Fast cooling in neutron stars

Basic tests to check your EOS

Melissa Mendes* [1]
Farrukh Fattoyev [2], Andrew Cumming [1], Charles Gale [1]

[1]McGill University, Montreal, CA

[2]Manhattan College, Riverdale, US

*melissa.mendessilva@mail.mcgill.ca

Neutron stars

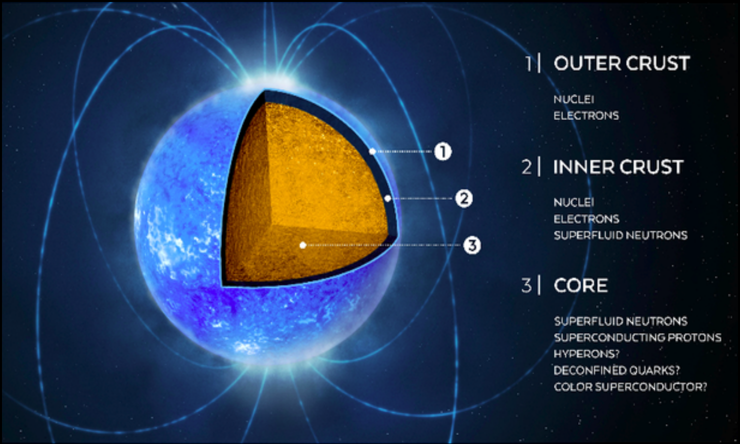


Figure: From Watts et al, 2016 (Rev. Mod. Phys) [ArXiv: 1602.01081]

Data constraints

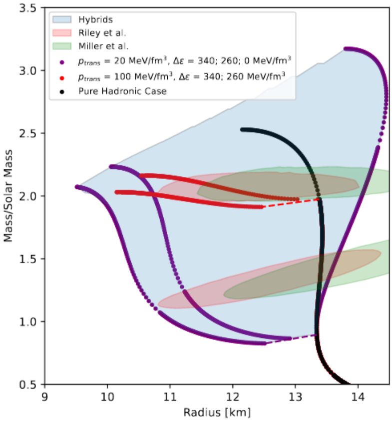


Figure: From Christian et al (Phys. Rev. D) [ArXiv: 2109.04191]

Probe structure, limited prediction power of microphysics

Cooling data to complement

Transiently-accreting sources

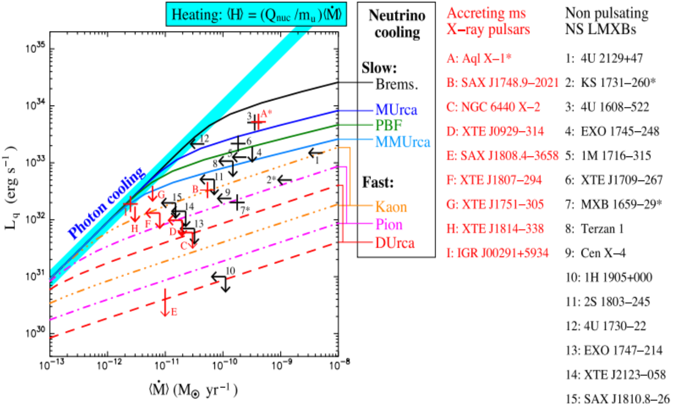


Figure: From Wijnands, Degenaar and Page, 2017 (J. Astrophys. Astr.) [ArXiv:1709.07034]

Fast cooling rates

Main neutrino processes in $npe\mu\Lambda\Sigma^-$ matter*)

(I) Baryon direct Urca		$Q \sim (10^{23}\text{–}10^{27})\, T_9^6$ erg cm^{-3} s^{-1}	
(1) $n \to pl\bar{\nu}_l$	$pl \to n\nu_l$	(2) $\Lambda \to pl\bar{\nu}_l$	$pl \to \Lambda\nu_l$
(3) $\Sigma \to nl\bar{\nu}_l$	$nl \to \Sigma\nu_l$	(4) $\Sigma \to \Lambda l\bar{\nu}_l$	$\Lambda l \to \Sigma\nu_l$
(II) Baryon modified Urca		$Q \sim (10^{18}\text{–}10^{21})\, T_9^8$ erg cm^{-3} s^{-1}	
(1) $nB \to pBl\bar{\nu}_l$	$pBl \to nB\nu_l$	(2) $\Lambda B \to pBl\bar{\nu}_l$	$pBl \to \Lambda B\nu_l$
(3) $\Sigma B \to nBl\bar{\nu}_l$	$nBl \to \Sigma B\nu_l$	(4) $\Sigma B \to \Lambda Bl\bar{\nu}_l$	$\Lambda Bl \to \Sigma B\nu_l$

Model	Process		Q, erg cm^{-3} s^{-1}
Pion condensate	$\tilde{n} \to \tilde{p}l\bar{\nu}_l$	$\tilde{p}l \to \tilde{n}\nu_l$	$(10^{22}\text{–}10^{26})\, T_9^6$
Kaon condensate	$\tilde{n} \to \tilde{p}l\bar{\nu}_l$	$\tilde{p}l \to \tilde{n}\nu_l$	$(10^{22}\text{–}10^{24})\, T_9^6$
Quark matter	$d \to ue\bar{\nu}_e$	$ue \to d\nu_e$	$(10^{25}\text{–}10^{26})\, T_9^6$

*) l stands for e or μ; $\tilde{n}$ and $\tilde{p}$ are quasinucleons (mixed n and p states); u and d are quarks.

