Inferring the dense matter equation of state

from a minimal number of constraints

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IN NUCLEAR PHYSICS AND RELATED AREAS

The dense matter equation of state (EOS)

- Recently, several EOS metamodels constrained by *ab-initio* theoretical calculations for both low and high density have been proposed:
  1) Nucleon-nucleon chiral potentials for the low density neutron and nuclear matte
     Hebeler et all (arxiv 1303.4662)
     Drischler et al arxiv: 1510.06728
  2) Perturbative Quantum Chromodynamics for asymptotically high-density regimes
     Kurkela et al arxiv 0912.1856

In order to account for all possible EOS compatible with these two constraints, the EOS at the two extreme densities are connected using a piecewise polytropic interpolation, a speed-of-sound interpolation or a spectral interpolation, and causality is imposed when necessary.

L. Lindblom et al, Phys. Rev. D 86, 084003 (2012), arXiv:1207.3744.
A. Kurkela et al, Astrophys. J. 789, 127 (2014), arXiv:1402.6618.
E. R. Most et al, Phys. Rev. Lett. 120, 261103 (2018), arXiv:1803.00549.
E. Lope Oter et al, J. Phys. G 46, 084001 (2019), arXiv:1901.05271.
E. Annala et al, Nature Phys. 16, 907 (2020), arXiv:1903.09121.
E. Annala et al, arXiv:2105.05132
Rahul Somasundaram et al, arXiv : 2112.08157
Sinan Altiparmak et al, arXiv: 2203.14974

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Motivation

We will look deep into one of the unsolved problem and try to find the answer to the following question.

- How can we determine the chemical constitution of matter from the NS EOS?
- Can we determine the proton fraction at a given density?

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Objective

- One of the main objective of the study is the determination of the domain of nucleonic NS matter EOS based on a relativistic approach with minimal constraints.

- To analyze the behavior of the density dependence of the symmetry energy, the high-density behavior of the EOS, and the upper and lower limits for several NS properties.

- To understand which are the limitations of the acceptable range of values for higher-order nuclear matter parameters.

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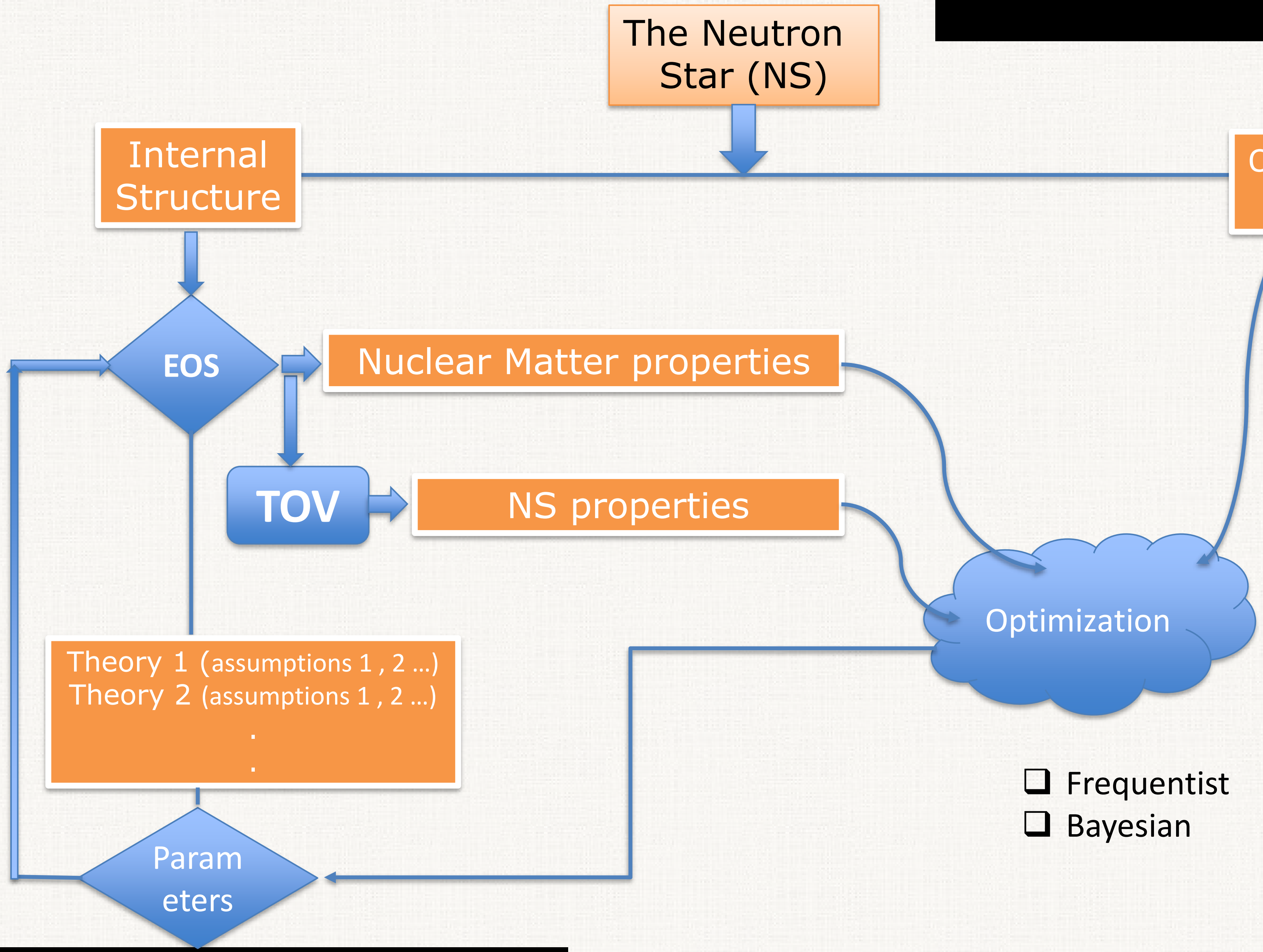

- ❑ Frequentist
- ❑ Bayesian

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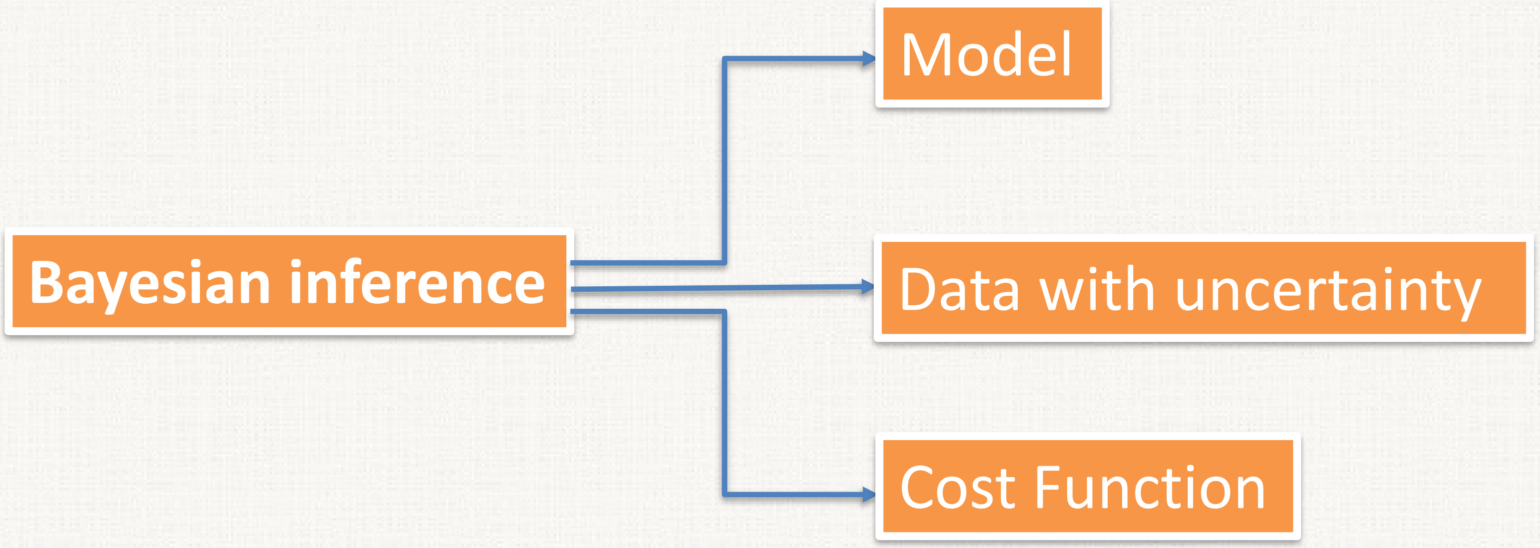

