Dinucleons in infinite nuclear matter at sub-saturation densities

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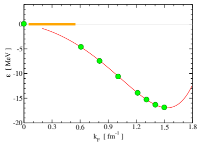
Itinerary

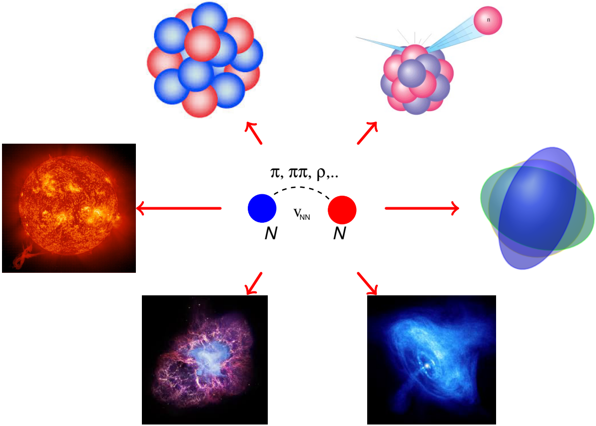
Problem & motivation

1. Genesis of the problem (personal):
   Collaboration w/Eric Bauge (CEA - Bruyères-le-Châtel) led to the conclusion that leading intrinsic medium effects in *NA* scattering take place at the surface of the target. [PRC 76,014613(2007) & PRC 78, 014608 (2008)]
2. Needed of accurate *g* matrices (BHF) at low densities...
3. but standard strategies resulted useless to get them due to unexpected instabilities.
4. Local *NN* effective interactions exclude parametrizations for $0 < k_F < 0.6$ fm^{-1}:

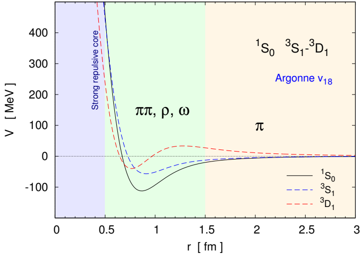


5. This (puzzling) situation led to investigate further the origin of such instabilities and physics behind them.

Many-nucleon systems from the bare interaction



About the bare *NN* interaction



NN data constraints

- Deuteron static properties
- Scattering amplitudes (E_{Lab} up to $\sim$300 MeV)
- Static properties ^{3}H, ^{3}He, ...

$$M_D c^2 = m_p c^2 + m_n c^2 + (-2.22 \text{ MeV}) \quad (\text{exp})$$

$$\longleftarrow \begin{cases} S=1 & (spin) \\ T=0 & (isospin) \\ J^\pi = 1^+ & (parity) \end{cases}$$

$J=1$

$L=0 \quad {}^3S_1 \quad |u\rangle$

$L=2 \quad {}^3D_1 \quad |w\rangle$

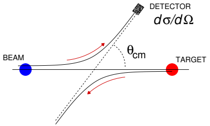
$P_D = 4-6$ %

$$\int_0^\infty (u^2+w^2)\, dr = 1$$

$$\int_0^\infty r^2 (u^2+w^2)\, dr = 4 R_D^2$$

$R_D = 2.14$ fm (exp)

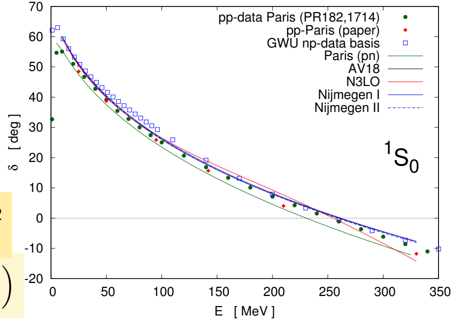
Realistic *NN* potential models



$$\frac{\hat{p}^2}{m}\varphi + V_{NN}\varphi = \frac{k^2}{m}\varphi$$

$$\frac{d\sigma}{d\Omega} = \left|\sum_{L=0}^{\infty}(2L+1)f_L(k)P_L(\cos\theta)\right|^2$$

