# Quantum Many-Body Physics Near Decay Thresholds

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Florida State University

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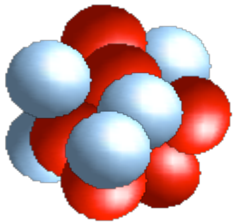
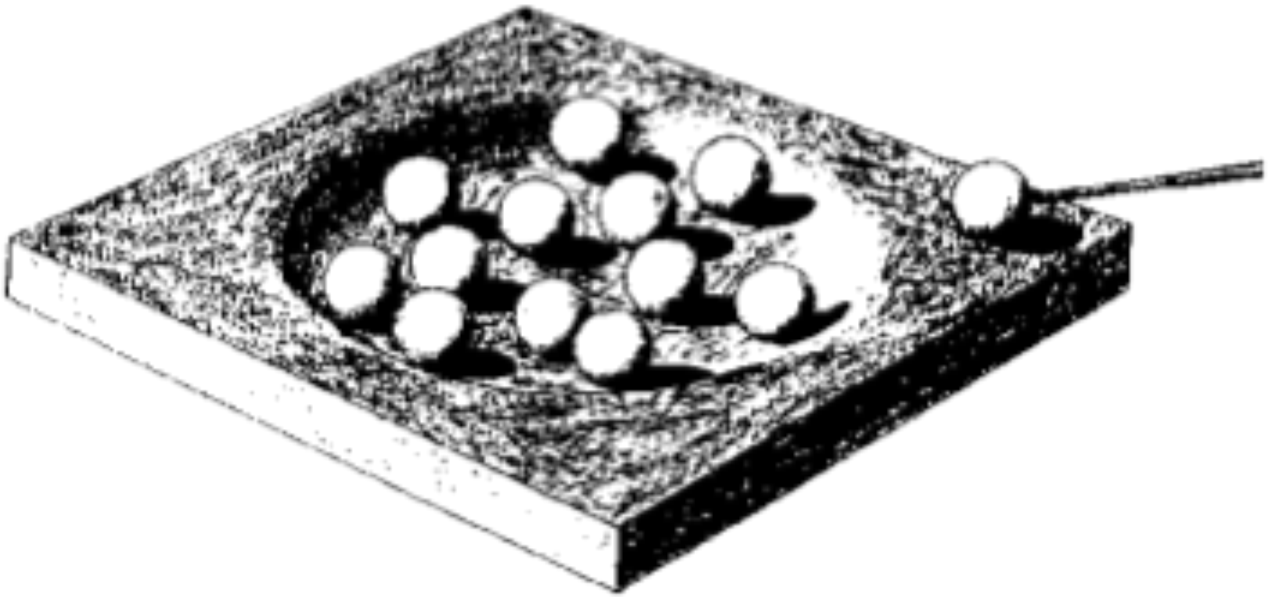
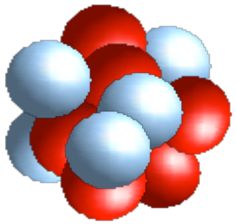
# Nucleus is an open quantum system

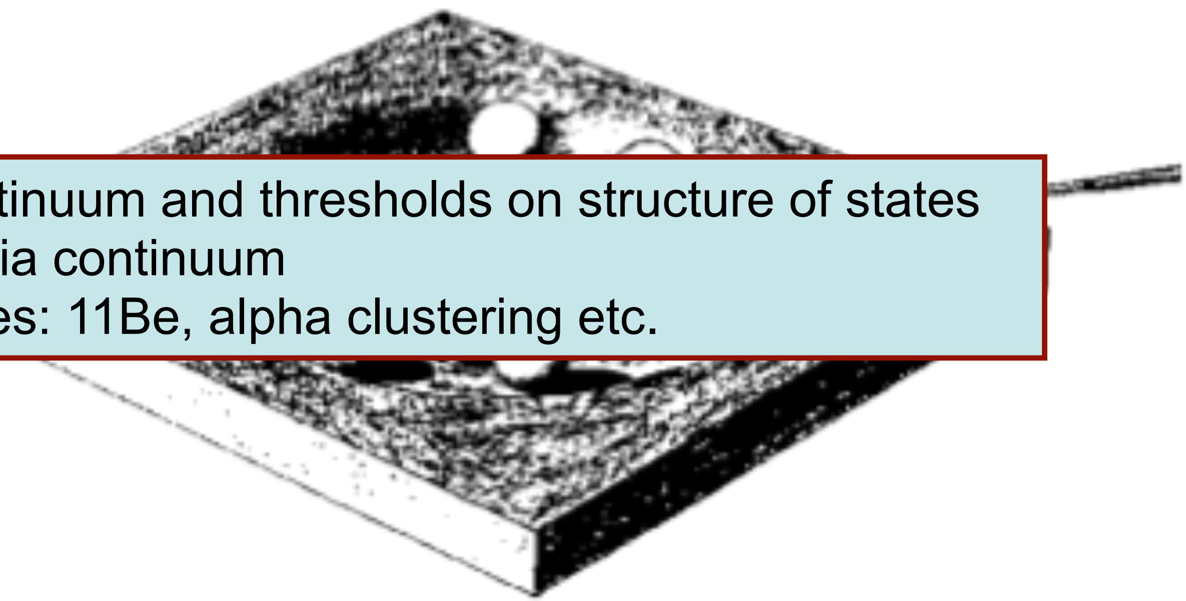




Wooden toy model illustrating Bohr's
compound nucleus, from Nature **137**, 351 (1936)

# Nucleus is an open quantum system

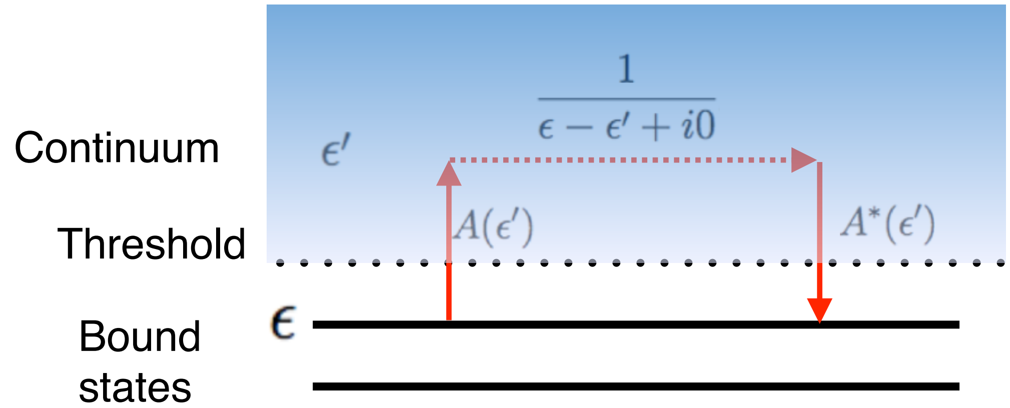




- Effect of continuum and thresholds on structure of states
- Transitions via continuum
- Specific cases: 11Be, alpha clustering etc.

Wooden toy model illustrating Bohr's
compound nucleus, from Nature **137**, 351 (1936)

# Physics of coupling to continuum



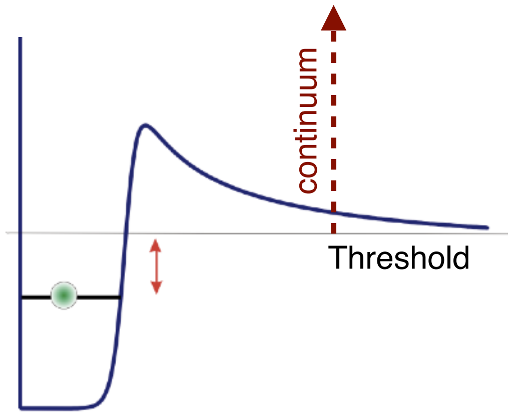

## The role of continuum-coupling

$$H'(\epsilon) = \int_0^\infty d\epsilon' \, A^*(\epsilon') \frac{1}{\epsilon - \epsilon' + i0} A(\epsilon') \qquad A(\epsilon') \equiv \langle I_2, \epsilon' | H_{PQ} | I_1 \rangle$$


[1] C. Mahaux and H. Weidenmüller, *Shell-model approach to nuclear reactions*, North-Holland Publishing, Amsterdam 1969

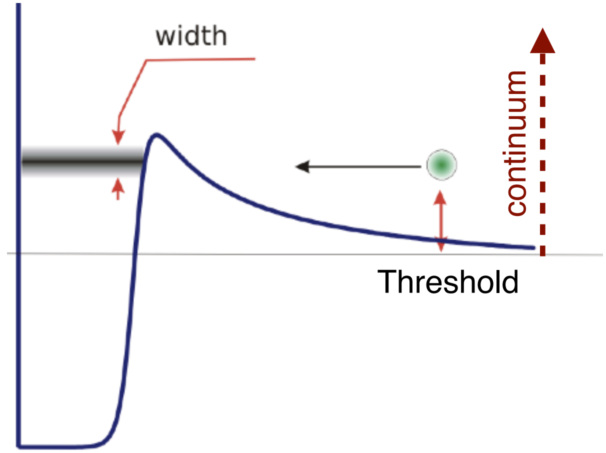
# Physics of coupling to continuum

$$H'(\epsilon) = \int_0^\infty d\epsilon' \frac{|A(\epsilon')|^2}{\epsilon - \epsilon' + i0}.$$




**Integration region involves no poles**

$$H'(\epsilon) = \Delta(\epsilon) \qquad \Delta(\epsilon) = \fint d\epsilon' \frac{|A(\epsilon')|^2}{\epsilon - \epsilon' + i0}$$

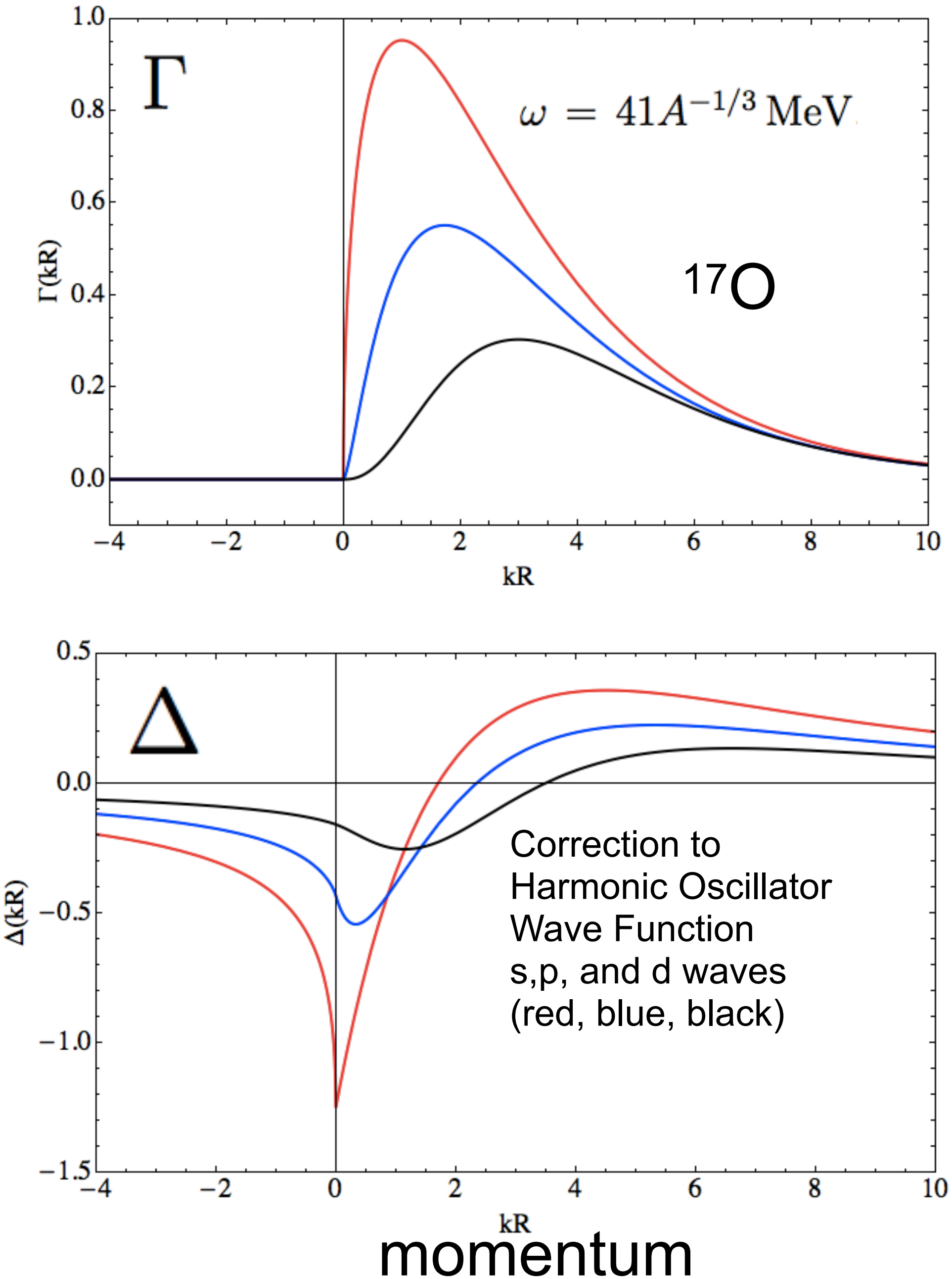



**State embedded in the continuum**

$$\frac{1}{x \pm i0} = \text{p.v.}\frac{1}{x} \mp i\pi\delta(x)$$

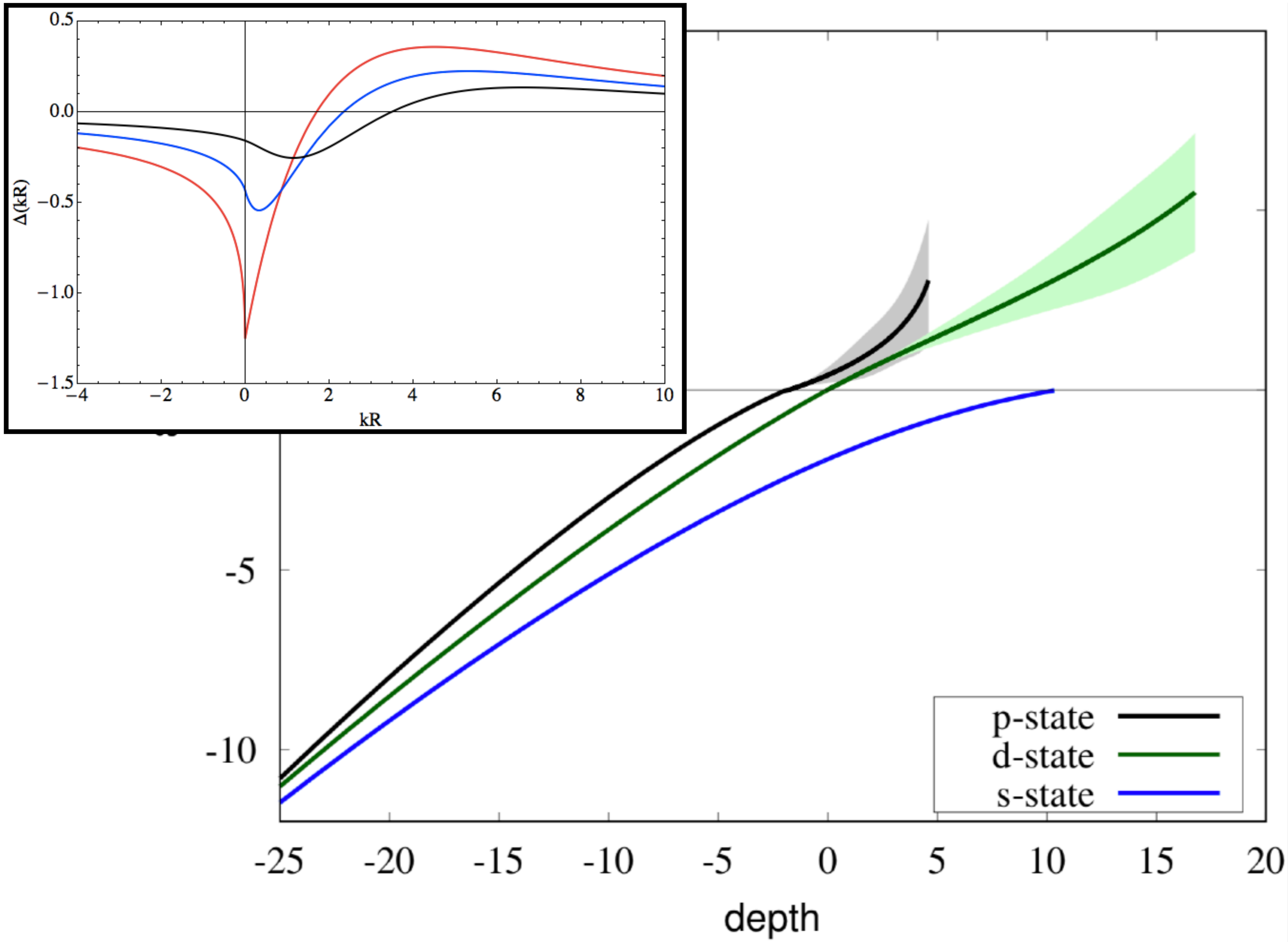
$$H'(\epsilon) = \Delta(\epsilon) - \frac{i}{2}\Gamma(\epsilon) \quad \Gamma(\epsilon) = 2\pi\,|A(\epsilon)|^2$$