Figure: From Yakovlev et al, 2000 (Phys. Rep.) [Arxiv: astro-ph/0012122]

Hadronic direct Urca cooling

Reactions like $n \to p + e^- + \bar{\nu}_e, \quad p + e^- \to n + \nu_e$ and
$n \to p + \mu^- + \bar{\nu}_{\mu^-}, \quad p + \mu^- \to n + \nu_{\mu^-},$

which can only happen when energy and momentum are conserved, that is, ignoring muons:

$$P_{Fn} \leq P_{Fp} + P_{Fe} \Rightarrow \quad Y_p \geq \frac{[(\rho Y_n)^{1/3} - \rho Y_e)^{1/3}]^3}{\rho}$$

such that $\epsilon_0^{dUrca} = \frac{457\pi}{10080} G_\mathrm{F}^2 \cos^2\theta_\mathrm{C} \left(1 + 3g_\mathrm{A}^2\right) \frac{m_n^* m_p^* m_e}{h^{10} c^3} (k_\mathrm{B} T)^6 \Theta_{npe}$

Don't forget superfluidity

When superfluidity or superconductivity are considered,

$$\epsilon^{dUrca} = \epsilon_0^{dUrca} R$$

where, for neutron triplets,

$$\tau = T/T_c$$

$$v_{\mathrm{T}} = \sqrt{1-\tau}\left(0.7893 + \frac{1.188}{\tau}\right)$$

$$R_{\mathrm{L}} = \left[0.2546 + \sqrt{(0.7454)^2 + (0.1284\, v_{\mathrm{T}})^2}\right]^5 \exp\left(2.701 - \sqrt{(2.701)^2 + v_{\mathrm{T}}^2}\right)$$

When proton and neutron pairing are simultaneously present, $R_L \sim \min\left(R_{L\,\mathrm{singlet}}, R_{L\,\mathrm{triplet}}\right)$

Summary of general picture

- Cooling data can provide microphysics constraints in a way that other measurements cannot
- Make sure your EOS accommodates slow and fast cooling sources
  - high proton fractions in fully hadronic EOS
  - kaons, pions, quarks + hadrons EOS
- Careful with suppression factors

Realistic hadronic EOS, with e^- and μ^-

Parametrizing
$E(\rho, \delta) = E_0(\rho) + E_{\rm sym}(\rho) \cdot \delta^2 + \mathcal{O}\left(\delta^4\right)$, where $\delta = (\rho_{\rm n} - \rho_{\rm p})/\rho$,

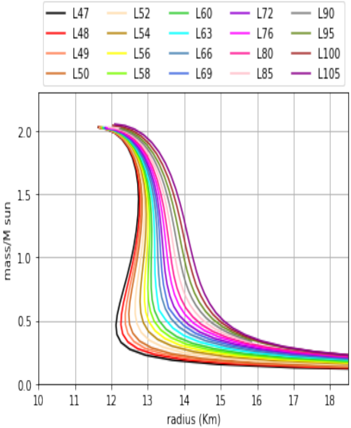
Around $\rho_{\rm sat} \approx 0.15\ \mathrm{fm}^{-3}$,

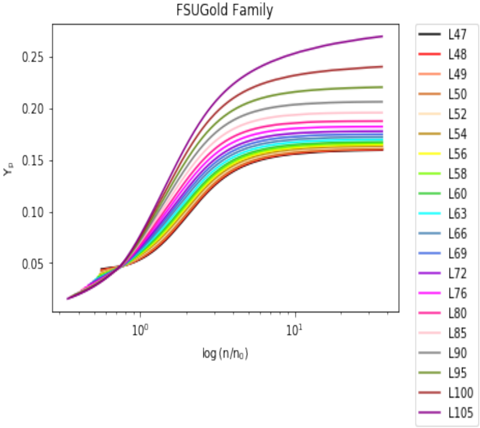
$$E_{\rm SNM}(\rho) = B + \frac{1}{2} K \left(\frac{\rho - \rho_{\rm sat}}{3\rho_{\rm sat}}\right)^2 + \cdots$$

$$E_{\rm sym}(\rho) = J + L \left(\frac{\rho - \rho_{\rm sat}}{3\rho_{\rm sat}}\right) + \frac{1}{2} K_{\rm sym} \left(\frac{\rho - \rho_{\rm sat}}{3\rho_{\rm sat}}\right)^2 + \cdots$$

$B =$ energy per nucleon, $K =$ incompressibility coefficient of SNM, $J =$ symmetry energy, $L =$ slope of symmetry energy, $K_{\rm sym} =$ incompressibility coefficient of symmetry energy at saturation density.

