Universal properties of two-neutron halo nuclei

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Nuclear physics at the edge of stability

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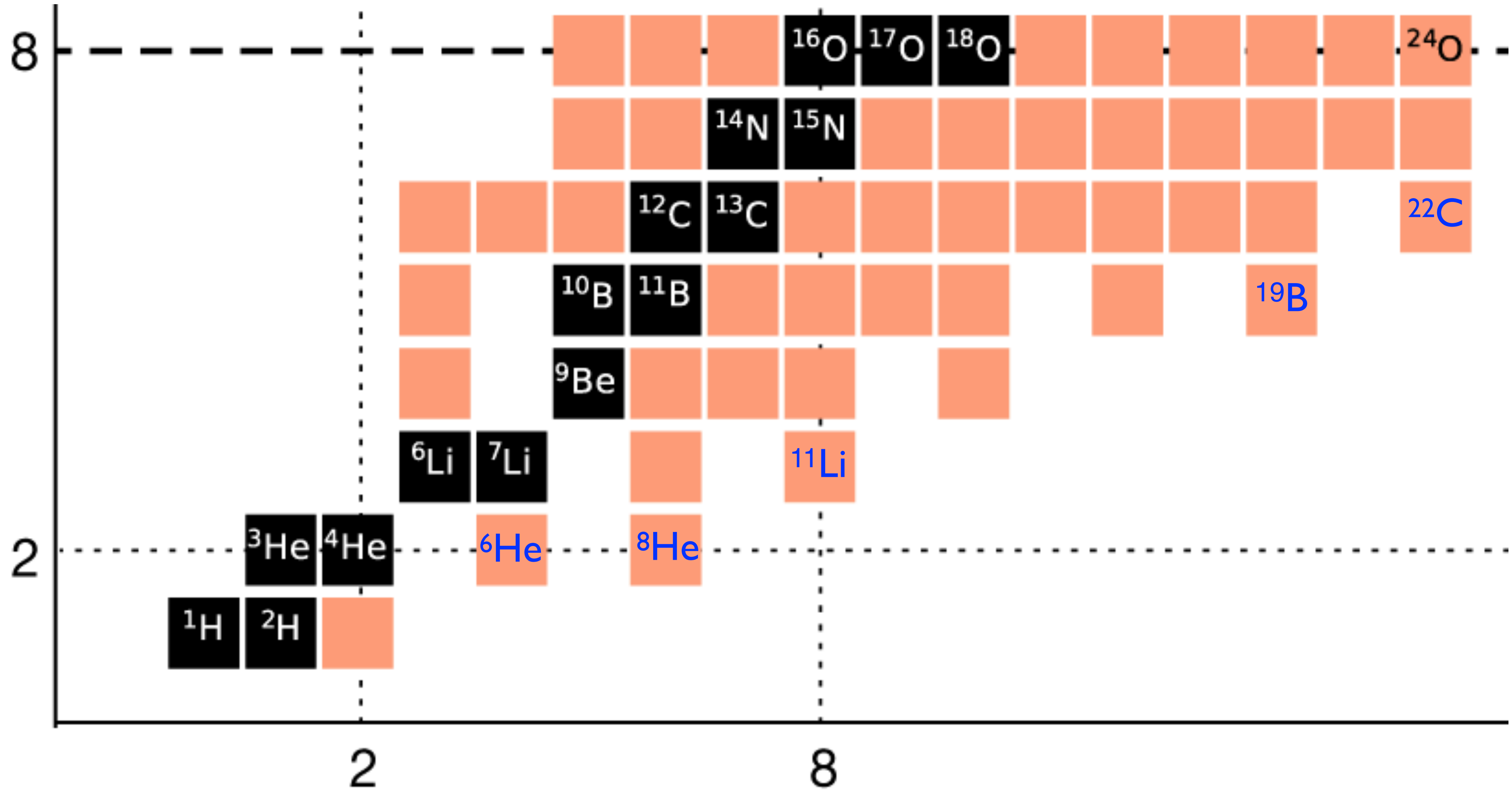
References

Masaru Hongo and DTS, PRL **128**, 212501 (2022)
[arXiv:2201.09912]

