

# The resonance of ${}^7\text{H}$ with $t+4n$ cluster model

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# ${}^7\text{H}$ ground state as a $t+4n$ resonance

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## Abstract

We investigated possible existence of  ${}^7\text{H}$  resonant state  $J^\pi=1/2^+$ , representing it as a five-body system consisting of  ${}^3\text{H}$  core and four valence neutrons. To this aim an effective  $n$ - ${}^3\text{H}$  potential has been constructed in order to reproduce low energy elastic neutron scattering on  ${}^3\text{H}$  phase shifts and the  ${}^5\text{H}$  resonant ground state in terms of  ${}^3\text{H}$ - $n$ - $n$  system. The variational Gaussian Expansion Method was used to solve the 5-body Schrödinger equation, while the resonant state parameters were estimated by means of the stabilization method. We have not found any signature of a narrow low energy resonance in the vicinity of  ${}^3\text{H}+4n$  threshold. However we have identified a very broad structure at  $E_R \approx 9$  MeV above this threshold, which corresponds to the  ${}^7\text{H}$   $J^\pi=1/2^+$  ground state. In the vicinity of this state, we have also identified a broad structure corresponding to ground state of  ${}^6\text{H}$  isotope with  $J^\pi = 2^-$ .

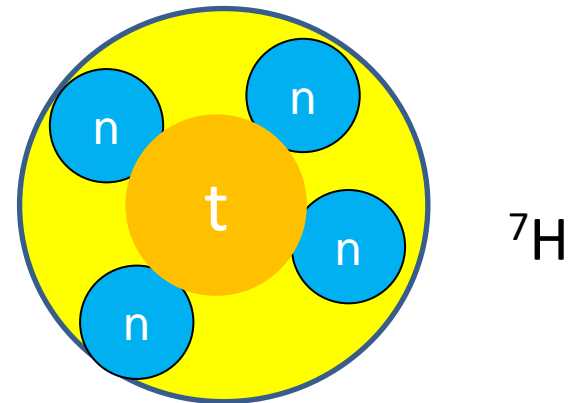
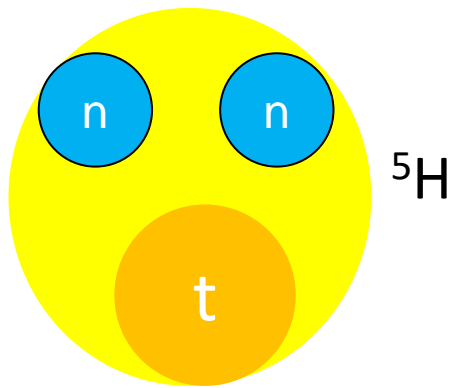
*Keywords:*  ${}^4\text{H}$ ,  ${}^5\text{H}$ ,  ${}^6\text{H}$  and  ${}^7\text{H}$ , Gaussian Expansion Method, Stabilization method, Few-Nucleon problem, *ab initio* calculations

Submitted in PLB

# Outline

- Introduction

- ${}^5\text{H}$  and  ${}^7\text{H}$



# Motivation why I study ${}^7\text{H}$

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Selected for a Viewpoint in Physics  
PHYSICAL REVIEW LETTERS

week ending  
5 FEBRUARY 2016



## Candidate Resonant Tetraneutron State Populated by the ${}^4\text{He}({}^8\text{He}, {}^8\text{Be})$ Reaction

K. Kisamori,<sup>1,2</sup> S. Shimoura,<sup>1</sup> H. Miya,<sup>1,2</sup> S. Michimasa,<sup>1</sup> S. Ota,<sup>1</sup> M. Assie,<sup>3</sup> H. Baba,<sup>2</sup> T. Baba,<sup>4</sup> D. Beaumel,<sup>2,3</sup> M. Dozono,<sup>2</sup> T. Fujii,<sup>1,2</sup> N. Fukuda,<sup>2</sup> S. Go,<sup>1,2</sup> F. Hammache,<sup>3</sup> E. Ideguchi,<sup>5</sup> N. Inabe,<sup>2</sup> M. Itoh,<sup>6</sup> D. Kameda,<sup>2</sup> S. Kawase,<sup>1</sup> T. Kawabata,<sup>4</sup> M. Kobayashi,<sup>1</sup> Y. Kondo,<sup>7,2</sup> T. Kubo,<sup>2</sup> Y. Kubota,<sup>1,2</sup> M. Kurata-Nishimura,<sup>2</sup> C. S. Lee,<sup>1,2</sup> Y. Maeda,<sup>8</sup> H. Matsubara,<sup>1,2</sup> K. Miki,<sup>5</sup> T. Nishi,<sup>9,2</sup> S. Noji,<sup>10</sup> S. Sakaguchi,<sup>11,2</sup> H. Sakai,<sup>2</sup> Y. Sasamoto,<sup>1</sup> M. Sasano,<sup>2</sup> H. Sato,<sup>2</sup> Y. Shimizu,<sup>2</sup> A. Stolz,<sup>10</sup> H. Suzuki,<sup>2</sup> M. Takaki,<sup>1</sup> H. Takeda,<sup>2</sup> S. Takeuchi,<sup>2</sup> A. Tamii,<sup>5</sup> L. Tang,<sup>1</sup> H. Tokieda,<sup>1</sup> M. Tsumura,<sup>4</sup> T. Uesaka,<sup>2</sup> K. Yako,<sup>1</sup> Y. Yanagisawa,<sup>2</sup> R. Yokoyama,<sup>1</sup> and K. Yoshida<sup>2</sup>

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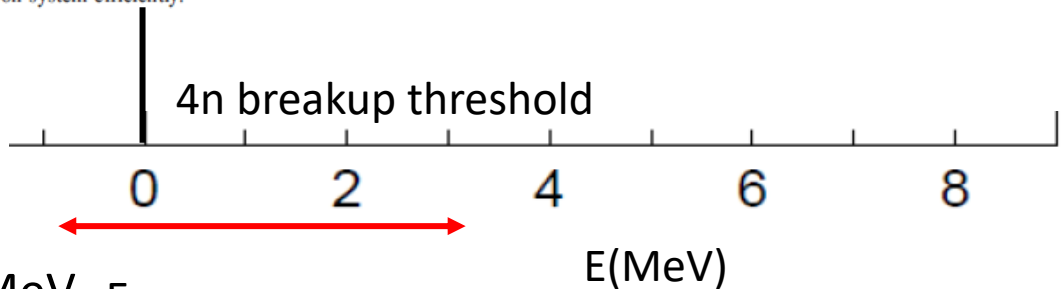
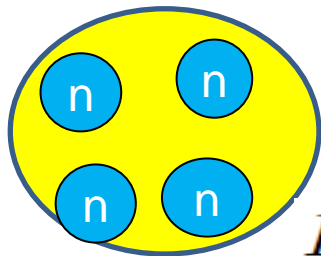
<sup>11</sup>Department of Physics, Kyushu University, 6-10-1 Hakozaeki, Higashi, Fukuoka 812-8581, Japan

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A candidate resonant tetraneutron state is found in the missing-mass spectrum obtained in the double-charge-exchange reaction  ${}^4\text{He}({}^8\text{He}, {}^8\text{Be})$  at 186 MeV/u. The energy of the state is  $0.83 \pm 0.65(\text{stat}) \pm 1.25(\text{syst})$  MeV above the threshold of four-neutron decay with a significance level of  $4.9\sigma$ . Utilizing the large positive  $Q$  value of the  $({}^8\text{He}, {}^8\text{Be})$  reaction, an almost recoilless condition of the four-neutron system was achieved so as to obtain a weakly interacting four-neutron system efficiently.

DOI: 10.1103/PhysRevLett.116.052501



$E_R = 0.83 \pm 0.65 \pm 1.25$  MeV Exp.  $\sim 3$  MeV

Observation of  $4n$  state by RIBF in 2016

If this observation is reliable,  
We observe 'no isotope nucleus'.

$\Gamma = 2.6$  MeV (Upper limit)

# Summary of the 4n calculation, currently

Authors	Method	$V_{NN}$	resonance
A.M. Shirokov et al.	Non-core shell model + phase shift analysis	JISP16	$E_r=0.8$ MeV $\Gamma=1.4$ MeV
S. Gandolfi et al.	Quantum Monte Carlo extrapolation	chiral(NNLO)	$E_r \sim 2.1$ MeV
K. Fossez et al.,	no-core Gamow shell model	N3LO, JISP16,	$E_r \sim 7$ MeV $\Gamma \sim 3.5$ MeV
E. Hiyama, R. Lazauskas et al.,	Gaussian Expansion + CSM Faddeev Yakubovsky	AV8	No resonance
Deltuva,	Faddeev Yakubovsky + AGS	SRG(AV18),NLO,	No resonance
M. D. Higgins et al.,	Hyperspherical harmonics phase shift analysis	AV8, AV18,	no resonance

Theoretically, we come to negative conclusion, no resonant state for 4n.

How do we understand 4n system?

# Four neutrons might form a transient isolated entity

Lee G. Sobotka & Maria Piarulli

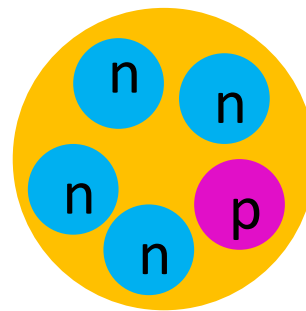
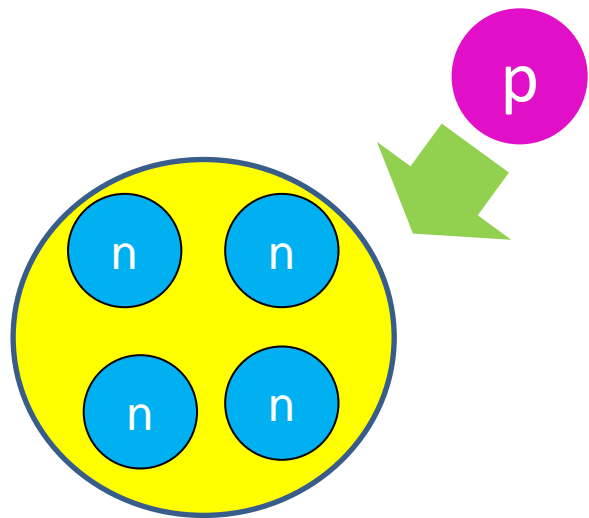
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An experiment firing helium-8 nuclei at a proton target has generated evidence that four neutrons can exist transiently without any other matter. But doubts remain, because the existence of such systems is at odds with theory. **See p.678**

