>0>y_*}{=} \int d^2z_1 d^2z_2\, \frac{\mathrm{tr}\{\not X_1 \not p_2 \not Y_1\, \gamma^\nu \not Y_2\, \not p_2 \not X_2 \gamma^\mu\}}{4\pi^6\, x_*^4 y_*^4\, \mathcal{Z}_1^3 \mathcal{Z}_2^3}\, \mathrm{tr}\{[x_*,y_*]_{z_1}[y_*,z_*]_{z_2}\}$$

$$-g\int d^2z_1 d^2z_2\, \frac{\mathrm{tr}\{\not X_1 \not p_2 \not Y_1 \gamma^\nu \not Y_2 \not p_2 \sigma^{\rho\sigma} \not X_2 \gamma^\mu\}}{64\pi^6\, x_*^4 y_*^4\, \mathcal{Z}_1^3 \mathcal{Z}_2^2}$$

$$\times\int_{y_*}^{x_*} d\omega_* \Big[\mathrm{tr}\{[x_*,y_*]_{z_1}[y_*,\omega_*]_{z_2} F^{\perp}_{\rho\sigma}[\omega_*,x_*]_{z_2}\} + \mathrm{tr}\{[x_*,\omega_*]_{z_2} F^{\perp}_{\rho\sigma}[\omega_*,y_*]_{z_2}[y_*,x_*]_{z_1}\}\Big]$$

- Leading Order Impact Factor for unpolarized case known for 40 years
- Leading Order Impact Factor for polarized case next slide

Photon Impact Factor for sub-eikonal spin corrections

$$\frac{\mathrm{tr}\{\not{X}_1 \not{p}_2 \not{Y}_1 \gamma^\nu \not{Y}_2 \not{p}_2 \sigma^{ij} \not{X}_2 \gamma^\mu\}}{64\pi^6 x_*^4 y_*^4 \mathcal{Z}_1^3 \mathcal{Z}_2^2} = \frac{i}{8\pi^6 x_*^4 y_*^4 \mathcal{Z}_1^3 \mathcal{Z}_2^2} \Big\{ \Phi_1^{\mu\nu,ij}(x,y;z_1,z_2) + \Phi_2^{\mu\nu,ij}(x,y;z_1,z_2)$$

$$+\Phi_3^{\mu\nu,ij}(x,y;z_1,z_2) + \Phi_4^{\mu\nu,ij}(x,y;z_1,z_2) - (\mu \leftrightarrow \nu, x \leftrightarrow y)$$

$$+\Phi_5^{\mu\nu,ij}(x,y;z_1,z_2) + \Phi_6^{\mu\nu,ij}(x,y;z_1,z_2) \Big\} - i \leftrightarrow j$$

$$\Phi_1^{\mu\nu,ij}(x,y;z_1,z_2) = p_2^\nu x_* \Big[\big(Y_{1\perp}^\mu Y_2^j - Y_{2\perp}^\mu Y_1^j \big)(z_1^i - z_2^i) + Y_2^j Y_1^i (X_{2\perp}^\mu + X_{1\perp}^\mu) \Big]$$

$$\Phi_2^{\mu\nu,ij}(x,y;z_1,z_2) = \frac{4}{s} p_1^\mu p_2^\nu x_*^2 Y_1^i Y_2^j$$

$$\Phi_3^{\mu\nu,ij}(x,y;z_1,z_2) = 2x_* y_* g^{j\nu}(z_1^i - z_2^i)\Big(\frac{2}{s} x_* p_1^\mu + X_{2\perp}^\mu\Big)$$

$$\Phi_4^{\mu\nu,ij}(x,y;z_1,z_2) = x_* p_2^\nu g^{\mu i}\Big(Y_2^j (Y_1, z_1 - z_2) - Y_1^j (Y_2, z_1 - z_2) + (Y_1, Y_2)(z_1^j - z_2^j) \Big)$$

$$\Phi_5^{\mu\nu,ij}(x,y;z_1,z_2) = p_2^\nu p_2^\mu \Big[(X_2, Y_1) Y_2^i X_1^j + (X_1, Y_2) X_2^j Y_1^i + (X_2, X_1) Y_2^j Y_1^i + (Y_1, Y_2) X_2^i X_1^j$$

$$+(X_2, Y_2) Y_1^j X_1^i + (Y_1, X_1) X_2^i Y_2^j \Big]$$

$$\Phi_6^{\mu\nu,ij}(x,y;z_1,z_2) = x_* y_* g_\perp^{i\mu} g_\perp^{j\nu} (z_1 - z_2)_\perp^2$$

