DIPOLE FACTORIZATION FOR DIS AT NLO:

Towards the full mass renormalized cross section

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Outline

- Goal/Motivation
- Dipole factorization framework for DIS
- DIS at NLO: part 1: massless quarks
  - One-loop virtual correction to the $\gamma^*_{T/L} \to q\bar{q}$ LCWF's
  - DIS cross section: UV subtraction between the $q\bar{q}$ and $q\bar{q}g$ terms
- DIS at NLO: part 2: massive quarks
  - One-loop virtual correction to the $\gamma^*_L \to q\bar{q}$ LCWF's
    - quark mass renormalization
  - Full DIS cross section for γ^*_L at NLO
  - Some discussion of γ^*_T case

Goal/Motivation

Goal: Study the regime of low Bjorken x_{B_j} (= high energy) for DIS on a proton/nucleus target:

- Probes the nonlinear regime of gluon saturation
- Outside the validity range of collinear factorization
  - Described within the dipole factorization picture

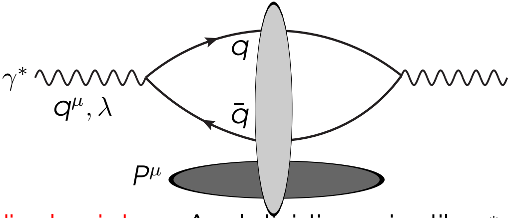
Rich phenomenology:

- Fits to HERA data in the dipole factorization at LO+LL with BK

Fast theoretical progress towards NLO/NLL accuracy:

- DIS structure functions at NLO in the dipole factorization
  - massless quarks: done (this talk)
  - massive quarks: work in progress (this talk)

Dipole factorization for DIS



DIS in the dipole picture: A relativistic projectile γ^* fluctuates into a $q\bar{q}$ pair which then scatters on a very dense target

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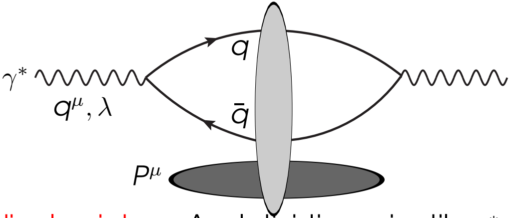


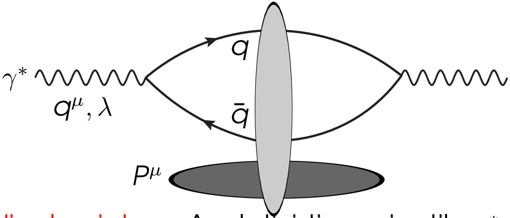
DIS in the dipole picture: A relativistic projectile γ^* fluctuates into a $q\bar{q}$ pair which then scatters on a very dense target

- Kinematics in LC coordinates $(+, -, \perp)$

$$q^\mu = (q^+, \frac{q^2}{2q^+}, \mathbf{q} = 0), \quad q^2 = -Q^2 > 0, \quad q^+ = \text{very large}$$

$$P^\mu = \frac{1}{\sqrt{2}}(m_p, m_p, \mathbf{0}), \quad \text{Bjorken} \quad x_{B_j} = \frac{Q^2}{2P \cdot q}$$

Dipole factorization for DIS



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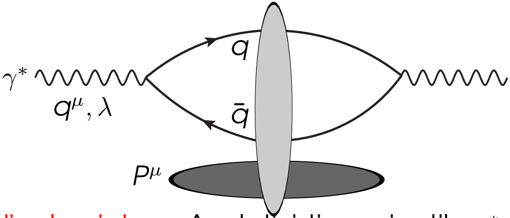

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- High energy limit is $x_{B_j} \to 0$ and Q^2 is "fixed"
  - DIS structure functions

$$F_2(x_{B_j}, Q^2) = \frac{Q^2}{(2\pi)^2 \alpha_{\rm em}} \left(\sigma_T^{\gamma^*} + \sigma_L^{\gamma^*} \right), \quad F_L(x_{B_j}, Q^2) = \frac{Q^2}{(2\pi)^2 \alpha_{\rm em}} \sigma_L^{\gamma^*}$$

At high energy: The total cross section for γ^* scattering from a classical gluon field (optical theorem + LC quantization)

$$\sigma_\lambda^{\gamma^*} = \frac{2}{2q^+(2\pi)\delta(q'^+ - q^+)} \Re e \left[{}_i\langle \gamma_\lambda^*(\vec{q}, Q^2) | 1 - \hat{S}_E | \gamma_\lambda^*(\vec{q}', Q^2) \rangle_i \right]$$

- Full perturbative Fock state decomposition for γ^*

$$|\gamma_\lambda^*(\vec{q}, Q^2)\rangle_i = \sqrt{Z_{\gamma^*}} \Big\{ |QED\rangle + \int PS_{(q\bar{q})} \psi^{\gamma^* \to q\bar{q}} |q\bar{q}\rangle + \int PS_{(q\bar{q}g)} \psi^{\gamma^* \to q\bar{q}g} |q\bar{q}g\rangle + \cdots \Big\}$$

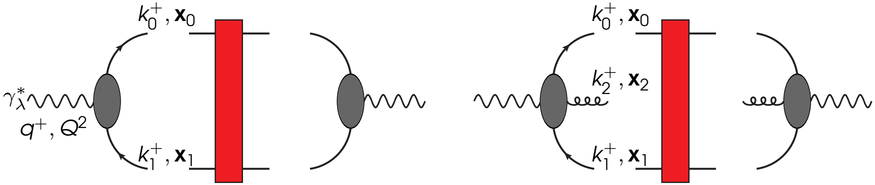
  - $\psi^{\gamma^* \to q\bar{q}}, \ldots$ are the light cone w-functions (LCWFs)
  - At LO of $\alpha_{\rm em}$ the WF renormalization $Z_{\gamma^*} = 1 + \mathcal{O}(\alpha_{\rm em})$ can be neglected

