

Recent applications of the Gamow shell model at proton and neutron driplines

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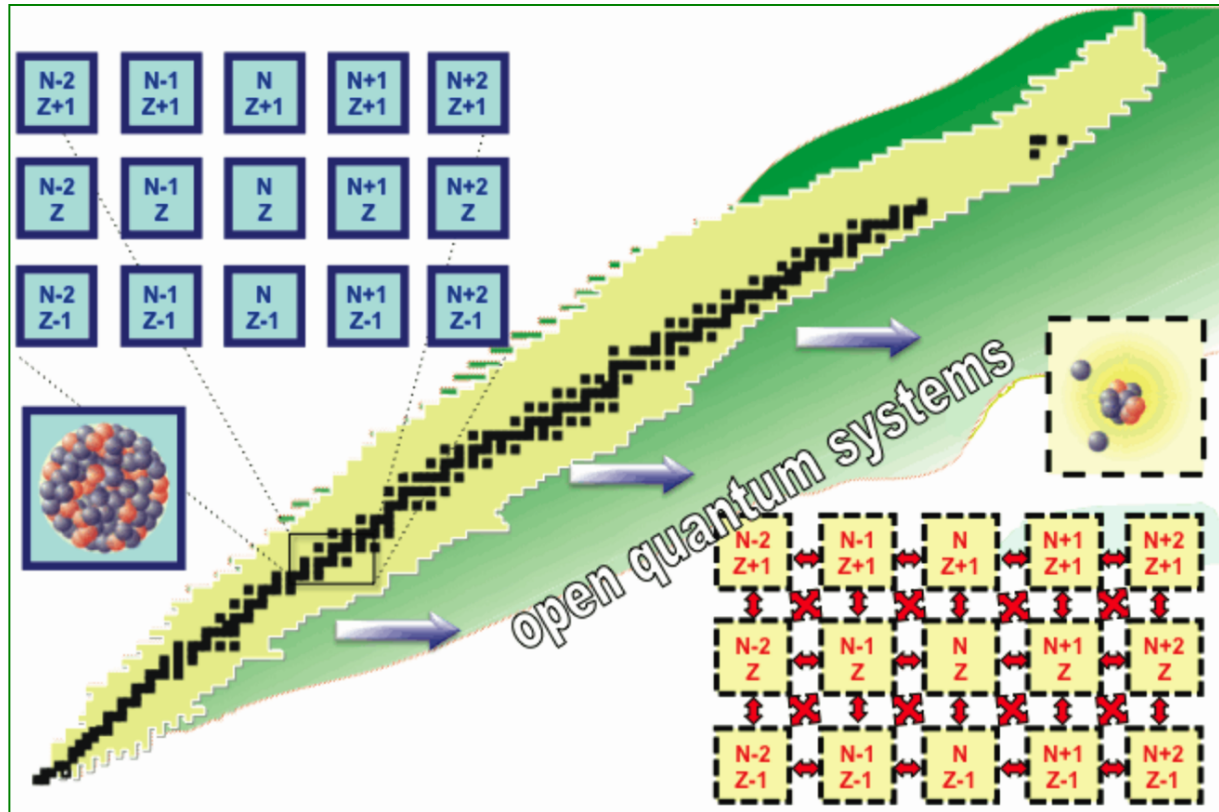
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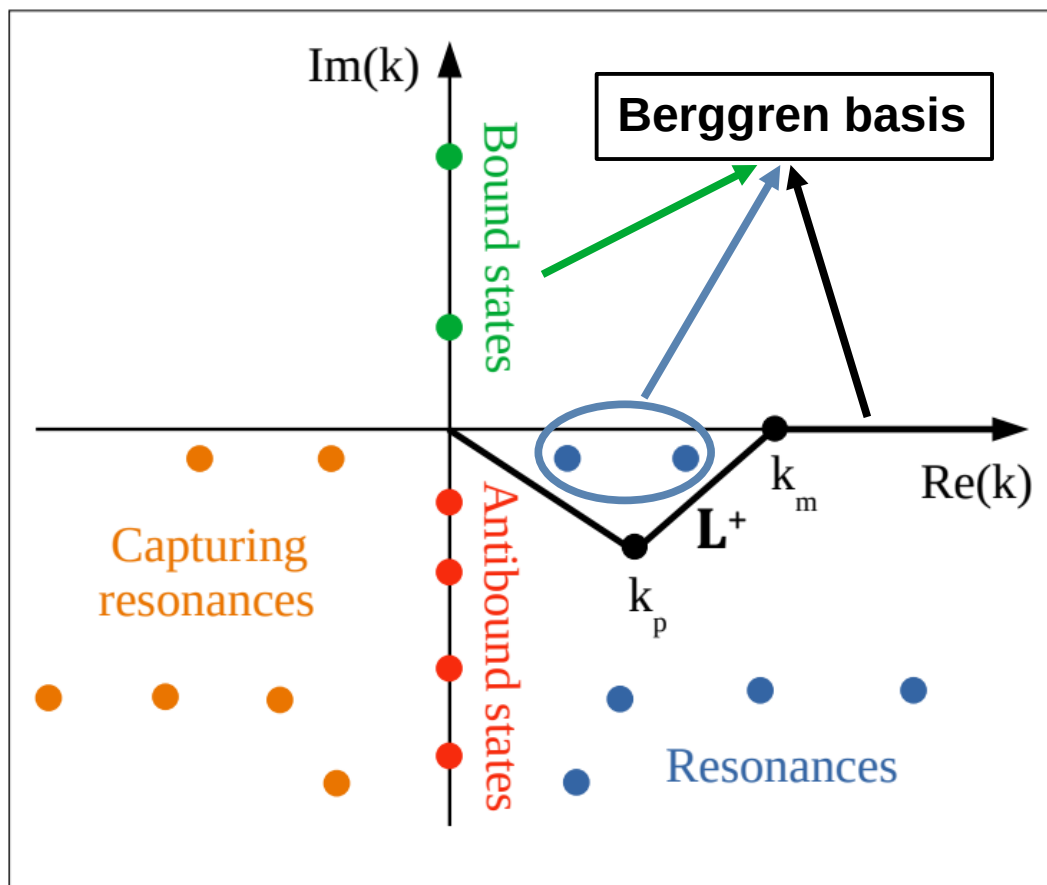
Scientific context



Experimental interest

Study of nuclei far from the valley of stability
 Many efforts made to study drip-line nuclei

