EFT-inspired two-body model of ^{11}Be accounting for the core excitations

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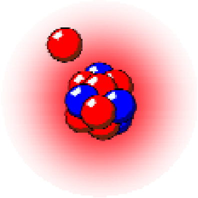
July 6, 2022

Overview

1. Introduction/motivation
   - Halo Nuclei
   - Halo EFT
   - Breakup reactions with halo nuclei
2. Coupled-channels calculations of ^{11}Be
   - $\frac{1}{2}^{+}$ states
   - $\frac{1}{2}^{-}$ states
3. Summary/perspectives

Halo nuclei

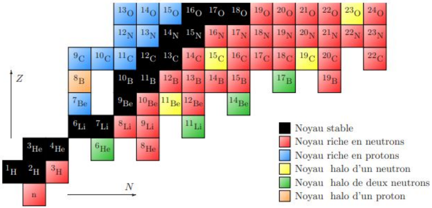
- Neutron-rich nuclei
- Light nuclei, large matter radius
  $\rightarrow$ due to one or two loosely-bound neutrons (low S_n or S_{2n})
- Clusterised structure: neutrons can tunnel far from the core
  $\rightarrow$ halo-nucleus $\equiv$ a compact core + valence neutron(s)



- Halos when low centrifugal barrier (low ℓ)

Where to find them ? How to study them ?

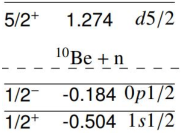
- Exotic nuclear structures found far from stability



- One-neutron, two-neutron and proton halos (less probable)
- **Our case study** : ^{11}Be $\equiv$ ^{10}Be + n
- Short-lived $[\tau_{1/2}(^{11}\text{Be}) = 13\text{ s}] \rightarrow$ study via reactions (e.g. **breakup**)
- **Breakup** of ^{11}Be $\equiv$ dissociation of halo (**n**) from core (**^{10}Be**) by interaction with target

$^{11}Be \equiv {}^{10}Be(0^+) + n$

- **Assumption**: ^{10}Be always in its 0^+ ground state



^{11}Be spectrum

- angular momenta ℓ and j fixed the spin and parity of ^{11}Be states:
  - $\frac{1}{2}^+$ ground state in s1/2
  - $\frac{1}{2}^-$ first excited state in p1/2
  - $\frac{5}{2}^+$ first excited state in d5/2

Single-particle description:

$$H_0(\mathbf{r}) = T_{\mathbf{r}} + V_{cn}(\mathbf{r})$$

Halo EFT description of ^{11}Be

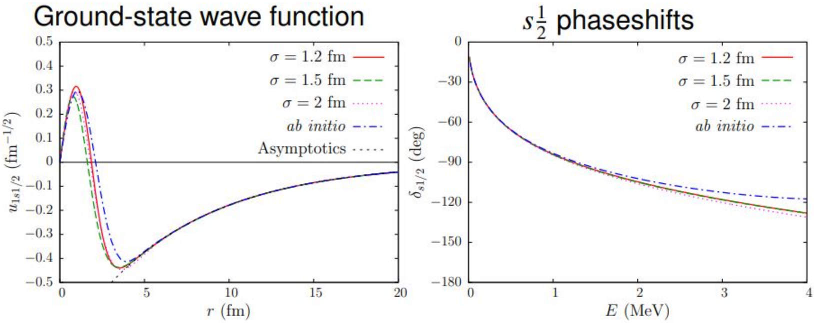
- Halo EFT : separation of scales [in energy/distance]
  $\rightarrow$ expansion parameter $\epsilon = \frac{R_{core}}{R_{halo}}$ or $\sqrt{\frac{S_{1n}}{E_{2^+}}} \simeq 0.4$
  $\rightarrow$ no inclusion of 2^+ state of ^{10}Be core at E(2^+)=3.4 MeV
  $\rightarrow$ expansion along ϵ of low-energy behaviour (long-range physics)
  [C. Bertulani, et al. NPA 712, 37 (2002)]
  [Hammer, Ji, Phillips, JPG 44, 103002 (2017)]
- Effective potentials in each partial wave **lj** - narrow Gaussians @NLO

$$V_{cn}(r) = V_{lj}^{(0)} e^{-\frac{r^2}{2\sigma^2}} + V_{lj}^{(2)} r^2 e^{-\frac{r^2}{2\sigma^2}}$$

