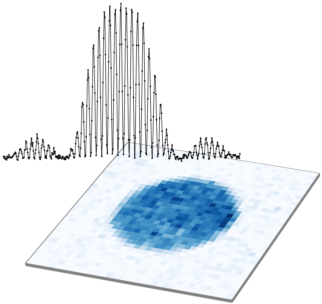
Measurements of Tan's contact in quantum gases



Jérôme Beugnon

Laboratoire Kastler Brossel. Collège de France

COLLÈGE DE FRANCE 1530 · SORBONNE UNIVERSITÉ · ENS · PSL RESEARCH UNIVERSITY PARIS · cnrs · QUANTERA · Agence Nationale de la Recherche ANR · erc

Outline

A - Quantum gases

B - Tan's contact

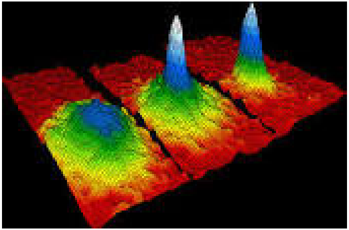
C - Overview of contact measurements in dilute gases

D - Measurement of Tan's contact in a 2D Bose gas

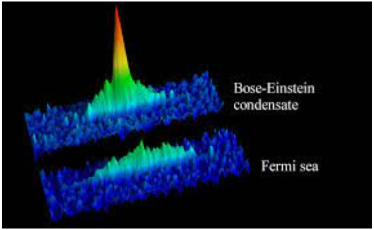
(Nat. Commun. **12** 760 (2021))

Quantum gases

Bose-Einstein condensates (1995)



Degenerate Fermi gases (2001)



Typical scales:

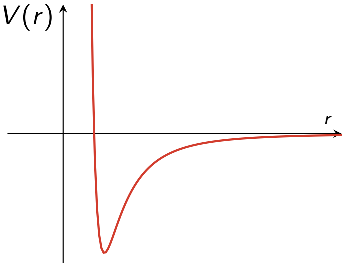
$n \sim 10^{20}$ atoms/m^3, $T_c \sim 1\,\mu\mathrm{K} \sim 0.1\,\mathrm{neV}$

$N \sim 1 \rightarrow 10^5$ atoms, $L \sim 10\,\mu\mathrm{m}$

Probing tools:

Density distribution, momentum distribution, internal state control ($\uparrow$, $\downarrow$,...)

Quantum gases: Microscopic physics



Mostly two-body interactions
Fermions $\equiv (|\uparrow\rangle, |\downarrow\rangle)$

van der Waals potential (range b)

Low-energy collisions ($kb \ll 1$):
Only $s-$wave $\rightarrow$ scattering length a

$a \sim b \sim \text{nm} \ll d \equiv n^{-1/3} \sim \mu\text{m}$

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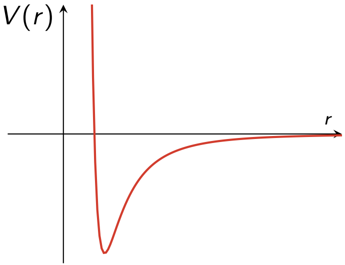

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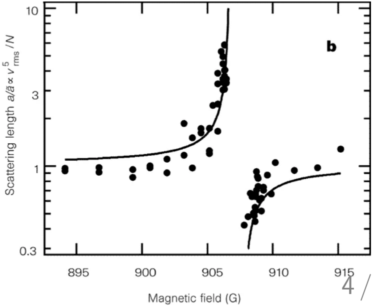
a tunable by Feshbach resonances
(Magnetic tuning of a resonance scattering).

Strongly interacting regime
$a \gg b$, $a \sim d \equiv n^{-1/3}$
$a \rightarrow \infty \equiv$ unitarity

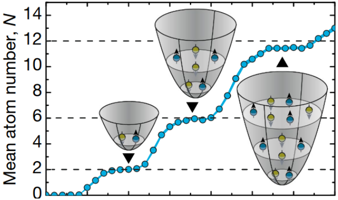
Nature **392** 151 (1998)



Quantum gases: Few-body physics

Few-fermion system:

Nature 587, 583 (2020)



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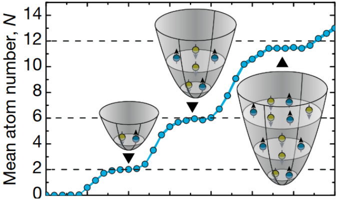