- G. Ashton et al., Astrophys. J. Suppl. 241, 27 (2019). arxiv 1811.02042
- A. Gelman, J. B. Carlin, H. S. Stern, D. B. Dunson, A. Vehtari, D. B. Rubin, J. Carlin, H. Stern, D. Rubin, and D. Dunson, Bayesian Data Analysis Third edition, November (CRC Press, 2013)

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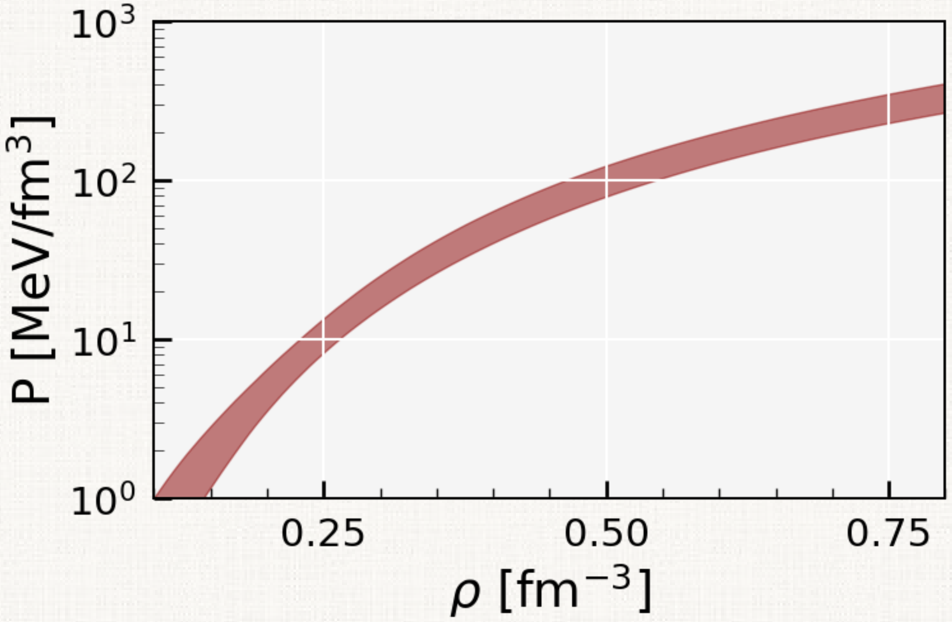
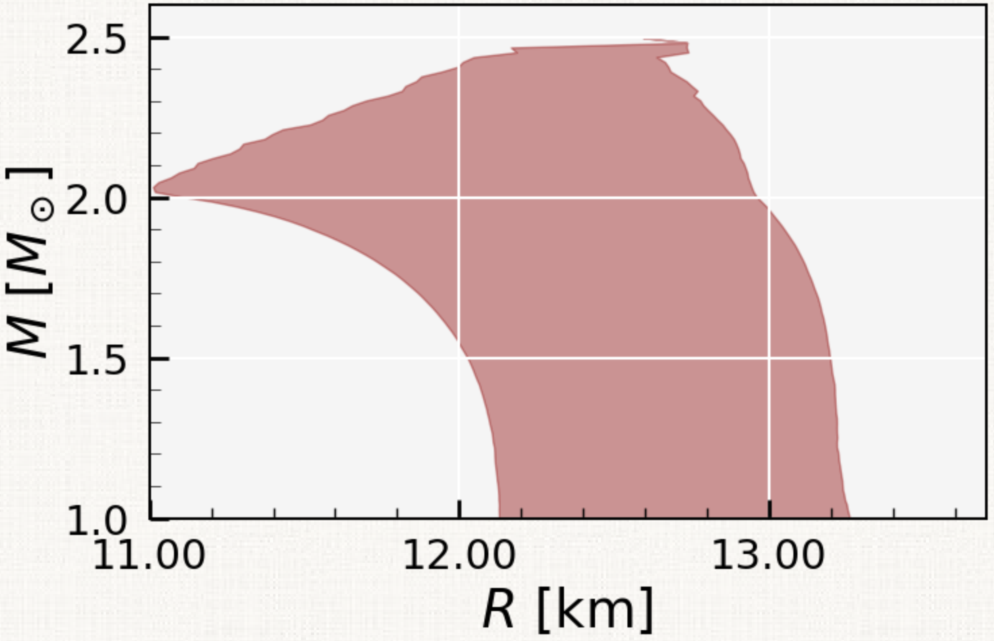
One-to-one correspondence





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The Nuclear Matter Properties (NMP)

$$\epsilon(\rho,\delta) \simeq \epsilon(\rho,0) + S(\rho)\delta^2$$

$$X_0^{(n)} = 3^n \rho_0^n \left(\frac{\partial^n \epsilon(\rho,0)}{\partial \rho^n}\right)_{\rho_0}, \ n = 2, 3, 4;$$

$$X_{\mathrm{sym},0}^{(n)} = 3^n \rho_0^n \left(\frac{\partial^n S(\rho)}{\partial \rho^n}\right)_{\rho_0}, \ n = 1, 2, 3, 4.$$

- Imam et al, Phys.Rev.C 105 (2022) 1, 015806 • e-Print: 2110.15776
- Márcio Ferreira et al, Phys.Rev.D 101 (2020) 4, 043021 • e-Print: 1912.11131

K_0	$J_{\mathrm{sym},0}$
Q_0	$L_{\mathrm{sym},0}$
Z_0	$K_{\mathrm{sym},0}$
	$Q_{\mathrm{sym},0}$
	$Z_{\mathrm{sym},0}$

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Table 1. The list of data/constraints considered in the Bayesian estimation of the model parameters which generate the DDB set. The ϵ_0 is the binding energy per nucleon, K_0 the incompressibility coefficient and $J_{\mathrm{sym},0}$ the symmetry energy evaluated at the nuclear saturation density ρ_0. The nuclear saturation properties are listed including an 1σ uncertainty. The PNM indicates the pressure of pure neutron matter for the densities 0.08, 0.12 and 0.16 fm^{-3} from N^{3}LO calculation in χEFT Hebeler et al. (2013). We consider $2 \times$ N^{3}LO data in the likelihood of the present calculation.

