Multi-component spectra and excitation functions with Gamow Shell Model

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with
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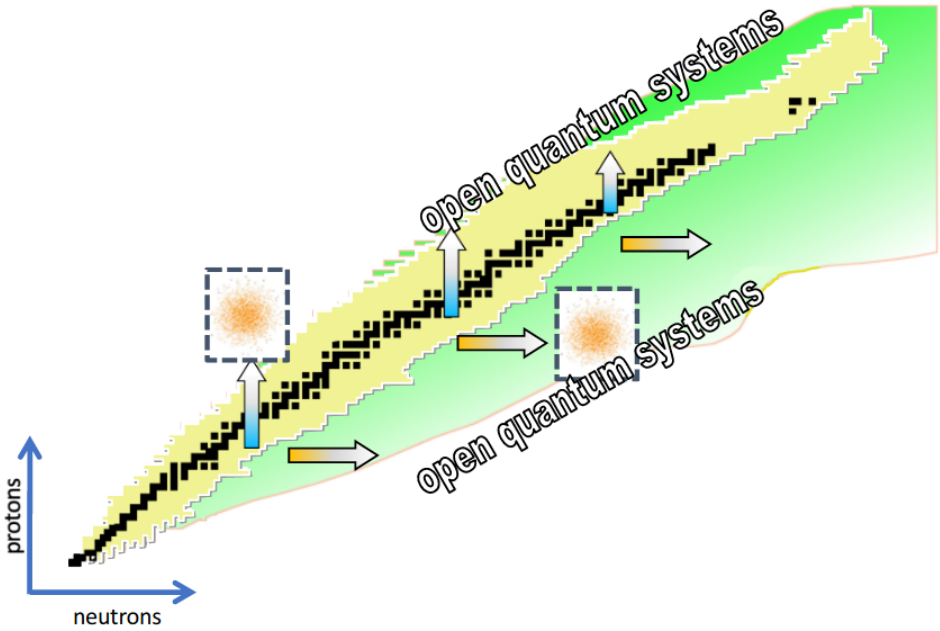
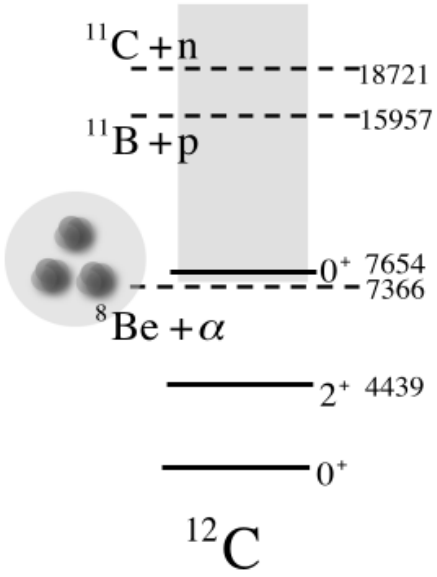
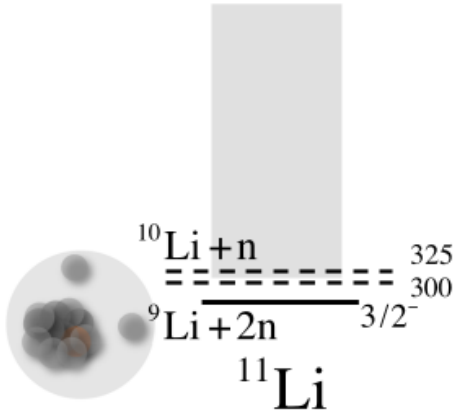
Introduction

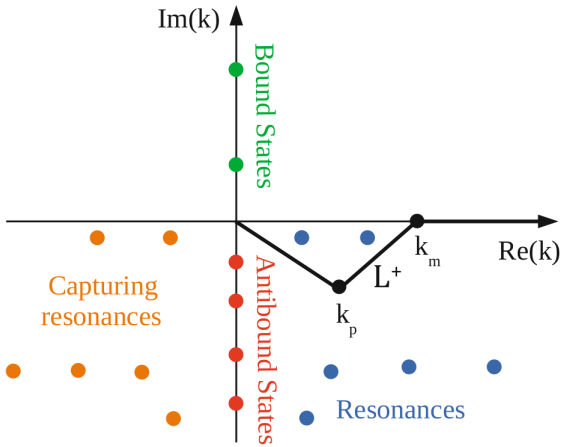


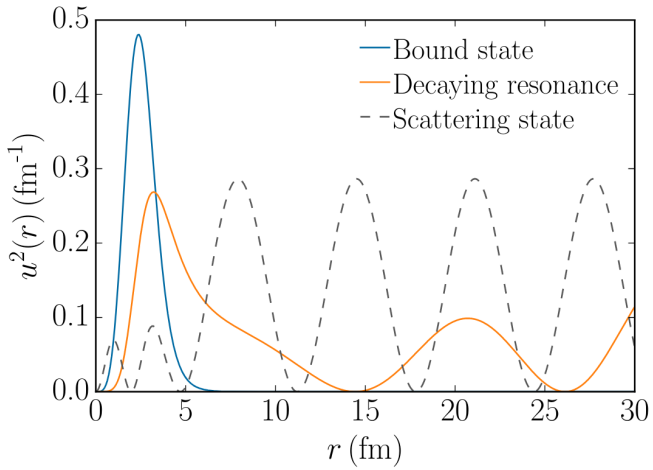




Low separation energies and clusterization → Open quantum system framework → Unification of nuclear structure and reactions

Berggren basis





- Bound states. $k \to i\kappa$
- Resonant/Gamow states: poles of the S-matrix. $k \to \kappa_1 + i\kappa_2$
- Scattering states: nonresonant continuum states.

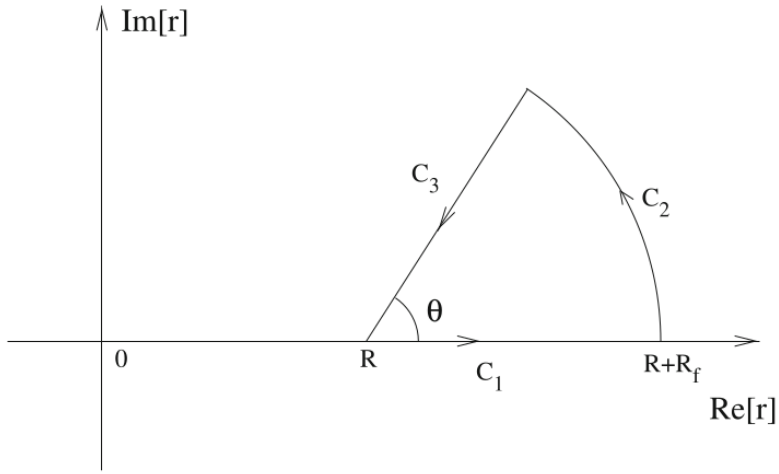
Berggren completeness relation:

T. Berggren, Nucl.Phys.A **109**, 265 (1968)

$$\sum_n u_n(r)u_n(r') + \int_{L^+} u_k(r)u_k(r')dk = \delta(r-r')$$

Resonant states regularized via complex scaling.

$$\int_0^\infty f(r)dr = \int_0^R f(r)dr + \int_0^\infty f(R+x\cdot e^{i\theta})e^{i\theta}dx$$



Gamow Shell Model (GSM)

Practical applications:

$$\int_{L^+} u_k(r)u_k(r') \approx \sum_i^{N_d} u_i(r)u_i(r')$$

Discretized contour in k-plane

Normalized discrete single particle Berggren basis $\{\phi_{\ell,\eta}\}$, which can be used to build a many-body basis

$$\sum_n |SD_n\rangle\langle\widetilde{SD_n}| \approx 1 \qquad |SD\rangle = |\phi_1\phi_2...\phi_M\rangle$$

Many-body completeness relation

N. Michel et al., Phys.Rev.Lett. **89**, 042502 (2002)

$$H|\Psi\rangle = E|\Psi\rangle \qquad |\Psi\rangle = \sum_n c_n|SD\rangle$$

Individual reaction channels cannot be defined in the Slater determinant representation of GSM.

