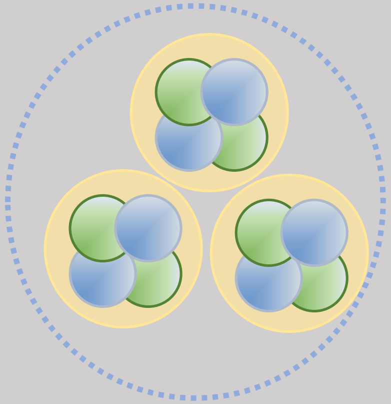
Resonance Ghost Anomaly in 3α-system



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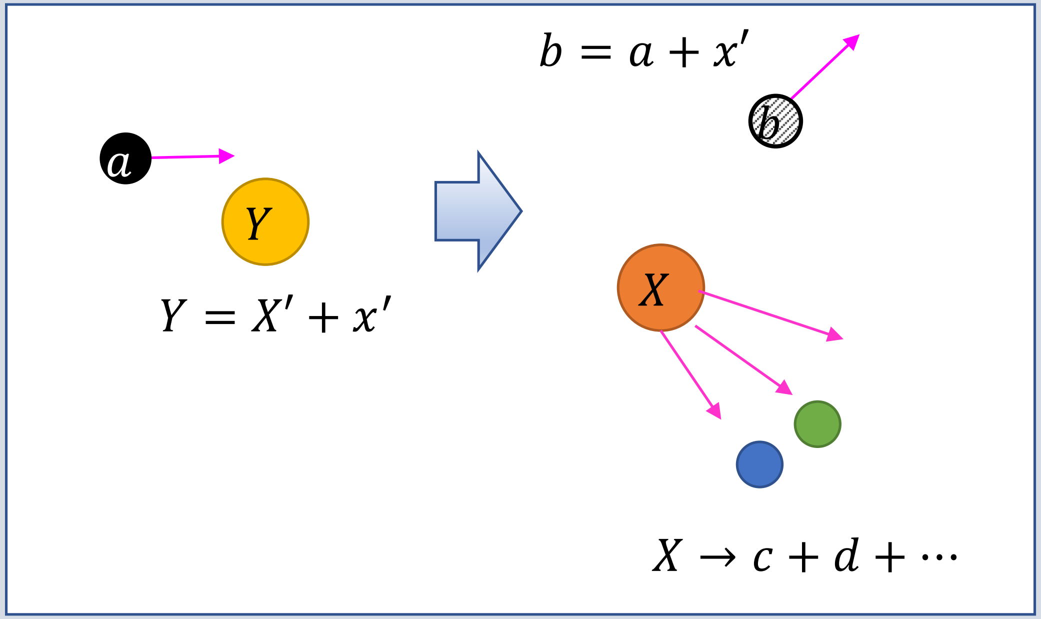
1. Introduction (1)



- Information of 'unstable' object X from $Y(a,b)X$

$$a + Y \to b + X, \qquad X \to c + d + \cdots$$

- Combination of $a + x' \to b$ & $X' \to X$

  Transition amplitude:

$$T \propto f(a + x' \to b)\langle \Psi_{X\to c+d+\cdots} | \hat{O} | \Psi_{X'} \rangle$$

- Transition strength function

$$S(E) = \int df \left| \langle \Psi_{X\to c+d+\cdots} | \hat{O} | \Psi_{X'} \rangle \right|^2 \delta(E - E_f) = -\frac{1}{\pi} \mathrm{Im} \left\langle \Psi_{X'} \middle| \hat{O}^\dagger \frac{1}{E + i\varepsilon - H_X} \hat{O} \middle| \Psi_{X'} \right\rangle$$

$$H_X | \Psi_X \rangle = E | \Psi_X \rangle, \quad E > 0$$

- If the system X has a complex energy eigen value, $E_r - \frac{i}{2}\Gamma$: → $S(E) = \frac{S_r}{\pi} \frac{\frac{\Gamma}{2}}{(E - E_r)^2 + \left(\frac{1}{2}\Gamma\right)^2}$

- When the complex energy is close to real axis (i.e. Γ is small enough) so that $S(E)$ has a peak around $E = E_r$, it is called as a resonance peak.

1. Introduction (2)

Does a peak in $S(E)$ always indicate the existence of a resonance ?

- A resonance close to the threshold → a narrow main peak and a broad but weaker ghost peak
  F.C. Barker & P. B. Treacy, NP**38** (1962) 33
- Example: ^{8}Be (2α system): $^9\mathrm{Be}(p,d)\ ^8\mathrm{Be}^*$

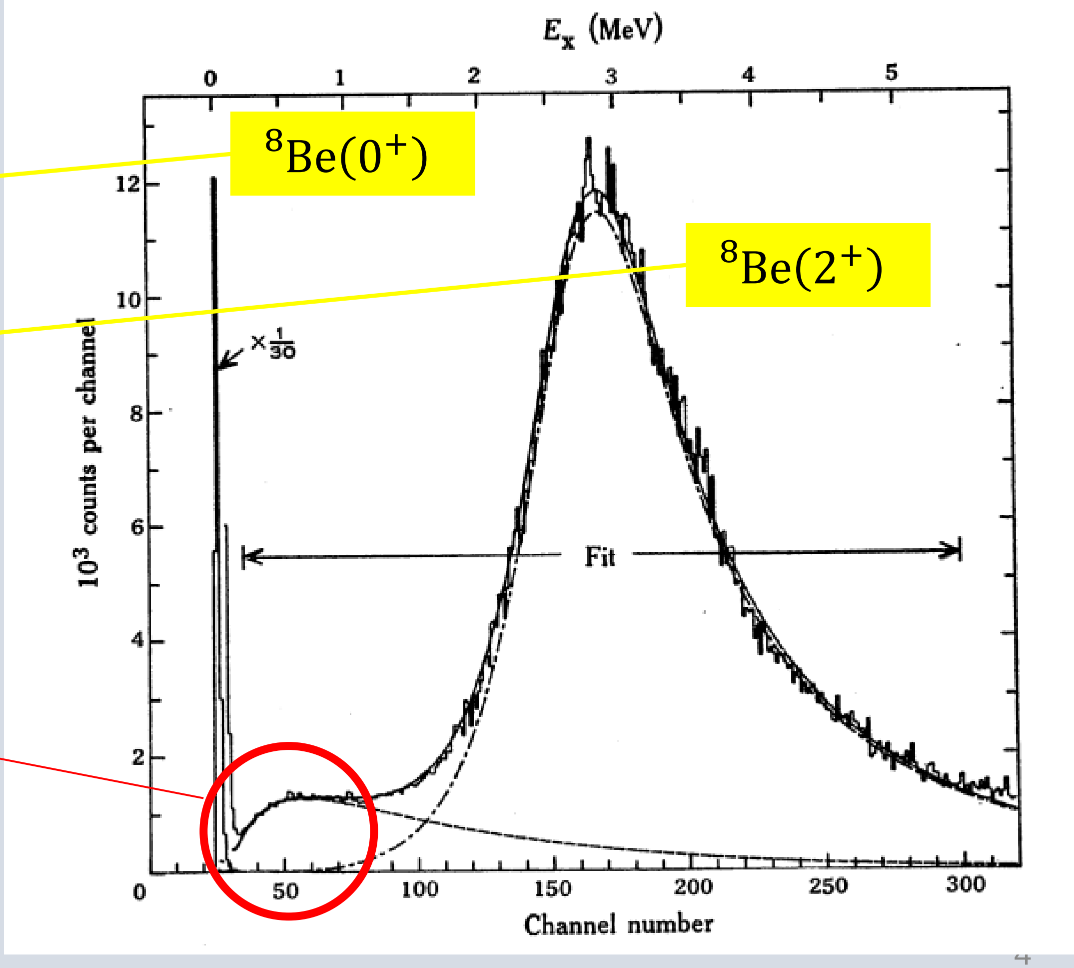
$^8\mathrm{Be}(0^+)$

$E_r = 0.092$ MeV, $\Gamma = 5.6 \times 10^{-6}$ MeV

$^8\mathrm{Be}(2^+)$

$E_r = 3.18$ MeV, $\Gamma = 1.5$ MeV

"Ghost peak at $E \approx 0.6$ MeV"



$^9\mathrm{Be}(p,d)\ ^8\mathrm{Be}^*$, $E_p = 39.91$ MeV, $\theta = 10°$
F.C. Baker et al., Aust. J. Phys. **29** (1976) 245

