Fast Rotating Neutron Stars: Spectra and Stability without Approximation

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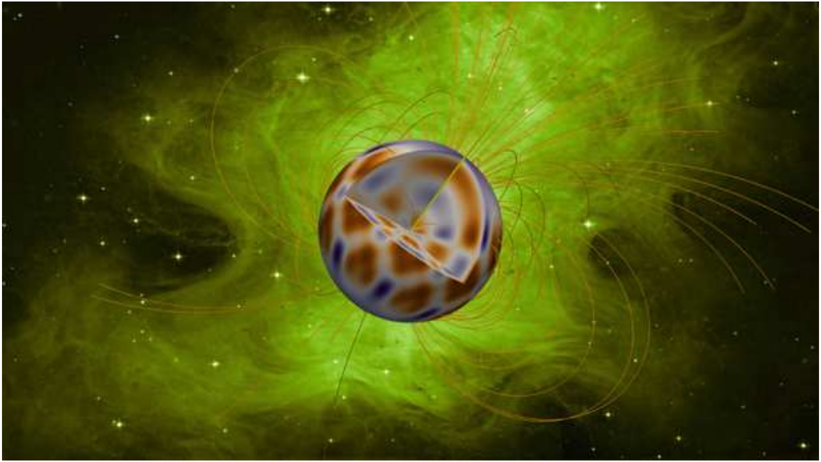
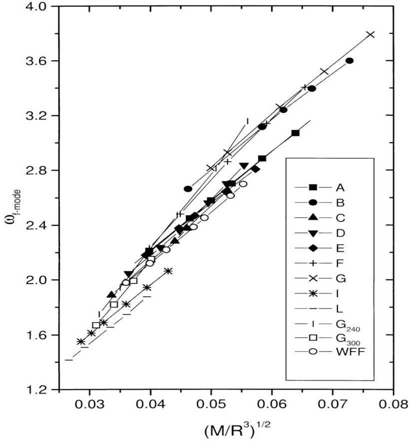
Neutron Stars & their Oscillations



Gravitational Wave Asteroseismology

The Spectrum of Neutron Stars

- Oscillation modes classified by restoring force
- f/p-modes: pressure (acoustic waves)
- w-modes: spacetime modes with no Newtonian counterpart
- Various other modes: g-, s-, i-, r-modes, ...



Andersson, Kokkotas (1998)+

Relevant Astrophysical Scenarios

- Collapse scenarios: excitation of f- and g-modes.
- Tidal effects during inspiral phase of binary mergers:
  - Love numbers (and f-Love-relations)
  - Impact on phase from f-mode resonance
- Early Post-Merger Phase
  - Useful for asteroseismology and constraining EoS
  - Need to implement differential rotation and hot EoS
  - Detection expected towards end of next decade
- Late Post-Merger Phase
  - f-mode instability

Previous Studies

- Cowling (1942): Mode classification
- The CFS-Instability: Chandrasekhar-Friedman-Schutz 1970+
- Spacetime (w-)modes: Kokkotas-Schutz 1986-90+
- Gravitational Wave Asteroseismology: Andersson-Kokkotas 1996+
- *r*-mode instability: Andersson, Friedman-Morsink 1998+
- Determination of QNMs: Detweiler-Lindblom 1983+, Andersson et al. 1995+
- Equilibrium Configurations of fast rotating NSs: Bonazzola (1974), Komatsu-Eriguchi-Hachisu (1988), Cook-Shapiro-Stergioulas-Friedman (1995)
- Onset of CFS-instability: Stergioulas-Friedman (1995)
- Oscillations of fast rotating NS (Cowling): Gaertig-Kokkotas (2008)

Time Evolution of Perturbation Equations

- Perturbed Einstein Equations
  & Conservation of Energy-Momentum

$$\delta G_{\mu\nu} = 8\pi\delta T_{\mu\nu},$$
$$\delta\left(\nabla_\nu T^{\mu\nu}\right) = 0,$$

  with neutron star modelled as perfect fluid

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + pg^{\mu\nu}.$$

- Choose Hilbert Gauge:

$$\nabla^\mu h_{\mu\nu} = 0.$$

  $\rightarrow$ wave equations for the metric components.

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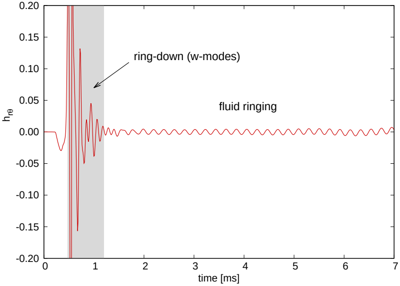
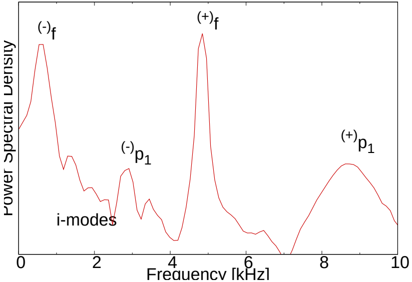
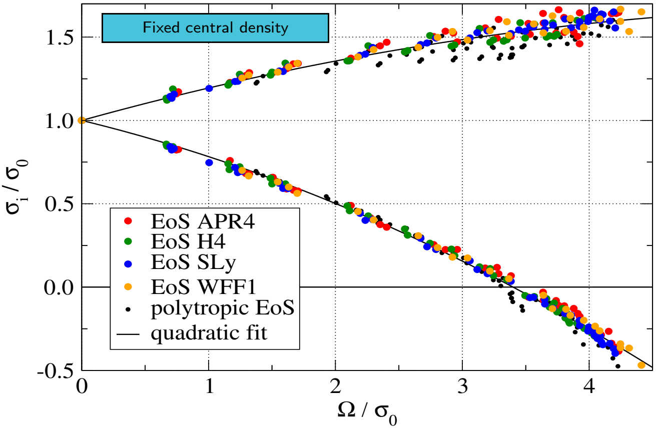
Characteristic example of time signal