- $\hat{S}_E$ describes the eikonal scattering on the gluon field

$$\hat{S}_E |q(k_0^+, \mathbf{x}_0, \alpha) \bar{q}(k_1^+, \mathbf{x}_1, \beta)\rangle = U_F(\mathbf{x}_0)_{\alpha\bar{\alpha}} U_F^\dagger(\mathbf{x}_1)_{\beta\bar{\beta}} |q(\ldots, \bar{\alpha}) \bar{q}(\ldots, \bar{\beta})\rangle$$

DIS cross section at NLO

In mixed space factorization between $|\psi|^2$ & target $\mathcal{S}$



$$\sigma_\lambda^{\gamma^*} = 2N_c \int \widetilde{PS}_{q\bar{q}} |\widetilde{\psi}^{\gamma_\lambda^* \to q\bar{q}}|^2 \Re e[1 - \mathcal{S}_{01}] + 2N_c C_F \int \widetilde{PS}_{q\bar{q}g} |\widetilde{\psi}^{\gamma_\lambda^* \to q\bar{q}g}|^2 \Re e[1 - \mathcal{S}_{012}]$$

- Dipole target: $\mathcal{S}_{01} \equiv \frac{1}{N_c} \mathrm{Tr}\left[U_F(\mathbf{x}_0) U_F^\dagger(\mathbf{x}_1)\right]$
- Dipole/g target: $\mathcal{S}_{012} \equiv \frac{1}{N_c C_F} \mathrm{Tr}\left[t^b U_F(\mathbf{x}_0) t^a U_F^\dagger(\mathbf{x}_1)\right] U_A(\mathbf{x}_2)_{ba}$

One-loop QCD correction to the $\gamma^*_{T/L} \to q\bar{q}$

Calculation done using the Light cone perturbation theory (LC gauge $A^+_a = 0$)

- Outline of 1-loop LCPT computation

$$\psi^{\gamma^* \to q\bar{q}}_{1-loop} \sim e g_s^2 \int \frac{\mathrm{d}k^+}{k^+} \int \frac{\mathrm{d}^{d-2}\mathbf{k}}{(2\pi)^{d-2}} \frac{num(k^+, \mathbf{k}, q^+, Q^2, m_q, \dots)}{ED^-_{i,1} ED^-_{i,2} \cdots ED^-_{i,f}}$$

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- Cut-off $k^+ > \alpha q^+$ introduced to regulate the $k^+ \to 0$ div's
  - Associated with low-x_{B_j} LL (resummed with BK/JIMWLK)
  - Brakes rotational invariance $\to$ problems in $m_q \neq 0$ case

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- Cut-off $k^+ > \alpha q^+$ introduced to regulate the $k^+ \to 0$ div's
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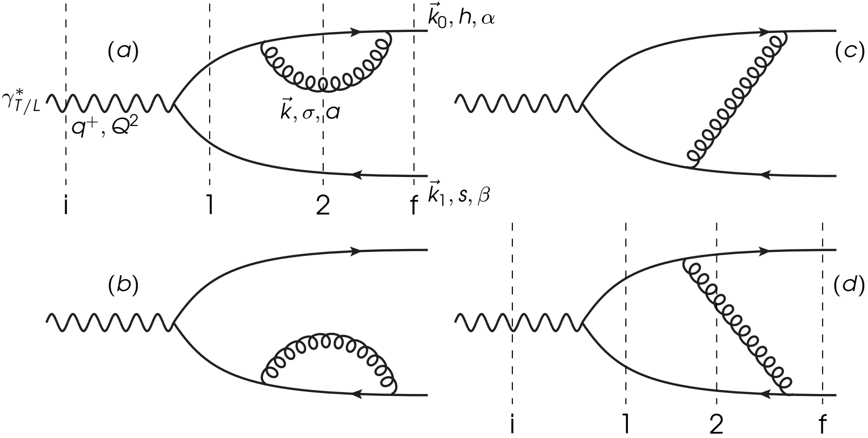
  - Brakes rotational invariance $\to$ problems in $m_q \neq 0$ case
- UV divergences from transverse $\mathbf{k}$ integrals: No UV renormalization at this order
  - UV divergences (and finite regularization artifacts) have to cancel at full cross section level
  - Transverse dimensional regularization in $d = 4 - 2\varepsilon$
    - $m_q = 0$ + CDR (G. Beuf PRD94 (2016) 054016)
    - $m_q = 0$ + FDH (Hänninen, Lappi, RP AoP 393 (2018) 358)

Part 1: One-loop correction to the $\gamma^*_{T/L} \to q\bar{q}$: $m_q = 0$ case

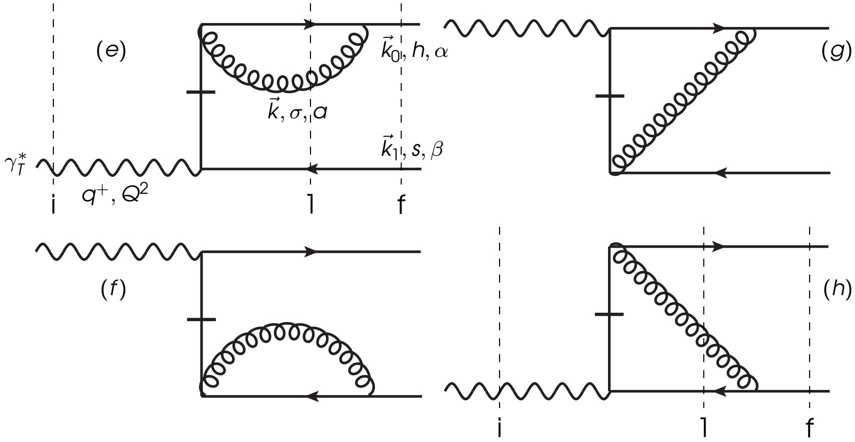
One-loop diagrams for the $\gamma^*_{T/L} \to q\bar{q}$

