# Slow antiproton collisions with deuterium

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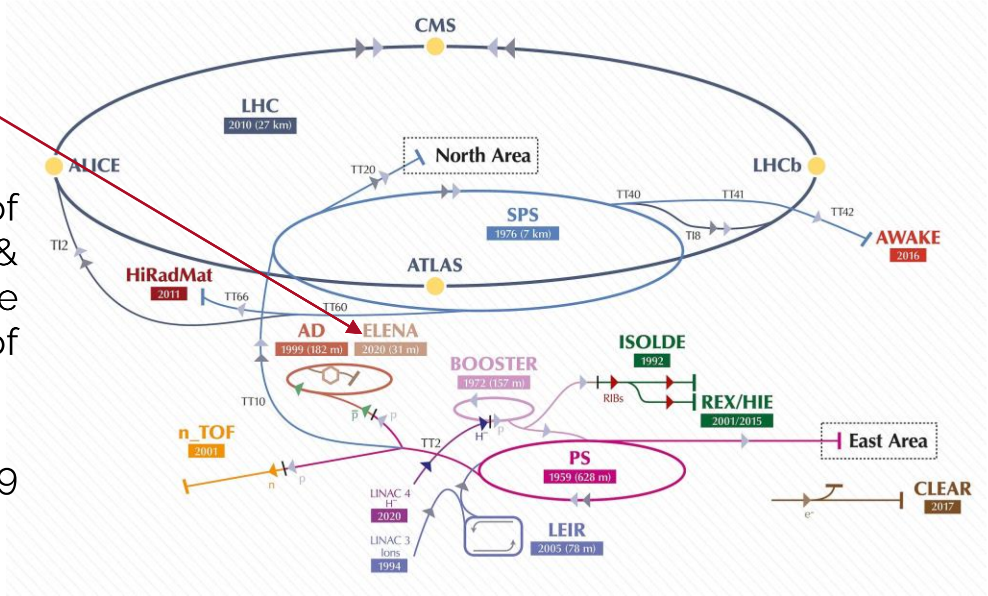
# 1.
# Introduction

# Revival of Low-energy antiproton physics at CERN

ELENA

Introduction of ELENA antiproton decelerator revives the physics of interface between the antimatter & standard matter, and offers unique possibility to study properties of the antimatter.

Rich experimental program with 9 dedicated experimental programs



# Antiproton interactions with atomic nucleus PUMA project

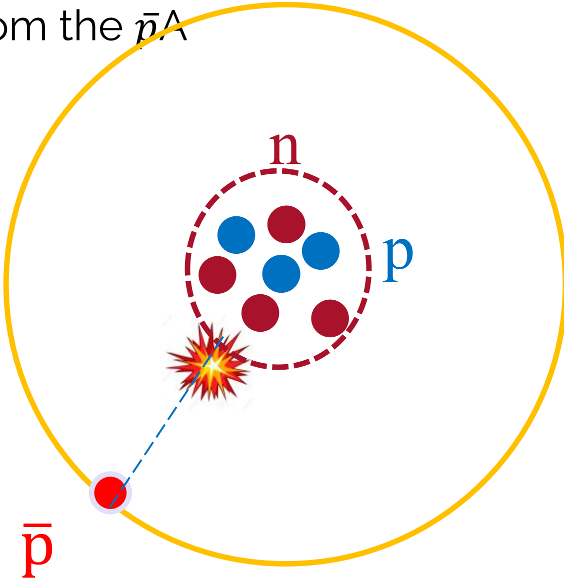
*T. Aumann, …,A Obertelli et al., Eur.Phys.J.A **58** (2022) 5, 88*

# PUMA project

*T. Aumann, …,A Obertelli et al., Eur.Phys.J.A **58** (2022) 5, 88*

The goal of PUMA project is to measure nuclear neuron skins from the $\bar{p}$A annihilation data:

*1)* $\bar{\mathrm{p}}\mathrm{n}/\bar{\mathrm{p}}\mathrm{p}$ annihilation ratio
2) Orbit from which $\bar{\mathrm{p}}$ is captured



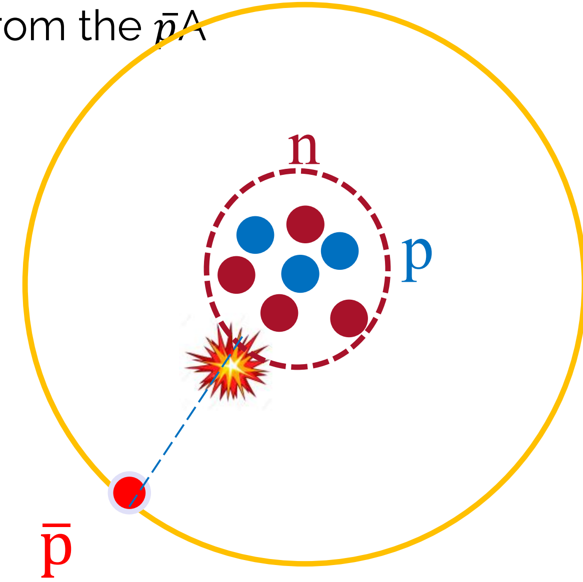
# PUMA project

*T. Aumann, ..,A Obertelli et al., Eur.Phys.J.A **58** (2022) 5, 88*

The goal of PUMA project is to measure nuclear neuron skins from the $\bar{p}$A annihilation data. We have to answer:

- If the exp. data lead to unambiguous conclusion?
- Can we interpret the data?
- If yes, how and how well?

Accuracy of the solutions, quality of the input, model dependence, ..