Constraints				
Quantity		Value/Band	Ref	DDB
NMP [MeV]	ρ_0	0.153 ± 0.005	Typel & Wolter (1999)	✓
	ϵ_0	-16.1 ± 0.2	Dutra et al. (2014)	✓
	K_0	230 ± 40	Shlomo, S. et al. (2006); Todd-Rutel & Piekarewicz (2005)	✓
	$J_{\mathrm{sym},0}$	32.5 ± 1.8	Essick et al. (2021a)	✓
PNM [MeV fm^{-3}]	$P(\rho)$	$2\times$ N^{3}LO	Hebeler et al. (2013)	✓
NS mass [$M_\odot$]	M_{max}	> 2.0	Fonseca et al. (2021)	✓

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Bayesian estimation of Model Parameters

$$P(\boldsymbol{\theta}|D) = \frac{\mathcal{L}(D|\boldsymbol{\theta})P(\boldsymbol{\theta})}{\mathcal{Z}},$$

$$P(\theta_i|D) = \int P(\boldsymbol{\theta}|D) \prod_{k \neq i} d\theta_k.$$

$$\mathcal{L}(D|\boldsymbol{\theta}) = \prod_j \frac{1}{\sqrt{2\pi\sigma_j^2}} e^{-\frac{1}{2}\left(\frac{d_j - m_j(\boldsymbol{\theta})}{\sigma_j}\right)^2}$$

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RMF

- Interactions between two nucleons → Exchange of virtual particles known as mesons.

$$\begin{aligned}\mathcal{L} =&\bar{\Psi}\Big[\gamma^\mu\left(i\partial_\mu - \Gamma_\omega A_\mu^{(\omega)} - \Gamma_\varrho \boldsymbol{\tau}\cdot\boldsymbol{A}_\mu^{(\varrho)}\right)\\ &- (m - \Gamma_\sigma \phi)\Big]\Psi + \frac{1}{2}\left\{\partial_\mu\phi\partial^\mu\phi - m_\sigma^2\phi^2\right\}\\ &- \frac{1}{4}F_{\mu\nu}^{(\omega)}F^{(\omega)\mu\nu} + \frac{1}{2}m_\omega^2 A_\mu^{(\omega)}A^{(\omega)\mu}\\ &- \frac{1}{4}\boldsymbol{F}_{\mu\nu}^{(\varrho)}\cdot\boldsymbol{F}^{(\varrho)\mu\nu} + \frac{1}{2}m_\varrho^2\boldsymbol{A}_\mu^{(\varrho)}\cdot\boldsymbol{A}^{(\varrho)\mu},\end{aligned}$$

$$h_\varrho(x) = \exp[-a_\varrho(x-1)]$$

$$h_M(x) = \exp[-(x^{a_M}-1)]$$

$$\Gamma_M(\rho) = \Gamma_{M,0}\, h_M(x)\ , \quad x = \rho/\rho_0$$

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Prior preparation

Table 2
The Considered Uniform Prior Distributions (P) of the DDH Model Parameters

No	Parameters	P Min	P Max
1	$\Gamma_{\sigma,0}$	7.5	13.5
2	$\Gamma_{\omega,0}$	8.5	14.5
3	$\Gamma_{\varrho,0}$	2.5	8.0
4	a_σ	0.0	0.30
5	a_ω	0.0	0.30
6	a_ϱ	0.0	1.30

Note. The parameters "min" and "max" denote the minimum and maximum values for the uniform distribution.

- The uniform prior has been taken with a reasonable boundary.
- We initially do a random sampling test with the Latin hypercube sampling (LHS) to get the overall idea about a reasonable boundary of the parameter space, i.e. the sub-domain for which we get a physical equation of state.
- The effective prior for NMP are reasonable wider.

**On Latin hypercube sampling, Wei-Liem Loh, Ann. Statist. 24(5): 2058-2080 (October 1996). DOI: 10.1214/aos/1069362310

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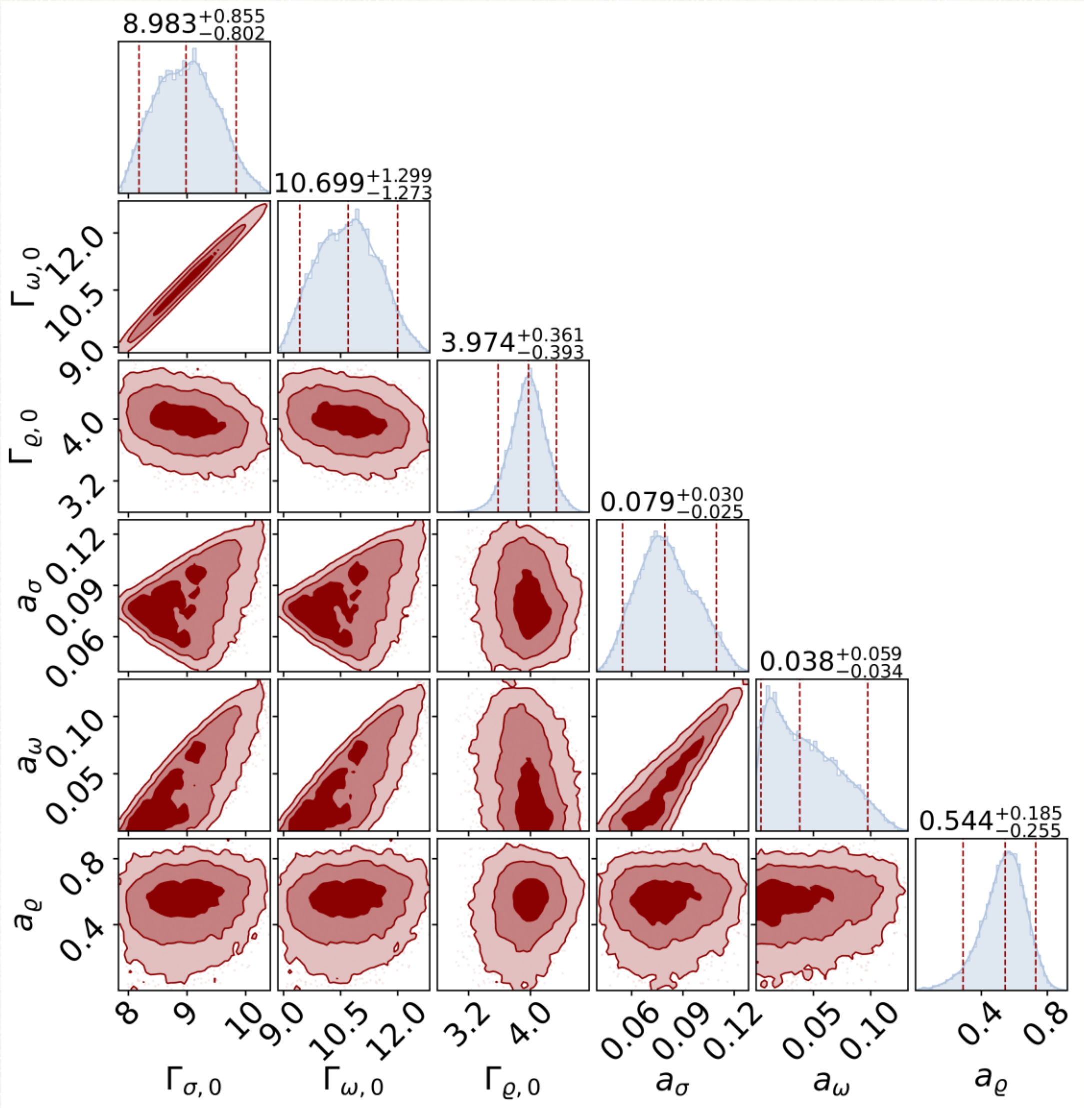
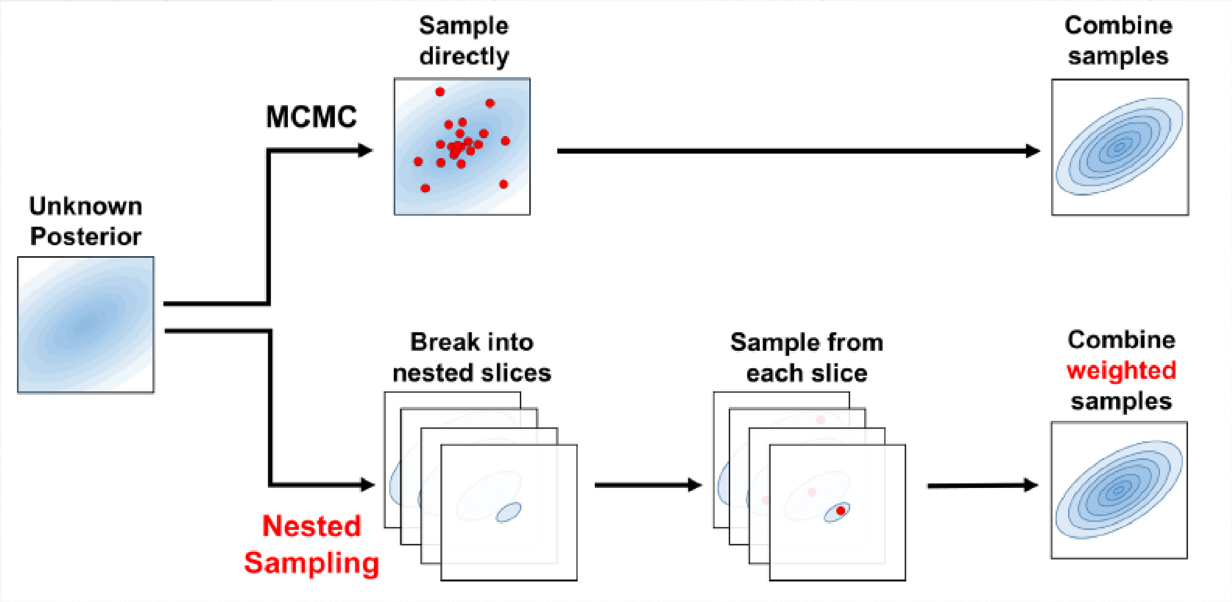
Marginalized posterior distributions obtained for the DDH parameters