4 "normal" x^+-ordered one-loop diagram



- Note the notation: $\vec{k} = (k^+, \mathbf{k})$ conserved

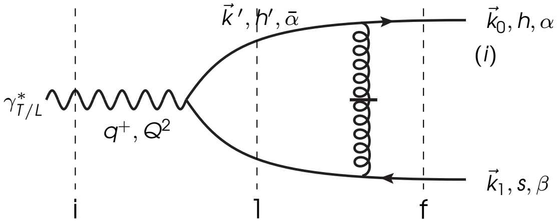
One-loop instantaneous diagrams for $\gamma_T^* \to q\bar{q}$



- All four diagrams vanishes in dimensional regularization

$$\psi_{1-loop}^{\gamma_T^* \to q\bar{q}}\bigg|_{(e),(f),(g),(h)} \sim \frac{1}{ED_{if}} \int \frac{\mathrm{d}^{d-2}\mathbf{k}}{(2\pi)^{d-2}} \frac{\mathbf{k}^i}{\mathbf{k}^2 + \Delta} = 0$$

One-loop instantaneous diagram for $\gamma^*_{T/L} \to q\bar{q}$



- γ_T^* case: vanishes in dim regularization

$$\psi_{1-loop}^{\gamma_T^* \to q\bar{q}}\Big|_{(i)} \sim \frac{1}{ED_{if}} \int \frac{\mathrm{d}^{d-2}\mathbf{k}'}{(2\pi)^{d-2}} \frac{\mathbf{k}'^i}{\mathbf{k}'^2 + \Delta} = 0$$

- γ_L^* case: non-zero & cancels the unphysical small $1/(k^+)^2$ divergence of the vertex correction diagrams

Results: NLO $\gamma^*_{T/L} \to q\bar{q}$ LCWFs in momentum space

$$\psi^{\gamma^*_{T/L}\to q\bar{q}} = \psi_{LO}^{\gamma^*_{T/L}\to q\bar{q}}\left[1 + \left(\frac{\alpha_s C_F}{2\pi}\right)\mathcal{K}^{T/L}\right] + \mathcal{O}(e\alpha_s^2)$$

where the NLO kernels

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where the NLO kernels

$$\mathcal{K}^L = 2\left[\log\left(\frac{\alpha}{\sqrt{z(1-z)}}\right) + \frac{3}{4}\right]\left\{\frac{1}{\varepsilon_{\overline{MS}}} + \log\left(\frac{\mu^2}{\overline{Q}^2}\right) - 2\log\left(\frac{\mathbf{P}^2 + \overline{Q}^2}{\overline{Q}^2}\right)\right\} + \frac{1}{2}\log^2\left(\frac{z}{1-z}\right) - \frac{\pi^2}{6} + 3 - \frac{1}{2} + \mathcal{O}(\varepsilon)$$

- Notations: $z = k_0^+/q^+$, $z \in [0,1]$, $\overline{Q}^2 = z(1-z)Q^2$ and $\mathbf{P} = \mathbf{k}_0 - z\mathbf{q} = \mathbf{k}_0$
- Regularization artifact: the factor 3 in CDR & 3 - 1/2 in FDH

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$$\mathcal{K}^T = \mathcal{K}^L + 2\left[\log\left(\frac{\alpha}{\sqrt{z(1-z)}}\right) + \frac{3}{4}\right]\left(\frac{\mathbf{P}^2 + \overline{Q}^2}{\mathbf{P}^2}\right)\log\left(\frac{\mathbf{P}^2 + \overline{Q}^2}{\overline{Q}^2}\right) + \mathcal{O}(\varepsilon)$$

Results: NLO $\gamma^*_{T/L} \to q\bar{q}$ LCWFs in mixed space

Fourier-transform (note: in CDR $d \neq 4$ and in FDH $d = 4$)

$$\psi^{\gamma^*_{T/L}\to q\bar{q}}(q^+, \mathbf{r}_{01}) = \int \frac{\mathrm{d}^{d-2}\mathbf{P}}{(2\pi)^{d-2}} e^{+i\mathbf{P}\cdot\mathbf{r}_{01}} \psi^{\gamma^*_{T/L}\to q\bar{q}}(q^+, \mathbf{P})$$

where $\mathbf{r}_{01} \equiv \mathbf{x}_0 - \mathbf{x}_1$ gives

$$\widetilde{\psi}^{\gamma^*_{T/L}\to q\bar{q}} = \widetilde{\psi}^{\gamma^*_{T/L}\to q\bar{q}}_{LO}\left[1 + \left(\frac{\alpha_s C_F}{2\pi}\right)\widetilde{\mathcal{K}}^{T/L}\right] + \mathcal{O}(e\alpha_s^2)$$

$$\widetilde{\mathcal{K}}^L = \widetilde{\mathcal{K}}^T = 2\left[\log\left(\frac{\alpha}{\sqrt{z(1-z)}}\right) + \frac{3}{4}\right]\left\{\frac{1}{\varepsilon_{\overline{MS}}} + \log\left(\frac{\mathbf{r}_{01}^2\mu^2}{4}\right) - 2\Psi_0(1)\right\} + \frac{1}{2}\log^2\left(\frac{z}{1-z}\right) - \frac{\pi^2}{6} + 3 - \frac{1}{2} + \mathcal{O}(\varepsilon), \quad \Psi_0(1) \equiv -\gamma_E$$

- In mixed space $\widetilde{\mathcal{K}}^L$ and $\widetilde{\mathcal{K}}^T$ are the same
- Remaining UV and small k^+ div's are cancelled at the cross section level (against $q\bar{q}g$ term)