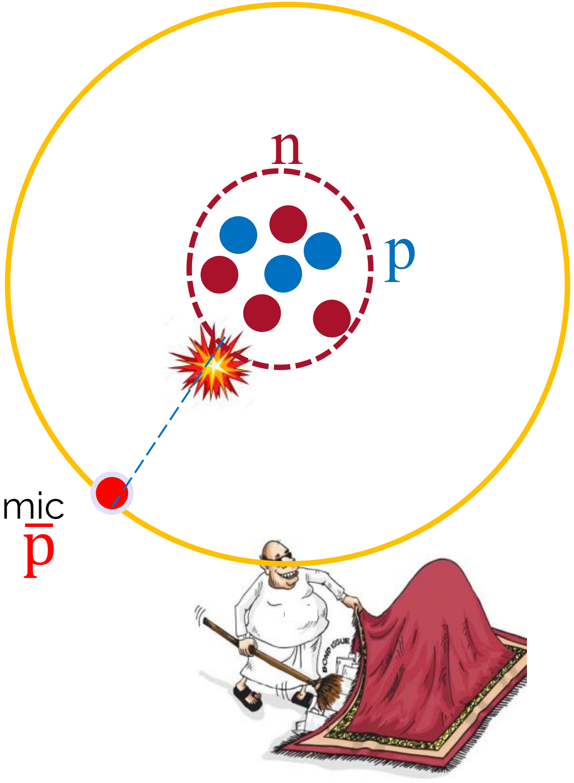



Our aim is to provide the «best» solutions for the accessible systems and use this knowledge to build «antiproton-nucleus» potentials for the rich-neutron systems of experimental realm

# PUMA project

Provide the « best possible » solution for the NR Schrödinger eq.

$$\hat{H}|\Psi\rangle = E|\Psi\rangle; \;\; \hat{H} = \widehat{H_0} + V$$




The problem is extremely complex:

- ~~Relativity~~ and *annihilation dynamics*
- Complexity of the $\bar{p}$N interaction and $\bar{p}$A dynamics
- Presence and coupling between the very different physical scales: atomic (Coulomb), nuclear ($\bar{p}$A), subatomic (annihilation) !!

# THE FACTS

There are two main sources of experimental info: scattering and protonium

SCATTERING

from $\bar{p}p$ one can measure three contributions to the total cross section

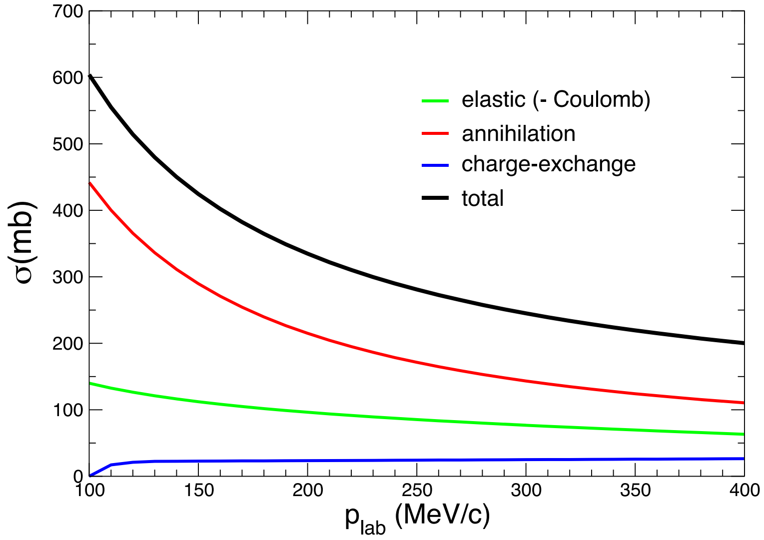
$$\sigma_t = \sigma_e + \sigma_a + \sigma_{ce}$$

- $\sigma_e$ elastic
- $\sigma_a$ annihilation: everything produced beyond $\bar{p}p$ ($\bar{n}n$ ) channels
- $\sigma_{ce}$ charge-exchange

$$\bar{p} + p \rightarrow \bar{n} + n$$

from $\bar{n}p$ one gain some interesting low energy results on $\sigma_e$ and $\sigma_a$. One is able to isolate the isospin T=1 component of the interaction and study it by avoiding complications brought by Coulomb interaction. Difficult measurement for it uses the ce to produce the secondary antineutron beam.

# THE FACTS




- At low energy ($p_L$<400 MeV/c) dominated by annihilation $\boldsymbol{\sigma}_a/\boldsymbol{\sigma}_e \simeq 2$
- Partial wave cross sections close to unitary limit $\boldsymbol{\sigma}_a^{(L)}=(2L+1)\boldsymbol{\pi}/k^2_{cm}$
- Cannot be reduced to a black sphere model (for which $\boldsymbol{\sigma}_a=\boldsymbol{\sigma}_e$) :