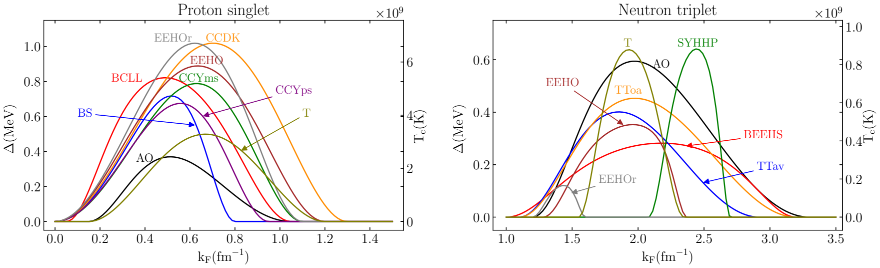
Realistic hadronic EOS, with e^- and μ^-





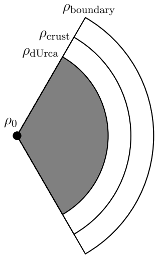
Varied nuclear pairing gap models

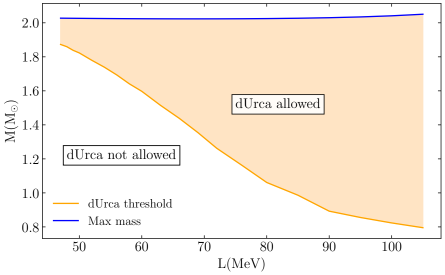
Many possible parametrizations, Ho et al, 2015 (PRC) [ArXiv:1412.7759]

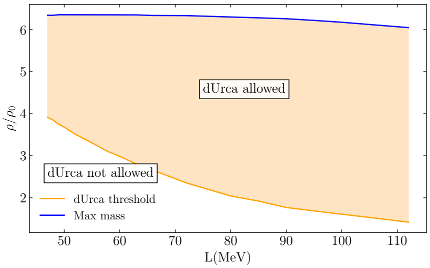


dUrca threshold

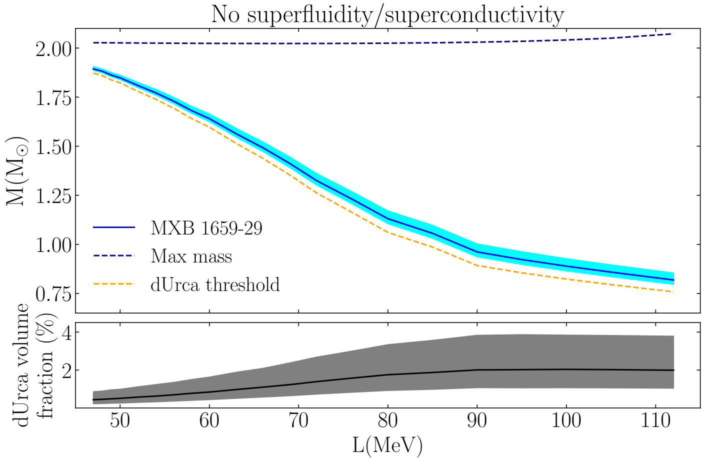
Without nuclear pairing







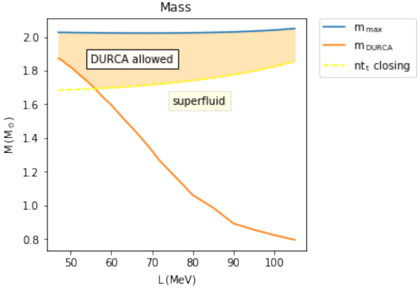
Can we reproduce MXB 1659-29 inferred luminosity?

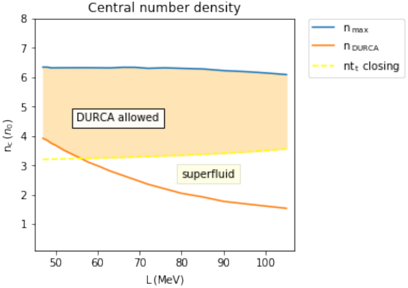


Without superfluidity, yes! Note the volume fraction

Nuclear pairing suppression

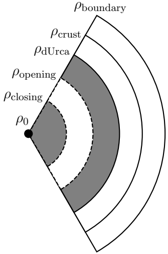
When the gap closes after the dUrca threshold, those reactions are strongly suppressed