Results: full $q\bar{q}$ contribution to $\sigma_L^{\gamma^*}$ at NLO

For γ_L^* (in FDH)

- Fourier-transformed LO w-function in mixed space

$$\widetilde{\psi}_{LO}^{\gamma_L^*\to q\bar{q}} = \frac{4q^+ e e_f Q \delta_{\alpha\beta}}{(2\pi)} [z(1-z)]^{3/2} K_0\left(|\mathbf{r}_{01}|\overline{Q}\right)$$

- $q\bar{q}$ contribution to σ^{γ^*} at NLO

$$\sigma_L^{\gamma^*}\Big|_{q\bar{q}} = 4N_c \frac{4\alpha_{em} e_f^2 Q^2}{(2\pi)^2} \int_{\mathbf{x}_0,\mathbf{x}_1} \int_0^1 \mathrm{d}z [z(1-z)]^2 \left[1 + \left(\frac{\alpha_s C_F}{\pi}\right) \widetilde{\mathcal{K}}^L\right] \times \left[K_0\left(|\mathbf{r}_{01}|\overline{Q}\right)\right]^2 \Re e[1-\mathcal{S}_{01}] + \mathcal{O}(\alpha_{em}\alpha_s^2)$$

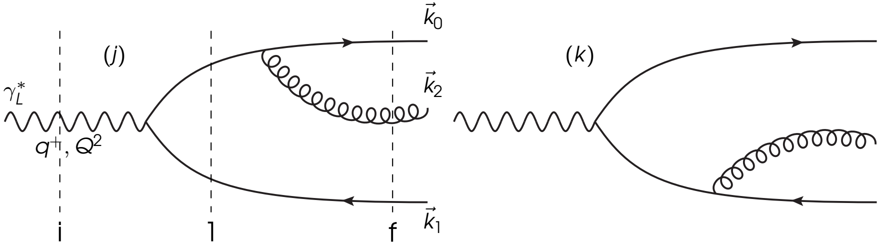
Similar result for γ_T^*, but longer expressions

Tree-level $\gamma^*_{T/L} \to q\bar{q}g$ LCWFs in momentum space

For the full NLO $\sigma^{\gamma^*}_\lambda$ cross section : we need to compute the $\sigma^{\gamma^*}_\lambda\Big|_{q\bar{q}g}$ contribution

- 2 diagrams contribute to $\gamma^*_L \to q\bar{q}g$ & 4 to $\gamma^*_T \to q\bar{q}g$

Result for γ^*_L case (γ^*_T goes in the same way)



Note: $k_i^+ = z_i q^+$ for $i = 1, 2, 3$ and $z_1 + z_2 + z_3 = 1$

$$
\begin{aligned}
\sigma_L^{\gamma^*}\Big|_{q\bar{q}g} = 4N_c \frac{4\alpha_{em} e_f^2 Q^2}{(2\pi)^3}\left(\frac{\alpha_s C_F}{\pi}\right)\int_0^\infty \mathrm{d}z_0\,\mathrm{d}z_1\,\mathrm{d}z_2 \delta(z_0+z_1+z_2-1)\frac{z_0 z_1}{z_2} \\
\times \frac{8^2\pi^4}{2}\int_{\mathbf{x}_0,\mathbf{x}_1,[\mathbf{x}_2]}\Bigg\{\left(\frac{z_1}{z_0}\right)\left[z_0(z_0+z_2)+\frac{z_2^2}{2}\right]|\mathcal{I}_{(j)}(\mathbf{r}_{102},\mathbf{r}_{20},\overline{Q}^2_{(j)},\omega_{(j)})|^2 \\
+(z_0\leftrightarrow z_1)|\mathcal{I}_{(k)}(\mathbf{r}_{012},\mathbf{r}_{21},\overline{Q}^2_{(k)},\omega_{(k)})|^2 \\
-\Big[z_1(z_0+z_2)+z_0(z_1+z_2)\Big]\Re e(\mathcal{I}_j\mathcal{I}_k^*)\Bigg\}\Re e[1-\mathcal{S}_{012}]
\end{aligned}
$$

where

$$
\mathcal{I}^m(\mathbf{b},\mathbf{r},\overline{Q}^2,\omega) = \mu^{2-d/2}\int\frac{\mathrm{d}^2\mathbf{P}}{(2\pi)^2}\int\frac{\mathrm{d}^{d-2}\mathbf{K}}{(2\pi)^{d-2}}\frac{\mathbf{K}^m e^{i\mathbf{P}\cdot\mathbf{b}}e^{i\mathbf{K}\cdot\mathbf{r}}}{\left[\mathbf{P}^2+\overline{Q}^2\right]\left[\mathbf{K}^2+\omega\left(\mathbf{P}^2+\overline{Q}^2\right)\right]}
$$

with the notation

$$
\mathbf{r}_{102}\equiv\mathbf{r}_{10}-\frac{z_2}{z_0+z_2}\mathbf{r}_{20},\quad \mathbf{r}_{012}\equiv\mathbf{r}_{01}-\frac{z_2}{z_1+z_2}\mathbf{r}_{21}
$$

$$
\omega_{(j)}\equiv\frac{z_0 z_2}{z_1(z_0+z_2)^2},\quad \overline{Q}^2_{(j)}\equiv z_1(1-z_1)Q^2,\quad \overline{Q}^2_{(k)},\omega_{(k)}\quad \text{with}\quad (z_0\leftrightarrow z_1)
$$

Fourier-transform integral simplifies to

$$\mathcal{I}^m(\mathbf{b},\mathbf{r},\overline{Q}^2,\omega) = i\frac{\mu^{2-d/2}}{8\pi^{d/2}}\mathbf{r}^m(\mathbf{r}^2)^{1-d/2}\int_0^\infty \frac{\mathrm{d}u}{u}e^{-u\overline{Q}^2}e^{-\frac{\mathbf{b}^2}{4u}}\Gamma\left(\frac{d}{2}-1,\frac{\omega\mathbf{r}^2}{4u}\right)$$