# Self energy, interaction with continuum

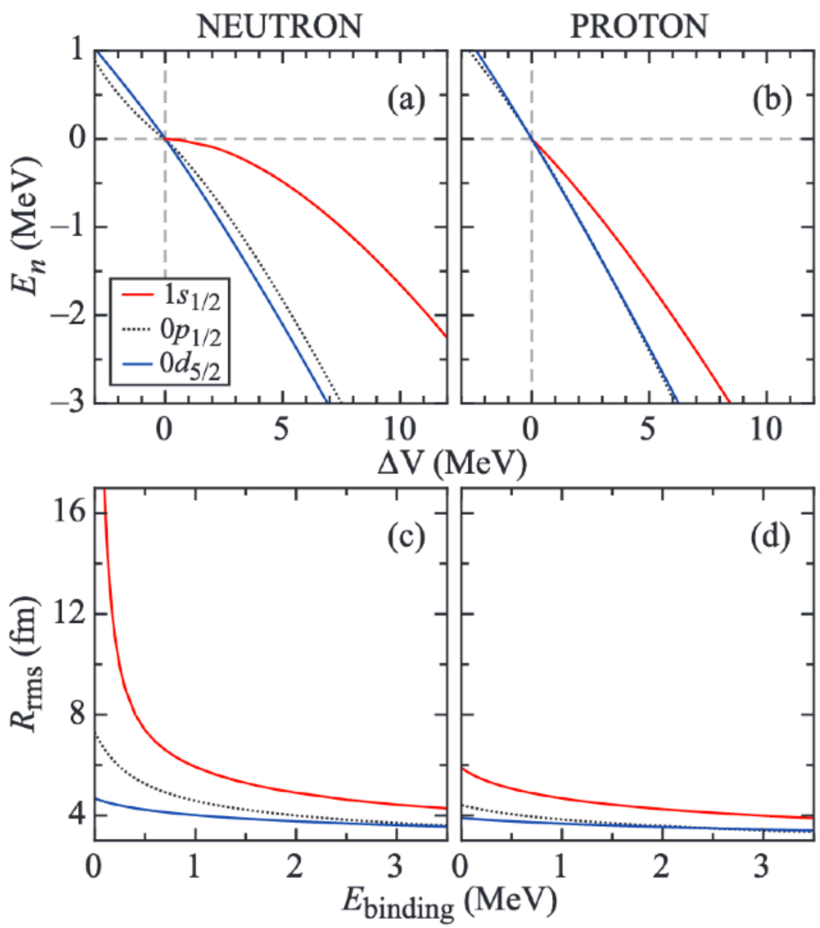


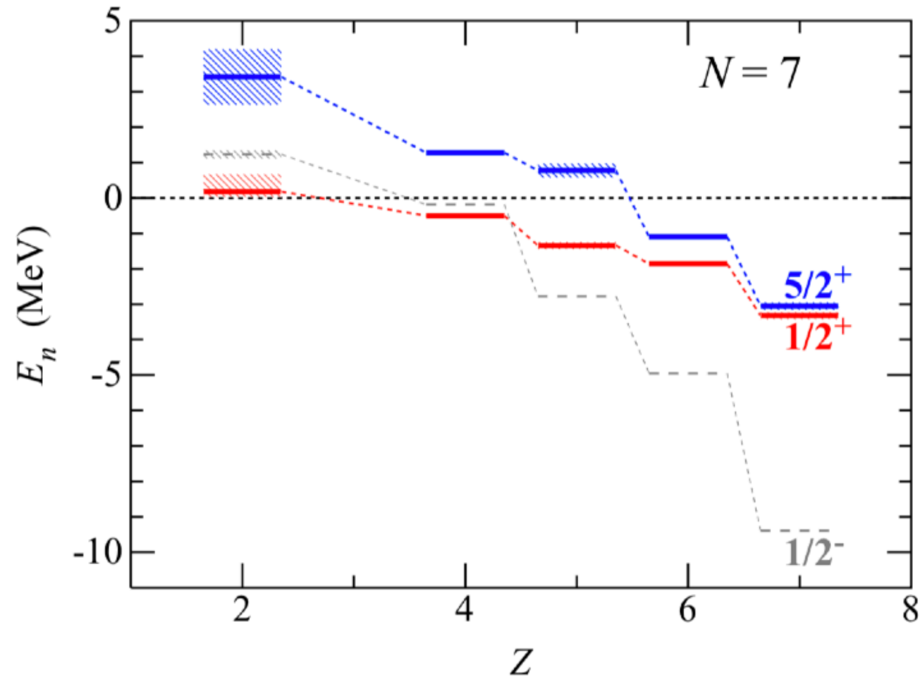

D. Abrahamsen, A. Volya, and I. Wiedenhoever, *Effective R-matrix parameters of the Woods-Saxon nuclear potential,* APS Volume 57, Number 16, section KA 26 (2012).

# Effect of weak binding



# Effect of weak binding



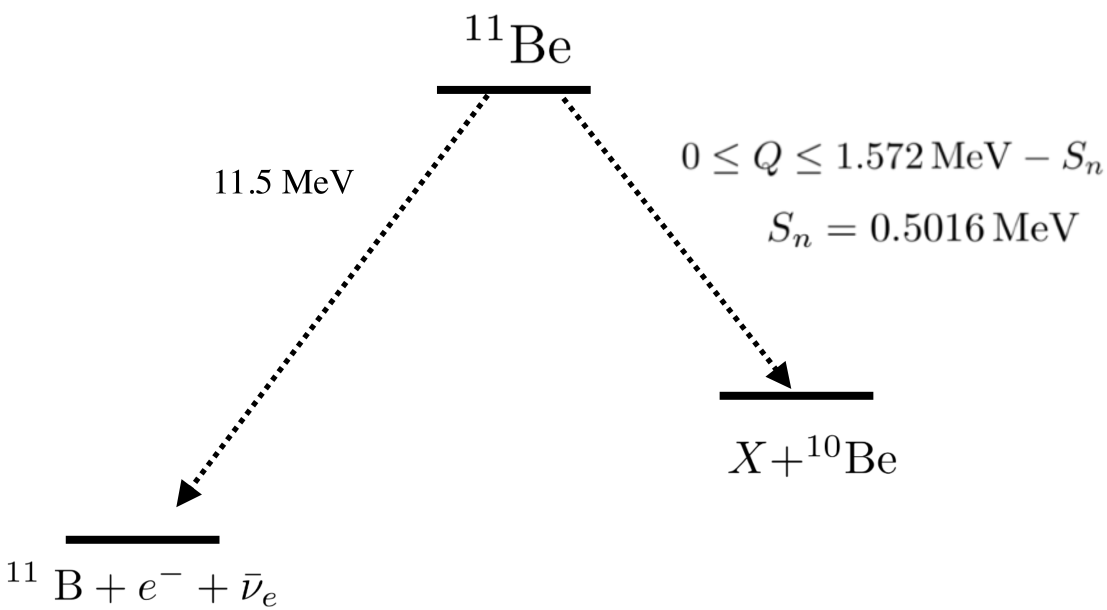




C. R. Hoffman, B. P. Kay, and J. P. Schiffer Phys. Rev. C 89, 061305(R)
B. P. Kay, C. R. Hoffman, and A. O. Macchiavelli Phys. Rev. Lett. 119, 182502

# 11Be beta decya

$$^{11}\mathrm{Be} \nearrow {}^{11}\mathrm{B} + e^- + \bar{\nu}_e \quad t_{\mathrm{Be}} = 13.76\ \mathrm{s}$$

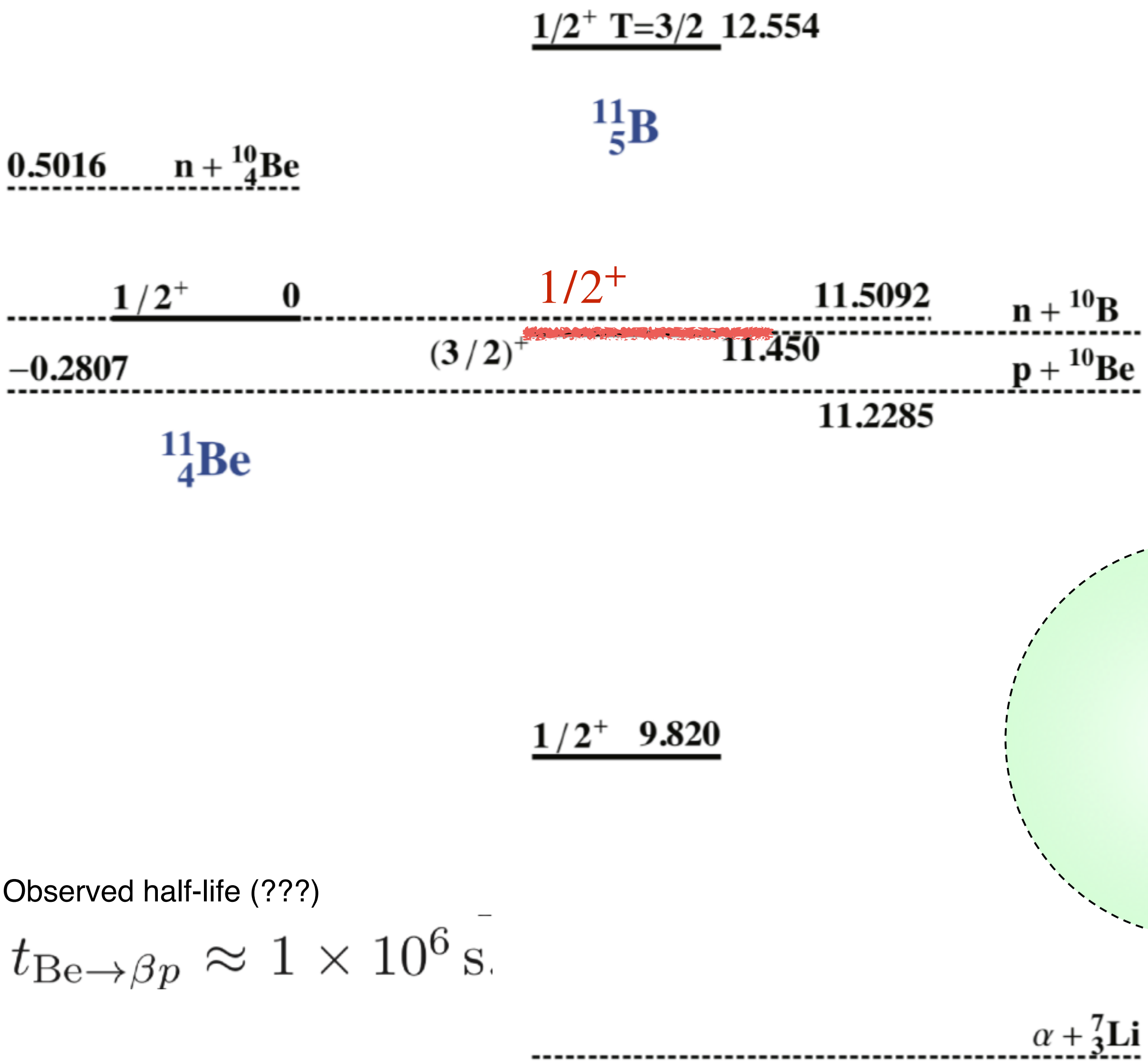
$$^{11}\mathrm{Be} \searrow X + {}^{10}\mathrm{Be} \quad t_{\mathrm{Be}\to X} > 10^5\ \mathrm{s}$$

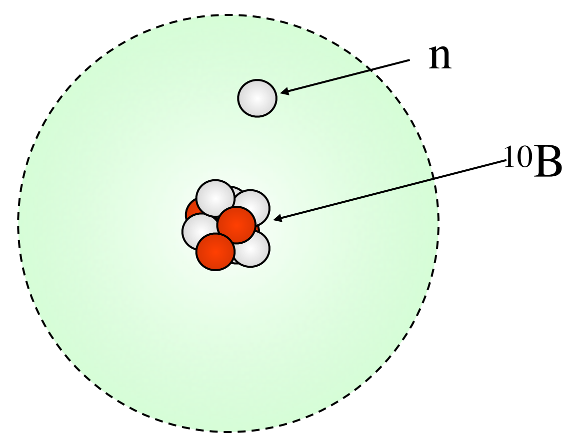




M. Pfützner, K. Riisager, *Examining the possibility to observe neutron dark decay in nuclei,* Phys. Rev. C **97** (2018) 042501.
F. E. Wietfeldt, G. L. Greene, *Colloquium: The neutron lifetime,* Rev. Mod. Phys. **83** (2011) 1173.

# 11Be beta-delayed proton decay



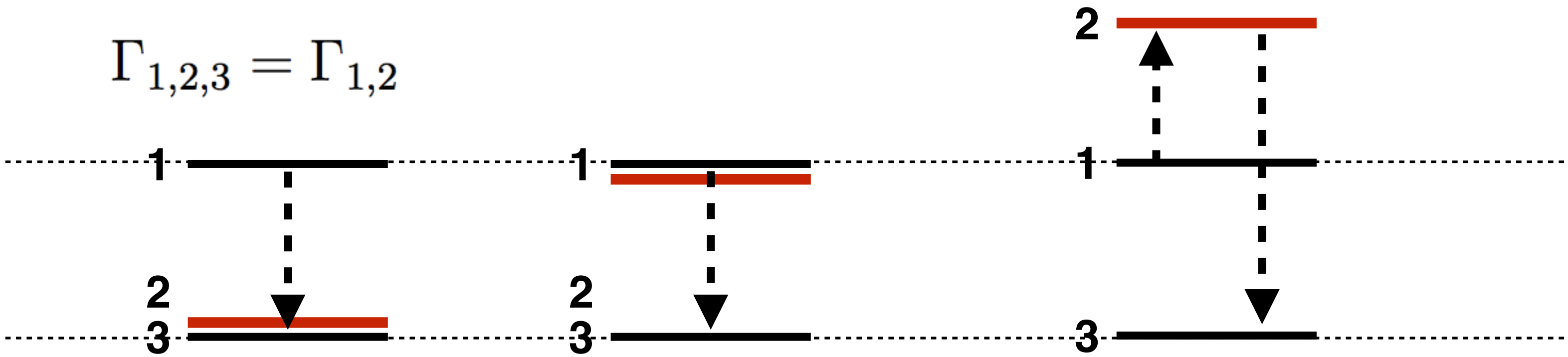




Observed half-life (???)