the strong force of nuclear origin plays a crucial role

# THE FACTS

There are two main sources of experimental info: scattering and protonium

PROTONIUM

In absence of strong interaction $\bar{p}p$ would form an H-like

$$E_c = -\frac{1}{4}\frac{m_p\alpha^2}{n^2} = -\frac{12.5\ keV}{n^2}$$

*with Bohr radius $a_0$= 57 fm (a 1000x reduced H-atom)*

Strong interaction shifts and broadens the pure Coulomb levels

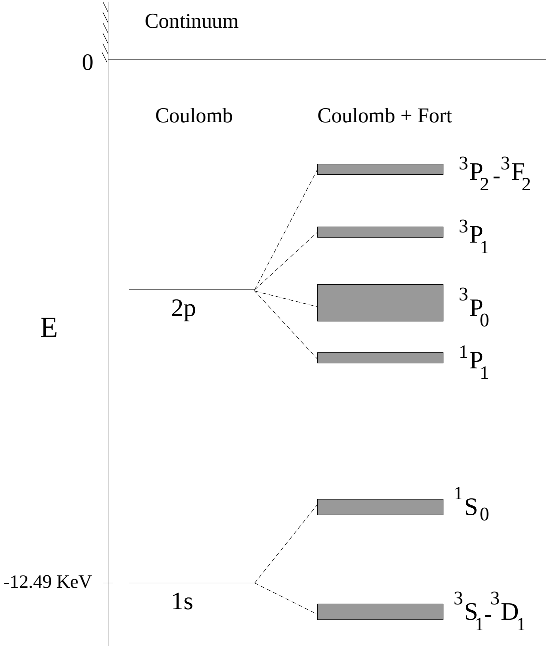
- Difference $\Delta E = \Delta E_R + i\Gamma/2$ is meeasured for low lying states (1s,2p)
- This difference is related to the scattering length $a_{\bar{p}p} = f_{\bar{p}p}$(E=0)

A priviledged open door to $\bar{N}N$ forces at low energy (controlled initial state)

Many other $\bar{\text{p}}$A atoms have been measured. It is however very difficult to extract useful information to construct $\bar{\text{p}}$N models;

# $\bar{N}N$ interaction

## THE FACTS




- Coulomb levels are shifted up/down w.r.t. QED, depending on the state:
- Energy shifts $\Delta E = \Delta E_R$ and lifetime $i\Gamma/2$ (energy spread) are measured

# $\bar{N}N$ interaction: theory

There are two approaches for $\bar{N}N$ interaction:

- « traditional » meson exchange approach of Nuclear Physics
- $\chi$EFT

In order to account for $\bar{N}N$ annihilation

- Add phenomenological negative imaginary part (optical potetnial)
- Introduce coupled $\bar{N}N/\bar{X}X$ channels

# $\bar{N}N$ interaction: theory

The « traditional » meson exchange approach in Nuclear Physics

$$V_{NN} = +V_\pi + V_\eta + V_\rho + V_\omega + V_{\sigma_0} + V_{\sigma_1}$$

Thouhg outfashioned – it is still remains the most employed model (most of existing calculations are performed based on these models).

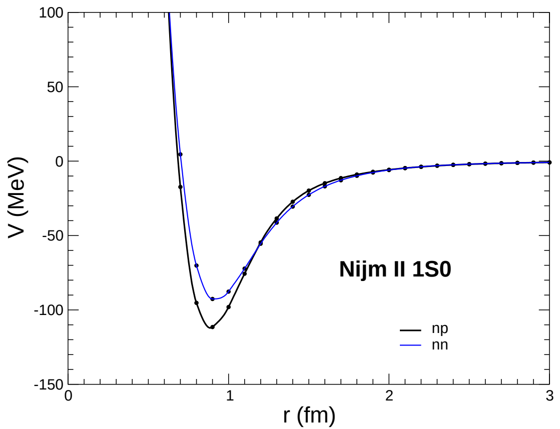
$V_{N\bar{N}}$ (real part, T-symmetry) follows from $V_{NN}$ by a G-parity transfomation of the meson-N vertex, providing multiplicative factor:

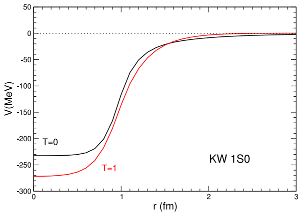
$$G = C(-)^T$$
$$V_{N\bar{N}} = -V_\pi + V_\eta + V_\rho - V_\omega + V_{\sigma_0} - V_{\sigma_1}$$

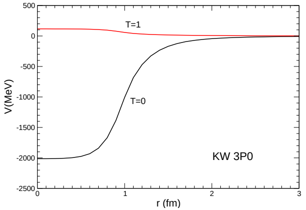
Consequences are dramatic: **$V_{NN}$** repulsive core – due to $\boldsymbol{\omega}$ - change its sign and becomes strongly attractive (in most of the S-T channels) and the tensor force becomes huge There should exist a rich « quasi-bound » and resonant states ....that have never been directly observed during LEAR time (specificallly built to this aim !!!) despite some intriguing « evidences » ...just before it closes.

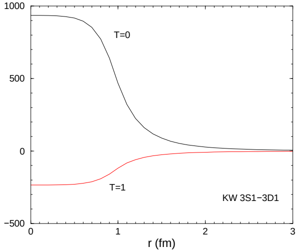
# $\bar{N}N$ interaction: theory

PW examples of G-parity transform for a meson exchange $V_{NN}$



# $\bar{N}N$ interaction: theory

$\chi$**EFT approach**: at first glance EFT philosophy seems to contradict $V_{\bar{N}N}$ physics (Q>M), still some models based on $\chi$EFT have been succesfully developped in the recent years:

X. W. Kang, J. Haidenbauer and U.-G. Meißner, JHEP 1402 (2014) 113 (N2LO)
L.Y.. Dai, J. Haidenbauer, Ulf-G. Meißner, JHEP 2017 (2017) 78 (N3LO)

These potentials are built in p-space and are strongly non-local what makes difficult direct comparison.

In $\chi$EFT, one retains only $\pi$ (at most!) and so the G-parity rule does not apply here in its full glory. The other terms are regularized contact terms whose constants are fited to $N\bar{N}$ phase shifts.

As in traditional models fully phenomenological imaginary part (optical pot.) is added in order to account for $N\bar{N}$ annihilation

Big advantage: possibility of the systematic error estimation

# $\bar{N}N$ interaction: theory

$V_{\bar{N}N}$ constructed in this way does not account the annihilation part:. There are two phenomenological ways to incorporate it: optical and/or coupled channel models

Optical models:

Add to $\mathbf{V_{NN}}$ a complex potential $\boldsymbol{V \rightarrow V_{NN} + W_R - iW_i}$
Which allows us to compute the « annihilation density »

In this description, $N\bar{N}$ particles disapear from the flux, go nowhere and never return: $\mathbf{SS^{+}<1}$ (not unitary approach)
The form of W is « guessed » and its parameters detemined by phenomenology.

