Thermal effects on nuclear matter properties

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Outlook

- Why finite-temperature EOSs in NS physics?
- Nuclear Hamiltonian
  - Correlated basis functions – effective interaction
- Generalisation to finite temperature
- Thermal effects on single-particle properties
  - Fermi distribution, spectrum and effective mass
- Average quantities: exact calculation VS fit
- Conclusions and perspectives

Hot EOS

- Merger and post-merger phases of a BNS coalescence
- Proto-NS evolution
- Supernova modelling
- Important phenomena involved
  - Thermal contribution to the pressure
  - Bulk viscosity
  - Neutrino emission and absorption

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Nuclear Many-Body Theory (NMBT)

- The basis for Nuclear Many-Body Theory is the Hamiltonian

$$H = \sum_{i=1}^{A} \frac{\mathbf{p}_i^2}{2m} + \sum_{j>i=1}^{A} v_{ij} + \sum_{k>j>i=1}^{A} V_{ijk}$$

where, in general, $v_{ij} = \sum_p v^p(r_{ij}) O_{ij}^p$

- Nucleon-nucleon (NN) potential: Argonne $v_6{}'$(AV6P)
- Three-nucleon (3N) potential: Urbana IX (UIX)
- AV6P+UIX results are very close to AV18+UIX [Lovato+ (2022), arXiv:2202.10293]
  - Repulsive part of UIX → additional repulsion → lower equilibrium density of SNM

CBF Effective Interaction

- NN forces are strongly repulsive at short distance → standard many-body perturbation theory cannot be used

- The CBF effective interaction is defined through

$$\langle H \rangle = \langle \Psi_0 | H | \Psi_0 \rangle = T_F + \langle \Phi_0 | \sum_{i<j} v_{ij}^{\text{eff}} | \Phi_0 \rangle$$

