Constraining the EOS from Rotating Neutron Stars

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ECT* Villazzano TRENTO
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Based on:

SV and Christian J. Krüger[+], **arXiv:2203.05555**
[+]Theoretical Astrophysics, University of Tübingen, Germany

ERC-2018-COG GRAMS 815673

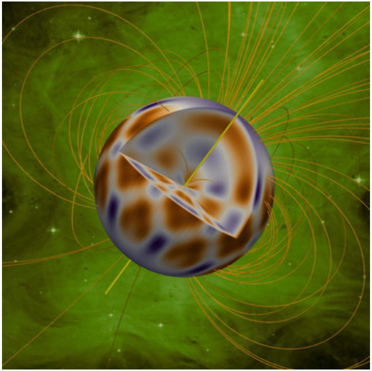
NEUTRON STARS IN A NUTSHELL

- most **compact** (M/R) stars:
  $1 \sim 2 M_\odot$ in radius of $\approx 10\,\mathrm{km}$
- **extreme** matter and gravity:
  "nutshell" of matter can weigh $10^{15}\,\mathrm{g}$
- for some time most precise clocks:
  extremely stable rotation
- have extremely flat surface:
  tiny "mountains" (few mm)

NEUTRON STAR PHYSICS

Many disciplines and aspects required (not complete!):

- strong gravity, requires full **general relativity**
- full composition unconstrained, **nuclear physics** (and beyond?)
- "**cosmic laboratory**" for QCD at low temperature/high density
- stability/formation of extreme **magnetic fields** (magnetars)
- **nucleosynthesis** of heavy elements (mergers)

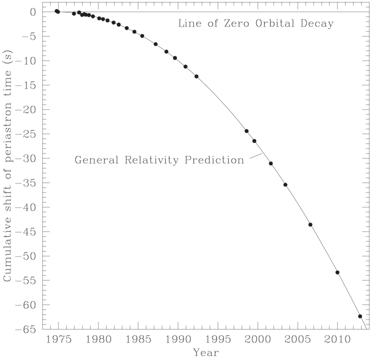


Credit: Textbook Tipler/Mosca; Kokkotas, Gaertig; ESA/Hubble & NASA

OBSERVATIONAL ASPECTS

From radio to X-ray and now gravitational waves (GWs)!

- first pulsar found by Jocelyn Bell Burnell in 1967 (radio)
- hot spots on surface (e.g. via NICER, X-ray)
- low mass/high mass X-ray binaries (X-ray)
- indirect GWs via Hulse-Taylor binary pulsar
- binary mergers via direct GWs (LIGO/Virgo)



Weisberg and Huang, ApJ 829 55, 2016

NEUTRON STAR THEORY

For spherically symmetric (static) stars:

- stellar structure given by **TOV**[1] **equations**

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{Gm}{r^2}\varepsilon\left(1+\frac{p}{\varepsilon c^2}\right)\left(1+\frac{4\pi r^3 p}{mc^2}\right)\left(1-\frac{2Gm}{rc^2}\right)^{-1}, \quad (1)$$

  mass m contained inside r is given by

$$m(r) = 4\pi\int_0^r r'^2 \varepsilon \mathrm{d}r'. \quad (2)$$

- first law of thermodynamics connects with energy density

$$\rho\, \mathrm{d}\varepsilon = (\varepsilon + p)\, \mathrm{d}\rho. \quad (3)$$

To close TOV equations, **equation of state (EOS)** $P(\rho)$ **is needed**!

[1] Tolman-Oppenheimer-Volkoff

EQUATIONS OF STATE

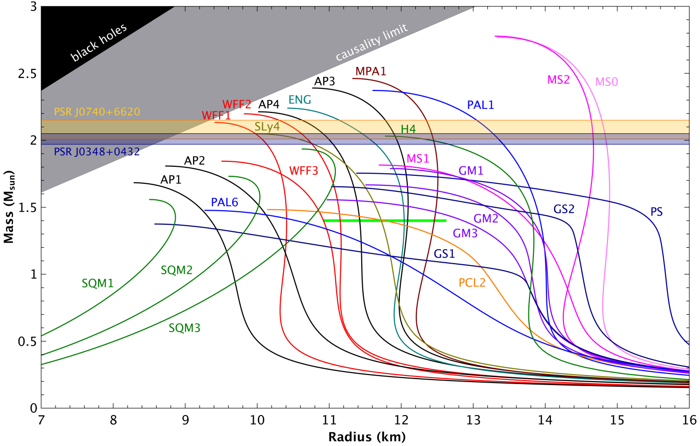


Figure created by Norbert Wex, taken from website of Paulo Freire.