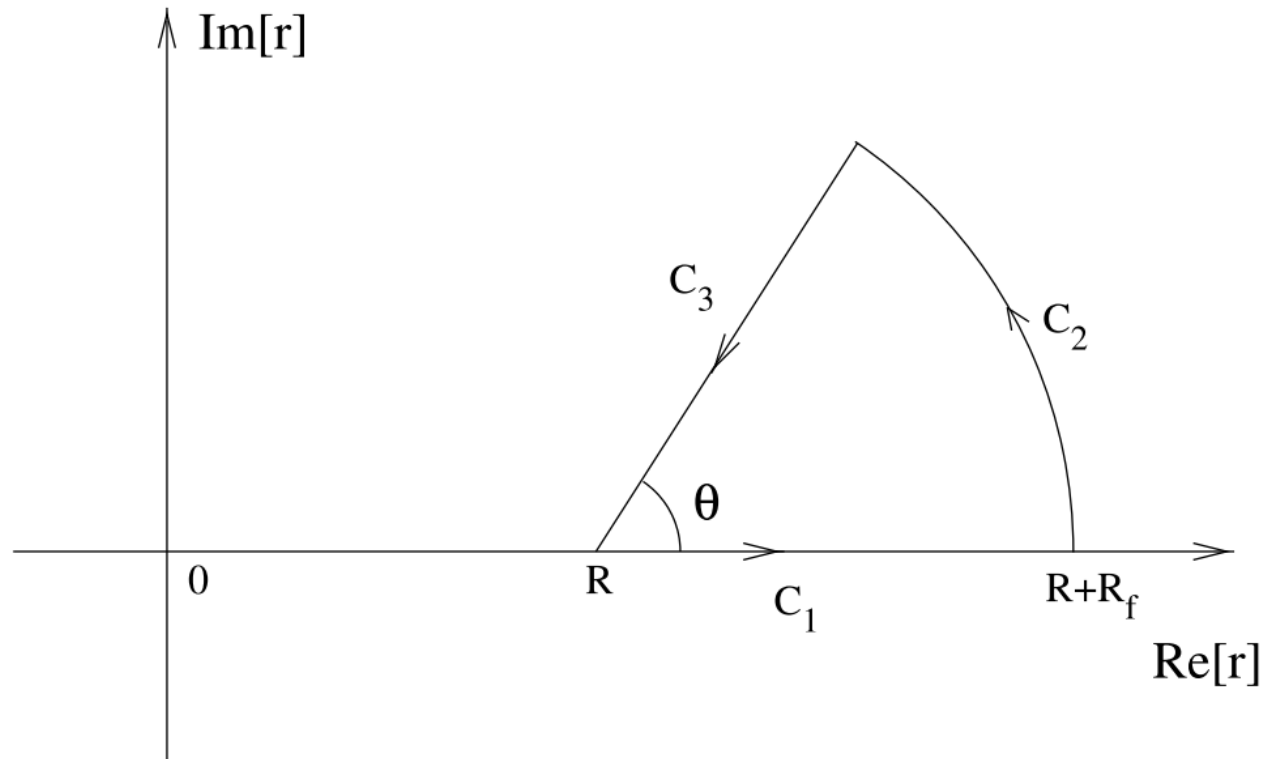
The Berggren basis



Berggren basis : bound, resonance and scattering states

Efficient discretization of the L^+ contour with Gauss-Legendre quadrature

Complex scaling

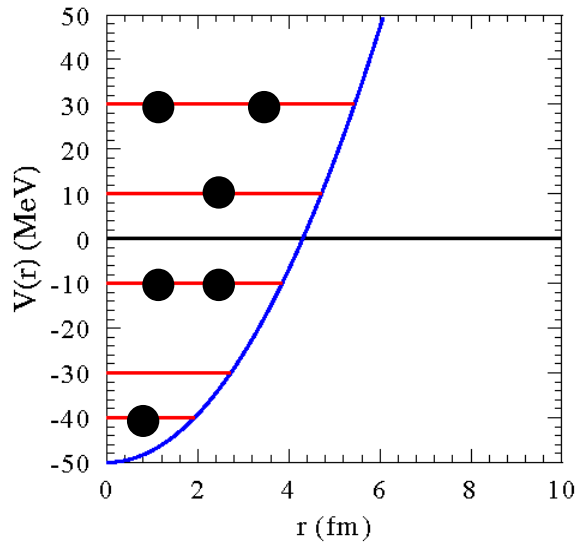


Divergence of unbound states on the real axis
Resonance states: localized in the complex plane
Complex scaling method to calculate matrix elements
Bound and resonance states: normalized
Scattering states: normalized with a Dirac delta

Gamow Shell Model (GSM)

Standard shell model

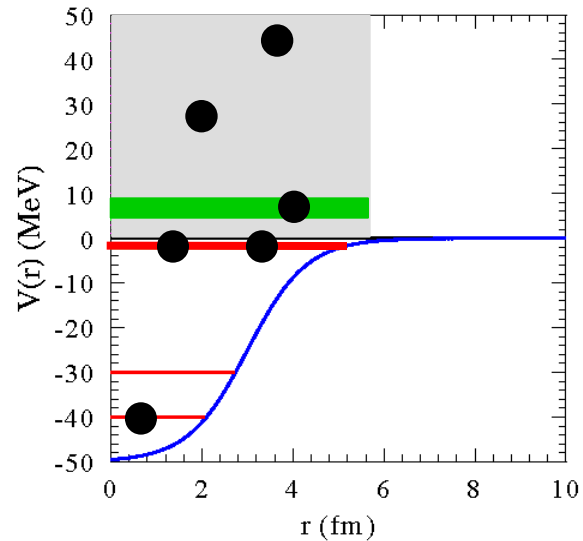
Closed quantum system description



Localized states

Gamow Shell Model

Open quantum system description



Localized states
Weakly bound/resonant states
Scattering states

Diagonalization of GSM matrices

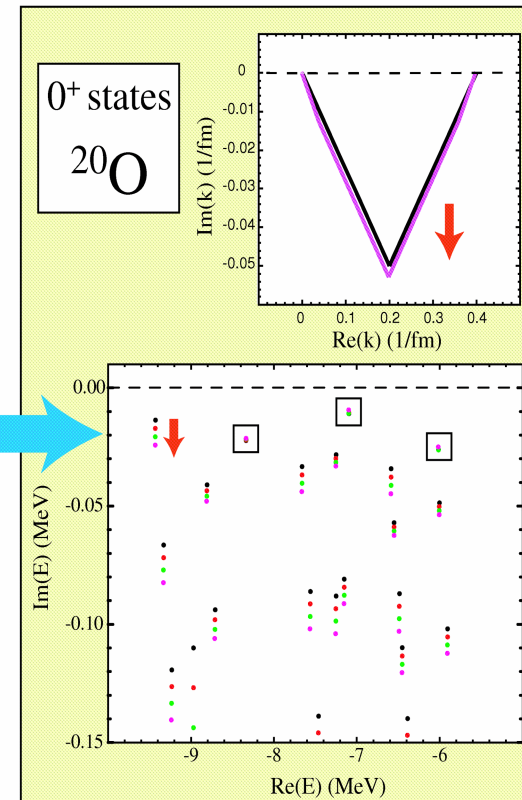
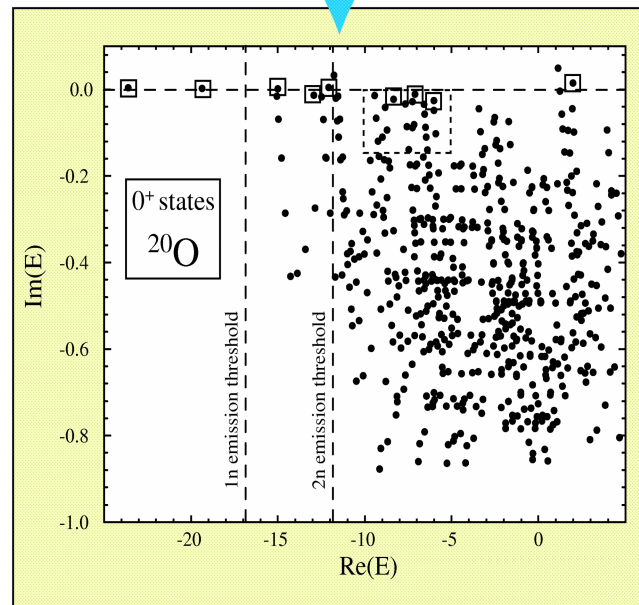
GSM Hamiltonian matrix

Resonant eigenstates
representing resonances
hidden among scattering eigenstates

The overlap method

- 1) Diagonalization in pole space
- 2) Diagonalization in full space
- 3) Identification of physical states

Stability of the 'physical' states



Cluster Orbital Shell Model (COSM)

Problematic

3A degrees of freedom (particles coordinates)

3(A-1) physically (translational invariance) → spurious states

Standard shell model

Calculation in a major shell (core + valence nucleons)

Lawson method (no-core shell model)

Harmonic oscillator basis only

Cluster orbital shell model (COSM)

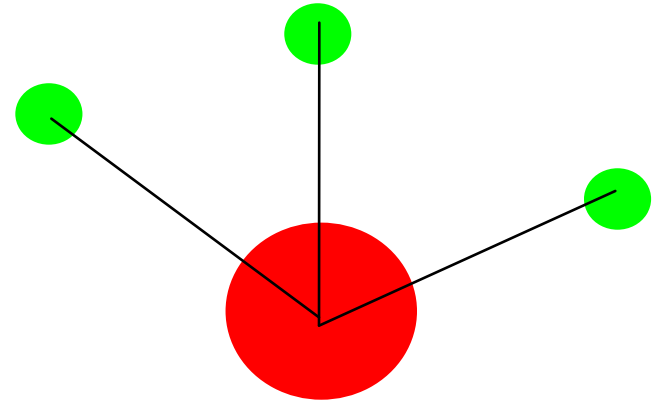
Relative core coordinates → no center of mass excitation

Center of mass handled by a recoil term in the Hamiltonian

+: Formal use identical to laboratory coordinates

–: Inferior to Jacobi coordinates

Pauli principle approximately treated with a Pauli operator on the core



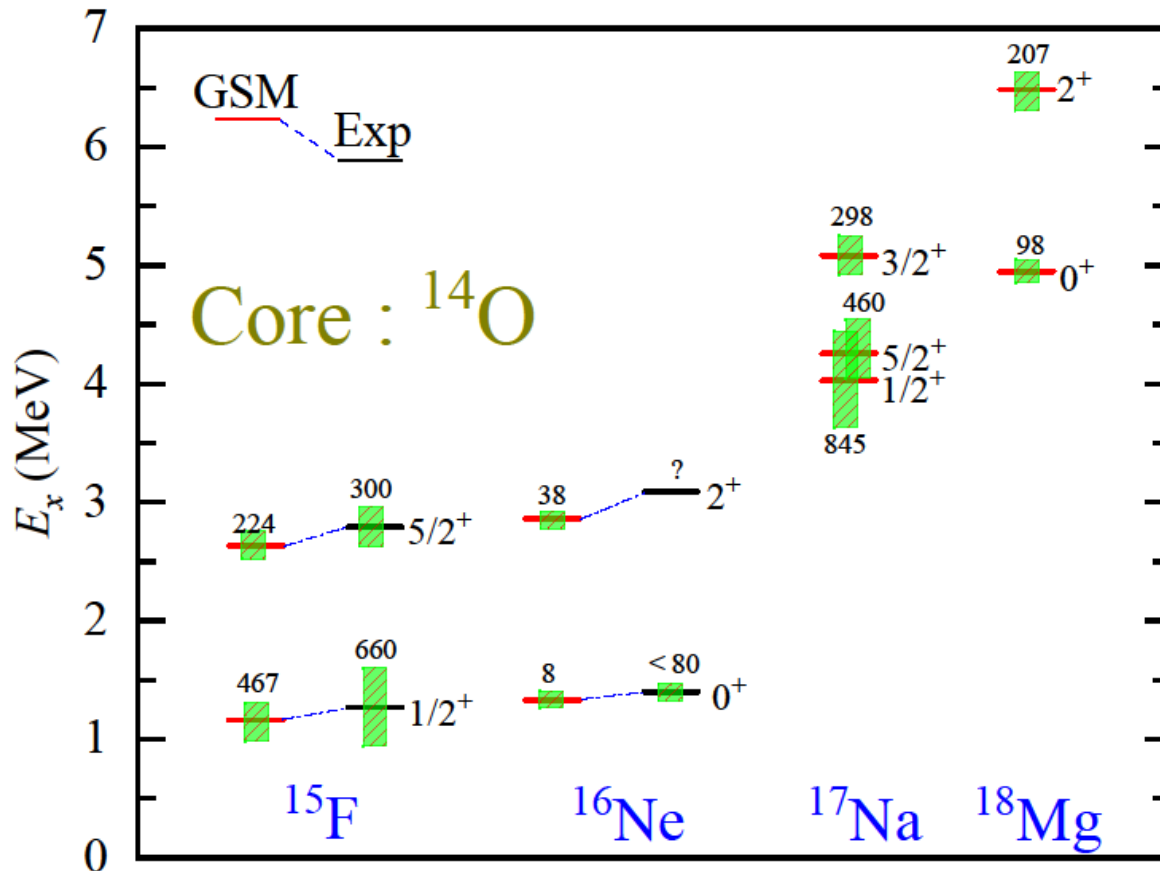
$$\vec{r} = \vec{r}_{lab} - \vec{R}_{core}$$