$V_{lj}^{(0)}$ and $V_{lj}^{(2)}$ fitted to reproduce [for bound states]:
$\rightarrow \epsilon_{nlj}$ (@ LO)
$\rightarrow \epsilon_{nlj}$ (@ LO) and ANC$_{nlj}$ (@ NLO)
σ:= unfitted parameter $\rightarrow$ evaluates sensitivity to short-range physics
[Capel, Phillips, Hammer, PRC 98, 034610 (2018)]

$s_{\frac{1}{2}}$: @NLO - single-particle calculations of ^{11}Be

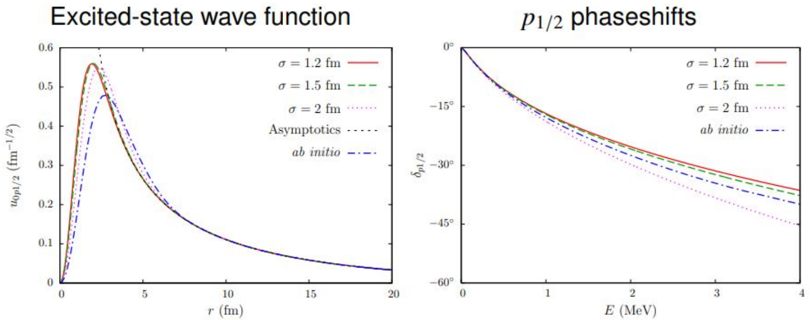
- $s_{\frac{1}{2}}$: @NLO potentials **fitted to** reproduce *ab initio* data:
  → **[Calci et al., PRL 117, 242501 (2016)]**
  → $\epsilon_{1s\frac{1}{2}}$=-0.503 MeV and $\text{ANC}_{1s\frac{1}{2}} = 0.786\ \text{fm}^{-1/2}$



- Wave functions: different interiors but **same asymptotics** as *ab initio*
- $\delta_{s1/2}$: $\forall\ \sigma$, **good agreement** with *ab initio* up to 1.5 MeV

$p_{\frac{1}{2}}$: @NLO - single-particle calculations of ^{11}Be

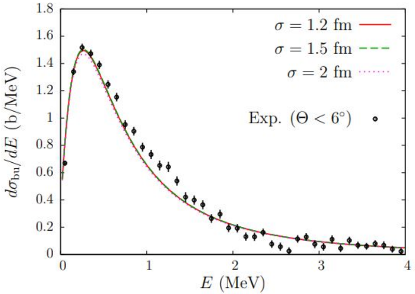
- $p_{\frac{1}{2}}$: @NLO potentials **fitted to** reproduce *ab initio* data:
  $\rightarrow \epsilon_{0p\frac{1}{2}}$=-0.184 MeV and $\mathrm{ANC}_{0p\frac{1}{2}} = 0.129\ \mathrm{fm}^{-1/2}$



- Wave functions: different interiors but **same asymptotics** as *ab initio*
- Larger variation in $\delta_{p1/2} \rightarrow \sigma$-dependency !!
  $\delta_{p1/2}$: $\forall\ \sigma$, **fair agreement** with *ab initio* only up to 0.5 MeV !
  $\rightarrow$ Is there a model to do better ?

@NLO - Coulomb breakup: ^{11}Be+Pb $\to$ ^{10}Be+n+Pb

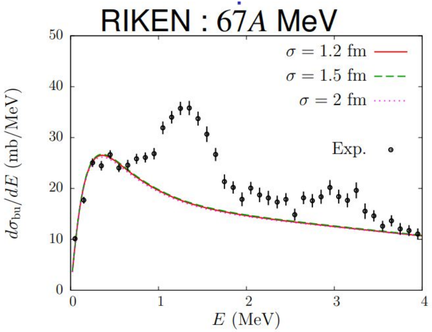
RIKEN : 69A MeV



Exp : [Fukuda *et al.* PRC 70, 054606 (2004)]
Th. : [P.C., Phillips & Hammer, PRC 98, 034610]