$$t_{Be \to \beta p} \approx 1 \times 10^{6}\ s.$$

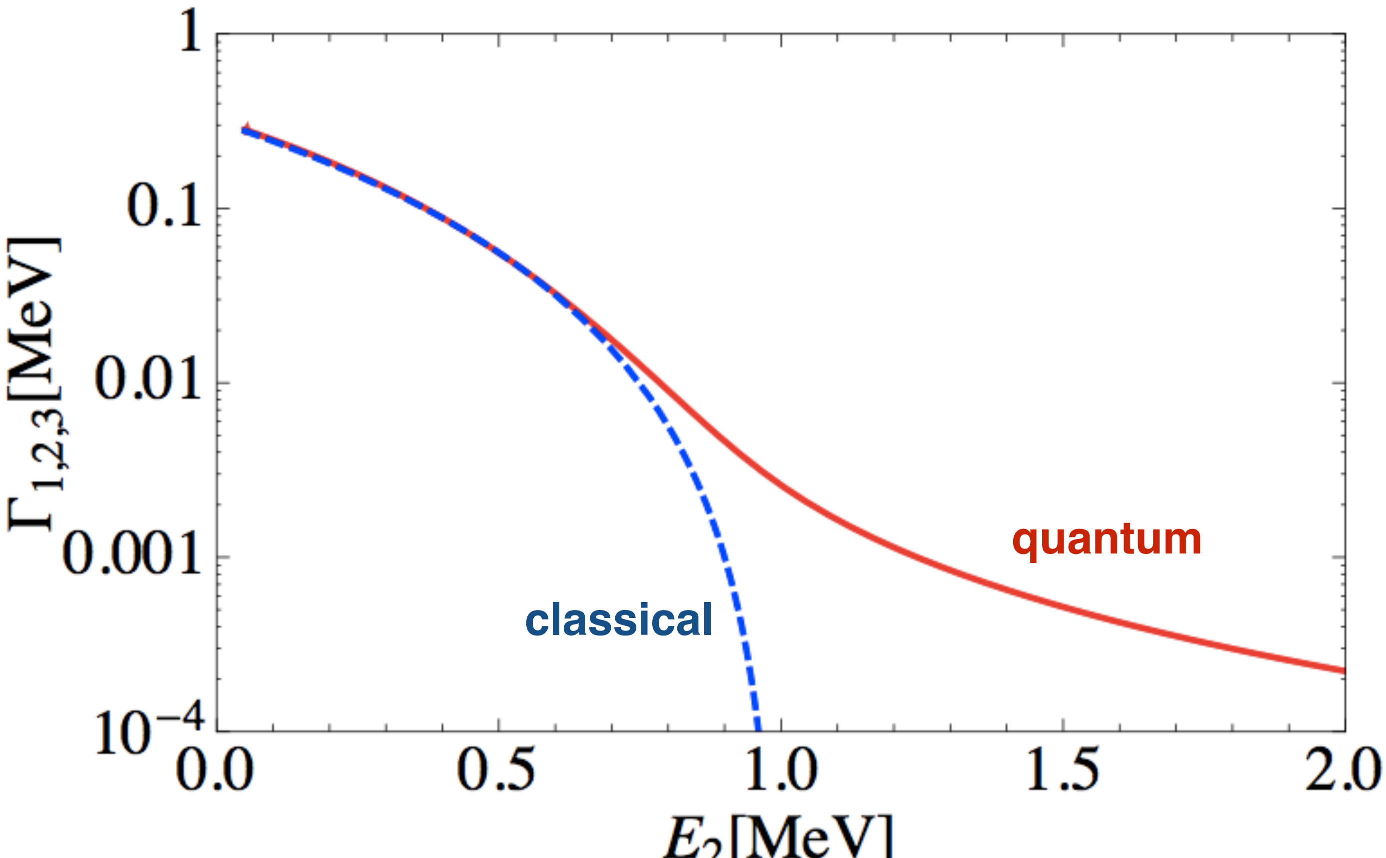
# sequential mechanism

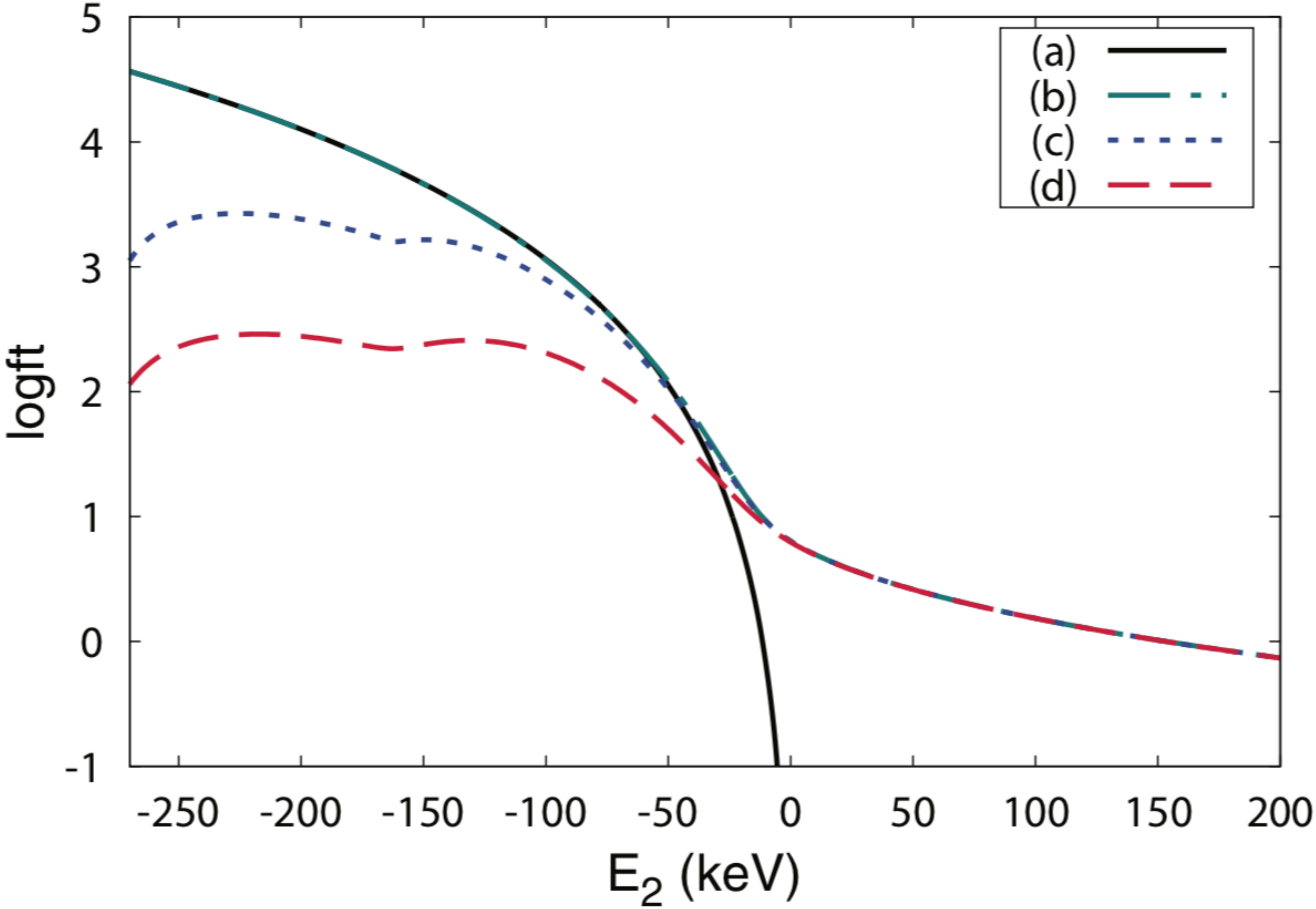
$\Gamma_{1,2,3} = \Gamma_{1,2}$



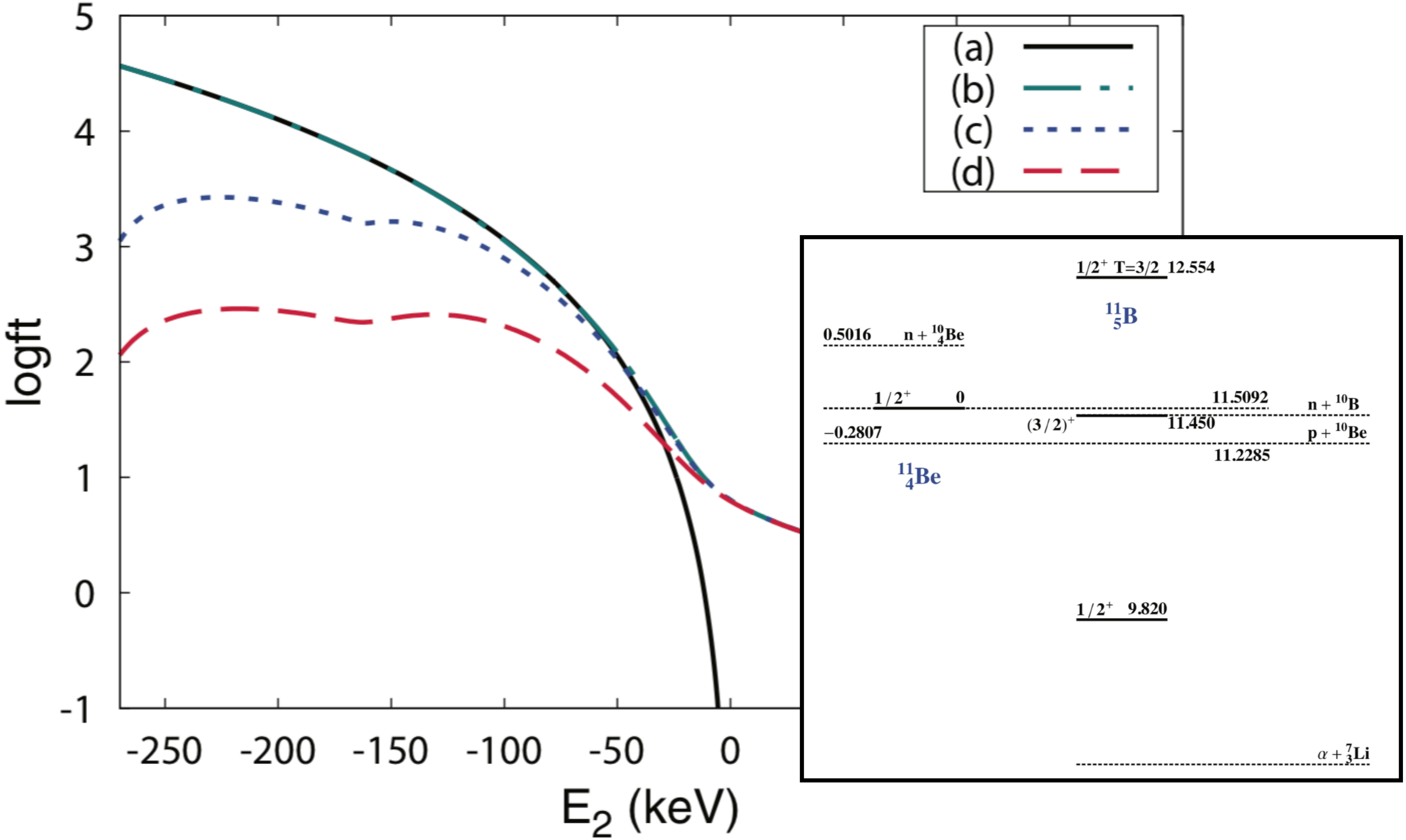

Classical limit

Virtual process

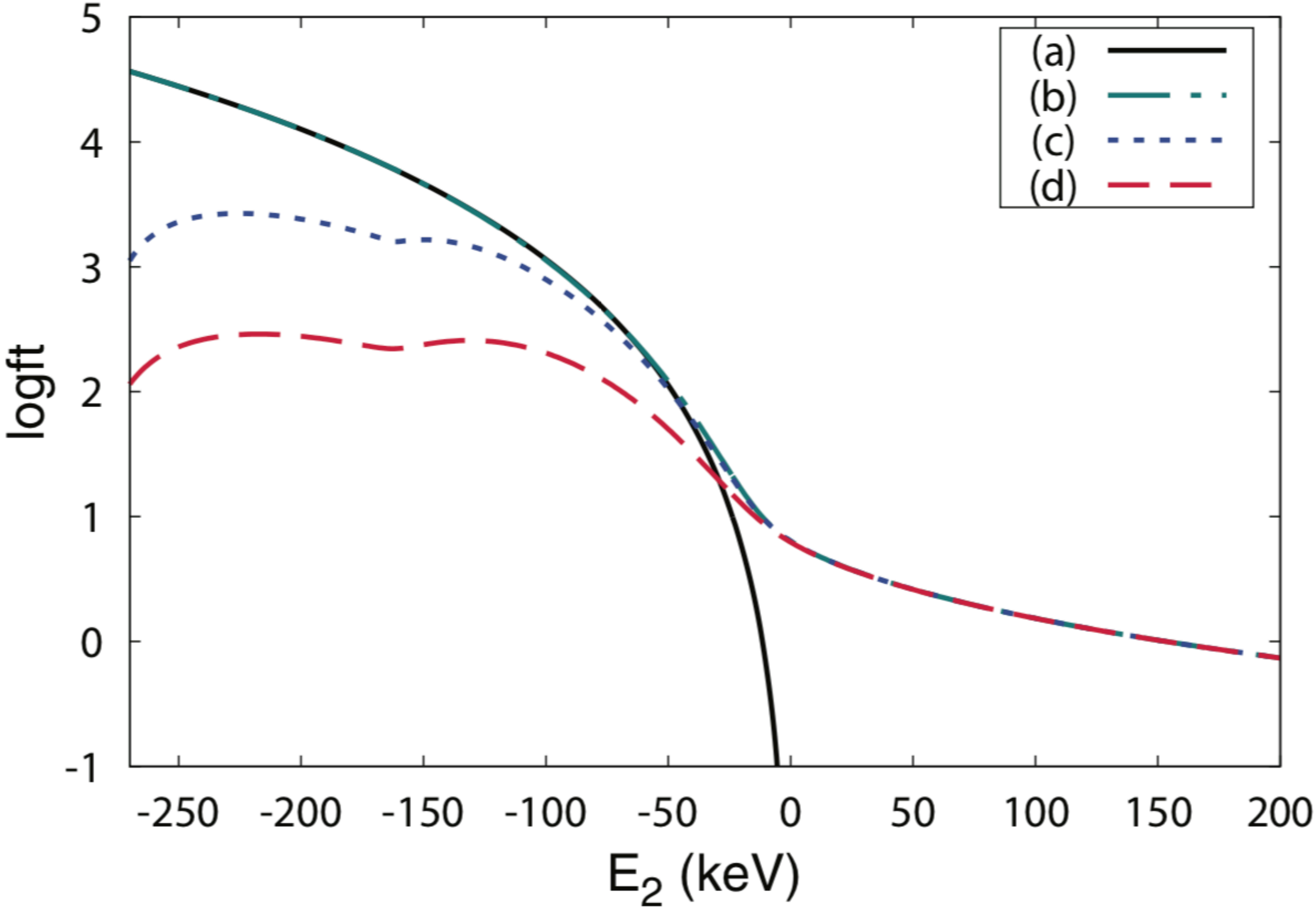




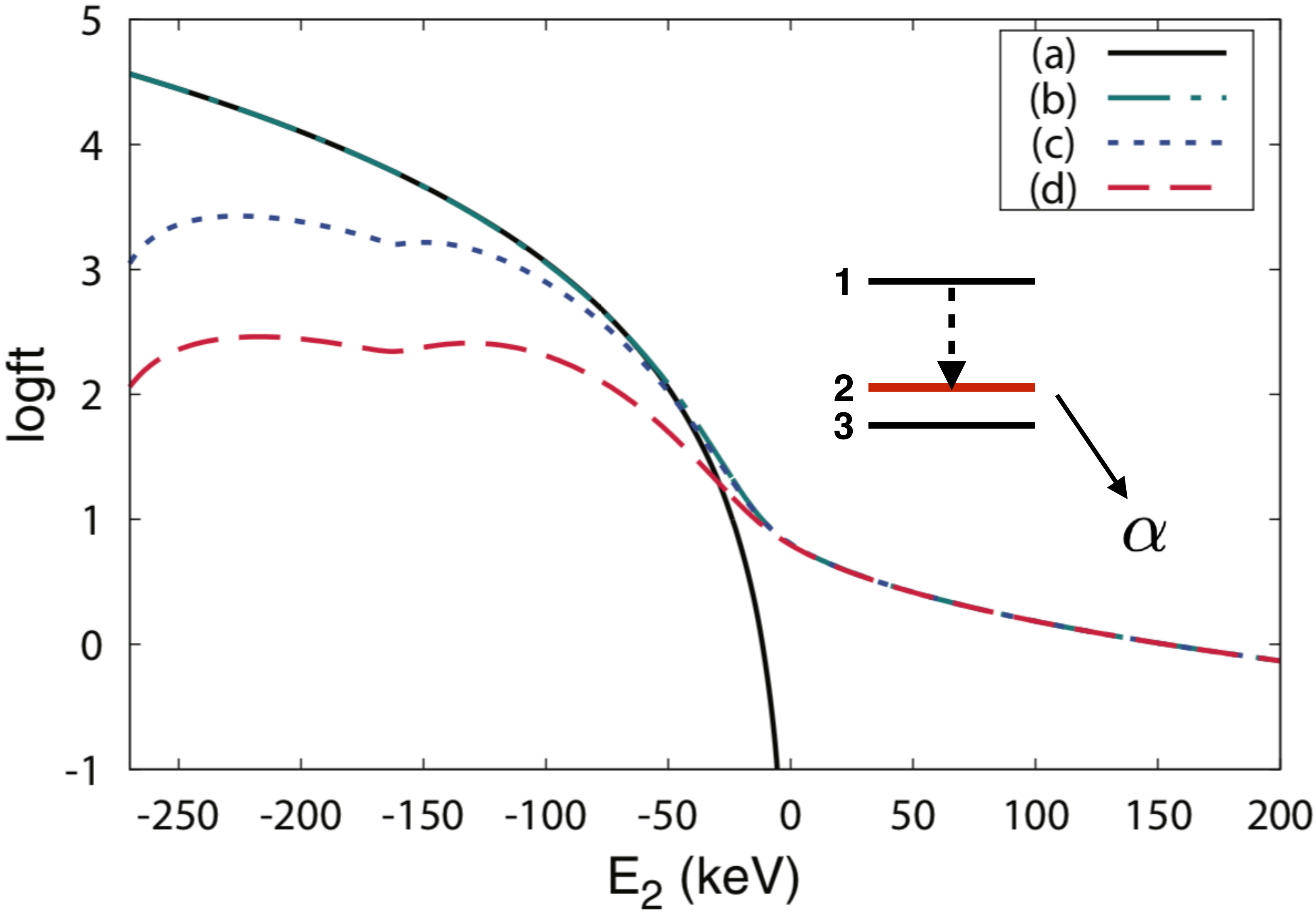

$t_{\mathrm{Be}\to\beta p} \approx 1 \times 10^6\,\mathrm{s}.$



5
4
3
2
1
0
-1
logft
-250
-200
-150
-100
-50
0
E2 (keV)
(a)
(b)
(c)
(d)
1/2+ T=3/2 12.554
11 5 B
0.5016
n + 10 4 Be
1/2+
0
11.5092
n + 10B
(3/2)+
11.450
−0.2807
p + 10Be
11.2285
11 4 Be
1/2+ 9.820
α + 7 3 Li



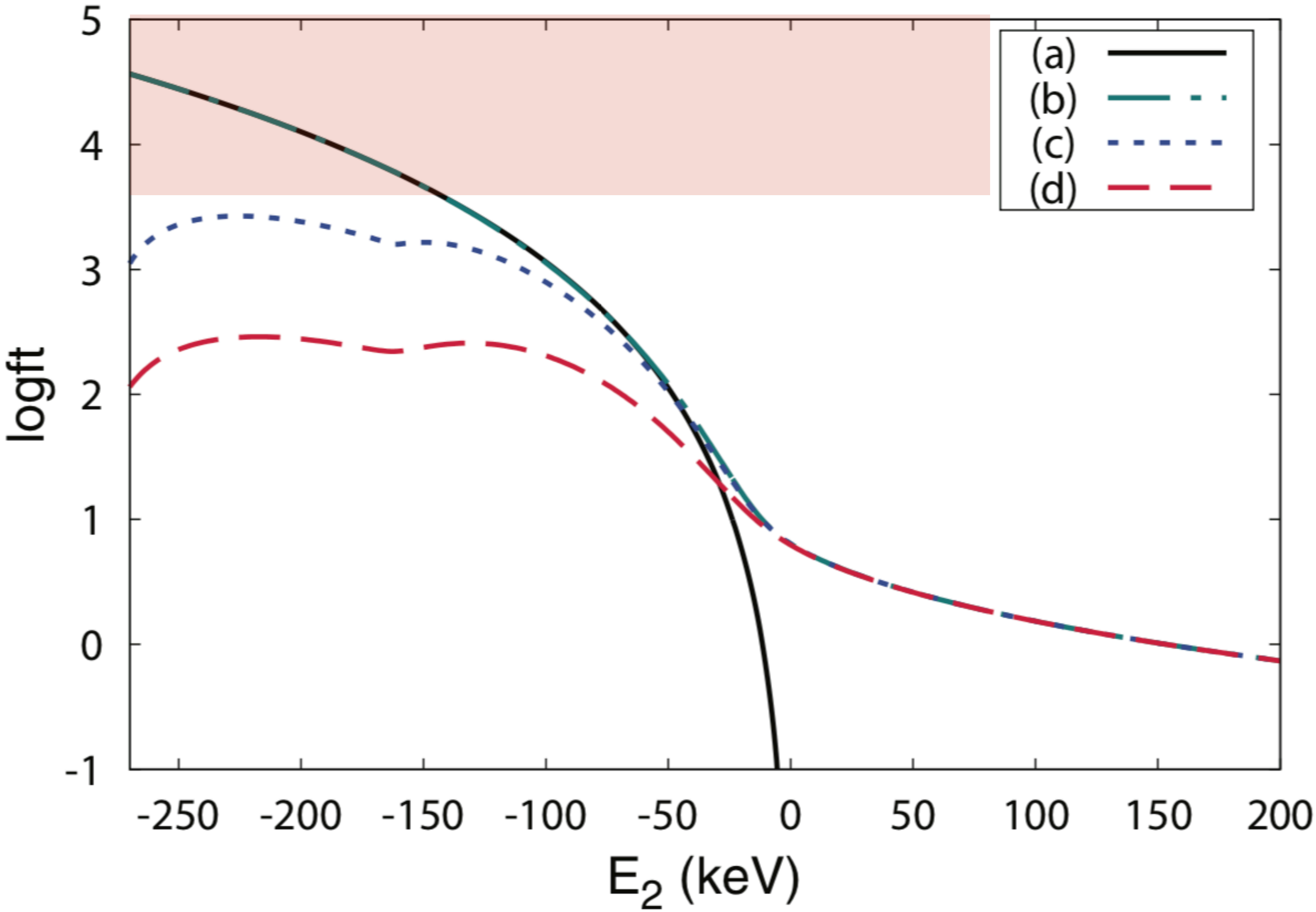
5
4
3
2
1
0
-1
logft
-250
-200
-150
-100
-50
0
50
100
150
200
E$_2$ (keV)