Photon Impact Factor for sub-eikonal spin corrections

$$\frac{\mathrm{tr}\{\not{X}_1 \not{p}_2 \not{Y}_1 \gamma^\nu \not{Y}_2 \not{p}_2 \sigma^{ij} \not{X}_2 \gamma^\mu\}}{64\pi^6 x_*^4 y_*^4 \mathcal{Z}_1^3 \mathcal{Z}_2^2} = \frac{i}{8\pi^6 x_*^4 y_*^4 \mathcal{Z}_1^3 \mathcal{Z}_2^2} \Big\{ \Phi_1^{\mu\nu,ij}(x,y;z_1,z_2) + \Phi_2^{\mu\nu,ij}(x,y;z_1,z_2)$$

$$+\Phi_3^{\mu\nu,ij}(x,y;z_1,z_2) + \Phi_4^{\mu\nu,ij}(x,y;z_1,z_2) - (\mu \leftrightarrow \nu, x \leftrightarrow y)$$

$$+\Phi_5^{\mu\nu,ij}(x,y;z_1,z_2) + \Phi_6^{\mu\nu,ij}(x,y;z_1,z_2) \Big\} - i \leftrightarrow j$$

- Impact factor is
  - Gauge invariant
  - Conformal invariant in SL(2,C)

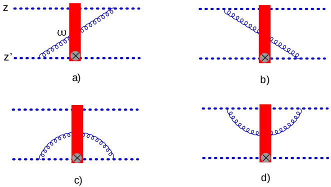
Evolution of sub-eikonal operator

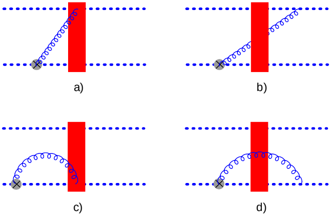
Consider, for example, the following sub-eikonal operator

$$\int_{y_*}^{x_*} d\omega_* \, \mathrm{tr}\{U_z[-\infty,\omega_*]_{z'} \, gF_{ij}(\omega_*, z'_\perp)[\omega_*, +\infty]_{z'}\}$$

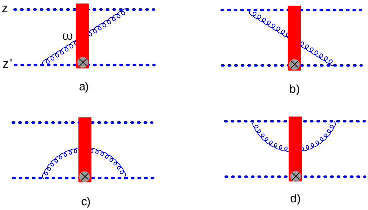
Background field method: split fields in quantum and classical and integrate out the quantum fields

Sample of diagrams:





sample of BK-type diagrams



- $\int_0^{+\infty} \frac{d\alpha}{\alpha}$ rapidity divergence
- if $\omega_\perp \rightarrow z_\perp$ divergence cancel out.
- if $\omega_\perp \rightarrow z'_\perp$ divergence does not cancel out.
  - we have $\left(\alpha_s \ln^2 \frac{1}{x}\right)$ type of contribution

Summing real and virtual diagrams we get

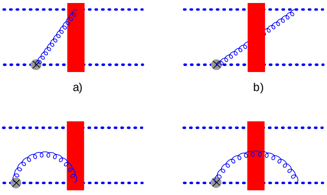
$$\int_{-\infty}^{+\infty} d\omega_* \langle \mathrm{tr}\{[\infty, -\infty]_z [-\infty, \omega_*]_{z'} gF_{ij}(\omega_*)[\omega_*, \infty]_{z'}\}\rangle_{\mathrm{BK-type}}$$

$$= \frac{\alpha_s}{2\pi^2} \int_{-\infty}^{+\infty} d\omega_* \int_0^{+\infty} \frac{d\alpha}{\alpha} \int d^2\omega \, \frac{(z-z')^2_\perp}{(z-\omega)^2_\perp (z'-\omega)^2_\perp}$$

$$\times \Big[\mathrm{tr}\{U_z U^\dagger_\omega\}\mathrm{tr}\{U_\omega [-\infty, \omega_*]_{z'} \, gF_{ij}(\omega_*, z'_\perp)[\omega_*, \infty]_{z'}\} - N_c \mathrm{tr}\{U_z [-\infty, \omega_*]_{z'} \, gF_{ij}(\omega_*, z'_\perp)[\omega_*, \infty]_{z'}\}\Big]$$