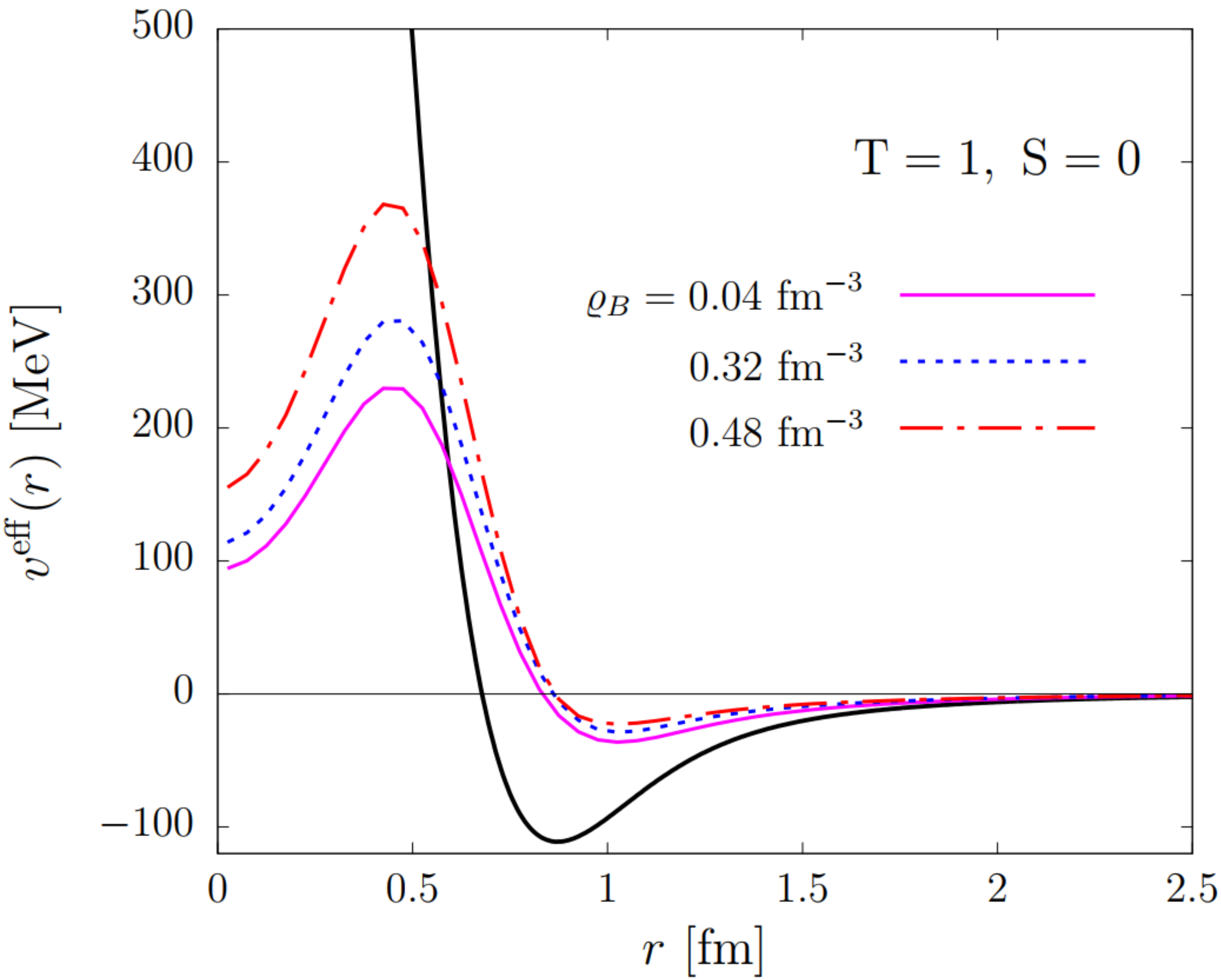
Correlated basis

Fermi gas basis

- Include NN and 3N forces and correlation effects

$$\langle H \rangle = T_F + \sum_n (\Delta E)_n = T_F + \langle \Phi_0 | \sum_{i<j} v_{ij}^{\text{eff}} | \Phi_0 \rangle$$

CBF Effective Interaction

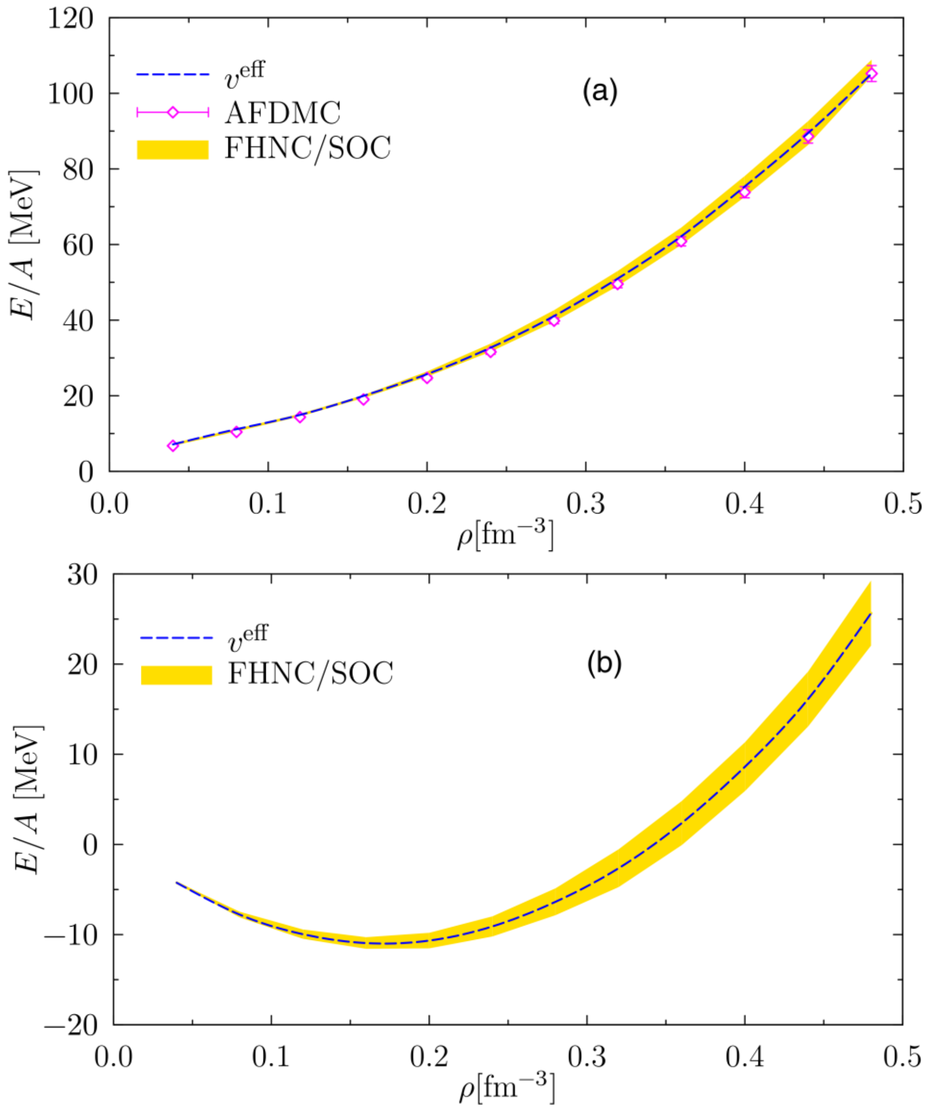


Radial dependence of the CBF effective potential in the $S = 0,\ T = 1$ channel

CBF Effective Interaction - Results

PNM

SNM



Lovato & Benhar, **PRC** 96 054301 (2017)

