# Reweighting the JIMWLK ensemble and biased event samples

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based on: A.D., V. Skokov, 1704.05917; 1710.05041
A.D., G. Kapilevich, V. Skokov, 1802.06111

# Small-x: ensemble of random classical fields

distribution: $$W[A^+] = \exp(-S[A^+])$$

expectation values: $$\langle O[A^+]\rangle = \frac{1}{Z}\int \mathcal{D}A^+\, W[A^+]\, O[A^+]$$

$$Z = \int \mathcal{D}A^+\, W[A^+]$$

$$k^2 A^+(k) = g\,\rho(k)$$

(MV: 2 x PRD '94
Kovchegov: PRD 94)

Example: MV model

$$S_{\rm MV} = \int \frac{d^2q}{(2\pi)^2}\, q^4\, \frac{{\rm tr}\, A^+(q)A^+(-q)}{g^2\mu^2}$$

$$\rightarrow \qquad g^2 \left\langle A^{+a}(p)\, A^{+b}(-q)\right\rangle =$$

$$\delta^{ab}\,(2\pi)^2\delta(p-q)\,g^4\mu^2/p^2q^2$$

# Examples for observables:

Dipole S-matrix: $V(x) = \mathcal{P}\, e^{-ig \int dx^- A^{+a}(x^-, x)\, t^a}$

$$\frac{1}{N_c}\left\langle \mathrm{tr}\, V(x) V^\dagger(y) \right\rangle = \exp\left\{ -\frac{1}{2N_c} \int dx^- dy^- \frac{d^2q}{(2\pi)^2}\frac{d^2p}{(2\pi)^2} (1 - e^{iq\cdot x - ip\cdot y}) \right.$$
$$\left. \times g^2 \left\langle A^{+a}(x^-, q)\, A^{+a}(y^-, -p) \right\rangle \right\}$$

Quadrupole: $\frac{1}{N_c}\left\langle \mathrm{tr}\, V(x)\, V^\dagger(y)\, V(u)\, V^\dagger(w) \right\rangle$

Gluon spectrum in pA collisions:

$$\left\langle \frac{1}{(2\pi)^3 k^2} (\delta_{ij}\delta_{lm} + \epsilon_{ij}\epsilon_{lm})\, \Omega^b_{ij}(k) \left[\Omega^b_{lm}(k)\right]^* \right\rangle_{p,A}$$

$$\Omega^a_{ij}(x) = -\left[ g \frac{\partial_i}{\partial^2} \rho^b_p(x) \right] \partial_j W^{ab}(x)$$

$$W(x) = \mathcal{P}\, e^{-ig \int dx^- A^{+a}_A(x^-, x)\, T^a}$$

## How else can we learn about the ensemble W[A$^+$] ?

→ reweighting !  (→ biased averages)

$$\langle O \rangle = \frac{\sum_i w_i O_i}{\sum_i w_i} \quad , \quad w_i = e^{-S_i} \; ,$$

$$\to \quad \langle O \rangle_{\rm rw} = \frac{\sum_i w_i' O_i}{\sum_i w_i'} \quad , \quad w_i' = w_i b_i$$

Example:

$$b[X] = \exp\left\{ \frac{1}{2} A_\perp N_c^2 \, \eta_0 \int_{Q_{s\,A}^2}^{Q^2} \frac{d^2\ell}{(2\pi)^2} \frac{X(\ell) - X_s(\ell)}{X_s(\ell)} \left( \frac{{q_0}^2}{\ell^2} \right)^a \right\}$$

$$X(q) \equiv g^2 {\rm tr}\, A^+(q) A^+(-q)$$

* reweights towards configurations with addtl gluons above Qs, and with “distorted” gluon distribution (if a ≠ 0)

effective percentile of configurations :

$$\nu_r = \frac{(\sum w_i')^2}{N \sum (w_i')^2}$$

→ choose $\eta_0$ such that $\nu_r$ = 5%, for example

generating function for correlators of gluon distribution:

$$b[X] = \exp\left(\int d^2\mathbf{q}\, t(\mathbf{q}) X(\mathbf{q})\right)$$

$$Z[t] = \int \mathcal{D}X(\mathbf{q})\, e^{-V_{\rm eff}[X]}\, b[X]$$

$$\frac{1}{Z}\, \frac{\delta^n Z[t]}{\delta t(\mathbf{k}_1)\cdots\delta t(\mathbf{k}_n)}\Big|_{t\equiv 0} = \langle X(\mathbf{k}_1)\cdots X(\mathbf{k}_n)\rangle$$

To understand what b[X] does, we first need to compute the distribution of gluon distributions X(q):

## Constraint effective action:

$$e^{-V_{\rm eff}[X(q)]} = \frac{1}{Z}\int \mathcal{D}A^+(q)\, W[A^+(q)]\, \delta(X(q) - g^2 \mathrm{tr}\, |A^+(q)|^2])$$

$$\int \mathcal{D}X(q)\, e^{-V_{\rm eff}[X(q)]} = 1$$

$$\frac{\delta V_{\rm eff}[X(k)]}{\delta X(q)} = 0 \quad \to \quad X_s(q) \equiv \langle X(q) \rangle$$

note implicit integration over impact parameter:

$$\mathrm{tr}\, |A^+(q)|^2 = \int_{A_\perp} d^2b \int d^2r\, e^{iq\cdot r}\, \mathrm{tr}\, A^+(b+\frac{r}{2})\, A^+(b-\frac{r}{2})$$