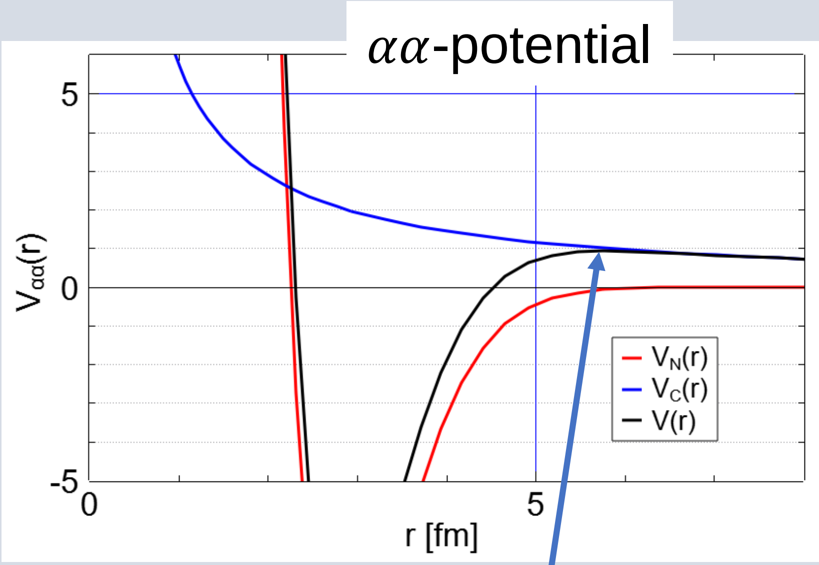
1. Introduction (3)

An explanation of the ghost peak

- Resonance formula (R-matrix theory) with energy-dependent width: $\Gamma(E) = 2P(E)\gamma^2$

$$\rho(E) = \frac{\frac{\Gamma(E)}{2}}{(E-\Delta(E)-E_r)^2+\left(\frac{1}{2}\Gamma(E)\right)^2},$$

$P(E)$: Penetration factor, γ^2:Reduced width



$V(r = 5.8\ \mathrm{fm}) = 0.93\ \mathrm{MeV}$

- Sharp resonance at low energy: $\mathrm{E} \gg E_r, \Gamma(E_r)$

$$\rho(E) \approx \frac{P(E)\gamma^2}{E^2}$$ at higher energies

- $P(E)$ increases as E goes through the Coulomb barrier

- Competition of the energy dependence of $P(E)$ vs. $\frac{1}{E^2}$ → a broad peak

1. Introduction (4)

- In this presentation:

1. Experiments that show the existence of ghost peak in 3α-system

2. Calculations in 3α-model
   - Monopole strength function
   - $^{12}\mathrm{C}(\alpha,\alpha')3\alpha$ reaction

2. Ghost Peak in 3α-system (1)

- Experimental studies on low-energy excited states of $^{12}\mathrm{C}\ [3\alpha]$

1. M. Itoh et al. PRC**84** (2011) 054308

   $^{12}\mathrm{C}\ (\alpha, \alpha')^{12}\mathrm{C}^{*}$@ $386\ \mathrm{MeV}$

   Multipole Decomposition Analysis

   $N(E) = \sum_{\lambda} a_{\lambda} S_{\lambda}(E)$ $\quad$ $S_{\lambda}(E)$: transition-strength function

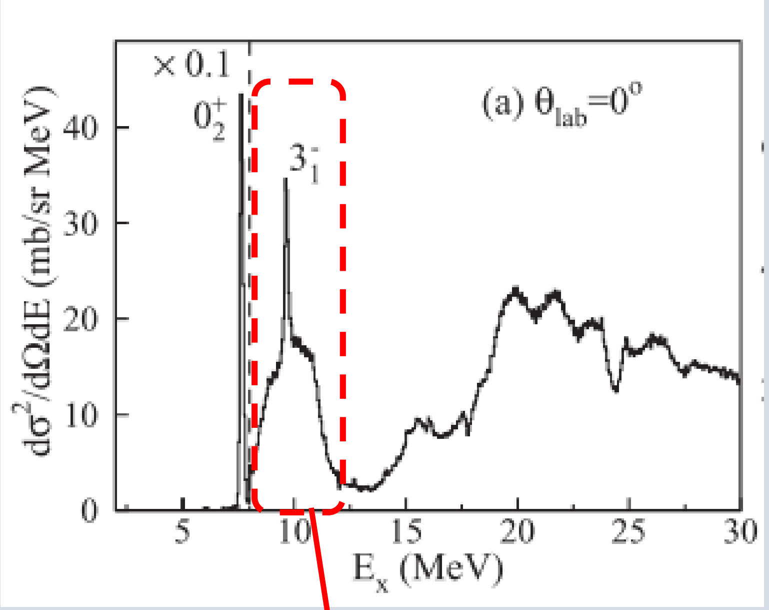
2. K. C. W. Li et al., PRC**105** (2022) 024308; PLB**827** (2022) 136928

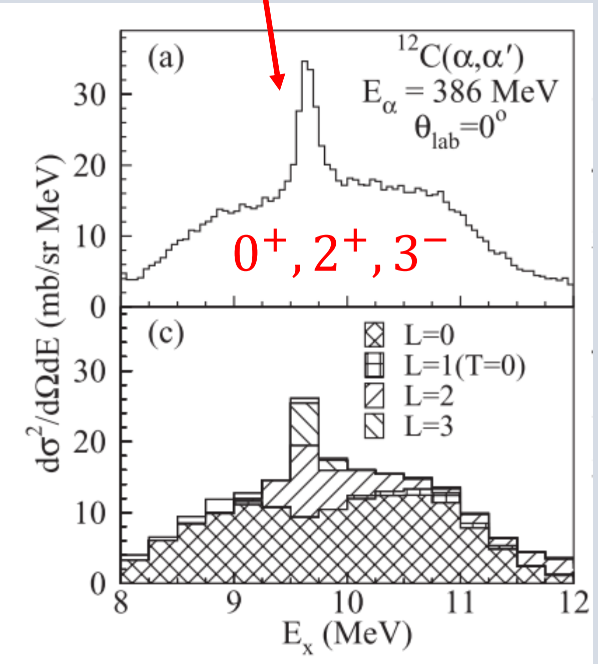
   $^{12}\mathrm{C}(\alpha, \alpha')^{12}\mathrm{C}^{*}$ @ $118\ \mathrm{MeV}, 160\ \mathrm{MeV}, 196\ \mathrm{MeV}$

   $^{14}\mathrm{C}(p, t)^{12}\mathrm{C}^{*}$@ $67.5\ \mathrm{MeV}, 100\ \mathrm{MeV}, 196\ \mathrm{MeV}$

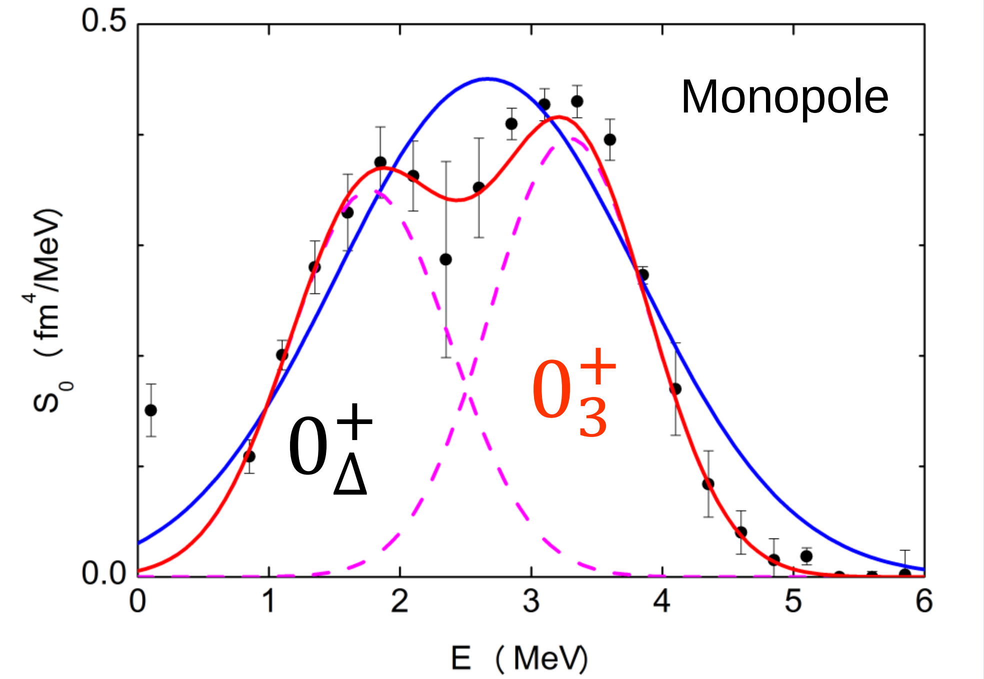
   Multi-level, multi-channel R-matrix theory