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Generalisation to finite temperature

- Our basic assumption: at $T \neq 0$ and $T \ll m_\pi \approx 140$ MeV, the Hamiltonian is largely unaffected by thermal effects

- Thermodynamic consistency is not easily achieved in finite-temperature perturbation theory, which can be tested by comparing the two, a priori equivalent, definitions of pressure

$$P = \rho\left(\mu - \frac{F}{N}\right) \qquad P = -\frac{\partial F}{\partial V} = \rho^2 \frac{\partial}{\partial \rho}\frac{F}{N}$$

- Variational approach: minimisation of a trial gran canonical potential

Variational principle

- The minimisation yields the solution for the Fermi distribution

$$\boxed{n(k,T) = \{1 + e^{\beta[e(k,T)-\mu]}\}^{-1}}$$

where the single-particle energy $e(k,T)$ is given by

$$\boxed{e(k,T) = \frac{k^2}{2m} + U_k + \delta e}$$

where

$$U_k = \sum_{k'} \langle kk'|v|kk'\rangle_A \, n(k',T)$$

$$\delta e = \frac{1}{2} \sum_{k,k'} \langle kk'|\frac{\partial v}{\partial \rho}|kk'\rangle_A \, n(k,T) n(k',T)$$

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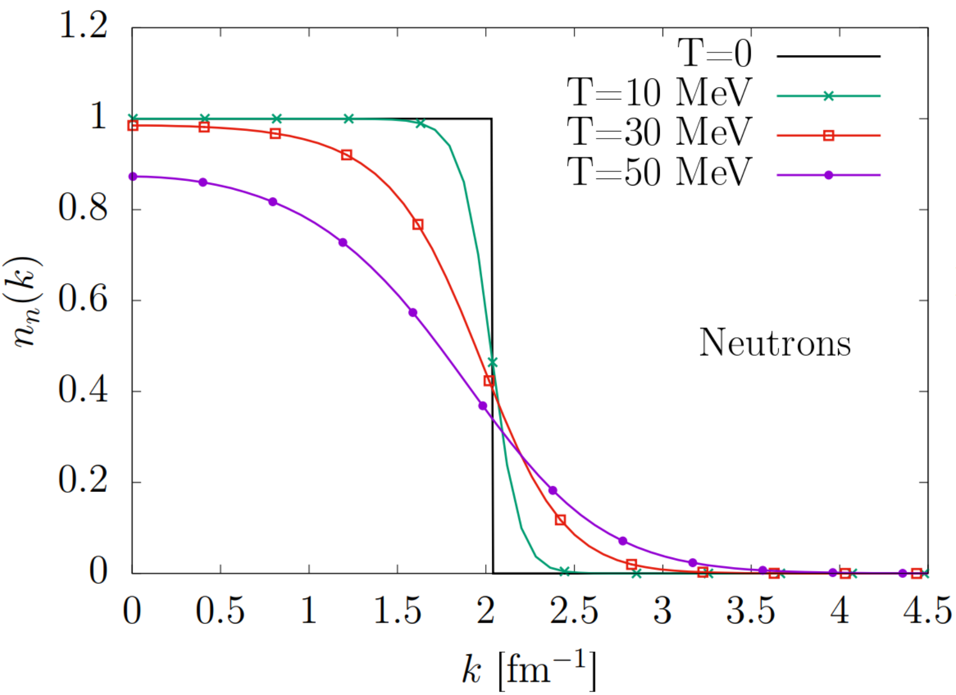
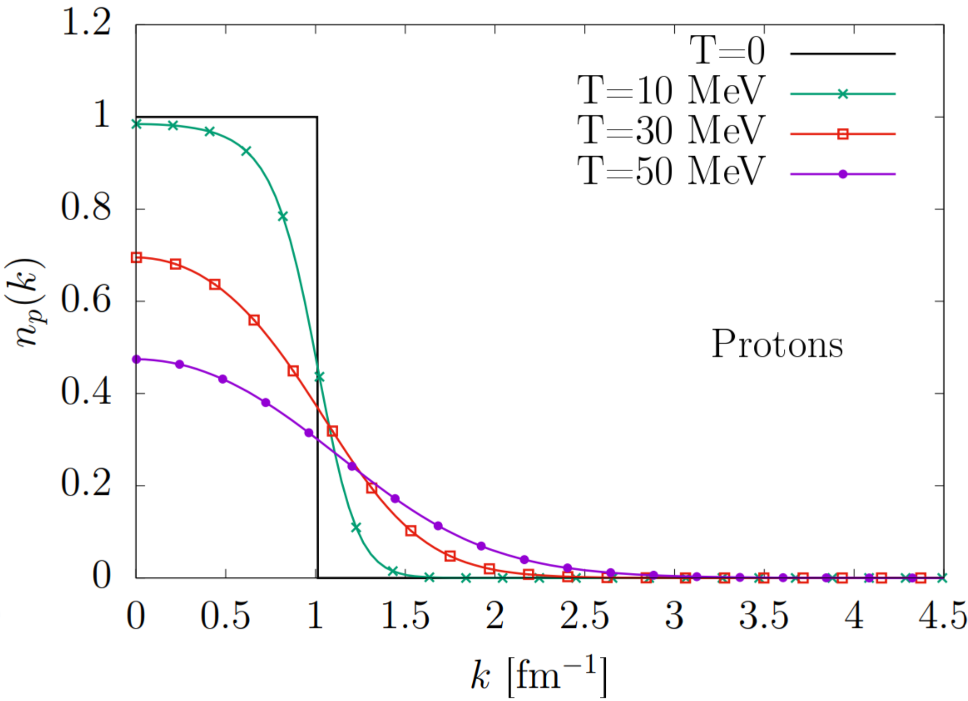
Fermi distributions