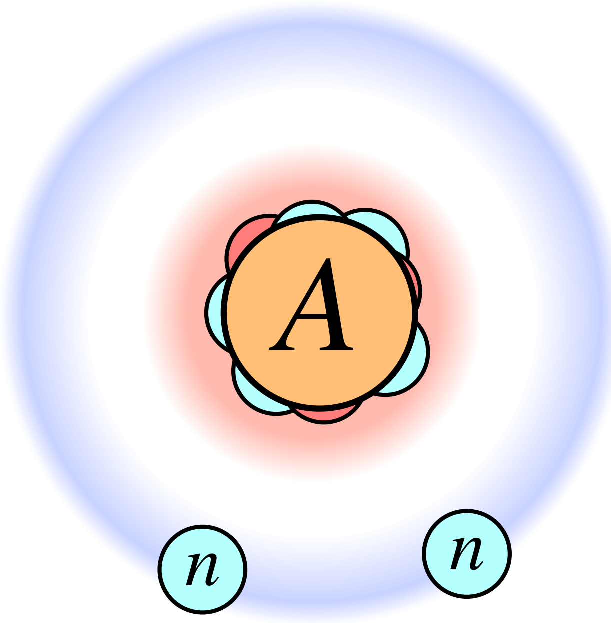
Plan

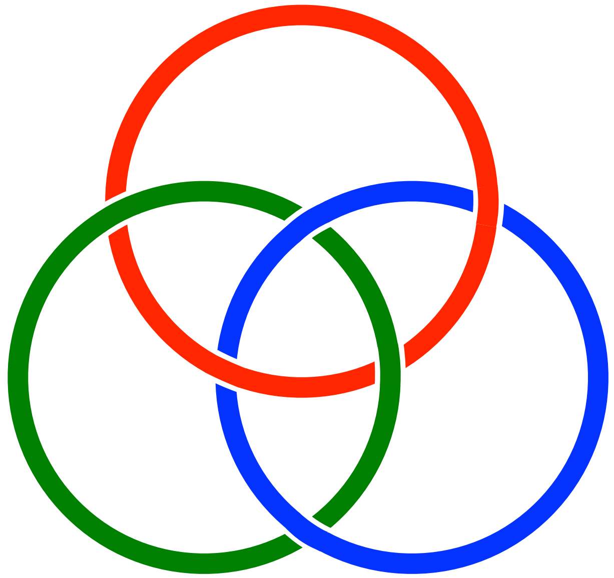
- Neutron-rich nuclei
- Two-neutron Borromean halo nuclei
- Neutrons as near-unitarity fermions: scaling dimensions of operators
- Coupling of neutron sector to the core nucleus: a renormalizable field theory



Tsunoda et al. Nature 587, 66 (2020)

Two-neutron halo nuclei





- Near the neutron drip line, sometimes one cannot add one neutron but can add 2
  - (Z, A) is bound (core)
  - $(Z, A+1)$ is unbound
  - $(Z, A+2)$ is bound
- Examples: ^{6}He, ^{8}He, ^{11}Li, ^{22}C

Two small energies

- Interaction between neutrons fine-tuned:

$$a \approx -19 \text{ fm} \qquad \epsilon_n = \frac{\hbar^2}{m_n a^2} \approx 0.12 \text{ MeV}$$

- 3-body binding energy (2n separation energy)

$$B(^6\text{He}) = 0.975 \text{ MeV}$$
$$B(^{11}\text{Li}) = 0.369 \text{ MeV}$$
$$B(^{22}\text{C}) < 0.18 \text{ MeV?}$$ Hammer Ji Phillips 2017

- Compare to the more typical energy scale

$$r_0 \approx 2.75 \text{ fm} \qquad \frac{\hbar^2}{m_n r_0^2} \approx 5.5 \text{ MeV}$$

Questions

- Is the 3-body system universal?

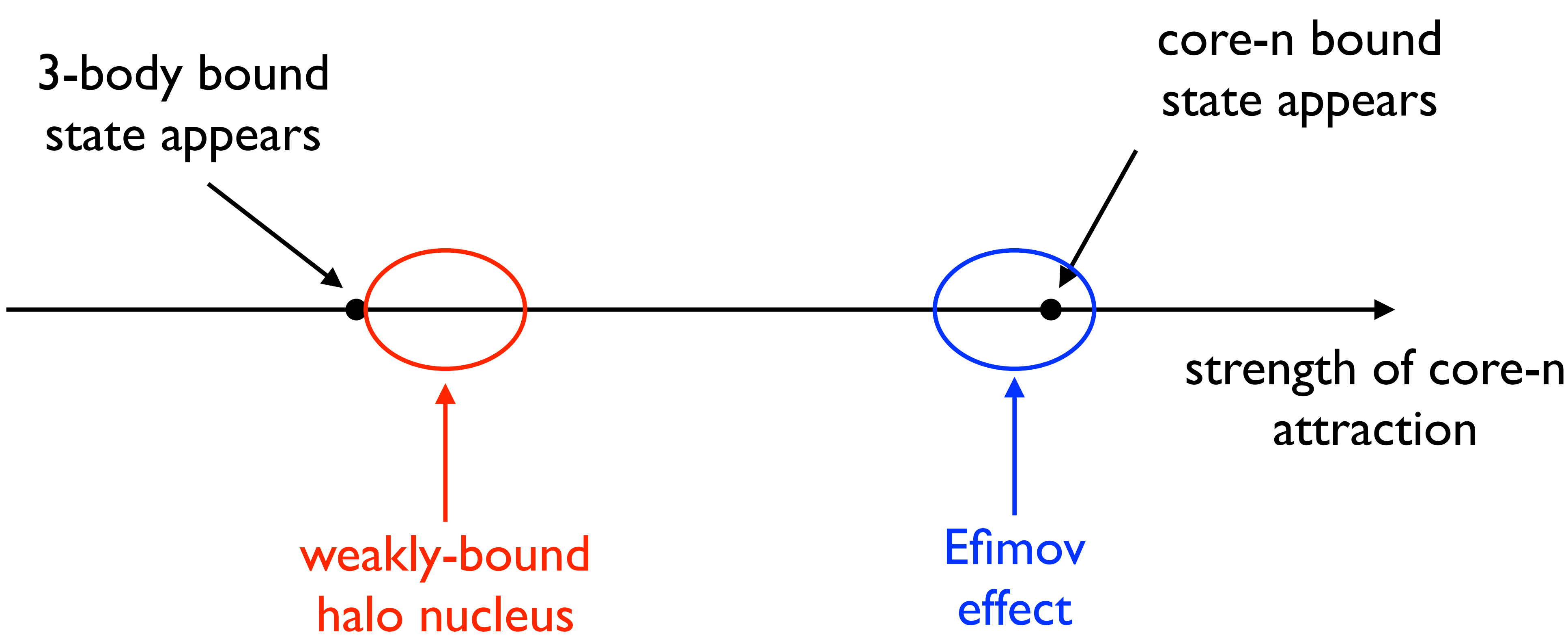
  Can any physical quantity can be written as