2. Ghost Peak in 3α-system (2) - $^{12}\mathrm{C}\,(\alpha,\alpha')3\alpha$ @ 386 MeV, $\theta_{\mathrm{Lab}} = 0°$





- $^{12}\mathrm{C}\,(\alpha,\alpha')3\alpha$ @ 386 MeV, $\theta_{\mathrm{Lab}} = 0°$
  M. Itoh et al. PRC**84** (2011) 054308
  Multipole Decomposition Analysis



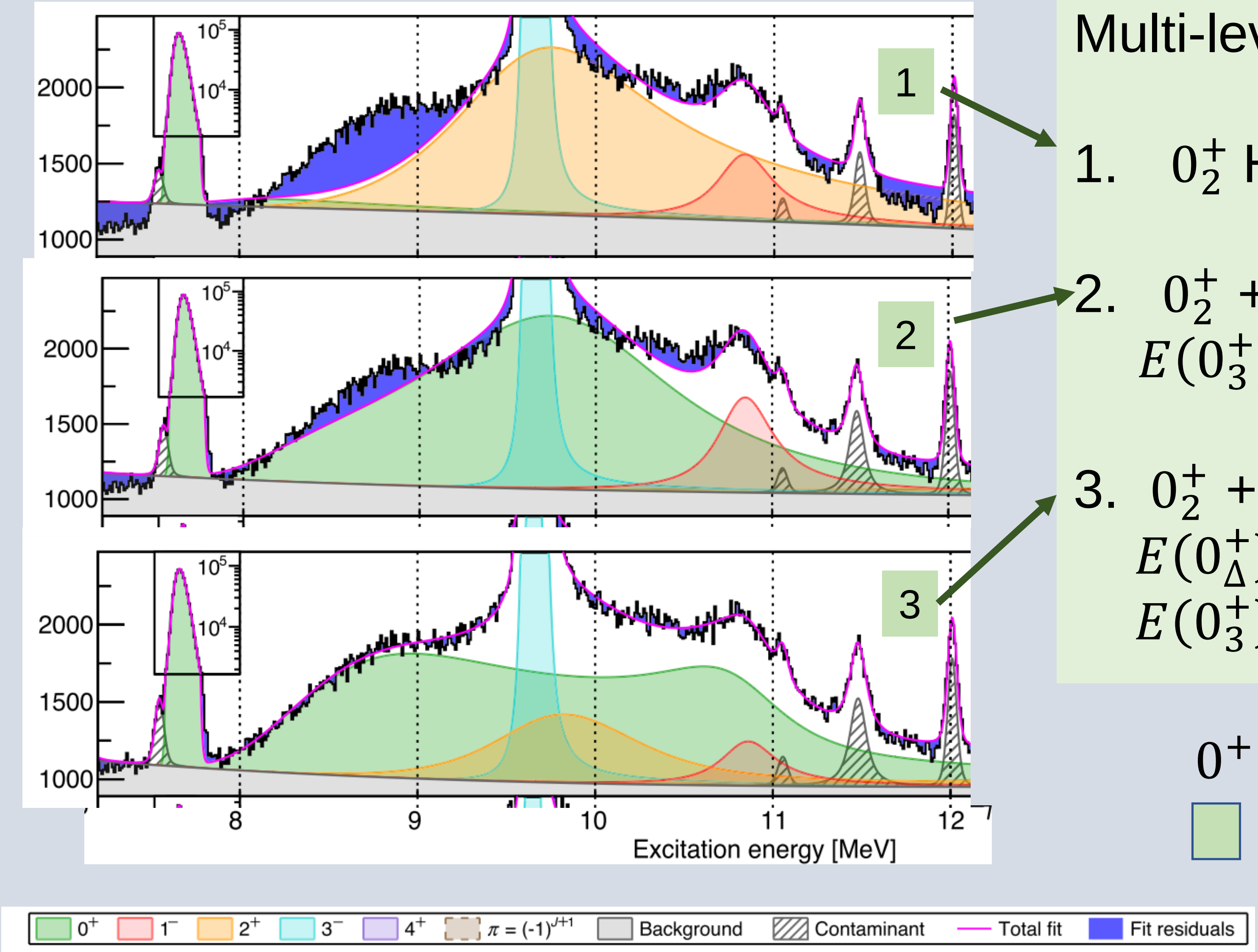
1-peak-fit	0_3^+	E_r=2.26(3) MeV	Γ=2.71(8)MeV

2-peak-fit	0_Δ^+	E=1.77(9)MeV	Γ=1.45(18)MeV
	0_3^+	E=3.29(6) MeV	Γ=1.42(8)MeV

$0_\Delta^+, 0_3^+$: Following the notation of K. C. W. Li et al., PRC**105**, 024308 (2022)

2. Ghost Peak in 3α-system (3) - $^{12}C(\alpha,\alpha')3\alpha$ @ 118 MeV, $\theta_{Lab} = 0°$

$^{12}C(\alpha,\alpha')^{12}C^*$ $E_\alpha = 118$ MeV $\theta_{Lab} = 0°$



Multi-level, multi-channel R-matrix theory

1. 0_2^+ Hoyle state (with ghost peak)

2. $0_2^+ + 0_3^+$(1-peak)
   $E(0_3^+) = 2.92$ MeV $\Gamma(0_3^+) = 2.28$ MeV

3. $0_2^+ + 0_\Delta^+ + 0_3^+$
   $E(0_\Delta^+) = 2.28$ MeV $\Gamma(0_\Delta^+) = 3.38$ MeV
   $E(0_3^+) = 3.64$ MeV $\Gamma(0_2^+) = 1.45$ MeV

0^+ 1^- 2^+ 3^- Fit residuals

K. C. W. Li et al., PRC**105** (2022) 024308; PLB**827** (2022) 136928

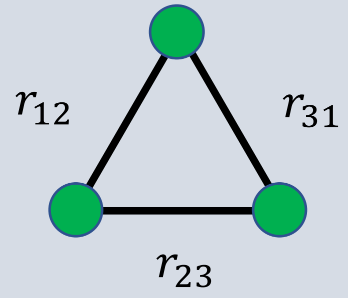
3. 3α model (1) - Hamiltonian

- Alpha particle is treated as a structureless 0^+ boson.
- 3α Hamiltonian $H_{3\alpha}$

$$H_{3\alpha} = K + V_{12} + V_{23} + V_{31} + V_{3\alpha}$$

- Phenomenological $\alpha\alpha$ potential: Ali-Bodmer Model D (L=0,2,4)

  Ref.: NP**80** (1966) 99



- Gaussian 3α potential

$$V_{3\alpha} = W_3 \exp\left[-\frac{{r_{12}}^2 + {r_{23}}^2 + {r_{31}}^2}{a^2}\right]$$

3. 3α model (2) - Transition Strength Function

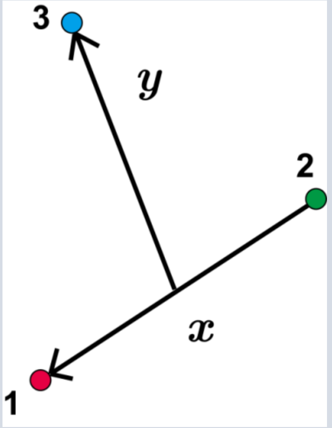
- Transition strength function for the bound state Ψ_b to 3α continuum states by operator $\hat{O}$:

$$S_{\hat{O}}(E) = \int df \left| \left\langle \Psi_f^{(-)} \middle| \hat{O} \middle| \Psi_b \right\rangle \right|^2 \delta(E - E_f) = -\frac{1}{\pi} \mathrm{Im} \left\langle \Psi_b \middle| \hat{O}^\dagger \frac{1}{E + i\varepsilon - H_{3\alpha}} \hat{O} \middle| \Psi_b \right\rangle$$