Coupled channel formulation of GSM (GSM-CC)

The channel wave-function is defined as

$$|(c,r)\rangle = \mathcal{A}\left\{|\Psi_T^{J_T}\rangle \otimes |\Psi_P^{J_P}\rangle\right\}_{M_A}^{J_A}.$$

$|\Psi_T^{J_T}\rangle = \sum_m c_m |SD_m\rangle$ from GSM

The quantum number $c \rightarrow \{Z-z, N-n, J^T; z, n, \ell, J_{int}, J_P\}$.
One can define entrance and exit channels with correct asymptotics.
The wave functions are

$$|\Psi_{M_A}^{J_A}\rangle = \sum_c \int_0^\infty \left(\frac{u_c(r)}{r}\right) |(c,r)\rangle r^2 dr.$$

The Schrödinger equation becomes the coupled-channel equation

$$\sum_c \int_0^\infty r^2 (H_{cc'}(r,r') - E N_{cc'}(r,r')) \frac{u_c(r)}{r} = 0.$$

Previous studies using GSM-CC

What has been achieved so far with the channel representation:

- $^{18}Ne(p,p')$

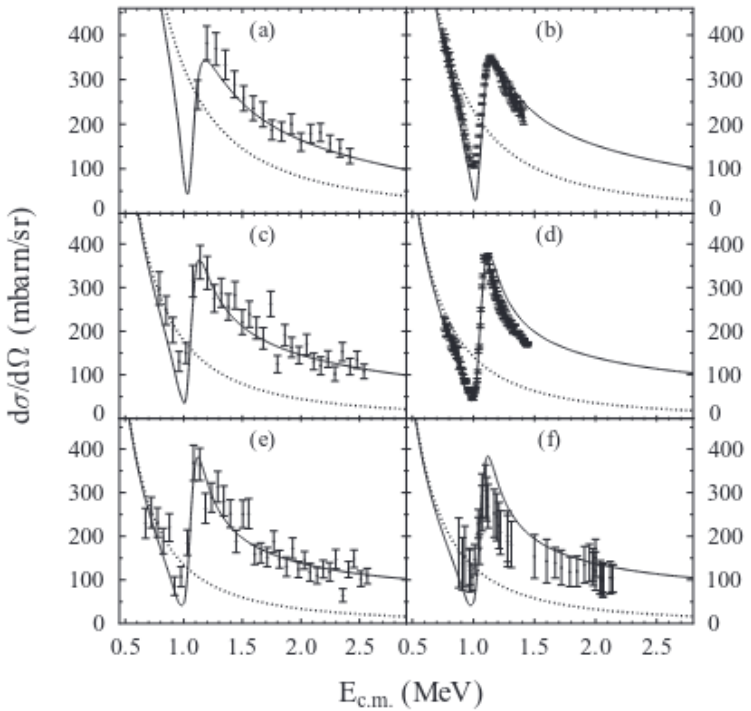
  Y Jaganathen et al., Phys.Rev.C **89**, 034624 (2014)

- $^{7}Be(p,\gamma)^{8}B$

  K Fossez et al., Phys.Rev.C **91**, 034609 (2015)

- $^{4}He(d,d)$

  A Mercenne et al., Phys.Rev.C **99**, 044606 (2019)



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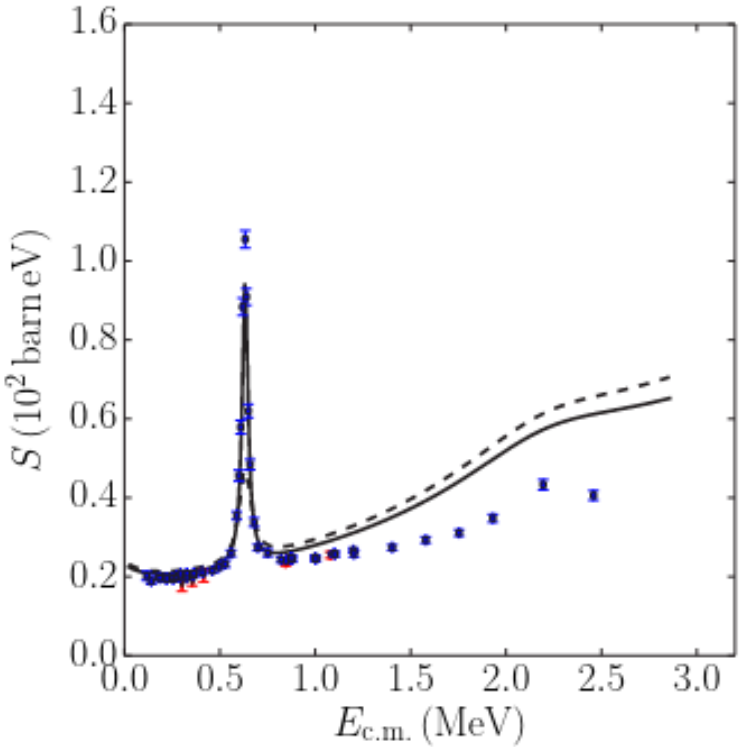

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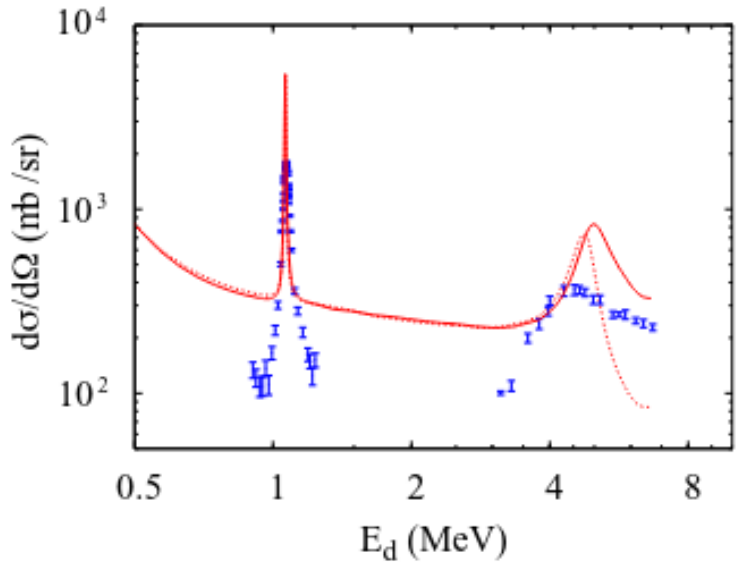
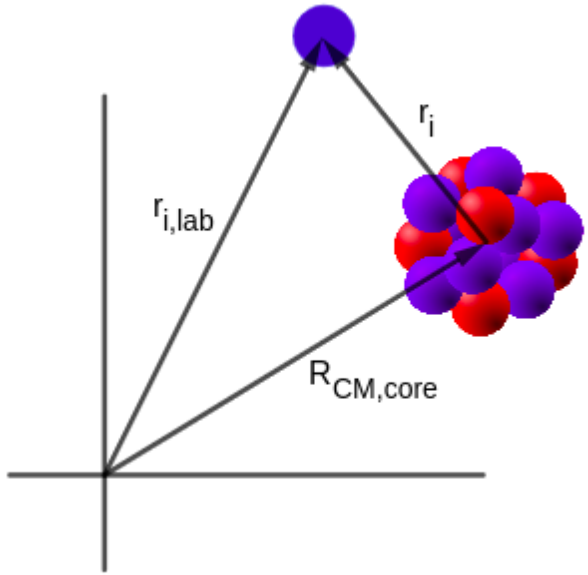


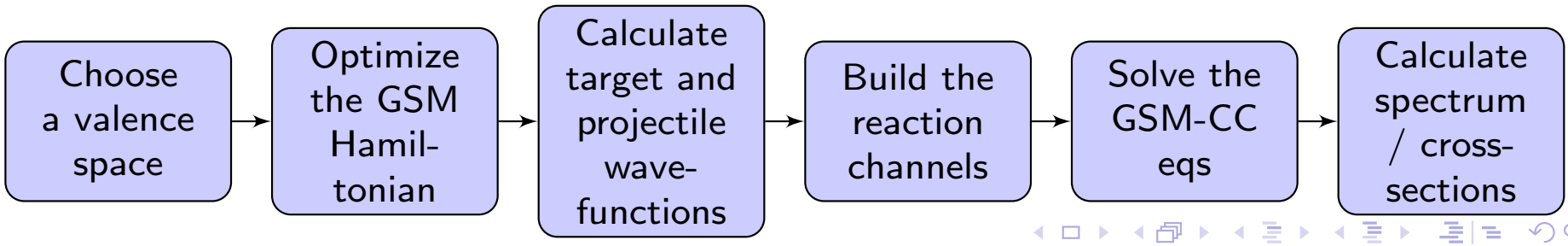
Methodology

- Model space: inert core + valence particles $\Rightarrow$ studies of heavier systems are possible.
- Calculations in cluster orbital shell model coordinates.
  Y Suzuki and K Ikeda, Phys.Rev.C **38**, 410 (1988)
- Two-body part: FHT interaction + Coulomb.
  H. Furutani et al., Prog.Theor.Phys. **62**, 981 (1979)
- Cluster part with a N^3LO interaction.