$$f_L(k) = \frac{1}{2ik}\left(e^{i\delta_L(k)} - 1\right)$$



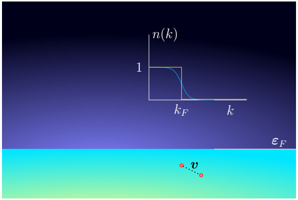
- Chiral ($\lesssim$ 290 MeV) — Entem *et al.*, PRC68, 041001 (2003); Holt et al. PRC81, 024002 (2010)
- CD Bonn ($\lesssim$ 350 MeV) — Machleidt, PRC63, 024001 (2001)
- Argonne v_{18} ($\lesssim$ 350 MeV) — Wiringa *et al.*, PRC51, 38 (1995)
- Nijmegen I and II ($\lesssim$ 350 MeV) — Stoks *et al.*, PRC49, 2950 (1994)
- Bonn A and B ($\lesssim$ 300 MeV) — Machleidt *et al.* PhysRep149, 1 (1987)
- Paris ($\lesssim$ 330 MeV) — Lacombe *et al.* PRC21, 861 (1980)

Brueckner-Hartree-Fock (BHF) approach for infinite NM

i.- In Brueckner-Bethe-Goldstone theory: lowest order in hole-line expansion for the ground-state energy.

ii.- In self-consistent Green's function (SCGF) theory: self-energy without hole-hole propagation.

iii.- In either case *in-medium* 2-body scattering matrix calculated self-consistently with the s.p. energy spectrum $e(k)$.



Infinite nuclear matter at density ρ:

$$\rho = \nu_{si} \sum_k n_k \to \nu_{si} \int \frac{d\mathbf{k}}{(2\pi)^3} n(k)$$

n Matter: $\rho = \frac{k_F^3}{3\pi^2}$

SNM: $\rho = \frac{2k_F^3}{3\pi^2}$

BHF g matrix

$$g(\omega) = v + v \frac{Q}{\omega + i\eta - \hat{h}_1 - \hat{h}_2} g(\omega)$$

$$Q|k_1 k_2\rangle = (1-n_{k_1})(1-n_{k_2})|k_1 k_2\rangle$$

$$\hat{h}_{1,2}|k_1 k_2\rangle = \left[\frac{k_{1,2}^2}{2m} + U(k_{1,2})\right]|k_1 k_2\rangle$$

Nonlinear structure for the g matrix in BHF

Integral equation for g

$$\langle \vec{\kappa}' | g_K(\omega) | \vec{\kappa} \rangle = \langle \vec{\kappa}' | v | \vec{\kappa} \rangle + \int d\vec{q}\, \langle \vec{\kappa}' | v | \vec{q} \rangle \frac{\Theta(k_+ - k_F)\Theta(k_- - k_F)}{\omega + i\eta - \frac{K^2}{4m} - \frac{q^2}{m} - \Sigma(K,q)} \langle \vec{q} | g_K(\omega) | \vec{\kappa} \rangle$$

Angular average:

$$\Sigma(K,q) = \left\langle U(|\tfrac{1}{2}\vec{K} + \vec{q}|) + U(|\tfrac{1}{2}\vec{K} - \vec{q}|) \right\rangle_{\hat{q}\cdot\hat{K}}$$

Self-consistency requirement

$$U(k) = \mathrm{Re}\left\{ \sum_p n_p \langle \tfrac{\mathbf{k}-\mathbf{p}}{2} | g_{\mathbf{k}+\mathbf{p}}(e_k + e_p) | \tfrac{\mathbf{k}-\mathbf{p}}{2} \rangle \right\}$$

$v = v_{NN}$ throughout!

What can we learn from the $v_{NN} \leftrightarrow g$ link?

(a) Binding energy of interacting Fermi system (nucleon):

$$\frac{B}{A} = \frac{\varepsilon}{\rho} = \frac{\sum_k n_k \frac{\hbar^2 k^2}{2m} + \frac{1}{2}\sum_k n_k U(k)}{\sum_k n_k}$$

$k_F^{sat} = 1.36 \pm 0.05\ \mathrm{fm}^{-1}$ $\qquad (B/A)_{sat} = 16 \pm 1$ MeV

(b) Equation of State (EoS) for nuclear matter:

$$p(\rho) = \rho^2 \frac{\partial(\varepsilon/\rho)}{\partial\rho})$$

TOV + EoS → hydrostatic equilibrium of neutron stars

(c) Fully off-shell g matrices for microscopic optical-model potentials

$U(\vec{k}', \vec{k}) = \langle \hat{\rho} \otimes g \rangle \qquad p + A \to p + A$