Diagrams with F_{ij} quantum Work in progress

- Diagram with gluon propagator without sub-eikonal corrections has no rapidity divergence



- $\Rightarrow$ Need sub-eikonal corrections also in gluon propagator.

Gluon propagator in the light-cone gauge with sub-eikonal corrections is

$$\langle \mathrm{T}A_\mu(x)A_\nu(y)\rangle = \Big[-\int_0^{+\infty}\frac{d\!\!{}^{-}\alpha}{2\alpha}\theta(x_*-y_*) + \int_{-\infty}^{0}\frac{d\!\!{}^{-}}{2\alpha}\theta(y_*-x_*)\Big]$$

$$\times\langle x_\perp|\, e^{-i\frac{p_\perp^2}{\alpha s}x_*}\,\mathcal{O}(x_*,y_*)\,e^{i\frac{p_\perp^2}{\alpha s}y_*}\,|y_\perp\rangle + i\langle x|\frac{p_{2\mu}p_{2\nu}}{p_*^2}|y\rangle$$

$$\mathcal{O}(x_*,y_*) = [x_*,y_*] - \frac{2ig}{\alpha s^2}\int_{y_*}^{x_*}dz_*\,\big(z_*\{p^j,[x_*,z_*]F_{\bullet j}[z_*,y_*]\} + \ldots\big)$$

see Balitsky – Tarasov 2016

Conclusions

- Quark propagator with sub-eikonal corrections is good for
  - spin-dependent TMDs: SIDIS, Weizsäcker-Williams TMD at low-x
  - spin g_1 structure function at low-x
- New operators appears if we consider spin
- OPE at high-energy extended to include sub-eikonal spin corrections
- LO Impact Factor for sub-eikonal spin correction is calculated
- Sub-eikonal corrections to BK-equation are now also possible
  - Although these are suppressed in the unpolarized case
- Future...include NLO corrections with spin

What it is the local operator whose anomalous dimension analytically continued to the *unphysical* point reproduces the Odderon intercept?

- Twist-2 gluon operator and anomalous dimension of light-ray operators.
- Odderon in dipole-Wilson lines formalism.
- Odderon and twist-3 operator.
- Scale dependence of Twist-3 operator and spinor formalism.
- Conclusions and Outlook.

Anomalous dimension of local Operators

- Scale dependence of hadronic-cross section is driven by anomalous dimension of local operator
- Application of OPE in DIS: expand the moments of structure function in inverse power of the hard scale Q

$$F(j,Q^2) = \int_0^1 dx_B\, x_B^{j-2} F(x_B,Q^2) = \sum_{n=2}^{\infty} \frac{1}{Q^n} \sum_a C_n^a(j,\alpha_s(Q^2)) \langle P|\mathcal{O}^a_{n,j}(0)|P\rangle$$

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After diagonalization of mixing matrix we get the multiplicatively renormalizable operators

$$Q^2 \frac{d}{Q^2} \langle P|O^a_{n\,,j}(0)|P\rangle = \gamma_n^a(j) \langle P|O^a_{n,j}(0)|P\rangle$$

Perurbative series of anomalous dimension:

$$\gamma_n^a(j) = \sum_{k=1}^{\infty} \gamma_n^a(k,j) \left(\frac{\alpha_s(Q^2)}{\pi}\right)^k$$

Moments of structure function at low-x region

- Regge behavior of structure function: $F(x_B, Q^2) \sim x_B^{1-\alpha}$ ($\alpha - 1$ is the intercept)

- Scale dependence of structure function at low-x_B
  - for $x_B \to 0$, $F(j, Q^2)$ has poles at $j \sim 1$: *unphysical* point
  - analytically continue the anomalous dimension $\gamma_n^a(j)$ from integer $j \geq 2$ to $j \sim 1$ and then invert the moments of structure functions

- Twist $n = 2$: use DGLAP expression for $\gamma_{n=2}^a(j)$

- Twist $n \geq 3$ case is difficult already at $j \geq 2$
  - Number of operators increases with twist
  - size of corresponding matrices depends on j
  - $\Rightarrow$ analytical continuation to *unphysical* point of the anomalous dimension of twist $n = 3$ operator is even more difficult.