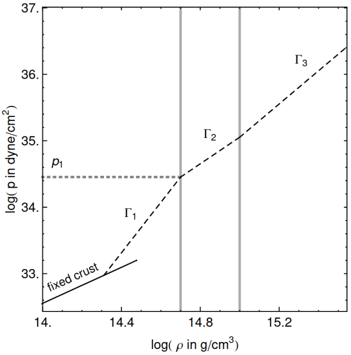
DESCRIBING THE EQUATION OF STATE

"Nuclear physics specific approach":

- compute from first principles
- clear connection to microscopic physics
- hard to compute without assumptions
- yields tabulated "realistic EOS"

"Theory agnostic parametrization":

- simple polytropic EOS: $P = K\rho^\Gamma$
- extension to multi-piecewise EOS
- no microscopic physics, but easy to use
- can be mapped to realistic EOS



Read et al., PRD 79, 124032, 2009

NEUTRON STAR "OBSERVABLES"

Direct problem: Given an EOS, what are (in principle) "observable" properties?

- mass
- radius
- spin/spindown
- moment of inertia
- magnetic fields
- oscillation modes
- tidal deformability/Love number
- surface temperature/cooling
- ...

Some properties are **static/stationary**, others are **perturbative/dynamical**

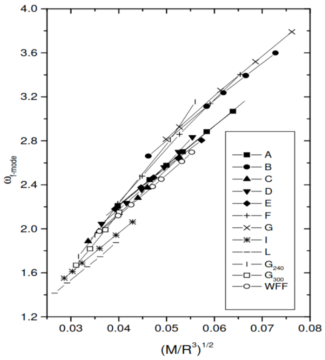
STELLAR OSCILLATIONS AND UNIVERSAL RELATIONS

Stellar Oscillations:

- perturbed stars have characteristic frequencies/damping times
- rich families of modes: f-modes, p-modes, g-modes, r-modes, w-modes,...
- details depend on underlying EOS and stellar properties

Universal Relations:

- oscillations scale with stellar properties, e.g., average density
- EOS insensitive relations can be found empirically
- "asteroseismology", relate modes directly with bulk properties



Andersson and Kokkotas, MNRAS 299:1059-1068, 1998

MOTIVATION OF OUR WORK

Main aspect of our work is the **inverse problem:**

- Given (simulated) observables, how can one **recover the underlying EOS**?
- How **does rotation impact the reconstruction**?
- One framework to **combine different observables** from different stars
- **Analysis via "post processed" data**, directly from observed properties

OVERVIEW

Summary main parts:

- **Mock Observations:** mass, radius, spin, co- & counter-rotating f-mode[2]
  full spin dependency!
- **Model Observations:** bulk properties via TOV, f-modes via universal relations
  here is the “slow spin” part
- **EOS:** piecewise polytropic EOS by Read et al. (4 paramters)
  pre-compute large TOV dataset on grid
- **Inference:** Bayesian approach via Markov-chain Monte Carlo (MCMC)
  using EMCEE sampler

[2] quadrupolar $l = m = 2$ fluid

Modeling neutron star observables:

- leading corrections to mass and radius enter $\sim \Omega^2$
- oscillation modes, e.g. f-mode(s) are modified $\sim \Omega$
- universal relations (UR) connect f with bulk properties
- for **slow rotation** describe (M, R, Ω, f^i) via TOV and UR!