# Non-local Gaussian approximation to JIMWLK:

$$S = \int d^2x d^2y \, \frac{\mathrm{tr}\, \nabla^2 A^+(x)\, \nabla^2 A^+(y)}{g^2 \mu^2(x-y)}$$

$$= \int \frac{d^2q}{(2\pi)^2}\, q^4 \frac{\mathrm{tr}\, A^+(q)\, A^+(-q)}{g^2\mu^2(q)}$$

$$\mu^2(q) = \mu_0^2 \left(\frac{q^2}{Q_s^2}\right)^{1-\gamma(k)}$$

at q>$Q_s$(Y); Iancu, Itakura, McLerran: NPA 724 (2003)

γ = BFKL anom. dim.

$$V_{\mathrm{eff}}[X(q)] = \int \frac{d^2q}{(2\pi)^2} \left[\frac{q^4}{g^4\mu^2(q)} X(q) - \frac{1}{2} A_\perp N_c^2 \log X(q)\right]$$

$$X_s(q) = \frac{1}{2} N_c^2 A_\perp \frac{g^4 \mu^2(q)}{q^4}$$ ✓

(cov. gauge gluon distribution at q>Qs)

$$1 = \int \prod_q d\lambda_q \, \delta\left(\lambda_q - \frac{g^4}{q^4}\mathrm{tr}\,|\rho_q|^2\right) \qquad , \qquad A_q^+ = \frac{g}{q^2}\rho_q$$

$$\begin{aligned} Z &= \prod_q \int d\lambda_q \frac{d\omega_q}{2\pi} \left(\prod_a d\rho_q^a\right) e^{-i\omega_q \lambda_q + i\omega_q \frac{g^4}{q^4}\mathrm{tr}\,|\rho_q|^2} e^{-\frac{d^2q}{(2\pi)^2}\frac{q^4}{g^4}\frac{\lambda_q}{\mu^2}} \\ &= \left[\prod_q \int d\lambda_q \frac{d\omega_q}{2\pi} e^{-i\omega_q\lambda_q} e^{-\frac{d^2q}{(2\pi)^2}\frac{q^4}{g^4}\frac{\lambda_q}{\mu^2}}\right] \underbrace{\prod_q \int \left(\prod_a d\rho_q^a\right) e^{i\omega_q \frac{g^4}{q^4}|\rho_q|^2}}_{\widetilde{Z}[\omega_q]} \end{aligned}$$

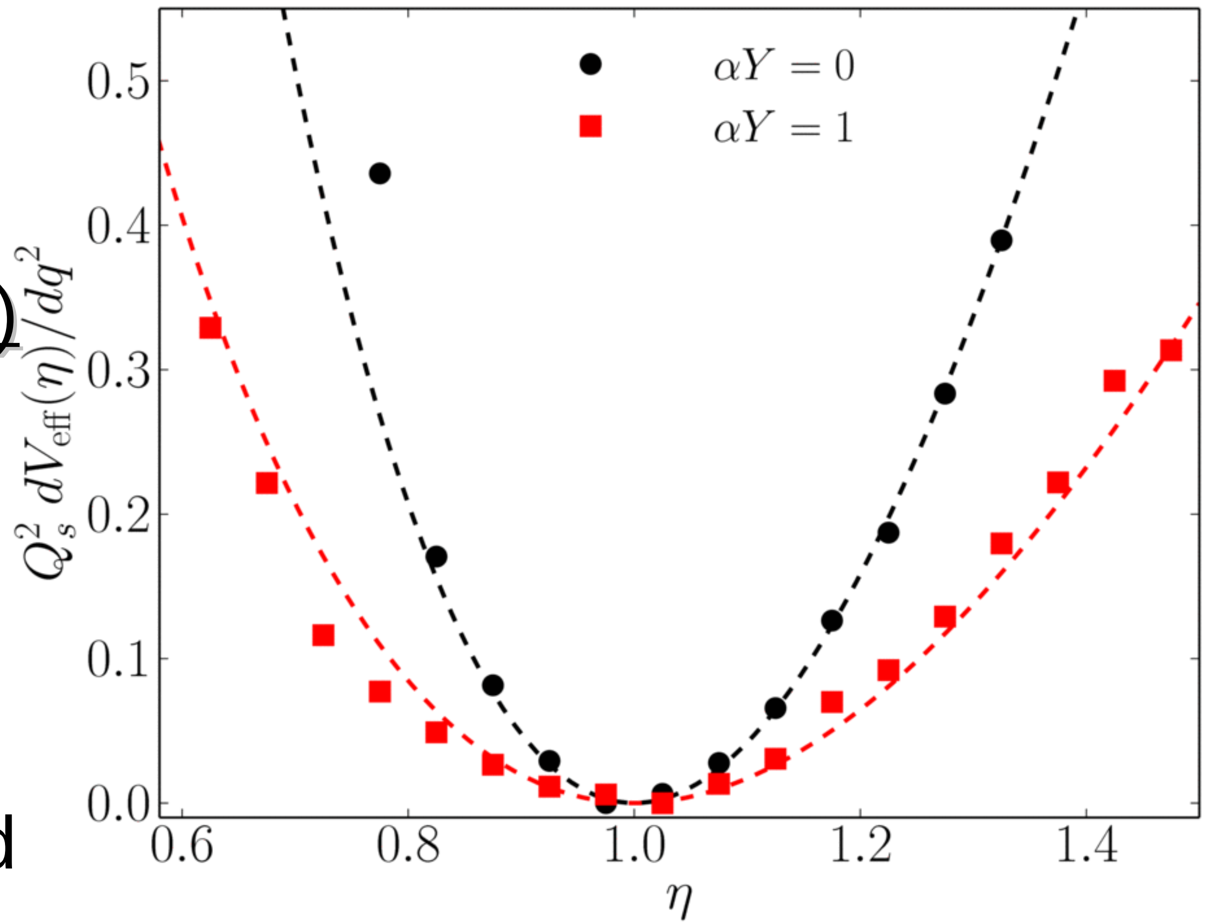
$$\begin{aligned} \widetilde{Z}[\omega_q] &= \int \prod_q dX_q \left(\prod_a d\rho_q^a\right) \delta\left(X_q - \frac{g^4}{q^4}\mathrm{tr}\,|\rho_q|^2\right) e^{i\omega_q \frac{g^4}{q^4}\mathrm{tr}\,|\rho_q|^2} \\ &\sim \prod_q \int dX_q \, X_q^{\frac{N_c^2}{2}} e^{i\omega_q X_q} \end{aligned}$$