(a)
(b)
(c)
(d)



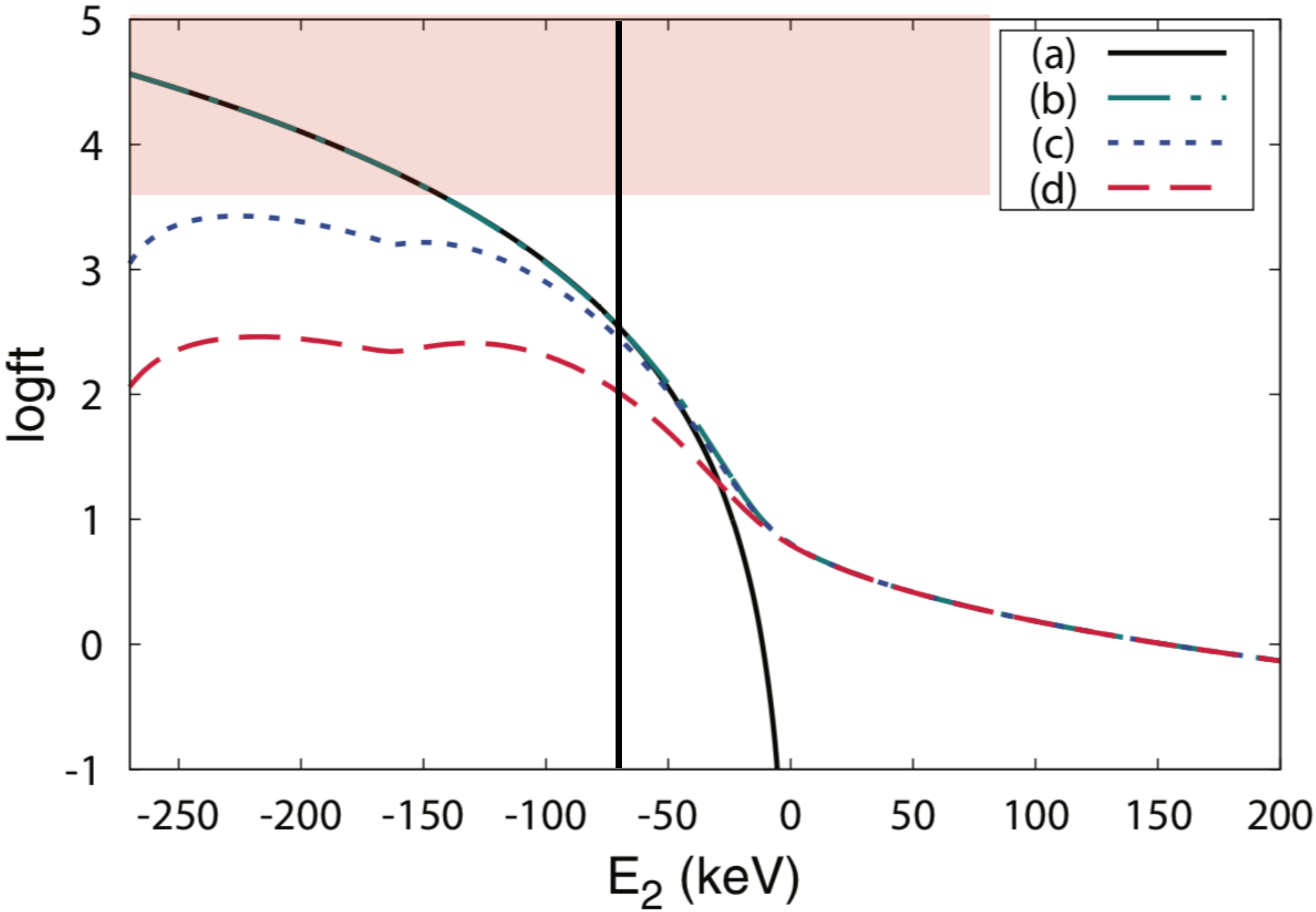
(a)
(b)
(c)
(d)
1
2
3
α
logft
E2 (keV)
5
4
3
2
1
0
-1
-250
-200
-150
-100
-50
0
50
100
150
200

1/2+ states in 11B

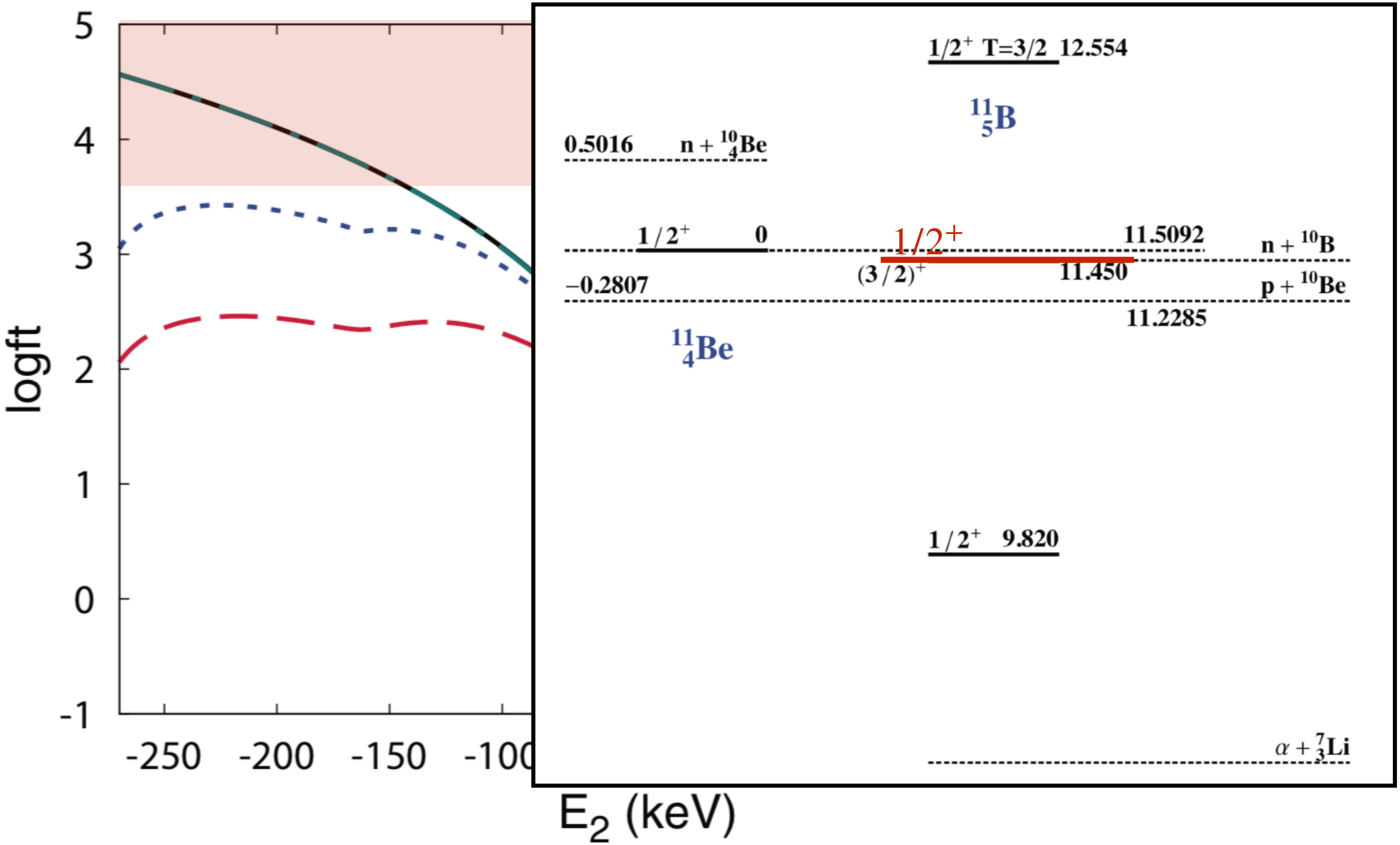
| | Theory (FSU) | | | Experiment | |
|---|---|---|---|---|---|
| **J** | **E(MeV)** | **log(ft)** | **SF(p)** | **E(MeV)** | **SF(p)** |
| **1/2+(1)** | 5.709 | 5.5 | 0.262 | 6.792 | |
| **1/2+(2)** | 10.545 | 3.4 | 0.117 | 9.820 | |
| **1/2+(3)** | 11.952 | 3.5 | 0.134 | **11.44** | **0.27(6)** |
| **1/2+(4) T=3/2** | 12.181 | | 0.274 | 12.554 | |
| **1/2+(5)** | 12.827 | 4.0 | 0.028 | | |
| **1/2+(6)** | 14.105 | 5.4 | 0.001 | | |



(a)
(b)
(c)
(d)
logft
5
4
3
2
1
0
-1
-250
-200
-150
-100
-50
0
50
100
150
200
$E_2$ (keV)