Quite successful despite its bare simplicity (probably thanks to the poor data)

- Annihilation dynamics is the same for all (T,S,L,J) states !
- Bad analytic properties (mainly in resonances)
- Depressed wave function due to absence of « re-annihilation »

# $\bar{N}N$ interaction: theory

$V_{\bar{N}N}$ constructed in this way does not account the annihilation part:. There are two phenomenological ways to incorporate it: optical and/or coupled channel models

## Coupled channel models:

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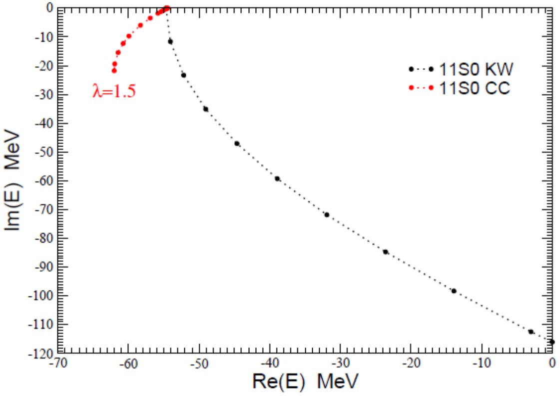


**Fig. 4** Complex energy trajectory of a $^{11}S_0$ state as a function of the annihilation strength in optical a unitary coupled channel models

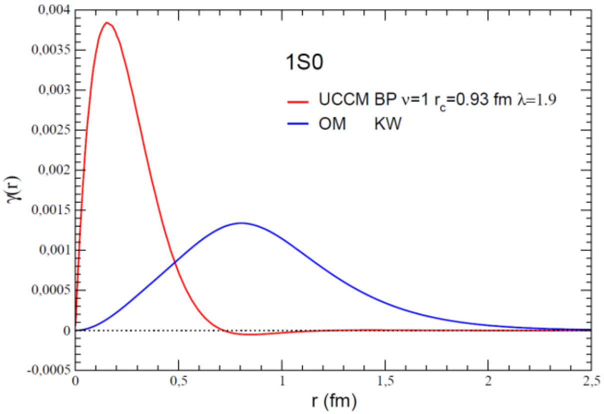


**Fig. 14** Protonium annihilation density for the $^1S_0$ state described with the UCCM (in red) and with OM (in blue). Both models reproduce the same experimental complex level shift $\Delta E$ value of Table 3.

E. Ydrefors, J.C : Eur. Phys. J. **A. 57** (2021) 303

# Protonium: level shifts/widths

| | 1S0 | | 3SD1 | | 1P1 | | 3P0 | | 3P1 | | 3PF2 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | keV | | keV | | meV | | meV | | meV | | meV | |
| | DER | -EI | DER | -EI | DER | -EI | | | | | | |
| DR1 | 0.54 | 0.51 | 0.77 | 0.45 | -26 | 13 | -74 | 57 | 36 | 10 | -4.8 | 15 |
| DR2 | 0.58 | 0.52 | 0.82 | 0.46 | -24 | 14 | -62 | 40 | 36 | 9 | -5.9 | 16 |
| KW | 0.50 | 0.63 | 0.78 | 0.49 | -29 | 13 | -69 | 48 | 29 | 11 | -8.5 | 18 |
| Paris 09 | 0.78 | 0.52 | 0.69 | 0.39 | -29 | 13 | -67 | 60 | 64 | 45 | +7.2 | 13 |
| EFT | 0.44 | 0.59 | 0.77 | 0.58 | / | | -8 | 188 | / | | / | |

Quite good agreement within the models!! And an acceptable comparison with data

## S-waves (eV) (*)

| | Exp [34] | KW | DR1 | DR2 |
|---|---|---|---|---|
| $\Delta E_{^1S_0}$ | 440±075 | 500 | 540 | 580 |
| $\Gamma_{^1S_0}$ | 1200±250 | 1260 | 1020 | 1040 |
| $\Delta E_{^3S_1}$ | 785±035 | 780 | 770 | 820 |
| $\Gamma_{^3S_1}$ | 940±080 | 980 | 900 | 920 |

*J. Carbonell, G. Ihle, J.M. Richard, Z. Phys. **A 334** (1989) 329

# Protonium: level shifts/widths

| | 1S0 | | 3SD1 | | 1P1 | | 3P0 | | 3P1 | | 3PF2 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | keV | | keV | | meV | | meV | | meV | | meV | |
| | DER | -EI | DER | -EI | DER | -EI | | | | | | |
| DR1 | 0.54 | 0.51 | 0.77 | 0.45 | -26 | 13 | -74 | 57 | 36 | 10 | -4.8 | 15 |
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Quite good agreement within the models!! And an acceptable comparison with data

## P-waves (meV) (*)

| | KW | | DR1 | | DR2 | | Exp | |
|---|---|---|---|---|---|---|---|---|
| State | $\Delta E_R$ | $\frac{\Gamma}{2}$ | $\Delta E_R$ | $\frac{\Gamma}{2}$ | $\Delta E_R$ | $\frac{\Gamma}{2}$ | $\Delta E_R$ | $\frac{\Gamma}{2}$ |
| $^1P_1$ | -29. | 13. | -26. | 13. | -24. | 14. | | |
| $^3P_0$ | -69 | 48 | -74 | 57 | -62 | 40 | -139±28 | 60±13 |
| $^3P_1$ | +29. | 11. | +36. | 10. | +36. | 8.8 | | |
| $^3PF_2$ | -8.5 | 18. | -4.8 | 15. | -5.9 | 16. | | |