$$\to Z = \prod_q \int dX_q \, e^{-\frac{d^2q}{(2\pi)^2}\frac{q^4}{g^4\mu^2}X_q + \frac{1}{2}N_c^2 \log X_q}$$

can rewrite in terms of $\eta(q) = X(q) / X_s(q)$ :

$$\begin{aligned}\Delta V_{\rm eff}[\eta(q)] &\equiv V_{\rm eff}[\eta(q)] - V_{\rm eff}[\eta(q)=1] \\ &= \frac{1}{2} N_c^2 A_\perp \int \frac{d^2q}{(2\pi)^2} \left[\eta(q) - 1 - \log\eta(q)\right]\end{aligned}$$

<u>$V_{\rm eff}[\eta]$ extracted from MC (histogram of fluctuations)</u>




* linear minus log indeed appears to work

field redefinition: $e^{\Phi(q)} \equiv X(q) / X_s(q)$ → Liouville action/potential

$$V_{\rm eff}[\phi(q)] = \frac{1}{2} A_\perp N_c^2 \int \frac{d^2q}{(2\pi)^2} \left[ e^{\phi(q)} - \phi(q) - 1 \right]$$

to compute averages over X(q):

$$g^2 \left\langle A^{+a}(q) A^{+b}(-k) \right\rangle = \delta^{ab} \, (2\pi)^2 \delta(q-k) \, \frac{g^4 \mu^2(q)}{q^2 k^2}$$

→

$$g^2 \left\langle A^{+a}(q) A^{+b}(-k) \right\rangle = \delta^{ab} \, (2\pi)^2 \delta(q-k) \, \frac{g^4 \mu^2(q)}{q^2 k^2} \times \int \mathcal{D}X(\ell) \, e^{-V_{\rm eff}[X(\ell)]} \, X(q)/X_s(q)$$

# Example: correlator of adj. Wilson lines

$$\left\langle \frac{1}{N_c^2-1} \mathrm{tr}\, W^\dagger(x) W(y) \right\rangle = \int \mathcal{D}X(q)\, e^{-V_{\mathrm{eff}}[X(q)]}$$

$$\times \exp\left( -\frac{2}{N_c A_\perp} \int \frac{d^2 s}{(2\pi)^2} \left(1 - e^{is\cdot r}\right) X(s) \right)$$

* If X(s) = $X_s$(s) one recovers the standard result

* But if one reweights with b[X] given on p.4:

$$b[X] = \exp\left\{ \frac{1}{2} A_\perp N_c^2\, \eta_0 \int_{Q_{sA}^2}^{Q^2} \frac{d^2\ell}{(2\pi)^2} \frac{X(\ell) - X_s(\ell)}{X_s(\ell)} \left(\frac{{q_0}^2}{\ell^2}\right)^a \right\}$$

*shifts the stationary point*

$$\frac{\delta}{\delta X(k)} \left(-V_{\mathrm{eff}}[X(\ell)] + \log b[X(\ell)]\right) = 0$$

* Now

$$\frac{X(q)}{X_s(q)} = 1 + \eta_0 \left( \frac{{q_0}^2}{q^2} \right)^a \Theta(Q^2 - q^2)\, \Theta(q^2 - Q_{s,A}^2) \ + \mathcal{O}(\eta_0^2)$$

$$\rightarrow \frac{1}{N_c^2 - 1} \left\langle \mathrm{tr}\, W^\dagger(x) W(y) \right\rangle \sim \exp\left( -\# \left( r^2 Q_{s,A}^2 \right)^\gamma - \# \eta_0 q_0^{2a} Q_{s,A}^{2\gamma} (r^2)^{\gamma + a} \right)$$

A.D., G. Kapilevich & V. Skokov, NPA (2018)

** like a shift of Qs but not quite, different power of $r^2$ **

# pA collisions

$$\left\langle E\frac{dN}{d^3k}\right\rangle_{\mathrm{high}-k} = \frac{g^2N_c^2\mu_A^2(k)A_\perp}{(2\pi)^3}\frac{Q_{s,p}^2}{k^4}\log\left(\frac{k^2}{Q_{s,p}^2}\right)\int\mathcal{D}X(q)\,e^{-V_{\mathrm{eff}}[X(q)]}\,\frac{X(k)}{X_s(k)}$$