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Modeling the EOS

- common approach fit multi-piecewise polytropic EOS
- Read et al. model has 4 parameters ($\theta = (p_1, \Gamma_1, \Gamma_2, \Gamma_3)$)
- very accurate to (sub-)percent level compared to realistic EOS table

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Bayesian approach

- define likelihood (uncorrelated Gaussian for each observables D)
- going from EOS parameters θ to $(M(\theta), R(\theta), ...)$ is **not trivial**
- parametric minimization problem in each likelihood evaluation, “expensive”

COMMENTS ON LIKELIHOOD

- our likelihood is given by

$$\log\left(\mathcal{P}(D|\theta)\right) = -\frac{1}{2}\sum_{i=1}^{N_D}\sum_{j=1}^{N_O}\left(\frac{D_j^i - M_j^i(\theta)}{\bar{\sigma}_j^i}\right)^2, \tag{4}$$

- model values are determined via

$$\Delta^2_{k_{\min}} \equiv \min\left(\Delta_k^2\right) \tag{5}$$

- $k \in N_{\text{seqgrid}}$ labels the values for the discretized EOS sequence and Δ_k^2 is defined as

$$\Delta_k^2 \equiv \sum_{j=0}^{N_{\text{dim}}}\left(D_j^{\text{k,model}} - D_j^{\text{data}}\right)^2. \tag{6}$$

WORKFLOW

Before MCMC sampling:

- like LIGO, build template bank (here for M-R curves)
- θ grid range includes realistic EOS models
- at the moment $50^4 = 6.25 \times 10^6$ M-R curves, each with 50 stars
- in total 312.5×10^6 TOV integrations
- on 50 CPUs, $1 \sim 2$ days with efficient C-code

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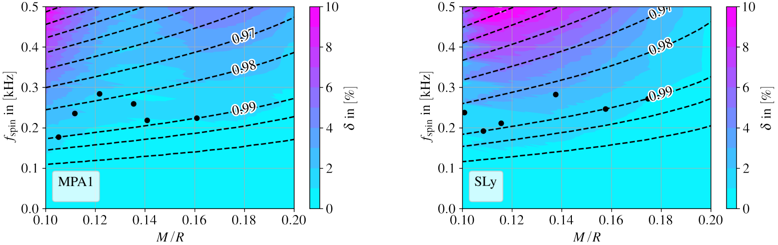
During MCMC sampling:

- for each θ proposal find closest dataset
- compute f-modes with UR along discrete Ω
- provide model proposal $M(\theta), R(\theta), \ldots$ via minimization scheme
- reasonable sampling takes few hours on laptop/workstation

APPLICATION

Our simulated data:

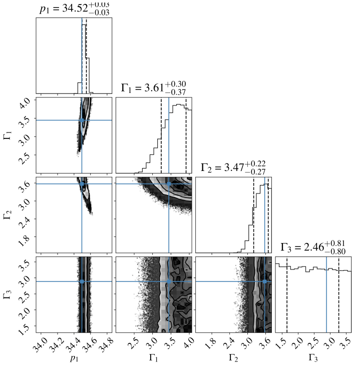
- bulk properties $R(\Omega), M(\Omega)$ full rotational dependency (RNS-code)
- f-modes from precise time evolution code (Krüger and Kokkotas, PRL 125, 111106, 2020)
- injected realistic EOS (MPA1 or SLy) using Read et al. model



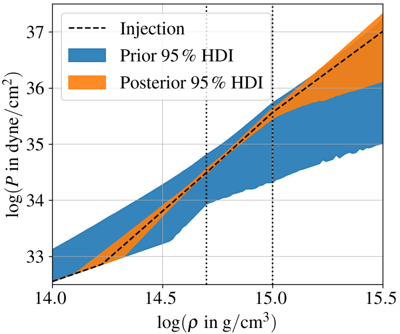
Compactness vs spin frequency. Color coded a measure for breakdown of slow spin approximation for M and R. Black dots our simulated data (f-modes not shown). Dotted lines different constant axis ratios (polar over equatorial radius).

RESULTS MPA1

Model works well with percent level data, here 3 % relative error.



MCMC sampling of the four EOS parameters. Blue Read et al. best fit injection for MPA1 EOS.



Prior vs posterior sampling of the EOS with 95 % highest probability density intervals. Black dashed injected EOS.