$$O = B^{\Delta_O} F_O\left(\frac{B}{\epsilon_n}\right), \quad O(\omega) = B^{\Delta_O} F_O\left(\frac{\omega}{B}, \frac{B}{\epsilon_n}\right)$$

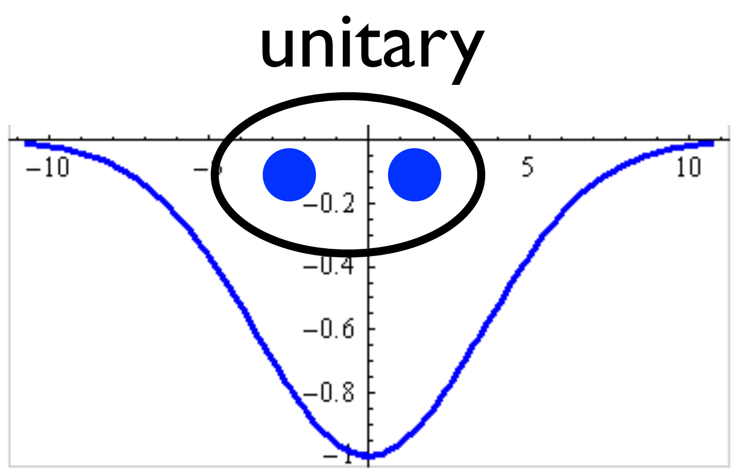
- Answer: almost

Efimov effect?

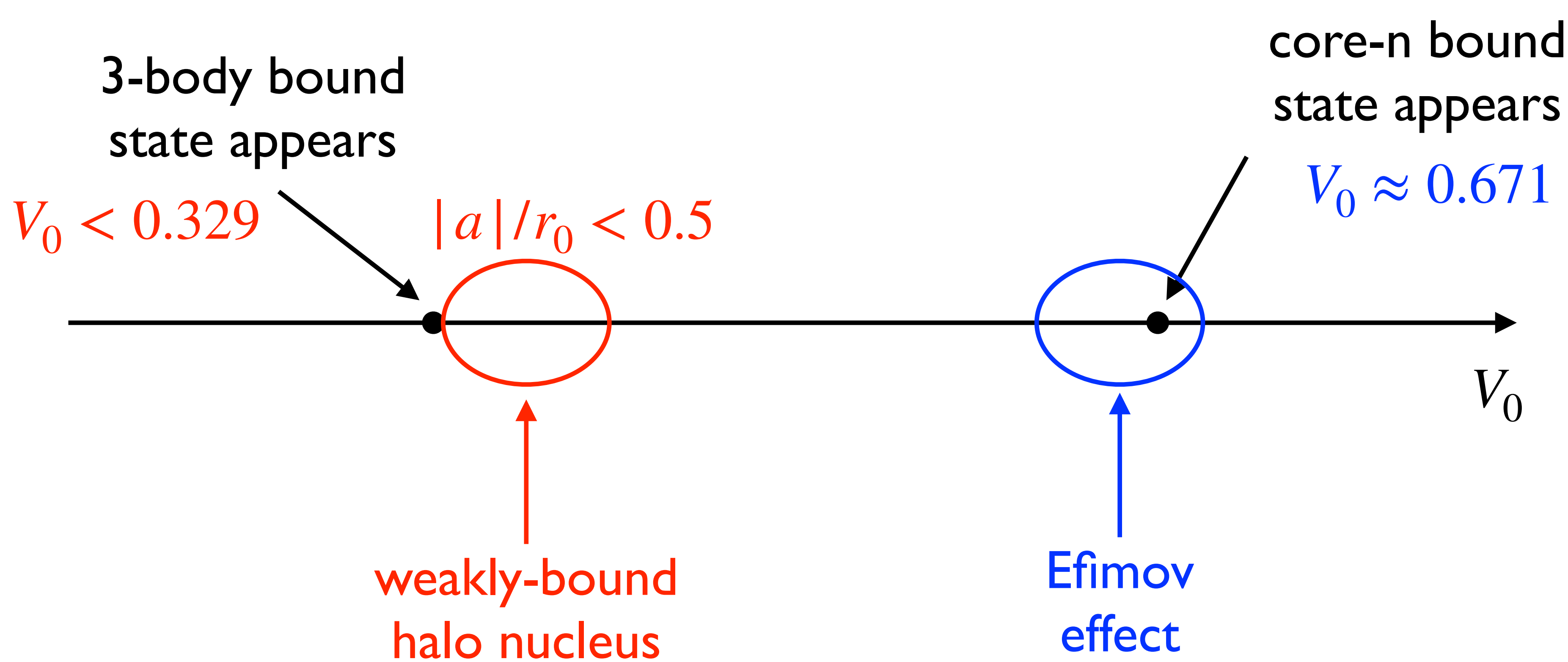
- When the core-neutron scattering length is also large: Efimov effect, Borromean bound state inevitable
- But 3-body bound state can exist without the Efimov effect