Without reweighting / bias,

$$R_{pA}(k) = \frac{\mu_A^2(k)}{N_{\mathrm{coll}}^{\mathrm{m.b.}}\,\mu_p^2(k)} \simeq \left(\frac{k^2}{Q_{s,p}^2}\right)^{\gamma_p(k)-\gamma_A(k)} \frac{1}{(N_{\mathrm{coll}}^{\mathrm{m.b.}})^{1-\gamma_A(k)}}$$

* applies at fixed point of small-x RG
  (memory of initial condition erased)

* this is the predicted suppression of $R_{pA}$ at small-x !
  aka “leading twist shadowing”
  [Kharzeev, Levin, McLerran: PLB (2003) ]

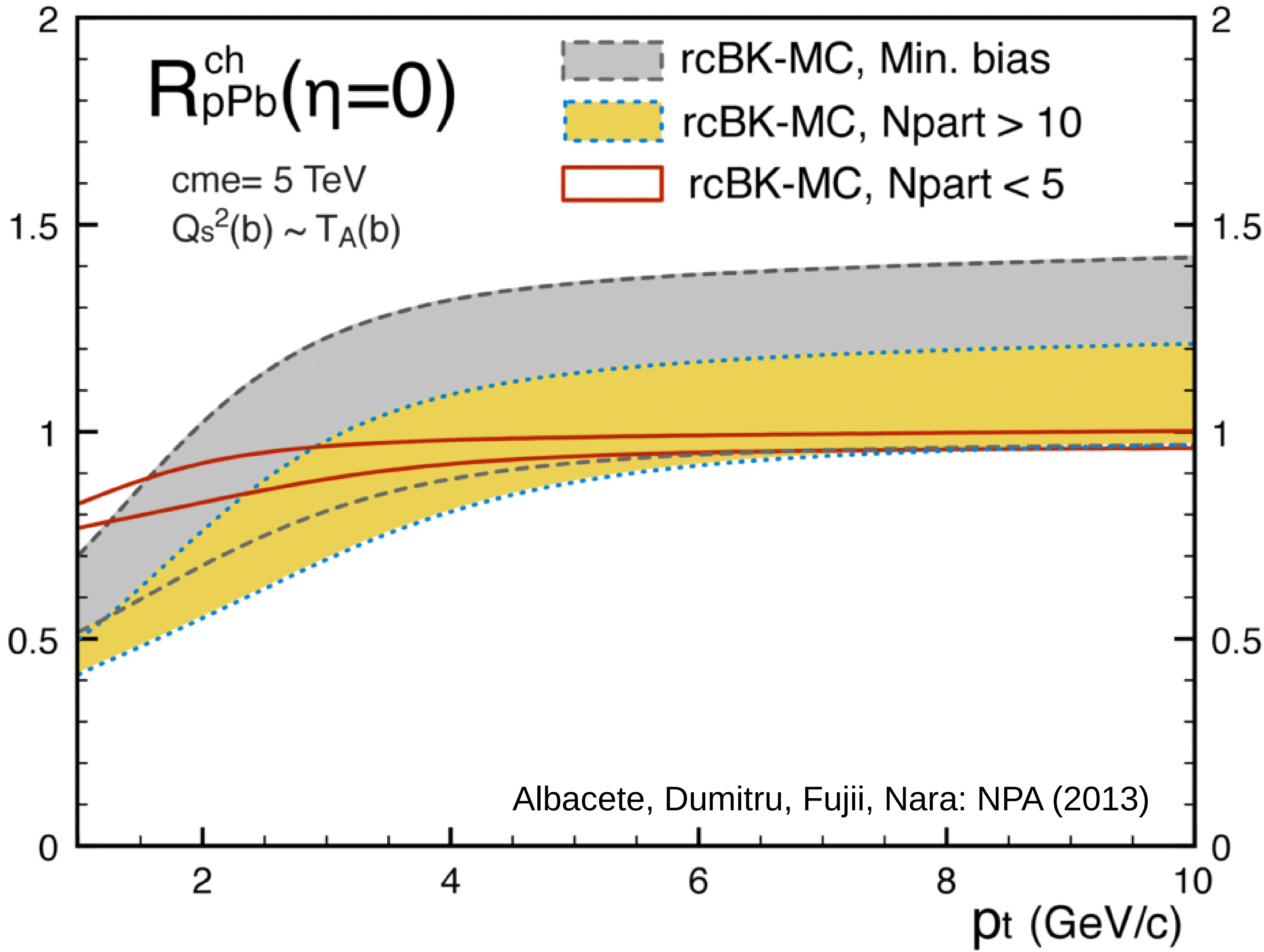
* More suppression for thicker target (say A=1000)