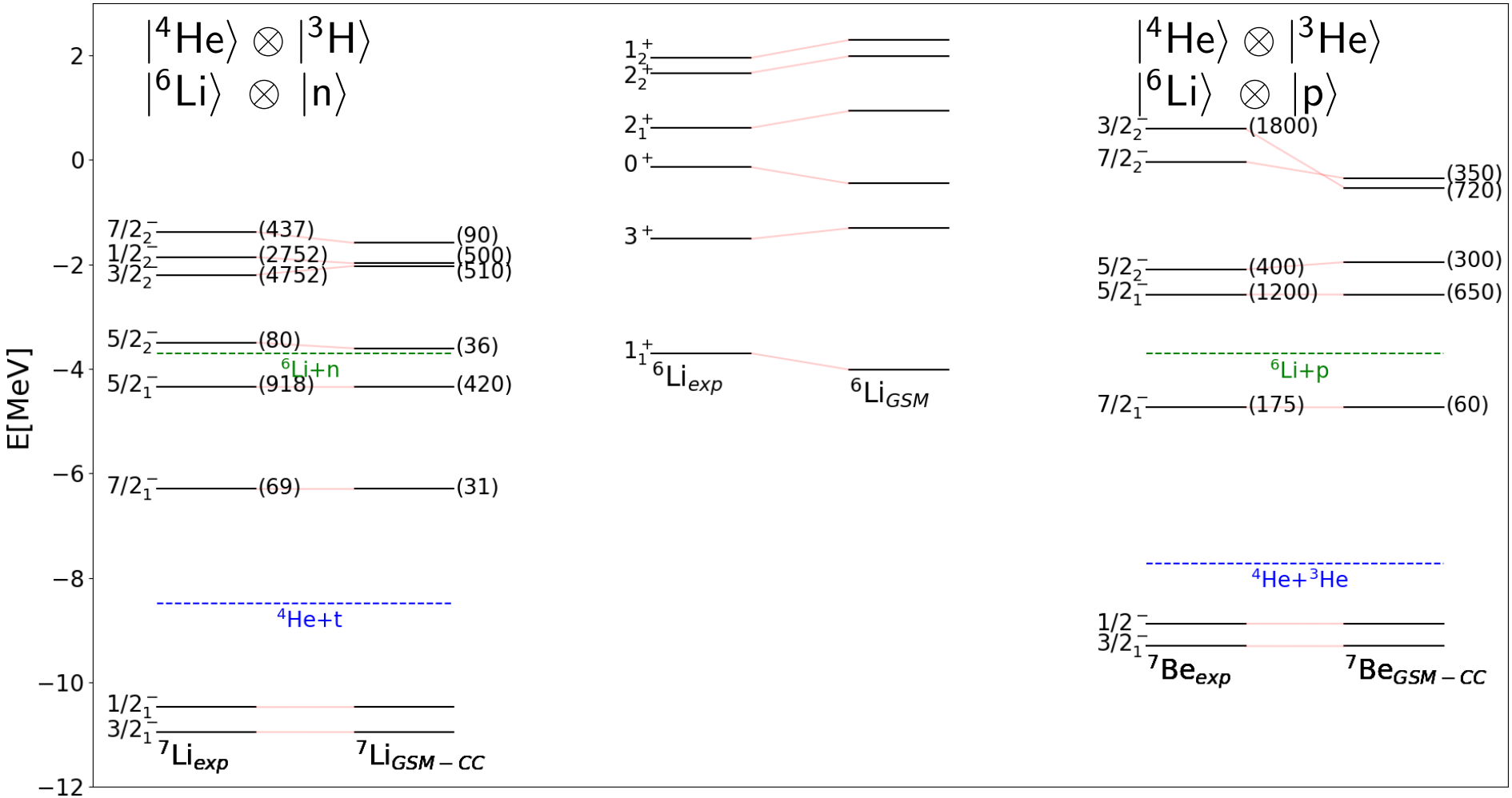


$$\hat{H}_{lab} \Rightarrow \hat{H}_{COSM} = \sum_{i \in val} \left(\frac{\mathbf{p}_i^2}{2\mu_i} + \hat{U}_i(\mathbf{r}_i) \right) + \sum_{(i<j) \in val} \hat{V}_{i,j} + \sum_{(i<j) \in val} \frac{\mathbf{p}_i \cdot \mathbf{p}_j}{M_{core}}$$