# General remarks

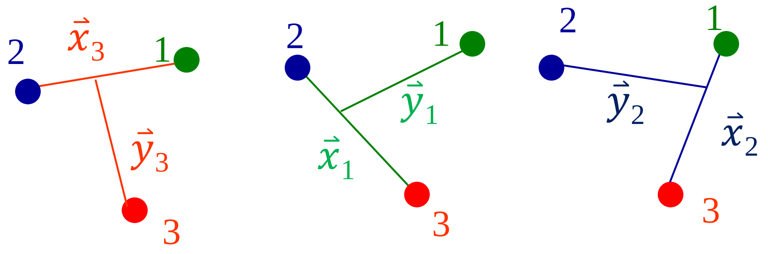
- Trueman (Nucl. Phys. **26** (1961) 57 ) relation allows to express $\bar{p}A$ level shifts in terms of $\bar{p}A$ scattering lengths
  Simple and practical. Energy shifts of excited orbits are interrelated
  This relation supposes $r_{strong} \sim a_{\bar{p}A} \ll a_{Coulomb}$: might be easily broken for heay nuclei but perfectly holds for protonium.
- The $\bar{n}n$ component is present together with $\bar{p}p$ one in wave function. But its effect on the energy is usually small.
- Non-perturbative! Despite the fact $\Delta E \ll E_{Coulomb}$ the strength of the annihilation potential strngly modifies the Coulomb wave funcion in the overlap region with nucleus ($r_{strong} \sim a_{\bar{p}A}$ domain).

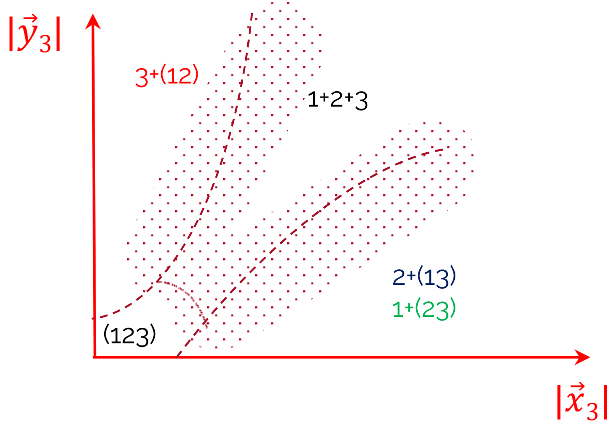
| | Exact | Perturbative |
|---|---|---|
| $^1S_0$ (eV) n=1 | 524-602i | -3030-3150i |
| n=2 | 65.1-77.8i | -379.-394i |
| n=3 | 19.3-23.3i | -112-117i |
| $^1P_1$ (meV) n=2 | -28.1-13.0i | -34.5-7.3i |
| n=3 | -9.9-4.6i | -12.1-2.6i |
| 1D2 (neV) n=3 | -378.-9.9i | -363-6.1i |

*Perturbative result is given by the overlap with the pure Coulomb wf.:* $\Delta E = \langle \Psi_C | V^{st}_{\bar{p}N} | \Psi_C \rangle$

# 2.
# Formalism

# Formalism (3-body)




$$\begin{cases} \psi_1 = G_0 V_1 \Psi \\ \psi_2 = G_0 V_2 \Psi \\ \psi_3 = G_0 V_3 \Psi \end{cases}$$

**or** $(E - H_0 - V_i)\psi_i = V_i \;(\psi_j + \psi_k); \quad (ijk) = (123)$

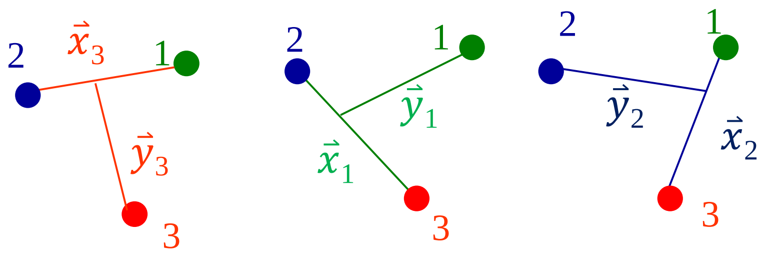
**Faddeev equations: [**L.D. Faddeev, Sov. Phys.—JETP 12, 1014 (1961)**]**

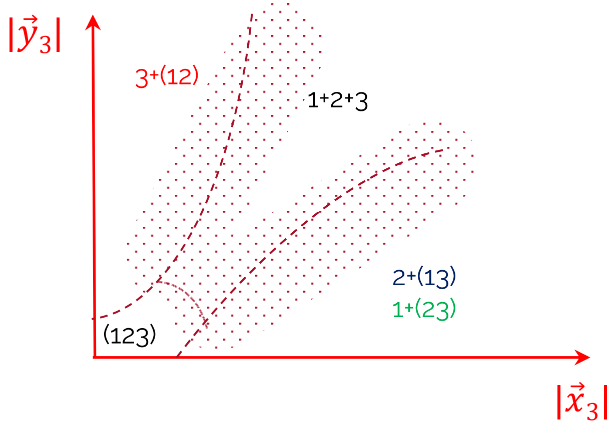
- One gets Schrödinger equation by summing these three eqs with $\Psi = \psi_1 + \psi_2 + \psi_3$
- If particle 3 goes away $V_2$ & $V_1 \to 0$ and thus $\psi_2$ & $\psi_1 \to 0$; therefore $\psi_3 \to \Psi$.