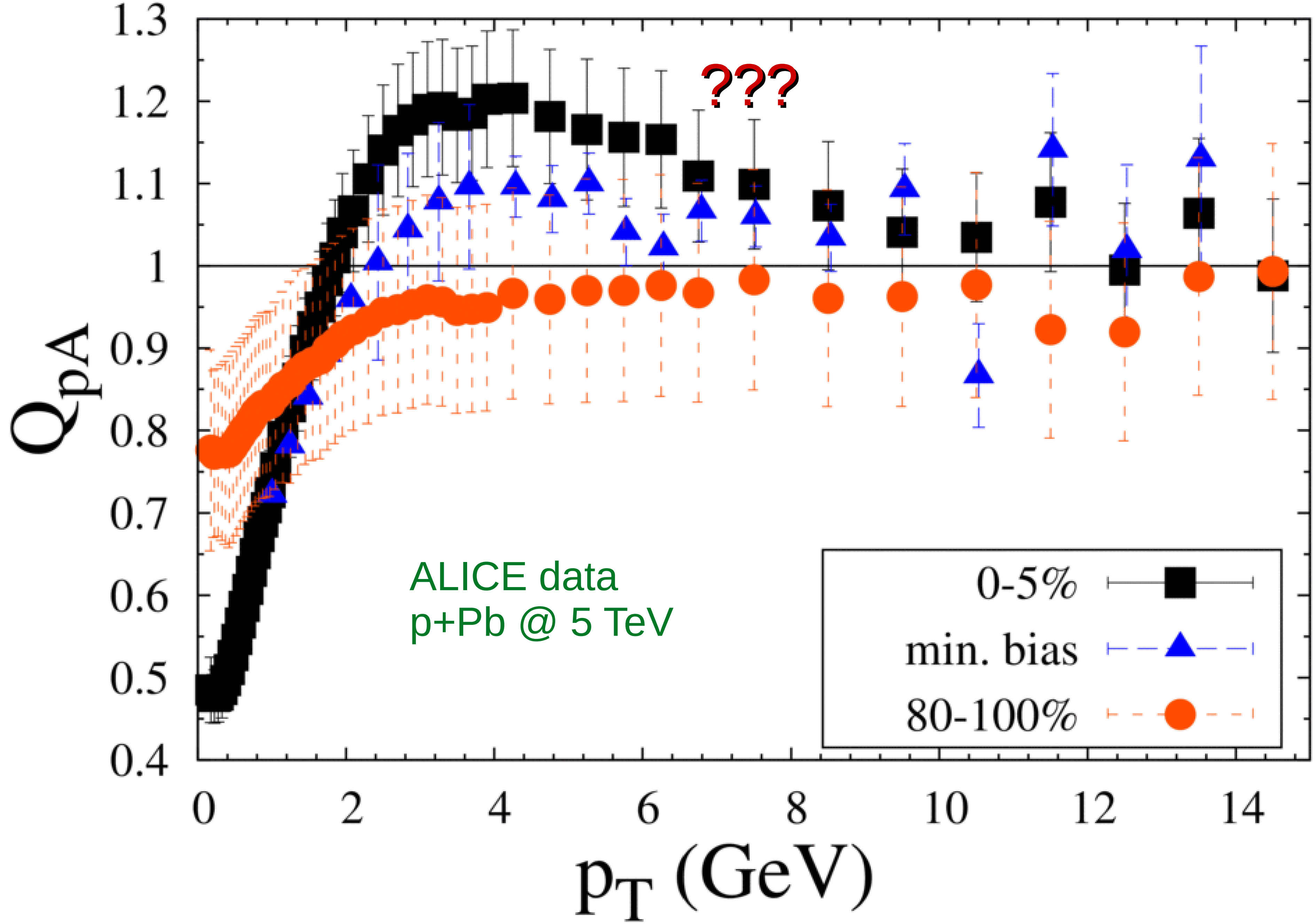
* Can't do but one *can* select "central" pA
  ATLAS: based on $E_T$ at $-4.9 < \eta < -3.1$
  ALICE: based on zero degree calorimeter &
  estimate of $N_{coll}$ from $dN_{ch}/d\eta$ (all / high $p_T$)

[→ biased $Q_{pA}$ instead of $R_{pA}$ ! ]

* But there is a problem... A big one.

numerical confirmation:





???
ALICE data
p+Pb @ 5 TeV
0-5%
min. bias
80-100%
$Q_{pA}$
$p_T$ (GeV)
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0
2
4
6
8
10
12
14

* we can't replicate ALICE centrality selection (zero degree calorimeter ?)

* but we can reweight towards configurations with more gluons at $p_T > Q_{gs}(Y) \sim Q_s^2(Y) / \Lambda$ (where anom. dim. $\gamma_A(p_T) \sim 1$, close to DGLAP limit)