**656** | Nature | Vol 606 | 23 June 2022

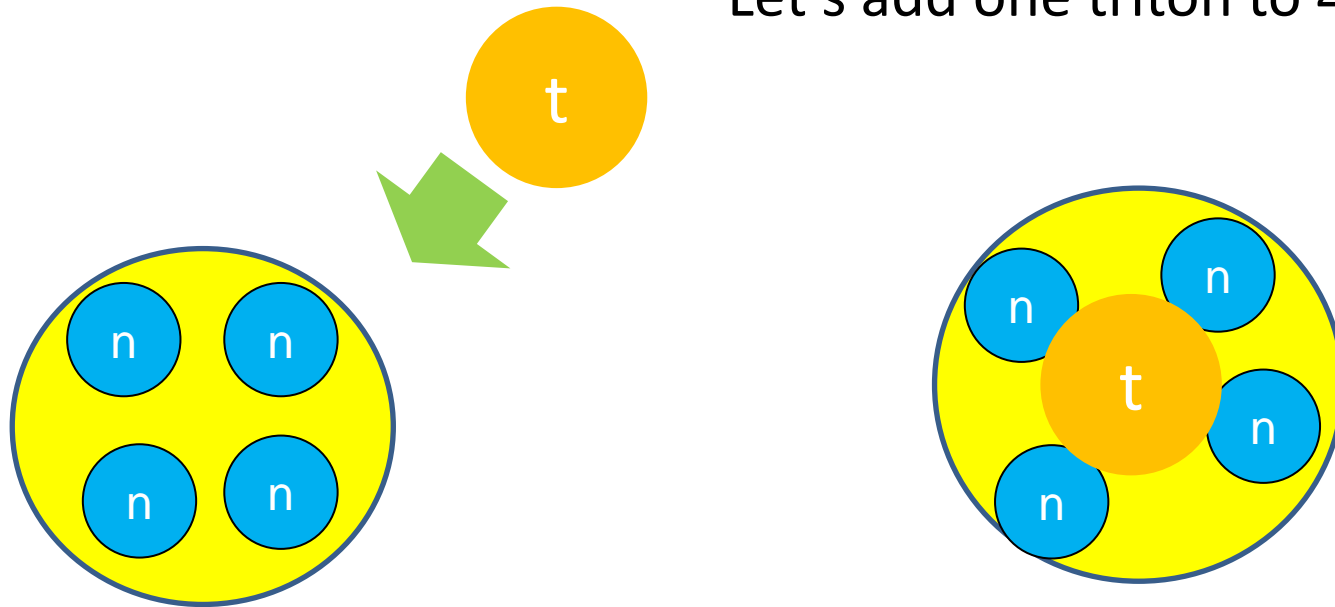
Just recently, experimentally,  $4n$  resonant state has been reported.

Now, is it time to consider this part theoretically?



${}^5\text{H}$

Let's add one triton to 4n.



${}^7\text{H}$

Neutron: 6  
Proton: 1 → Super heavy hydrogen

${}^7\text{H}$  is bound, resonance, nothing?

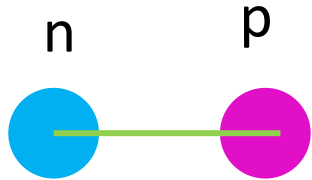
Talked by Caamano and Lenain

Let's explain about hydrogen Isotope before talking about  ${}^7\text{H}$ .

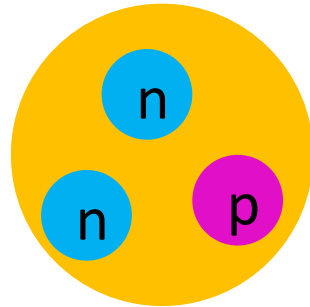


The lightest isotope is Hydrogen (H).

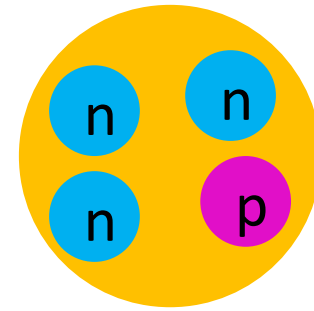
Exp.



${}^2\text{H}$   $J=1^+$  -2.22 MeV



${}^3\text{H}$   $J=1/2^+$  -8.48 MeV



${}^4\text{H}$

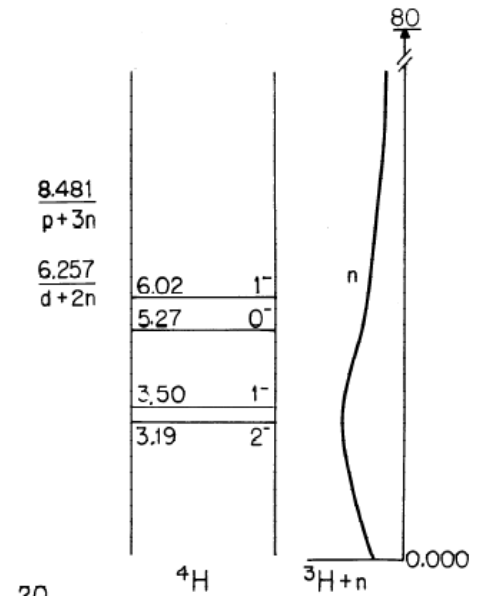


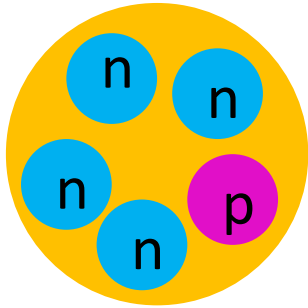
Table 4.1: Energy levels of  ${}^4\text{He}$  defined for channel radius  $a_n = 4.9$  fm. All energies and widths are in the cm system.

$E_x$ (MeV)	$J^\pi$	$T$	$\Gamma$ (MeV)	Decay	Reactions
g.s. <sup>a</sup>	$2^-$	1	5.42	$n, {}^3\text{H}$	1, 11
0.31	$1^-$	1	6.73 <sup>b</sup>	$n, {}^3\text{H}$	11, 12
2.08	$0^-$	1	8.92	$n, {}^3\text{H}$	
2.83	$1^-$	1	12.99 <sup>c</sup>	$n, {}^3\text{H}$	11, 12

<sup>a</sup> 3.19 MeV above the  $n + {}^3\text{H}$  mass.

<sup>b</sup> Primarily  ${}^3\text{P}_1$ .

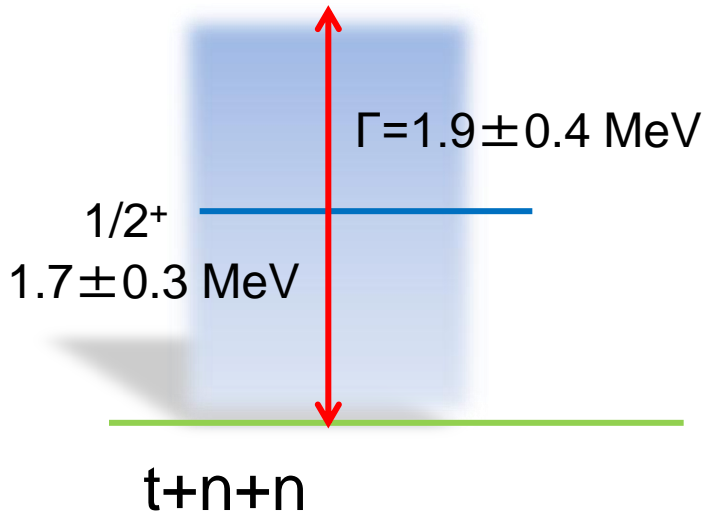
<sup>c</sup> Primarily  ${}^1\text{P}_1$ .



${}^5\text{H}$

transfer reaction  $p({}^6\text{He}, {}^2\text{He}){}^5\text{H}$

A. A. Korcheninnikov, et al. Phys. Rev. Lett.  
87 (2001) 092501.



Superheavy hydrogen

$(E_R, \Gamma_R)$ (MeV)	
$J^\pi$	$1/2^+$
${}^5\text{H}$ (full)	(1.57, 1.53)
${}^5\text{H}$ ( $d = 0$ )	(1.55, 1.35)
Theor. [16]	(2.26, 2.93)
Theor. [12]	(2.5–3.0, 3–4)
Theor. [13]	(3.0–3.2, 1–4)
Theor. [15]	(1.59, 2.48)
Exp. [3]	$(1.7 \pm 0.3, 1.9 \pm 0.4)$
Exp. [8]	$(1.8 \pm 0.1, < 0.5)$
Exp. [4]	(1.8, 1.3)
Exp. [5]	(2, 2.5)
Exp. [6]	(3, 6)
Exp. [9]	$(5.5 \pm 0.2, 5.4 \pm 0.6)$

[3] A.A. Koroshennikov et al., PRL87 (2001) 092501

[8] S.I. Sidorchuk et al., NPA719 (2003) 13

[4] M.S. Golovkov et al. PRC 72 (2005) 064612

[5] G. M. Ter-Akopian et al., Eur. Phys. J A25 (2005) 315.

Energy of  ${}^5\text{H}$  is similar. But decay width is dependent on experiment.

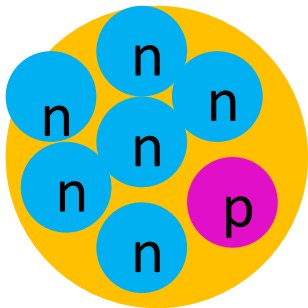
In 2017, we have a new data on  ${}^5\text{H}$ .

A. H. Wuosmaa, Phys. Rev. C95, 014310 (2017)

${}^6\text{He} (d, {}^3\text{He}) {}^5\text{H}$

$$E_r = 2.4 \pm 0.3 \text{ MeV} \quad \Gamma = 5.3 \pm 0.4 \text{ MeV}$$

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${}^7\text{H}$

A. A. Korshennikov et al., PRL 90, 082501 (2003)

M. Caamano et al., PRL99, 062502(2007)  
PRC 78, 044001 (2008)

$$E_r = 0.57^{+0.42}_{-0.21} \text{ MeV} \quad \text{from } t+4n \text{ threshold}$$

$$\Gamma = 0.09^{+0.94}_{-0.06} \text{ MeV}$$

${}^{12}\text{C}({}^8\text{He}, {}^7\text{H}){}^{13}\text{N}$  reaction

If we have narrow decay at lower energy, there could exist in have heavier H-hydrogen isotope such as  ${}^9\text{H}$ .

What is limit for H-isotope? Probably  ${}^7\text{H}$ ?