Adapted for scattering problems, since allows to separate assymptotes of (2)+1 particle channels

- Should be modified, when longue range interactions are present

# Formalism (3-body)



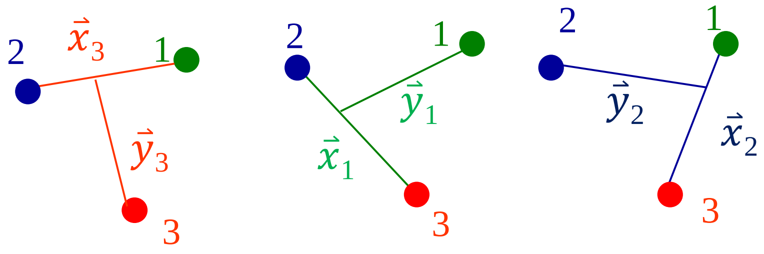

$$V_i = V_i^{short} + V_i^{long}$$

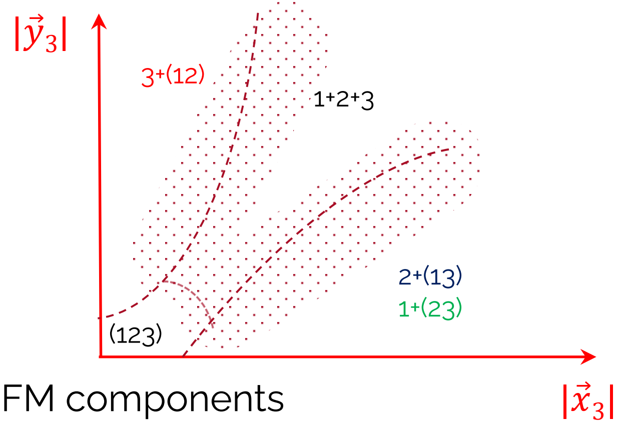
$$\psi_i = \left( H_0 + \sum_{j \neq i}^{3} V_j^{long} \right)^{-1} V_i^{short} \Psi; \quad (ijk) = (123)$$

**Faddeev-Merkuriev equations:[**S.P Merkuriev, S. P.,*Annals of Physics* 130.2 (1980): 395-426**]**

- Specific separation of Coulomb interaction into long and short range parts, in order to guarantee separability of channel assymptotes
- One gets Schrödinger equation by summing these three eqs with $\Psi = \psi_1 + \psi_2 + \psi_3$
- FM equations allow to separate assymptotes of the binary scattering channels
- In general, FM components $\psi_i$ are smoother functions than $\Psi$

# Numerical procedure




- Partial wave expansion to express angular dependence of FM components

$$\psi_i(\vec{x}_i, \vec{y}_i) = \frac{f_{i,l_x l_y}(x_i, y_i)}{x_i y_i} \left[ \mathrm{Y}_{\ell_x}(\hat{x}_i) \otimes \mathrm{Y}_{\ell_y}(\hat{y}_i) \right]_{\mathrm{LM}}$$

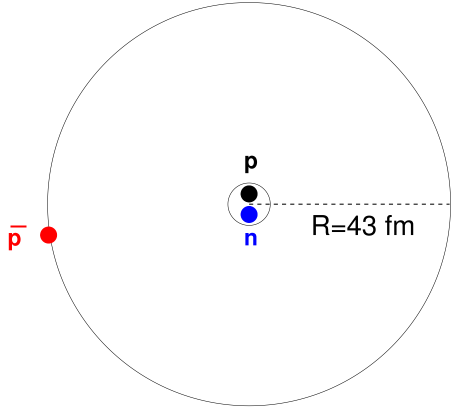
- Lagrange-mesh method to express radial dependence $f_{i,l_x l_y}(x_i, y_i)$ and impose proper boundary conditions
- Iterative linear algebra methods to solve resulting large scale problem of linear equations
- Resonance positions might be found directly by applying complex-scaling method

# 3-body problem: $^{2}\mathrm{H} - \bar{\mathrm{p}}$

NN model dependence:

| | MT13 | AV18 | INOY | I-N3LO | $-\epsilon_n^{(0)}$ (keV) |
|---|---|---|---|---|---|
| S-waves | | $\Delta E$ (keV) | | | |
| $^2S_{1/2}$, n=1 | 2.251-1.0045i | 2.147-1.0440i | 2.214-0.99433i | 2.209-1.0509i | 16.6662 |
| $^2S_{1/2}$, n=2 | 0.294-0.1406i | 0.279-0.1454i | 0.289-0.13892i | 0.288-0.1468i | 4.16655 |
| $^2S_{1/2}$, n=3 | 0.088-0.0433i | 0.084-0.0446i | 0.087-0.04271i | 0.086-0.0451i | 1.85180 |
| P-waves | | $\Delta E$ (meV) | | | |
| $^2P_{1/2}$, n=2 | 49.1-258.0i | -55.3-239.2i | -56.2-241.1i | -58.5-244.0i | 4.16655 |
| $^4P_{1/2}$, n=2 | 24.4-194.8i | 200.2-186.4i | 200.2-188.2i | 200.3-186.1i | 4.16655 |
| $^2P_{1/2}$, n=3 | 16.1-90.6i | -14.0-83.94i | -14.2-84.57i | -15.0-85.61i | 1.85180 |
| $^4P_{1/2}$, n=3 | 8.62-68.4i | 59.4-65.51i | 59.0-66.14i | 58.4-65.36i | 1.85180 |

Table 1: Complex p̄d energy shifts $\Delta E_n$ obtained for different NN interactions and the KW $\bar{N}N$ model.