- $\forall\ \sigma$ (effective potentials), very similar results:
  $\to$ wave functions, $\neq$ interiors, **same asymptotics** and **same results**
  $\to$ reaction sensitive to **ANC** and **phaseshifts**
  $\to$ reaction is **peripheral**
  $\to$ adequacy of Halo-EFT to reproduce long-range physics

Nuclear breakup: ^{11}Be+C $\rightarrow$ ^{10}Be+n+C



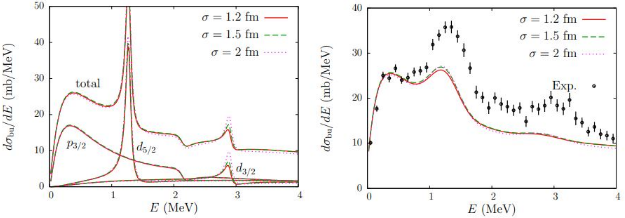
Exp : [Nakamura *et al.* PRC 70, 054606 (2004)]
Th. : [P.C., Phillips & Hammer, PRC 98, 034610]

- $\forall\ \sigma$ (effective potentials), very similar results once again: reaction **still peripheral**
- Good order of magnitude but missing breakup strength at the $\frac{5}{2}^+$ and $\frac{3}{2}^+$ resonances
  $\rightarrow$ need for a better description of the continuum

Beyond NLO - Nuclear breakup: ^{11}Be+C $\to$ ^{10}Be+n+C @67 AMeV

- Beyond NLO:= NLO + resonances
  $\to$ potentials fitted to reproduce *ab initio* data (E_R, Γ)
- $\forall\ \sigma$ (effective potentials), similar breakup cross sections

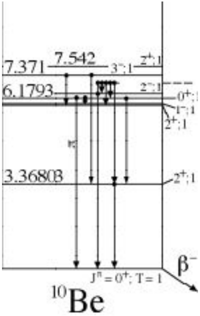
Total breakup cross section and dominant contributions | Folded with energy resolution [Fukuda *et al.* PRC 70, 054606 (2004)]



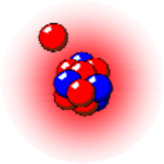
- Missing breakup strengths at resonances $\frac{5}{2}^+$ and $\frac{3}{2}^+$
- Single-particle description is not sufficient !
- ^{10}Be (2^+) = the missing degree of freedom !
  [Moro, Lay PRL 109, 232502 (2012)]

^{10}Be spectrum

- **Idea:=** include core excitation in Halo-EFT
  → allowing the ^{10}Be core to be excited to its 2^+ state
  → adding a new degree degree of freedom to Halo EFT



Adding core degrees of freedom



- **Two-body** Hamiltonian of the projectile:

$$H_0(\mathbf{r}, \xi) = T_{\mathbf{r}} + V_{cn}(\mathbf{r}, \xi) + h_c(\xi)$$

  where :
  - ξ : core internal d.o.f
  - $h_c(\xi)$: intrinsic Hamiltonian of the core
  - $V_{cn}(\mathbf{r}, \xi)$: effective c-n interaction

  $\rightarrow$ contains a non central part for the core admixtures
- Model for $h_c(\xi)$?

Rotational model - $V_{cn}(\mathbf{r}, \xi)$

- 1^{st} idea of rotational model for ^{11}Be using mean field potentials: $\rightarrow$ [**F.M. Nunes, et al. NPA, 596 (1996)**]
- **Particle-rotor model [Bohr and Mottelson (1975)]:** $\rightarrow$ core with a permanent quadrupole deformation
- Multipolar expansion of V_{cn} assuming **small deformation lengths**. **Coupling interaction:**

$$V_{cn}(\mathbf{r}, \xi) = V_{cn}(r; \sigma_0) + \beta_2 \sigma_c Y_2^0(\hat{r}) \frac{d}{d\sigma_c} V_{cn}(r; \sigma_c)$$