## Theoretical calculation for ${}^5\text{H}$ and ${}^7\text{H}$

[N. K. Timofeyuk](#), PRC65, 064306(2002), PRC69, 034336(2004)

Volkov NN potential, Hyperspherical harmonics method: 5-body and 7-body calculations

${}^5\text{H}$ : about 1 MeV above t+n+n threshold.

${}^7\text{H}$ : about 3MeV above t+4n threshold

She calculated the energies with bound state approximation.

Then, she did not give decay width for these nuclei.

[S. Aoyama and N. Itagaki](#), PRC80,021304 (R)

Volkov NN potential, AMD calculation

${}^7\text{H}$ : 4.2 MeV above t+4n threshold, no calculation for decay width

No report for the energy of  ${}^5\text{H}$

[H. H. Li et al.](#), PRC 104, L061306 (2021)

Gamow shell model calculation using Minnesota NN potential.

Energy and decay width of  ${}^5\text{H}$  is 1.4 MeV and 0.5 MeV, respectively.

Energy and decay width of  ${}^7\text{H}$  is about 2-3MeV and about 0.1 MeV, respectively.

They predicted to have very narrow decay width for  ${}^5\text{H}$  and  ${}^7\text{H}$ .

Experiment situation:

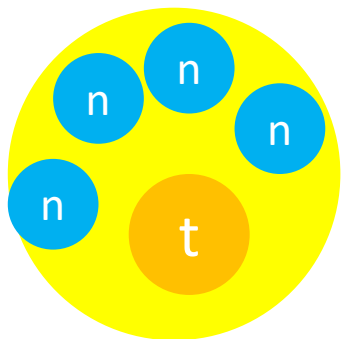
Recently,  ${}^8\text{He} (p,2p) {}^7\text{H}$  reaction has been done at RIBF.  
RIBF Experimental Proposal NP1512-SAMURAI34.  
The analysis is on going.

Then, it is timely to calculate  ${}^7\text{H}$  to obtain the energy and width theoretically.

Motivated by this situation, we study  ${}^7\text{H}$  structure within the framework of  $t+4n$  5-body problem. We also discuss on the energy and decay width of  ${}^5\text{H}$  within  $t+n+n$  three-body problem.

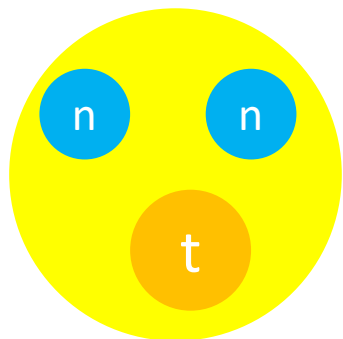
# Framework

NN: Minnesota potential (central potential)



${}^7\text{H} = t + 4n$  model

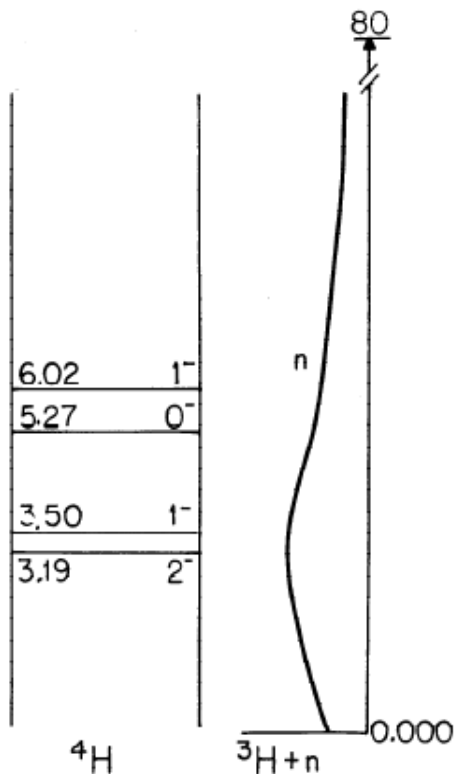
t-n potential => there is a large degree of ambiguity.  
Only several data for phase shift of t-n



${}^5\text{H}$

$\frac{8.481}{p+3n}$

$\frac{6.257}{d+2n}$



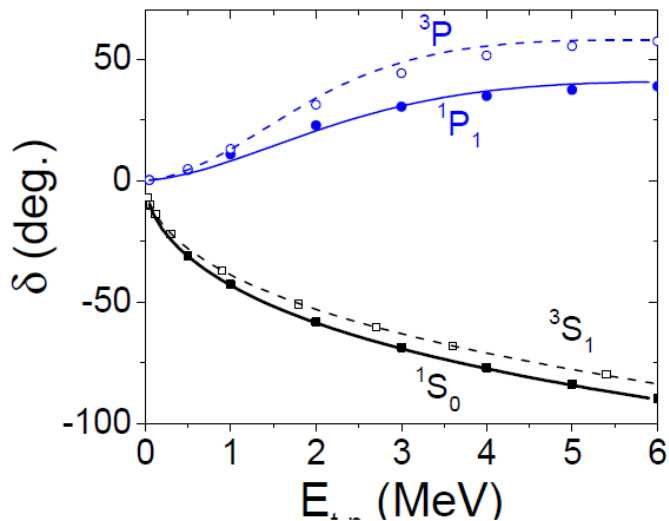
$$V(r, l, s)_{nt} = \delta_{l,0} |\varphi_0\rangle \lambda_\infty \langle \varphi_0| + \sum_{i=1}^2 (v_i^{(c)} + (-)^l v_i^{(P)} + \frac{\hat{s}^2}{2} v_i^{(s)} + (-)^l \frac{\hat{s}^2}{2} v_i^{(SP)}) \exp(-\alpha_i r^2)$$

$$|\varphi_0\rangle = \exp(-a_0 r^2)$$

$$\lambda_\infty = \infty$$

$i$	1	2
$\alpha_i (fm^{-2})$	0.471241	0.0549825
$v_i^{(c)} (MeV)$	-41.3619	1.22768
$v_i^{(P)} (MeV)$	-0.309720	6.89574
$v_i^{(s)} (MeV)$	-28.2483	-0.972465
$v_i^{(SP)} (MeV)$	10.3308	-1.25695

$$a_0 = 0.1979068 \text{ fm}^{-2}$$



Based on four-body calculation with MT I-III

$\alpha_i$	$V_{nt} (1)$	$4N [12]$
$L = 1^-, S = 0$	1.28-2.61 i	0.88(5)-2.20(5) i
$L = 1^-, S = 1$	1.33-1.84 i	1.08(3)-2.03(3) i

Two-body calculation of t-n is almost consistent with that of 4-body calculation.

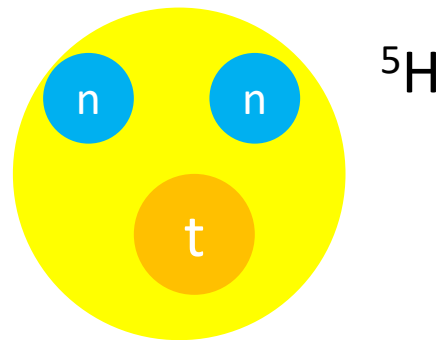


+ I introduce a phenomenological three-body t-n-n force to obtain energy trajectory.

$$V_{tnn}(\rho) = -V_0 e^{-\frac{\rho^2}{b_3^2}} \quad \rho^2 = \frac{m_n}{M} r_{nn}^2 + \frac{m_t}{M^2} r_{nt}^2 + \frac{m_t}{M^2} r_{nt}^2 \quad M = 2m_n + m_t$$

$V_0, b_3$  : parameters.  Fit so as to reproduce the data of  ${}^5\text{H}$

 apply



Our few-body calculation method

## Gaussian Expansion Method (GEM) , since 1987,

- A variational method using Gaussian basis functions
- Take all the sets of Jacobi coordinates

Developed by Kyushu Univ. Group,  
Kamimura and his collaborators.

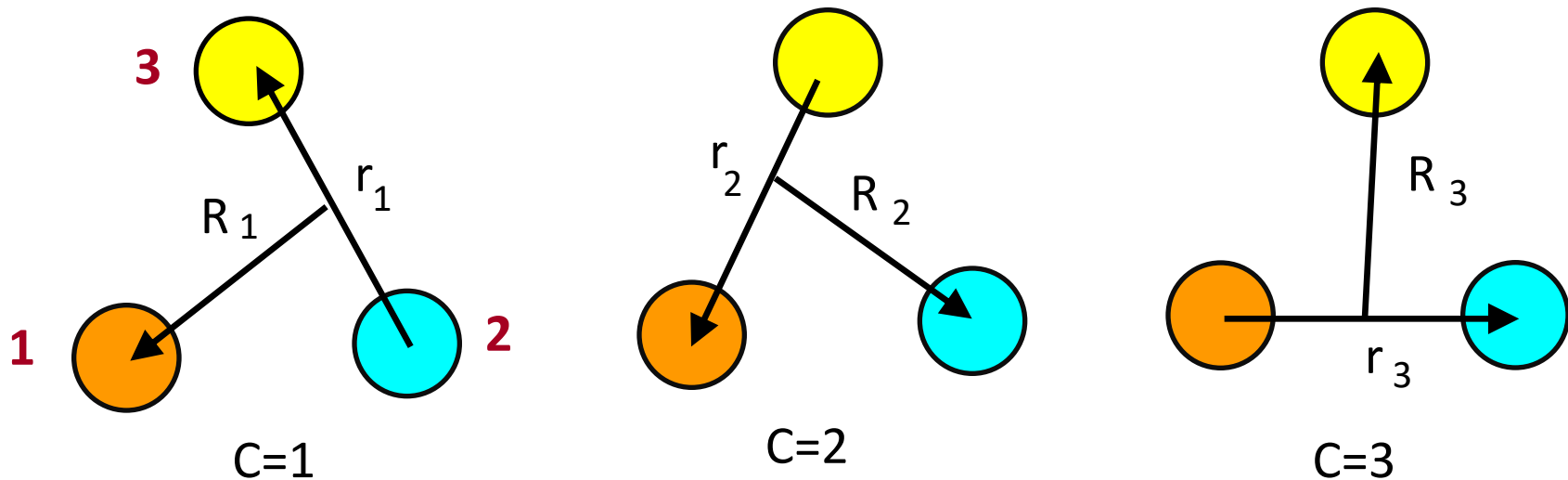
Review article :

E. Hiyama, M. Kamimura and Y. Kino,  
Prog. Part. Nucl. Phys. 51 (2003), 223.

**High-precision calculations** of various 3- and 4-body systems:

Exotic atoms / molecules ,  
3- and 4-nucleon systems,  
multi-cluster structure of light nuclei,