RESULTS SLY

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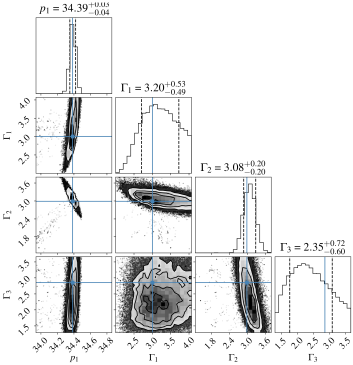
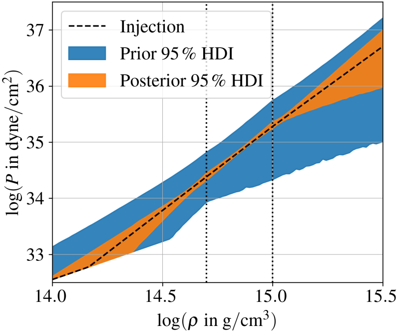


MCMC sampling of the four EOS parameters. Blue Read et al. best fit injection for SLy EOS.



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OUTLOOK / DISCUSSION

Aspects that need to be improved/extended in upcoming work:

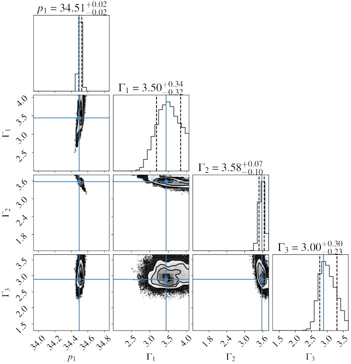
- include $M(\Omega), R(\Omega)$ also in the model
- include tidal Love numbers and moment of inertia (also Ω!)
- eventually beyond Read et al. EOS (more EOS pieces)
- include more realistic errors (correlations)

CONCLUSIONS

- approached **inverse problem** for **rotating** neutron stars
- simulated observations $M, R, \Omega, f_{2+}, f_{2-}$ take into account rotational effects
- model takes into account **(slow) rotation** (not common)
- assumptions are **reliable on percent level** for slow rotation ($\lesssim 200\,\mathrm{Hz}$)
- required precision calls for next generation detectors/experiments

Results MPA1: Consistency check

Inject model as data: here $1\,\%$ relative error.



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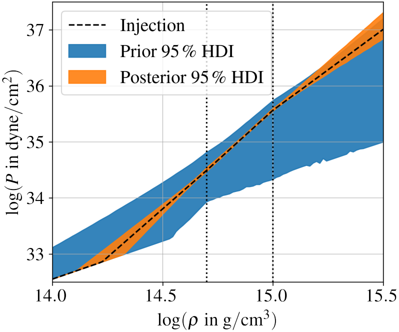

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DATA MPA1

n	M $[M_\odot]$	R [km]	$f_{\rm spin}$ [kHz]	$\sigma^{\rm u}$ [kHz]	$\sigma^{\rm s}$ [kHz]	ar
1	1.764	12.523	0.218	1.533	2.116	0.989
2	1.311	12.466	0.177	1.468	1.900	0.990
3	2.002	12.445	0.224	1.618	2.239	0.990
4	1.400	12.528	0.236	1.394	1.993	0.983
5	1.700	12.559	0.260	1.442	2.129	0.983
6	1.531	12.584	0.284	1.361	2.094	0.978

Simulated data used for the inverse problem. The assumed underlying EOS is MPA1 with PPA coefficients $\theta^{\rm MPA1} = (34.495, 3.446, 3.572, 2.887)$. The relative errors are discussed in the main text. The last column shows the axis ratio (ar), which we report for completeness, but which did not enter our analysis.

DATA SLY

n	M [$M_\odot$]	R [km]	$f_{\rm spin}$ [kHz]	$\sigma^{\rm u}$ [kHz]	$\sigma^{\rm s}$ [kHz]	ar
1	1.282	11.827	0.192	1.590	2.075	0.990
2	1.200	11.902	0.238	1.486	2.082	0.983
3	1.363	11.792	0.211	1.603	2.143	0.989
4	1.924	11.013	0.272	1.862	2.649	0.990
5	1.601	11.637	0.283	1.618	2.368	0.984
6	1.788	11.340	0.247	1.798	2.476	0.990

Simulated data used for the inverse problem. The assumed underlying EOS is SLy with PPA coefficients $\theta^{\rm SLy} = (34.384, 3.005, 2.988, 2.851)$. The relative errors are discussed in the main text.The last column shows the axis ratio (ar), which we report for completeness, but which did not enter our analysis.