- Def. $|\Psi\rangle$: wave function corresponding to the process ${}^{12}\mathrm{C}(0_1^+) \to 3\alpha$:

$$|\Psi\rangle = \frac{1}{E+i\varepsilon-H_{3\alpha}} \hat{O}|\Psi_b\rangle \to N \frac{e^{iKR}}{R^{5/2}} \left\langle \Psi_f^{(-)} \middle| \hat{O} \middle| \Psi_b \right\rangle$$

$$R = \sqrt{x^2 + \frac{4}{3} y^2} \qquad K = \sqrt{\frac{m}{\hbar^2} E}$$



- Calculate $|\Psi\rangle$ by Faddeev methods, from which 3-body breakup amplitude $\left\langle \Psi_f^{(-)} \middle| \hat{O} \middle| \Psi_b \right\rangle$ is obtained:

References: Faddeev calculations for $3\alpha(0^+)$ systems:
S. I. : PRC **87** (2013) 055804, PRC **90** (2014) 061604, PRC **94** (2016) 061603

4. Monopole strength function in 3α model (1)

- Monopole strength function: $\hat{O} = \sum_{i=1,3} r_i^2$
- Phenomenological $\alpha\alpha$ potential: Ali-Bodmer Model D (L=0,2,4)

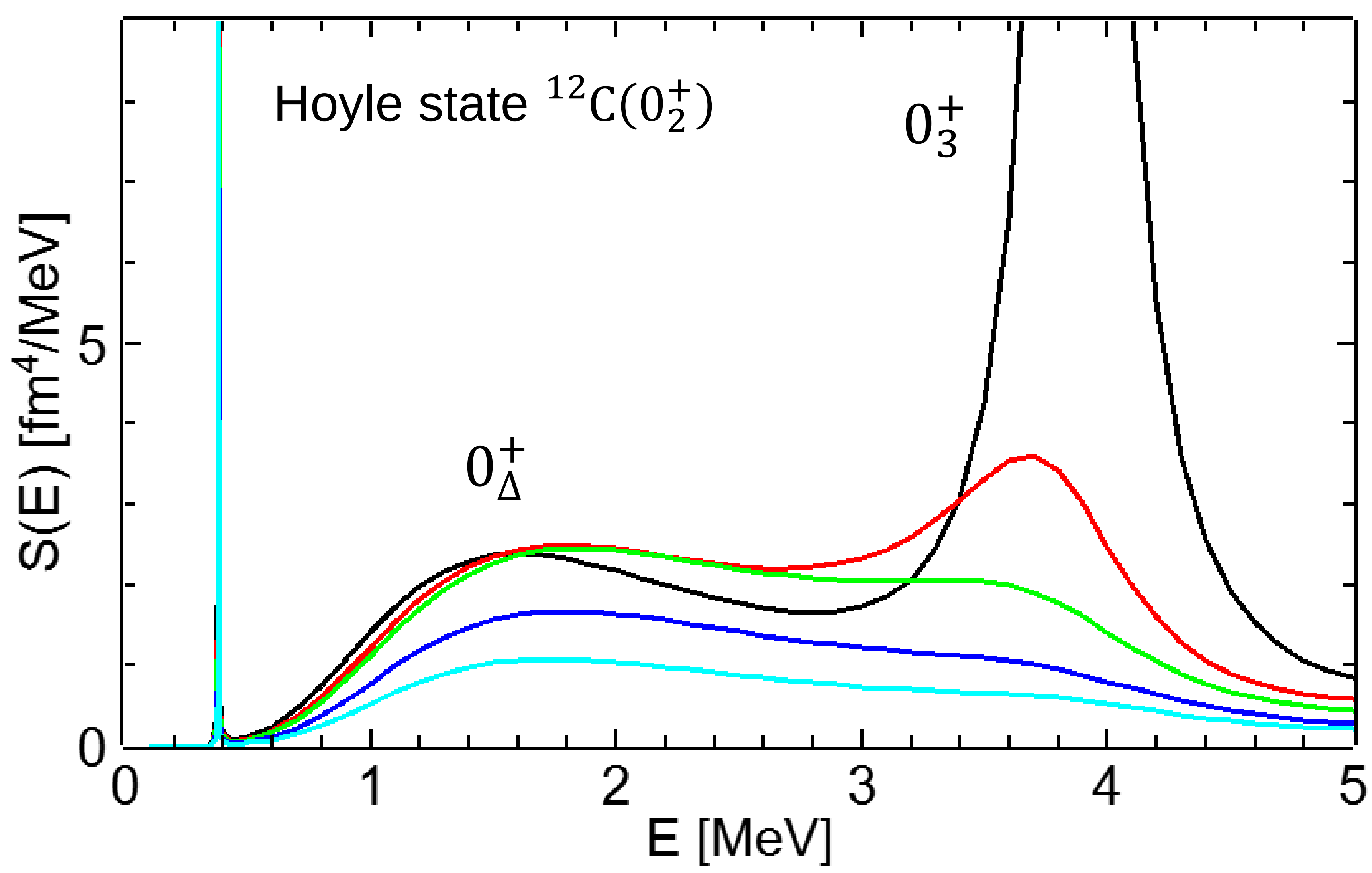
  Ref.: NP**80** (1966) 99
- Gaussian 3α potential $V_{3\alpha} = W_3 \exp\left[-\frac{{r_{12}}^2+{r_{23}}^2+{r_{31}}^2}{a^2}\right]$

  ・Model1~Model5: $a = 4.0\sim2.4$ [fm]

  ・Strength parameter W_3 : fitted to the Hoyle state energy

Model	a [fm]	W_3[MeV]
Model-1	4.0	-48.54
Model-2	3.4	-92.85
Model-3	3.0	-156.9
Model-4	2.6	-303.66
Model-5	2.4	-431.1

4. Monopole strength function in 3α model (2) - Calculations



$^{12}C(0_1^+) \to 3\alpha[0^+]$

Model

- 1: $a = 4.0$ fm
- 2: $a = 3.4$ fm
- 3: $a = 3.0$ fm
- 4: $a = 2.6$ fm
- 5: $a = 2.4$ fm

$0_\Delta^+, 0_3^+$: Following the notation of K. C. W. Li et al., PRC**105**, 024308 (2022);

5. Comparison with R-matrix theory (1)

- Comparison of the 3α calculations with he R-matrix theory
- R-matrix

$$S(E) = \frac{B}{\pi} \frac{\frac{\Gamma(E)}{2}}{(E - \Delta(E) - E_r)^2 + \left(\frac{1}{2}\Gamma(E)\right)^2}$$

$$\Gamma(E) = 2P(E)\gamma^2$$

$$P(E) = \frac{ka}{F_\ell(\eta, ka)^2 + G_\ell(\eta, ka)^2}$$

$$\Delta(E) = -[S(E) - S(E_r)]\gamma^2$$

$$S(E) = \frac{ka[F_\ell(\eta, ka)F'_\ell(\eta, ka) + G_\ell(\eta, ka)G'_\ell(\eta, ka)]}{F_\ell(\eta, ka)^2 + G_\ell(\eta, ka)^2}$$

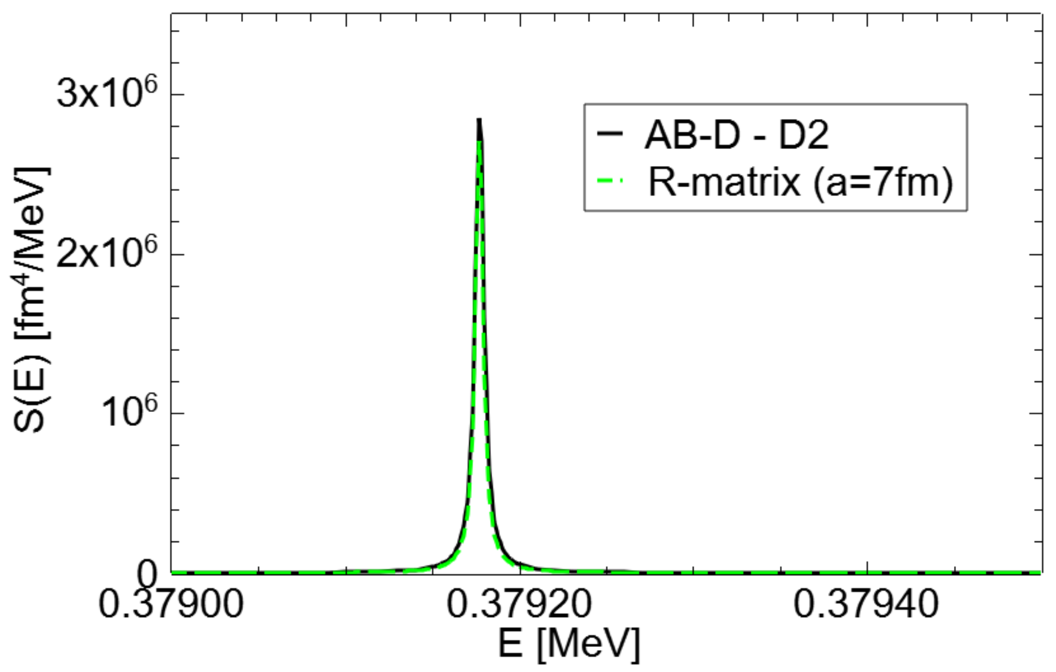
5. Comparison with R-matrix theory (2)