E. Lopez-Saavedra, et.al. resonance at 211(40) keV

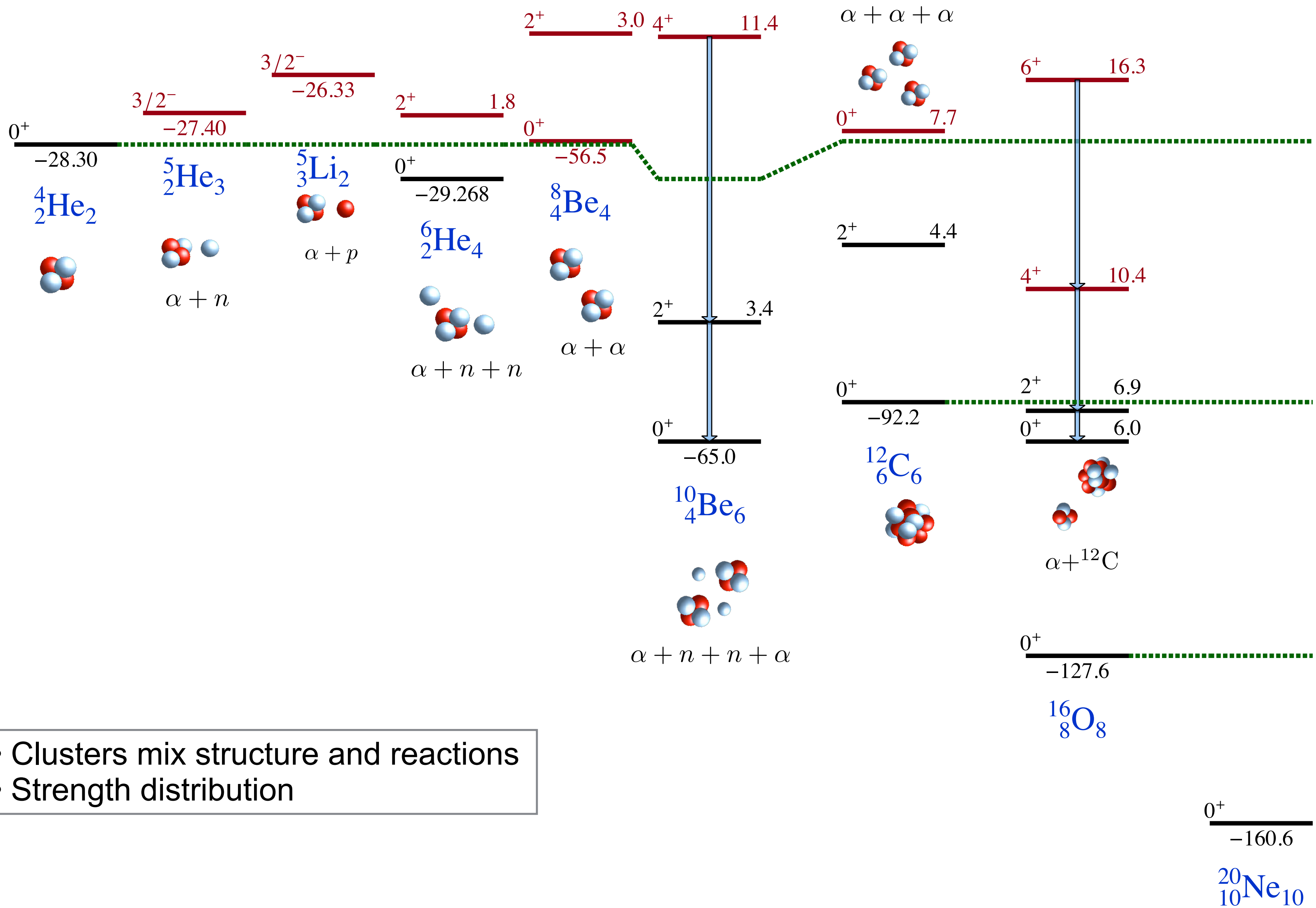



E. Lopez-Saavedra, et.al. resonance at 211(40) keV

# Questions

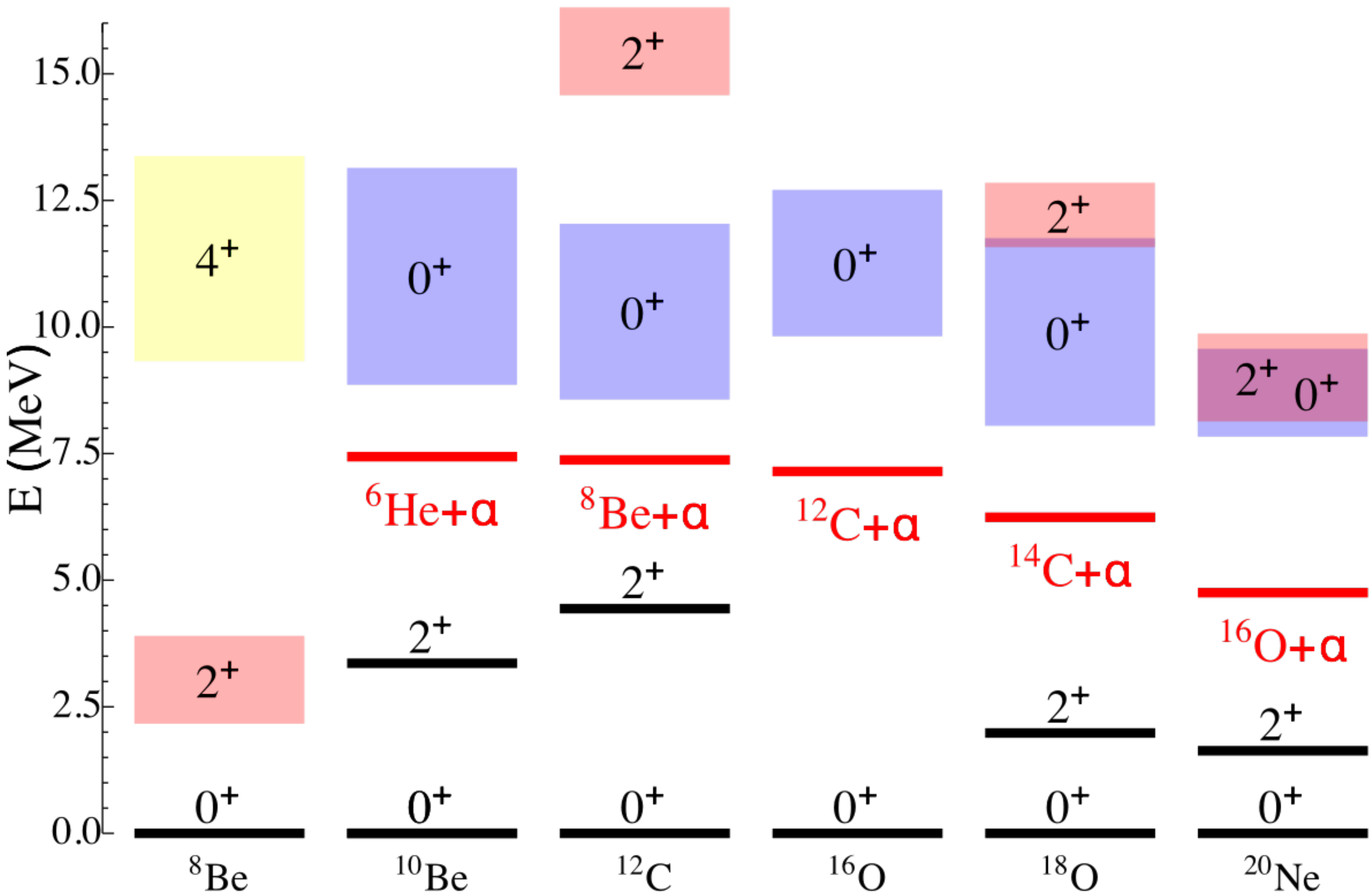
- Even with proton resonance, beta-delayed rate is hard to explain
- The proton resonance is likely 1/2+(3), why it is lowered (predicted 12.2 MeV, observed at 11.44 MeV
- Why proton SF is so large, while there is no alpha decay?
- Can exotic mechanisms, transition via continuum, isospin mixing etc explain the situation