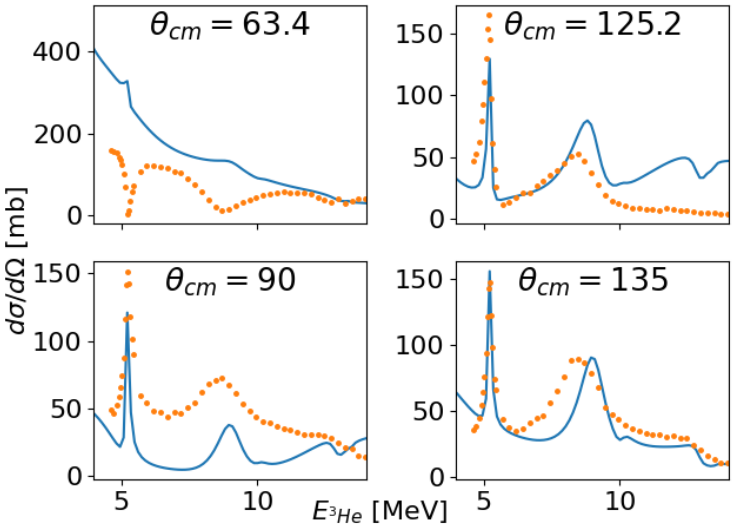
recoil term

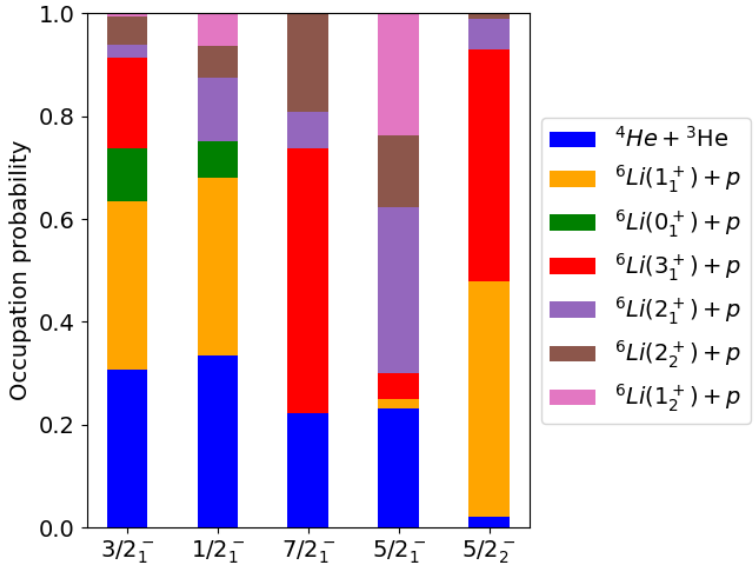


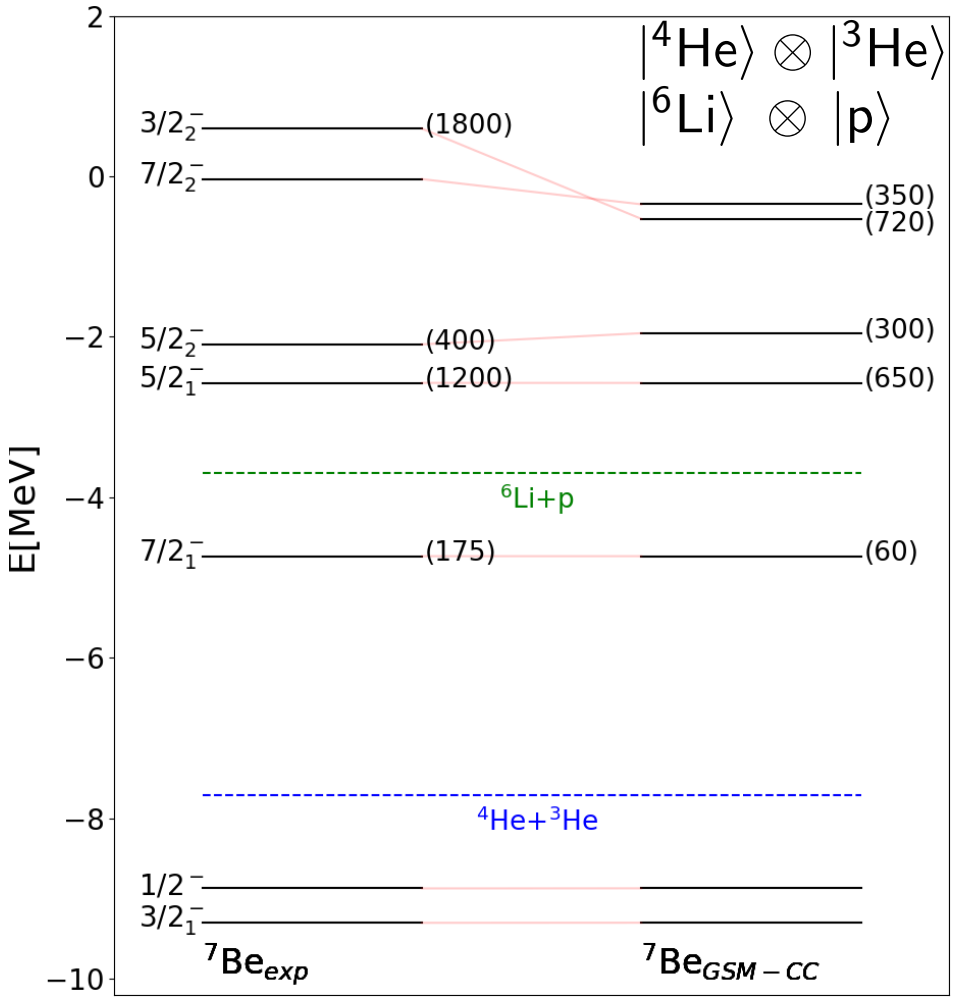
Spectra of ^{7}Be and ^{7}Li



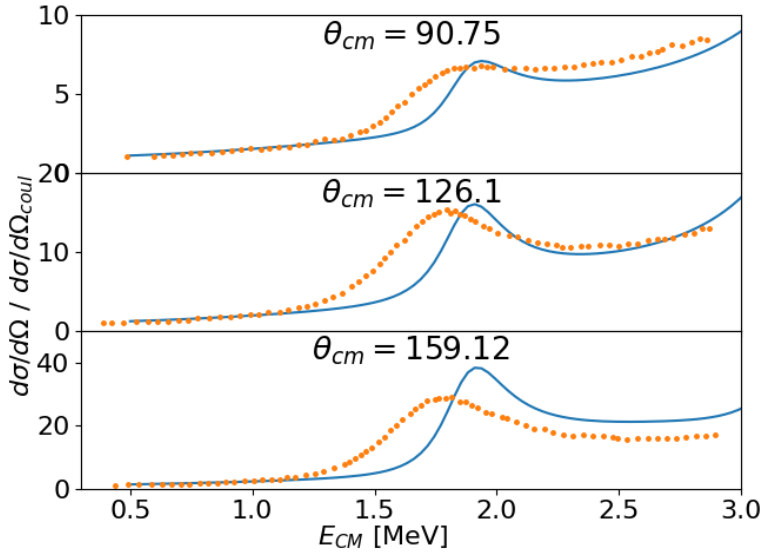
Structure of ^{7}Be and ^{4}He(^{3}He,^{3}He) cross-section