Light hypernuclei,  
3-quark systems,

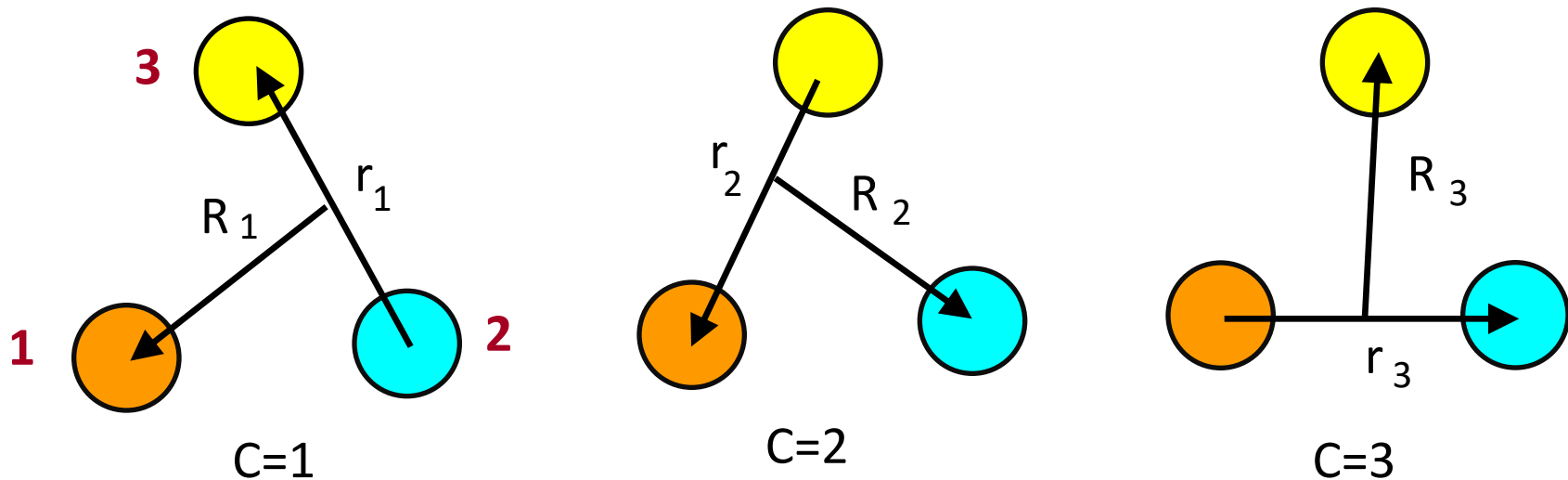


$$(H - E)\Psi_{JM} = 0$$

$$H = T + V_1(r_1) + V_2(r_2) + V_3(r_3)$$

$$T = -\frac{\hbar^2}{2\mu_{r_c}} \nabla_{r_c}^2 - \frac{\hbar^2}{2\mu_{R_c}} \nabla_{R_c}^2 \quad (c = 1, 2, \text{ or } 3)$$

$$\Psi_{JM} = \Phi_{JM}^{(1)}(r_1, R_1) + \Phi_{JM}^{(2)}(r_2, R_2) + \Phi_{JM}^{(3)}(r_3, R_3)$$



$$\Psi_{JM} = \Phi_{JM}^{(1)}(r_1, \mathbf{R}_1) + \Phi_{JM}^{(2)}(r_2, \mathbf{R}_2) + \Phi_{JM}^{(3)}(r_3, \mathbf{R}_3)$$

Basis functions of each Jacobi coordinate

$$\phi_{nl}^{(c)}(r_c) Y_{lm}(\hat{\mathbf{r}}_c), \quad \psi_{NL}^{(c)}(R_c) Y_{LM}(\hat{\mathbf{R}}_c), \quad (c = 1, 2, 3)$$

$\downarrow$   $\downarrow$   
 $(\theta, \phi)$   $(\Theta, \Phi)$

$$\Phi_{JM}^{(c)}(r_c, \mathbf{R}_c) = \sum_{nl, NL} \underbrace{A_{nl, NL}^{(c)}}_{\uparrow} \phi_{nl}^{(c)}(r_c) \psi_{NL}^{(c)}(R_c) [Y_l(\hat{\mathbf{r}}_c) \otimes Y_L(\hat{\mathbf{R}}_c)]_{JM}$$

Determined by diagonalizing H

For this purpose, we use the following basis function:

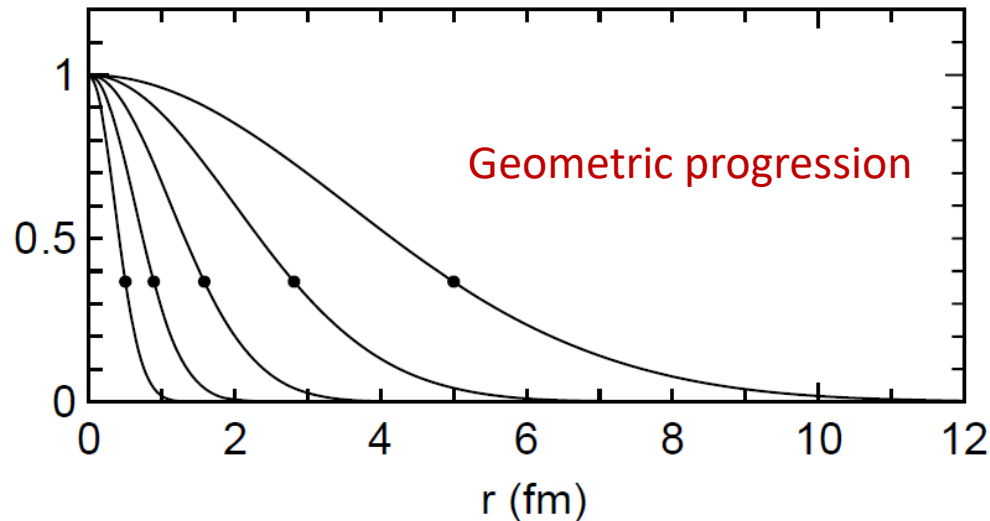
$$\phi_{nlm}(\mathbf{r}) = r^l e^{-\nu_n r^2} Y_{lm}(\hat{\mathbf{r}})$$

$$\nu_n = \frac{1}{r_n^2}$$

$$r_n = r_1 a^{n-1} \quad (n = 1, \dots, n_{\max})$$

Geometric progression

The Gaussian basis function is suitable not only for the calculation of **the matrix elements** but also for describing **short-range** correlations and **long-range** tail behaviour.




Where the energy and overlap matrix elements are given by

$$H_{in} = \langle \Phi_i | H | \Phi_n \rangle \quad (i, n = 1, \dots, N)$$

$$N_{in} = \langle \Phi_i | 1 | \Phi_n \rangle \quad \text{--- non-orthogonal basis}$$

Next, we get eigenenergy  $E$  and coefficients  $C_n$  by solving generalized matrix eigenvalue problem,

$$(\mathbf{H} - E) \Psi = 0 \quad \Psi = \sum_{n=1}^N C_n \Phi_n$$


$$\left[ \begin{array}{c} (H_{in}) - E (N_{in}) \end{array} \right] \left[ \begin{array}{c} C_n \end{array} \right] = 0$$

solution  $\Psi = \Psi_0, \Psi_1, \Psi_2, \dots, \Psi_N$

$$E = E_0, E_1, E_2, \dots, E_N$$

The calculation is for the bound states.

# Benchmark-test 4-body calculation : Phys. Rev. C64 (2001), 044001

## Benchmark test calculation of a four-nucleon bound state

by 7 groups  ${}^4\text{He}$

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② E. Hiyama

*High Energy Accelerator Research Organization, Institute of Particle and Nuclear Studies, Tsukuba 305-0801, Japan*

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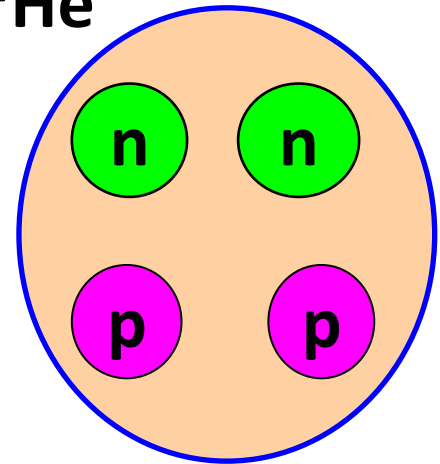
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4 nucleon  
bound state

Realistic NN  
force: AV8'

# Benchmark-test calculation of the 4-nucleon bound state

Good agreement among 7 different methods

In the binding energy, r.m.s. radius and wavefunction density

H. KAMADA *et al.*

PHYSICAL REVIEW C **64** 044001

TABLE I. The expectation values  $\langle T \rangle$  and  $\langle V \rangle$  of kinetic and potential energies, the binding energies  $E_b$  in MeV, and the radius in fm.

Method	$\langle T \rangle$	$\langle V \rangle$	$E_b$	$\sqrt{\langle r^2 \rangle}$
FY	102.39(5)	-128.33(10)	-25.94(5)	1.485(3)
<b>GEM</b> <i>7</i>	102.30	-128.20	-25.90	1.482
SVM	102.35	-128.27	-25.92	1.486
HH	102.44	-128.34	-25.90(1)	1.483
GFMC	102.3(1.0)	-128.25(1.0)	-25.93(2)	1.490(5)
NCSM	103.35	-129.45	-25.80(20)	1.485
EIHH	100.8(9)	-126.7(9)	-25.944(10)	1.486

very different techniques and the complexity of the nuclear force chosen. Except for NCSM and EIHH, the expectation values of  $T$  and  $V$  also agree within three digits. The NCSM results are, however, still within 1% and EIHH within 1.5% of the others but note that the EIHH results for  $T$  and  $V$  are