Sampling



@ MNRAS 493, 3132–3158 (2020)

We generate samples for starting 3000 *"n-live"* points with both samplers, separately. The *Pymultinest* selects around 14000 final models by calling 5.78×10^5 models and the *Dynesty* selects around 13000 final models by calling 5×10^7 models. The evidence obtained in both samplers are similar. In the next section, we will present the results sets obtained in *Pymultinest*.

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Low density PNM EOS

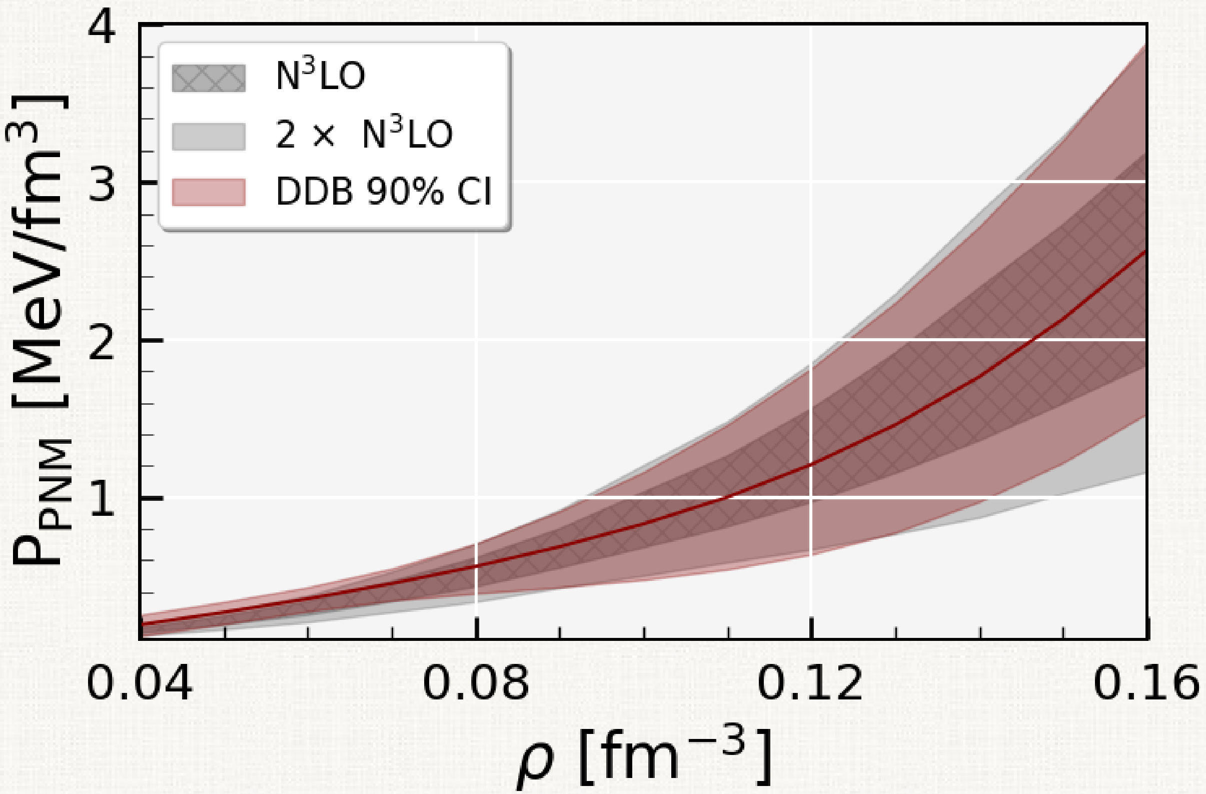
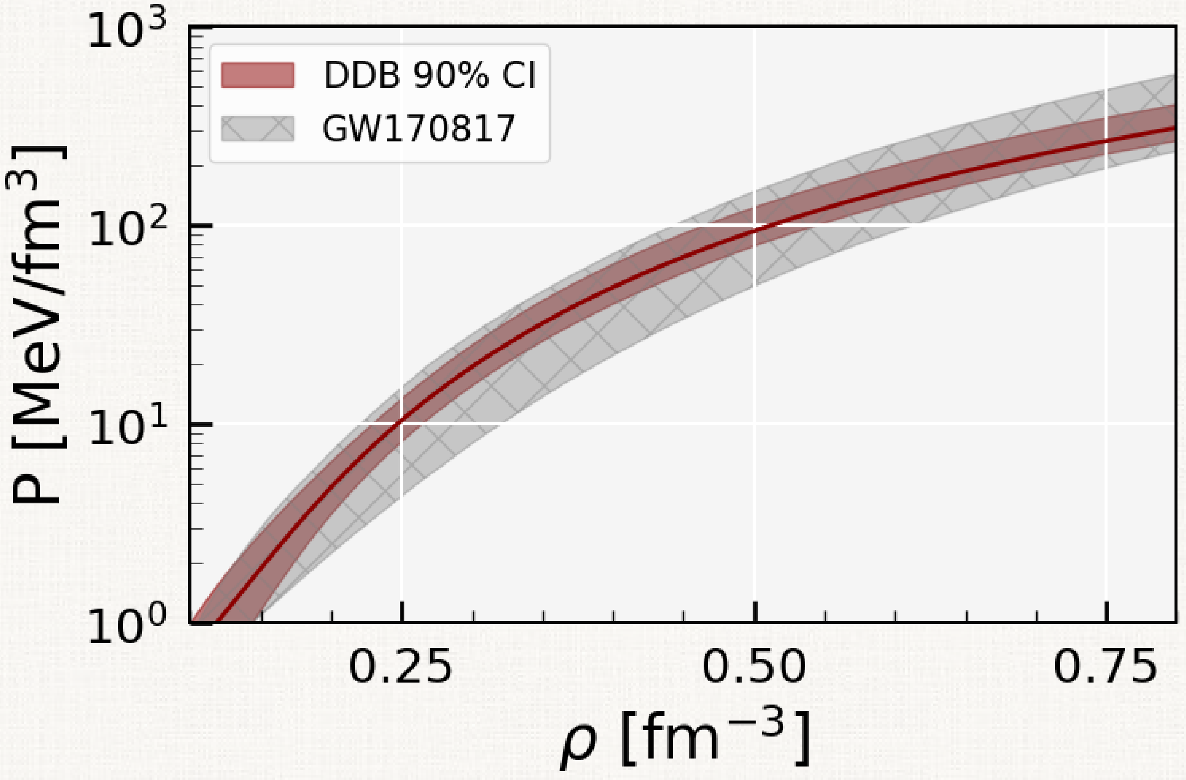
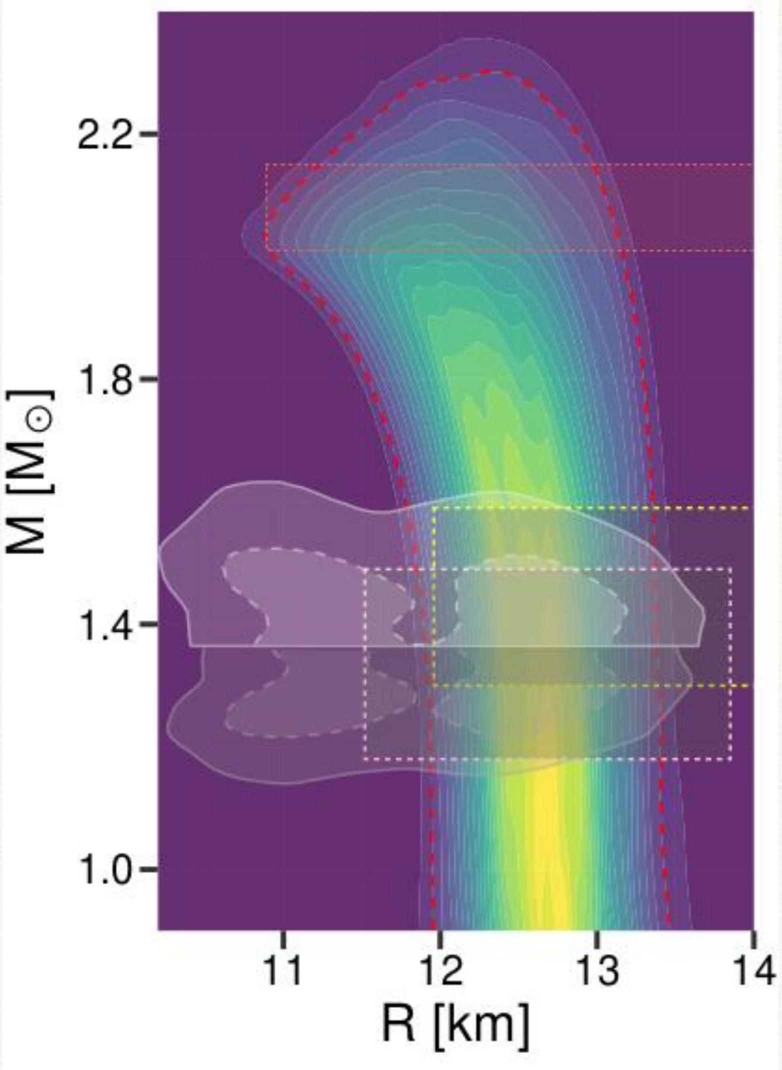