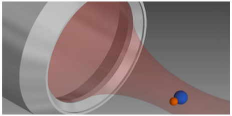

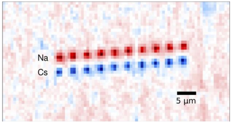

J. Zhang, APS arXiv:2112.00991 (Harvard)

- Formation of dimers
- Atom-molecule, molecule-molecule interactions
- Quantum chemistry

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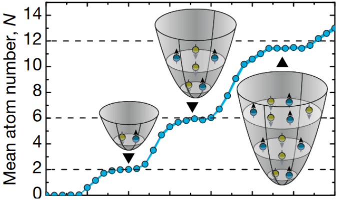

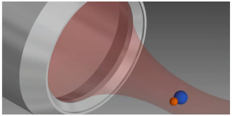

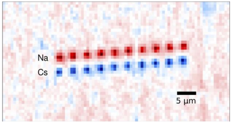

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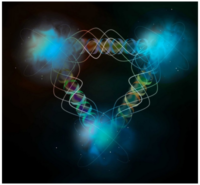
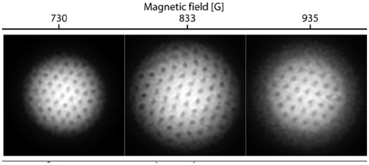
Three-body physics: Efimov states

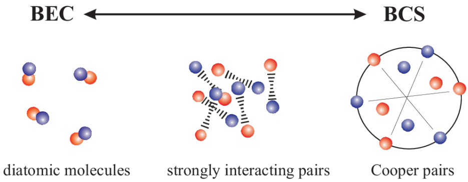


Quantum gases: Many-body physics

Strongly interacting fermions: BEC-BCS crossover



Nature **435**, 1047 (2005).



PhD Regal

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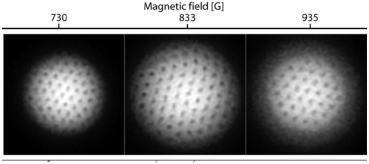

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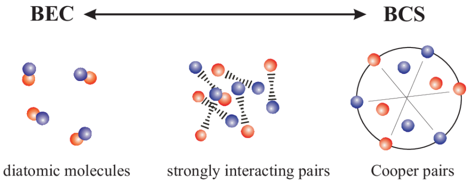

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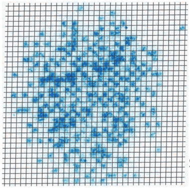
Fermi-Hubbard/Bose-Hubbard physics

Quantum gas microscopes



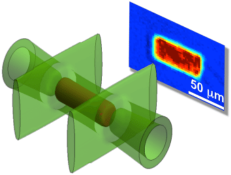
Science **353**, 1253 (2016)

Quantum gases: Uniform gas in box potentials

Recently developped optical box potentials ⇒ Homogeneous Fermi or Bose gases

Bose 3D

Phys. Rev. Lett. **112**, 040403 (2014)



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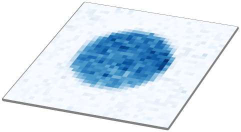
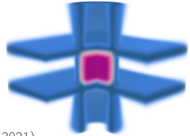
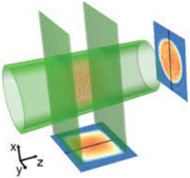


Molecules 3D



Phys. Rev. Research **3**, 033013 (2021)

Fermi 3D



Phys. Rev. Lett. **118**, 123401 (2017)

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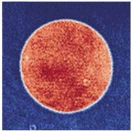

Phys. Rev. Lett. **120**, 060402 (2018)

Outline

Tan's contact

Contribution of short-range physics to thermodynamics given by Tan's contact C

Valid if short-range physics only described by a scattering length a.
Same form whatever is the interaction strength, temperature