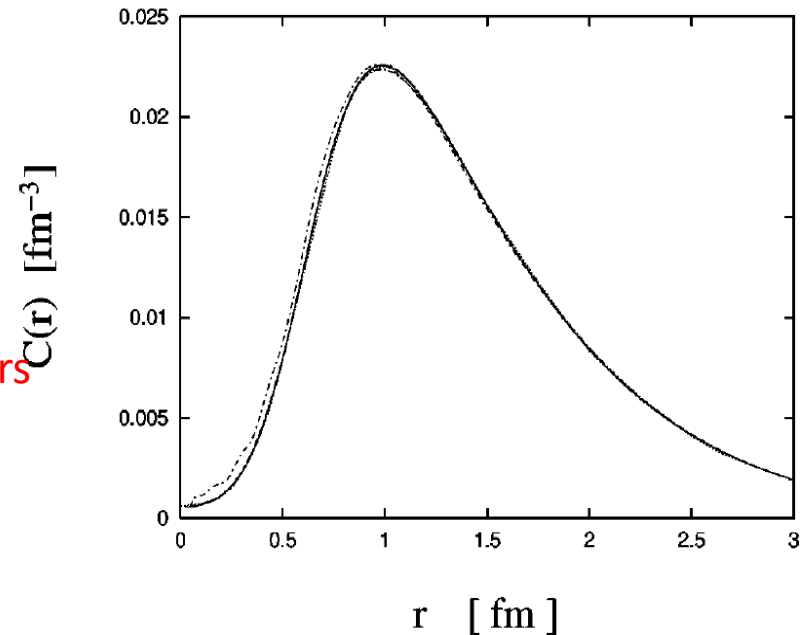


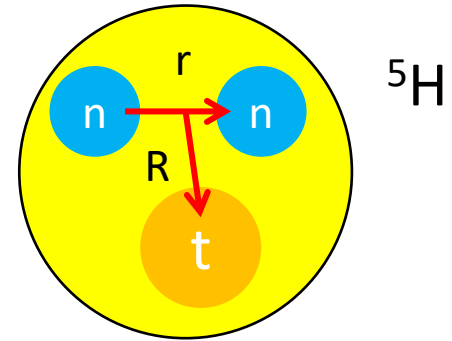
FIG. 1. Correlation functions in the different calculational schemes: EIHH (dashed-dotted curves), FY, CRCGV, SVM, HH, and NCSM (overlapping curves).



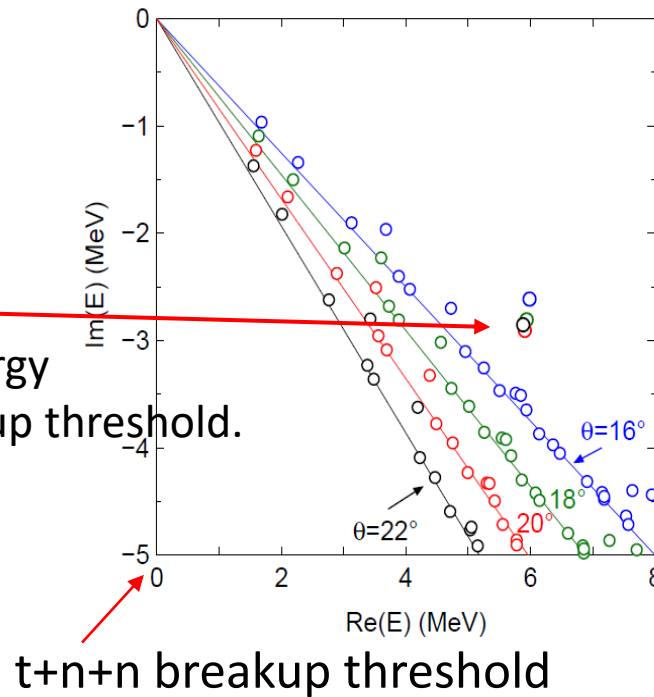
Observed data of  ${}^5\text{H}$  is resonant state.

To obtain resonant state of  ${}^5\text{H}$ , we use complex scaling method.

$$r_c \rightarrow r_c e^{i\theta}, R_c \rightarrow R_c e^{i\theta},$$



The energy pole is stable with respect to  $\theta$ .  
 Re(E) corresponds to energy with respect to  $4n$  breakup threshold.  
 Im(E) corresponds to  $\Gamma/2$ .

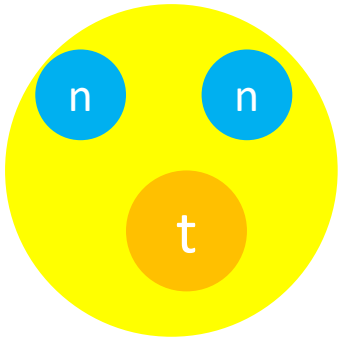


+ I introduce a phenomenological three-body t-n-n force to obtain energy trajectory.

$$V_{tnn}(\rho) = -V_0 e^{-\frac{\rho^2}{b_3^2}} \quad \rho^2 = \frac{m_n}{M} r_{nn}^2 + \frac{m_t}{M^2} r_{nt}^2 + \frac{m_t}{M^2} r_{nt}^2 \quad M = 2m_n + m_t$$

$V_0, b_3$  : parameters. →

Fit so as to reproduce the data of  ${}^5\text{H}$



${}^5\text{H}$

Question: Which experimental data of  ${}^5\text{H}$  should we fit?

$(E_R, \Gamma_R)$ (MeV)	
$J^\pi$	$1/2^+$
${}^5\text{H}$ (full)	(1.57, 1.53)
${}^5\text{H}$ ( $d = 0$ )	(1.55, 1.35)
Theor. [16]	(2.26, 2.93)
Theor. [12]	(2.5–3.0, 3–4)
Theor. [13]	(3.0–3.2, 1–4)
Theor. [15]	(1.59, 2.48)
Exp. [3]	$(1.7 \pm 0.3, 1.9 \pm 0.4)$
Exp. [8]	$(1.8 \pm 0.1, < 0.5)$
Exp. [4]	(1.8, 1.3)
Exp. [5]	(2, 2.5)
Exp. [6]	(3, 6)
Exp. [9]	$(5.5 \pm 0.2, 5.4 \pm 0.6)$

[3] A.A. Koroshennikov et al., PRL87 (2001) 092501

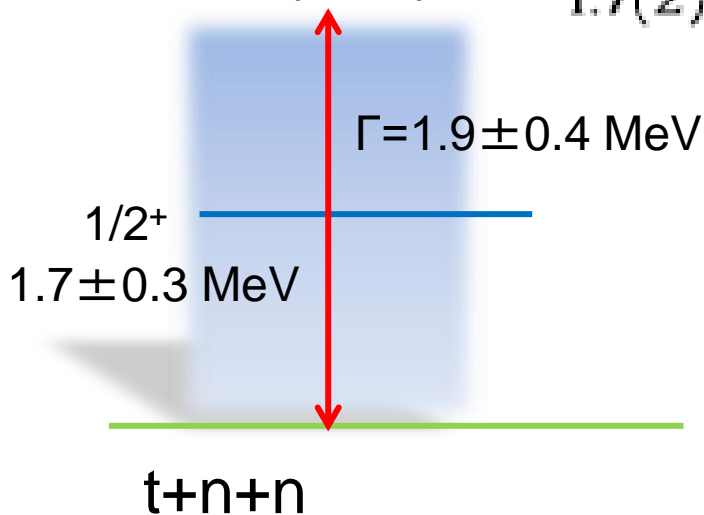
[8] S.I. Sidorchuk et al., NPA719 (2003) 13

[4] M.S. Golovkov et al. PRC 72 (2005) 064612

[5] G. M. Ter-Akopian et al., Eur. Phys. J A25 (2005) 315.

Energy of  ${}^5\text{H}$  is similar. But decay width is dependent on experiment.

$J=1/2^+$		
	$E_R$	$\Gamma$
N3LO (ACCC)	1.8(1)	2.4(2)
(SECS)	1.9(2)	2.4(2)
INOY (ACCC)	1.7(1)	2.4(2)
(SECS)	1.8(1)	2.4(2)
MT13 (ACCC)	1.4(1)	2.4(2)
(SECS)	1.7(2)	2.4(2)



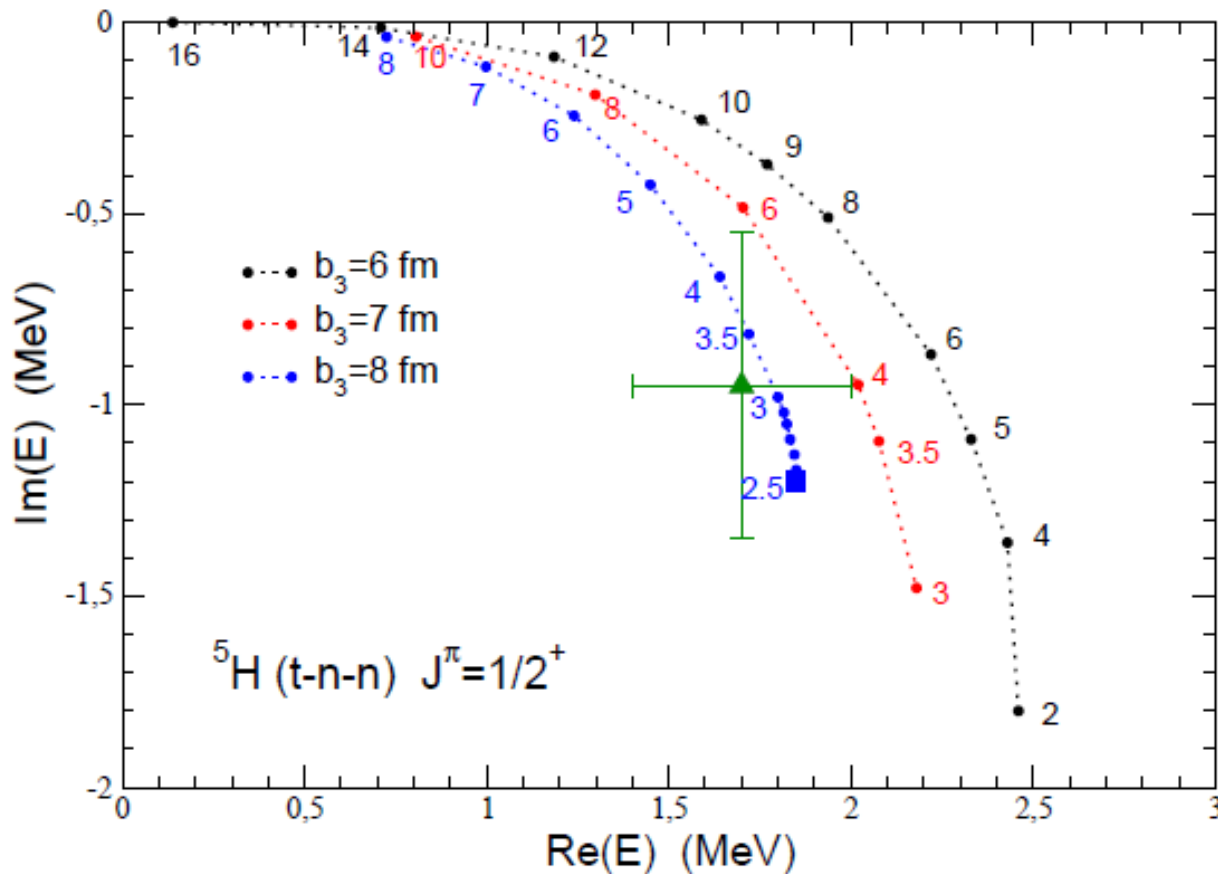
We take this result as 'exp.' data.