Practical use of COSM

Definition of Hamiltonian directly in COSM frame

Calculations with COSM and Jacobi coordinate models very close

Proton-rich carbon isotones with GSM



Interest

p, 2p emission in light nuclei

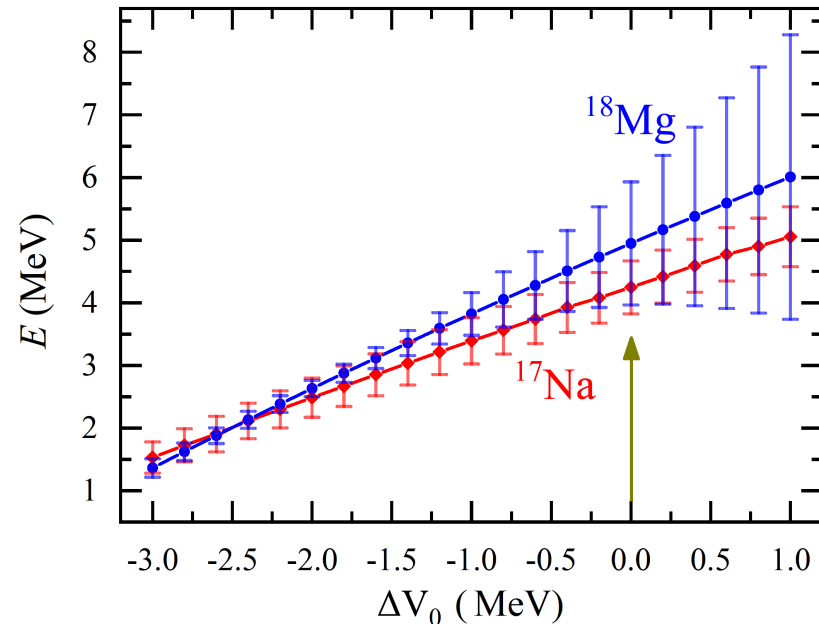
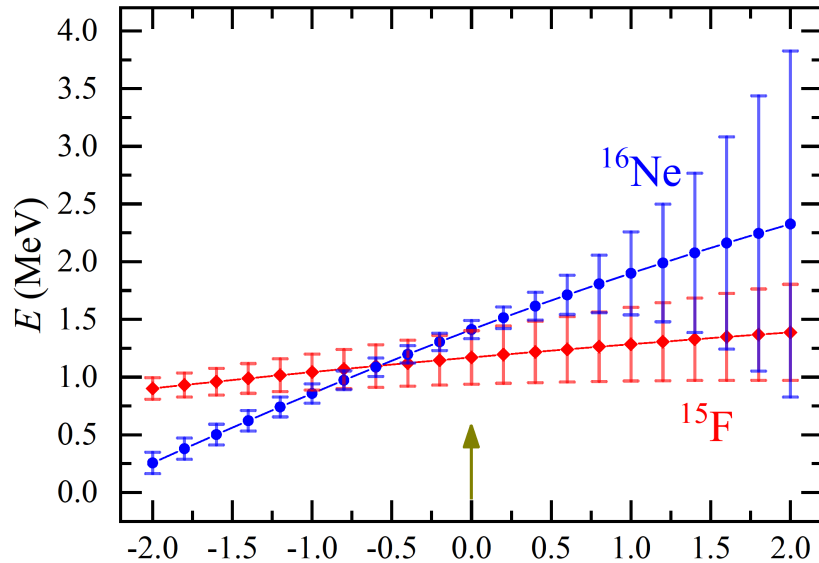
Model

GSM with a ^{14}O core
EFT interaction
psd Berggren basis

Spectra

Energies and widths well reproduced

Diproton emission in ^{16}Ne and ^{18}Mg



Proton and diproton emission

Two emission channels : proton and diproton

GSM width : sum of all partial widths

How to separate proton from diproton emission ?

Channel separation

Well binding ^{14}O core potential : only 2p emission

Extrapolation to physical case : $\Gamma_{2p} \sim \text{const}$

Γ_{1p} from Γ_{2p} and Γ

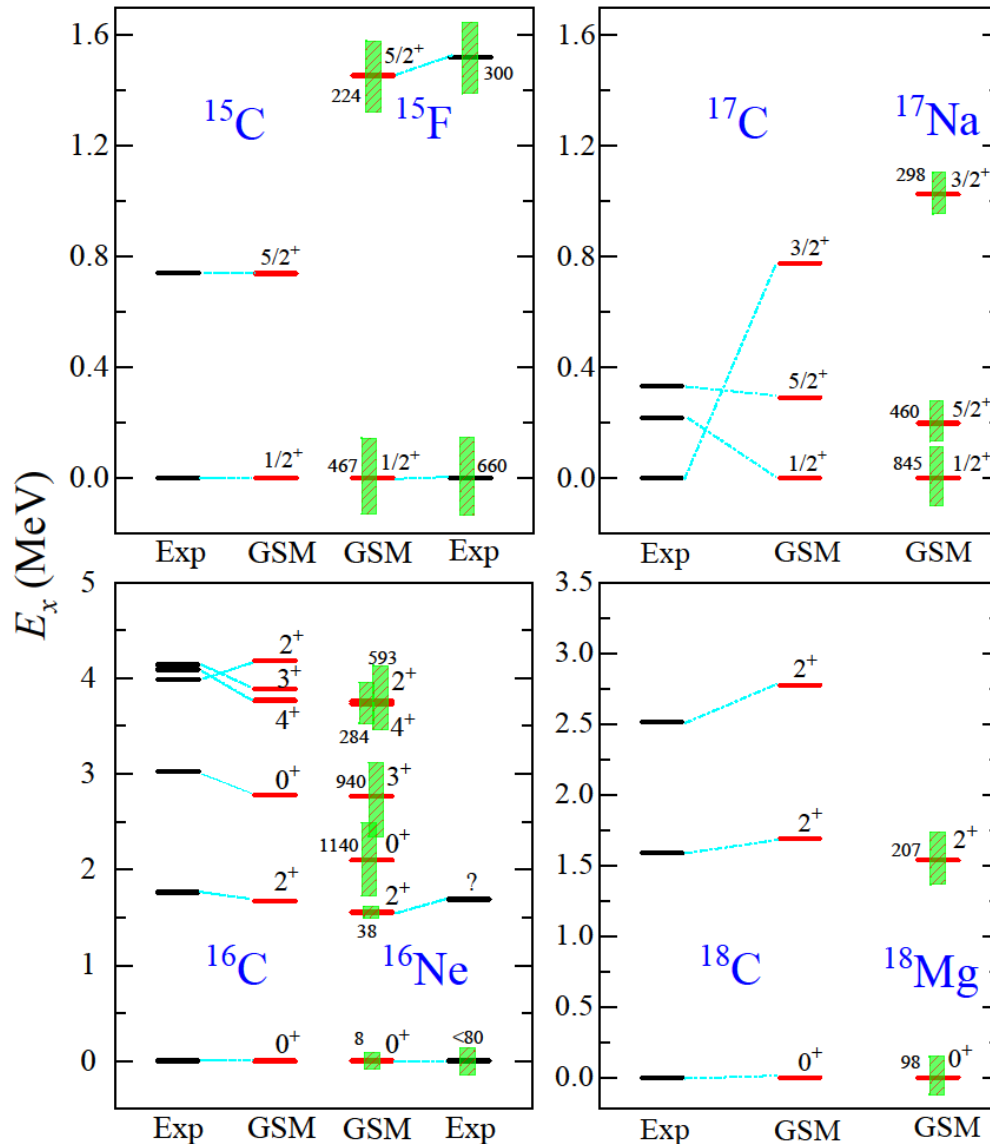
Results

Diproton width : ~ 10 keV.

Proton width : ~ 100 keV.