Simple model

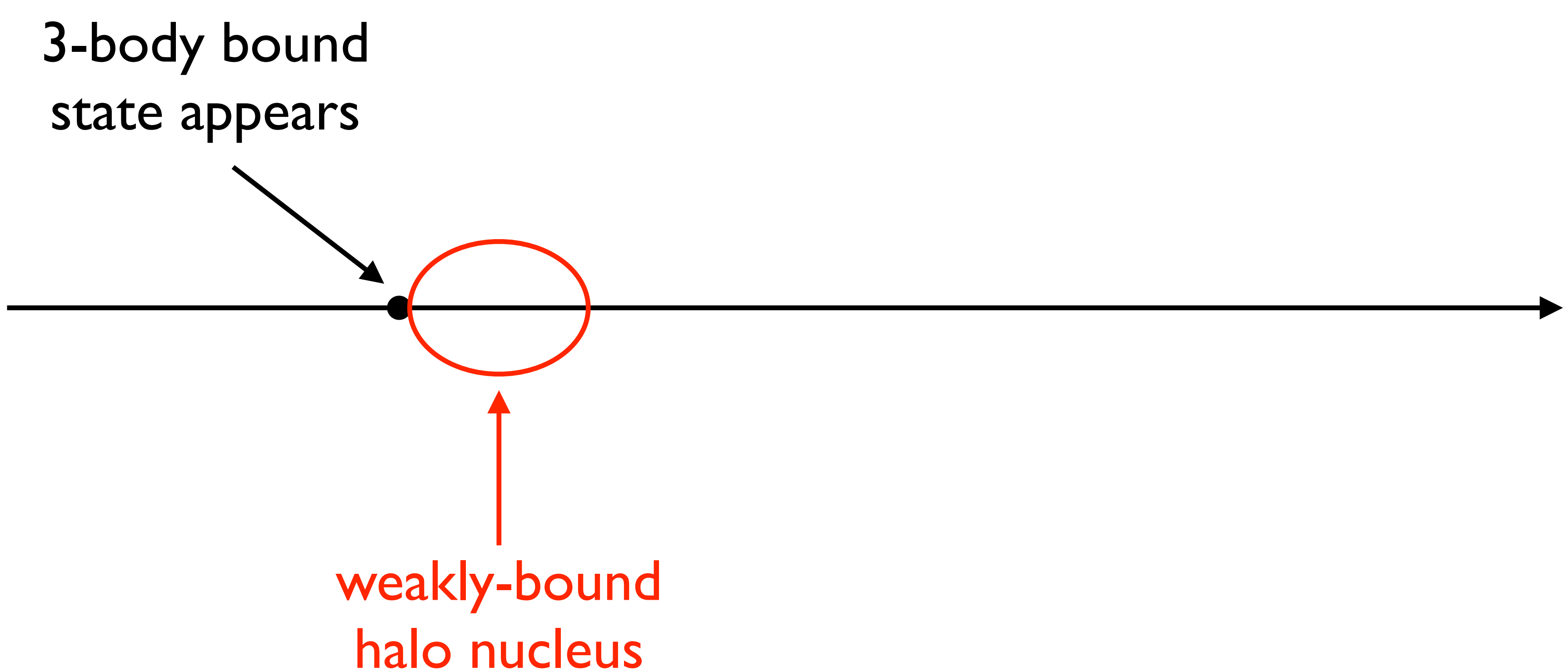


$$H = -\frac{1}{2}\nabla_{\mathbf{x}}^2 - \frac{1}{2}\nabla_{\mathbf{y}}^2 - V_0\left(e^{-x^2/2} + e^{-y^2/2}\right)$$