Close to the below exp.data

A. A. Korcheninnikov, et al. Phys. Rev. Lett.  
 87 (2001) 092501.

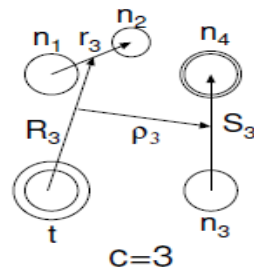
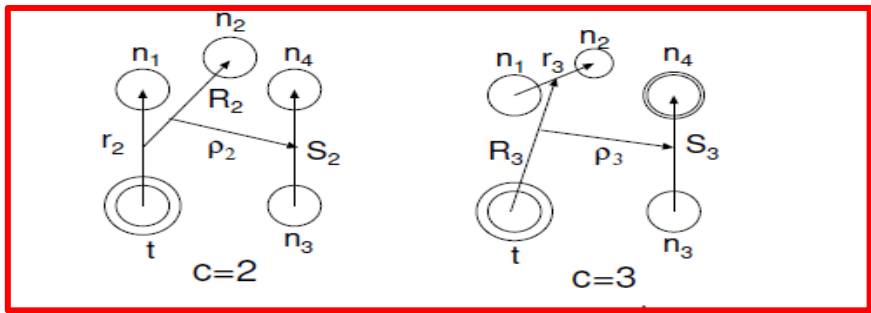
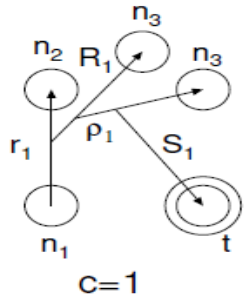
$$V_{tnn}(\rho) = -V_0 e^{-\frac{\rho^2}{b_3^2}} \quad \rho^2 = \frac{m_n}{M} r_{nn}^2 + \frac{m_t}{M^2} r_{nt}^2 + \frac{m_t}{M^2} r_{nt}^2 \quad M = 2m_n + m_t$$

When  $b_3=8$  fm and  $V_0=3$  to 2.5 MeV, the energy pole of  ${}^5\text{H}$  is close to exp. data. If we have this potential parameter, what is energy pole of  ${}^7\text{H}$ ?

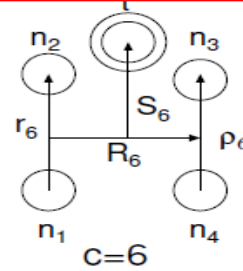
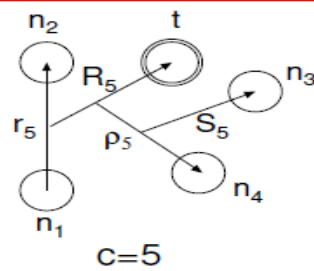
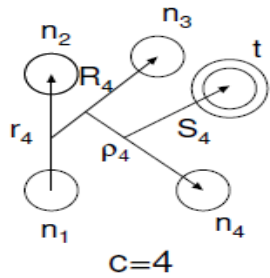


# Framework of ${}^7\text{H}$

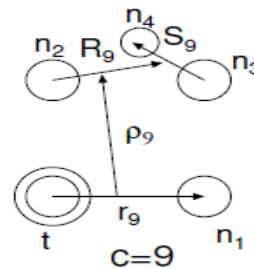
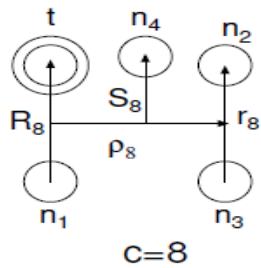
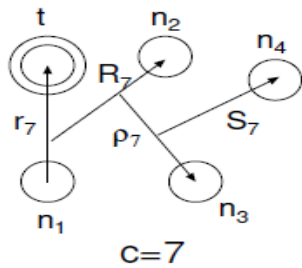
The Hamiltonian is the same as the case of  ${}^5\text{H}$ .



$$(t+n+n)+nn$$



Totally 120 Jacobi coordinates



$$\begin{aligned} \Psi_{JM}({}^7\text{H}) &= \left[ \left[ \left[ \left[ \eta_{\frac{1}{2}}(n) \eta_{\frac{1}{2}}(n) \right]_t \eta_{\frac{1}{2}}(n) \right]_{T_0} \eta_{\frac{1}{2}}(n) \right]_{T_4} \eta_{\frac{1}{2}}(t) \right]_{TT_z} \\ &\times \left[ \left[ \left[ \left[ \chi_{\frac{1}{2}}(n) \chi_{\frac{1}{2}}(n) \right]_t \chi_{\frac{1}{2}}(n) \right]_{\Sigma} \chi_{\frac{1}{2}}(n) \right]_{S_4} \chi_{\frac{1}{2}}(t) \right]_S \\ &\times \left[ \left[ \left[ \left[ \phi_{\ell}(r_c) \psi_L(R_c) \right]_{\Lambda} \phi_{\lambda}(\rho_c) \right]_I \phi_{\xi}(s_c) \right]_K \right]_{JM} \end{aligned}$$

## Form of each basis function

5-body spatial function

$$\left[ \left[ \left[ \phi_{nl}^{(c)}(\mathbf{r}_c) \psi_{NL}^{(c)}(\mathbf{R}_c) \right]_I \varphi_{n'l'}^{(c)}(\boldsymbol{\rho}_c) \right]_K \Phi_{N'L'}^{(c)}(\mathbf{S}_c) \right]_L$$

Gaussian for radial part :

$$\phi_{nlm}(\mathbf{r}) = r^l e^{-(r/r_n)^2} Y_{lm}(\hat{\mathbf{r}})$$

geometric progression  
for Gaussian ranges :

$$r_n = r_1 a^{n-1} \quad (n = 1 - n_{\max})$$

Similarly for the  
other basis :

$$\psi_{NLM}^{(c)}(\mathbf{R}_c) \quad \varphi_{n'l'm'}^{(c)}(\boldsymbol{\rho}_c) \quad \Phi_{N'L'M'}^{(c)}(\mathbf{S}_c)$$

Use of this type gaussian basis is known to be very suitable  
for describing simultaneously both the **short-range** correlations and  
**long-range** tail behaviour of few-body systems;

This is precisely  
shown in



Gaussian Expansion Method (GEM)

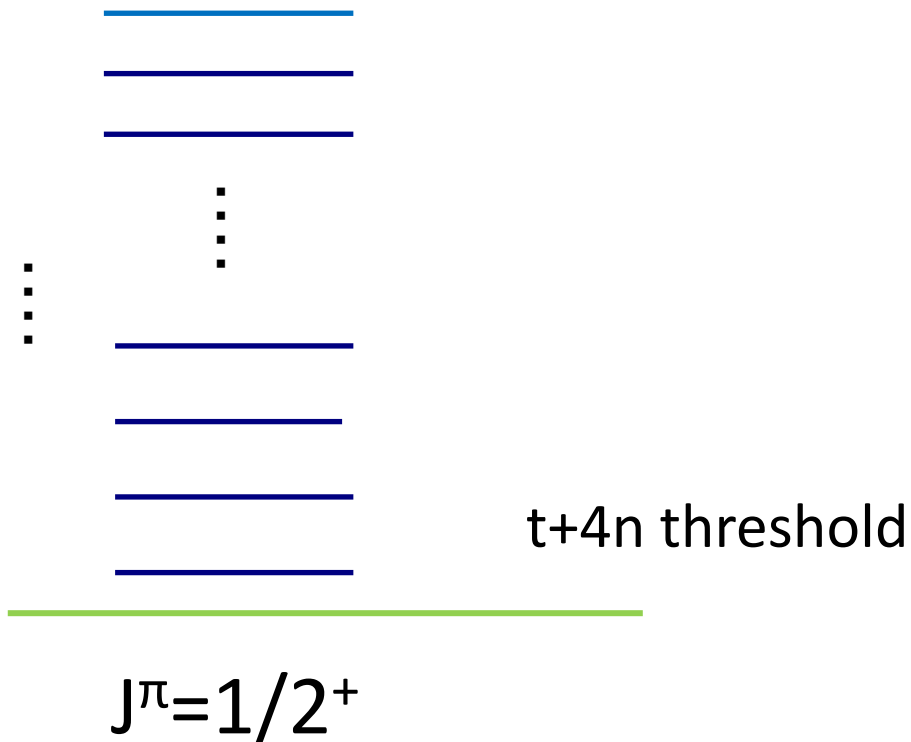
(review paper) E. H., Y. Kino and M. Kamimura,  
Prog. Part. Nucl. Phys., 51 (2003) 223.

$$(H-E)\Psi=0$$

By the diagonalization of Hamiltonian, we obtain N eigenstates for each  $J^\pi$ .

Here, we use about 56,000 basis functions.

Then, we obtained 56,000 eigenfunctions for  $J^\pi=1/2^+$ .





For the calculation of  ${}^7\text{H}$ , it would be difficult to apply complex scaling method for 5-body calculation. Then, for this calculation, I used real scaling method.

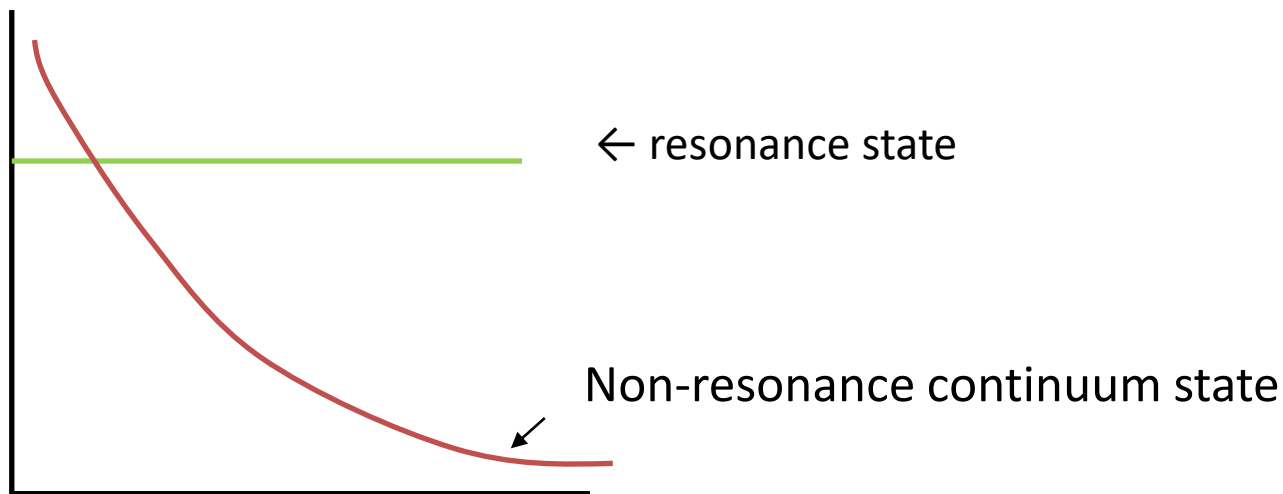
useful method: real scaling method

often used in atomic physics

In this method, we artificially scale the range parameters of our Gaussian basis functions by multiplying a factor  $\alpha$ :

$$r_n \rightarrow \alpha r_n \text{ in } r^l \exp(-r/r_n)^2 \quad \text{for exmple } 0.8 < \alpha < 1.5$$

and repeat the diagonalization of Hamiltonian for many value of  $\alpha$ .