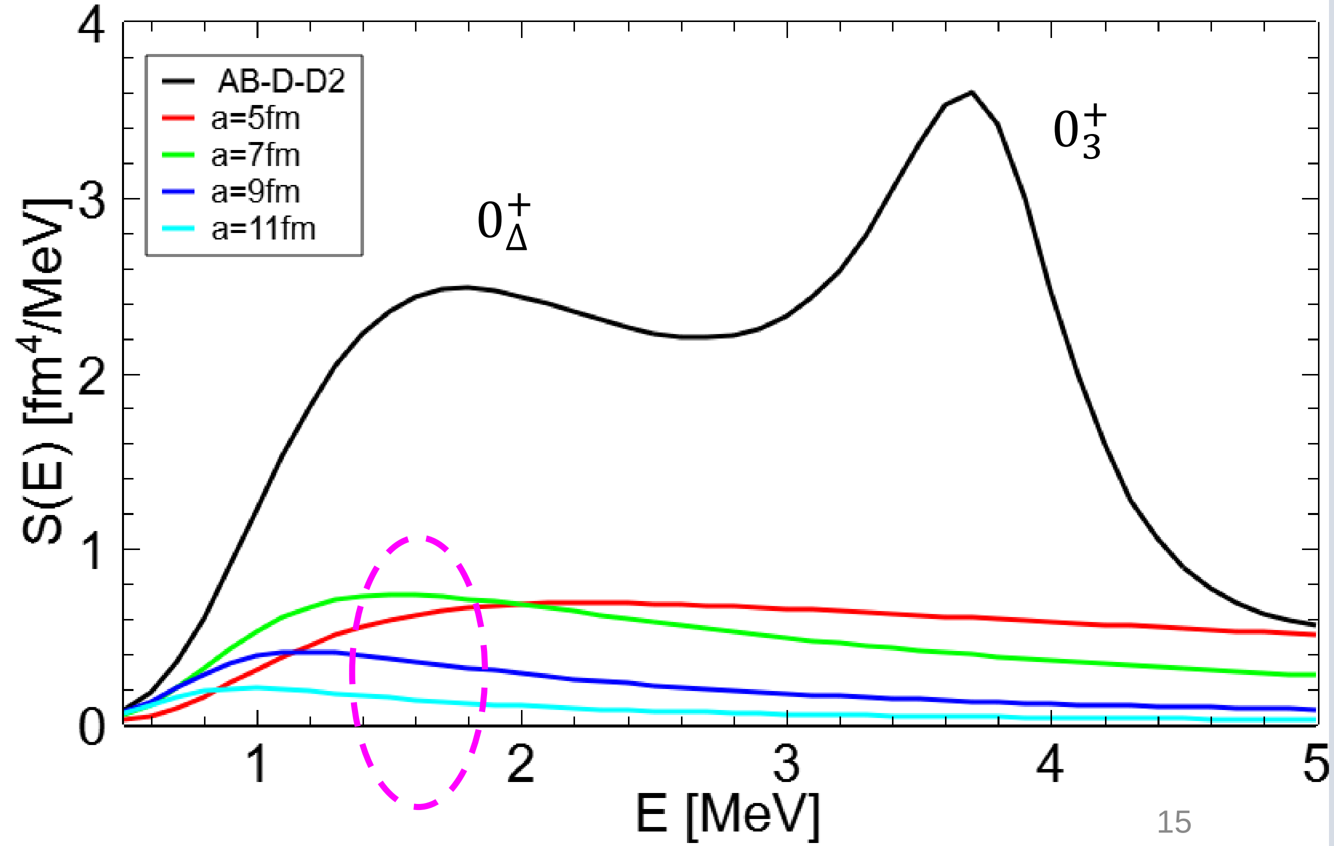
Input parameters: $E_r, \Gamma(E_r), B, a$

$E_r, \Gamma(E_r), B \rightarrow$ taken from the calculated values of the model-2

$$E_r = 0.379177\ \mathrm{MeV}, \Gamma(E_r) = 6.8\ \mathrm{eV}, B = 28.9\ \mathrm{fm}^4$$

Channel radius $a = 5\mathrm{fm}\ \ldots\ 11\ \mathrm{fm}$





5. Comparison with R-matrix theory (3)

- The monopole strength function in 3α-model gives the 0^+_Δ-peak, which is higher than one by R-matrix theory.

In the next:

- Is the 0^+_Δ-peak corresponding to a resonance?

- Does the 0^+_Δ-peak have enough strength to explain the experimental data ?

1. Volume integral of the absolute square of the wave-function $|\Psi\rangle$,

$$|\Psi\rangle = \frac{1}{E + i\varepsilon - H_{3\alpha}} \hat{O} |\Psi_b\rangle$$

with the integration range being restricted as $x \leq 12$ fm and $y \leq 12$ fm.

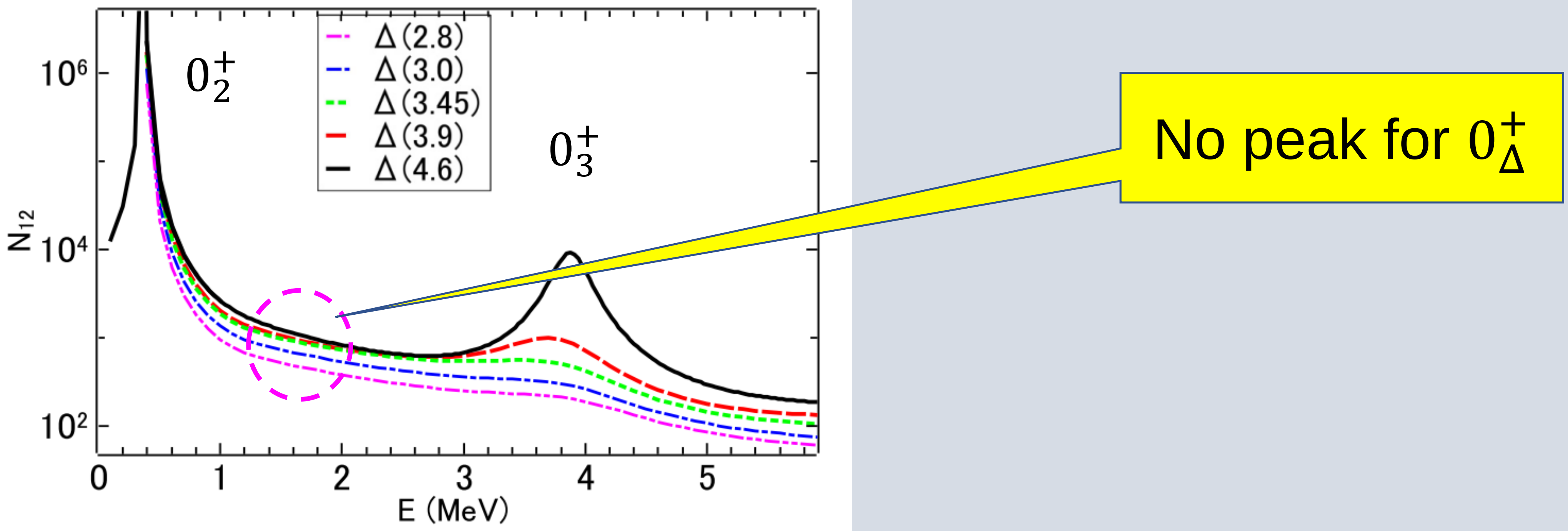


FIG. 2. The volume integral $N_{12}(E)$ defined in the text.