Figure 2. The pressure of low density neutron matter from a N^{3}LO calculation in χEFT Hebeler et al. (2013). The 90% CIs of the pressure of the low density neutron matter for DDB is also compared. It is to be noted that we consider 2× N^{3}LO uncertainty and three intermediate points in the likelihoods of the present calculation .

The NS matter EOS and joint probability for mass-radius





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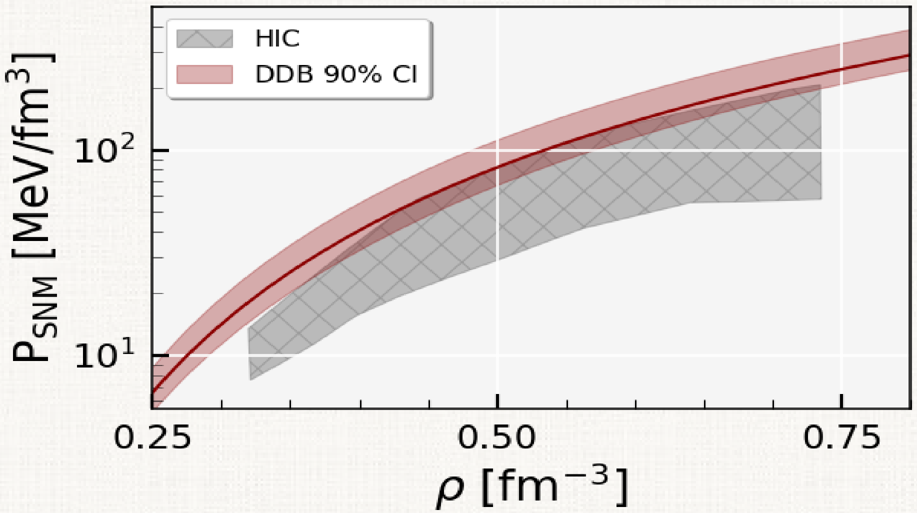
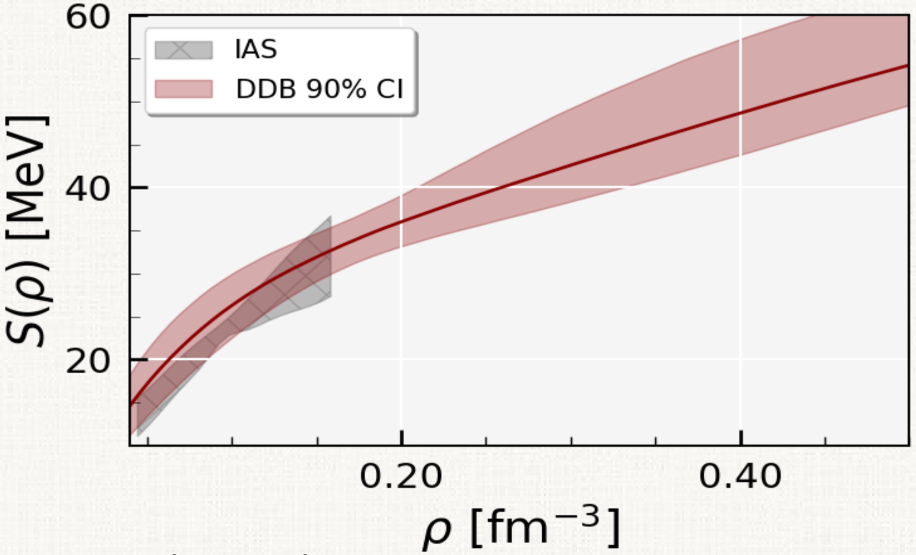
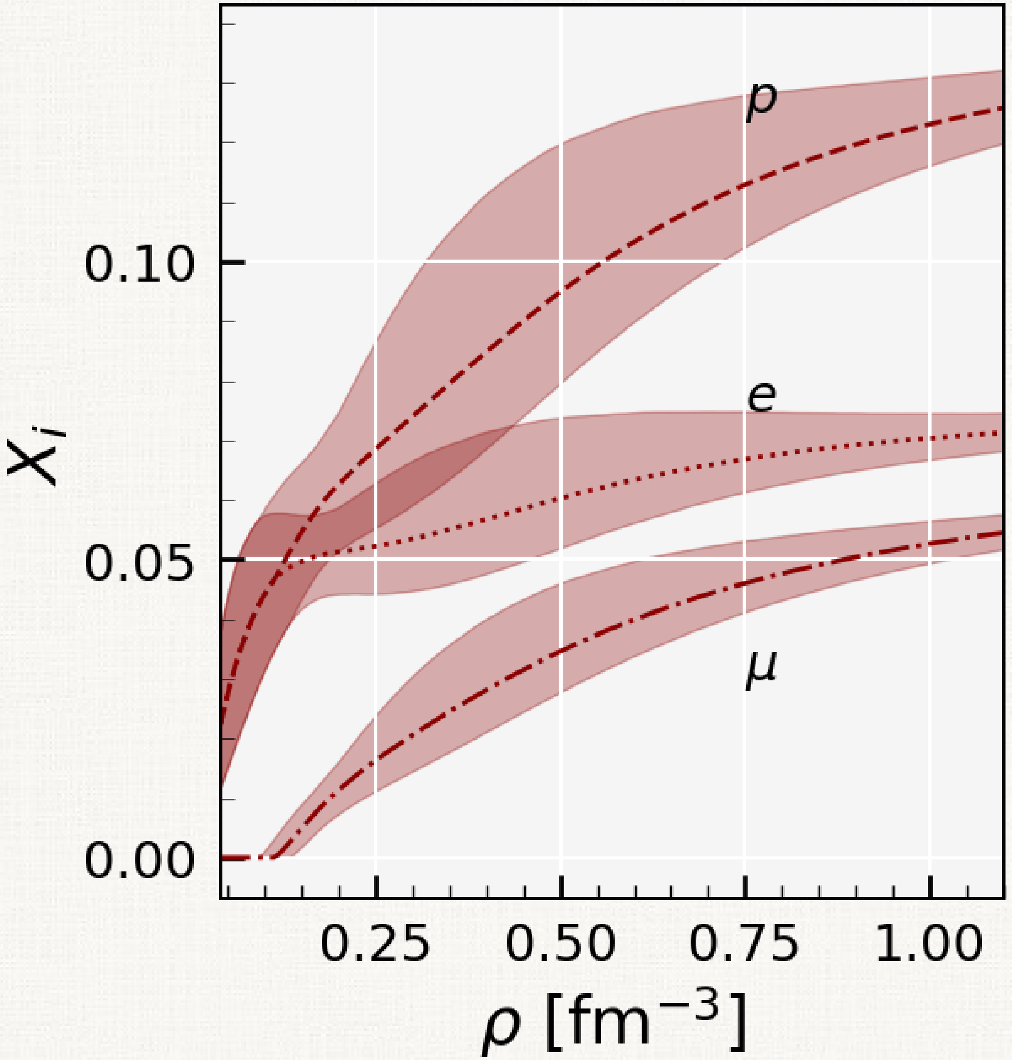
The uncertainty on NS matter EOS components