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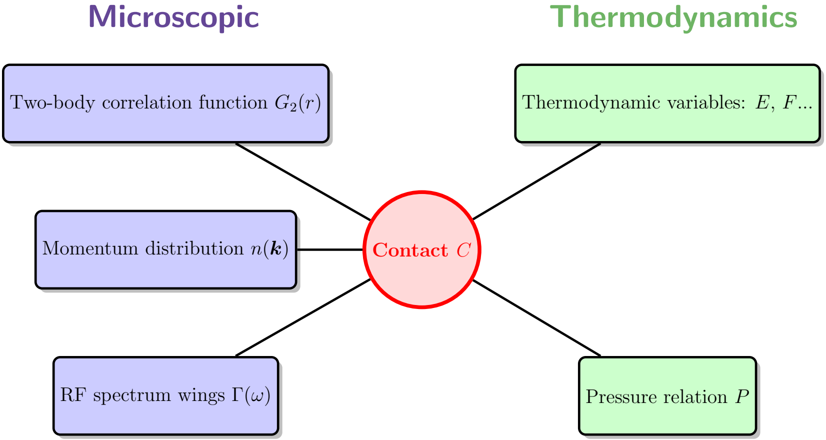

Contact: thermodynamics

Thermodynamic approach for a simple fluid (S, V, N):

$$\mathrm{d}E = T\mathrm{d}S - P\mathrm{d}V + \mu\mathrm{d}N$$

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$$\mathrm{d}E = T\mathrm{d}S - P\mathrm{d}V + \mu\mathrm{d}N + \mathcal{C}\mathrm{d}a$$

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A definition of Tan's contact: (fermions) Tan Ann. Phys. **323** 2971 (2008)

$$C = \frac{4\pi m a^2}{\hbar^2}\left.\frac{\partial E}{\partial a}\right|_{S,V,N} = \frac{4\pi m a^2}{\hbar^2}\left.\frac{\partial F}{\partial a}\right|_{T,V,N}$$

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Pressure relation

$$PV = \frac{2}{3}E + \frac{\hbar^2 C}{12\pi m a}$$

Contact: microscopic properties

Two-body correlation function:

$b \ll r \ll a$. Two-body scattering state $\psi(r) \propto \frac{1}{r}$

$$G_{2,\uparrow,\downarrow}(r) = \langle \hat{\Psi}^\dagger_\uparrow(r) \hat{\Psi}^\dagger_\downarrow(0) \hat{\Psi}_\downarrow(0) \hat{\Psi}_\uparrow(r) \rangle \propto \frac{C}{r^2}$$

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Radiofrequency spectrum wings:

Radiofrequency spectroscopy to an auxiliary state.
Transfer rate:

$$\Gamma(\omega) \propto \frac{C}{(\omega-\omega_0)^{3/2}}$$

(equivalent to $1/k^4$ law. $\hbar|\omega-\omega_0| \gg \frac{\hbar^2}{ma^2}$)

Contact: how to link macro and micro ?

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Hellmann-Feynman theorem

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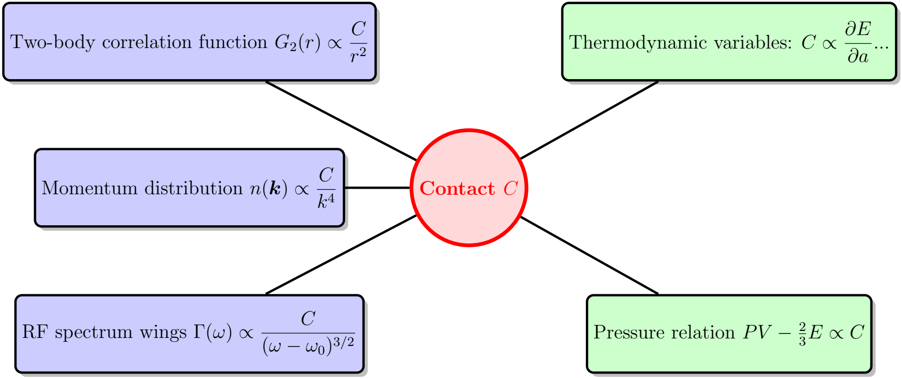
In our case :

$$\langle \frac{\partial \hat{H}}{\partial a} \rangle = \langle \frac{\partial \hat{H}_{\mathrm{int}}}{\partial a} \rangle \propto \int \delta U_{\mathrm{int}} G_{2,\uparrow,\downarrow}(r) \mathrm{d}^3 r \propto C$$