S.I., PRC **94** (2016) 061603(R)

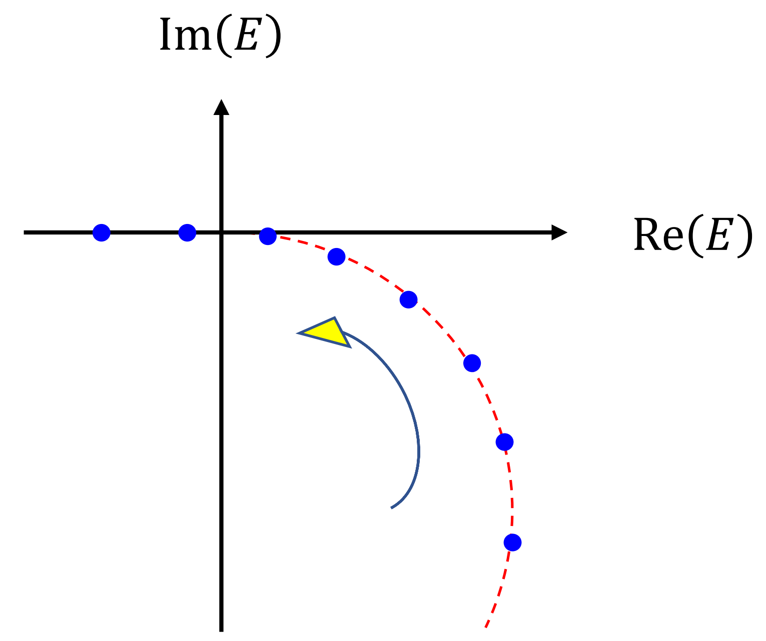
6. Nature of the 0_Δ^+ peak (2)

2. When fictional attractive effects are added, in general, the energy of a resonance decreases and the resonance becomes a bound state.

   In this work, attractive effects will be given by 3αP of the same range parameter a with the original Hamiltonian

$$\Delta V_{3\alpha} = \delta_3 \exp\left[-\frac{{r_{12}}^2+{r_{23}}^2+{r_{31}}^2}{a^2}\right]$$

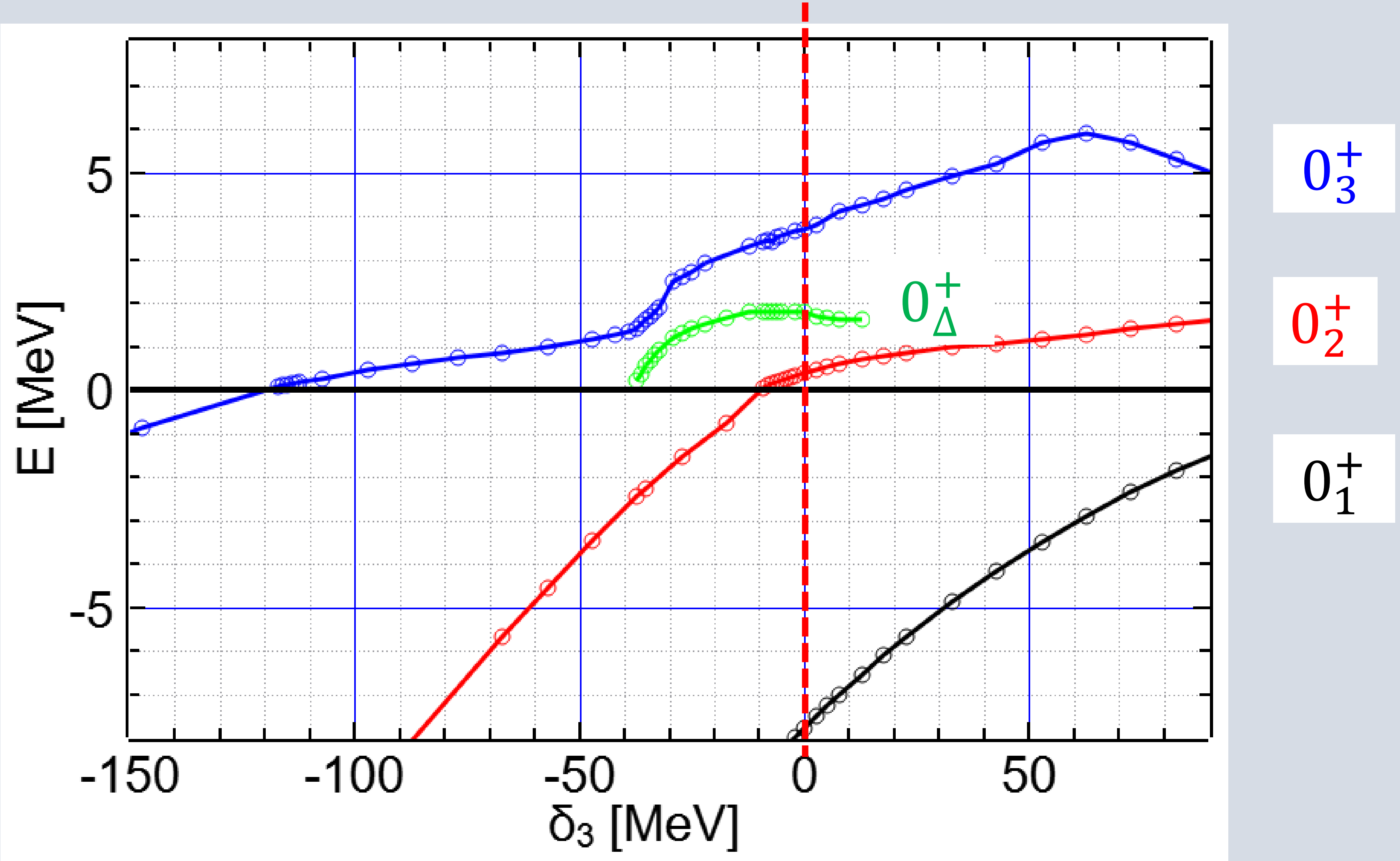
   No change for the nuclear and Coulomb parts of $\alpha\alpha$-potential to keep $^8\mathrm{Be}(0_1^+)$ (2α-resonance).