The symmetric nuclear matter EOS



The nuclear symmetry energy



The particle fraction

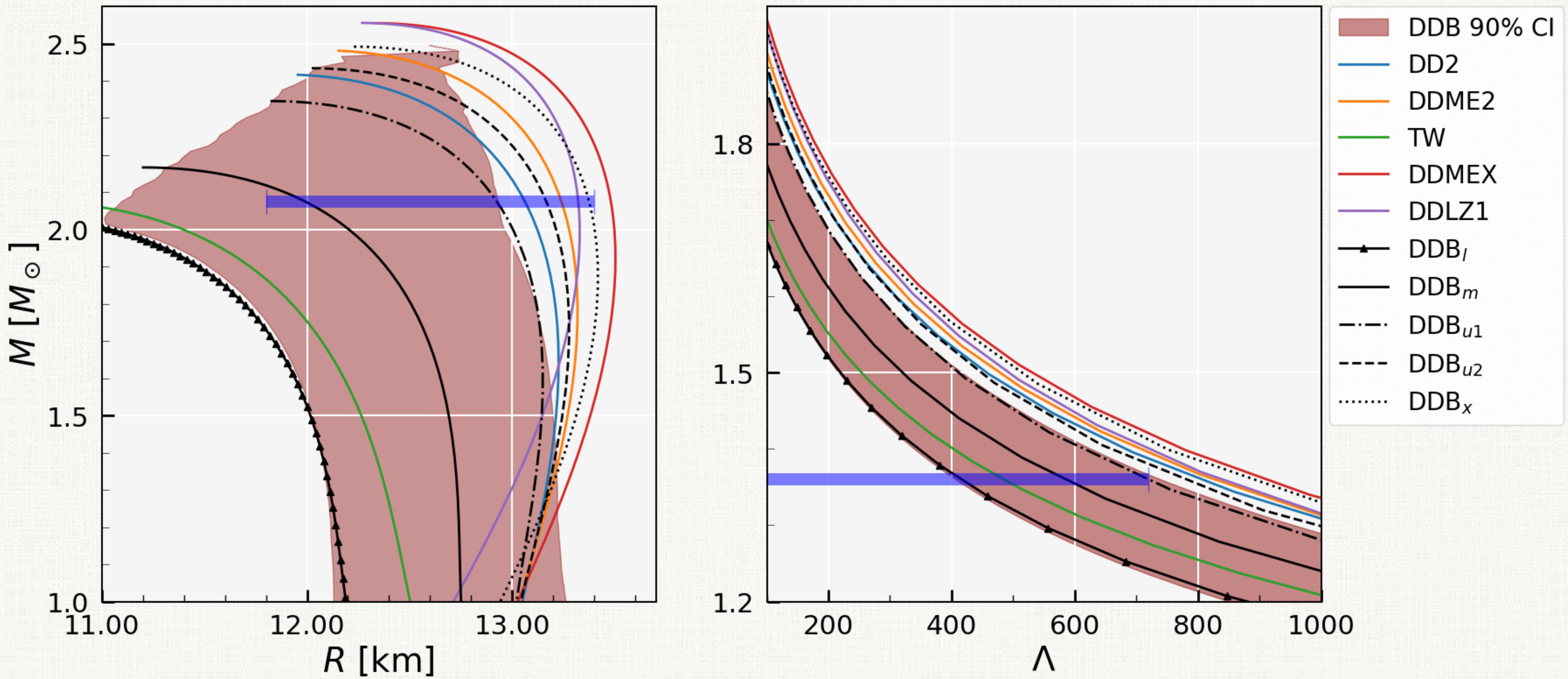


Figure 8. The 90% CI for the conditional probabilities $P(R|M)$ (left) and $P(\Lambda|M)$ (right) for DDB (dark red). For reference on the left panel, results for models TW Typel & Wolter (1999), DD2 Typel et al. (2010), DDME2 Lalazissis et al. (2005), DDMEX Taninah et al. (2020); Huang et al. (2020), DD-LZ1 Wei et al. (2020) and DDBl, DDBm, DDBu1, DDBu2 and DDBx are also shown. The blue horizontal bars indicate: on the left panel the 90% CI radius for a $2.08M_\odot$ star determined in Miller et al. (2021) combining observational data from GW170817 and NICER as well as nuclear data, on the right panel the 90% CI obtained for the tidal deformability of a $1.36M_\odot$ star in Abbott et al. (2018).