Alternative strategy

- Calculate small-x behavior of structure functions $F(x_B, Q^2)$ within the BFKL formalism.

- and compare it with twist expansion of the moments of the structure functions $F(j, Q^2)$ (Jaroszewicz (1982))

Gluonic structure function at low-x_B. DIS: $\Lambda_{QCD} \ll P^2 = M^2 \ll Q^2 \ll s$

- Q^2 behavior of gluon structure function are driven by the anomalous dimension of twist-2 gluonic operator

$$\mu \frac{d}{\mu} F^a_{\mu+} \nabla_+^{j-2} F_+^{\mu a} = \gamma(\alpha_s, j) F^a_{\mu+} \nabla_+^{j-2} F_+^{\mu a}$$

- $\gamma^*\gamma^*$ scattering amplitude

$$A(s) = \int d\nu f(\nu) \left(\frac{s}{Q_1 Q_2}\right)^{N(\nu)} \left(\frac{Q_1}{Q_2}\right)^{i\nu}$$

- DIS structure function: $Q_1^2 = Q^2 = -q^2$ and $p^2 = -Q_2^2$
  - DIS at small-x: $s \gg Q^2 \gg p^2$

- Gluonic structure function obtained in the BFKL formalism

$$F(x_B, Q^2) = \frac{1}{2\pi} \int d\nu\, f(\nu)\, x_B^{-1-N(\nu)} \left(\frac{Q^2}{P^2}\right)^{\frac{1}{2}+i\nu}$$

$N(\nu)$ pomeron intercept;

Alternative strategy

Gluonic structure function at low-x_B. DIS: $\Lambda_{QCD} \ll P^2 = M^2 \ll Q^2 \ll s$

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$N(\nu)$ pomeron intercept;

- To write the low-x_B structure function in this form at NLO one has to
  - use the NLO impact factor obtained with the composite Wilson line formalism (I. Balitsky G.A.C (2012))
  - and use the NLO BFKL eigenfunctions (G.A.C, Yu. Kovchegov (2013))

Alternative strategy

$\gamma = \frac{1}{2} + i\nu$

$$F(j, Q^2) = \int_0^1 dx_B\, x_B^{j-2} F(x_B, Q^2) = \frac{1}{2\pi i} \int\limits_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} d\gamma \, \frac{f(\gamma)}{j-2-N(\gamma)} \left(\frac{Q^2}{P^2}\right)^{\gamma}$$

- analytic continuation: $j-2 \to \omega$ complex continuous variable;
- Residues $\omega = N(\gamma)$; expand $N(\gamma)$ for small γ and solve for γ

$$\gamma(\alpha_s, \omega) = \frac{\alpha_s N_c}{\pi\omega} + \mathcal{O}(\alpha_s^2)\,, \qquad\qquad F(\omega, Q^2) \sim \left(\frac{Q^2}{P^2}\right)^{\frac{\alpha_s N_c}{\pi\omega}}$$

We get the analytic continuation of anomalous dimension at the *unphysical* point $j \to 1$ of twist-2 operators: $F^a_{\mu+}\nabla_+^{-1}F_+^{\mu a}$

Light-ray operator

- This procedure does not tell us the explicit form of the operator $F^a_{\mu+}\nabla_+^{\omega-1}F_+^{\mu a}$ at the *unphysical* point $\omega \to 0$.
- the operator is a light-ray operator (Balitsky-Kazakov-Sobko (2013-2016), see also Balitsky (2014))

$$\mathcal{F}_\omega(x_\perp) = \int_0^\infty dL_- L_-^{-\omega} \int dx_- \, F^a_{+i}(L_- + x_- + x_\perp)[L_- + x_-, x_-]^{ab} \, F_+^{bi}(x_- + x_\perp)$$

The anomalous dimension of this operator is the analytic continuation of the anomalous dimension of local operator $F^a_{\mu+}\nabla_+^{j-2}F_+^{\mu a}$ with j integer.