$V_{cn}(\mathbf{r}, \xi)$: from deforming the NLO Gaussian Halo-EFT potentials
σ_0, σ_c: widths of Gaussian coupling
$\beta_2 \sim$ low-energy constant (LEC)

Coupled-channels equations to be solved: with **R-Matrix method on a Lagrange mesh**

- Core eigenfunctions satisfy the equations:

$$h_c(\xi)\phi^I_{MK}(\xi) = \epsilon\phi^I_{MK}(\xi)$$

with $\phi^I_{MK}(\xi) \propto D^I_{MK}$:= **Wigner rotational matrices**

- $\psi_\alpha(r)$ = solutions of the set of coupled-channel equations, $\alpha = \{n\ell jI\}$

$$(\epsilon^{^{11}Be}_{J^\pi} - \epsilon^{^{10}Be}_{\alpha})\psi_\alpha(r) = [T^\ell_r + V_{\alpha\alpha}(r)]\psi_\alpha(r) + \sum_{\alpha'\neq\alpha} V_{\alpha\alpha'}(r)\psi_{\alpha'}(r)$$

with

$$V_{\alpha\alpha'}(\mathbf{r}) = \langle Y_\alpha(\hat{\mathbf{r}})\phi_\alpha(\xi)|V_{cn}(\mathbf{r},\xi)|Y_{\alpha'}(\hat{\mathbf{r}})\phi_{\alpha'}(\xi)\rangle$$

the coupling interaction being:

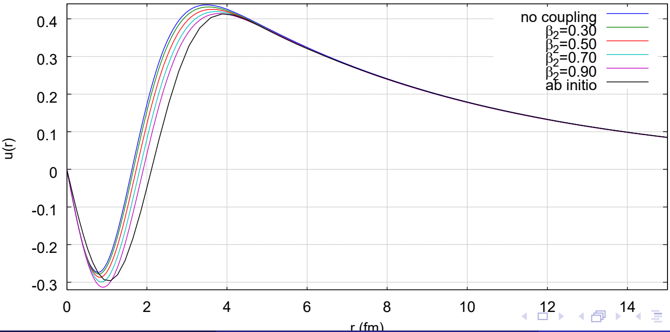
$$V_{cn}(\mathbf{r},\xi) = V^{NLO}_{cn}(r;\sigma_0) + \beta_2\sigma_c Y^0_2(\hat{r})\frac{d}{d\sigma_c}V^{NLO}_{cn}(r;\sigma_c)$$

- **2 $\neq$ types of calculations**: $\sigma_c = \sigma_0 = \sigma_c$ and $\sigma_c \neq \sigma_0$

^{11}Be: $\frac{1}{2}^{+}$ states

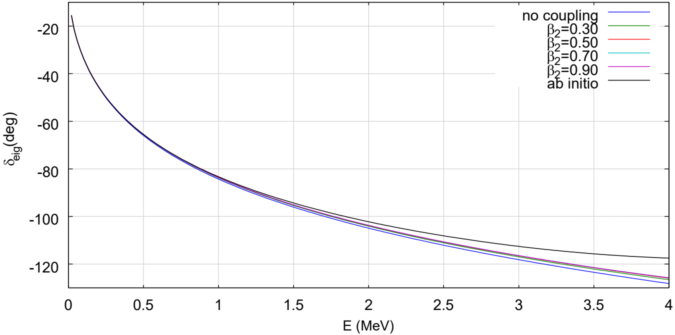
Bound state - coupled-channels ^{11}Be - σ_c=σ_0 [=1.5 fm]

- Ground state of ^{11}Be
- $s_{\frac{1}{2}}$: β_2, @NLO potentials **fitted to** reproduce *ab initio* data:
  $\rightarrow$ **[Calci et al., PRL 117, 242501 (2016)]**
  $\rightarrow$ $\epsilon_{1s\frac{1}{2}}$=-0.503 MeV and $\text{ANC}_{1s\frac{1}{2}} = 0.786$ fm$^{-1/2}$
- Wave functions: different interiors, **same asymptotics** as *ab initio* but **no real improvement** compared to single-particle description
- Similar results obtained for σ_0=1.2 and 2.0 fm