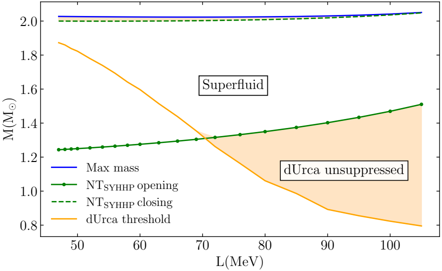


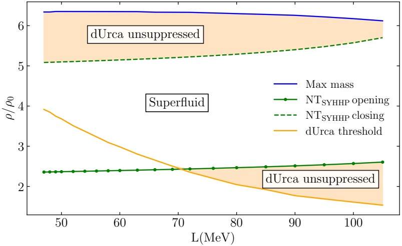


Nuclear pairing suppression

Late opening example







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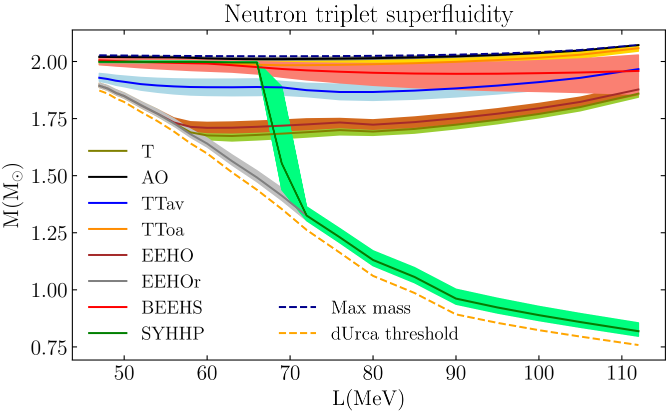
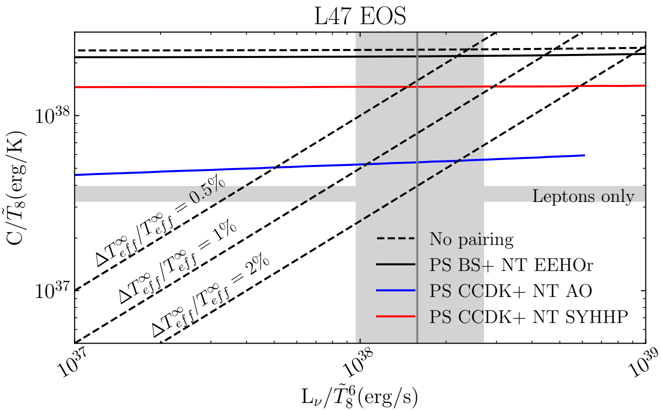


With superfluidity, yes too! Note the still small range of masses for a given EOS

Ask me about PS+NT combinations :)

A word about heat capacity

Knowing this source's temperature in 10 years, we could measure or constraint its heat capacity

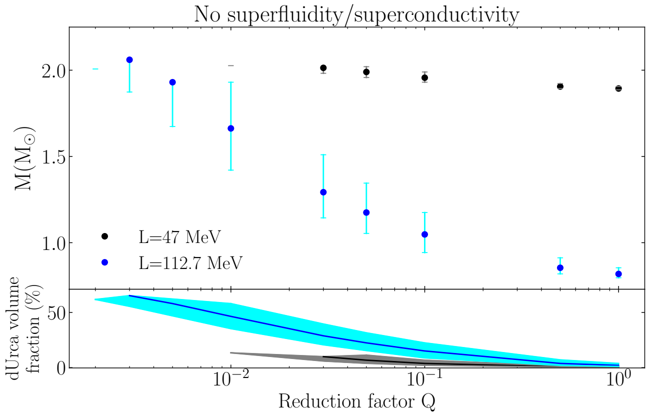


Conclusions pt 1

- Central proton fraction directly proportional to L, thus large L EOS needs strong superfluidity
- Fully hadronic EOS can describe MXB 1659-29 and potentially hotter and colder sources
- “Strength” of superfluidity better observed with core volume submitted to it than critical temperature

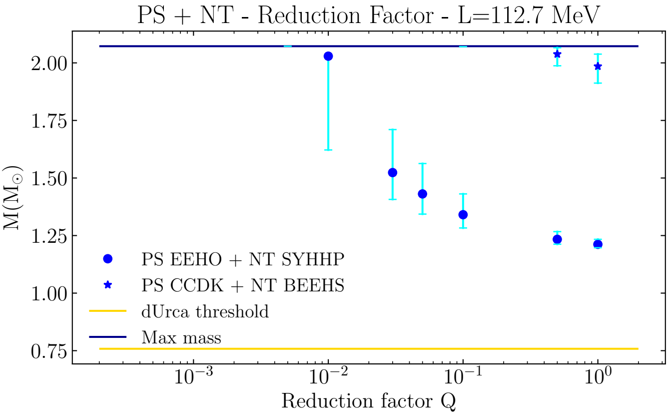
What if... there are less effective dUrca reactions active?

Upper limit



Up to $10^{-3} < Q \leq 10^{-2}$, depending on L

What if... there are less effective dUrca reactions active?