(d) Nuclear superfluid states: pairing, condensates

Self-consistent search

1. Make a guess for $U(k) \leftarrow U_0$
2. Then evaluate mass operator

$$M(k;e_k) = \sum_p n_p \langle \tfrac{1}{2}(\boldsymbol{k}-\boldsymbol{p})| g_{\boldsymbol{K}}(\underbrace{e_k+e_p}_{\omega})|\tfrac{1}{2}(\boldsymbol{k}-\boldsymbol{p})\rangle$$

   by solving

$$g(\omega) = v + v\frac{Q}{\omega + i\eta - h_1 - h_2} g(\omega)$$

   Continuous choice; $k \leq 5.5$ fm^{-1}; with $J \leq 7$

3. Take the real part of on-shell mass operator: $U(k) = \mathrm{Re}\, M(k;e_k) \to U_1$
4. Compare U_0 with U_1:

If $U_1 \simeq U_0$ $\longrightarrow$ self-consistency fullfilled

If $U_1 \neq U_0$ $\longrightarrow$ set $U_1 \to U_0$ and start over

Difficulties at subsaturation densities...

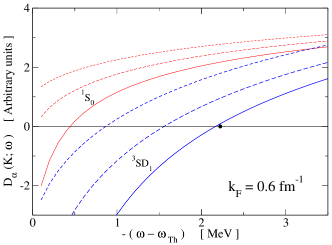
- Instabilities:
  zigg-zagging $U(k)$ in SNM $0.15 \lesssim k_F \lesssim 0.25$ fm^{-1} (feedback ambiguity)
- Sporadic but huge [±1E60] contributions in $^3SD_1$ and $^1S_0$ channels ($k_F \lesssim 1$ fm^{-1})
  Calculated $U(k)$ becomes meaningless!
- Problem worsens when Fermi-motion integrals ($\sum_k \cdots$) are made with thinner mesh (convergence dubious)

Cooper-pair eigenstates

$$g(\omega) = v + v\Lambda_K(\omega)\, g(\omega)$$

$$[1 - v\Lambda_K(\omega)]\, g(\omega) = v$$

$$\det[1 - v\Lambda_K(\omega)] = 0$$



Search at sub-saturation densities ($k_F < 1$ fm^{-1})

Control on Cooper eigenstates:

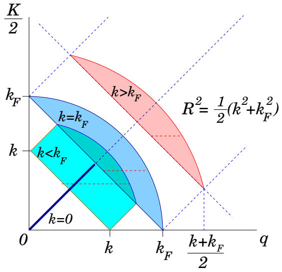
$$U(k) = \sum_p n_p \, g_{k+p}(e_k + e_p)$$

$$\to \int dK \int_{q_i}^{q_f} dq \; n_p \, g_K(e_k + e_p)$$

Whenever ω_C are found at K apply

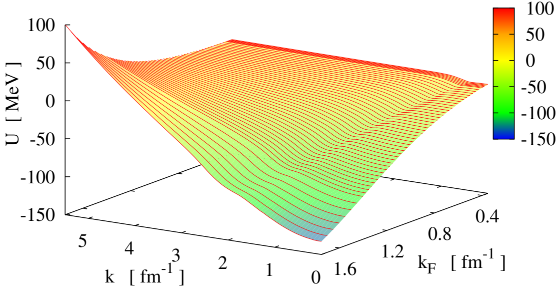
$$g_K(\omega) \to g_K(\omega) \, \frac{(\omega - \omega_C)^2}{(\omega - \omega_C)^2 + \eta^2}$$

- $\eta = 100$ keV adequate



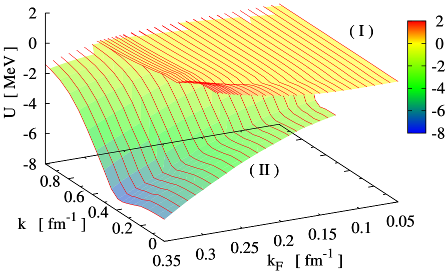
Results for $U(k)$ at $0.35 \leq k_F \leq 1.75$ fm^{-1}

Symmetric nuclear matter based on AV18



Coexisting solutions (low densities)