EoS SLy	$M = 2.02\,M_\odot$
$\epsilon_c = 1.2e15\ \mathrm{g/cm^3}$	$\Omega = 1.3\,\mathrm{kHz}$
$r_e/r_p = 0.56$	$\Omega/\Omega_K = 0.98$

Fitting formulae – σ/σ_0 vs. Ω/σ_0



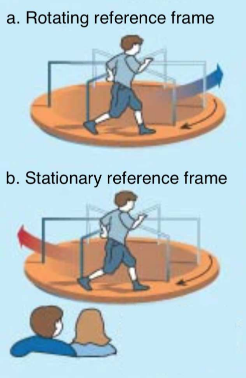
Zeeman-like splitting of f-mode, here shown for $l = |m| = 2$.

$$\frac{\sigma^u}{\sigma_0} = 1 - 0.193 \left(\frac{\Omega}{\sigma_0}\right) - 0.0294 \left(\frac{\Omega}{\sigma_0}\right)^2$$

$$\frac{\sigma^s}{\sigma_0} = 1 + 0.220 \left(\frac{\Omega}{\sigma_0}\right) - 0.0170 \left(\frac{\Omega}{\sigma_0}\right)^2$$

f-mode "changes sign", i.e., becomes CFS-unstable when $\Omega \gtrsim 3.4\sigma_0$.

CFS-Instability



Fryer, Woosley (2001)

- Retrograde pattern of non-axisymmetric modes has $J_{comov} < 0$ as measured by a **comoving** observer.
- For sufficiently fast rotating stars, an **inertial** observer (parents) will also see the retrograde pattern propagate along with the star's rotation, i.e. $J_{inert} > 0$.
- Emission of GW will radiate angular momentum, $\rightarrow$ mode becomes unstable (Chandrasekhar 1969, Friedman & Schutz 1978).

This is the CFS-Instability.

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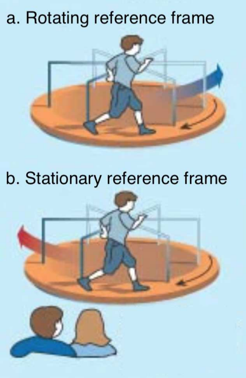

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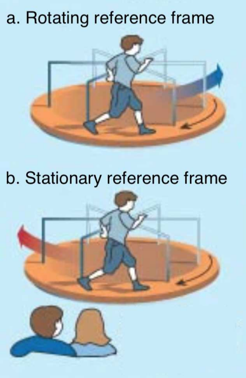

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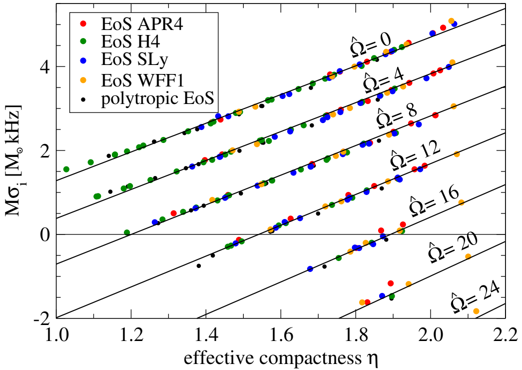
Fitting formulae – $M\sigma$ vs. $\hat{\Omega}$ vs η

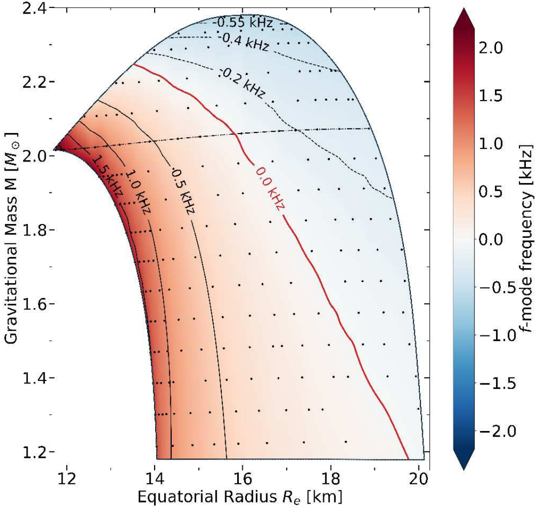


$$M\sigma_i^u = \left(-2.14 - 0.201\hat{\Omega} - 7,68 \cdot 10^{-3}\hat{\Omega}^2\right) + \left(3.42 + 1.75 \cdot 10^{-3}\hat{\Omega}^2\right)\eta \qquad \hat{\Omega} = M\Omega$$

$$M\sigma_i^s = \left(-2.14 + 0.220\hat{\Omega} - 14.6 \cdot 10^{-3}\hat{\Omega}^2\right) + \left(3.42 + 6.86 \cdot 10^{-3}\hat{\Omega}^2\right)\eta \qquad \eta = \sqrt{M^3/I}$$

Non-rotating case: Tsui, Leung (2005) **Cowling case:** Doneva, Kokkotas (2015)

Overview of Entire (Cold) EoS H4



Summary

- f-mode frequencies determined in full GR
- Provide various universal relations (EoS independent)
- Important for a number of astrophysical scenarios
  - Inspiral, post-merger, collapse, glitches, ...

Thank you for your attention!

Questions?