- UV behaviour for $\mathbf{r}\to 0$ (IR $|\mathbf{b}|\to\infty$ safe)

$$\mathcal{I}^m(\mathbf{b},\mathbf{r},\overline{Q}^2,\omega)_{UV} = i\frac{\mu^{2-d/2}}{4\pi^{d/2}}\mathbf{r}^m(\mathbf{r}^2)^{1-d/2}K_0\left(|\mathbf{b}|\overline{Q}\right)\Gamma(d/2-1)e^{-\frac{\mathbf{r}^2}{2\mathbf{b}^2e^{\gamma_E}}}$$

UV Subtraction procedure: UV div's

- At $\mathbf{x}_2\to\mathbf{x}_{0/1}$ for $|\mathcal{I}_{(j/k)}|^2$ contribution

$$\left\{|\mathcal{I}_{(j/k)}|^2\Re e[1-\mathcal{S}_{012}]-|\mathcal{I}_{(j/k),UV}|^2\Re e[1-\mathcal{S}_{01}]\right\}+|\mathcal{I}_{(j/k),UV}|^2\Re e[1-\mathcal{S}_{01}]$$

- Term in $\{...\}$ leads to a UV & IR finite integral in $\mathbf{x}_2$: $d=4$ limit can be taken
- Second term gives the UV divivergence

Full result for the $\sigma_L^{\gamma^*}\Big|_{q\bar{q}g}$ term in FDH

Full result $\sigma_L^{\gamma^*}\Big|_{q\bar{q}g} = \sigma_L^{\gamma^*}\Big|_{q\bar{q}g,fin} + \sigma_L^{\gamma^*}\Big|_{q\bar{q}g,UV}$

$$\sigma_L^{\gamma^*}\Big|_{q\bar{q}g,fin} = 4N_c \frac{4\alpha_{em} e_f^2 Q^2}{(2\pi)^3} \left(\frac{\alpha_s C_F}{\pi}\right) \int_{\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \int_0^1 \mathrm{d}z_0 \int_0^{1-z_0} \frac{\mathrm{d}z_2}{z_2}$$
$$\times \Bigg\{ F_{(j)} \left([K_0(|\mathbf{R}|Q)]^2 \Re e[1 - \mathcal{S}_{012}] - [K_0(|\mathbf{r}_{01}|\overline{Q}_{(j)})]^2 e^{-\frac{\mathbf{r}_{20}^2}{\mathbf{r}_{01}^2 e^{\gamma_E}}} \Re e[1 - \mathcal{S}_{01}] \right)$$
$$+ (z_0 \leftrightarrow z_1, \mathbf{x}_0 \leftrightarrow \mathbf{x}_1) - G_{(j,k)} \frac{(\mathbf{r}_{20} \cdot \mathbf{r}_{21})}{(\mathbf{r}_{20}^2)(\mathbf{r}_{21}^2)} K_0(|\mathbf{R}|Q)]^2 \Re e[1 - \mathcal{S}_{012}] \Bigg\}$$

where

$$F_{(j)} \equiv z_1^2 \Big[2z_0(z_0 + z_2) + z_2^2 \Big], \quad G_{(j,k)} \equiv 2\Big[z_0 z_1^2 (z_0 + z_2) + z_1 z_0^2 (z_1 + z_2) \Big]$$
$$\mathbf{R}^2 \equiv z_0 z_1 \mathbf{r}_{01}^2 + z_0 z_2 \mathbf{r}_{20}^2 + z_1 z_2 \mathbf{r}_{21}^2$$

Full result for the $\sigma_L^{\gamma^*}\Big|_{q\bar{q}g}$ term in FDH

UV divergent contribution (Note: extra -1/2 term in CDR)

$$\sigma_L^{\gamma^*}\Big|_{q\bar{q}g,UV} = -4N_c\frac{4\alpha_{em}e_f^2Q^2}{(2\pi)^2}\left(\frac{\alpha_s C_F}{\pi}\right)\int_{\mathbf{x}_0,\mathbf{x}_1}\int_0^1 \mathrm{d}z[z(1-z)]^2$$
$$\times 2\left[\log\left(\frac{\alpha}{\sqrt{z(1-z)}}\right)+\frac{3}{4}\right]\left\{\frac{1}{\varepsilon_{\overline{MS}}}+\log\left(\frac{\mathbf{r}_{01}^2\mu^2}{4}\right)-2\Psi_0(1)\right\}\Re e[1-\mathcal{S}_{01}]$$

- This term cancels the remaining UV & small k^+ div's in $\sigma_L^{\gamma^*}\Big|_{q\bar{q}}$

Full NLO result for $\sigma_L^{\gamma^*}$

Full NLO result for γ_L^*:

$$\sigma_L^{\gamma^*} = \sigma_L^{\gamma^*}\Big|_{q\bar{q},fin} + \sigma_L^{\gamma^*}\Big|_{q\bar{q}g,fin}$$

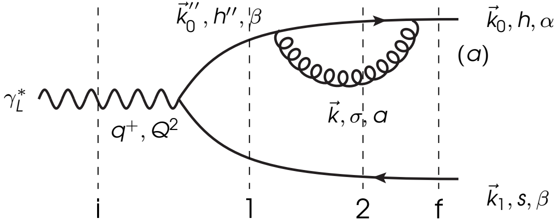
where the UV subtracted dipole contribution

$$\sigma_L^{\gamma^*}\Big|_{q\bar{q},fin} = 4N_c \frac{4\alpha_{em} e_f^2 Q^2}{(2\pi)^2} \int_{\mathbf{x}_0,\mathbf{x}_1} \int_0^1 \mathrm{d}z [z(1-z)]^2 [K_0\left(|\mathbf{r}_{01}|\overline{Q}\right)]^2$$
$$\times \left[1 + \left(\frac{\alpha_s C_F}{\pi}\right)\left\{\frac{1}{2}\log^2\left(\frac{z}{1-z}\right) - \frac{\pi^2}{6} + \frac{5}{2}\right\}\right] \Re e[1 - \mathcal{S}_{01}] + \mathcal{O}(\alpha_{em}\alpha_s^2)$$