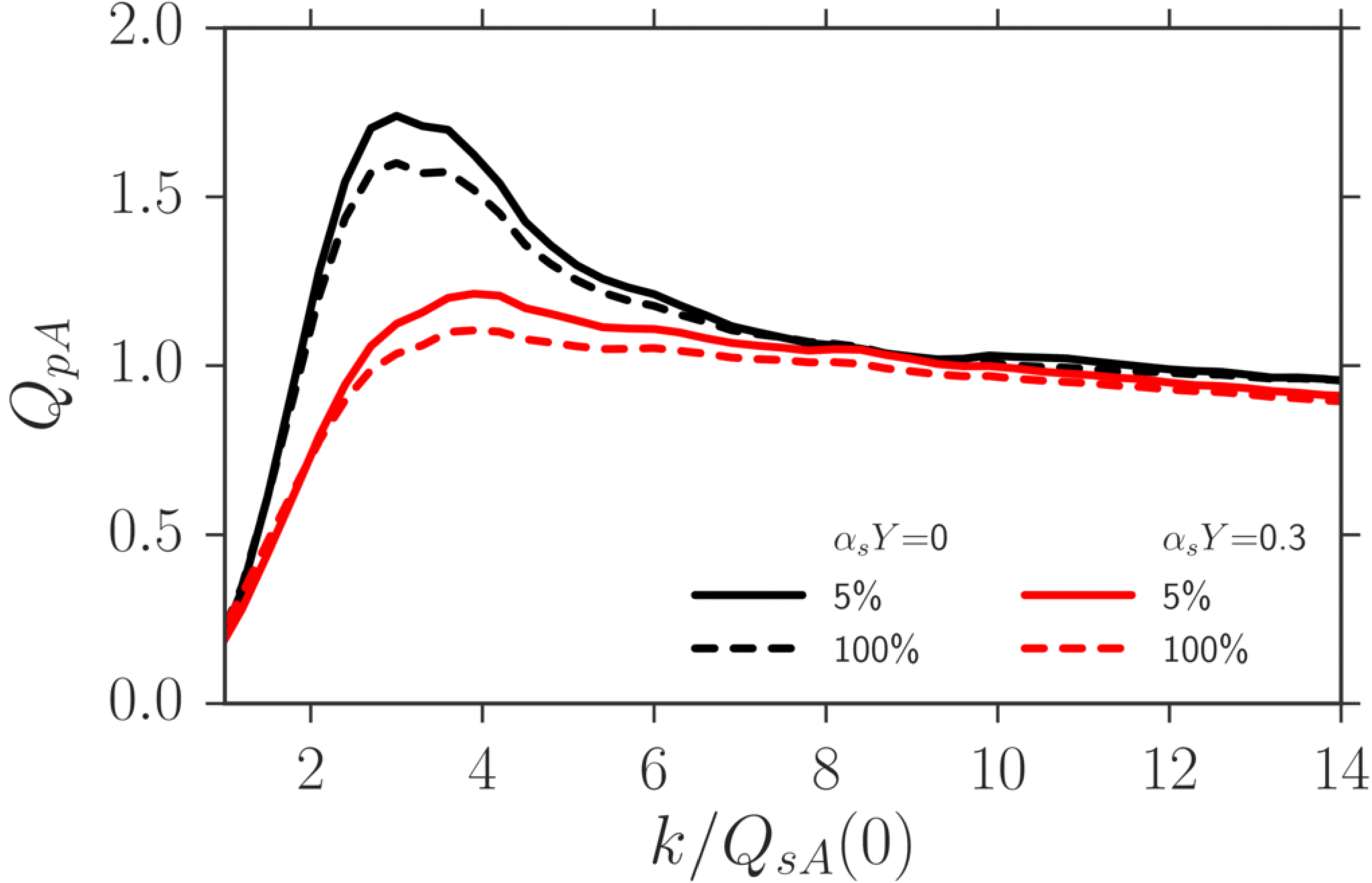
* take

$$N_{\rm coll} = \frac{\int\limits_{Q_{\rm gs}} \left\langle \frac{dN_{pp,pA}}{d^2p_T\,dy} \right\rangle_{\rm rw}}{\int\limits_{Q_{\rm gs}} \left\langle \frac{dN_{pp}}{d^2p_T\,dy} \right\rangle}$$

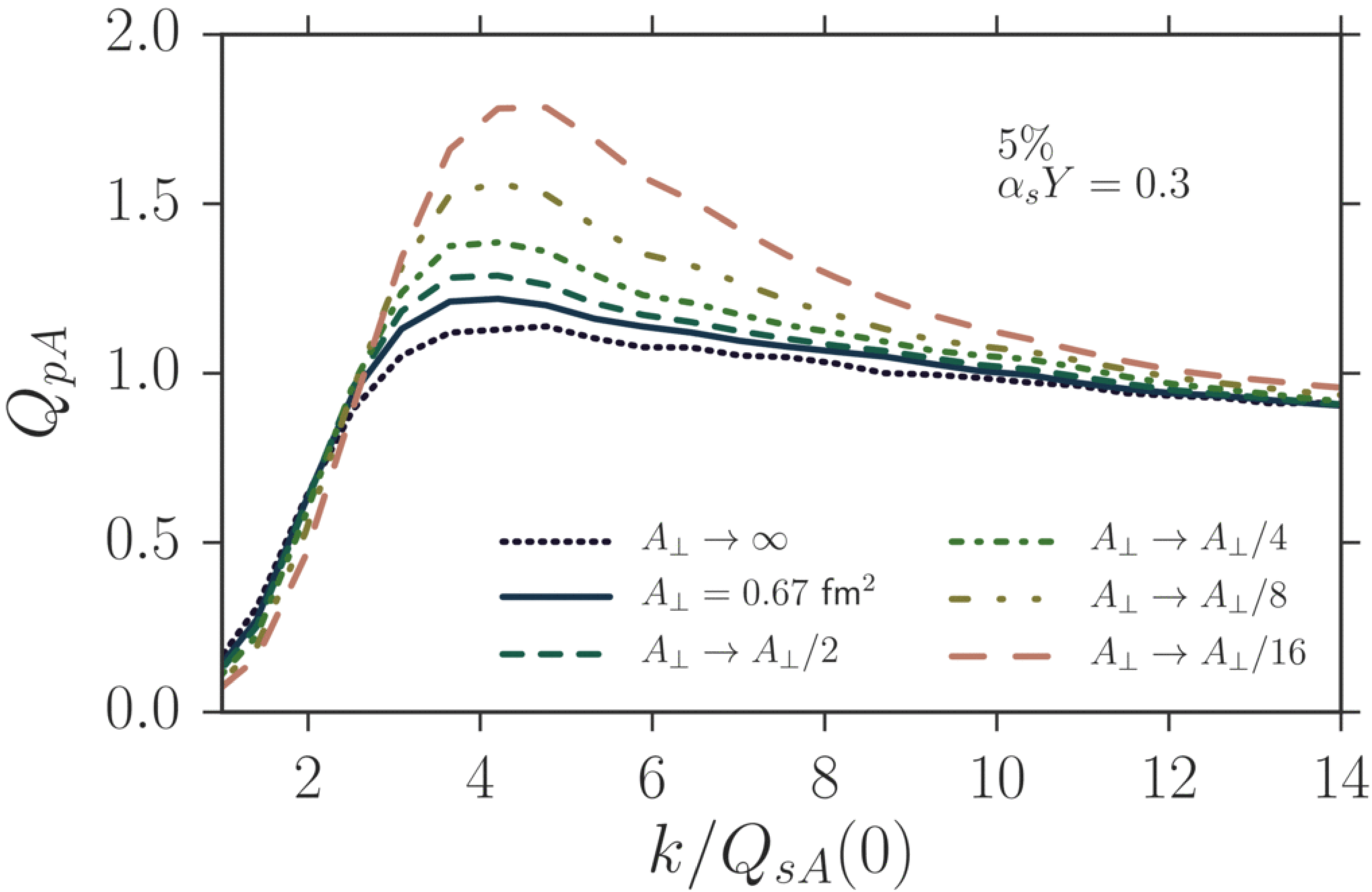
$$\rightarrow Q_{pA}(k) \simeq \left( 1 + \frac{8\pi \log p_r^{-1}}{N_c^2 A_\perp k^2 \log \frac{Q^2}{Q_{s,A}^2}} \right) R_{pA}(k)$$