Contact: summary



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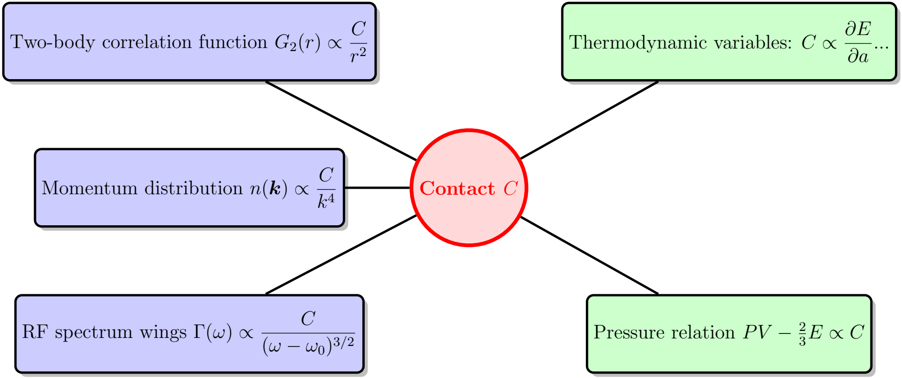

Generalization

- Three-body contact
- *p*-wave contact
- ...

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(Nat. Commun. **12** 760 (2021))

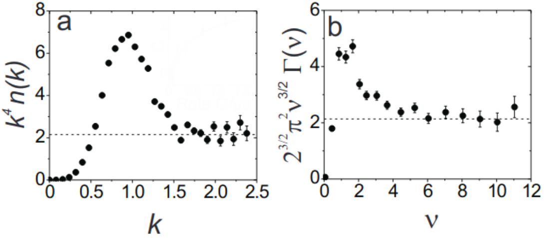
Tan's contact in 3D Fermi gases

First experiments:

Momentum distribution

+ RF spectroscopy:

Phys. Rev. Lett. **104**, 235301 (2010)



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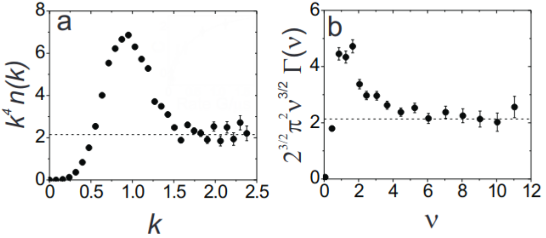
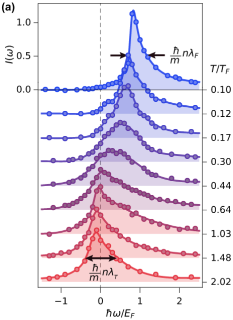
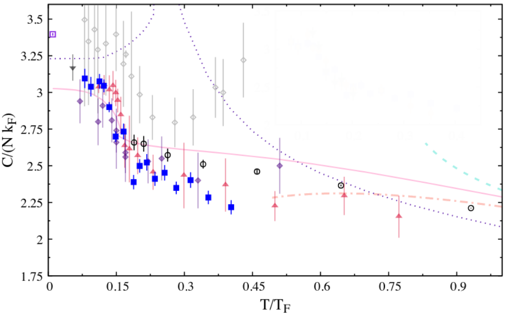


RF spectroscopy at unitarity in a box potential:





Phys. Rev. Lett. **122**, 203402 (2019) & Phys. Rev. Lett. **122**, 203401 (2019) Phys. Rev. Lett. **125**, 043402 (2020)

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Three-body physics (Efimov) could contribute
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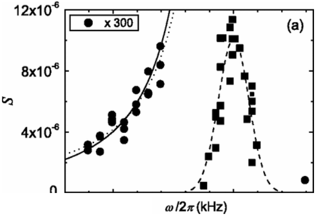
Phys. Rev. Lett. **108**, 145305 (2010)



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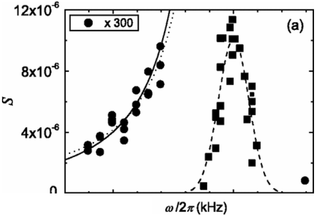
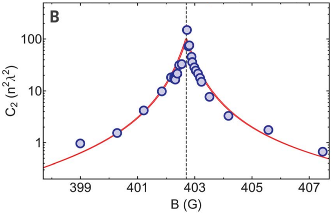


Uniform thermal unitary

Ramsey spectroscopy

Science **355**, 377 (2017)