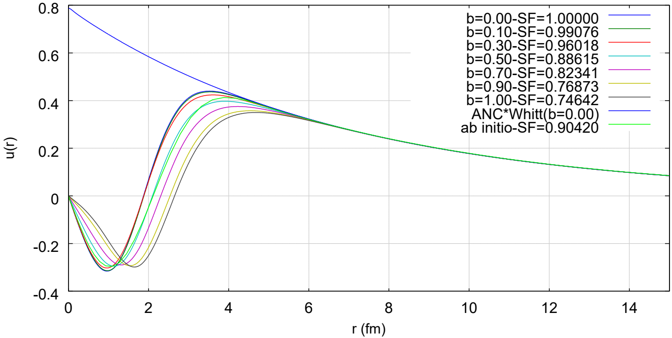
Phaseshift - coupled-channels ^{11}Be - σ_c=σ_0 [=1.5 fm]

- $s_{\frac{1}{2}}$: β_2, @NLO potentials **fitted to** reproduce *ab initio* data:
  $\rightarrow$ **[Calci et al., PRL 117, 242501 (2016)]**
- $\delta_{s1/2}$: $\forall\ \sigma$, **good agreement** up to 1.5 MeV with *ab initio* but **no improvement** compared to single-particle description
- Similar results obtained for σ_0=1.2 and 2.0 fm



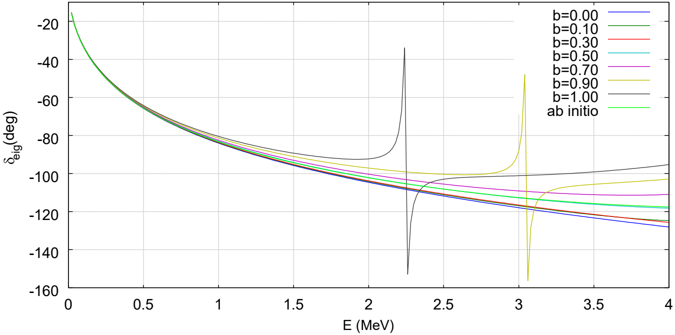
Bound state - coupled channels ^{11}Be - $\sigma_c \neq \sigma_0$

- Let us try $\sigma_c \neq \sigma_0$, i.e. **coupling acting at larger distances**
- $s_{\frac{1}{2}}$: β_2, @NLO potentials **fitted to** reproduce *ab initio* data:
  $\rightarrow \epsilon_{1s\frac{1}{2}}$=-0.503 MeV and $\text{ANC}_{1s\frac{1}{2}} = 0.786$ fm$^{-1/2}$
- Here, σ_c=2.2fm, σ_0=1.2fm
- $\forall$ β_2, $\neq$ interiors, **same asymptotics** as *ab initio*
- $\exists$ β_2 (=0.5) which gives really good agreement with *ab initio*



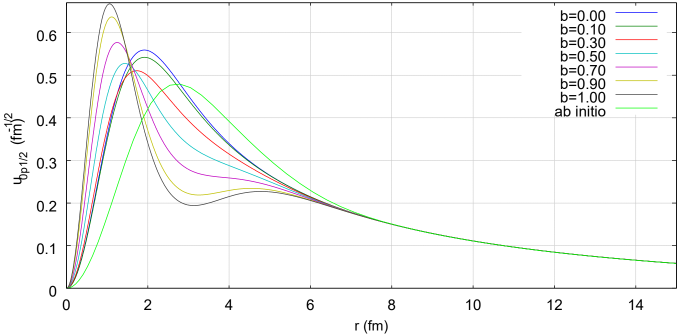
Phaseshift - coupled-channels ^{11}Be - $\sigma_c \neq \sigma_0$

- $s_{\frac{1}{2}}$: β_2, @NLO potentials **fitted to** reproduce *ab initio* data:
- $\sigma_c \neq \sigma_0 \rightarrow$ significative improvement of the phaseshift:
  $\rightarrow \exists\ \beta_2$ (=0.5) which reproduces **ab initio** up to 4MeV ! (see below)
  Also, best $\beta_{wf} \simeq$ best $\beta_{\delta_{eig}}$
  $\rightarrow \sigma_0$=1.2fm $\rightarrow \simeq$ 1.8fm : same results, agreement up to 3 MeV
- Zone of interest: $\sigma_0 \ll \sigma_c \sim 2.2$fm [**rms radius of ^{10}Be core**]