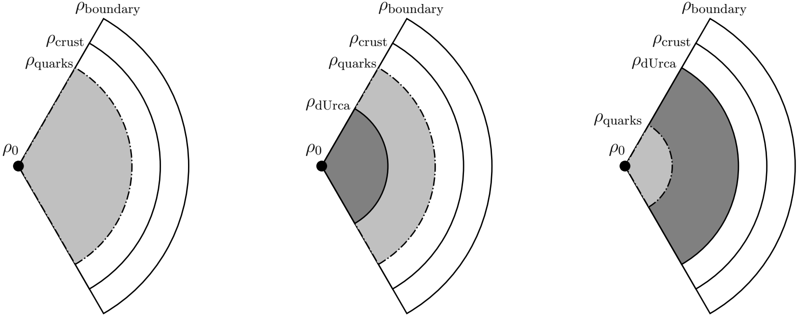


Note the decreasing mass range for smaller Q

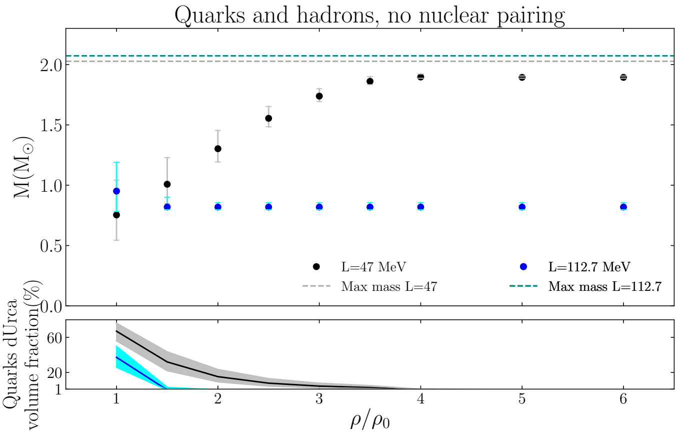
Conclusions pt 2

- Increase in mass range is not significant for all L, oversimplified picture?
- 100 times less efficient process would have difficulty reproducing much colder sources, large L disfavored?
- Some gap model combinations unable to reproduce data if Q small, are they disfavored?

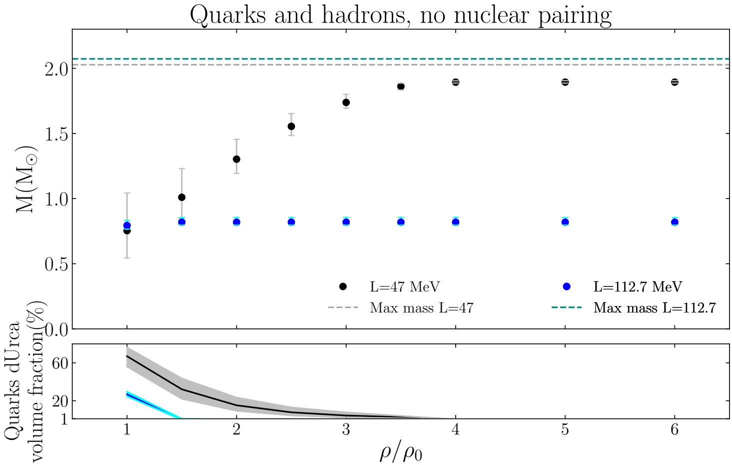
Simulating hybrid EOS with quarks



What if... there are quarks in the core?



What if... there are quarks in the core? - Mixed phase



Conclusions pt 3

- Quark dUrca cooling dominant only if transition before dUrca threshold, careful calculations for small L
- Possibility of constraining quark-hadron transition with cooling data
- The possible presence of mixed phase does not change the picture significantly

Summary

- Fully hadronic EOS can describe MXB 1659-29 and potentially colder sources
- Less effective cooling processes not necessarily favored (cooler sources would need even larger volume fractions)
- If quarks present, careful study of the star's cooling is needed (including nuclear and color condensate(?) pairing)
- Let's talk about your favorite EOS so we can investigate these details together!

Thank you!

melissa.mendessilva@mail.mcgill.ca

References I

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[R. Wijnands N. Degenaar and D. Page]
J. Astrophys. Astr., 38:49 (2017)

[W. Ho et al]
Physical Review C 91, 015806 (2015)

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Phys. Rev. Lett. 120, 182701 (2018)

TOV equations

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{\mathcal{E}(r)}{c^2}\frac{Gm(r)}{r^2}\left[1+\frac{P(r)}{\mathcal{E}(r)}\right]\left[1+\frac{4\pi r^3 P(r)}{m(r)c^2}\right]\left[1-\frac{2Gm(r)}{c^2 r}\right]^{-1},$$

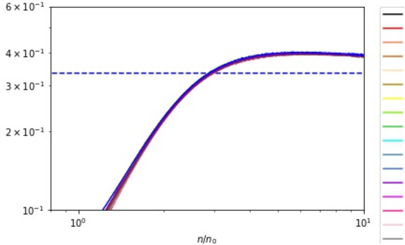
$$\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2\frac{\mathcal{E}(r)}{c^2},$$

$$\frac{\mathrm{d}\phi}{\mathrm{d}r} = -\frac{1}{\mathcal{E}(r)+P(r)}\frac{\mathrm{d}P}{\mathrm{d}r}$$

with $\phi(R) = 1/2\ln\left(1-\frac{2GM}{Rc^2}\right)$ and $T(r) = \tilde{T}\exp(-\phi(r))$