- Similar result for γ_T^* (but longer expressions):
  (G. Beuf PRD 96 (2017) 074033)
  (Hänninen, Lappi, RP AoP 393 (2018) 358)
- Small-x BK/JIMWLK resummation/numerical implementation
  (E. Iancu et al. PLB 744 (2015) 293-302)
  (G. Beuf PRD 89 (2014) 074039)
  (B. Ducloué et al. PRD 96 (2017) no.9, 094017)

Part 2: One-loop correction to the $\gamma_L^* \to q\bar{q}$: $m_q \neq 0$ case

One-loop quark self-energy contribution:



$$\psi_{(a),UR}^{\gamma_L^*\to q\bar{q}} = \frac{1}{8(2\pi)} \int_0^{k_0^+} \frac{\mathrm{d}k^+}{k^+(k_0^+ - k^+)k_0^+} \int \frac{\mathrm{d}^{d-2}\mathbf{k}}{(2\pi)^{d-2}} \frac{num_{(a)}(\mathbf{k}^2, m_q^2, \dots)}{ED_{i1}ED_{i2}ED_{if}}$$

- On-shell mass renormalization scheme
  - Require the cancellation of all contribution which are singular in the on-shell limit $ED_{i1} \to 0$ i.e. $q^- = k_0''^- + k_1^-$

$$\frac{1}{ED_{i1}ED_{i2}} = \frac{1}{ED_{i1}} \left\{ \frac{1}{ED_{i2}}\Big|_{q^-=k_0''^- + k_1^-} + \frac{\partial}{\partial q^-} \frac{1}{ED_{i2}}\Big|_{q^-=k_0''^- + k_1^-} + \cdots \right\}$$

  - First term correspond to the mass correction and is subtracted by a mass counterterm δm^2

The mass renormalized quark self-energy contribution:

$$\psi^{\gamma_L^* \to q\bar{q}}_{(a),R} = \psi^{\gamma_L^* \to q\bar{q}}_{LO} \left(\frac{-g^2 C_F}{8\pi^2} \right) \left\{ \int_{\alpha/z}^{1} \mathrm{d}\xi [1 + (1-\xi)^2] A_0(\Delta_1) + \frac{2(1-z)m_q^2}{[\mathbf{P}^2 + \overline{Q}^2 + m_q^2]} \int_0^1 \mathrm{d}\xi A_0(\Delta_1) - \delta m^2_{(a)} \right\} \quad (1)$$

where $\xi = k^+/k_0^+$ & $\mathbf{P} = \mathbf{k}_0$ ($\mathbf{q} = 0$)

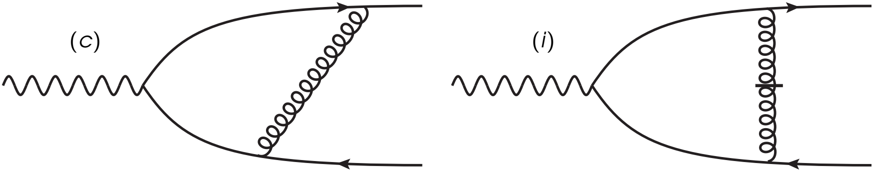
$$A_0(\Delta_1) = \frac{1}{\varepsilon_{\overline{MS}}} + \log\left(\frac{\mu^2}{\Delta_1}\right) + \mathcal{O}(\varepsilon), \quad \Delta_1 = \frac{\xi(1-\xi)}{z}[\mathbf{P}^2 + \overline{Q}^2 + m_q^2] + \xi^2 m_q^2$$

and

$$\delta m^2_{(a)} = \frac{2(1-z)m_q^2}{[\mathbf{P}^2 + \overline{Q}^2 + m_q^2]} \int_0^1 \mathrm{d}\xi A_0(\xi^2 m_q^2) \quad (2)$$

- Eq. (2) cancels the UV part from the mass term in (1)
- Only after the m-ren (1) can be F-transformed into mixed space
  - Finite part of $\delta m^2_{(a)}$ dramatically simplifies the F-transformation

One-loop vertex correction for γ_L^*: sum of diagrams (c) & (i)



$$\text{Diagram (c)} = \psi_{LO}^{\gamma_L^* \to q\bar{q}} \left(\frac{g^2 C_F}{8\pi^2}\right) \mathcal{V}^L + \psi_{(c)}^{\gamma_L^* \to q\bar{q}}\Big|_{hf}$$

where

- $\mathcal{V}^L$ quite tricky to compute, but can be done
- For γ_L^* helicity flip (hf) contribution is UV/IR finite

$$\psi_{(c)}^{\gamma_L^* \to q\bar{q}}\Big|_{hf} \sim m_q \left(\frac{g^2 C_F}{8\pi^2}\right) \left[\mathbf{k}_0^i \bar{u}_h(k_0)\gamma^+\gamma^i v_s(k_1)\right] \times \#\Big|_{UV/IR-finite}$$

At NLO $\psi_{(c)}^{\gamma_L^* \to q\bar{q}}\Big|_{hf}$ term vanishes on the cross section level, but will contribute at NNLO

Full NLO result for the $\sigma_L^{\gamma^*}$ with $m_q \neq 0$

Same steps as in the $m_q = 0$ case:

- F-transforming LC w-functions into mixed space
- UV subtraction in $q\bar{q}g$ term/adding the $q\bar{q}$ and $q\bar{q}g$ terms together
  - Cancellation of UV div's & regularization scheme dependence