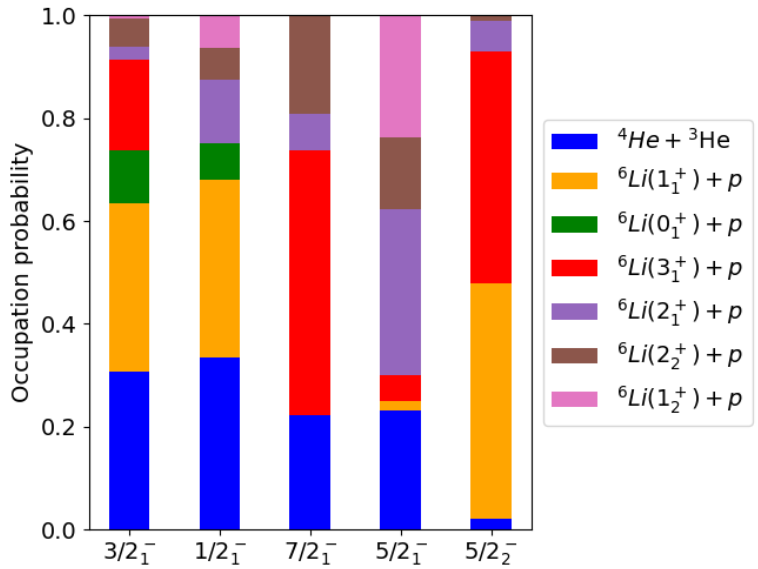


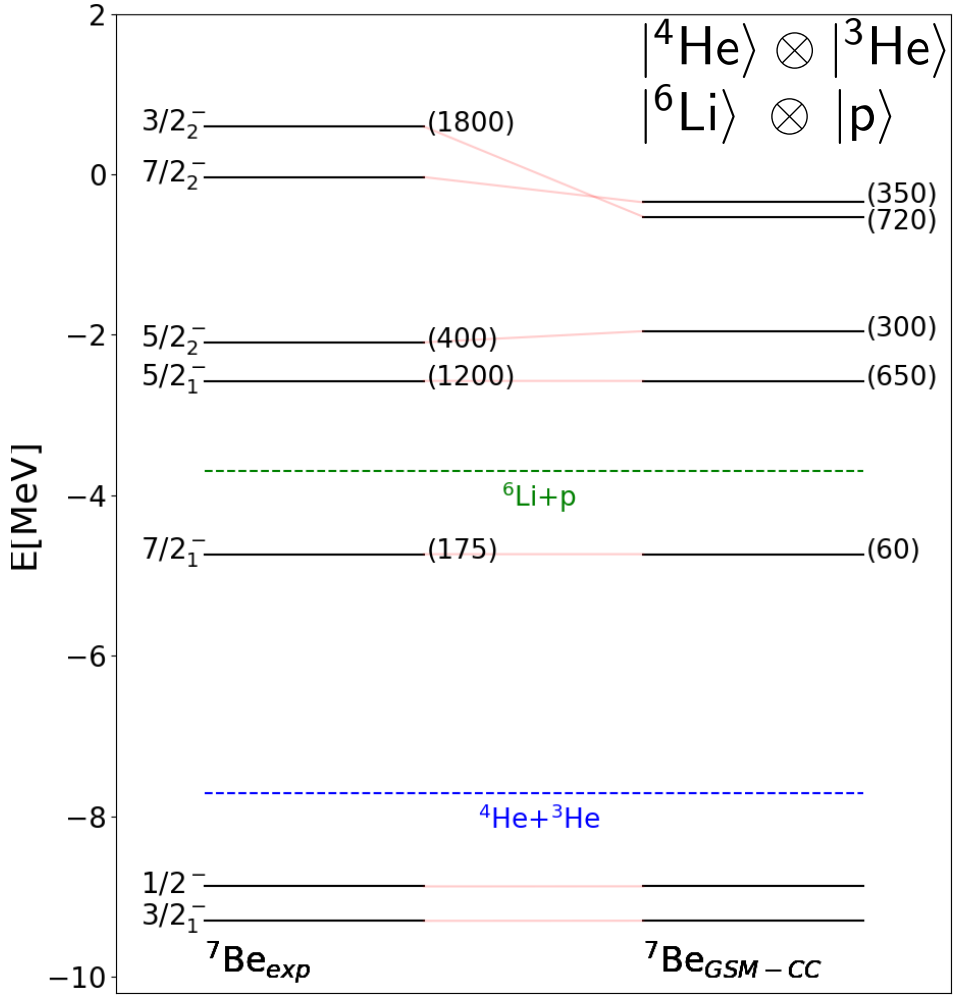




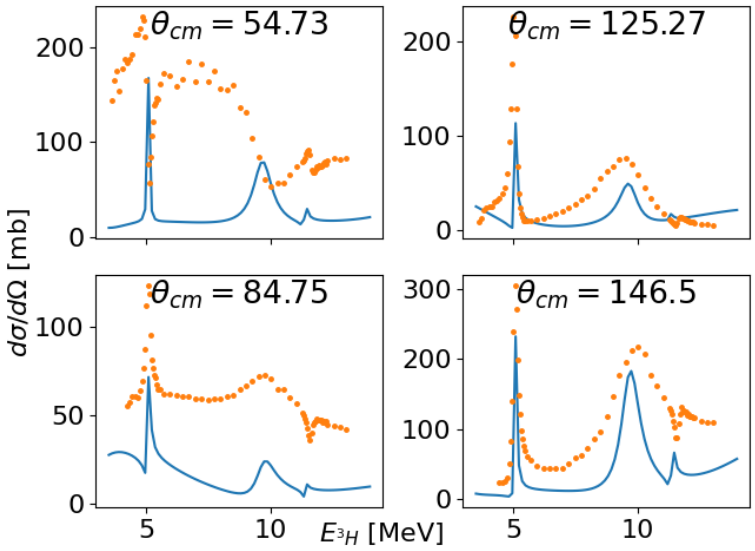
Structure of ^{7}Be and ^{6}Li(p,p) cross-section