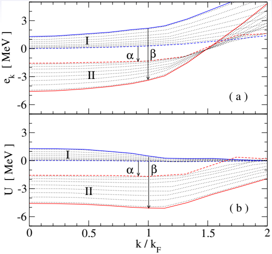
- At a same k_F two solutions satisfy BHF
- Two families of solutions are found:
  Phase I: $k_F \leq 0.285$ fm^{-1}
  Phase II: $k_F \geq 0.130$ fm^{-1}
- Range of overlap (coexistence): $0.130 \leq k_F \leq 0.285$ fm^{-1}

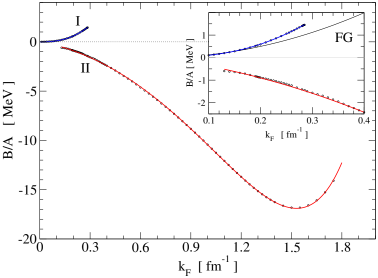


Features of s.p. solutions:

- s.p. energies $e(k)$ grow monotonically.
- Slope $(\partial U/\partial k)_{k_F}$ negative. $\Rightarrow m^* > m$
- Effective-mass approximation $(U \sim A + Bk^2)$ not valid at low densities

$$e(k) = \frac{k^2}{2m} + U(k) \nrightarrow \frac{k^2}{2m^*} + U_0$$





Properties at saturation

$k_F^{sat} = 1.53\ \mathrm{fm}^{-1}$ vs $1.36 \pm 0.05\ \mathrm{fm}^{-1}$

$(\frac{B}{A})_{sat} = -16.8$ MeV vs 16 ± 1 MeV

$K_\infty = 213$ MeV vs 220 ± 20 MeV

Incompressibility:

$$K_\infty = 9\rho^2 \frac{\partial^2(B/A)}{\partial^2\rho}$$

Masses and energies

- Effective k-mass:

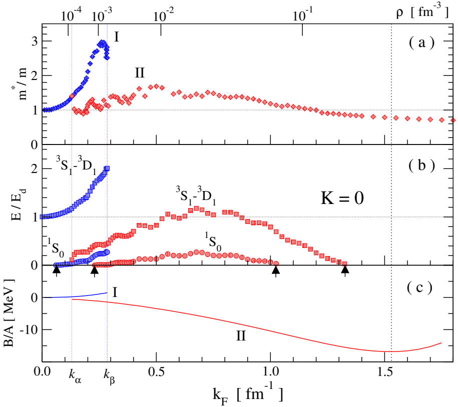
$$\frac{m^*}{m} = \left[1 + \frac{m}{k}\frac{\partial U(k)}{\partial k}\right]^{-1}_{k_F}$$

- Binding energies

$$E = \omega_C - 2\,e_F$$

(channels $^1S_0$ and $^3SD_1$)

- Pair c.m. motion $K = 0$



Eigenfunctions

- Condition for pair eigenstate: $\det[1 - v\Lambda_K(\omega)] = 0$
- Considering the spectral representation ... $g(\omega) = v + \sum_\alpha \frac{vQ|\alpha\rangle\langle\alpha|Qv}{\omega - \varepsilon_\alpha}$

  ... the g matrix near eigenenergy ε_β satisfies

$$\lim_{\eta\to 0} i\eta\, g(\varepsilon_\beta + i\eta) = vQ|\beta\rangle\langle\beta|Qv \equiv \hat{M}_\beta$$

- To get the eigenfunction (momentum space) do

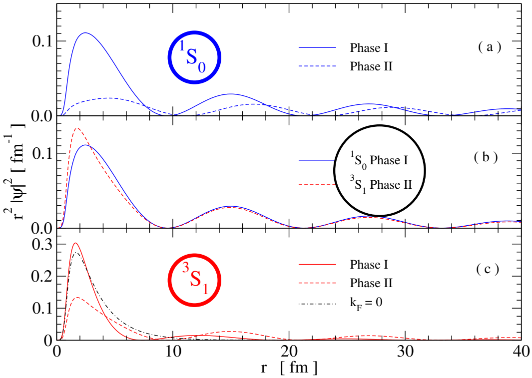
$$\langle k|(\varepsilon_\beta - 2e(k))|\beta\rangle = \langle k|vQ|\beta\rangle \quad \Rightarrow \quad \langle k|\beta\rangle = \mathrm{Sgn} \times \frac{\sqrt{\langle k|\hat{M}_\beta|k\rangle}}{\varepsilon_\beta - 2e(k)}$$