Carbon-22

- $|a(n^{20}C)| < 2.8$ fm Mosby et al 2013: non-Efimovian
- large matter radius Togano et al 2016 → small binding energy
- maybe it is here:



Unitarity limit: Zeldovich's 1960 paper

SOVIET PHYSICS JETP VOLUME 11, NUMBER 4 OCTOBER, 1960

THE EXISTENCE OF NEW ISOTOPES OF LIGHT NUCLEI AND THE EQUATION OF STATE OF NEUTRONS

Ya. B. ZEL'DOVICH

Submitted to JETP editor October 22, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **38**, 1123-1131 (April, 1960)

The limits of stability (relative to nucleon emission) of light nuclei are considered. The existence (in the sense of stability against decay with emission of a nucleon) of the following nuclei is predicted: He^8, Be^{12}, O^{13}, $B^{15,17,19}$, C^{16-20}, N^{18-21}, Mg^{20}. The problem of the possibility of existence of heavy nuclei composed of neutrons only is considered. The problem is reduced to that of a Fermi gas with a resonance interaction between the particles. The energy of such a gas is proportional to $\omega^{2/3}$, where ω is its density. The accuracy of the calculations is not sufficient to determine the sign of the energy and answer the question as to the existence of neutron nuclei.

Unitarity limit

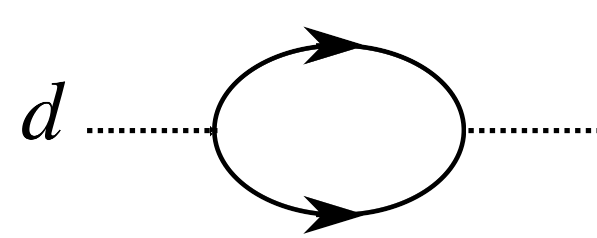
- 2 particles with opposite spins at $\mathbf{x}, \mathbf{y}$
- Wave function required to have asymptotics

$$\psi(\mathbf{x}, \mathbf{y}) = \frac{1}{|\mathbf{x} - \mathbf{y}|} f\left(\frac{\mathbf{x} + \mathbf{y}}{2}\right) + o(1)$$

- Kinetic energy

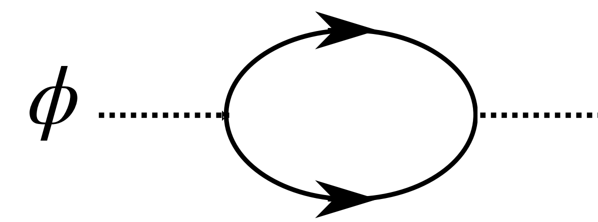
Neutrons sector: fermions near unitarity

- $L = i\psi^\dagger\left(\partial_t + \frac{\nabla^2}{2m}\right)\psi - c_0\psi_\uparrow^\dagger\psi_\downarrow^\dagger\psi_\downarrow\psi_\uparrow$
- Introducing auxiliary field d
- $L = i\psi^\dagger\left(\partial_t + \frac{\nabla^2}{2m}\right)\psi - \psi_\uparrow^\dagger\psi_\downarrow^\dagger d - d^\dagger\psi_\downarrow\psi_\uparrow + \frac{d^\dagger d}{c_0}$
- Compute full propagator of d



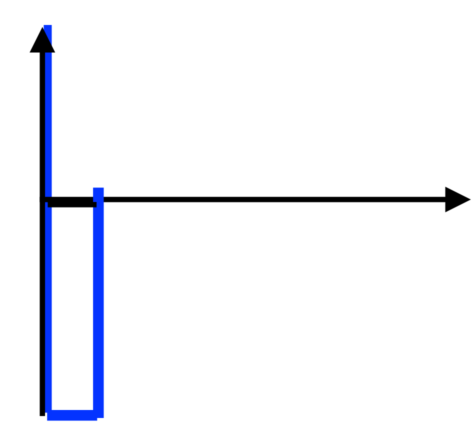
Renormalization

- $G_d^{-1}(\omega, \mathbf{p}) = c_0^{-1}$ + one-loop integral



- $= c_0^{-1} + \Lambda + \left(\frac{p^2}{4m} - \omega\right)^{1/2}$
- Unitarity: fine-tuning so that $c_0 + \Lambda = 0$
- (scattering length: $c_0 + \Lambda = \frac{1}{a}$)
- Physically: fine-tune the attractive short-range potential to have a bound state at threshold