# Clustering in light nuclei

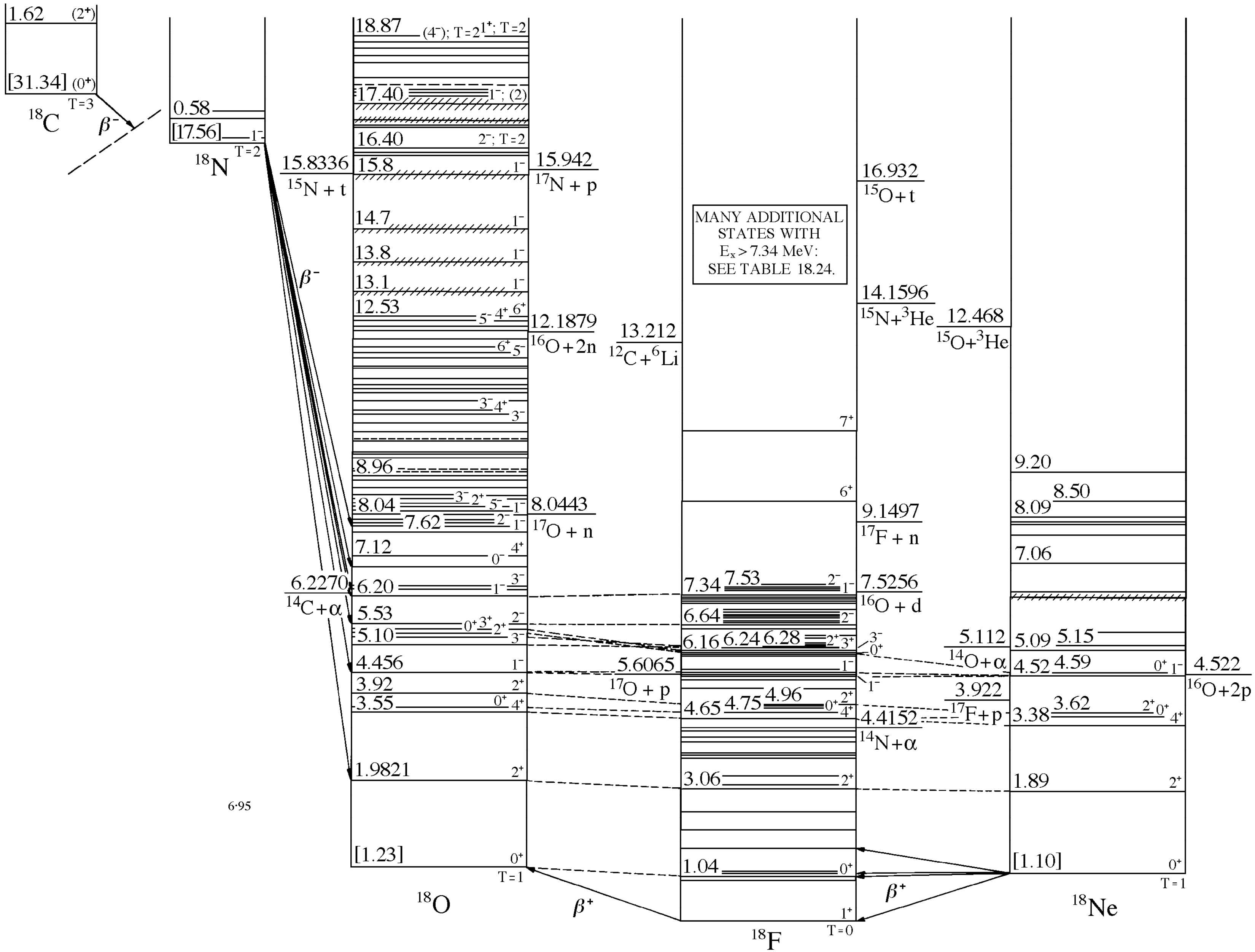



- Clusters mix structure and reactions
- Strength distribution

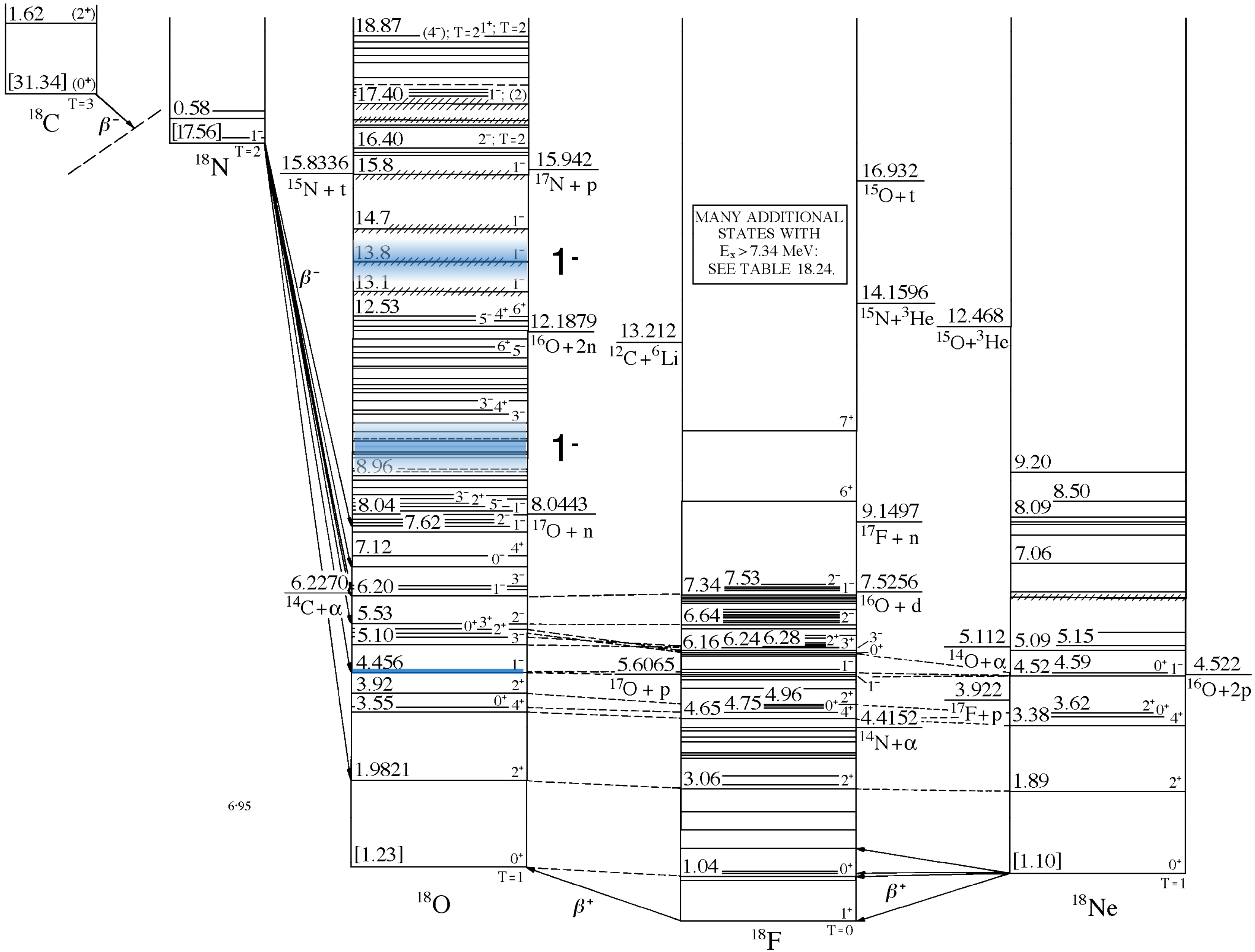
# Clustering and continuum

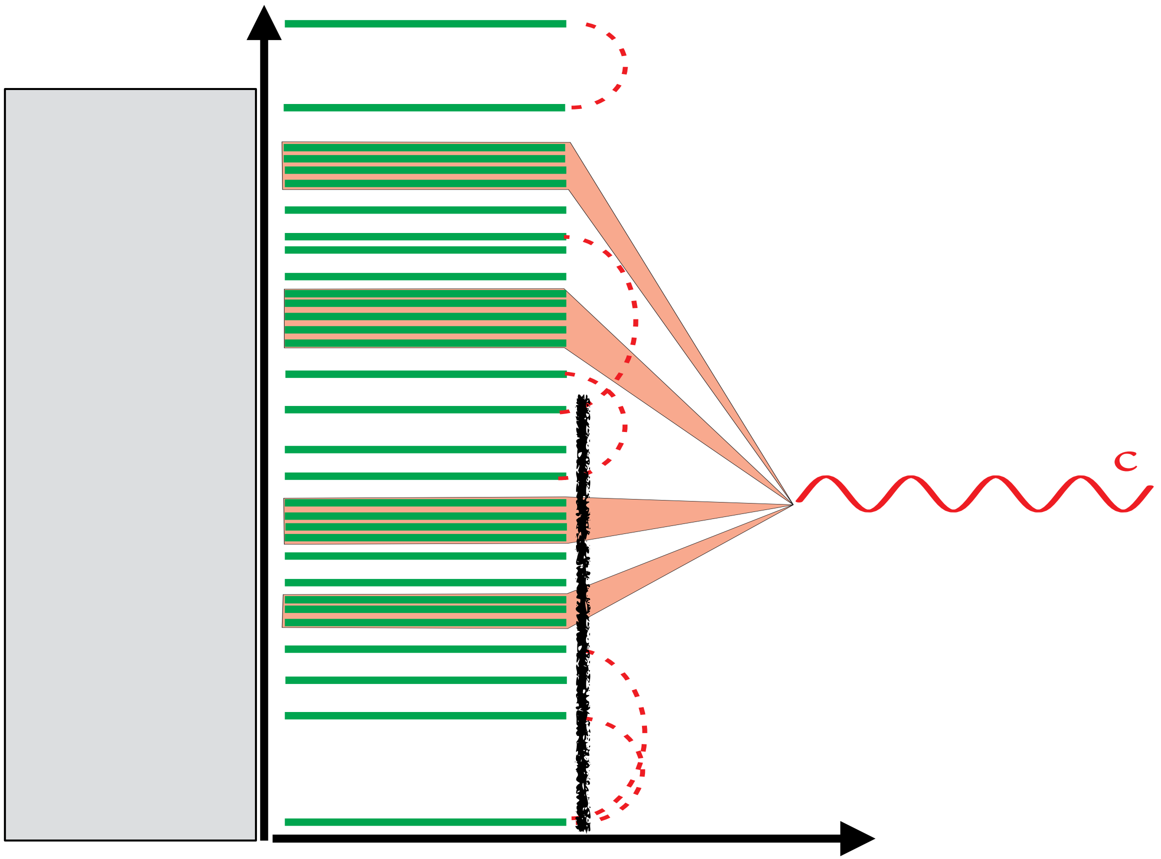


# Clustering in A=18 mirror systems



# Clustering in A=18 mirror systems



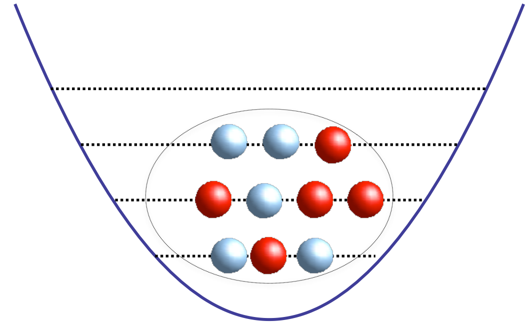

c

# Translational invariance and Center of Mass (CM)

**Shell model, Glockner-Lawson procedure**

$$\Psi_D = \phi_{000}(\mathbf{R}_D)\,\Psi'_D$$

SM state — Center-of-mass vibration — Intrinsic state



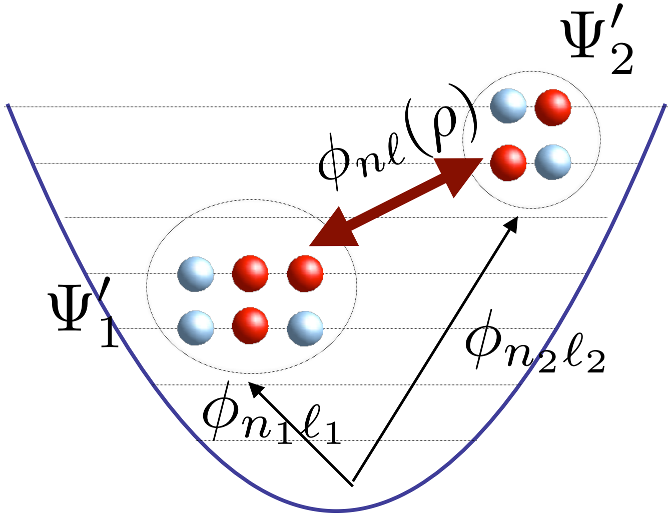
**Controlling CM with operator** $\mathbf{R}$

Control only CM quanta

$$D_\mu = \sqrt{\frac{4\pi}{3}}R_\mu \qquad R_\mu = \sqrt{\frac{\hbar}{2Am\omega}}(\mathcal{B}^\dagger_\mu + \mathcal{B}_\mu)$$

# Clustering reaction basis channel

(basis states for clustering)




$$\Psi = \phi_{000}(\mathbf{R})\Psi'$$

Boost

$$\Psi_{n\ell m} = \phi_{n\ell m}(\mathbf{R})\,\Psi'$$

CM-Recouple

$$\Phi_{n\ell m} = \mathcal{A}\left\{\phi_{000}(\mathbf{R})\phi_{n\ell m}(\boldsymbol{\rho})\Psi'^{(1)}\Psi'^{(2)}\right\}$$

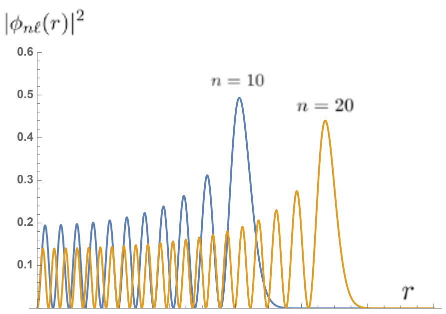
$$\Phi^{\dagger}_{n\ell} = \sum_{\substack{n_1\ell_1\\ n_2\ell_2}} \mathcal{M}^{n\ell 00;\ell}_{n_1\ell_1 n_2\ell_2}\left[\Psi^{\dagger}_{n_1\ell_1 m_1} \times \Psi^{\dagger}_{n_2\ell_2 m_2}\right]_{\ell}$$

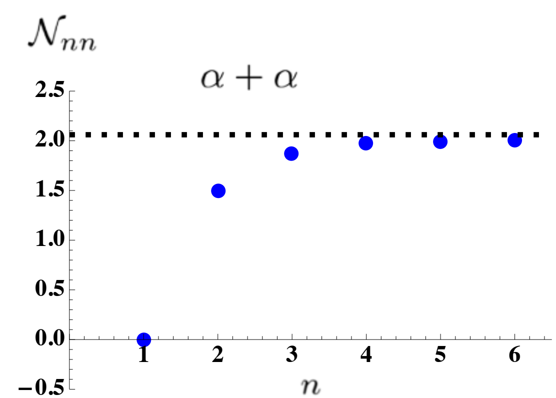
# Resonating group method and reactions

$$\sum_n \mathcal{H}^{(\ell)}_{nn'}\chi_{n'} = E\sum_n \mathcal{N}^{(\ell)}_{nn'}\chi_{n'}$$

$$\begin{pmatrix} \mathcal{H}_{00} & \dots & \mathcal{H}_{0n} & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & 0 & \vdots & \vdots \\ \mathcal{H}_{n0} & \dots & \mathcal{H}_{nn} & T_{nn+1} & 0 & \vdots \\ 0 & 0 & T_{n+1n} & T_{n+1n+1} & T_{n+1n+2} & 0 \\ 0 & \dots & 0 & T_{n+2n+1} & T_{n+2n+2} & \ddots \\ 0 & \dots & \dots & 0 & \ddots & \ddots \end{pmatrix} \begin{pmatrix} \chi_0 \\ \vdots \\ \chi_n \\ x_{n+1} \\ x_{n+2} \\ \vdots \end{pmatrix} = \mathrm{E} \begin{pmatrix} \mathcal{N}_{00} & \dots & \mathcal{N}_{0n} & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & 0 & \vdots & \vdots \\ \mathcal{N}_{n0} & \dots & \mathcal{N}_{nn} & 0 & 0 & \vdots \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \dots & 0 & 0 & 1 & 0 \\ 0 & \dots & \dots & 0 & 0 & \ddots \end{pmatrix} \begin{pmatrix} \chi_0 \\ \vdots \\ \chi_n \\ x_{n+1} \\ x_{n+2} \\ \vdots \end{pmatrix}$$

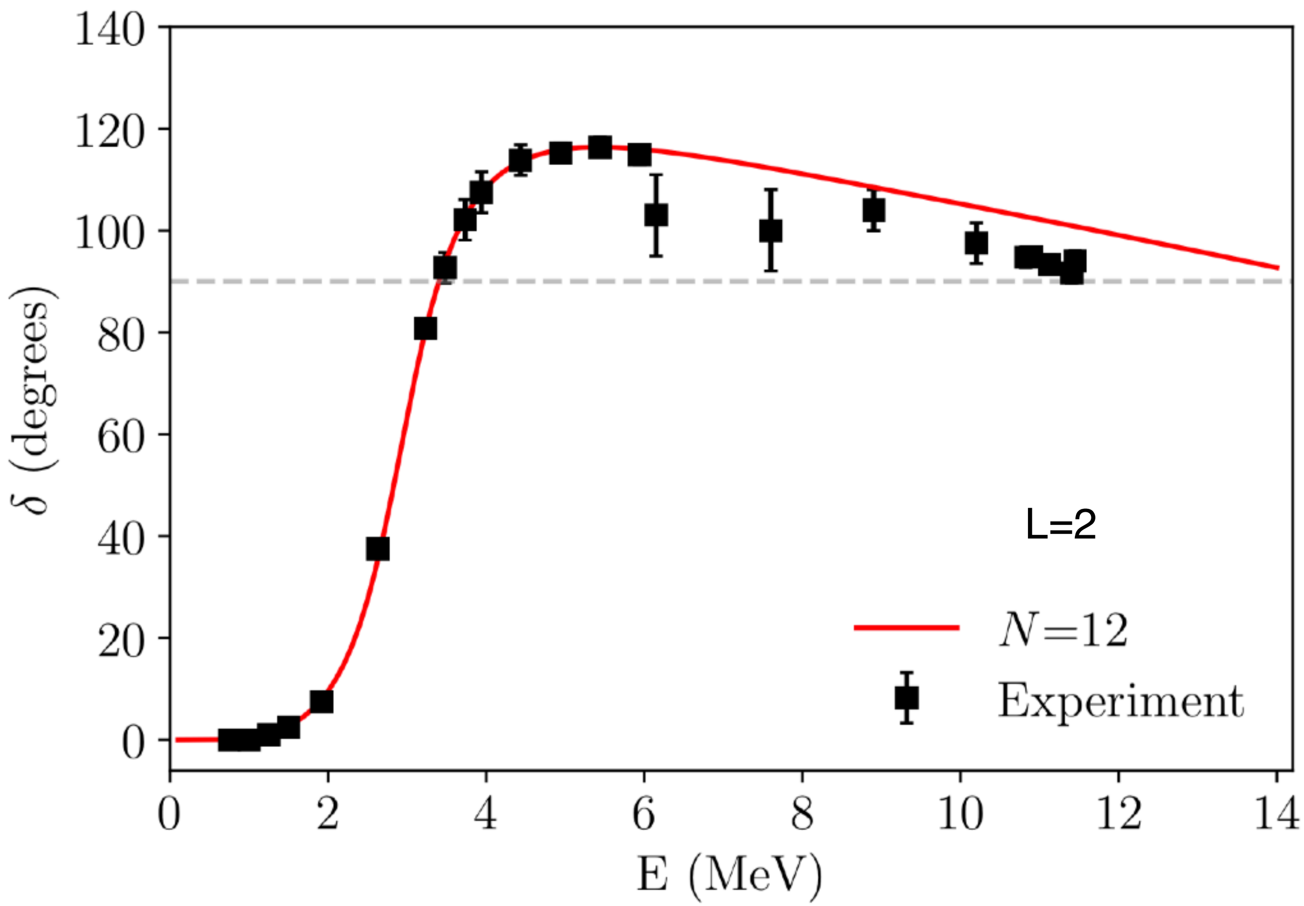
Asymptotic solution with phase shift



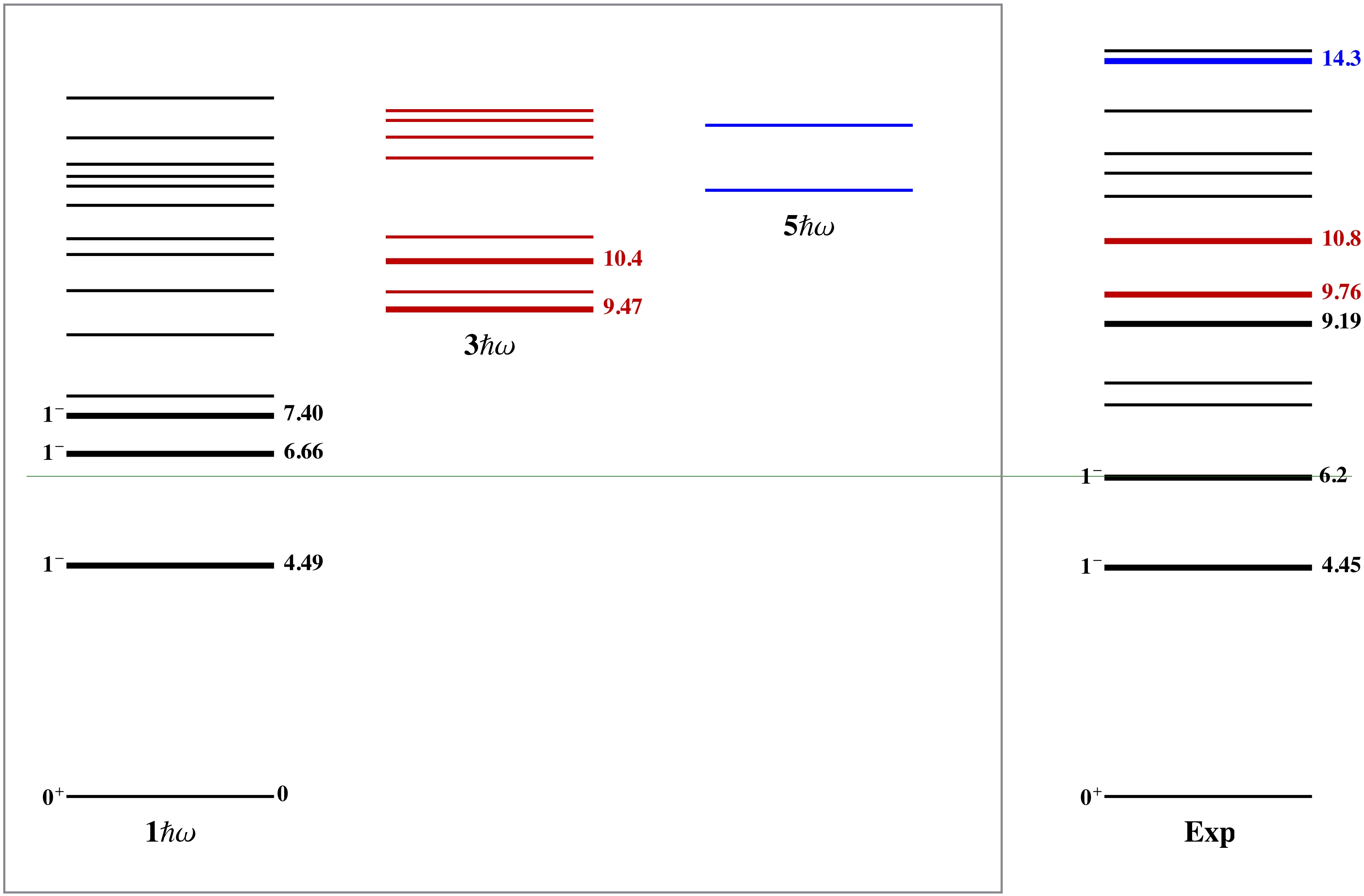

J-matrix (or HORSE) method: J. M. Bang, Annals of Physics **280**, 299 (2000)
Experimental data: Phys. Rev. 168, 1114 (1968); Nucl. Phys. **A287**, 317 (1977)
K. Kravvaris, A. Volya, Phys. Rev. C 100 (2019) 034321