- In coordinate space

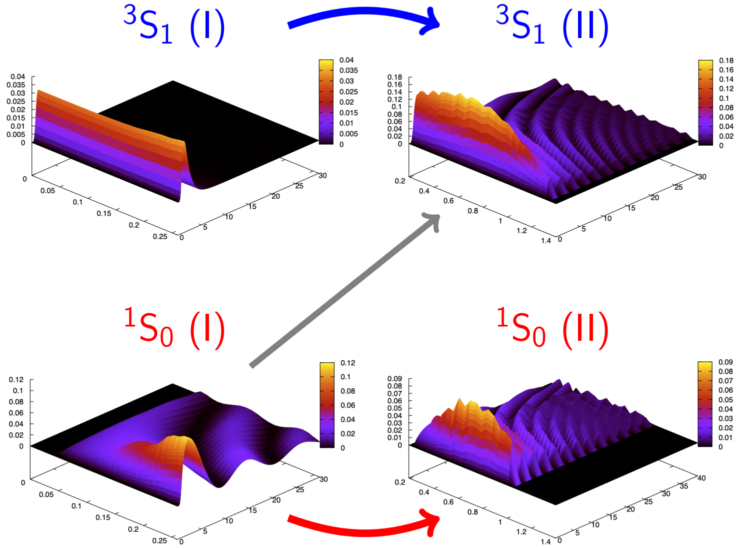
$$\langle\vec{r}|\beta\rangle = \int d\vec{k}\, e^{i\vec{k}\cdot\vec{r}} \langle\vec{k}|\beta\rangle$$

Probability density $|\Psi(r)|^2$ $\quad\langle r|\beta\rangle = \Psi(r)$

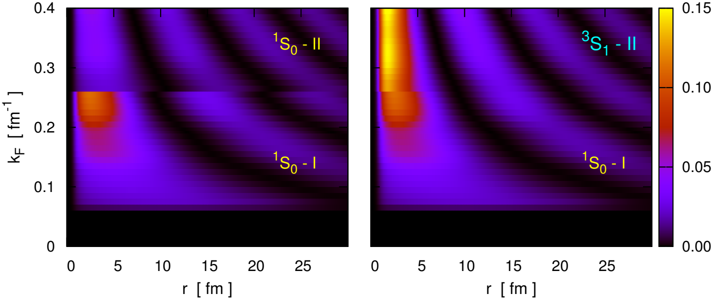
In-medium S-wave radial probability density $r^2|\Psi(r)|^2$ $\quad(k_F = 0.25\ \text{fm}^{-1})$.



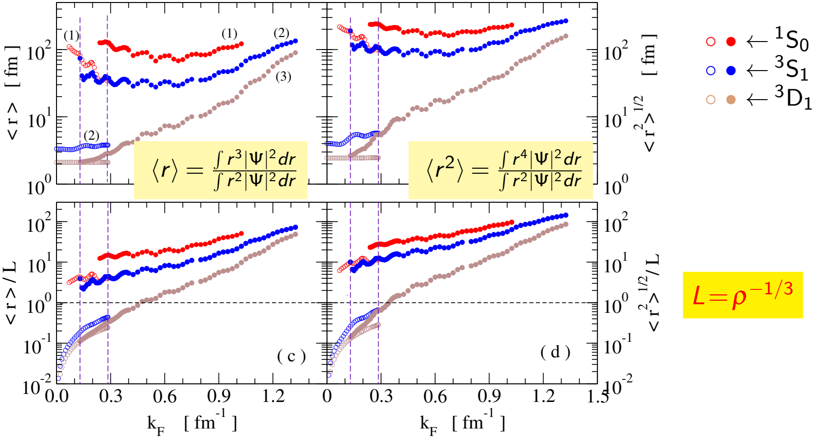
Eigenfunctions in the r-k_F plane



Transition ($^1S_0 \rightarrow {}^3SD_1$)



Correlation length



$L = \rho^{-1/3}$

We evaluate $F(s) = \int_0^\infty e^{-sr} |\Psi(r)|^2 r^2 dr$

Expand $F(s)$ for small s to extract $\langle r \rangle$, $\langle r^2 \rangle$, etc. [EPJA 57,7(2015)]