- How to see this:
  - The expansion on the light-cone of the evolution equation of non-local (light-ray) operator reproduces the anomalous dimension of local operator (Balitsky Braun (1982))
  - Due to this observation the identification of $\mathcal{F}_\omega(x_\perp)$ as the analytic continuation of local operator to the *unphysical* point is trivialized.

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$$\mathcal{F}_\omega(x_\perp) = \int_0^\infty dL_- L_-^{-\omega} \int dx_-\, F^a_{+i}(L_- + x_- + x_\perp)[L_- + x_-, x_-]^{ab}\, F_+^{bi}(x_- + x_\perp)$$

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- What can we say about twist-3 operators?

- What has to do Odderon with all this?

Low-x evolution of Odderon

- In the linear case the Odderon follows a dipole-BFKL type of evolution equation (Kovchegov-Szymanowski-Wallon (2003))
- To solve the evolution equation use BFKL eigenfunctions with odd n
  - $E^{n,\nu}(z_1, z_2) = \left(\frac{z_{12}}{z_1 z_2}\right)^{\frac{1+n}{2}+i\nu} \left(\frac{z_{12}^*}{z_1^* z_2^*}\right)^{\frac{1+n}{2}-i\nu} = (-1)^n E^{n\nu}(z_1, z_1)$
  - The leading high-energy intercept is $N_{odd} - 1 = 2\frac{\alpha_s N_c}{\pi}\chi(n=1, \nu=0) = 0$ (Bartels-Lipatov-Vacca (2000))
  - The odderon intercept is equal to 0 to all loop order in the planar Limit (Caron-Huot (2015))
  - Hint in this direction is also coming from integrability and AdS-CFT Correspondence (Alfimov Gromov Kazakov (2015)).

In Wilson line formalism Odderon is understood as

$$\mathrm{tr}\{U(x_\perp)U^\dagger(y_\perp)\} - \mathrm{tr}\{U^\dagger(x_\perp)U(y_\perp)\}$$

where $U(x_\perp) = \mathrm{P}\left\{ig\int dx^+ A^-(x^+ p_1 + x_\perp)\right\}$ with p_1^μ light-con vector

Odderon in Wilson lines formalism

- Let us find a relation between Odderon and light-ray operators
- choose $y_\perp = -x_\perp$ and expand Wilson lines for $x_\perp \to 0$ to the first non trivial order

$$\mathrm{tr}\{U(x_\perp)U^\dagger(-x_\perp)\} \sim x^i x^j x^k \int dx_1 dx_2 dx_3\, d^{abc}\, F^a_{+i}(x_1)\, F^b_{+j}(x_2)\, F^c_{+k}(x_3)$$

the resulting operator is a twist-3 operator. This is the light-ray operator for the Odderon:That is, the non local operator in the unphysical point

- Find its scale dependence
- Construct the local operator by analytic continuation

Introduce spinor notation (Braun-Manashov-Rohrwild (2009); Braun-Manashov-Pirnay (2012))

$$F_{\alpha\beta,\dot{\alpha}\dot{\beta}} = \sigma^{\mu}_{\alpha\dot{\alpha}}\sigma^{\nu}_{\beta\dot{\beta}}F_{\mu\nu} = 2\left(\epsilon_{\dot{\alpha}\dot{\beta}}f_{\alpha\beta} - \epsilon_{\alpha\beta}\bar{f}_{\dot{\alpha}\dot{\beta}}\right)$$

Here $f_{\alpha\beta}$ and $\bar{f}_{\dot{\alpha}\dot{\beta}}$ are chiral and antichiral symmetric tensors, $f^* = \bar{f}$, which belong to $(1,0)$ and $(0,1)$ representations of the Lorenz group, respectively.