# alpha+alpha scattering phase shifts



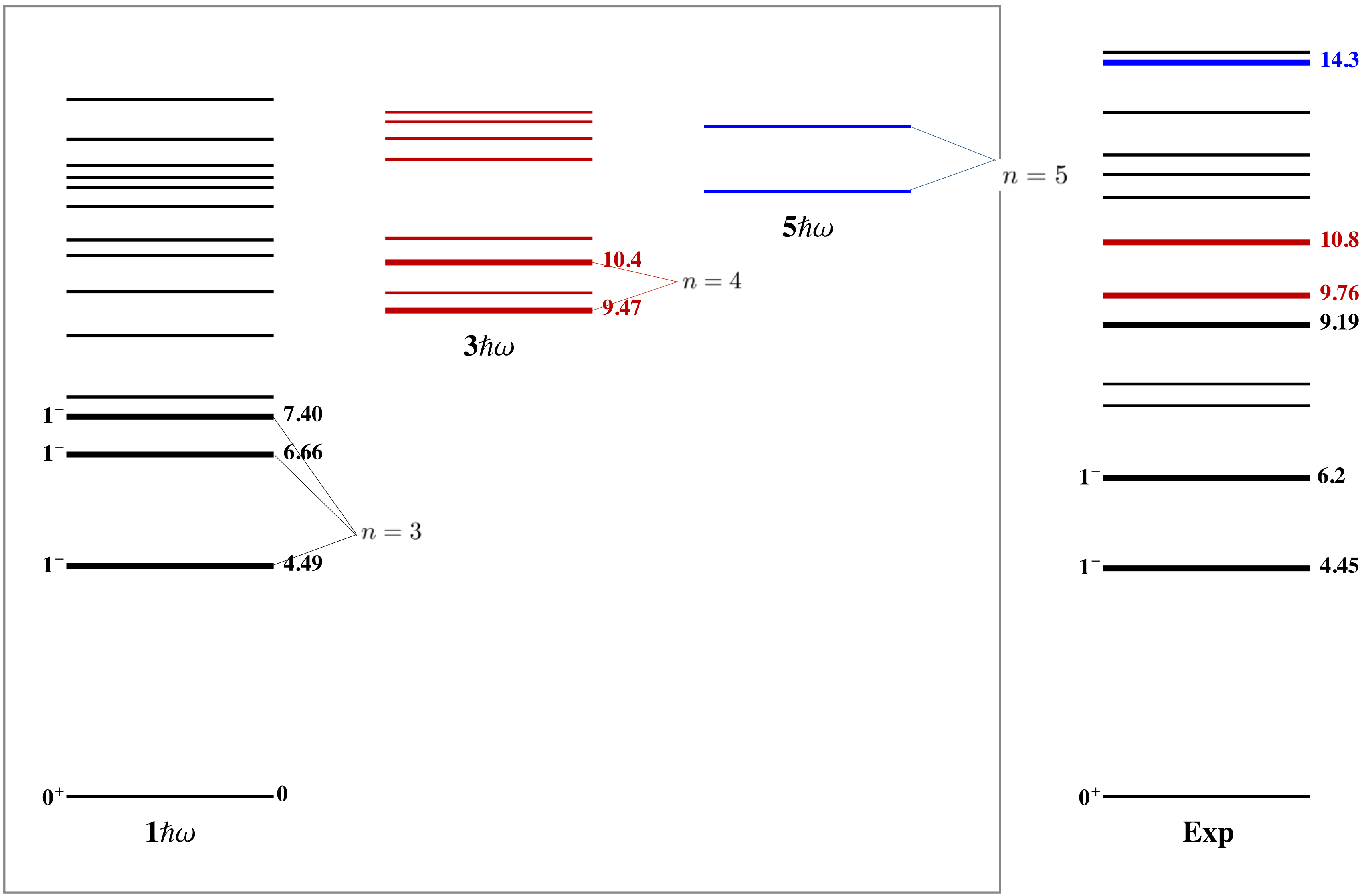

Experimental data from S. A. Afzal, A. A. Z. Ahmad, and S. Ali, Rev. Mod. Phys. 41, 247 (1969).
K. Kravvaris, A. Volya, Phys. Rev. C 100 (2019) 034321

# Channel coupling in $^{18}O$
# l=1 channel

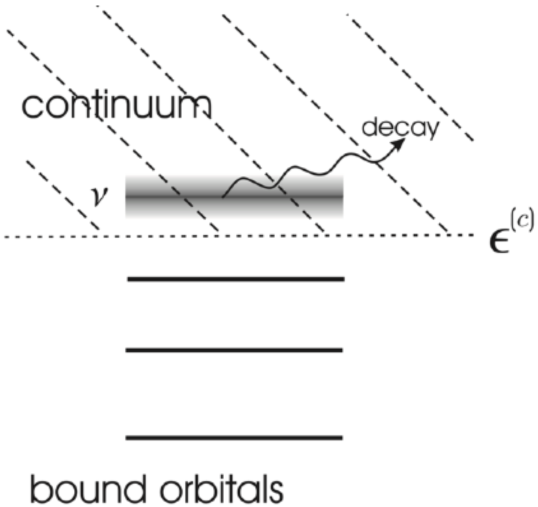



M. Barbui, et al.,(2022) https://arxiv.org/abs/2206.10659

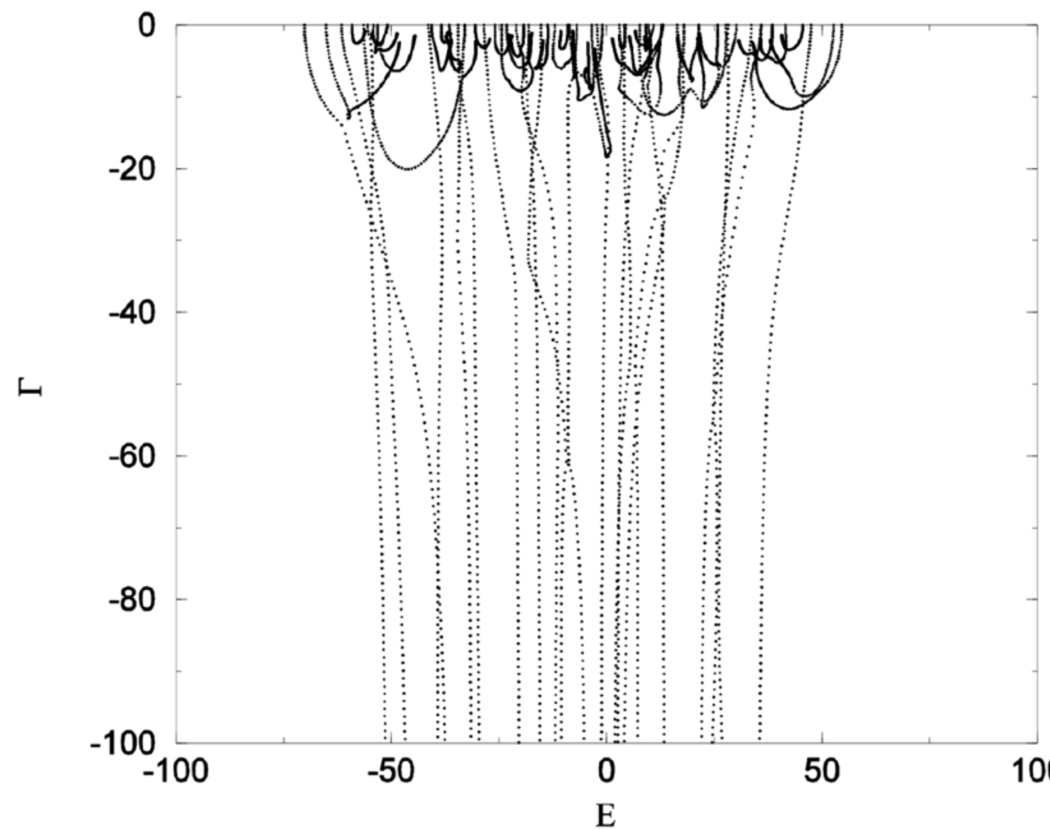
# Channel coupling in $^{18}O$
# l=1 channel



# Single-particle decay in many-body system



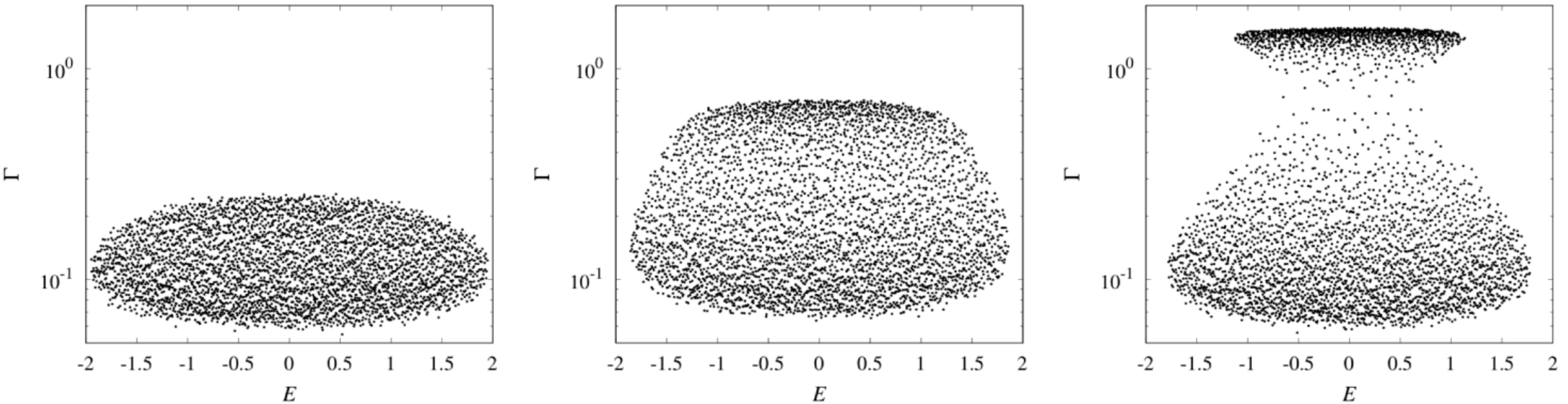

Evolution of complex energies E=E-i $\Gamma/2$ as a function of $\gamma$




- Assume energy independent W
- Assume one channel $\gamma=A^2$
- System 8 s.p. levels, 3 particles
- One s.p. level in continuum $e=\varepsilon - i\gamma/2$

Total states 8!/(3! 5!)=56; states that decay fast 7!/(2! 5!)=21

# Separation of states in the complex plane




(a) $\varkappa = 0.5$ (b) $\varkappa = 1$ (c) $\varkappa = 1.8$

# Observing superradiance

$$H = \begin{pmatrix} \epsilon - \frac{i}{2}\Gamma & v \\ v & 0 \end{pmatrix} = H_0 - \frac{i\Gamma}{2} A^\dagger A \qquad A = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Stationary system $\Gamma = 0$

Energies

$$E_{1,2} = \frac{1}{2}\left(\epsilon \pm \sqrt{\epsilon^2 + 4v^2}\right)$$

Spectroscopic Factors

$$\mathrm{SF}_{1,2} = \frac{1}{2}\left(1 \pm \frac{\epsilon}{\sqrt{\epsilon^2 + 4v^2}}\right)$$

# Observing superradiance

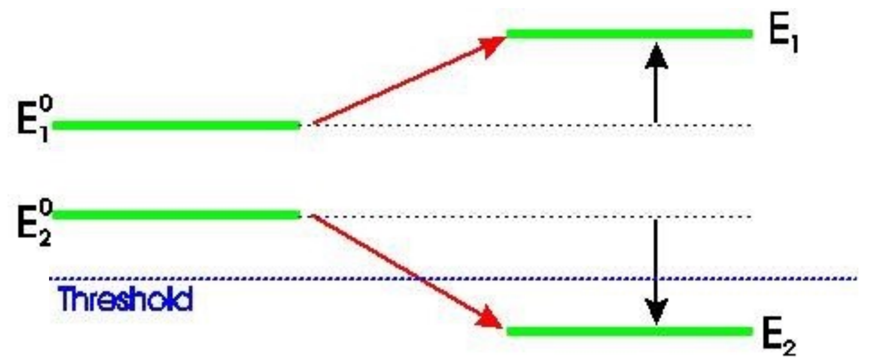
$$H = \begin{pmatrix} \epsilon - \frac{i}{2}\Gamma & v \\ v & 0 \end{pmatrix} = H_0 - \frac{i\Gamma}{2} A^\dagger A \qquad A = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Energies $$\mathcal{E}_{1,2} = \frac{1}{2}\left(\epsilon - \frac{i}{2}\Gamma \pm \sqrt{\left(\epsilon - \frac{i}{2}\Gamma\right)^2 + 4v^2}\right)$$

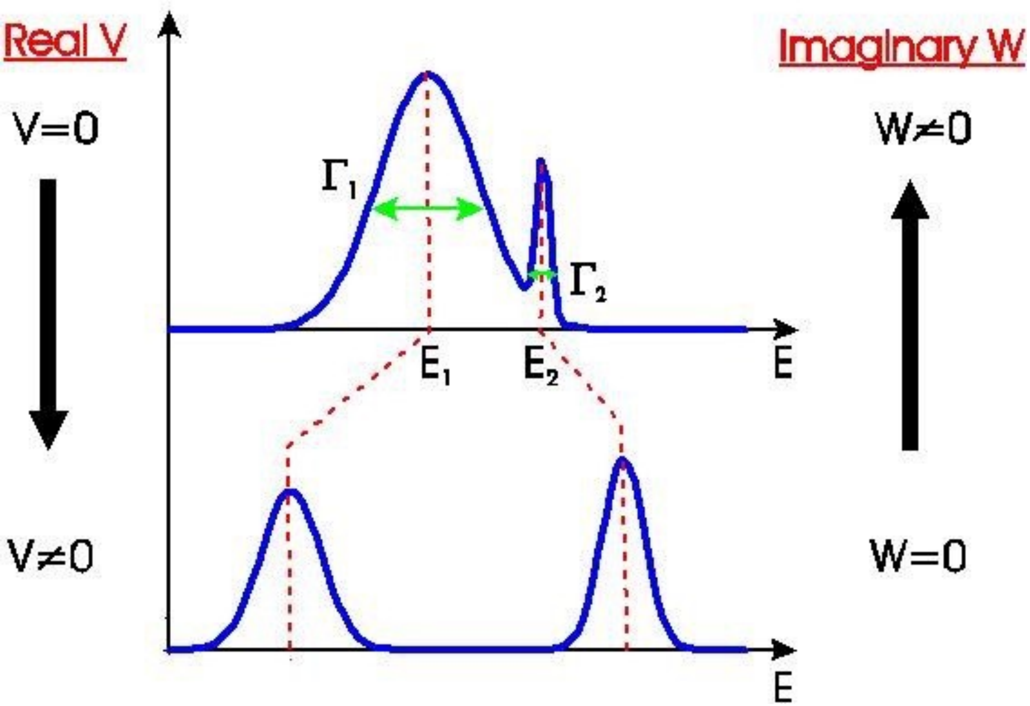
Width $$\Gamma_{1,2} = -2\,\mathrm{Im}\left(\mathcal{E}_{1,2}\right)$$