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- No long-range order at the thermodynamic limit at $T \neq 0$
- Interactions $\Rightarrow$ Normal-to-superfluid transition of Kosterlitz-Thouless type
- Scale-invariant system: s-wave interactions described by dimensionless $\tilde{g}$
  Thermodynamics with dimensionless functions: $\mathcal{D} = n\lambda_T^2 = f(\mu/k_B T, \tilde{g})$
- Critical point $\mathcal{D}_c \approx \ln(380/\tilde{g})$ (n: 2D density, λ_T: thermal wavelength)

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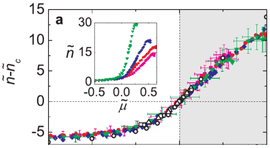
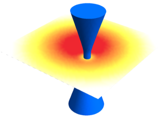
Some recent experiments

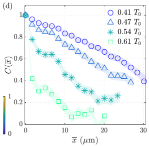
Equation of state	Superfluidity	Coherence	Superfluid fraction



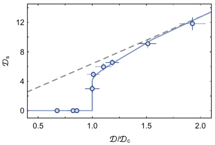
PRL **107** 130411
Nature **470** 236



Nat. Phys. **8** 645

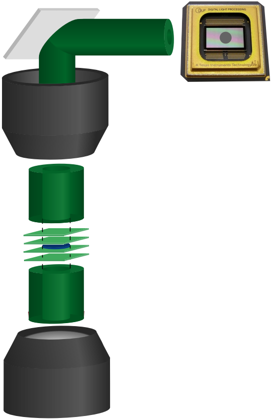


PRL **128** 250402



Nature **594** 191

Our setup



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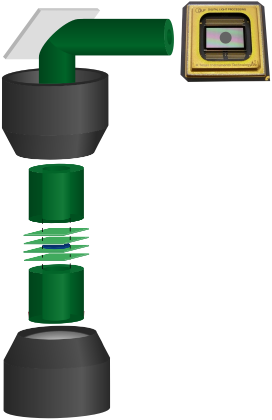

- ^{87}Rb
- $\omega_z \approx 2\pi \times 4$ kHz
  $\Rightarrow \tilde{g} \sim 0.16$
- $T \approx$ 10-100 nK
- $n_{2D} \approx 100\,\mu m^{-2}$
- Atom number $\approx 10^5$
- Spatial light modulator (DMD) to shape the potential. (10 kHz refresh rate)

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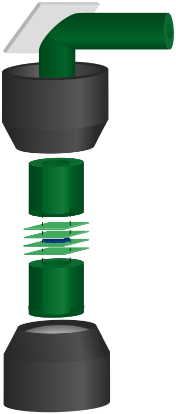

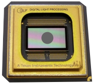

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Ville et al. PRA **95**, 013632 (2017)
Zou et al. J. Phys. B **54** 08LT01 (2021)

Theory for contact in 2D: State-of-the-art

- $T = 0$

$$C = \frac{8\pi m a^2}{\hbar^2} \frac{\langle \hat{H}_{\rm int} \rangle}{a} \xrightarrow{\text{Contact interactions } (\hat{\delta}(\boldsymbol{r}-\boldsymbol{r}'))} C = C_0 \equiv 4\pi \tilde{g} \bar{n} a N$$

$\bar{n}$: z-averaged density

Extension to finite T: Bogoliubov

Theory for contact in 2D: State-of-the-art

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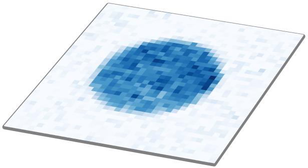
Extension to lower T: Virial expansion

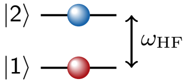
- Intermediate regime: $T \sim T_c$

Monte-Carlo simulations : Prokofev, Svistunov Phys. Rev. A 66, 043608 (2002)

Experimental protocol

Prepare uniform gas ($\bar{n}$, T) in $F = 1$





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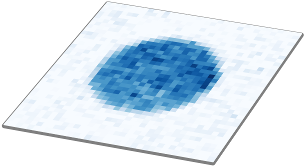
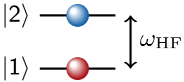