Results consistent with current experimental situation

Isospin symmetry breaking in carbon isotopes and isotones with GSM



Isospin symmetry breaking

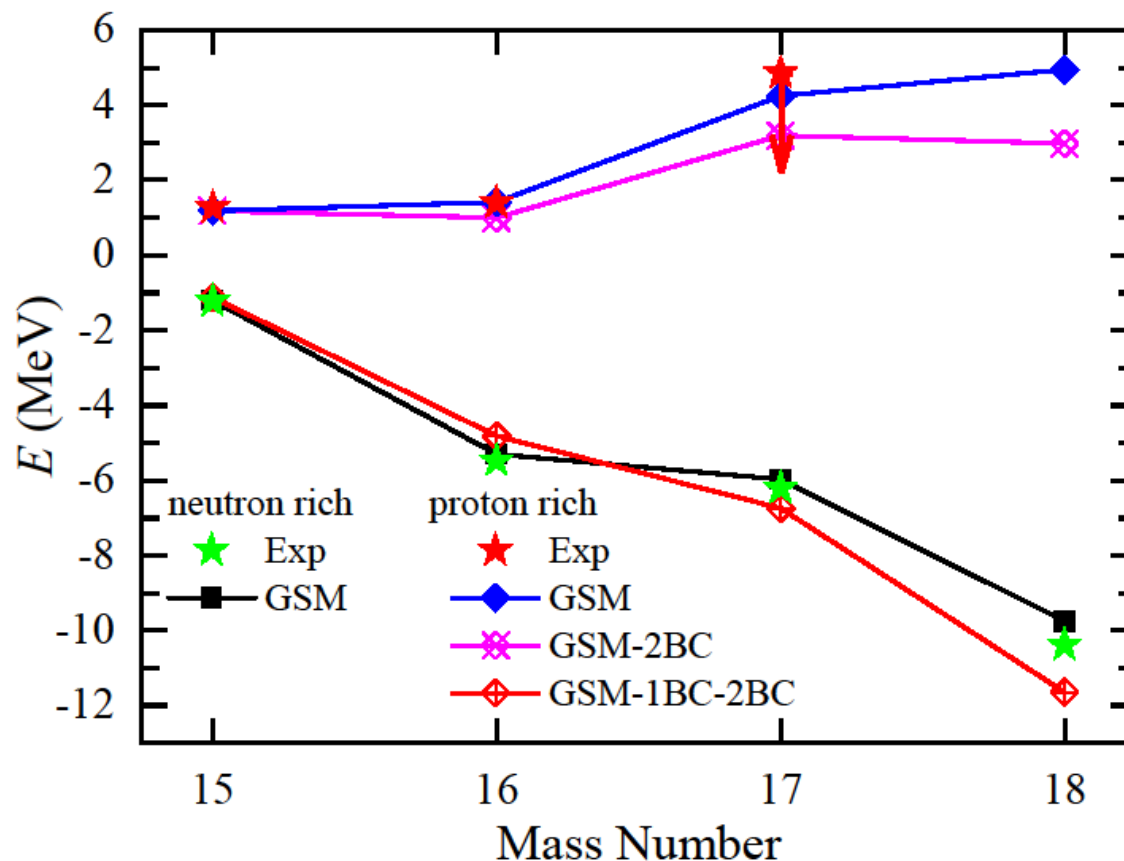
Carbon isotopes : well bound
Carbon isotones : unbound

Thomas-Ehrmann shift
induced by continuum coupling

Competing effects
Increasing Coulomb interaction
Continuum coupling
Nuclear structure

Large width not necessarily induces
large Thomas-Ehrmann shift

Coulomb contribution in proton-rich nuclei



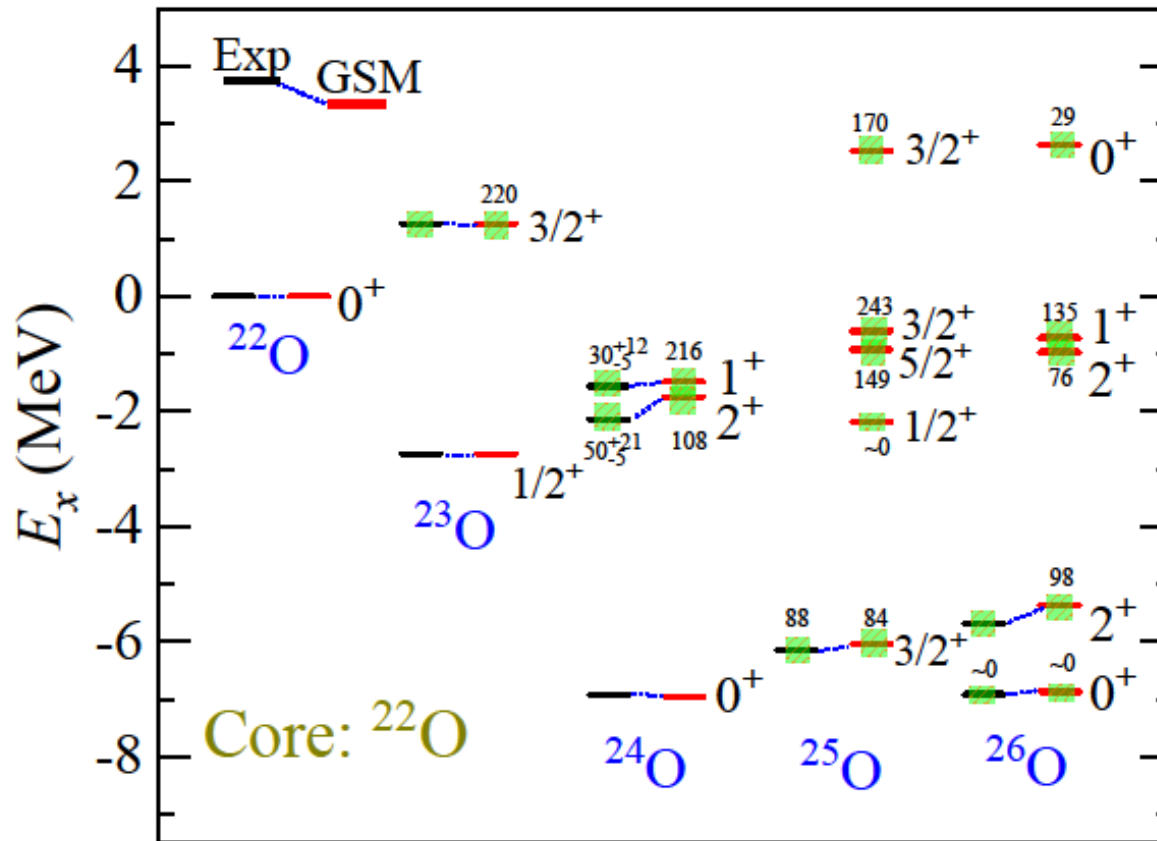
Coulomb contribution

^{14}O core potential : one-body Coulomb potential
Valence protons : two-body Coulomb interaction
Coulomb interaction maximal at proton drip-line

Comparison with carbon isotopes

Coulomb removal : similar to carbon isotopes
Energy difference : isospin symmetry breaking
Unintuitive behavior of energy : non monotonous

Unbound spectra of oxygens with GSM



Oxygen isotopes

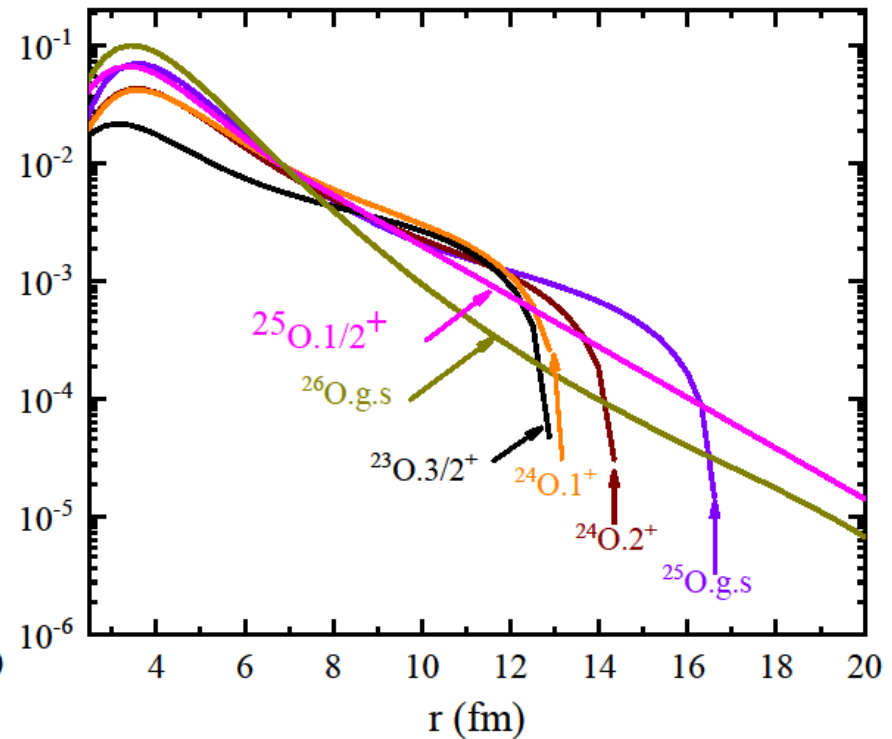
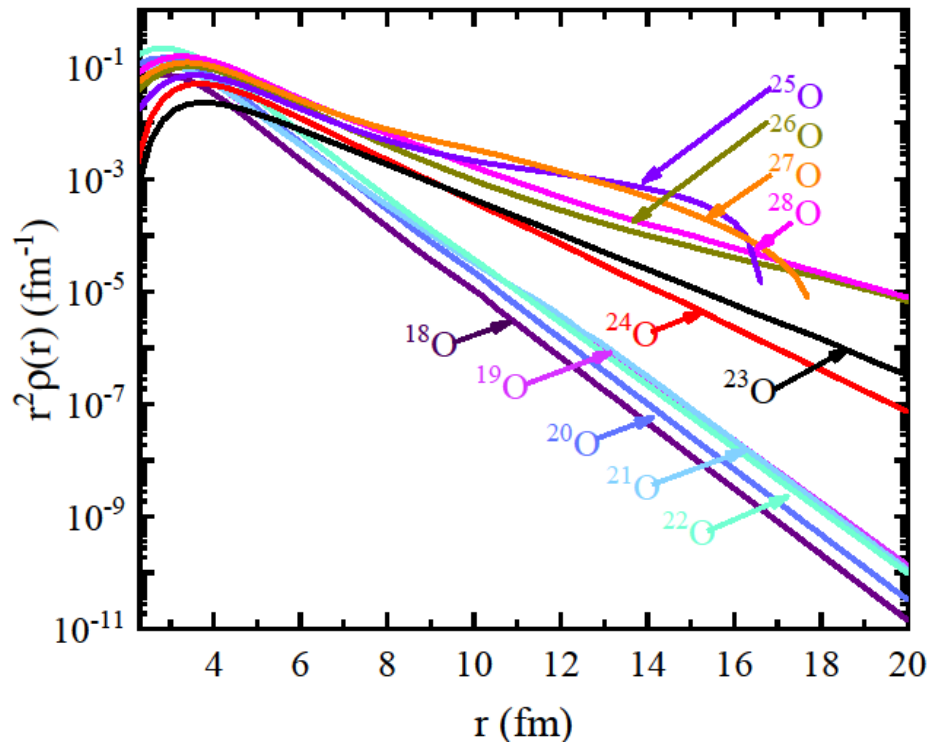
GSM with a core of ^{22}O