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...but now with a smaller font size 😄

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$$\sigma_L^{\gamma^*}\Big|_{q\bar{q},fin} = 4N_c \frac{4\alpha_{em} e_f^2 Q^2}{(2\pi)^2} \int_{\mathbf{x}_0,\mathbf{x}_1} \int_0^1 \mathrm{d}z [z(1-z)]^2 \Bigg\{ \left[K_0\left(|\mathbf{r}_{01}|\sqrt{M}\right)\right]^2$$
$$\times \left[1 + \left(\frac{\alpha_s C_F}{\pi}\right)\left\{\frac{1}{2}\log^2\left(\frac{z}{1-z}\right) - \frac{\pi^2}{6} + \frac{5}{2}\right\}\right]$$
$$+ K_0\left(|\mathbf{r}_{01}|\sqrt{M}\right) \mathcal{K}^L\Big|_{m_q} \Bigg\} \Re e[1 - \mathcal{S}_{01}] + \mathcal{O}(\alpha_{em}\alpha_s^2),$$

where

$$\mathcal{K}^L\Big|_{m_q} = K_0\left(|\mathbf{r}_{01}| M\right)\mathcal{R}^L + \mathcal{J}(z) + \mathcal{J}(z \to 1-z), \quad M \equiv \overline{Q}^2 + m_q^2$$

...and the coefficients

$$\mathcal{R}^L \equiv \Bigg\{ \frac{1}{2z}\left[\log(1-z) + \gamma\log\left(1 - \frac{1}{\xi_{(-)}}\right)\right] - Li_2\left(-\frac{z}{1-z}\right) + Li_2\left(\frac{1}{\xi_{(+)}}\right) + Li_2\left(\frac{1}{\xi_{(-)}}\right) + (z \to 1-z) \Bigg\} + \frac{(1-\gamma)}{4z(1-z)}\log\left(\frac{\overline{Q}^2 + m_q^2}{m_q^2}\right) + \frac{m_q^2}{2\overline{Q}^2}\log\left(\frac{\overline{Q}^2 + m_q^2}{m_q^2}\right) - \frac{\pi^2}{3}$$

where $\xi_{(\pm)} \equiv 1 - \frac{1}{2z} \pm \frac{\gamma}{2z}$ & $\gamma \equiv \sqrt{1 + \frac{4m_q^2}{Q^2}}$

$$\mathcal{J}(z) \equiv \int_0^1 \frac{\mathrm{d}\xi}{\xi}\left[\frac{-2\log(\xi)}{(1-\xi)} + \frac{(1+\xi)}{2}\right]\left\{K_0\left(|\mathbf{r}_{01}|\sqrt{M}\right) - K_0\left(|\mathbf{r}_{01}|\sqrt{M + \frac{\xi(1-z)m_q^2}{(1-\xi)}}\right)\right\} + m_q^2\int_0^1\int_0^1 \mathrm{d}x\,\mathrm{d}\xi \left\{K_0\left(|\mathbf{r}_{01}|\sqrt{M}\right) - K_0\left(|\mathbf{r}_{01}|\sqrt{\frac{M}{(1-x)} + \kappa}\right)\right\}\frac{(1-\xi)^{-1}(1-x)^{-1}C^L_{m_q}}{\left[x(1-\xi) + \frac{\xi}{(1-z)}\right]\left[\frac{xM}{(1-x)} + \kappa\right]}$$

...and the coefficients

$$\mathcal{R}^{L} \equiv \left\{ \frac{1}{2z}\left[\log(1-z) + \gamma \log\left(1 - \frac{1}{\xi_{(-)}}\right)\right] - Li_2\left(-\frac{z}{1-z}\right) + Li_2\left(\frac{1}{\xi_{(+)}}\right) + Li_2\left(\frac{1}{\xi_{(-)}}\right) + (z \to 1-z) \right\} + \frac{(1-\gamma)}{4z(1-z)} \log\left(\frac{\overline{Q}^2 + m_q^2}{m_q^2}\right) + \frac{m_q^2}{2\overline{Q}^2} \log\left(\frac{\overline{Q}^2 + m_q^2}{m_q^2}\right) - \frac{\pi^2}{3}$$

where $\xi_{(\pm)} \equiv 1 - \frac{1}{2z} \pm \frac{\gamma}{2z}$ & $\gamma \equiv \sqrt{1 + \frac{4m_q^2}{Q^2}}$

$$\mathcal{J}(z) \equiv \int_0^1 \frac{\mathrm{d}\xi}{\xi}\left[\frac{-2\log(\xi)}{(1-\xi)} + \frac{(1+\xi)}{2}\right]\left\{K_0\left(|\mathbf{r}_{01}|\sqrt{M}\right) - K_0\left(|\mathbf{r}_{01}|\sqrt{M + \frac{\xi(1-z)m_q^2}{(1-\xi)}}\right)\right\} + m_q^2 \int_0^1 \int_0^1 \mathrm{d}x\, \mathrm{d}\xi \left\{K_0\left(|\mathbf{r}_{01}|\sqrt{M}\right) - K_0\left(|\mathbf{r}_{01}|\sqrt{\frac{M}{(1-x)} + \kappa}\right)\right\} \frac{(1-\xi)^{-1}(1-x)^{-1} C^L_{m_q}}{\left[x(1-\xi) + \frac{\xi}{(1-z)}\right]\left[\frac{xM}{(1-x)} + \kappa\right]}$$

with more coefficients...