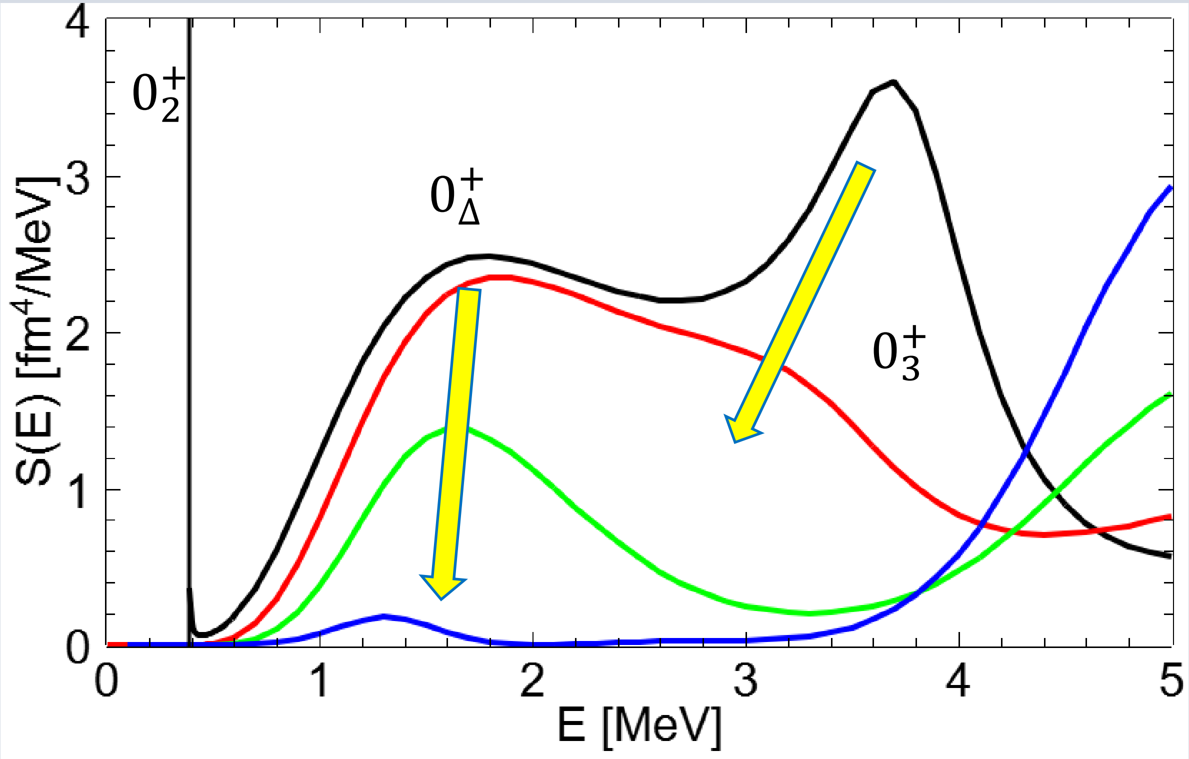
Pole trajectory

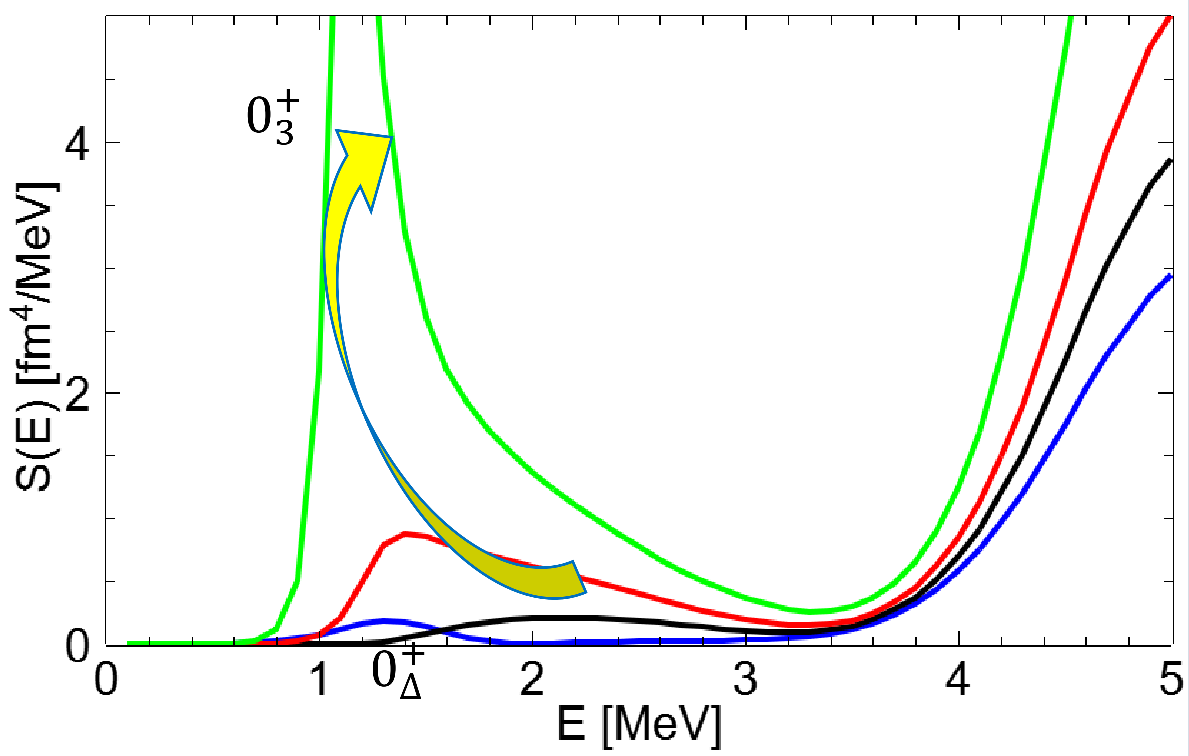
6. Nature of the 0^+_Δ peak (3) - 3α energy δ_3 -dependence

δ_3 -dependence of 3α (peak) energy



6. Nature of the 0^+_Δ peak (4)





$\delta_3 = -0$ MeV,
$\delta_3 = -7$ MeV,
$\delta_3 = -17$ MeV,
$\delta_3 = -27$ MeV

$\delta_3 = -27$ MeV,
$\delta_3 = -32$ MeV,
$\delta_3 = -37$ MeV,
$\delta_3 = -47$ MeV

6. Nature of the 0^+_Δ peak (5)

- The 0^+_Δ peak

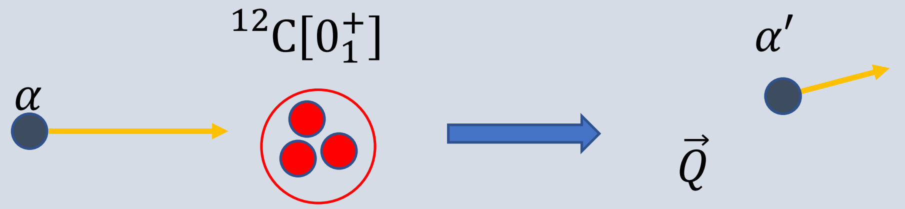
  No concentration of the wave function at interior region.
  No bound state as attraction of the interaction is enhanced.

  The 0^+_Δ peak may not be caused by a 3α resonant state.

- Is the 0^+_Δ peak in 3α-model enough to explain the experimental data ?

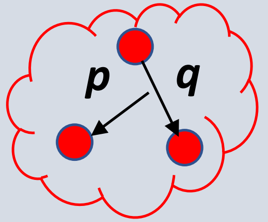
7. $^{12}\mathrm{C}(\alpha,\alpha')3\alpha$ reaction in 3α-model (1)

- Spectrum of inelastic scattering $^{12}\mathrm{C}(\alpha,\alpha')3\alpha$



$$\frac{d^2\sigma}{d\Omega dE} = \sum_{\lambda=0}^{3} a_\lambda \frac{c_\lambda(Q)}{Q^4} S_\lambda(E)$$

Q : Momentum transfer



- Multipole strength functions

$$S_\lambda(E) = \int dE' \int dq dp \left| \left\langle \Psi_{3\alpha}^{(-)}(E';q,p) \middle| \hat{O}_\lambda \middle| \Psi(^{12}\mathrm{C}) \right\rangle \right|^2 \delta(E-E')$$

$$\hat{O}_\lambda = \left[r^2,\ \ r^3 Y_1(\hat{r}),\ \ \ r^\lambda Y_\lambda(\hat{r}) \right] \quad (\lambda = 0, \lambda = 1, \lambda \geq 2)$$

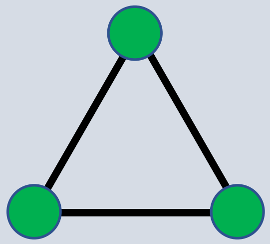
- Gauss convolution FWHM=100keV
- Fitting parameters (a_0, a_1, a_2, a_3)

7. $^{12}\mathrm{C}(\alpha,\alpha')3\alpha$ reaction in 3α-model (2) - Multipole strength functions

- Transition strength functions $S_\lambda(E)$ for

$$\hat{O}_\lambda = \left[r^2,\ \ r^3 Y_1(\hat{r}),\ \ \ r^\lambda Y_\lambda(\hat{r})\right] \quad (\lambda = 0, \lambda = 1, \lambda \geq 2)$$

- α-α-α Potentials



$$V_{3\alpha} = \sum_J \hat{P}_J\, W_3^{[J]} \exp\left[-\frac{{r_{12}}^2 + {r_{23}}^2 + {r_{31}}^2}{a^2}\right]$$