Speed of sound and other parameters

Saturation density, $\rho_0 = 0.1504\,\mathrm{fm}^{-3}$, binding energy per nucleon, $B = -16.26$ MeV, incompressibility, $K_0 = 237.7$ MeV



FSUGold2 Family		
J (MeV)	L (MeV)	K_sym (MeV)
30.604	47.000	53.983
30.773	48.000	45.666
30.938	49.000	37.754
31.099	50.000	30.228
31.407	52.000	16.268
31.702	54.000	3.671
31.984	56.000	-7.660
32.255	58.000	-17.802
32.516	60.000	-26.824
32.892	63.000	-38.383
33.250	66.000	-47.719
33.594	69.000	-54.969
33.924	72.000	-60.252
34.346	76.000	-64.406
34.750	80.000	-65.443
35.233	85.000	-62.616
35.694	90.000	-55.470
36.153	95.000	-44.243
36.560	100.000	-29.141
36.970	105.000	-10.341
37.573	112.680	25.383

Nuclear pairing parametrization

For the polynomial parametrization we use here,

$$\Delta(k_{\mathrm{Fx}}) = \Delta_0 \frac{(k_{\mathrm{Fx}} - k_0)^2}{(k_{\mathrm{Fx}} - k_0)^2 + k_1} \frac{(k_{\mathrm{Fx}} - k_2)^2}{(k_{\mathrm{Fx}} - k_2)^2 + k_3}$$

TABLE II. Superfluid gap parameters.

Gap model	Δ_0 (MeV)	k_0 (fm^{-1})	k_1 (fm^{-2})	k_2 (fm^{-1})	k_3 (fm^{-2})	Ref.
			Proton singlet (ps)			
AO	14	0.15	0.22	1.05	3.8	[57,58]
BCLL	1.69	0.05	0.07	1.05	0.16	[39,58]
BS	17	0.0	2.9	0.8	0.08	[59]
CCDK	102	0.0	9.0	1.3	1.5	[49,58]
CCYms	35	0.0	5.0	1.1	0.5	[60]
CCYps	34	0.0	5.0	0.95	0.3	[60]
EEHO	4.5	0.0	0.57	1.2	0.35	[58]
EEHOr	61	0.0	6.0	1.1	0.6	[42]
T	48	0.15	2.1	1.2	2.8	[61]
			Neutron triplet (nt)			
AO	4.0	1.2	0.45	3.3	5.0	[38]
BEEHS	0.45	1.0	0.40	3.2	0.25	[62]
EEHO	0.48	1.28	0.1	2.37	0.02	[41]
EEHOr	0.23	1.2	0.026	1.6	0.0080	[42]
SYHHP	1.0	2.08	0.04	2.7	0.013	[11]
T	1.2	1.55	0.05	2.35	0.07	[38,64]
TTav	3.0	1.1	0.60	2.92	3.0	[65]
TToa	2.1	1.1	0.60	3.2	2.4	[65]

Figure: Modified from W. Ho et al (2015)

Heat capacity calculation details

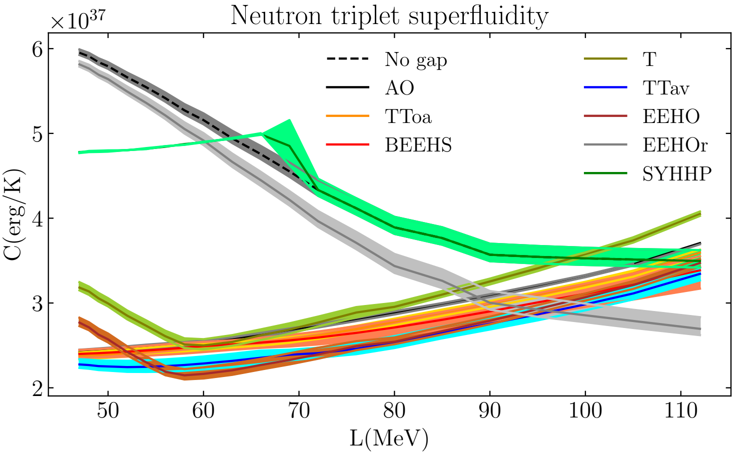
Given the individual heat capacities of each species C_x, we find the total heat capacity by

$$C_{\mathrm{total}}^{\mathrm{core}} = \int_0^{R_{\mathrm{core}}} \frac{4\pi r^2 \sum C_x}{\left(1 - (2Gm(r)/c^2 r)\right)^{1/2}} dr$$