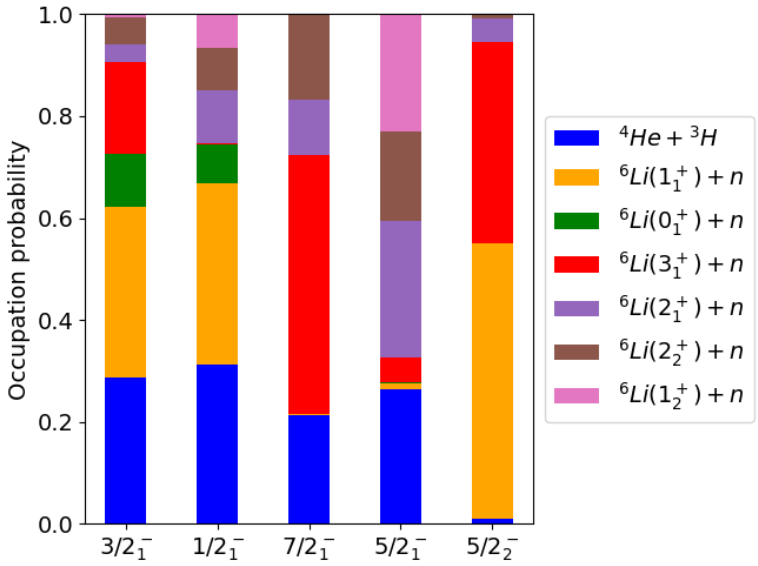


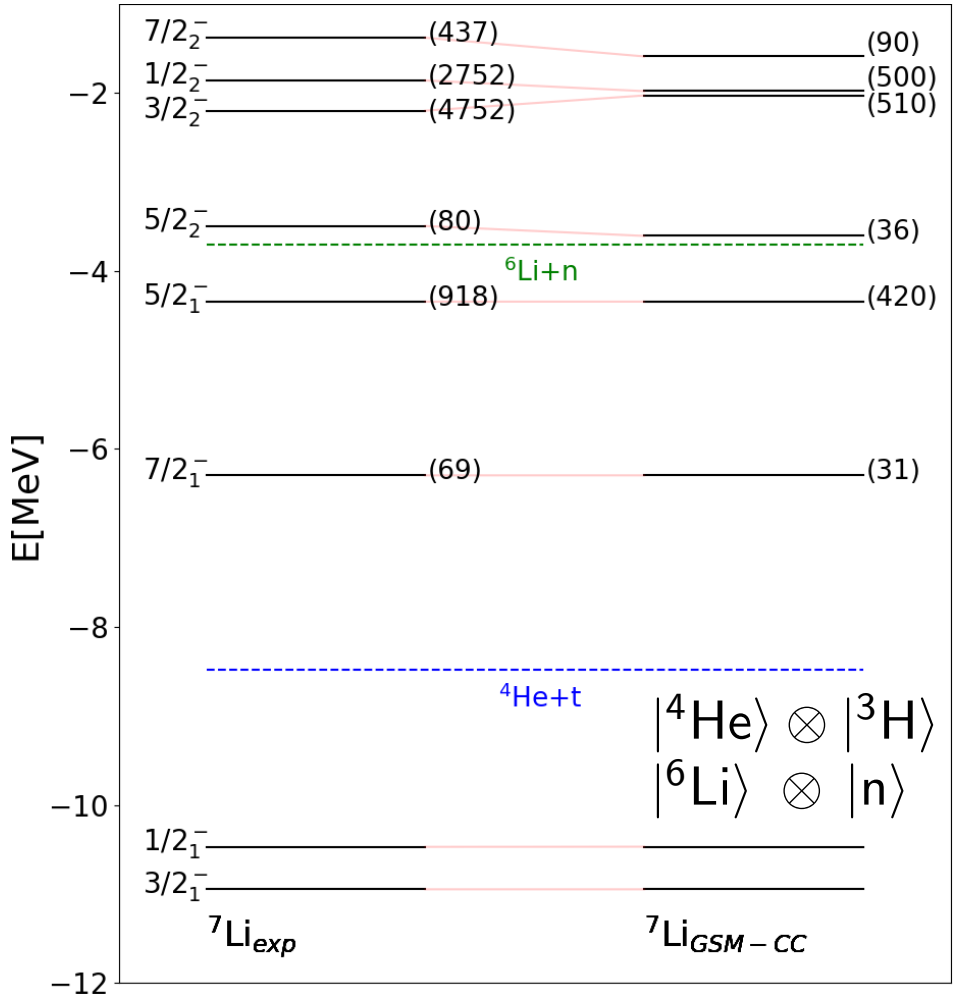




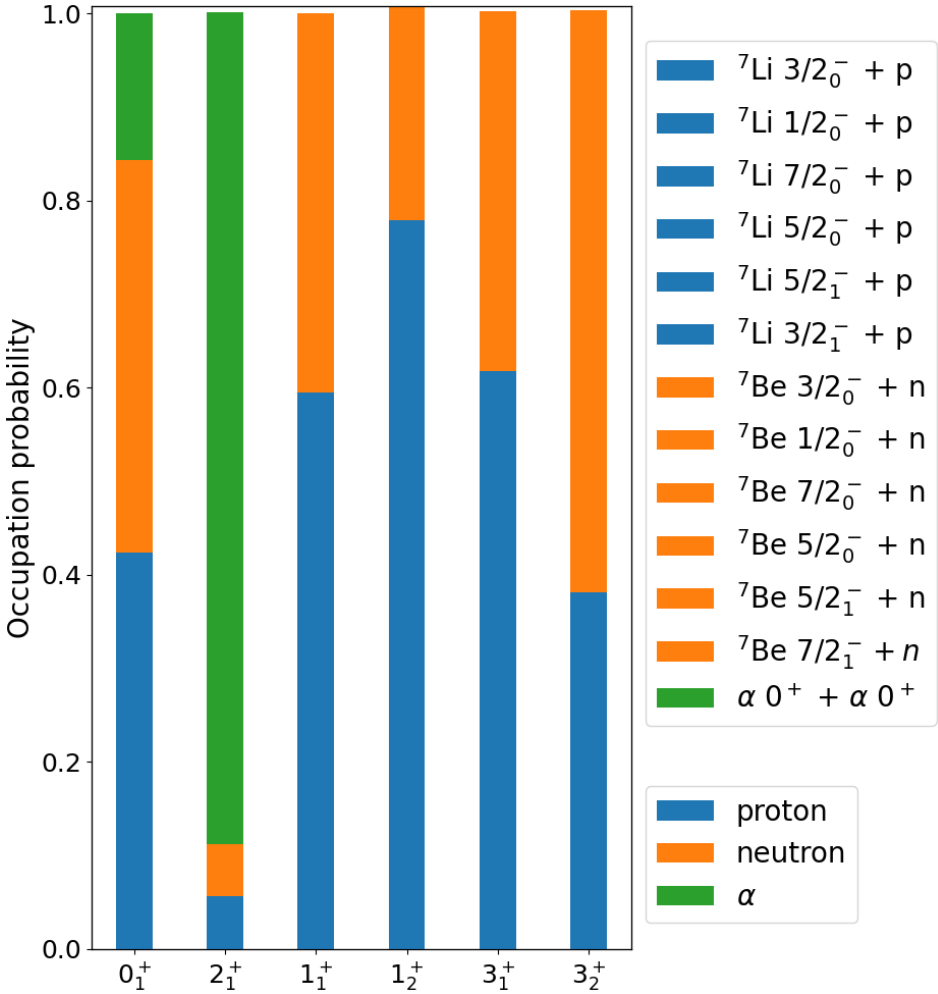
Structure of ^{7}Li and ^{4}He(^{3}H,^{3}H) cross-section