Induce Δa : $F = 1,\ a_{11} \to F = 2,\ a_{22}$

Measure $h\Delta\nu = \Delta E$
with Ramsey spectroscopy

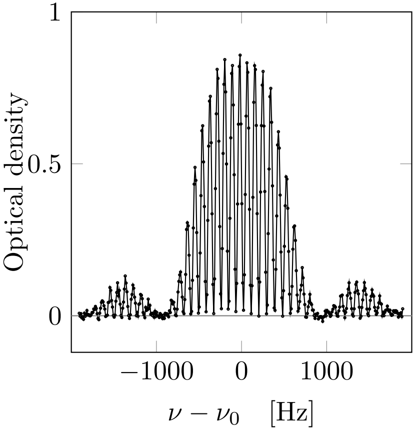
Compute $C \propto \dfrac{\Delta E}{\Delta a}$

(NB : valid only if $a_{11} \sim a_{12} \sim a_{22}$)

Ramsey spectroscopy

Protocol:

preparation in $|1\rangle$ - $\pi/2$ pulse - wait T - $\pi/2$ pulse - measure population in $|2\rangle$



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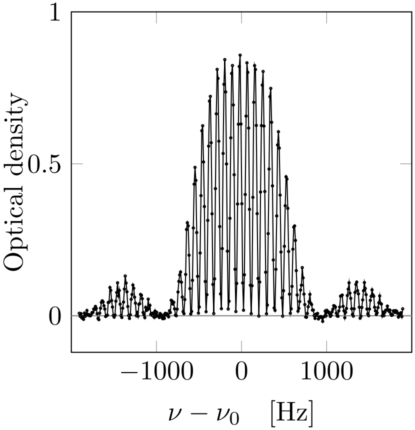
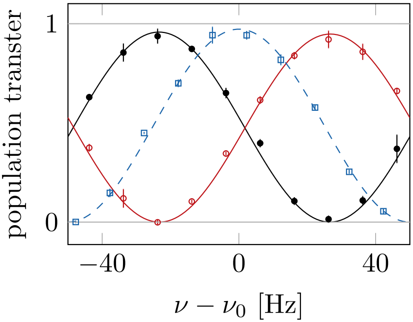


Zomm around $\nu = \nu_0$:

$|2\rangle$, $|1\rangle$, ToF-single particle



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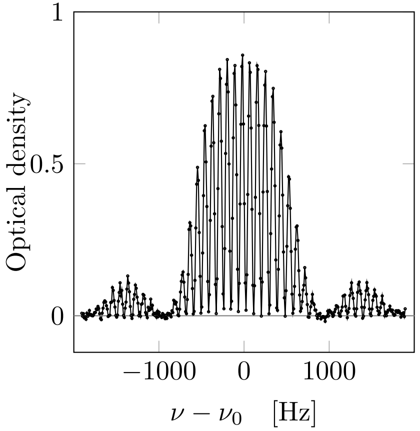

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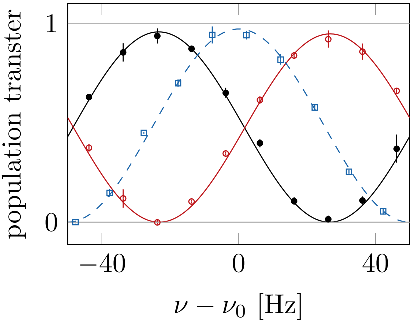
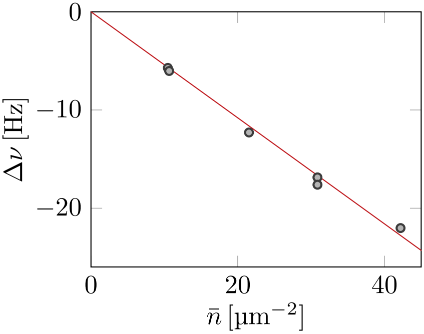


$\Rightarrow$ For a given $(\bar{n}, T)$, extract $\Delta\nu = \nu_{\rm res} - \nu_0$

Ramsey spectroscopy

Vary density for $T \sim 0$:



Linear behaviour with density

Mean-field description OK

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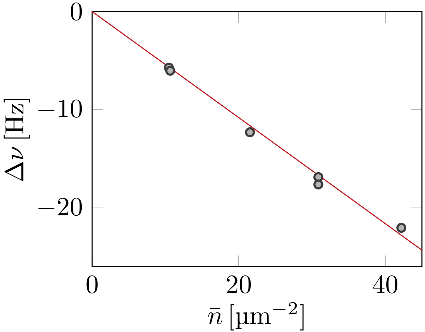