With superfluidity, it gets reduced such that $C_x^{pairing} = C_x R$. For neutron triplets,

$$\tau = T/T_c$$
$$u_{\mathrm{T}} = \sqrt{1-\tau}(5.596 + 8.424/\tau)$$
$$R_C = \left[0.6893 + \sqrt{(0.790)^2 + (0.03983\, u_{\mathrm{T}})^2}\right]^2 \exp\left(1.934 - \sqrt{(1.934)^2 + \frac{u_{\mathrm{T}}^2}{16\pi}}\right)$$

Heat capacities of the stars found



Temperature variation and heat capacity

When the core is isothermal, a neutron star in quiescence has

$$C\frac{d\tilde{T}}{dt} = -L_\gamma(\tilde{T}) - L_\nu(\tilde{T}),$$

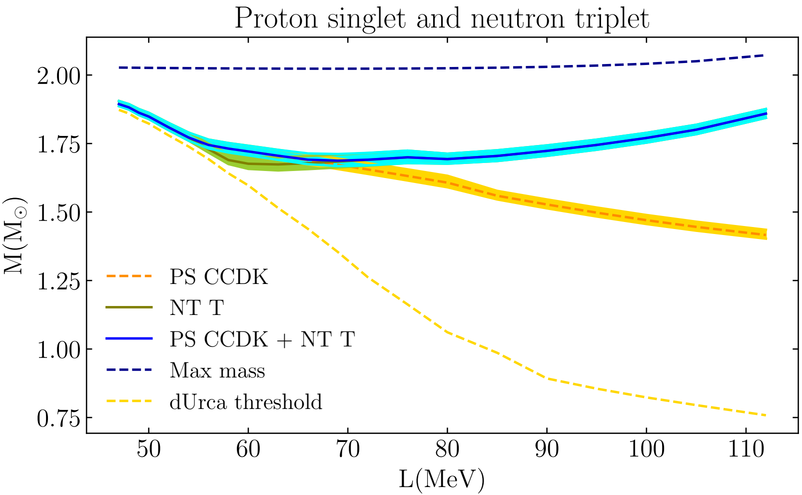
neglecting $L_\gamma(\tilde{T})$ and knowing $C \propto \tilde{T}$ and $L_\nu(\tilde{T}) \propto \tilde{T}^6$, for dUrca processes, it can be shown that

$$\left(\frac{\tilde{T}_i}{\tilde{T}_f}\right)^4 - 1 = 4\frac{\Delta t\, L_\nu}{C\tilde{T}}.$$

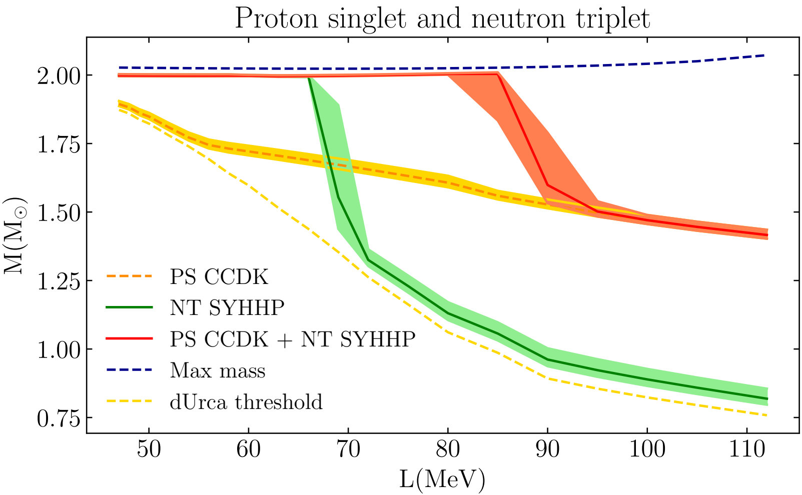
for $\Delta t << C\tilde{T}/L_\nu$, we expand in first order to get

$$\frac{C_{38}}{L_{v,35}} = \left(\frac{\Delta\tilde{T}/\tilde{T}}{0.3\%}\right)^{-1}\left(\frac{t_q}{10\,\mathrm{yr}}\right)\tilde{T}_8^{-1}$$