EFT interaction in $spdf$ space with the Berggren basis

A-dependence to simulate three-body effects

Prediction of narrow resonance states in spectra

Densities of oxygen states with GSM



Densities of oxygen many-body states

$^{18-22}\text{O}$: well bound many-body states

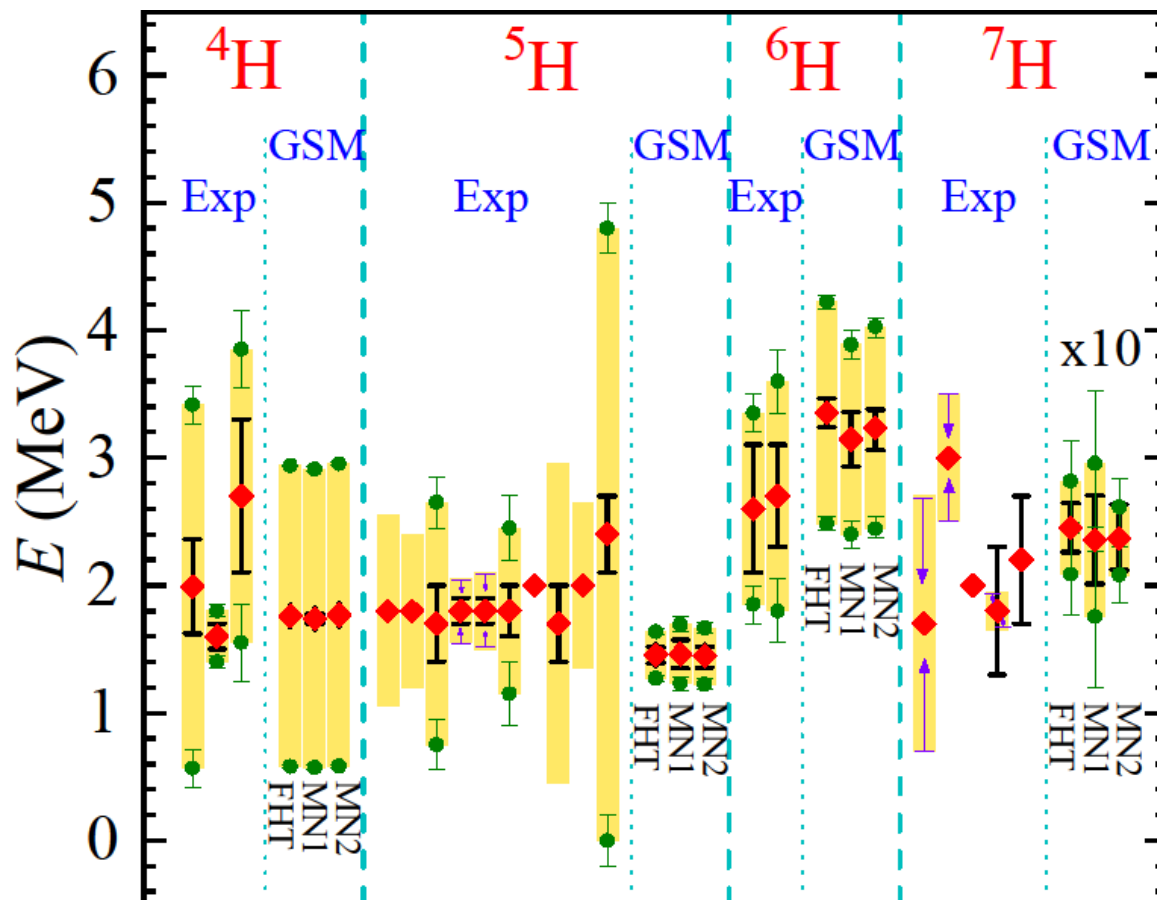
$^{23-28}\text{O}$: weakly bound or unbound many body states

Bound/resonance character seen in neutron densities

Exponential decay for bound/narrow resonances

Oscillations at large distance for broad resonances

Unbound hydrogen isotopes with GSM (1/2)



Hydrogen isotopes

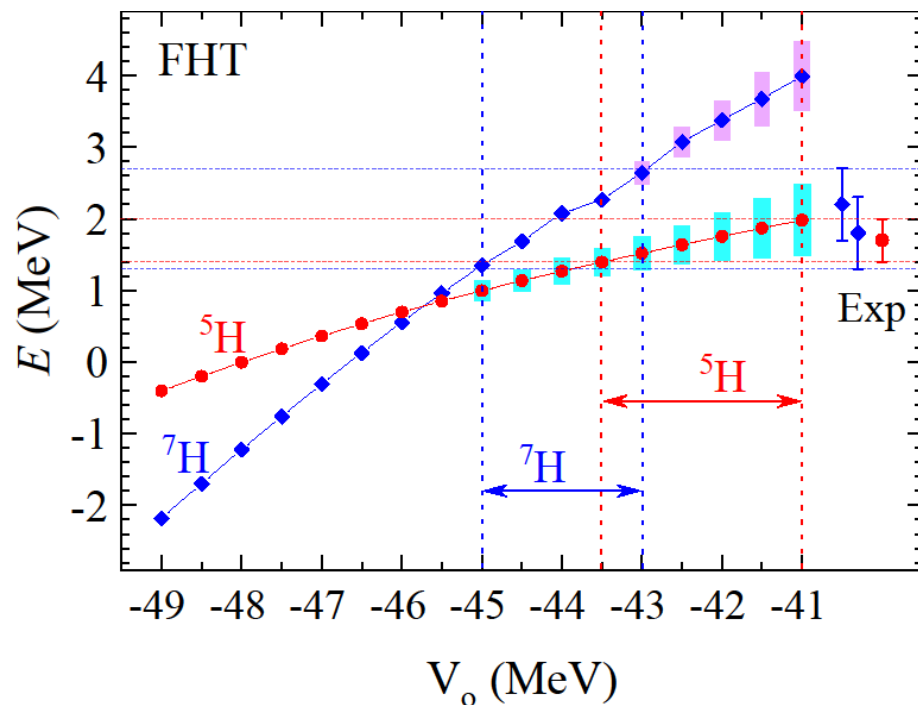
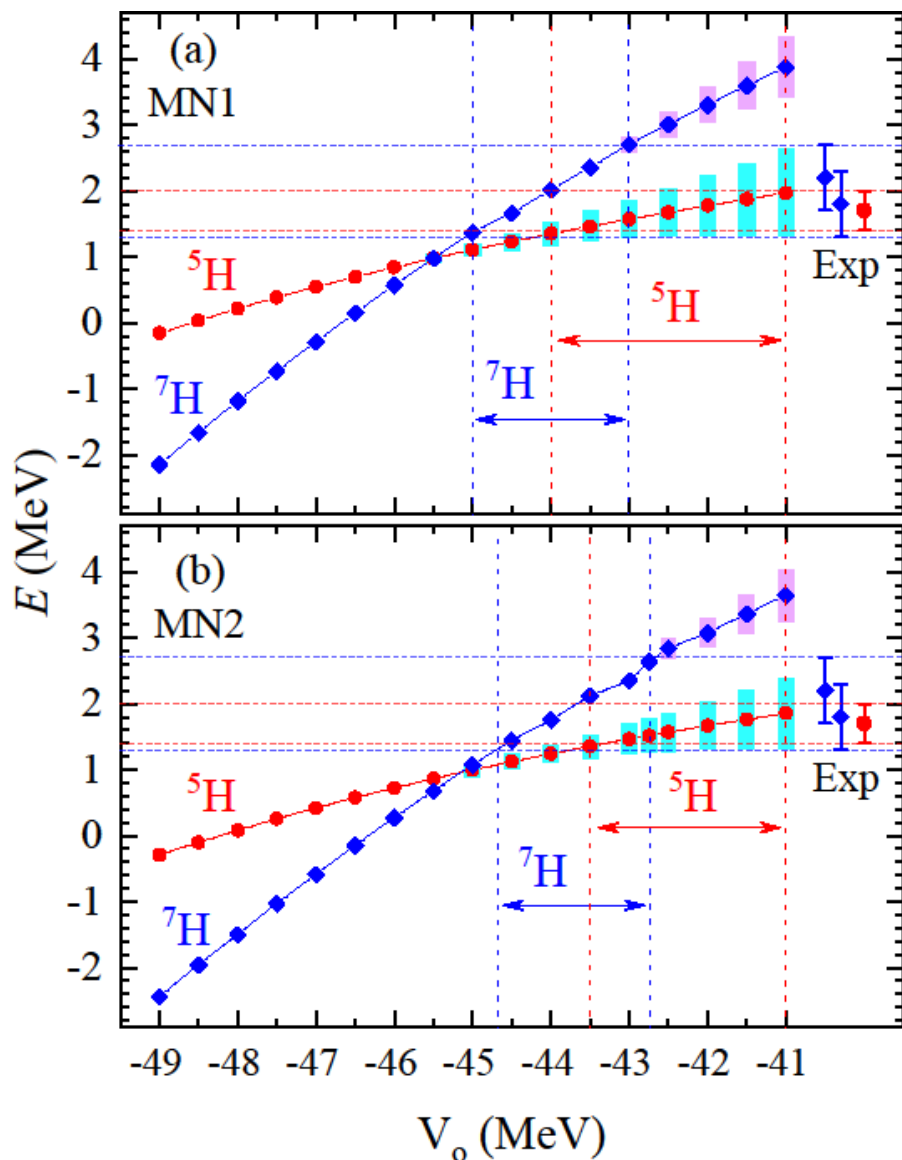
GSM with a core of ${}^3\text{H}$ (ab-initio GSM not applicable : model spaces too large)