$$G_d(\omega, \mathbf{p}) = \frac{1}{\sqrt{\frac{p^2}{4m} - \omega}}$$



Power counting

- Elementary exercise in QFT: counting operator dimensions. Set $m = 1$

$$S = \int dt\, d^3\mathbf{x}\, \psi^\dagger \left(i\partial_t + \frac{\nabla^2}{2} \right) \psi \qquad [x] = -1, \;\; [t] = -2$$

$$[\psi] = \frac{3}{2}$$

- Consistent with propagator:

$$\langle \psi(t, \mathbf{x}) \psi(0, \mathbf{0}) \rangle \sim \frac{e^{ix^2/2t}}{t^{3/2}}$$

Dimension of dimer operator

- $$G_d(\omega, \mathbf{p}) = \frac{1}{\sqrt{\frac{p^2}{4m} - \omega}}$$
- $$\langle d(t, \mathbf{x}) d^\dagger(0, \mathbf{0}) \rangle \sim \frac{e^{ix^2/4t}}{t^2} \quad \Rightarrow [d] = 2$$
- An operator in a nonrelativistic CFT
- a simplest "unnucleus" Hammer Son, arXiv:2103.12610

Dimension of operators

- Local two-body operator in theory of particles at unitarity:

$$d(\mathbf{x}) = \lim_{\mathbf{x}\to\mathbf{y}} |\mathbf{x}-\mathbf{y}|\,\psi_\uparrow(\mathbf{x})\psi_\downarrow(\mathbf{y})$$

- finite matrix element

$$\langle 0|\,d(\mathbf{x})\,|\Psi_{\text{2-body}}\rangle = \lim_{\mathbf{y}\to\mathbf{x}} |\mathbf{x}-\mathbf{y}|\,\Psi(\mathbf{x},\mathbf{y})$$

- dimension of d:

$$[d] = \frac{3}{2} + \frac{3}{2} - 1 = 2$$

Effective theory of weakly-bound halo nuclei

- Add two fields to the nonrelativistic CFT
  - the core ϕ
  - the halo nucleus h
- Interaction: $h^\dagger d\phi + d^\dagger\phi^\dagger h$
  - dimension: $\frac{3}{2} + \frac{3}{2} + 2 = 5$: marginal
  - leading-order EFT renormalizable

Effective Lagrangian

$$\mathscr{L} = h_0^\dagger\left(\mathrm{i}\partial_t + \frac{\nabla^2}{2m_h} + B_0\right)h_0 + \phi^\dagger\left(\mathrm{i}\partial_t + \frac{\nabla^2}{2m_\phi}\right)\phi + g_0(h_0^\dagger\phi d + \phi^\dagger d^\dagger h_0)$$
$$+\psi^\dagger\left(\mathrm{i}\partial_t + \frac{\nabla^2}{2m}\right)\psi - \psi_\uparrow^\dagger\psi_\downarrow^\dagger d - d^\dagger\psi_\downarrow\psi_\uparrow + \frac{d^\dagger d}{c_0}$$

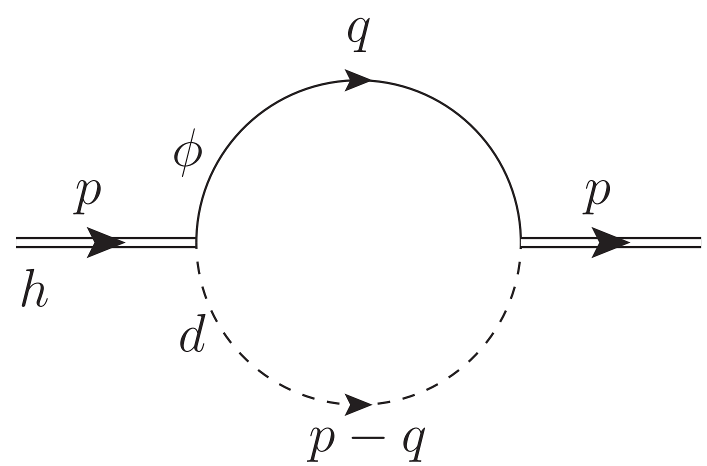
halo = bound state of core and dimer

Scale invariant theory except for:

- a_{nn} large but not infinite
- three-body binding energy $B \neq 0$

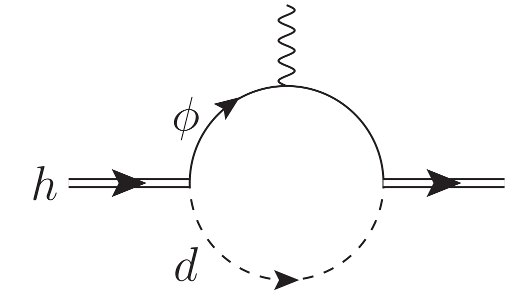
Logarithmic running of g ($g \to 0$ in the IR, Landau pole in UV)