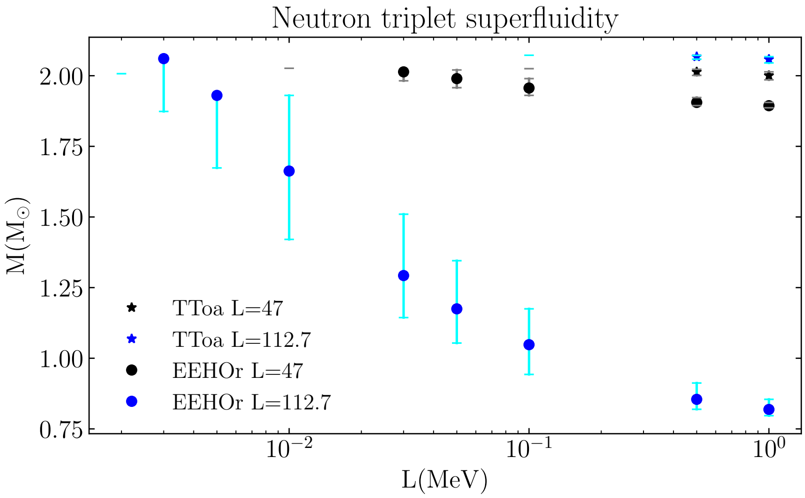
Some Ps+Nt combinations



Some Ps+Nt combinations



Different Q for other Ps+Nt combinations



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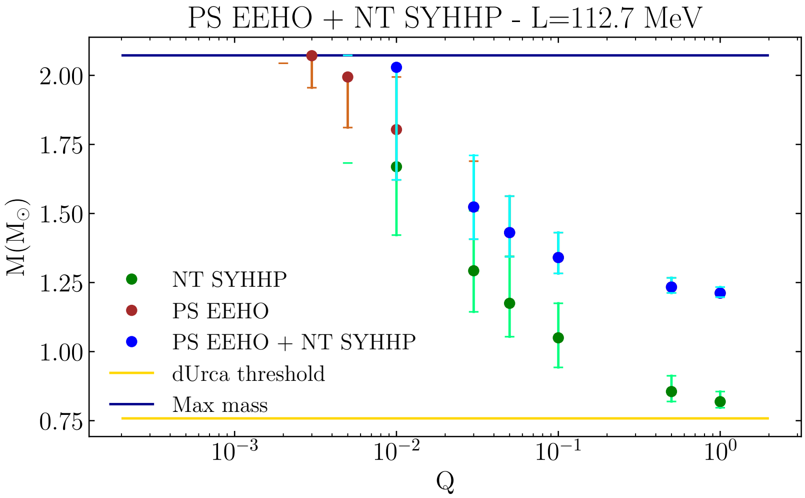

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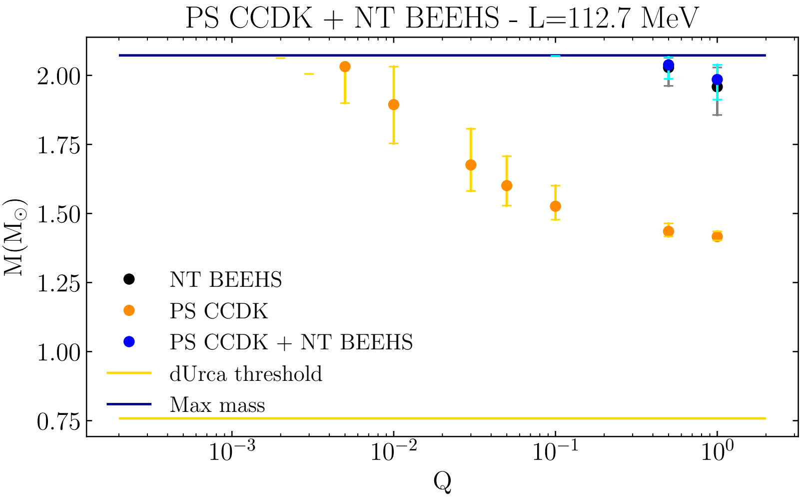
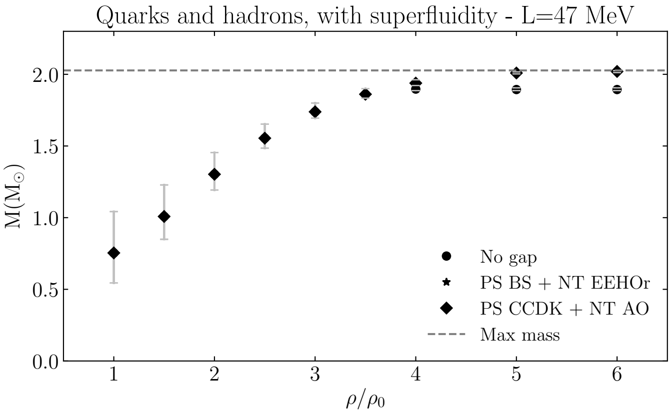
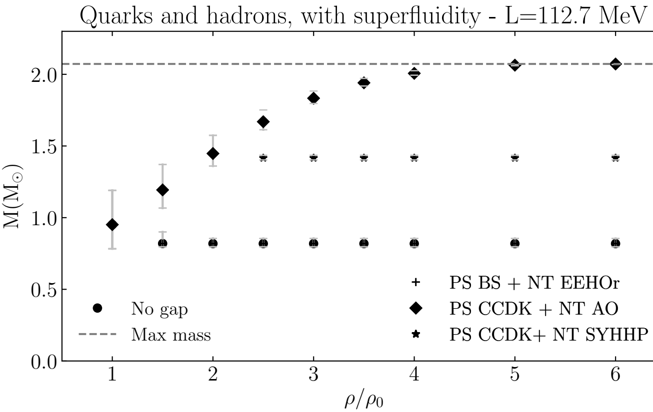


Quarks and superfluid hadrons



Quarks and superfluid hadrons



Quarks and hadrons mixed phase