FHT and Minnesota (MN1, MN2) interactions in $spdf/spd$ space with the Berggren basis

Two-body interactions obtained from a fit of the He chain

Large widths for ${}^4,6\text{H}$, smaller widths for ${}^5,7\text{H}$

Unbound hydrogen isotopes with GSM (2/2)



Properties of odd unbound hydrogen isotopes

Experimental data with large errors
 ${}^5, {}^7\text{H}$ widths estimated from energy plus/minus error
 Similar results obtained with FHT, Minnesota

${}^5\text{H}$: ~ 500 keV width, moderate resonance
 ${}^7\text{H}$: 10-250 keV width, narrow resonance

To be checked in future experiments

Application of GSM and DMRG to the tetraneutron (1/3)

Tetraneutron

Long standing question about its existence

Many-body effect of nuclear interaction

Pauli principle

Coupling to the neutron continuum

Most advanced many-body techniques used

Inter-nucleon correlations and continuum

No-core Gamow shell model with N^3LO

GSM with Jacobi-Davidson method

GSM with DMRG

Berggren basis and natural orbitals

Very difficult calculations:

GSM-Davidson with Berggren 2p-2h incomplete

Natural orbitals with 3p-3h quickly imprecise

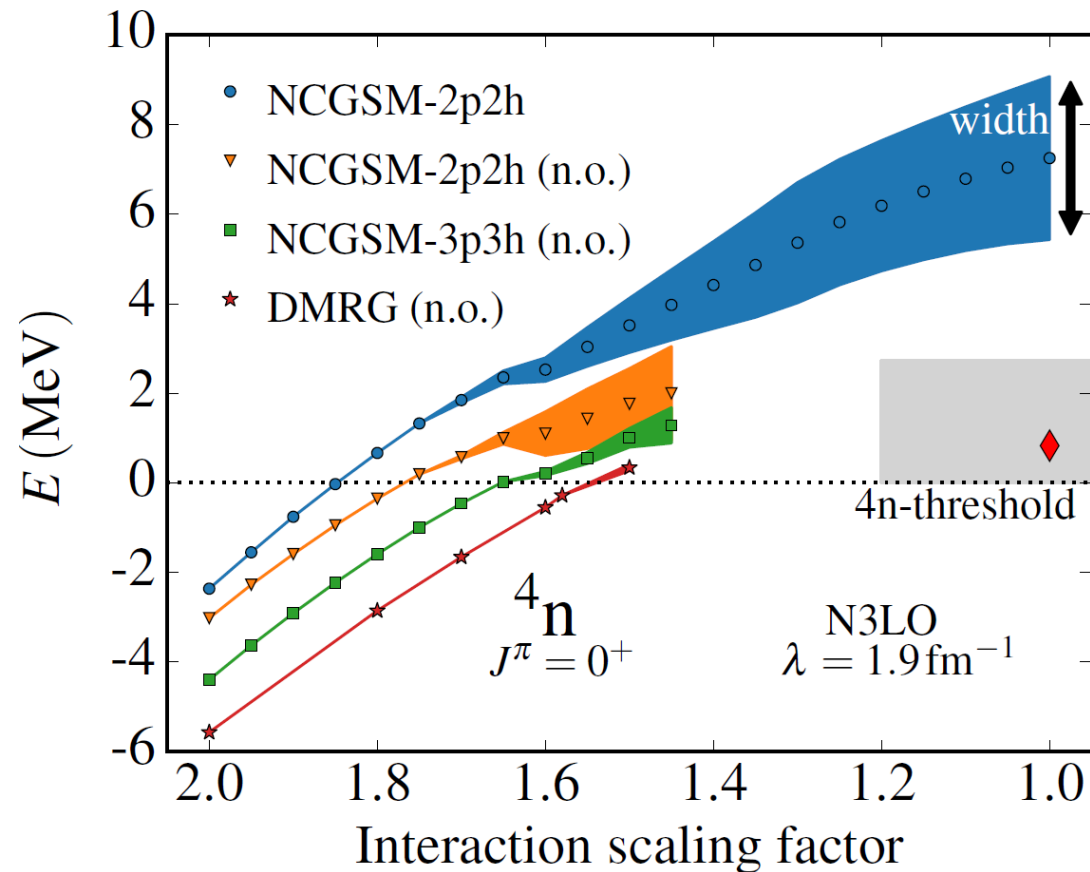
DMRG quickly unstable in the unbound region

DMRG extrapolates to the experimental region

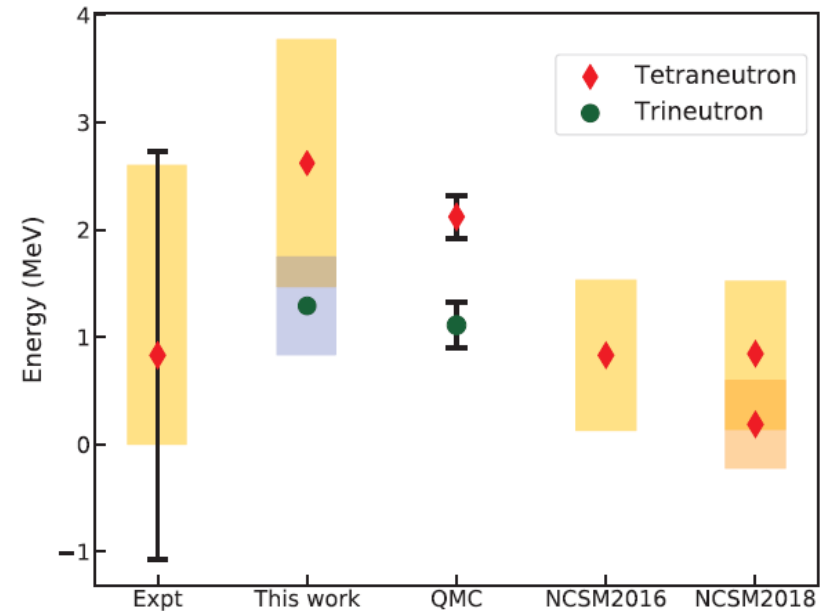
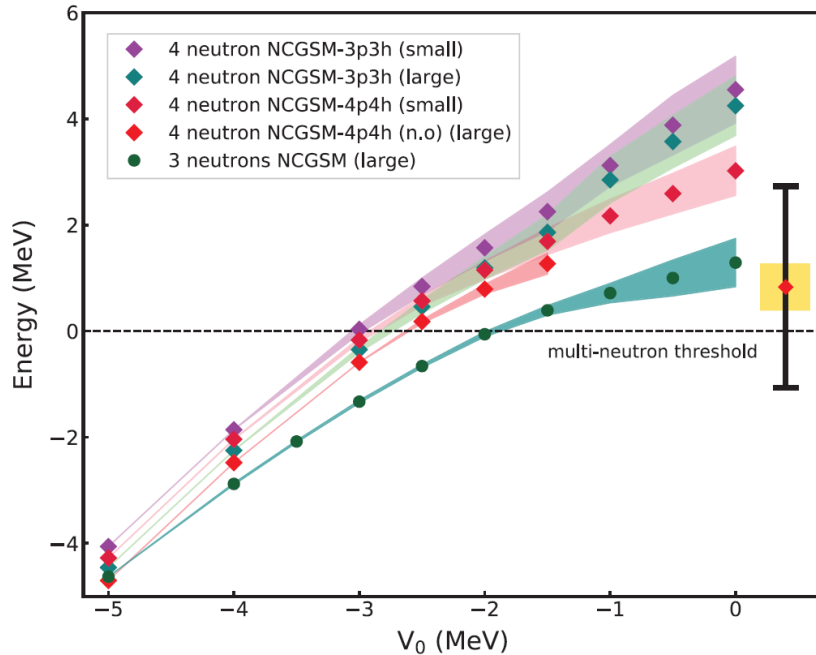
Important information gained