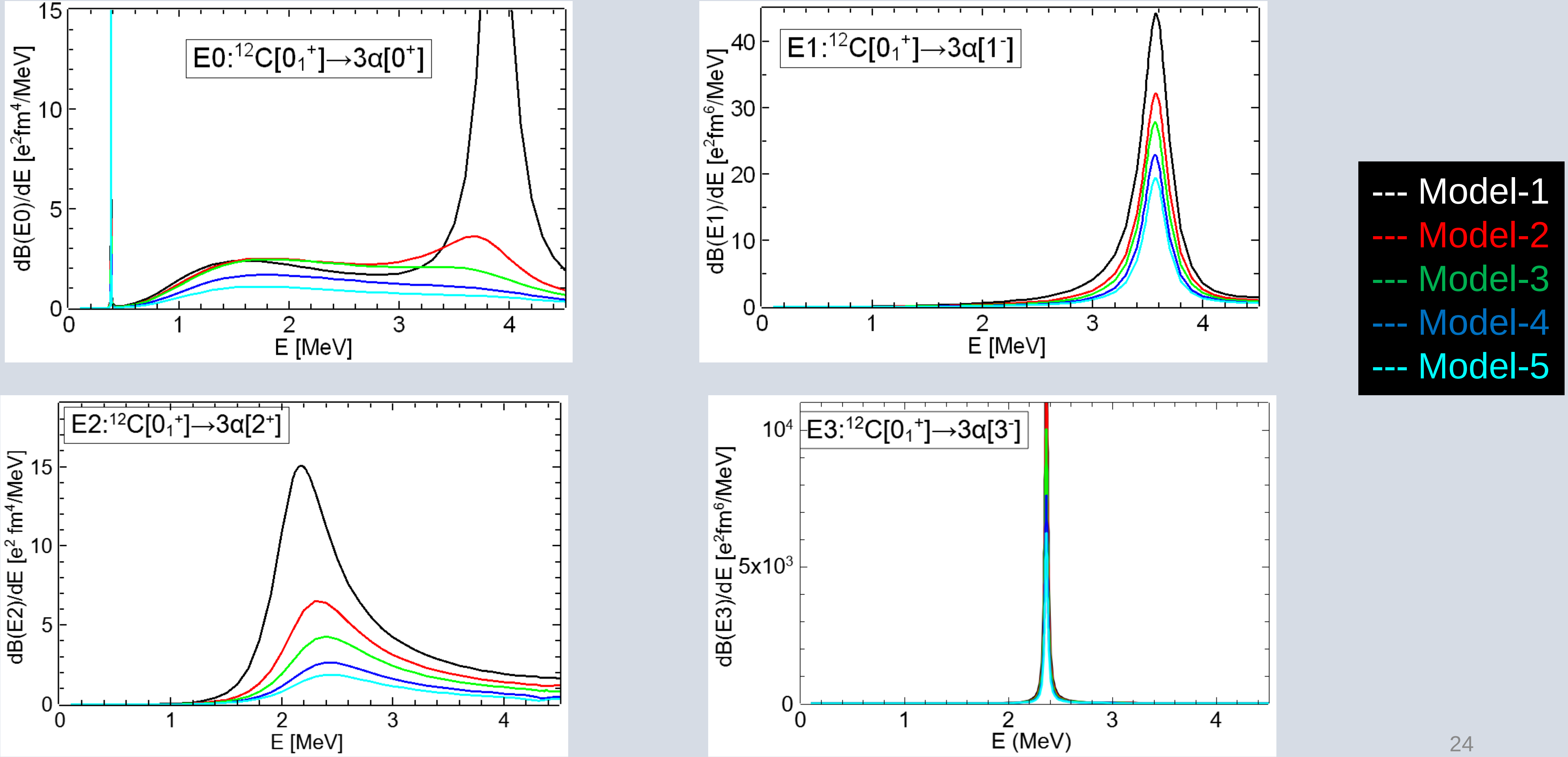
- Fitted to

$$E[0_2^+] = 0.379\ \mathrm{MeV}, \qquad E[1_1^-] = 3.569\mathrm{MeV}$$
$$E[2_1^+] = -2.836\ \mathrm{MeV}, \qquad E[3_1^-] = 2.336\ \mathrm{MeV}$$

Model	a [fm]	$W_3^{[0]}$[MeV]	$W_3^{[1]}$[MeV]	$W_3^{[2]}$[MeV]	$W_3^{[3]}$[MeV]
Model-1	4.0	-48.54	-15	-25.5	6.03
Model-2	3.39	-92.85	-37	-46.0	12.69
Model-3	3.0	-156.9	-79	-78.3	25.0
Model-4	2.61	-303.66	-193	-158.1	63.6
Model-5	2.43	-431.1	-300	-231.	108.3

7. $^{12}\mathrm{C}(\alpha,\alpha')3\alpha$ reaction in $\mathbf{3}\alpha$-model (3)Numerical results

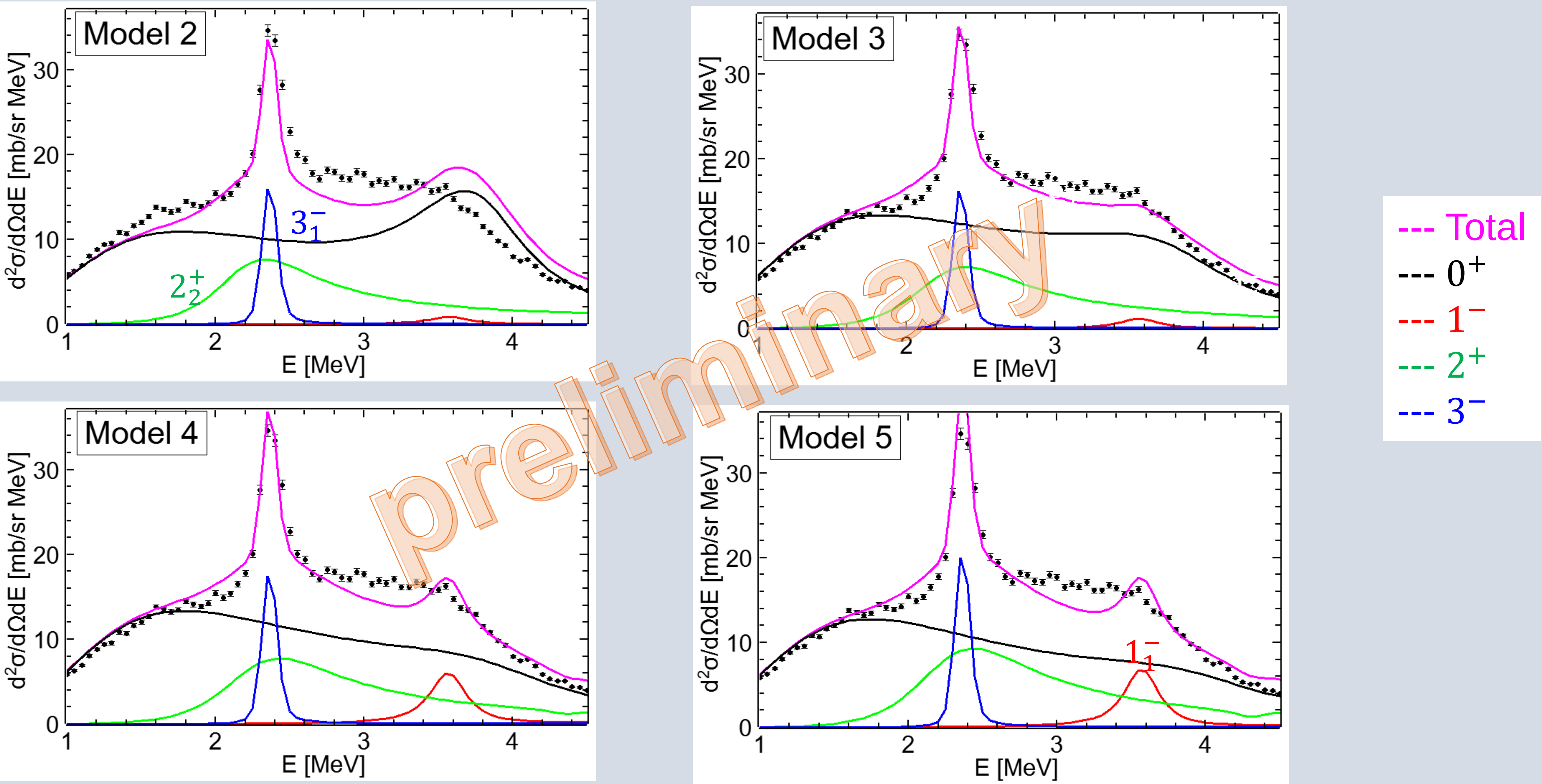
$^{12}\mathrm{C}(0_1^+) \rightarrow 3\alpha(\lambda)$ Transition Strength Function $S_\lambda(E)$



7. $^{12}\mathrm{C}(\alpha,\alpha')3\alpha$ reaction in 3α−model (4)

$$E_\alpha = 386\ \mathrm{MeV} \qquad \theta_{\mathrm{Lab}} = 0°$$

Model2～ Model5



Data from M. Itoh et al. PRC**84** (2011) 054308

8. Summary

1. A broad and weak peak in the monopole strength function for the transition, $^{12}\mathrm{C}(0_1^+) \to 3\alpha[0^+]$, reported in recent experiments is studied.
2. The monopole strength functions in 3α models have such peak (0_Δ^+-peak).
3. When (attractive) strength of the 3α-potential is increased, no bound state appeared corresponding to the 0_Δ^+-peak, while another peaks ($0_2^+, 0_3^+$) transfer to bound states.
4. The 0_Δ^+-peak in 3α model has enough strength to explain the $^{12}\mathrm{C}(\alpha,\alpha')3\alpha$ experimental data.
5. The 0_Δ^+-peak is Ghost of the Hoyle state.