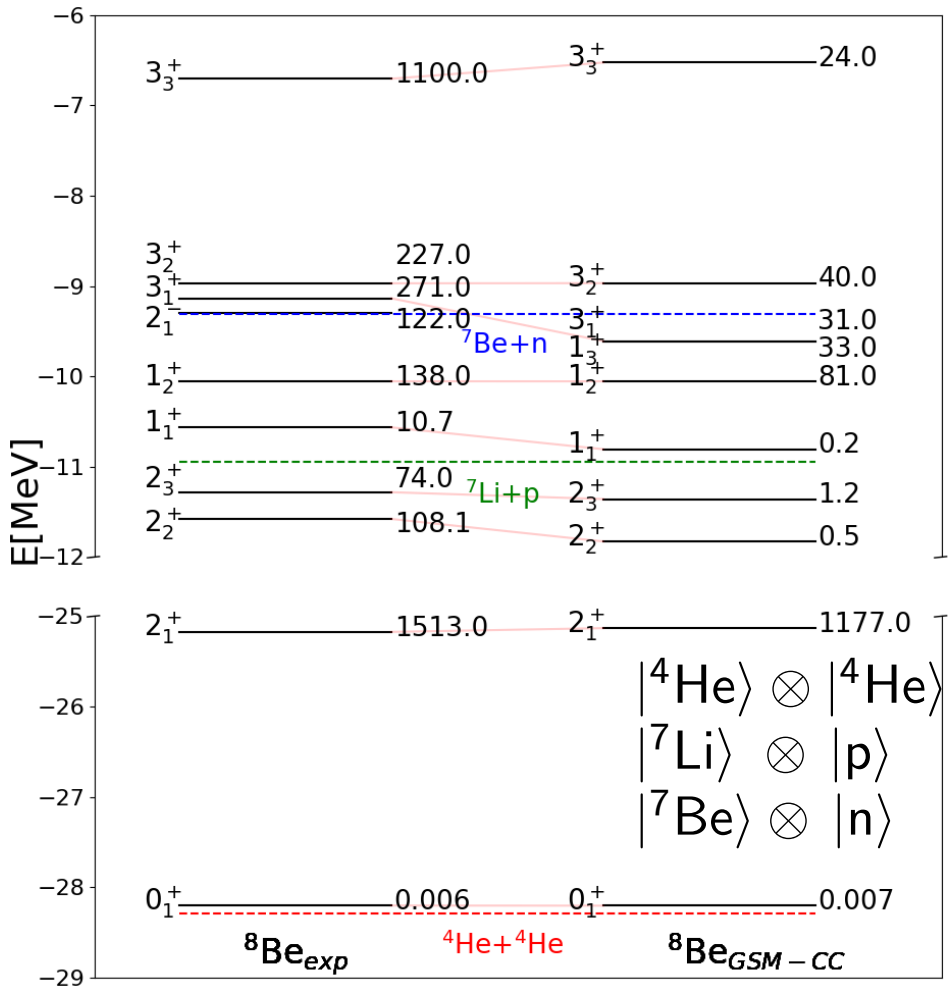






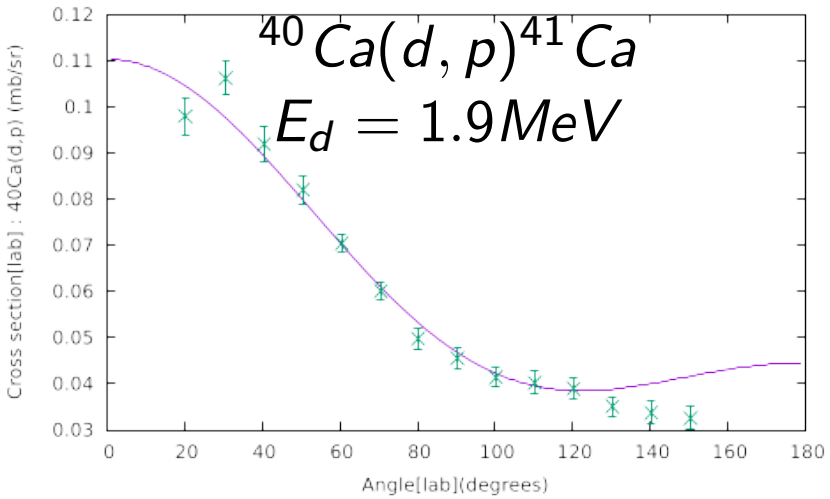
Structure of ^{8}Be

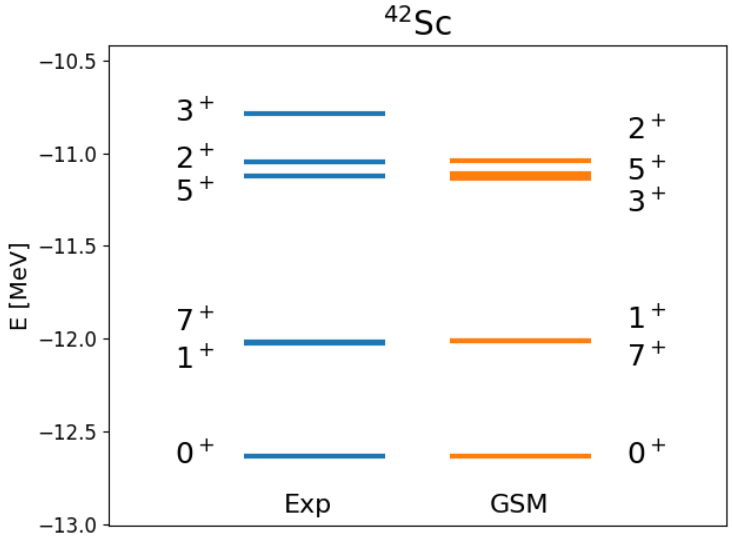




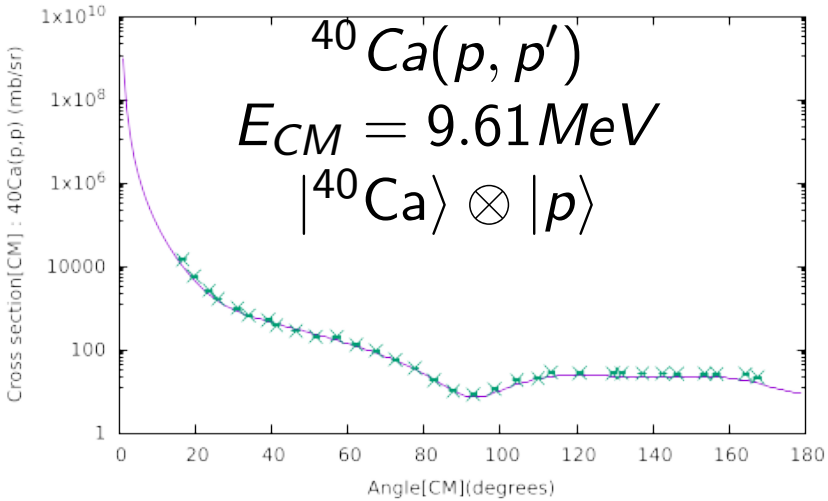
The case of a heavier core: ^{42}Sc

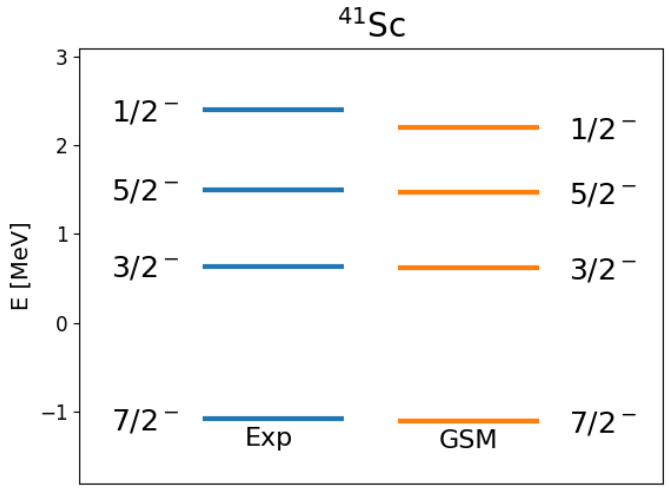
- ^{40}Ca core and valence particles in the *fp*-shell.
- The Hamiltonian consist of a 1-body potential + FHT interaction.
- Channels: $|^{40}\mathrm{Ca}\rangle \otimes |d\rangle$, $|^{41}\mathrm{Ca}\rangle \otimes |p\rangle$ and $|^{41}\mathrm{Sc}\rangle \otimes |n\rangle$

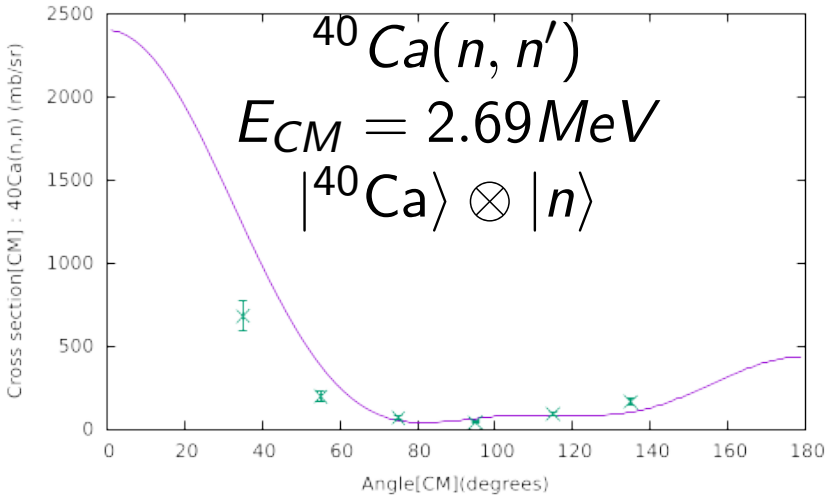


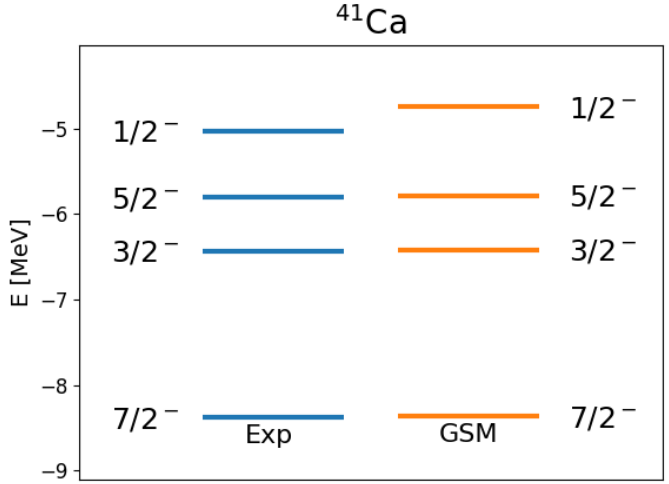


The case of a heavier core: ^{41}Sc and ^{41}Ca









Conclusions

- GSM-CC provides the microscopic open quantum system description of spectra and reactions in multichannel and multiple mass partition framework.
- First applications of GSM-CC in multichannel calculations with different mass partitions:
  - ^{7}Be (^{3}He + ^{4}He, ^{6}Li + p)
  - ^{7}Li (^{3}H + ^{4}H, ^{6}Li + n)
  - ^{8}Be (^{4}He + ^{4}He, ^{7}Li + p, ^{7}Be + n)
  - ^{42}Sc (^{40}Ca + d, ^{41}Ca + p, ^{41}Sc + n).
- Possible application in heavier systems: ^{42}Sc.

Outlook:

- Radiative capture ^{4}He(^{3}He,γ) using the same interaction of the previous calculations.

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Outlook:

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Thank you for your attention!

Back up

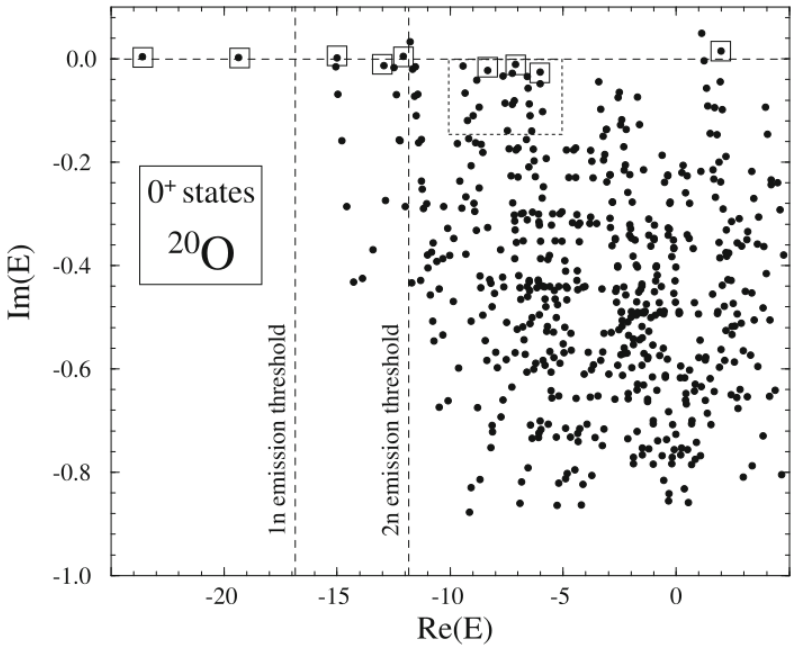
The overlap method

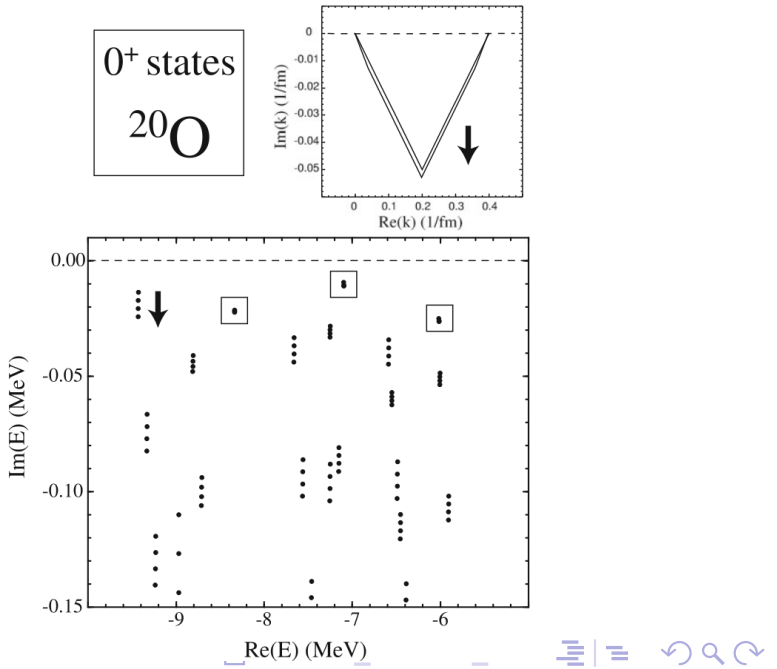
Why Lanczos doesn't work?