Renormalization



- Halo self-energy $\Sigma(p) \sim \int d^4q D(p-q) G(q)$ diverges quadratically
- Quadratic divergence almost cancelled by $B_0 h_0^\dagger h_0$: fine-tuning for shallow 3-body bound state
- Remaining logarithmic divergence: wave function renormalization of halo field: $h_0 = Z^{1/2} h$, leads to logarithmic running of the coupling g

Charge radius



- Charge radius $\langle r_c^2 \rangle = \dfrac{4}{\pi} \dfrac{A^{1/2}}{(A+2)^{5/2}} \dfrac{g^2}{B} f_c(\beta),$

$$f_c(\beta) = \frac{1}{1-\beta^2} - \frac{\beta \arccos \beta}{(1-\beta^2)^{3/2}}$$

$$\beta = \sqrt{\frac{\epsilon_n}{B}}$$

- not completely "universal": g^2 depends logarithmically on the UV cutoff

Charge and matter radii

- Charge radius $$\langle r_c^2 \rangle = \frac{4}{\pi} \frac{A^{1/2}}{(A+2)^{5/2}} \frac{g^2}{B} f_c(\beta),$$

$$f_c(\beta) = \frac{1}{1-\beta^2} - \frac{\beta \arccos \beta}{(1-\beta^2)^{3/2}}$$

$$\beta = \sqrt{\frac{\epsilon_n}{B}}$$

- Matter radius $$\langle r_m^2 \rangle = \frac{2}{2\pi} \frac{A^{3/2}}{(A+2)^{5/2}} \frac{g^2}{B} [f_c(\beta) + f_n(\beta)]$$

$$f_n(\beta) = \frac{1}{\beta^3} \left[\pi - 2\beta + (\beta^2 - 2) \frac{\arccos \beta}{\sqrt{1-\beta^2}} \right]$$

- Universal ratio

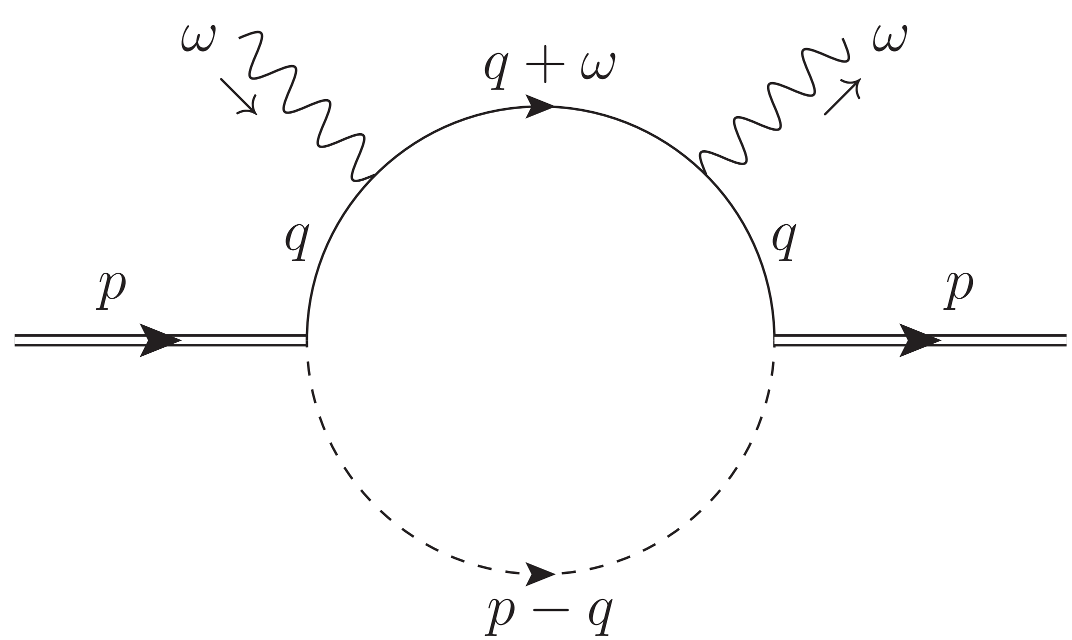
$$\frac{\langle r_m^2 \rangle}{\langle r_c^2 \rangle} = \frac{A}{2} \left[1 + \frac{f_n(\beta)}{f_c(\beta)} \right] = \begin{cases} \frac{2}{3}A & B \gg \epsilon_n \\ A & B \ll \epsilon_n \end{cases}$$

E1 dipole strength function

- $$\frac{dB(E1)}{d\omega}(\omega) \sim \sum_n |\langle n|(\mathbf{r}_c - \mathbf{R}_{cm})|0\rangle|^2$$
- can be mapped to current-current correlation

$$\frac{dB(E1)}{d\omega}(\omega) \sim \frac{1}{\omega^2}\mathrm{Im}\,\langle JJ\rangle(\omega)$$