$p_r$: suppression probability

Numerical results from f.c. JIMWLK evolution
w/ MV-model initial condition, $\mu_A/\mu_p = \sqrt{6}$ (p+Pb)



$\mu_A/\mu_p = \sqrt{6}$ (p+Pb), varying $A_T$



# Summary:

* biased gluon distribution from reweighting

* tool to investigate *ensemble* of small-x gluon distributions rather than just average (or most likely) gluon distribution

* Example: this can resolve the dilemma that $Q_{pA} > R_{pA}$ in “central” p+Pb collisions, and the re-appearance of the “Cronin peak”

# Backup slides

$$1 = \int \prod_q d\lambda_q \, \delta\left(\lambda_q - \frac{g^4}{q^4} \mathrm{tr}\, |\rho_q|^2\right) \qquad , \qquad A_q^+ = \frac{g}{q^2}\rho_q$$

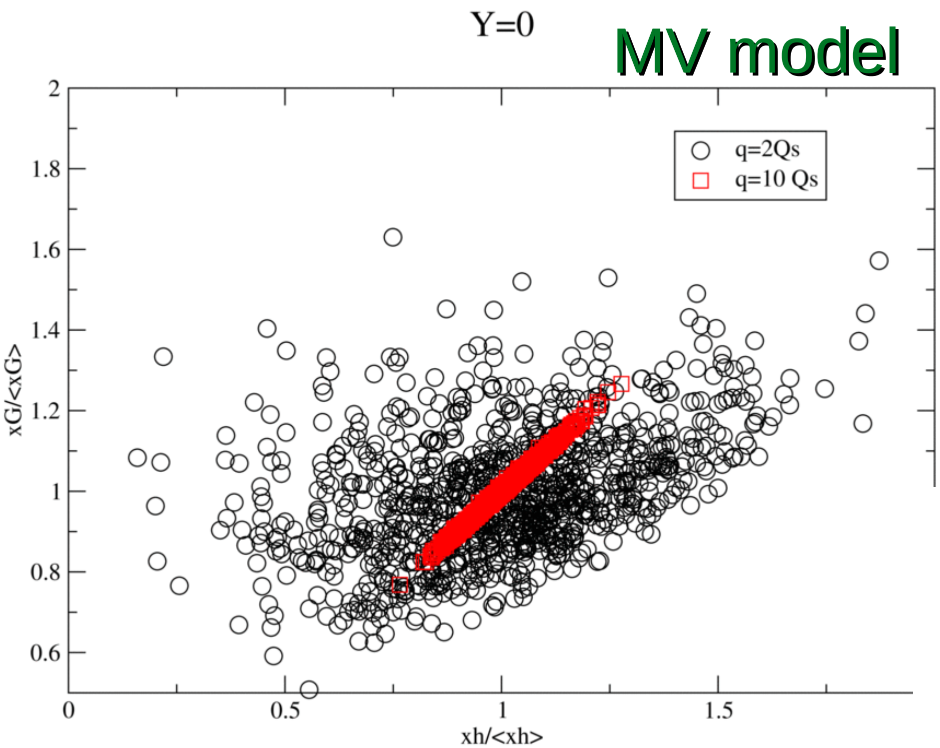
$$\begin{aligned} Z &= \prod_q \int d\lambda_q \frac{d\omega_q}{2\pi} \left(\prod_a d\rho_q^a\right) e^{-i\omega_q \lambda_q + i\omega_q \frac{g^4}{q^4} \mathrm{tr}\, |\rho_q|^2} e^{-\frac{d^2 q}{(2\pi)^2} \frac{q^4}{g^4} \frac{\lambda_q}{\mu^2}} \\ &= \left[\prod_q \int d\lambda_q \frac{d\omega_q}{2\pi} e^{-i\omega_q \lambda_q} e^{-\frac{d^2 q}{(2\pi)^2} \frac{q^4}{g^4} \frac{\lambda_q}{\mu^2}}\right] \underbrace{\prod_q \int \left(\prod_a d\rho_q^a\right) e^{i\omega_q \frac{g^4}{q^4} |\rho_q|^2}}_{\widetilde{Z}[\omega_q]} \end{aligned}$$