- Resonant A-body states are surrounded by many scattering A-body states.
- One cannot differentiate between resonant and scattering states given by the Lanczos method.

The overlap method consist of two steps:

1. Fully diagonalize with an incomplete basis of bound and resonant pole states. From this we extract a pivot $|\Psi_0\rangle$.
2. Using the full Berggren basis, find $|\Psi\rangle$ that optimizes the overlap $|\langle\Psi_0|\Psi\rangle|$.





Coupled channel formulation of GSM (GSM-CC)

One separates the Hamiltonian as

$$\hat{H} = \hat{H}_p + \hat{H}_t + \hat{H}_{tp} \rightarrow \begin{cases} \hat{H}_p[|\Psi_T^{J_T}\rangle \otimes |\Psi_P^{J_P}\rangle] = |\Psi_T^{J_T}\rangle \otimes \hat{H}_p|\Psi_P^{J_P}\rangle \\ \hat{H}_t[|\Psi_T^{J_T}\rangle \otimes |\Psi_P^{J_P}\rangle] = \hat{H}_t|\Psi_T^{J_T}\rangle \otimes |\Psi_P^{J_P}\rangle \end{cases}$$

$$|\Psi_p^{J_p}\rangle = [|K_{CM}, L_{CM}\rangle \otimes |K_{int}, L_{int}\rangle]_{M_p}^{J_p} \rightarrow |\Psi_p^{J_p}\rangle^{HO} = \left[|K_{CM}, L_{CM}\rangle^{HO} \otimes |K_{int}, L_{int}\rangle^{HO}\right]_{M_p}^{J_p}$$

(H_{CM} and H_{int})

Normalizing $\langle\Psi_p^{J_p}|\Psi_p^{J_p}\rangle = \delta(K_{CM} - K'_{CM})$ is difficult.

$$|\Psi_p^{J_p}\rangle^{HO} = \sum_N C_N^{HO}|SD_N\rangle^{HO} = \sum_n C_n|SD_n\rangle$$

Target SD generated by GSM

Old solution

We begin by tranforming the problem to a standard eigenvalue problem

- In matrix form: $\mathcal{HU} = \mathcal{EOU}$.
- If we do the substitutions $\mathcal{W} = \mathcal{O}^{1/2}\mathcal{U}$ and $\mathcal{H}_m = \mathcal{O}^{-1/2}\mathcal{H}\mathcal{O}^{-1/2}$.
- the CC equation becomes $\mathcal{H}_m\mathcal{W} = E\mathcal{W}$.

The CC-equation to solve becomes:

$$\left[\frac{\hbar^2}{2\mu_c}\left(-\frac{d^2}{dr^2} + \frac{l_c(l_c+1)}{r^2}\right) + V_c^{loc}(r)\right] w_c(r) + \sum_{c'}\int_0^\infty V_{cc'}^{non-loc} w_c'(r)dr' = (E - E_{T_c})w_c(r),$$

with

$$u_c(r) = w_c(r) + \sum_{c'}\int_0^\infty r'^2[\mathcal{O}^{1/2}(\mathcal{O}-1)\mathcal{O}^{1/2}]_{cc'}(r,r')\frac{w_{c'}(r')}{r'}.$$

Old solution

To tolve the CC-equation we use the method of modified equivalent potential.

$$W''(r) = M^{eq}(r)W(r) + S^{eq}(r)$$

where

- $M^{eq}_{cc'} \rightarrow M^{eq}_{cc'}(V^{loc}_{cc'}, V^{non-loc}_{cc'})$
- $S^{eq}_{cc'} \rightarrow S^{eq}_{cc'}(V^{non-loc}_{cc'})$

Solution via Green Functions.

For $\hat{H}|\Psi\rangle = E|\Psi\rangle$ we make the ansatz:

- $\hat{H} = \hat{H}^{(0)} + \hat{H}_{rest}$
- $|\Psi\rangle = |\Psi^{(0)}\rangle + |\Psi_{rest}\rangle$
- $\hat{H}^{(0)}|\Psi^{(0)}\rangle = E|\Psi^{(0)}\rangle$

We end up with

$$(\hat{H} - E)|\Psi_{rest}\rangle = -\hat{H}_{rest}|\Psi^{(0)}\rangle \rightarrow M_E|\Psi_{rest}\rangle = |S\rangle.$$

Generating a Berggren basis $\{|n, c\rangle\}$:

- $(\Psi_{rest})_{n,c} = \langle n, c|\Psi_{rest}\rangle$
- $(M_E)_{n',c',n,c} = \langle n', c'|(\hat{H} - E)|n, c\rangle$
- $S_{n'.c'} = \langle n', c'|S\rangle$

We now solve $\Psi_{rest} = M_E^{-1}S$

Interaction

The GSM Hamiltonian is:

$$H = \sum_i \left[\frac{p_i^2}{2m} + V_{WS}(r_i) + V_C(r_i) \right] + V_{res,12}.$$

The Woods-Saxon (WS) potential given by

$$V(r) = -V_{WS}\dot{f}(r) - 4V_{so}(\vec{l}\cdot\vec{s})\frac{1}{r}\left|\frac{df(r)}{dr}\right|,$$

and the WS form factor

$$f(r) = \left[1 + \exp\left(\frac{r - R_0}{d}\right)\right]^{-1}.$$

Interaction

The FHT H. Furutani et al., Prog.Theor.Phys. **62**, 981 (1979) interaction is written in terms of spin-isospin operators Π_{ST} A. DeShalit and H. Feshbach, (1974):

$$V_c(r) = \sum_{S,T=0,1} V_c^{ST} f_c^{ST}(r) \Pi_{ST}$$

$$V_{LS}(r) = (\vec{L} \cdot \vec{S}) V_{LS}^{11} f_{LS}^{11}(r) \Pi_{11}$$

$$V_T(r) = S_{ij} \sum_{T=0,1} V_T^{1T} f_T^{1T}(r) \Pi_{1T},$$

where $S_{ij} = 3(\vec{\sigma}_i \cdot \hat{r})(\vec{\sigma}_j \cdot \hat{r}) - \vec{\sigma}_i \cdot \vec{\sigma}_j$, $f_i^{ST}(r)$ are radial form factors and V_i^{ST} are parameters to be optimized.

Cross sections

We can use the standard cross-section formulas. The channel wave-function has the asymptotic behavior

$$u_c^{eJ_A^\pi}(r) \overset{r\to\infty}{\to} \delta_{ce} F_{\ell_e \eta_e}(k_e r) - T_{ec}^{J_A} H^+_{\ell_c \eta_c}(k_c r)$$

The cross-section is

$$\frac{d\sigma_{\tilde{e}\to\tilde{c}}}{d\Omega}(\theta) = \frac{1}{(2J_{int}+1)(2J_T^{\tilde{e}}+1)} \sum_{M_p^{\tilde{e}} M_t^{\tilde{e}} M_p^{\tilde{c}} M_t^{\tilde{c}}} \frac{K_{CM}^{\tilde{c}}}{K_{CM}^{\tilde{e}}} \left| f_{\tilde{e} M_p^{\tilde{e}} M_t^{\tilde{e}} \to \tilde{c} M_p^{\tilde{c}} M_t^{\tilde{c}}}(\theta) \right|^2$$

where the form factor is obtained from the T-matrix.