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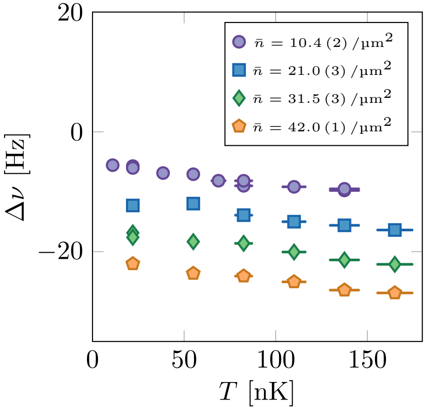

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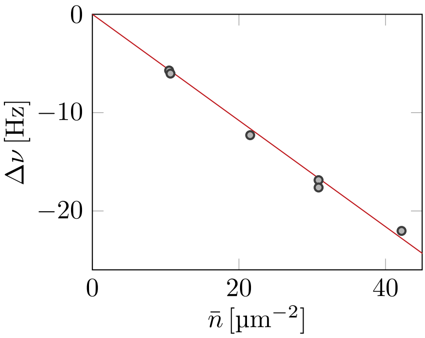

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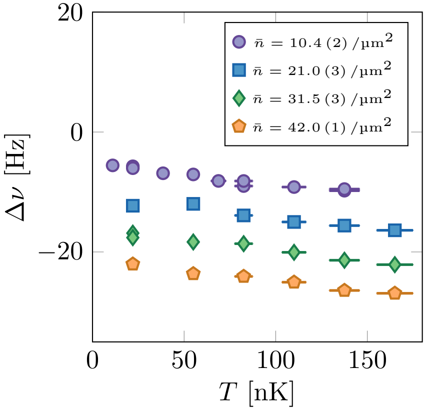
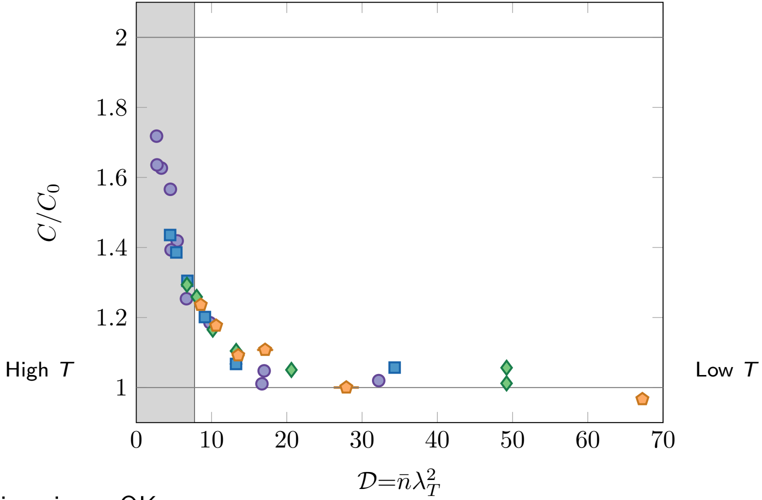


Scale-invariance ⇒ plot C/C_0 vs $\mathcal{D} = \bar{n}\lambda_T^2 \propto \bar{n}/T$

Contact measurement



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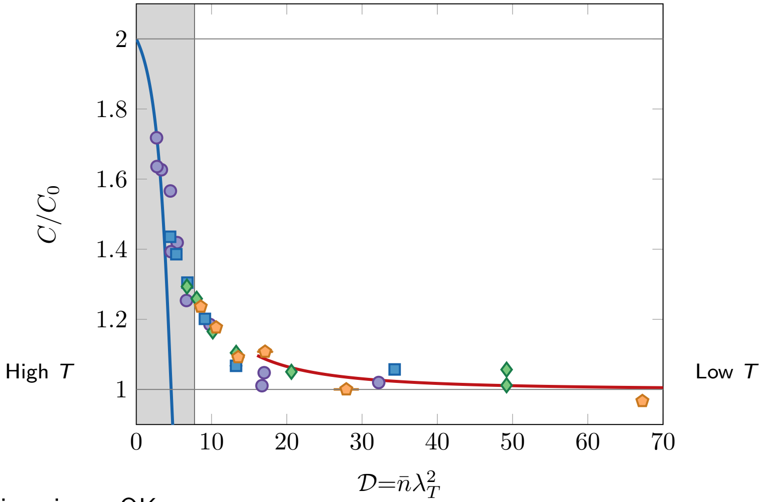