Bound tetraneutron very unlikely to exist

Width estimated to be of several MeV's



Application of GSM and DMRG to the tetraneutron (2/3)



Full space results estimated from 3p-3h and 4p-4h subspaces calculations

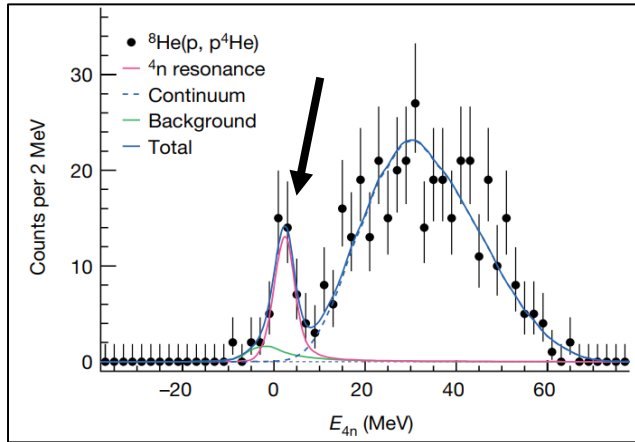
3n: $E \sim 1.29$ MeV , $\Gamma \sim 0.91$ MeV

4n: $E \sim 2.65$ MeV , $\Gamma \sim 2.38$ MeV

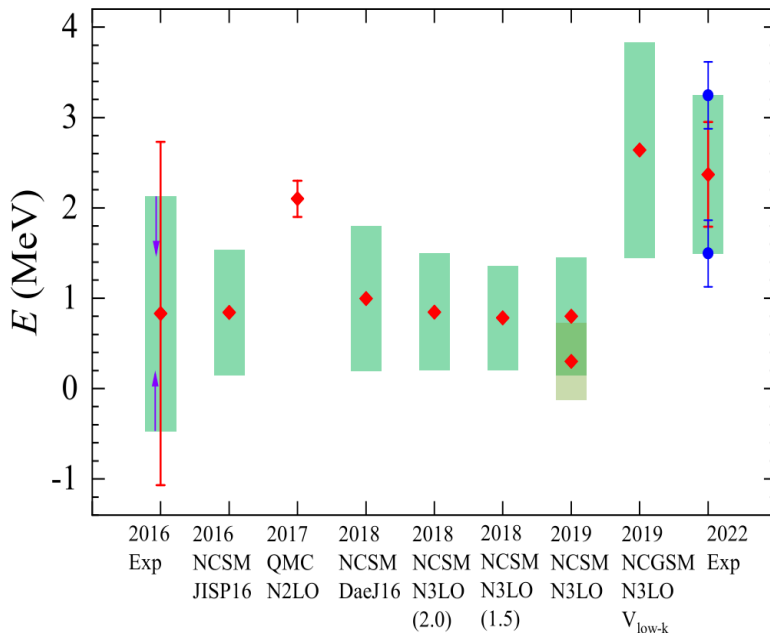
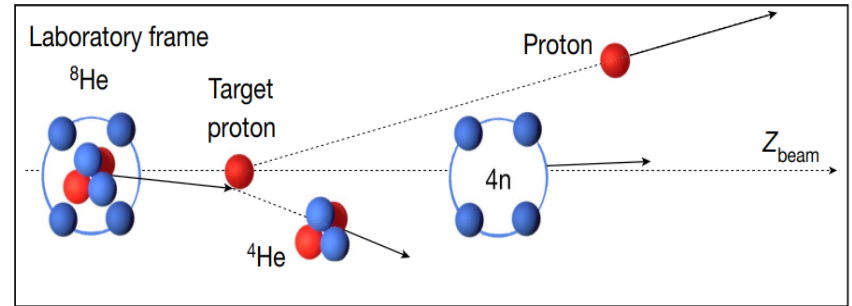
4n results close to recent experimental data (M.Duer et al., Nature **606**, 678–682 (2022))

Trineutron width smaller than that of tetraneutron: probably easier to measure than tetraneutron

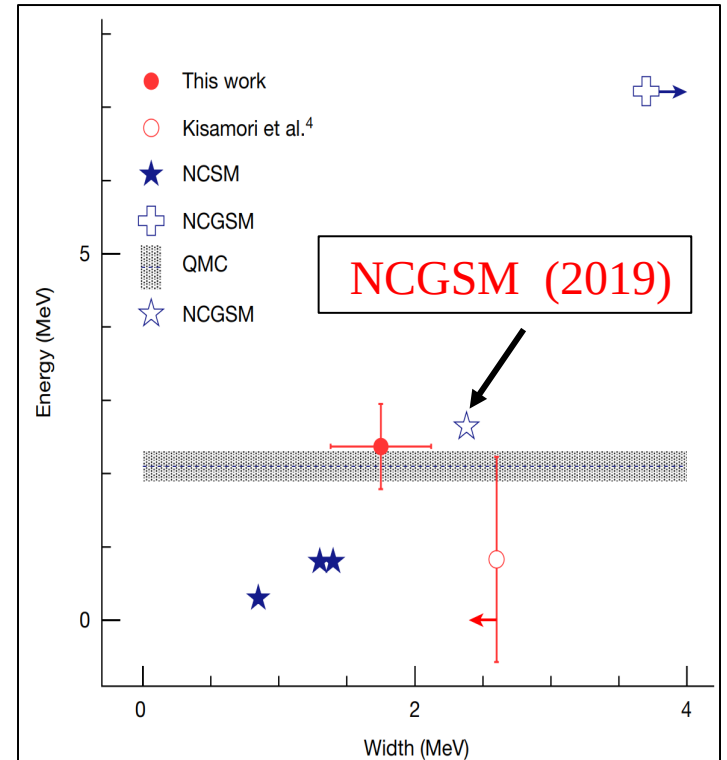
Application of GSM and DMRG to the tetraneutron (3/3)



M.Duer et al., Nature 606, 678–682 (2022)

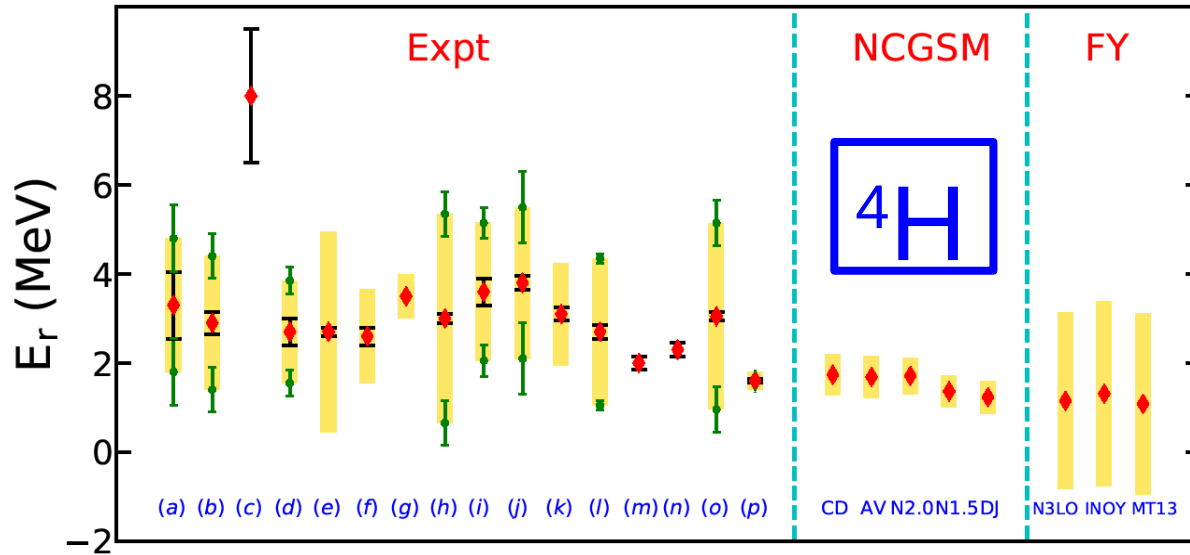


J.G. Li, N. Michel, B.S. Hu, W. Zuo, F.R. Xu, Phys. Rev. C **100**, 054313 (2019)