- similar to deep inelastic scatterings



Dipole strength in unitarity limit

- When neutrons are in the unitarity limit, the dipole strength has a very simple shape

$$\frac{\mathrm{d}B(E1)}{\mathrm{d}\omega} \sim \frac{(\omega - B)^2}{\omega^4}$$

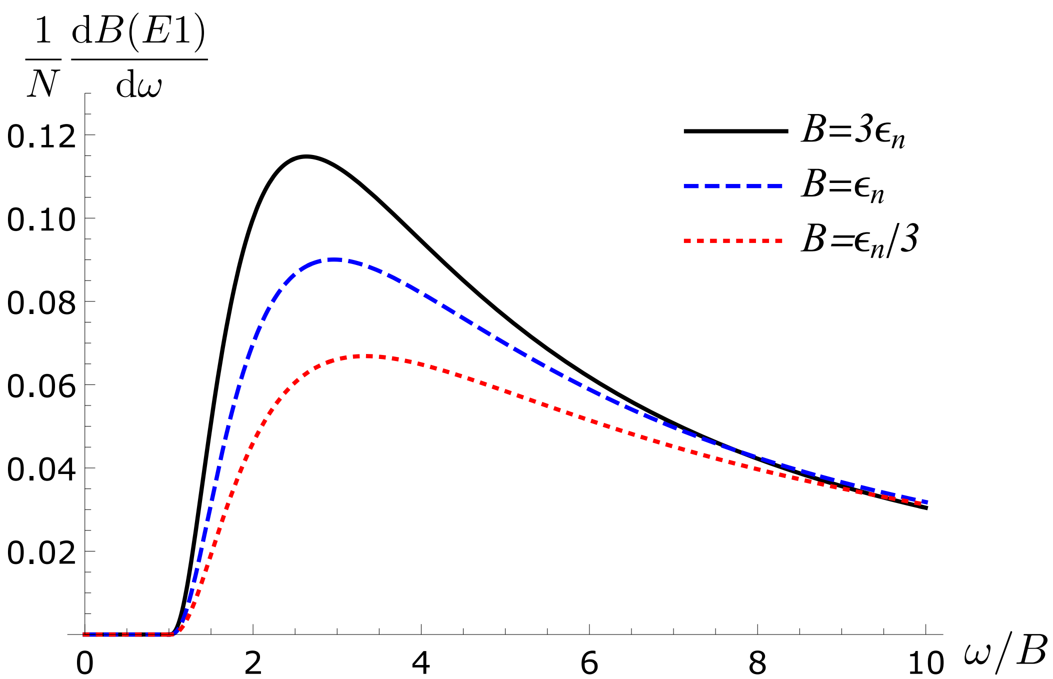
- Overall coefficient $\sim g^2$ and is logarithmically dependent on the UV cutoff

Result for dipole strength

$$\frac{\mathrm{d}B(E1)}{\mathrm{d}\omega} = \frac{3}{4\pi} Z^2 e^2 \frac{12 g^2}{\pi} \frac{A^{1/2}}{(A+2)^{5/2}} \frac{(\omega - B)^2}{\omega^4} \times f_{E1}\left(\frac{1}{-a\sqrt{\omega - B}}\right), \quad (29)$$

where

$$f_{E1}(x) = 1 - \frac{8}{3} x (1+x^2)^{3/2} + 4x^2 \left(1 + \frac{2}{3} x^2\right). \quad (30)$$



consistency check: sum rules

$$\int_0^\infty \mathrm{d}\omega \, \frac{\mathrm{d}B(E1)}{\mathrm{d}\omega} = \frac{3}{4\pi} Z^2 e^2 \langle r_c^2 \rangle,$$

$$\int_0^\infty \mathrm{d}\omega \, \omega \frac{\mathrm{d}B(E1)}{\mathrm{d}\omega} = \frac{3}{4\pi} Z^2 e^2 \frac{3}{A(A+2)},$$

Corrections to EFT

- Corrections to EFT are irrelevant terms EFT
- Effective range in *n*-*n* scattering: $r_0 d^\dagger (i\partial_t - \frac{1}{4}\nabla^2) d$
- *s*-wave core-neutron scattering $a_{cn}\phi^\dagger\psi^\dagger\psi\phi$
  - exp upper bound on n-^{20}C scattering length: correction is estimated to be < 25%
- *p*-wave core-neutron resonance (i.e., ^{5}He) can also be included

Conclusion

- Weakly bound two-neutron halo nuclei are next to simplest objects to described by EFT (after deuteron)
- Logarithmic running of coupling
- Ratios of lengths and shape of E1 dipole function are universal
- Next: *nn* scattering length correction (relatively easy), core-neutron scattering length or *p*-wave resonance (3-loop graphs)