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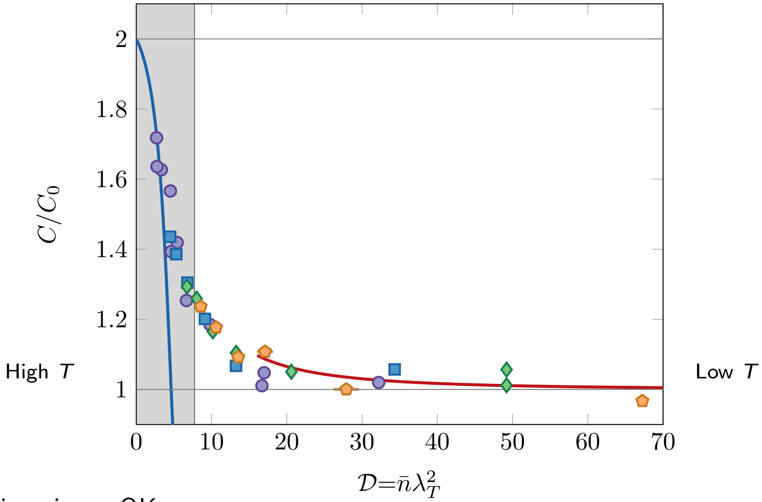

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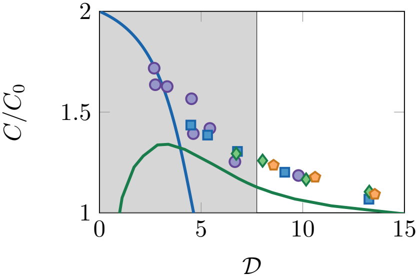
⇒ What about the critical region ?

Contact in the critical region



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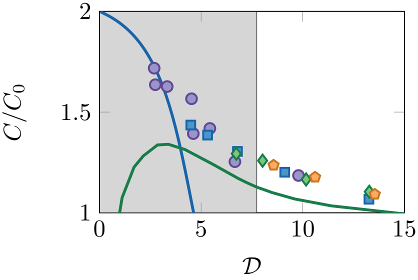

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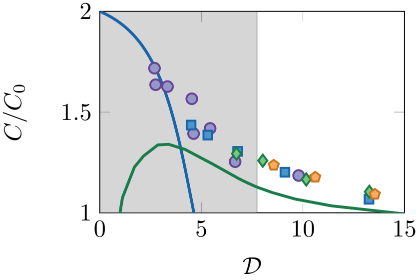

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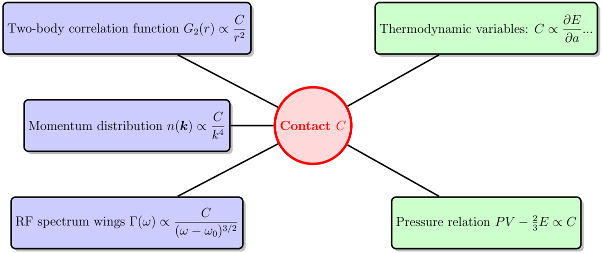
Outlook : beyond mean-field regime, breaking of scale invariance...

Conclusion

Contact has become a major tool for the understanding of quantum gas



Focus here on two-body s-wave contact

But *p*-wave, three-body, ... any interaction described by a single parameter

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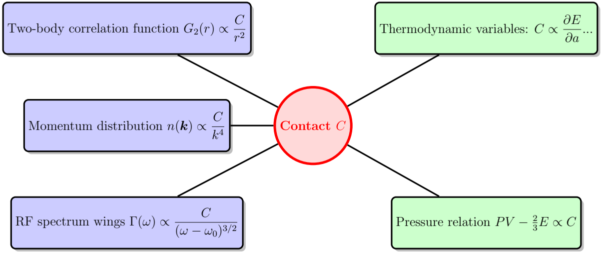

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Contact of a 2D Bose gas

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