- Consider the following twist-3 operator (P_{ij} permutation operators acting on the fields coordinates)

$$\mathcal{F}^{\pm}(z) = 2gC^{abc}_{\pm}\tilde{s}^{\rho}\left(1 \mp P_{23} \pm P_{12}\right)F_{+}{}^{\nu,a}(z_1)F^{b}_{+\rho}(z_2)F^{c}_{+\nu}(z_3)$$

  - rewrite it in spinor notation

$$\mathcal{F}^{\pm}(z) = -\frac{ig}{\sqrt{2}}C^{abc}_{\pm}\left\{s_{\mu\bar{\lambda}}\bar{f}^{a}_{++}(z_1)f^{b}_{++}(z_2)f^{c}_{++}(z_3) - s_{\lambda\bar{\mu}}f^{a}_{++}(z_1)\bar{f}^{b}_{++}(z_2)\bar{f}^{c}_{++}(z_3)\right\}$$

$C^{abc}_{+} = f^{abc}$ and $C^{abc}_{-} = d^{abc}$.

Scale dependence of twist-3 operator

We need d^{abc} since Odderon is odd under Charge conjugation.

$$\mathcal{F}^{-}(z) = -\frac{ig}{\sqrt{2}}\, d^{abc}\left\{s_{\mu\bar{\lambda}}\bar{f}^{a}_{++}(z_1)f^{b}_{++}(z_2)f^{c}_{++}(z_3) - s_{\lambda\bar{\mu}}f^{a}_{++}(z_1)\bar{f}^{b}_{++}(z_2)\bar{f}^{c}_{++}(z_3)\right\}$$

- Observation: $ff\bar{f}$ and $\bar{f}\bar{f}f$ types of operators do not mix under renormalization.

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} + \frac{\alpha_s}{4\pi}\mathbb{H}^{\pm}\right)\mathcal{F}^{\pm} = 0$$

where

$$\mathbb{H}^{\pm} = \begin{pmatrix} \mathbb{H}^{\pm}_{QQ} & \mathbb{H}^{\pm}_{QF} \\ \mathbb{H}^{\pm}_{FQ} & \mathbb{H}^{\pm}_{FF} \end{pmatrix}$$

we are interested in gluodynamics so we only need ($b_0 = \frac{11}{3}N_c - \frac{2}{3}n_f$)

$$\mathbb{H}^{-}_{FF} = N_c\left(\widehat{\mathcal{H}}_{12} + \widehat{\mathcal{H}}_{23} + \widehat{\mathcal{H}}_{31} - 6(\mathcal{H}^{-}_{12} + \mathcal{H}^{-}_{13})\right) - b_0$$

Evolution kernels

we also need the Bukhvostov, Frolov, Lipatov Kuraev (BFLK) kernels

$$[\widehat{\mathcal{H}}\ \varphi](z_1,z_2) = \int_0^1 \frac{d\alpha}{\alpha}\Big[2\varphi(z_1,z_2) - \bar{\alpha}^{2j_1-1}\varphi(z_{12}^{\alpha},z_2) - \bar{\alpha}^{2j_2-1}\varphi(z_1,z_{21}^{\alpha})\Big]\,,$$

$$[\mathcal{H}^d\varphi](z_1,z_2) = \int_0^1 d\alpha\,\bar{\alpha}^{2j_1-1}\alpha^{2j_2-1}\,\varphi(z_{12}^{\alpha},z_{12}^{\alpha})\,,$$

$$[\mathcal{H}^+\varphi](z_1,z_2) = \int_0^1 d\alpha\int_0^{\bar{\alpha}} d\beta\,\bar{\alpha}^{2j_1-2}\bar{\beta}^{2j_2-2}\,\varphi(z_{12}^{\alpha},z_{21}^{\beta})\,,$$

$$[\widetilde{\mathcal{H}}^+\varphi](z_1,z_2) = \int_0^1 d\alpha\int_0^{\bar{\alpha}} d\beta\,\bar{\alpha}^{2j_1-2}\bar{\beta}^{2j_2-2}\left(\frac{\alpha\beta}{\bar{\alpha}\bar{\beta}}\right)\varphi(z_{12}^{\alpha},z_{21}^{\beta})\,,$$

$$[\mathcal{H}^-\varphi](z_1,z_2) = \int_0^1 d\alpha\int_{\bar{\alpha}}^1 d\beta\,\bar{\alpha}^{2j_1-2}\bar{\beta}^{2j_2-2}\,\varphi(z_{12}^{\alpha},z_{21}^{\beta})\,,$$

$$[\mathcal{H}_{12}^{e,(k)}\varphi](z_1,z_2) = \int_0^1 d\alpha\,\bar{\alpha}^{2j_1-k-1}\,\alpha^{k-1}\varphi(z_{12}^{\alpha},z_2),$$