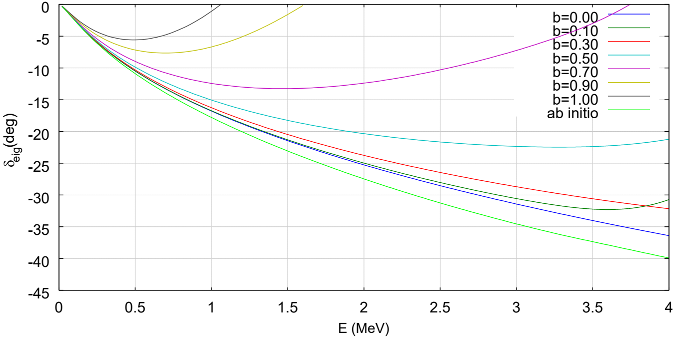
^{11}Be: $\frac{1}{2}^{-}$ states

- 1^{st} excited state of ^{11}Be
- $\sigma_c \neq \sigma_0$ (no improvement of wave functions/ phaseshift otherwise)
- $p_{\frac{1}{2}}$: β_2, @NLO potentials **fitted to** reproduce *ab initio* data:
  $\rightarrow \epsilon_{0p\frac{1}{2}}$=-0.184 MeV and $\text{ANC}_{0p\frac{1}{2}} = 0.129 \text{ fm}^{-1/2}$
- Here, σ_c=2.2fm, σ_0=1.2fm [similar results for other (σ_c, σ_0) values]
- **No improvement of the wavefunction**



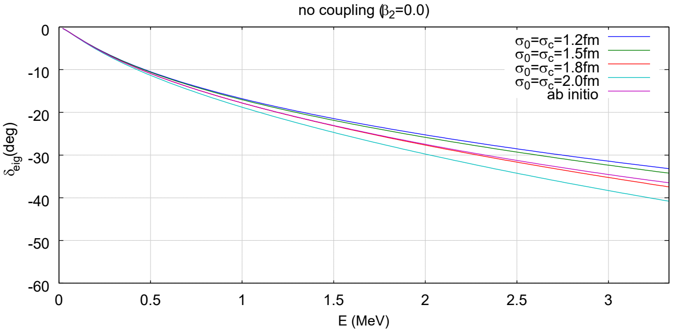
Phaseshift - coupled-channels ^{11}Be - $\sigma_c \neq \sigma_0$

- $p_{\frac{1}{2}}$: β_2, @NLO potentials **fitted to** reproduce *ab initio* data:
- Here, σ_c=2.2fm, σ_0=1.2fm [similar results for other (σ_c, σ_0) values]
- **No improvement of the phaseshift**
- What is happening for the $\frac{1}{2}^-$ states ? What can we improve here ?



σ-dependency in $\frac{1}{2}^-$ states ?

- $p_{\frac{1}{2}}$: @NLO potentials **fitted to** reproduce *ab initio* data
- Single-particle description
- σ=1.8fm, improvement phaseshift up to 3 MeV (0.5 MeV for other σ)
  $\rightarrow$ **σ-dependency**
  $\rightarrow$ **no improvement when adding the coupling with the core**
  $\rightarrow$ Work in progress to adress this question !



Summary/perspectives

We have developed a model that:

1. includes **core excitations** perturbatively in Halo-EFT
2. $\frac{1}{2}^{+}$**states:** $\sigma_c \simeq$ ^{10}Be rms radius
   $\rightarrow$ better description of *ab initio* wave functions and phaseshifts
3. $\frac{1}{2}^{-}$**states:** coupling with the core does not improve anything
   $\exists\ \sigma$ in agreement with *ab initio* till 3 MeV (σ-dependency)
   $\rightarrow$ Should we go to N2LO to remove this σ-dependency ?
4. **Future:** using our model for breakup calculations