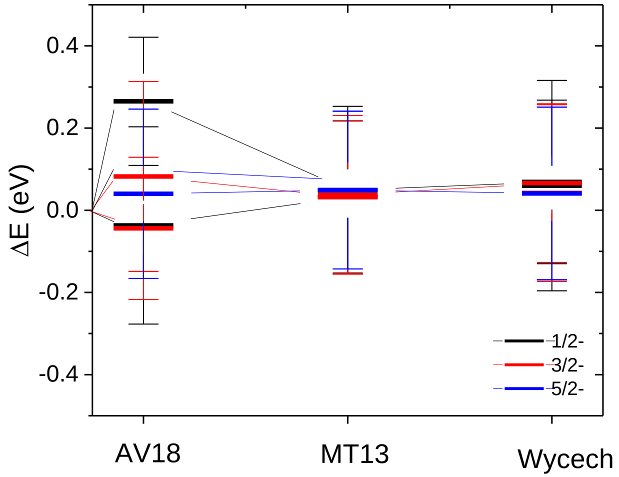



- Quite good agreement between the realistic interaction NN model predictions
- MT13 lacks tensor force (ignoring presence of deuterons quadrupole moment) falls short for P-states

# 3-body problem: $^{2}\mathrm{H} - \bar{\mathrm{p}}$

Comparison with experiment:

| | | MT13/ KW | AV18/ KW | Wycech[1] | Exp[2,3] |
|---|---|---|---|---|---|
| L=0 | $\Delta E\ (eV)$ | 2297 | 2194 | 2170 | 1050+/-250 |
| | $\Gamma\ (eV)$ | 1982 | 2129 | 1250 | |
| L=1 | $\Delta E\ (meV)$ | 26.6 | 22.5 | 52 | |
| | $\Gamma\ (meV)$ | 428 | 414 | 422 | |




Comparison with pionner work (separable approxim


[1]S. Wycech et al, Phys. Lett B152 (1985) 308


Experiment:


[2]D. Gotta et al., Nucl. Phys. A660 (1999) 283

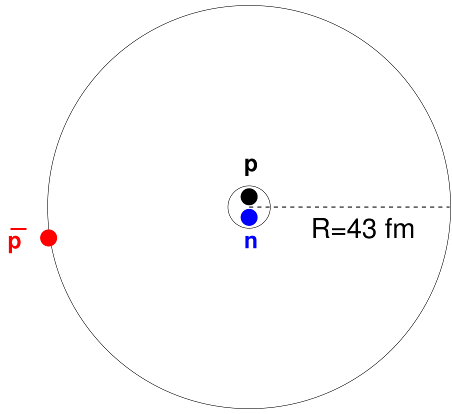
[3]M. Augsburger et al., Phys. Lett. B461 (1999) 417

# 3-body problem: $^2\mathrm{H} - \bar{\mathrm{p}}$

## $\bar{N}N$ model dependence :

| | I-N3LO +KW | | I-N3LO +Jülich | |
|---|---|---|---|---|
| | p̄p | p̄p + n̄n | p̄p | p̄p + n̄n |
| $^2S_{1/2}$, n=1 (keV) | 2.179-1.024i | 2.209-1.050 i | 2.028-0.928i | 2.108-1.085i |
| $^2S_{1/2}$, n=2 (eV) | 284-143i | 288-147 i | 264-128i | 274- 151i |
| $^2S_{1/2}$, n=3 (eV) | 85.3-43.9i | 86.4-45.1 i | 79.1-39.3 | 82.0-46.3i |
| $^4S_{3/2}$, n=1 (keV) | 2.206-0.970i | 2.306-1.045i | 2.027-0.916i | 2.321-1.216i |
| $^4S_{3/2}$, n=2 (eV) | 288-136i | 302-147i | 264-127i | 302- 171i |
| $^4S_{3/2}$, n=3 (eV) | 86.6-41.7i | 90.7-45.2i | 79.1-38.8 | 90.7-52.6i |
| $^2P_{1/2}$, n=2 (meV) | -61.6-210i | -58.5-244 i | -105-194i | 18.7-329i |
| $^4P_{1/2}$, n=2 (meV) | 214-158i | 200-186 i | 200-124i | 171-194i |
| $^2P_{1/2}$, n=3 (meV) | -16.3-73.8i | -15.0-85.6 i | -31.9-68.3i | 13.2-120i |
| $^4P_{1/2}$, n=3 (meV) | 63.5-55.5i | 58.4-65.4 i | 59.1-43.5i | 47.0-63.7i |
| $^2P_{3/2}$, n=2 (meV) | -60.3-201i | -76.2-226i | -81.2-144i | -108-207i |
| $^4P_{3/2}$, n=2 (meV) | 43.6-180i | 35.0-191i | 55.0-137i | 40.4-160i |
| $^2P_{3/2}$, n=3 (meV) | -17.3-68.6i | -21.4-79.5i | -23.3-50.6i | -32.7-72.7i |
| $^4P_{3/2}$, n=3 (meV) | 13.8-63.2i | 10.7-67.0i | 17.8-48.3i | 12.7-56.3i |
| $^4P_{5/2}$, n=2 (meV) | 57.6-185i | 34.7-208i | 7.1-132i | -21.6-205i |
| $^4P_{5/2}$, n=3 (meV) | 18.7-64.8i | 10.7-72.9i | 1.1-46.2i | -9.1-72.1i |

Table 2: Complex level shifts (18) of atomic p̄d states calculated with the same I-N3LO NN interaction (for deuteron) and two different N̄N models: KW [15] and Julich [17].