Nuclear physics at the edge of stability, Trento, July 4th 2022

A=4 T=1 resonances with ab-initio GSM

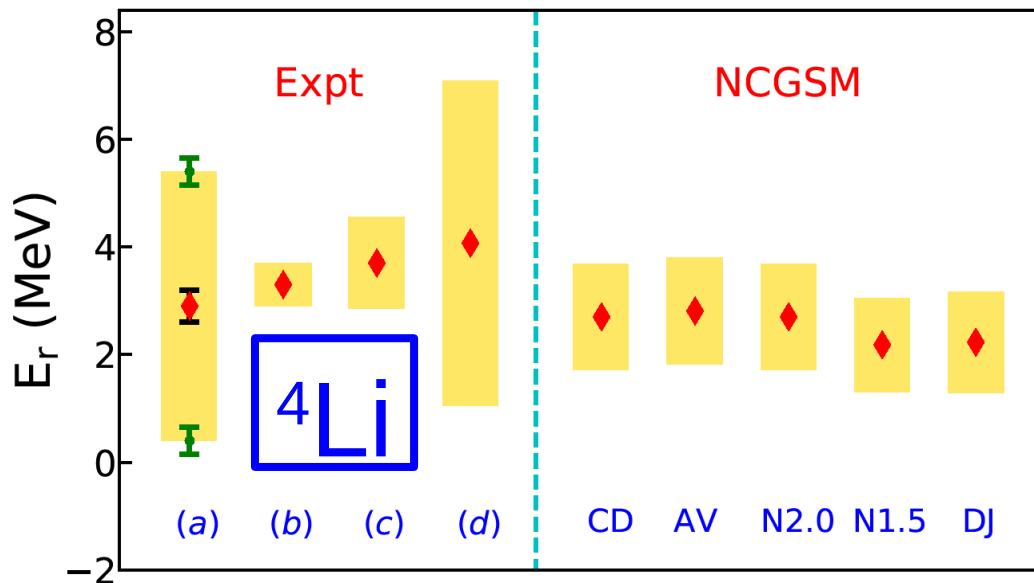


A=4 nuclei

T=1 ground/excited states
Broad resonances

Not well understood
Contradicting experimental data

No core Gamow shell model
Different realistic interactions used

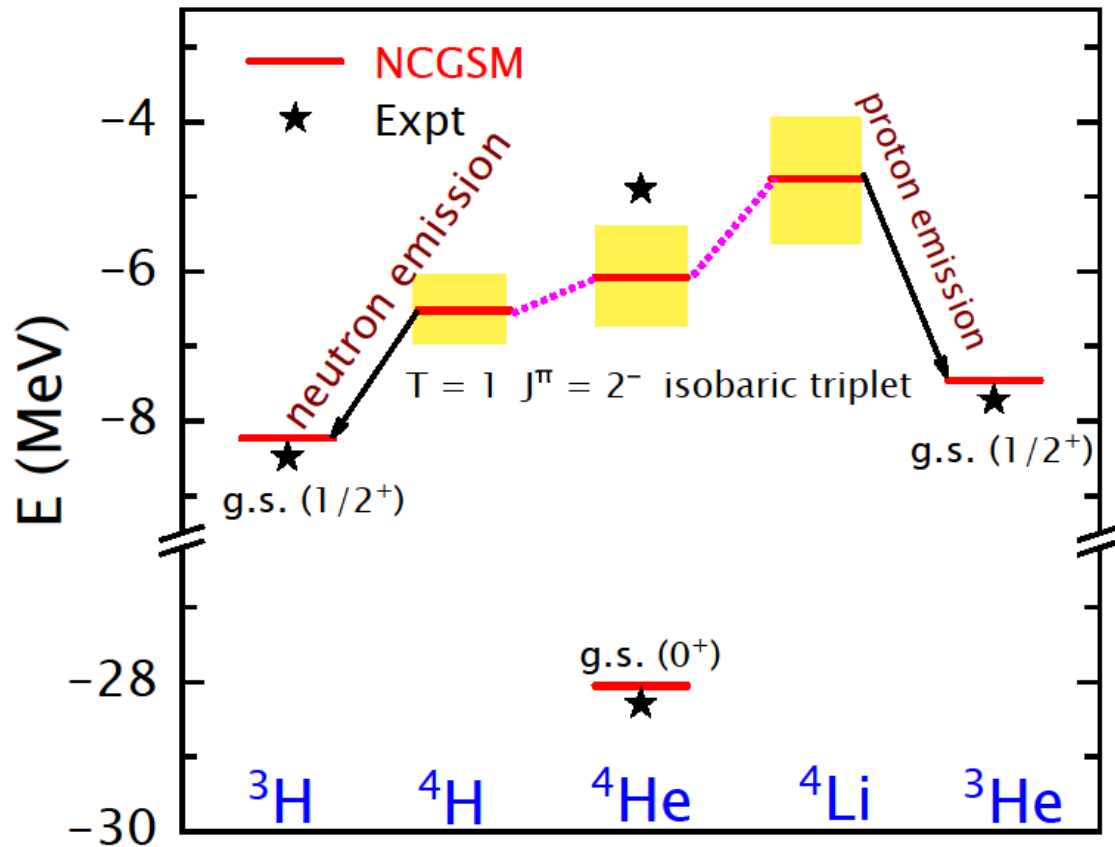


Results

GSM energies close to experimental data

Small GSM widths obtained : 1-2 MeV
Compatible with some experimental data

Isospin symmetry breaking in $A=4$ $T=1$ resonances with ab-initio GSM



Coulomb interaction

No Coulomb force in ^4H
 Coulomb force moderate in ^4He
 Coulomb force largest in ^4Li

Width increases from ^4H to ^4Li
 due to Coulomb force

Isospin symmetry breaking

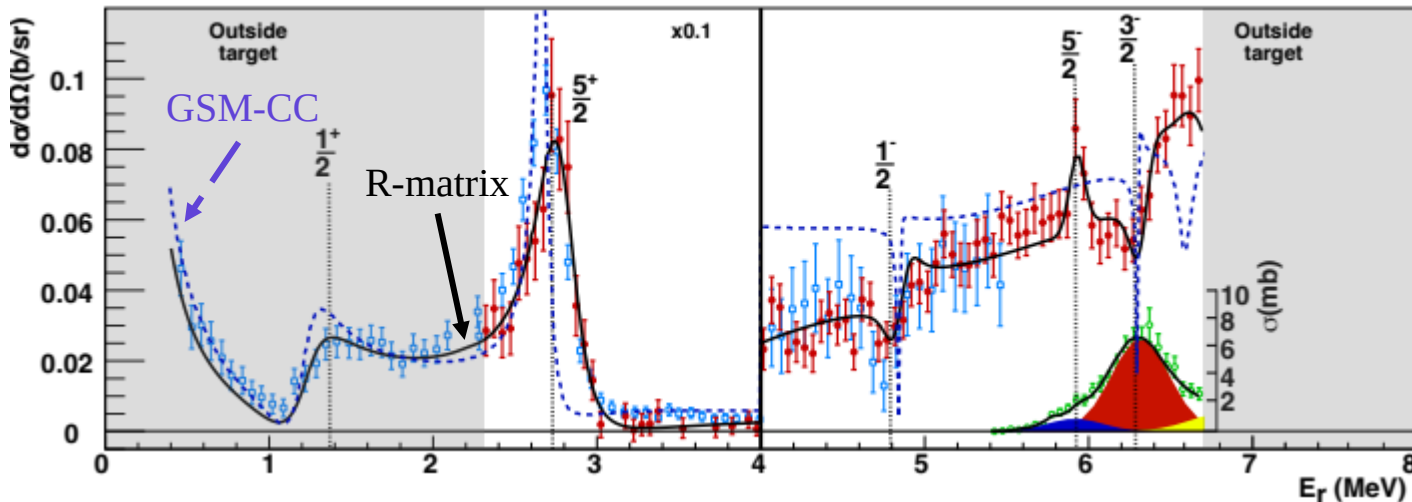
$T=1$ 2- many body states
 Isospin multiplet in $A=4$ states

Unique situation
 Broad resonances in $T=1$ multiplet

$T = 1$ in ^4H and ^4Li
 $T \sim 0.71$ in ^4He

Isospin symmetry strongly broken

New narrow resonances in ^{15}F



$^{12}\text{C} + 3\text{p}$		$^{13}\text{N} + 2\text{p}$		$^{14}\text{O} + \text{p}$	
$1/2^-$	6.514 (101)	$3/2^-$	6.33 (28)	2^+	6.790 (?)
$3/2^-$	6.225 (12)	$5/2^-$	5.93 (3)	3^+	6.590 (60)
$5/2^-$	6.093 (9)			0^-	6.272 (103)
	6.323	$1/2^-$	4.88 (30)	0^-	5.920 (50)
$1/2^-$	4.77 (18)		4.628	1^-	5.710 (400)
	4.400				5.173 (38)
		$5/2^+$	2.81 (251)		
		$5/2^+$	2.601 (94)		
		$1/2^+$	1.270 (500)		
		$1/2^+$	1.146 (250)		
				0^+	0 (0)

GSMCC ^{15}F exp

Narrow resonances and cross sections
 $1/2^-, 3/2^-, 5/2^-$ states found at high energy
 Small width : $\sim 15\text{-}30$ keV

GSM-CC : GSM + coupled channels
 Spectrum + cross sections with same Hamiltonian
 Fitted spectrum only: predictive model

^{14}O core + valence protons
 Hamiltonian : WS + FHT interaction
 Collective structure due to continuum coupling
 1p emission suppressed: small phase space

Book on Gamow shell model

The Gamow Shell Model: the unified theory of nuclear structure and reactions

Authors : N. Michel and M. Płoszajczak

Publisher : Lectures Notes in Physics (Springer)

Background

Functional analysis, linear algebra, differential equations, standard quantum mechanics

Main topics

Introduction with one-body and two-body systems

Many-body theory of complex-energy physics

Halos and resonances in molecules and nuclei

Examples of applications in nuclear structure and reactions

Exercises and codes

Theoretical details about used methods

Computational applications using codes available from internet

Conclusion

Current status

GSM: structure model including the continuum
Effective interactions with core + valence nucleons
Realistic interactions in ab-initio no-core Gamow shell model
GSM-CC: reaction model including nuclear structure

Energetics of the lightest nuclei
Emission channels separated by varying Hamiltonian parameters (p, 2p)
Nice agreement with experimental data
Fine tuning of Hamiltonian necessary

Book on GSM and GSM-CC published
Theory, exercises, GSM codes publicly available

Perspectives

Isotopic chains with effective and realistic interactions
Reactions cross sections using targets calculated in GSM
Many-body projectiles in direct, transfer and radiative capture reactions in GSM-CC