Spectroscopic Factors $$\mathrm{SF}_{1,2} = \Gamma_{1,2}/\Gamma$$

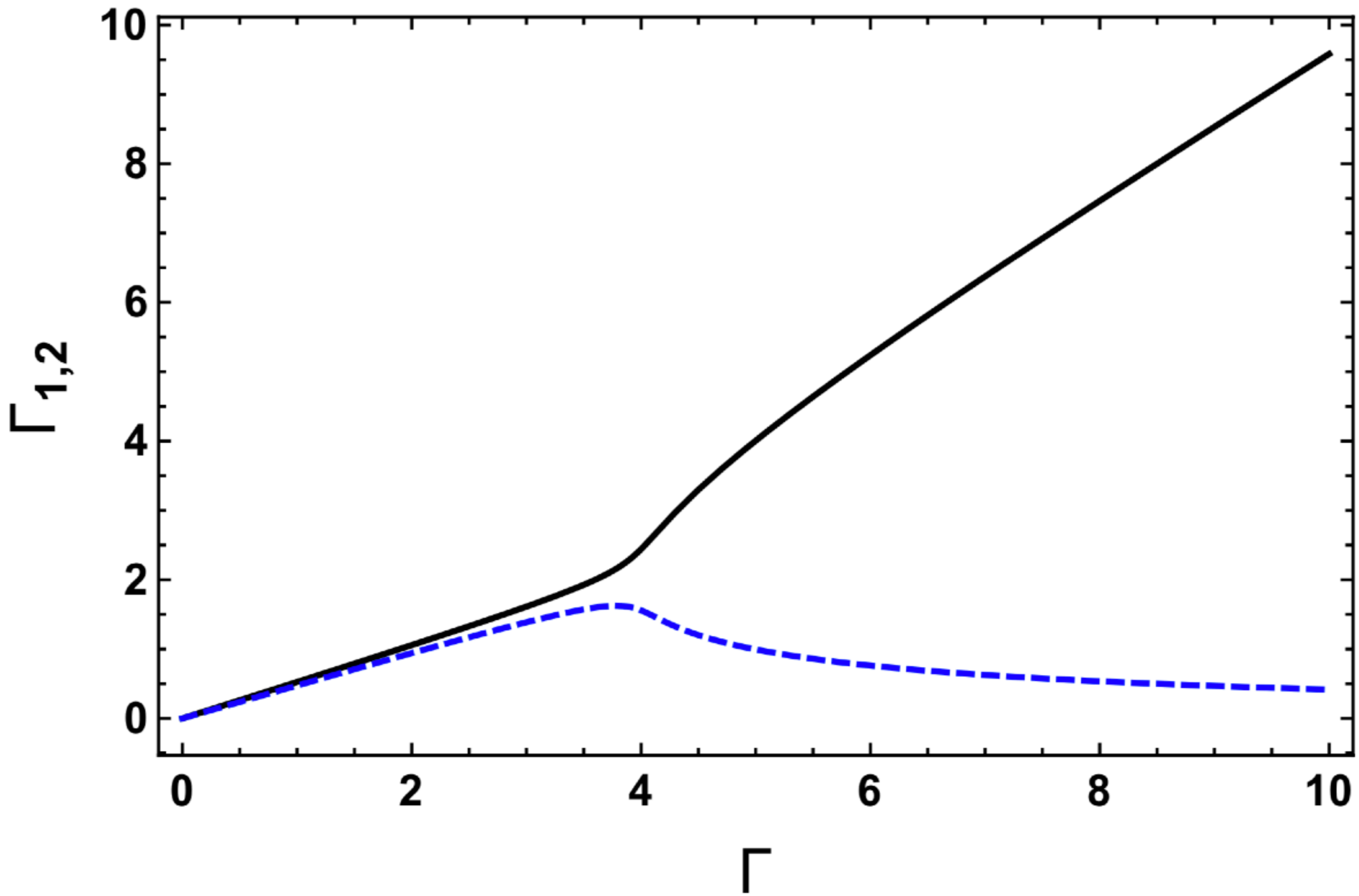
# Example of interacting resonances



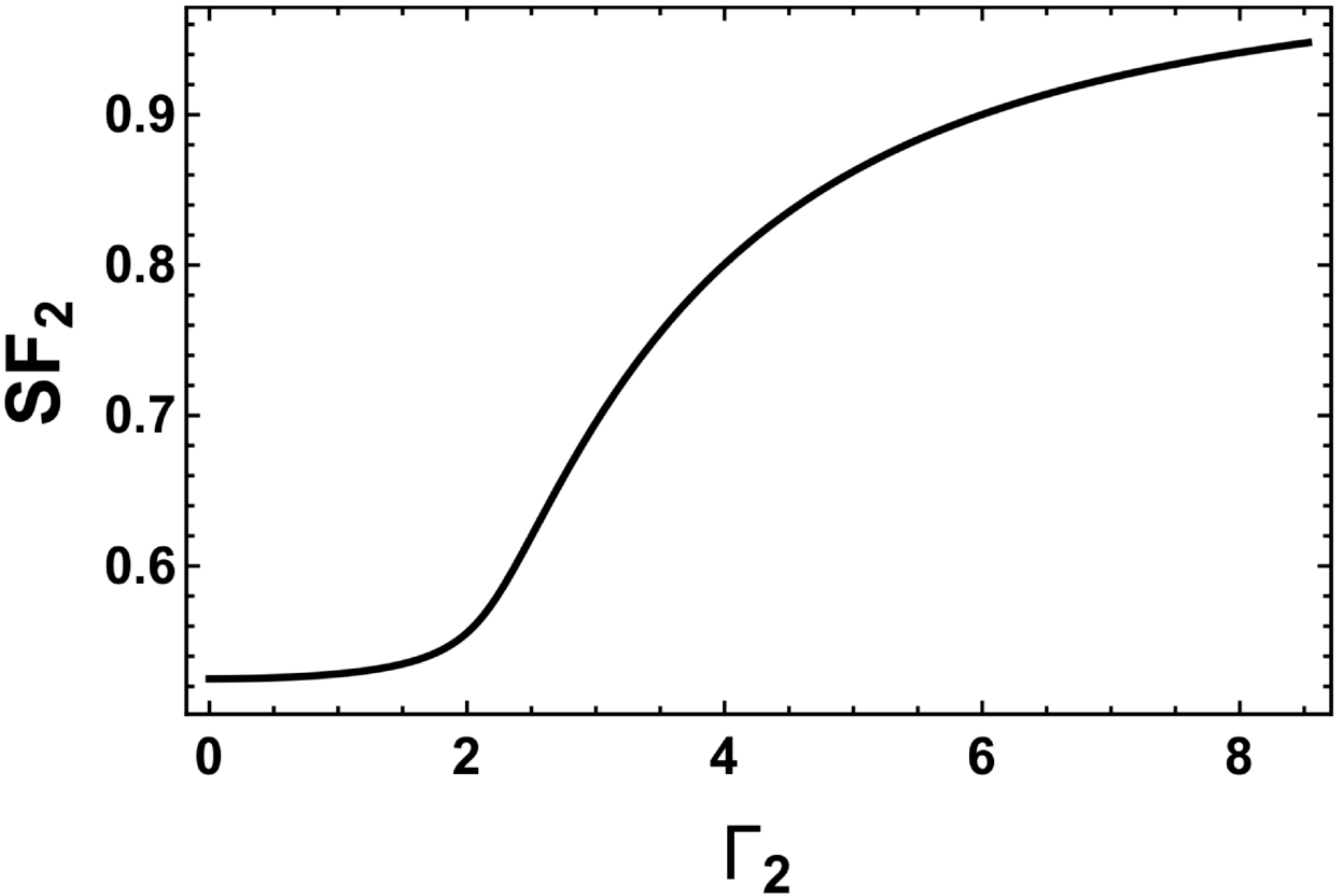

$$\mathcal{H} = H^0 + V - iW/2$$



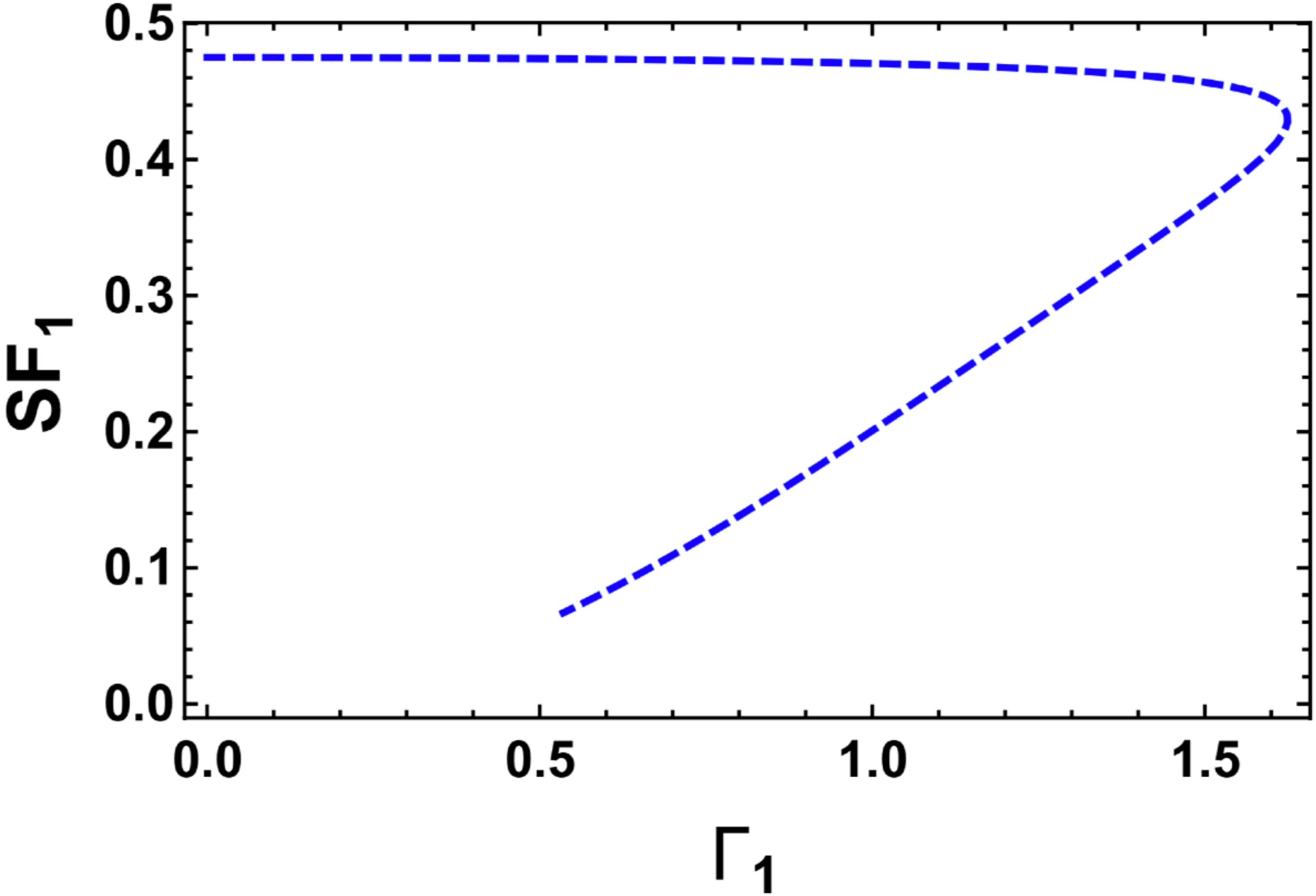
# Observing superradiance



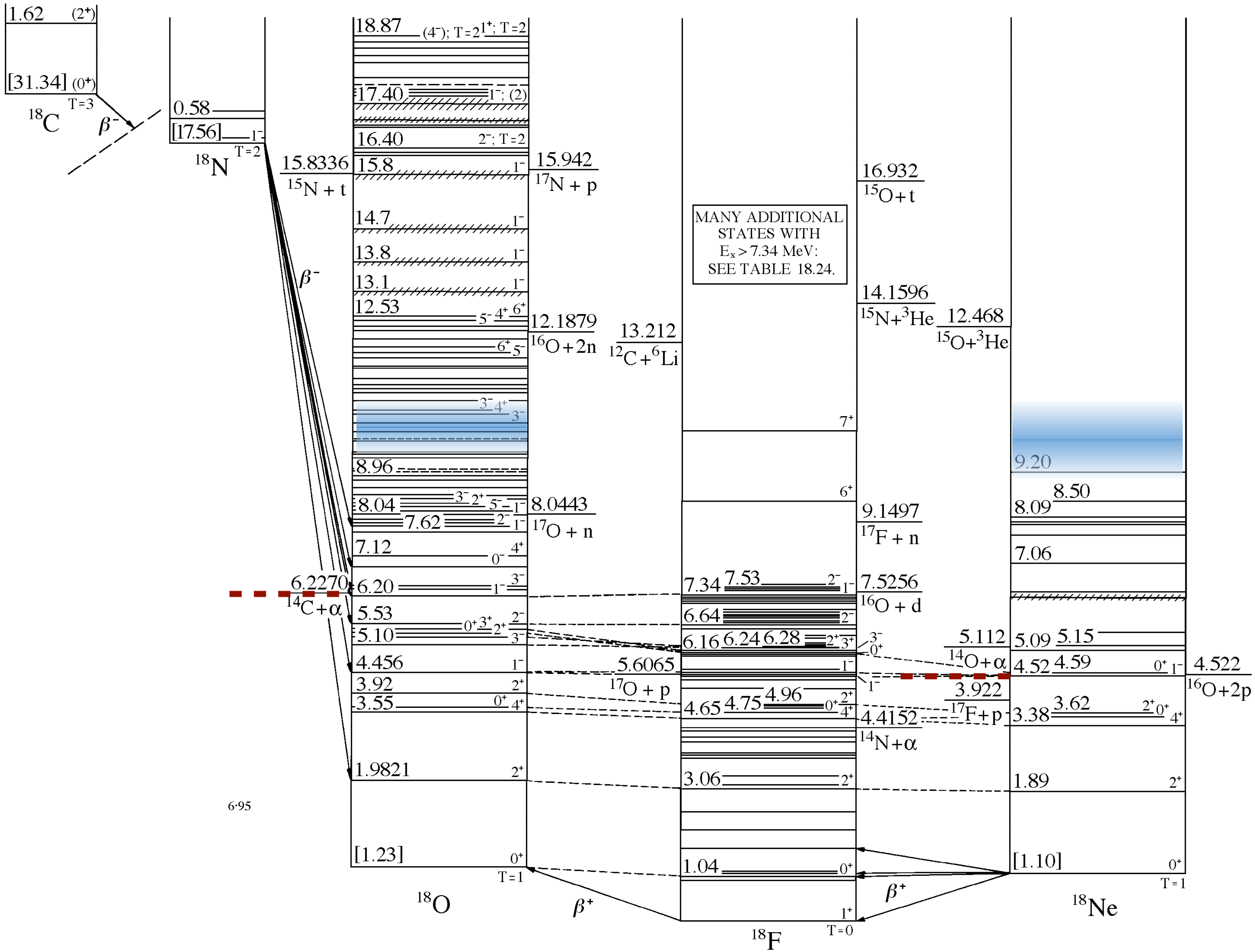
# Spectroscopic factor for superradiant state



# Spectroscopic factor for trapped state



# Isospin symmetry



| | $^{18}Ne$ | | | $^{18}O$ | | |
|---|---|---|---|---|---|---|
| J | E (MeV) | $\Gamma$ (keV) | SF | E (MeV) | $\Gamma$ (keV) | SF |
| $1^-$ | 9.08(1) | 357 | **0.21(1)** | 9.19(2) | 200 | **0.20(1)** |
| $1^-$ | 9.57(1) | 1062 | **0.51(5)** | 9.76(2) | 630 | **0.46(4)** |
| $1^-$ | 10.58(4) | 416 | **0.15(5)** | 10.8(3) | 630 | **0.29(4)** |
| $1^-$ | 13.730(2) | 780 | **0.2(1)** | 14.3(3) | 400 | **0.10(4)** |
| $2^+$ | 9.19(3) | 265 | **0.21(2)** | 9.79(6) | 90 | **0.10(3)** |
| $2^+$ | 10.94(6) | 1302 | **0.52(3)** | 12.21(8) | 1000 | **0.37(9)** |
| $2^+$ | 13.4 (2) | 1755 | **0.45(8)** | 12.8(3) | 4800 | **1.56(13)** |
| $2^+$ | 16.9(2) | 1515 | **0.3(2)** | | | |
| $3^-$ | | | | 8.29(6) | 2.9 | **0.18(1)** |
| $3^-$ | 8.77(8) | 419 | **1.0(4)** | 9.35(2) | 110 | **0.48(13)** |
| $3^-$ | 11.0(1) | 497 | **0.28(7)** | 11.95(1) | 300 | **0.17(2)** |
| $3^-$ | 12.7(2) | 2025 | **0.7(2)** | 12.98(4) | 770 | **0.32(5)** |
| $3^-$ | 14.8(2) | 3967 | **1.0(2)** | 14.0(2) | 2100 | **0.7(1)** |
| $4^+$ | 8.16 | 31 | **0.8(3)** | 7.11* | | |
| $4^+$ | 13.3(3) | 845 | **0.37(4)** | 13.46(2) | 210 | **0.12(1)** |
| $4^+$ | 14.15(21) | 375 | **0.14(10)** | 14.77(5) | 680 | **0.28(2)** |
| $5^-$ | 11.31(4) | 15 | 0.03(2) | 11.63(1) | 30 | **0.13(1)** |
| $5^-$ | 12.9(2) | 532 | **0.48(12)** | 13.08(1) | 120 | **0.17(1)** |
| $5^-$ | 13.79(8) | 219 | **0.14(10)** | 14.1(1) | 260 | **0.23(2)** |
| $5^-$ | 14.6(7) | 521 | **0.27(20)** | 14.7(1) | 230 | **0.16(6)** |
| $6^+$ | 11.8(2) | 54 | **0.30(7)** | 11.69(5) | 12 | **0.23(1)** |
| $6^+$ | 12.4(2) | 167 | **0.56(26)** | 12.57(1) | 50 | **0.38(8)** |

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**Publications:**

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