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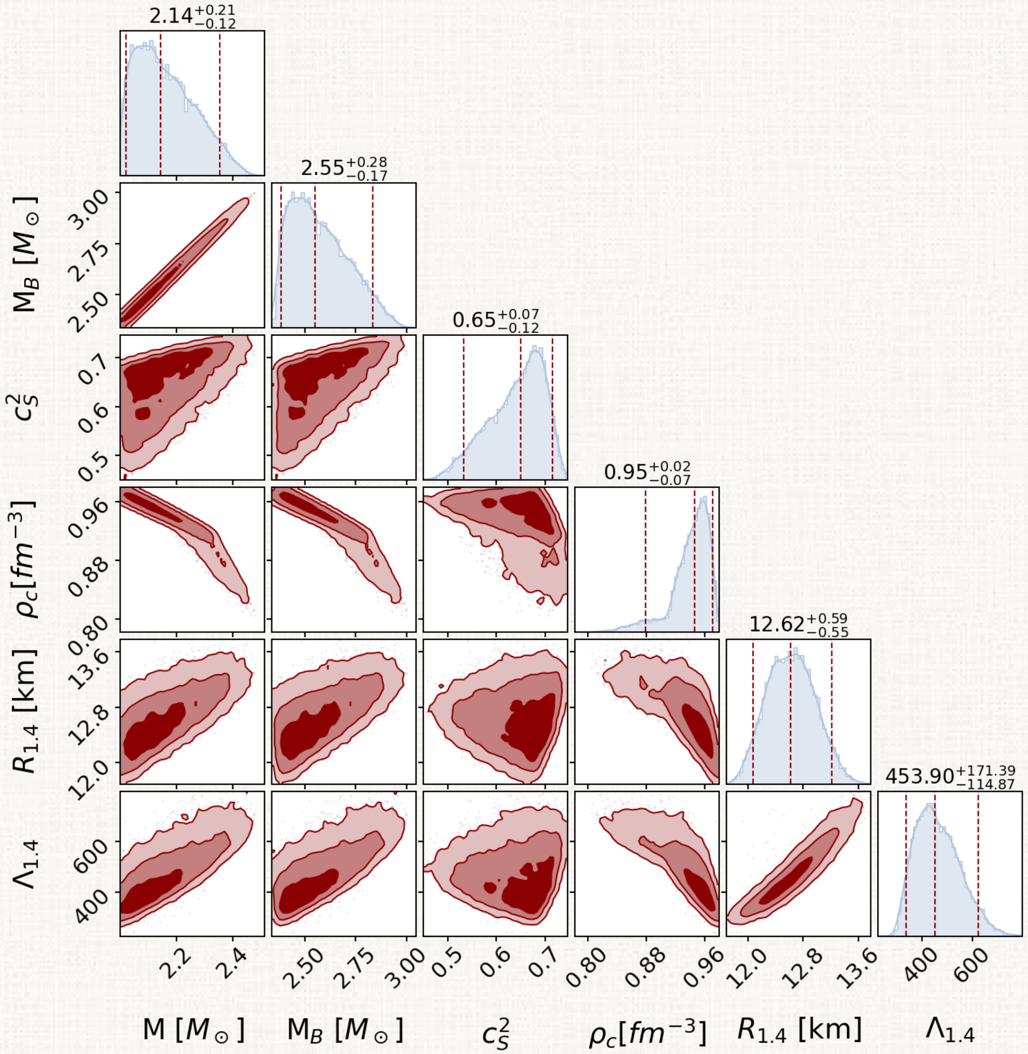
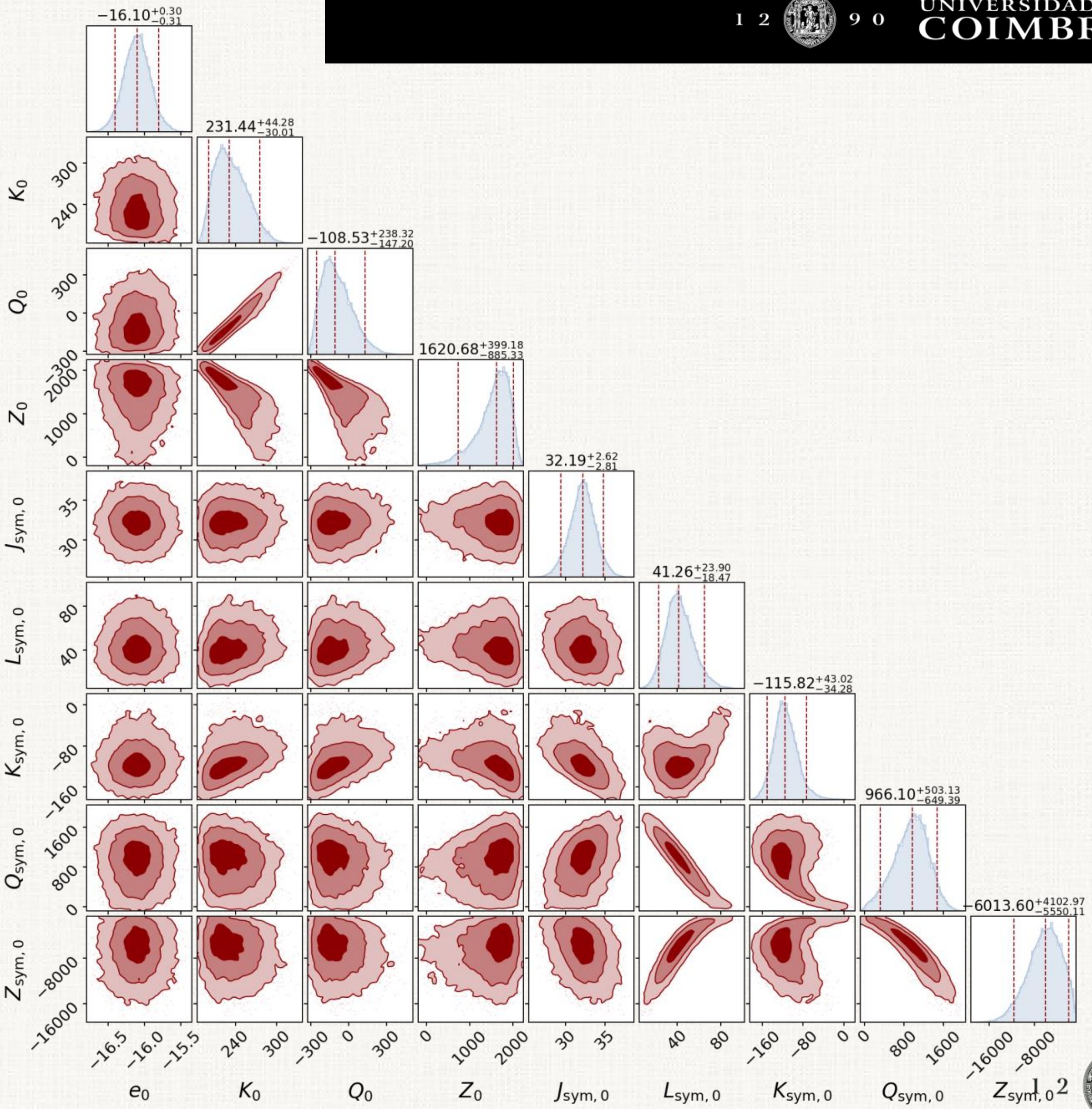
The NMP and NS properties distribution





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- Within a Bayesian approach, we have generated a set of models based on the RMF framework with density dependent coupling parameters and no non-linear mesonic terms.

- This set of EOS is compatible with recent observational and empirical constraints.

- The main objective of the study is the determination of the domain of nucleonic neutron star EOS based on a relativistic approach with minimal constraints.

- We have analysed the behavior of the density dependence of the symmetry energy, the high density behavior of the EOS and the upper and lower limits for several NS properties. We have verified that the posterior distribution of NS maximum mass, radii and tidal deformabilities are compatible with recent NS observables.

We also release our entire sets of ~14K NS matter EOS [Zenedo: 10.5281/zenodo.6342099].

Relativistic Description of Dense Matter Equation of State and Compatibility with Neutron Star Observables: A Bayesian Approach, Tuhin Malik, Márcio Ferreira, B.K. Agrawal, Constança Providência Astrophys.J. 930 (2022) 1, 17 • e-Print: 2201.12552