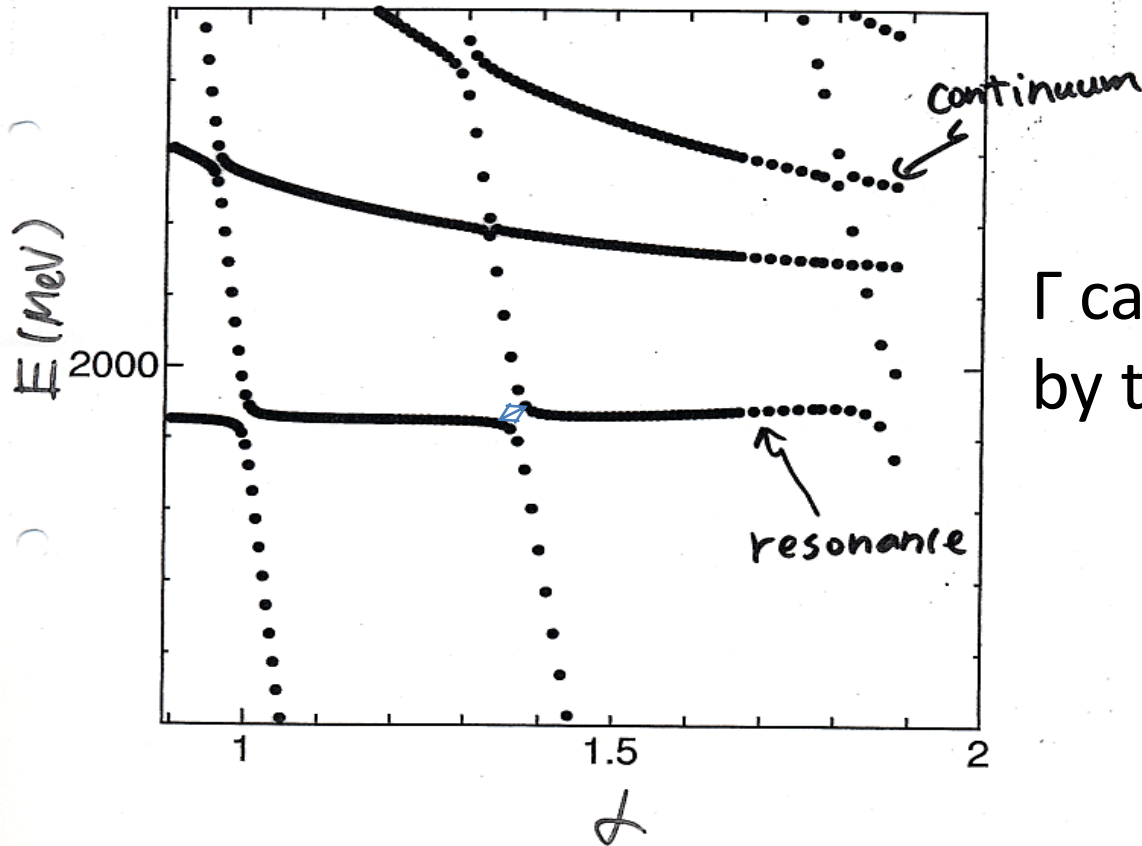
$\alpha$ : range parameter of Gaussian basis function

[schematic illustration of the real scaling]

What is the result in our pentaquark calculation?

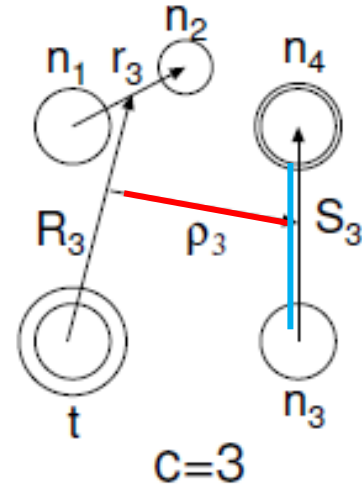
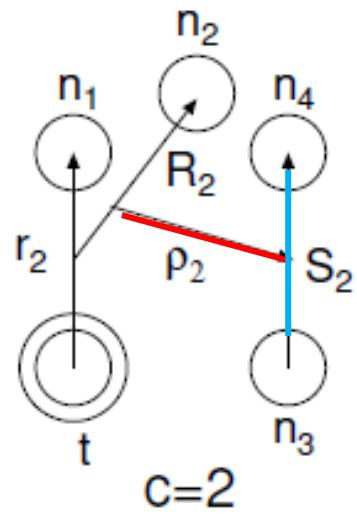
# Example of real scaling

Not result of penta quark system



$\Gamma$  can be estimated by the  $\Delta E$ .

What is the result of our pentquark calculation?