Pairing and superfluidity

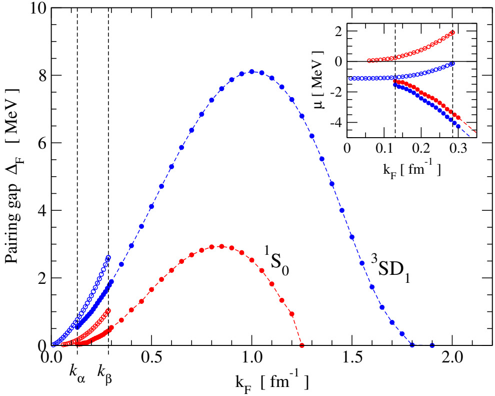
- Presence of Cooper eigenstates alter the s.p. picture of BHF approach.
- Beyond BHF $\rightarrow$ SCGF theory.The tools we have developed may help in doing so.
- Still, it becomes instructive to assess how important is the role of consensation.

- Gap equation with anisotropic kernel angle-averaged:

$$\Delta_L(k) = -\frac{2}{\pi}\int_0^\infty k'^2\,dk' \sum_{L'} i^{L-L'} v_{LL'}(k,k')\,\frac{\Delta_{L'}(k')}{2E(k')}$$

- Quasiparticle energy $\quad E(k)^2 = (e_k - \mu)^2 + \sum_L \Delta_L(k)^2$
- Normal density distribution $\quad n(k) = \frac{1}{2}\left[1 - \frac{e_k - \mu}{E(k)}\right]$
- Chemical potential μ must satisfy (SNM) $\quad \rho = 4\int \frac{d^3k}{(2\pi)^3} n(k)$

Energy gap $\Delta_F = \Delta(k_F)$ as function of k_F (SNM)



Normal vs superfluid matter

- Investigate energy per nucleon including the *condensation energy* [Lombardo et al, PRC59, 2927(1999).]

$$\frac{B}{A} = \frac{1}{\rho} \sum_k \left\{ 4n(k) \left[\frac{k^2}{2m} + \frac{1}{2} U(k) \right] - 2 \frac{\Delta^2(k)}{2E(k)} \right\}$$

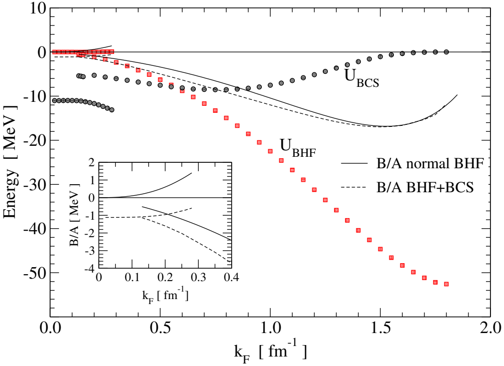
- Define:

$$U_{BHF} = \frac{2}{\rho} \sum_k n(k)\, U(k)$$

$$U_{BCS} = -\frac{1}{\rho} \sum_k \frac{\Delta^2(k)}{E(k)}$$

- Selfconsistency not met:

$$U(k) = \mathrm{Re} \sum_p n_p\, g_K(e_k + e_p)$$



Concluding remarks

- Cooper eigenstates are the cause of instabilities in BHF at subsaturation densities. Their presence is tractable.
- Effective-mass approximation for $U(k)$ inadequate at subsaturation densities.
- Coexisting self-consistent s.p. fields in the range $0.13 \leq k_F \leq 0.285\ \text{fm}^{-1}$, $10^{11.4} \lesssim \rho_{mass} \lesssim 10^{12.4}\ \text{g cm}^{-3}$.
- Size of Cooper eigenstates greater than internucleon separation. They could get as large as 100 fm!
- Condensate energy U_{BCS} 'small' at normal densities.
- Condensate energy comparable to that from normal state at sub-saturation densities $\Rightarrow$ need to include hole-hole propagation. (Matías Gutierrez, U Chile).
- The EoS for nuclear matter has to be a continuous function, even in the overlap of phases I and II.

Credits: Collaborators ... (alphabetical)

Jean-Paul Delaroche, CEA, France
Felipe Isaule, U Glashow, Scotland
Arnau Rios, U Barcelona, Spain

Grad students ...
Matías Gutierrez, U Chile (SCGF w/hh propagation)
Sebastián Vargas, U Chile (BHF at finite temperature)

Nelson Adriazola, U Chile (Neutron start stability)
José Fuentealba, U Chile (NA scattering with exotic nuclei)

Thank you all !