$$C^{\mathrm{L}}_{m_q} = \frac{z^2(1-\xi)}{(1-z)}\left\{-\xi^2 + x(1-\xi)\frac{\left[1 + (1-\xi)\left(1 + \frac{z\xi}{(1-z)}\right)\right]}{\left[x(1-\xi) + \frac{\xi}{(1-z)}\right]}\right\}$$

$$\kappa = \frac{\xi m_q^2}{(1-\xi)(1-x)\left[x(1-\xi) + \frac{\xi}{(1-z)}\right]}\left[\xi(1-x) + x\left(1 - \frac{z(1-\xi)}{(1-z)}\right)\right]$$

$$\sigma_L^{\gamma^*}\Big|_{q\bar{q}g,fin} = 4N_c \frac{4\alpha_{em}e_f^2Q^2}{(2\pi)^3}\left(\frac{\alpha_s C_F}{\pi}\right)\int_{\mathbf{x}_0\mathbf{x}_1\mathbf{x}_2}\int_0^1 \mathrm{d}z_0 \int_0^{1-z_0}\frac{\mathrm{d}z_2}{z_2}\Bigg\{$$
$$+ F_{(j)}\left[\frac{\mathbf{r}_{20}^2}{64}H_{(4)}^2(\{''(j)''\})\Re e[1-\mathcal{S}_{012}] - \left[K_0(|\mathbf{r}_{01}|\sqrt{\overline{Q}_{(j)}^2+m_q^2})\right]^2 \frac{e^{-\frac{\mathbf{r}_{20}^2}{(\mathbf{r}_{01}^2 e^{\gamma_E})}}}{\mathbf{r}_{20}^2}\Re e[1-\mathcal{S}_{01}]\right]$$
$$+ (z_0 \leftrightarrow z_1, \mathbf{x}_0 \leftrightarrow \mathbf{x}_1)$$
$$+ \frac{m_q^2}{16}\left[(z_1)^2\frac{(z_2)^4}{(z_0+z_2)^2}\overline{H}_{(4)}^2(\{''(j)''\}) + (z_0 \leftrightarrow z_1, \mathbf{x}_0 \leftrightarrow \mathbf{x}_1)\right]\Re e[1-\mathcal{S}_{012}]$$
$$- \frac{1}{64}\left[2\left[(z_0+z_2)z_0z_1^2 + (z_2+z_1)z_1z_0^2\right](\mathbf{r}_{20}\cdot\mathbf{r}_{21})H_{(4)}(\{''(j)''\})H_{(4)}(\{''(k)''\})\right.$$
$$\left. + 8m_q^2\frac{z_0z_1(z_2)^4}{(z_0+z_2)(z_2+z_1)}\overline{H}_{(4)}(\{''(j)''\})\overline{H}_{(4)}(\{''(k)''\})\right]\Re e[1-\mathcal{S}_{012}]\Bigg\}$$

where

$$H_{(4)}(\{''(j)''\}) = \int_0^\infty \frac{\mathrm{d}u}{u} e^{-u(\overline{Q}^2+m_q^2)} e^{-\mathbf{r}_{102}^2/(4u)} \int_0^{u/\omega_{(j)}} \frac{\mathrm{d}t}{t^2} e^{-t\omega_{(j)}\overline{m}_{q,(j)}^2} e^{-\mathbf{r}_{20}^2/(4t)}$$
$$\overline{H}_{(4)}(\{''(j)''\}) = \int_0^\infty \frac{\mathrm{d}u}{u} e^{-u(\overline{Q}^2+m_q^2)} e^{-\mathbf{r}_{102}^2/(4u)} \int_0^{u/\omega_{(j)}} \frac{\mathrm{d}t}{t} e^{-t\omega_{(j)}\overline{m}_{q,(j)}^2} e^{-\mathbf{r}_{20}^2/(4t)}, \quad \overline{m}_{q,(j)}^2 \equiv \frac{z_2z_1}{z_0}m_q^2$$

- The $m_q \to 0$ limit fully recovers the massless result
- Remaining UV/IR finite integrals computed numerically

...γ_L^* looks good, but "surprising" problem in γ_T^* case:

$$\text{Vertex correction diagram (c)} = \left(\frac{g^2 C_F}{8\pi^2}\right)\mathcal{V}^T + \psi_{(c)}^{\gamma_T^*\to q\bar{q}}\Big|_{hf}$$

$$\psi_{(c)}^{\gamma_T^*\to q\bar{q}}\Big|_{hf} \sim m_q\left(\frac{g^2 C_F}{8\pi^2}\right)\left[\left(\frac{3}{\varepsilon_{\overline{MS}}}+\ldots\right)\bar{u}_h(k_0)\gamma^+\gamma^m v_s(k_1)\varepsilon_\lambda^m + \cdots\right]$$

- Additional UV div appears in the vertex helicity flip term

The "vertex mass" correction $\qquad \delta m_v = \left(\frac{g^2 C_F}{4\pi^2}\right)\frac{3}{\varepsilon_{\overline{MS}}} + \text{finite}$

...but

The propagator mass correction $\qquad \delta m = \left(\frac{g^2 C_F}{4\pi^2}\right)\frac{2}{\varepsilon_{\overline{MS}}} + \text{finite}$

- REASON: our cut-off for k^+ brakes rotational invariance
- SOLUTION: additional renormalization condition for δm_v to restore rotational invariance ?
  - Similar problem in (M. Burkardt, A. Langnau PRD44 (1991) 3857): decay of heavy scalar particles

Conclusion

This talk:

- One-loop computation of $\gamma^*_{T/L} \to q\bar{q}$ LC w-functions, both in momentum and in mixed space
- Full NLO correction to DIS F_L and F_T from combination of $q\bar{q}$ and $q\bar{q}g$ contributions
- Extension to the case of massive quarks
  - Full NLO correction for F_L (**done**)
  - F_T case: problems in mass renormalization - vertex mass renormalization ? (**work in progress**)

Phenomenology/Theory outlook:

- Fits to HERA data at NLO+LL/NLL accuracy (massless and massive case)
- Application of the NLO $\gamma^*_{T/L} \to q\bar{q}$ LCWFs to calculate other DIS observables at NLO
- Understand mass renormalization $\Longleftrightarrow$ rot. invariance