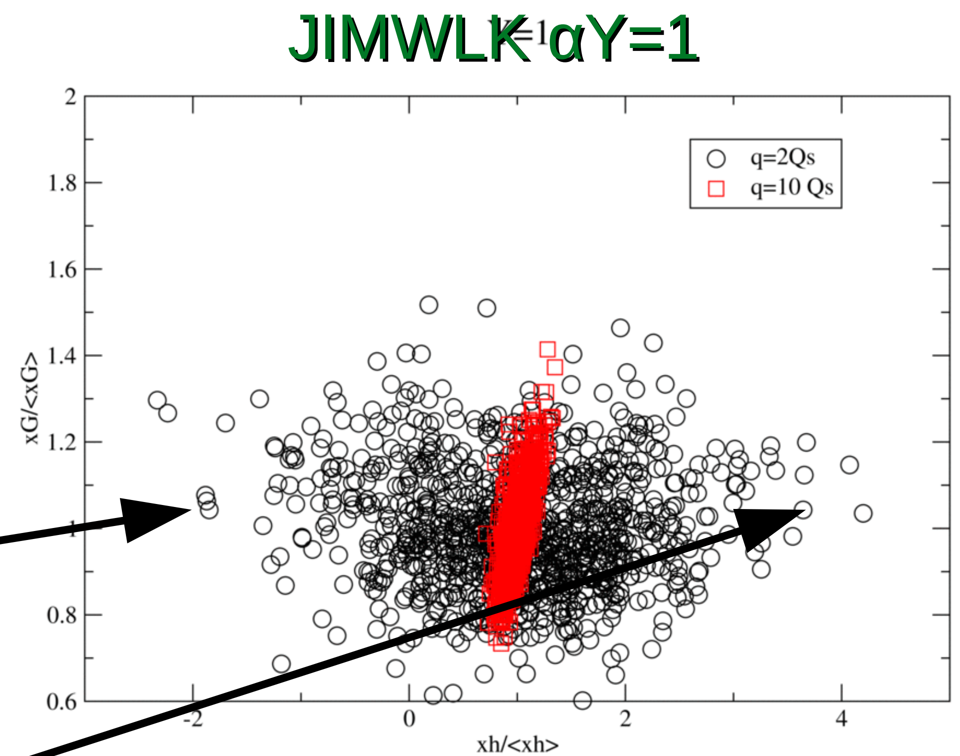
$$\begin{aligned} \widetilde{Z}[\omega_q] &= \int \prod_q dX_q \left(\prod_a d\rho_q^a\right) \delta\left(X_q - \frac{g^4}{q^4} \mathrm{tr}\, |\rho_q|^2\right) e^{i\omega_q \frac{g^4}{q^4} \mathrm{tr}\, |\rho_q|^2} \\ &\sim \prod_q \int dX_q \, X_q^{\frac{N_c^2}{2}} e^{i\omega_q X_q} \end{aligned}$$

$$\to Z = \prod_q \int dX_q \, e^{-\frac{d^2 q}{(2\pi)^2} \frac{q^4}{g^4 \mu^2} X_q + \frac{1}{2} N_c^2 \log X_q}$$

# Fluctuations of WW gluon distributions (MV vs. f.c. JIMWLK)

(all orders in $A^+$)




note negative xh here

and large positive xh here

# Resummation of boost-invariant quantum fluctuations (JIMWLK):

classical ensemble at Y = log $x_0$/x = 0:

$$P[\rho] \sim e^{-S_{\rm cl}[\rho]}\ ,\ S_{\rm MV} \quad = \quad \int d^2x_\perp\ dx^+\ \frac{1}{2\mu^2}\rho^a\rho^a\ ,$$

$$V(x_\perp) = \mathcal{P}\exp\ ig^2 \int dx^+ \frac{1}{\nabla_\perp^2}\rho(x_\perp)$$

JIMWLK quantum evolution: functional RG equation

$$\frac{\partial}{\partial Y} W[V] = -H\left[V, \frac{\delta}{\delta A^-}\right] W[V]$$

distribution in space of Wilson lines

# quantum evolution to Y>0: Langevin / random walk in space of Wilson lines

$$\partial_Y V(x_\perp) = V(x_\perp)\; it^a \left\{ \int d^2 y_\perp \; \varepsilon_k^{ab}(x_\perp, y_\perp)\; \xi_k^b(y_\perp) + \sigma^a(x_\perp) \right\} \; .$$

$$\varepsilon_k^{ab} = \left(\frac{\alpha_s}{\pi}\right)^{1/2} \; \frac{(x_\perp - y_\perp)_k}{(x_\perp - y_\perp)^2} \; \left[1 - U^\dagger(x_\perp) U(y_\perp)\right]^{ab}$$

$$\langle \xi_i^a(x_\perp)\; \xi_j^b(y_\perp)\rangle = \delta^{ab}\delta_{ij}\delta^{(2)}(x_\perp - y_\perp)$$

$$\sigma^a(x_\perp) = -i\frac{\alpha_{\rm s}}{2\pi^2}\int d^2 z_\perp \, \frac{1}{(x_\perp - z_\perp)^2} {\rm tr}\; \left(T^a U^\dagger(x_\perp)\, U(z_\perp)\right)$$