Scale dependence of odderon twist-3 operator

- Odderon twist-3 operator in spinor notation

$$\mathrm{tr}\{U(x_\perp)U^\dagger(-x_\perp)\} \sim x^i x^j x^k \int dx_1 dx_2 dx_3\, d^{abc}\, F^a_{+i}(x_1)\, F^b_{+j}(x_2)\, F^c_{+k}(x_3)$$
$$\propto \left[f^a_{++}(x_1) - \bar{f}^a_{++}(x_1)\right]\left[f^b_{++}(x_2) - \bar{f}^b_{++}(x_2)\right]\left[f^c_{++}(x_3) - \bar{f}^c_{++}(x_3)\right]$$

- $f\,f\,f$ and $\bar{f}\,\bar{f}\,\bar{f}$ are $3/2$ helicity operators and do not couple to the proton state,
- so in the forward matrix element only $f\,f\,\bar{f}$ and $\bar{f}\,\bar{f}\,f$ types of operator survive.
- It turns out that

$$x^i x^j x^k\, d^{abc}\, \langle P|F^a_{+i}(x_1)\, F^b_{+j}(x_2)\, F^c_{+k}(x_3)|P\rangle$$
$$\propto x^i d^{abc} \langle P|F^a_{+i}(u)F^b_{+j}(t)F^{c\,j}_{+}(x_3) + F^a_{+j}(x_1)F^b_{+i}(x_2)F^{c\,j}_{+}(x_3) + F^a_{+j}(x_1)F^{b\,j}_{+}(x_2)F^c_{+i}(x_3)|P\rangle$$

Scale dependence of odderon twist-3 operator

- This operator is not multiplicative renormalizable so we cannot compute its anomalous dimension and its analytic continuation to the *unphysical* point.

- Idea:
  - We know that the Odderon intercept is 0: $\chi(n=1,\nu=0)=0$
  - Consider the following forward light-ray operator and check whether it is 0 in the *unphysical* point.

$$\int_0^{+\infty} du\, u^{-\omega} \int_0^u dt\, [\mathbb{H}^-_{FF} + b_0] \otimes x_\perp^\mu \Big[F^a_{+\mu}(u) F^b_{+j}(t) F^{c\,j}_{+}(0) + F^a_{+j}(u) F^b_{+\mu}(t) F^{c\,j}_{+}(0) + F^a_{+j}(u) F^{b\,j}_{+}(t) F^c_{+\mu}(0) \Big]$$

$$\propto \int_0^{+\infty} du\, u^{-\omega} \int_0^u dt\, [\mathbb{H}^-_{FF} + b_0] \otimes \Big[f^a_{++}(u) f^b_{++}(0) \bar{f}^c_{++}(t) + f^c_{++}(t) f^a_{++}(u) \bar{f}^b_{++}(0) + f^c_{++}(t) f^b_{++}(0) \bar{f}^a_{++}(u)$$
$$- f^c_{++}(t) \bar{f}^a_{++}(u) \bar{f}^b_{++}(0) - f^a_{++}(u) \bar{f}^c_{++}(t) \bar{f}^b_{++}(0) - f^b_{++}(0) \bar{f}^c_{++}(t) \bar{f}^a_{++}(u) \Big] \stackrel{?}{=} \mathcal{O}(\omega)$$

- next construct by analytic continuation the corresponding local operator.
  - This is opposite to the twist-2 case: there we new the local operator and constructed the by analytic continuation the operator in the *unphysical* point.

- Work in progress ...

Conclusions and Outlook

- Light-ray operators are the analytic continuation to the *unphysical* point of twist-2 local operator.
- Expansion of the dipole-Wilson line Odderon generates twist-3 operators.
- Spinor formalism simplify the analysis of scale dependence of twist-3 operators.
- Outlook
  - Assemble the result for the scale dependence of Odderon-light ray operator at the unphysical point.
  - Construct by analytic continuation the local operator from the Odderon-light ray operator.