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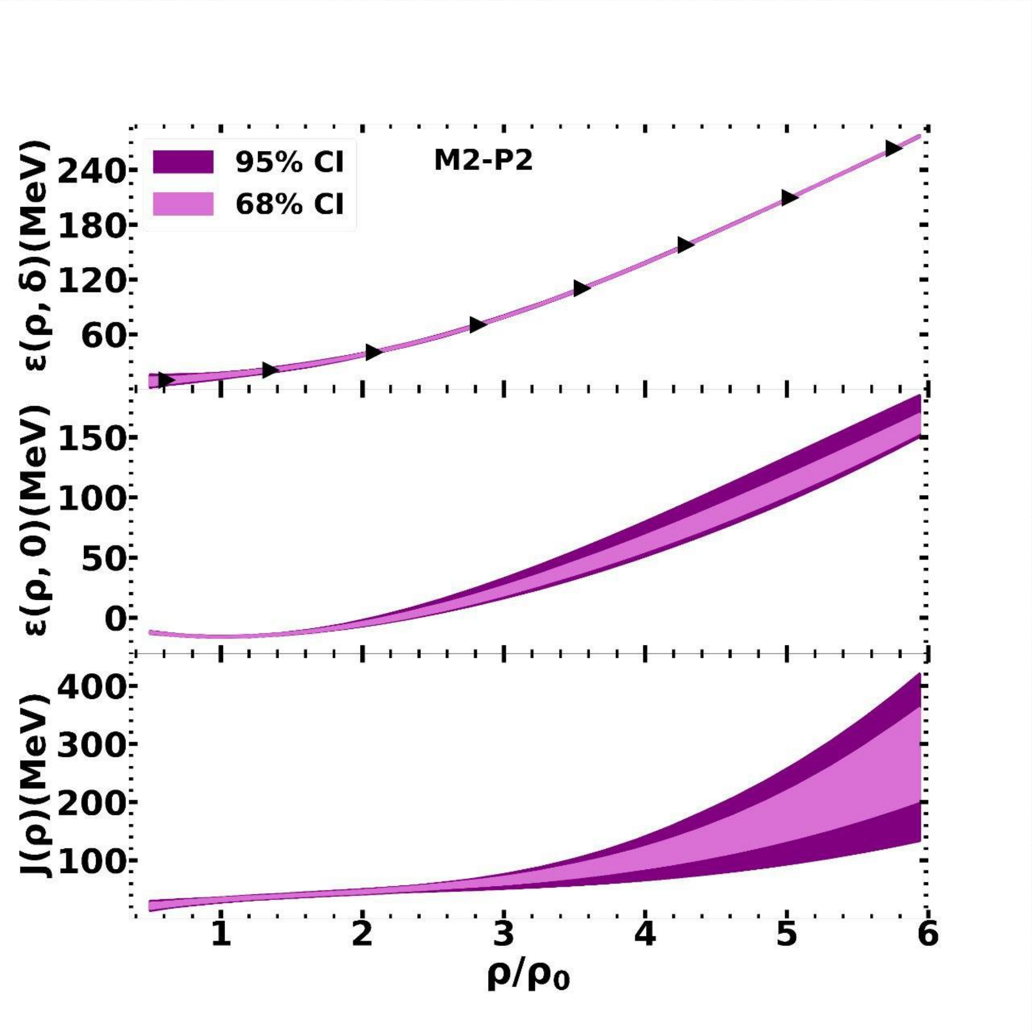
The Future challenges



Sk Md Adil Imam, N.K. Patra, C. Mondal, Tuhin Malik, B.K. Agrawal, Phys.Rev.C 105 (2022) 1, 015806 • e-Print: 2110.15776

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Need some other probe

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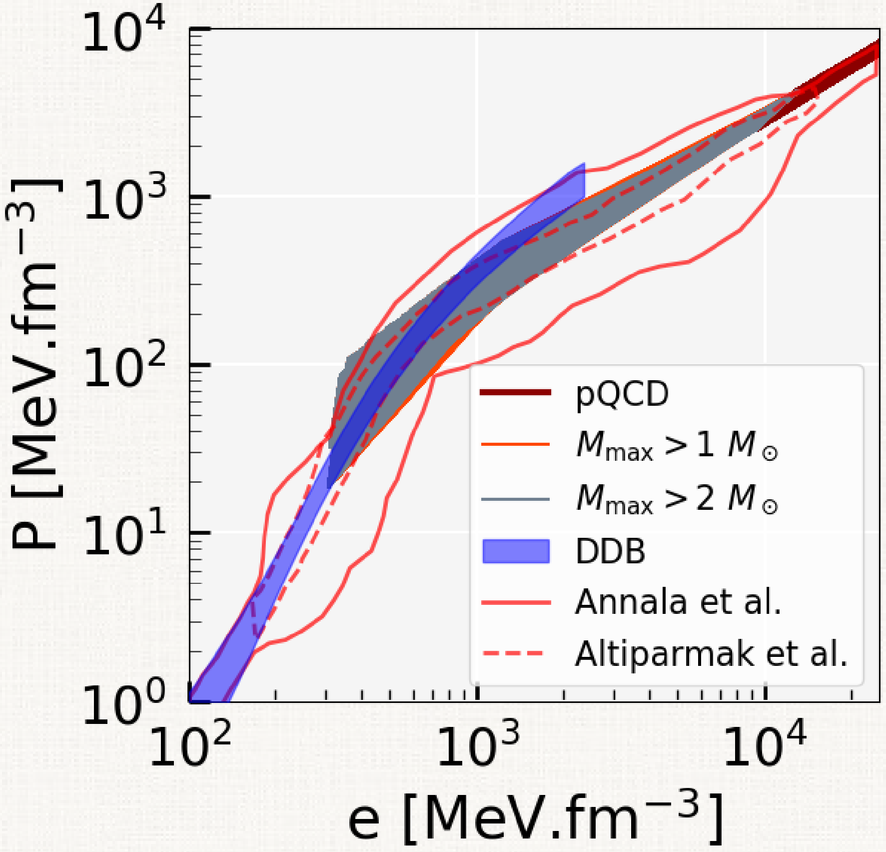
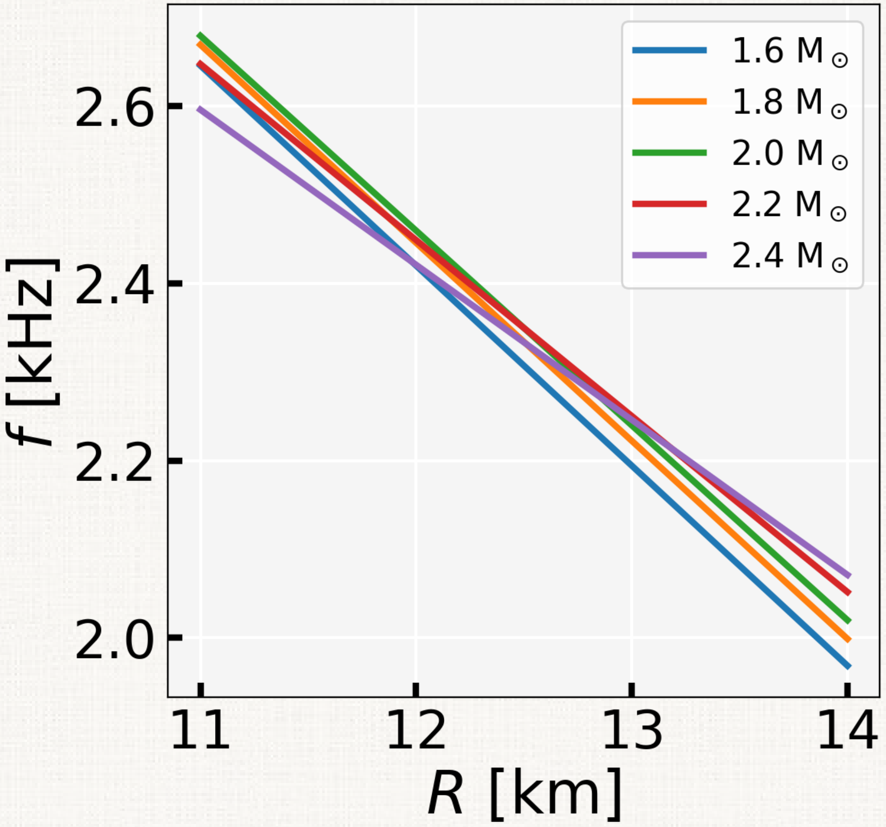
The future probe

- In the context of GWs, the non-radial oscillations of NS are particularly interesting as they can carry information of the internal composition of the stellar matter. These oscillations in the presence of perturbations (electromagnetic or gravitational) can emit the gravitational waves at the characteristic frequencies of its quasinormal modes (QNMs).
- The frequencies of QNM depend on the internal structure of the NS and it may be an another probe to get an insight regarding the composition of neutron star matter also known as asteroseismology.

Among QNM the important fluid modes related to GW emission include the fundamental (f) modes and the frequency can vary from 1-3 kHZ and expected to detect with 3G detector such as Einstein Telescope (ET).

The future probe





The f mode frequency and radii universal linear relations

Deepak Kumar, Tuhin Malik, Hiranmaya Mishra, and Constanc ̧a Provid ̂encia

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