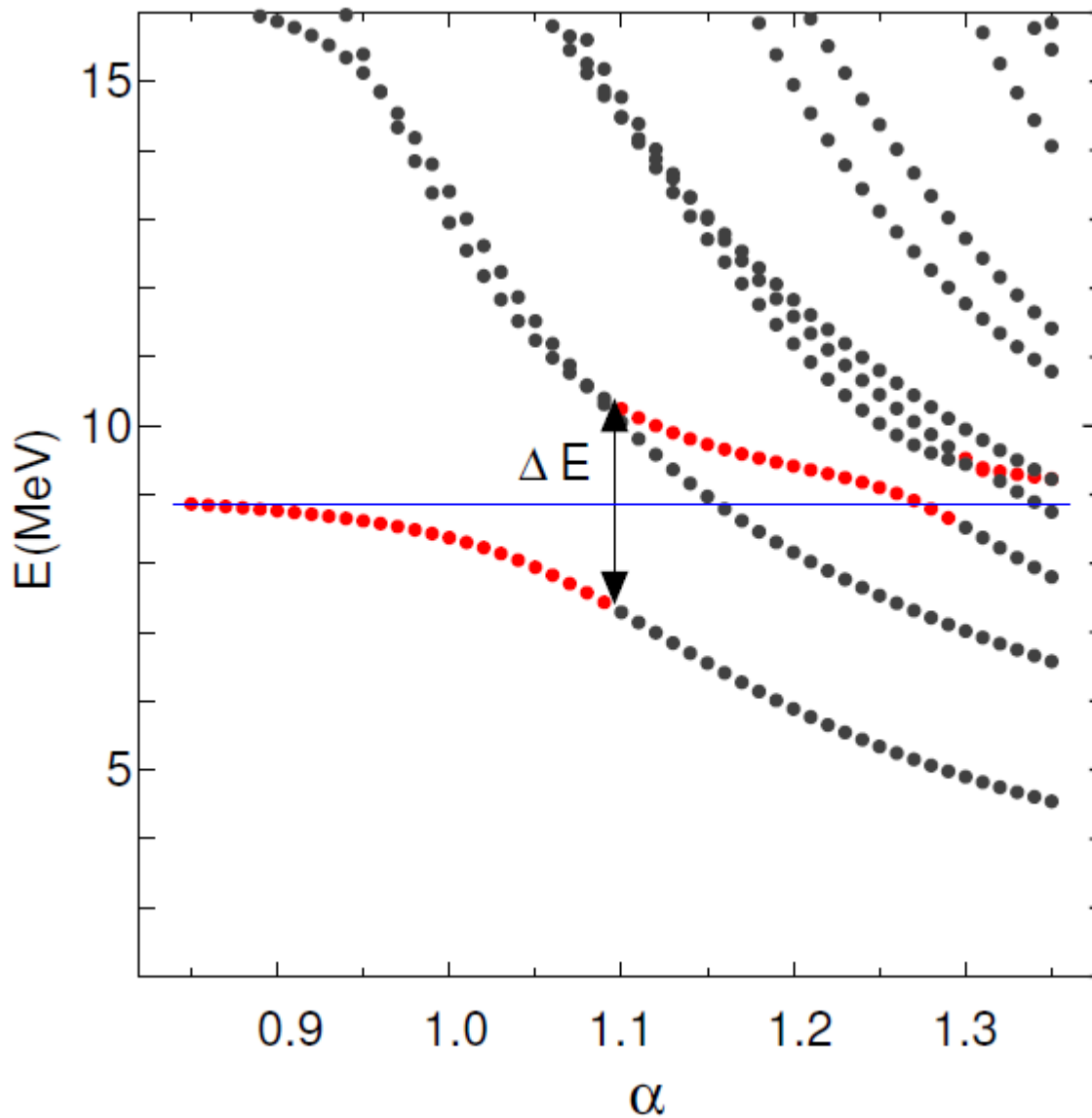


$$\rho_n \Rightarrow \alpha \rho_n$$

$$s_n \Rightarrow \alpha s_n$$

$$V_{tnn}(\rho) = -V_0 e^{-\frac{\rho^2}{b_3^2}}$$

$b_3=8.0\text{fm}$   $V_0=-3\text{ MeV}$



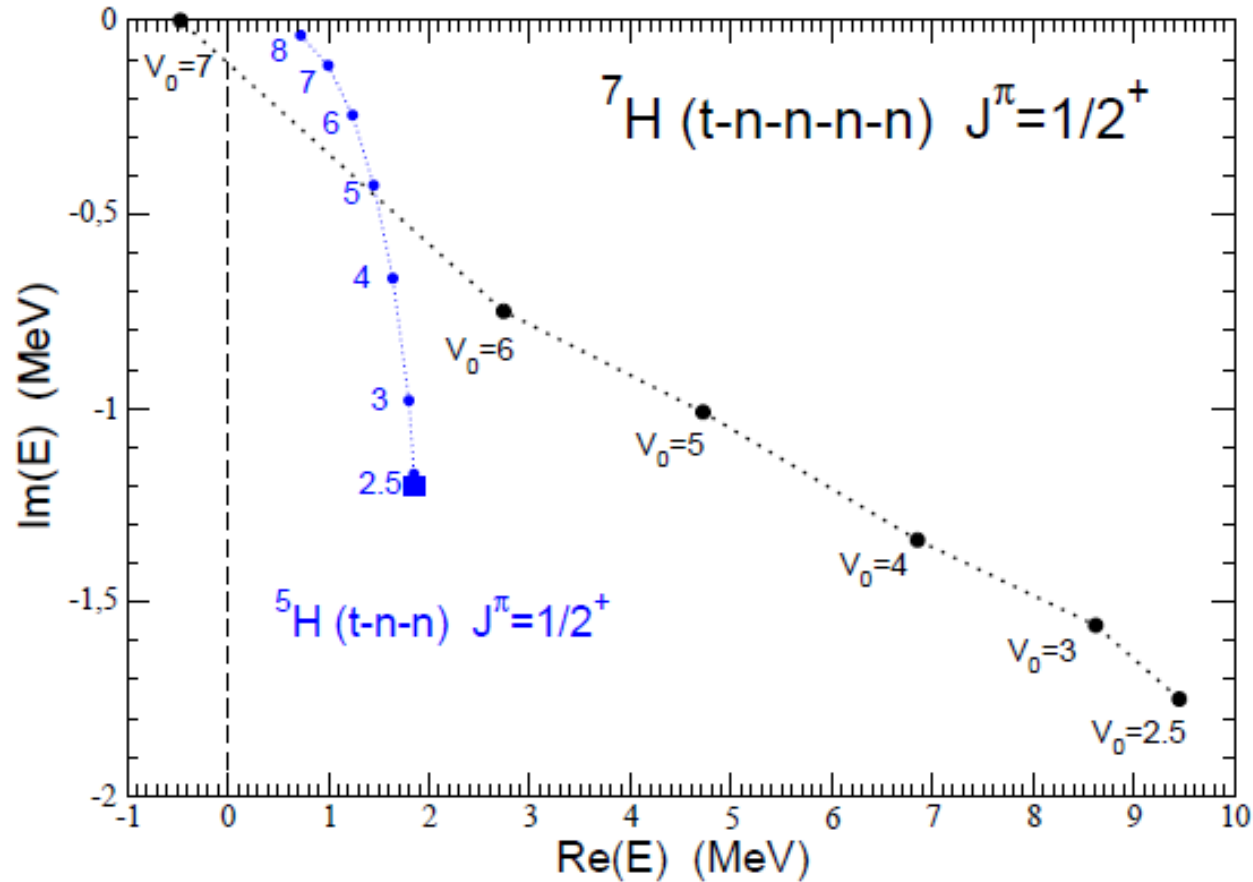
$E_r \sim 8.8\text{ MeV}$

$\Gamma \sim 3.1\text{ MeV}$

With respect to  
t+4n threshold

5H: close to  
Exp. data

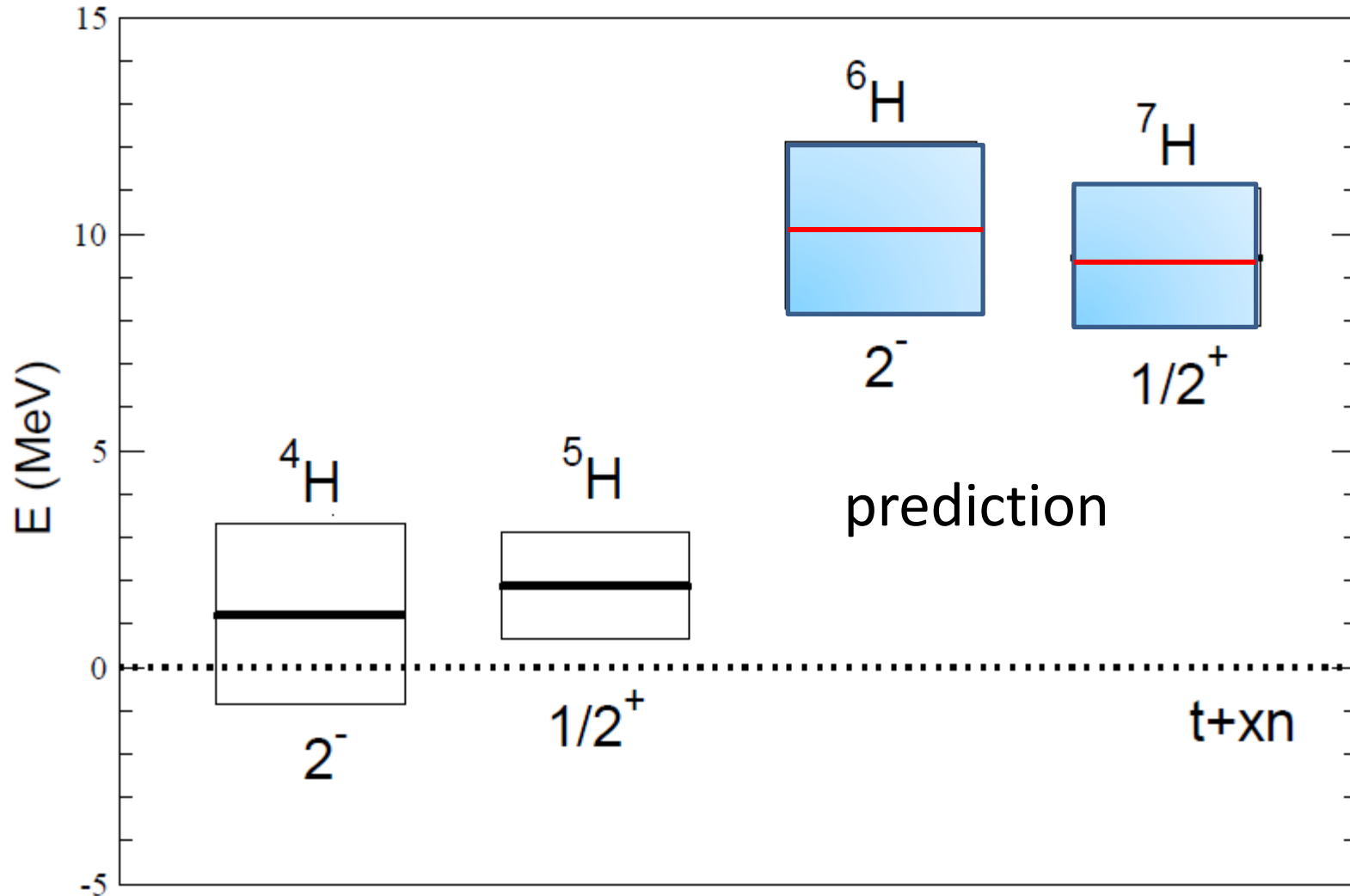
$$\text{Im}(E) = \Gamma/2$$



For  $V_0 = 2.5$ , we reproduce the data of  ${}^5\text{H}$  accurately.  
In this case, the energy pole of  ${}^7\text{H}$ ,  $E = 9.5$  MeV,  $\Gamma \sim 3.5$  MeV.  
Our energy of  ${}^7\text{H}$  is much higher and broad decay width.

# Summary of H-isotope (according to our calculation)

End of H-isotope



## Summary

Assuming  $E_r \sim 1.9$  MeV and  $\Gamma \sim 2.4$  MeV for  ${}^5\text{H}$ ,  
Our calculated energy and decay width of  ${}^7\text{H}$  are  
about  $E_r \sim 8$  to  $9$  MeV, and  $\Gamma \sim 3$  MeV.  
That is much higher than  ${}^5\text{H}+n+n$  threshold,  
broad decay width.

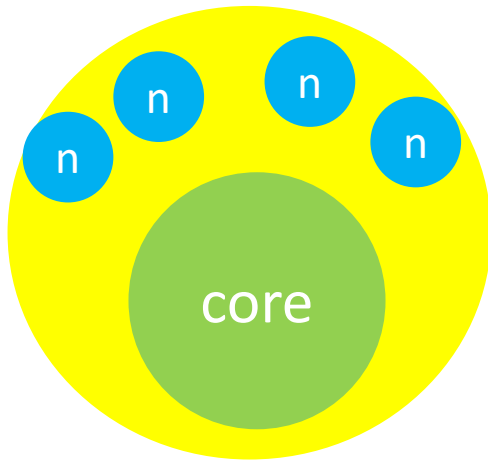
${}^8\text{He} (p,2p) {}^7\text{H}$  reaction was done at RIBF, recently.  
RIBF Experimental Proposal NP1512-SAMURAI34.  
The analysis is on going. =>The result will be reported by Lenain.

I am waiting for future experimental result.



Thank you!

Future prospect:



We have a code to calculate core+4n.

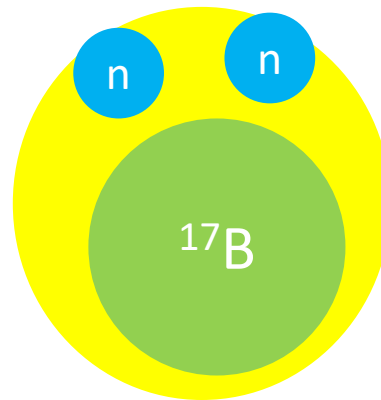


We could apply the method to many neutron-rich nuclei.

Example:  $^{19}\text{B} = ^{15}\text{B} + 4\text{n}$

Recent measurement of  $^{19}\text{B}$  (PRL 124, 212503 (2020))

At that time, E. Hiyama, R. Lazauskas, F.M. Marqu´es, and J. Carbonell, Phys. Rev. C 100, 011603(R) (2019).



Next, we plan to study  $^{17}\text{B} + 4\text{n}$ .

In order to solve few-body problem accurately,

## Gaussian Expansion Method (GEM) , since 1987

- A variational method using Gaussian basis functions
- Take all the sets of Jacobi coordinates

Developed by Kyushu Univ. Group,  
Kamimura and his collaborators.

Review article :

E. Hiyama, M. Kamimura and Y. Kino,  
Prog. Part. Nucl. Phys. 51 (2003), 223.

**High-precision calculations** of various 3- and 4-  
body systems:

Exotic atoms / molecules ,	Light hypernuclei,
3- and 4-nucleon systems,	3-quark systems,
multi-cluster structure of light nuclei,	$^4\text{He}$ -atom tetramer