# 3-body problem: $^2\mathrm{H} - \bar{\mathrm{p}}$

| | MT13 +KW | AV18 +KW | INOY +KW | I-N3LO +KW | I-N3LO +Jülich | Ref. [30] | Exp. |
|---|---|---|---|---|---|---|---|
| L=0 ΔE(eV) | 2297 | 2194 | 2268 | 2274 | 2250 | 2170 | 1050±250 [24, 25, 26] |
| L=0 Γ (eV) | 1982 | 2129 | 1971 | 2095 | 2344 | 1250 | 1100±750 [24, 25, 26] |
| | | | | | | | 2270±260 [25] |
| L=1 ΔE (meV) | 26.6 | 22.5 | 20.7 | 18.2 | -1.1 | 52 | 243±26 [25] |
| L=1 Γ (meV) | 428 | 414 | 420 | 420 | 416 | 422 | 489±30 [25] |

Table 4: Spin-averaged level shifts ($\Delta E_R$) and widths (Γ) compared to LEAR experimental results

- Coupling $\bar{p}p \leftrightarrow \bar{n}n$ has strong contribution for Jülich $\chi$EFT compared to meson exchange potentials
- There is significant $\bar{N}N$ interaction model dependence

# Trueman relation: $^2\text{H} - \bar{\text{p}}$

T.L. Trueman, Nucl. Phys. **26** (1961) 57

| | MT13 +KW | | | |
|---|---|---|---|---|
| | $a_0$ (fm) | $\Delta E_1$ (keV) | $\Delta E_2$ (keV) | $\Delta E$ (keV) |
| $^2S_{1/2}$ n=1 | 1.596-0.8569i | 2.463-1.322i | 2.259-1.014i | 2.251-1.004i |
| $^4S_{3/2}$ n=1 | 1.647-0.8419i | 2.541-1.299i | 2.316-0.987i | 2.321-0.984i |
| | $a_1$ (fm$^3$) | $\Delta E_1$ (meV) | $\Delta E_2$ (meV) | $\Delta E$ (meV) |
| $^4P_{5/2}$ n=2 | 0.450-2.68i | 34.8-207i | 34.8-207i | 26.2-215i |
| | AV18 +KW | | | |
| | $a_0$ (fm) | $\Delta E_1$ (keV) | $\Delta E_2$ (keV) | $\Delta E$ (keV) |
| $^2S_{1/2}$ n=1 | 1.505-0.8779i | 2.323-1.355i | 2.155-1.057i | 2.147-1.044i |
| $^4S_{3/2}$ n=1 | 1.59-0.8771i | 2.541-1.354i | 2.257-1.039i | 2.218-1.075i |
| | $a_1$ (fm$^3$) | $\Delta E_1$ (meV) | $\Delta E_2$ (meV) | $\Delta E$ (meV) |
| $^4P_{5/2}$ n=2 | 0.469-2.57i | 36.4-199i | 36.4-199i | 39.9-204i |

Table 5: Atomic level shifts, calculated from p̄d scattering lengths ($a_0$ and $a_1$) employing Trueman relations at first order ($\Delta E_1$) and second order ($\Delta E_2$) are compared with the values obtained from direct binding energy calculations ($\Delta E$).

- Trueman relation works well for spin uncoupled states, but is broken by long-ranged $r^{-3}$ interactions (quadrupole & magnetic moment) for spin-coupled states.
- Magmetic Moment interaction terms should be considered to describe level shifts in l>0 states.

# Annihilation densities: $^{2}\mathrm{H} - \bar{\mathrm{p}}$

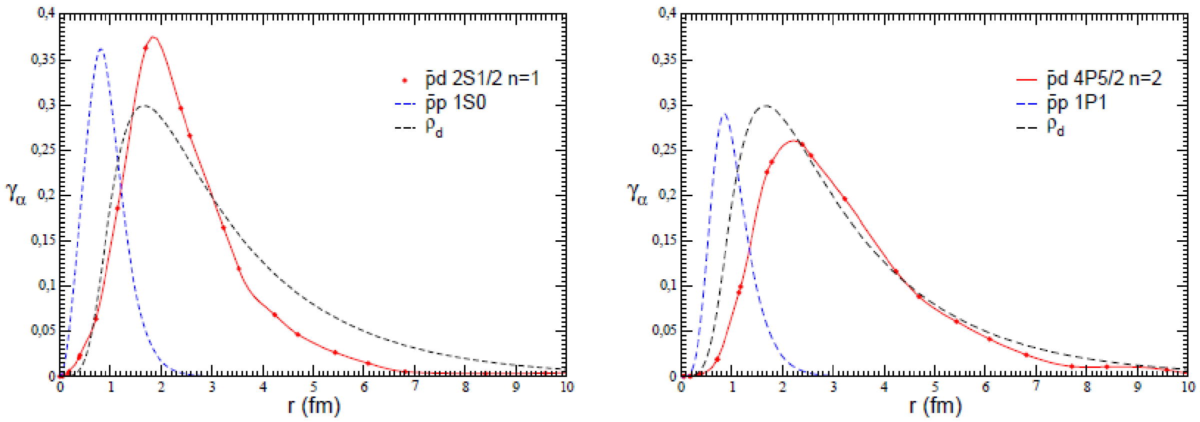



Figure 3: p̄d annihilation densities $\gamma_a$ for the 2S1/2 (left panel) and 4P5/2 (right panel) states calculated with the MT13+KW model. They are compared with the p̄p $^1S_0$ and $^1P_1$ $\gamma_a$'s in protonium and with corresponding deuteron matter density $\rho_d$.

- Annihilation is peripheral for P-wave, however it is not a case for S-wave.

# Conclusions perspectives

- Antiproton interface with matter is little explored and open field for exploration of new phenomena

- There are vast space for improving our understanding of $\bar{N}A$ interactions & dynamics, startind by $\bar{N}N$ interaction:
  ✓Inclusion of EM interaction in current models
  ✓Curing model dependence
  ✓More advanced models for annihilation

- Trueman relation might be applied to simplify calculation of level broadenings/shifts, but should be modified to take into account secondary EM long range interactions : quadrupole-charge, magnetic moments, ..
- Annihilation is peripheral for L>0 waves, however it is not a case for S-wave.