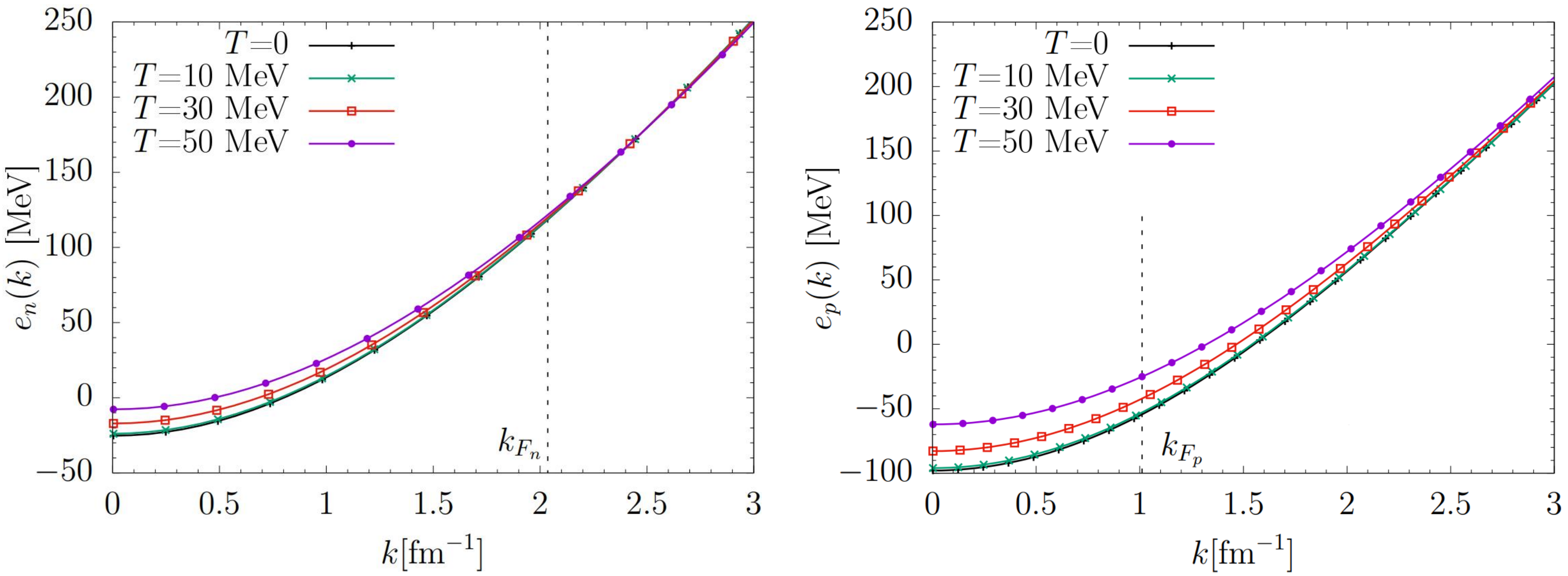
→ Contain all thermodynamic information





Beta-stable matter at $\rho = 2\rho_0$

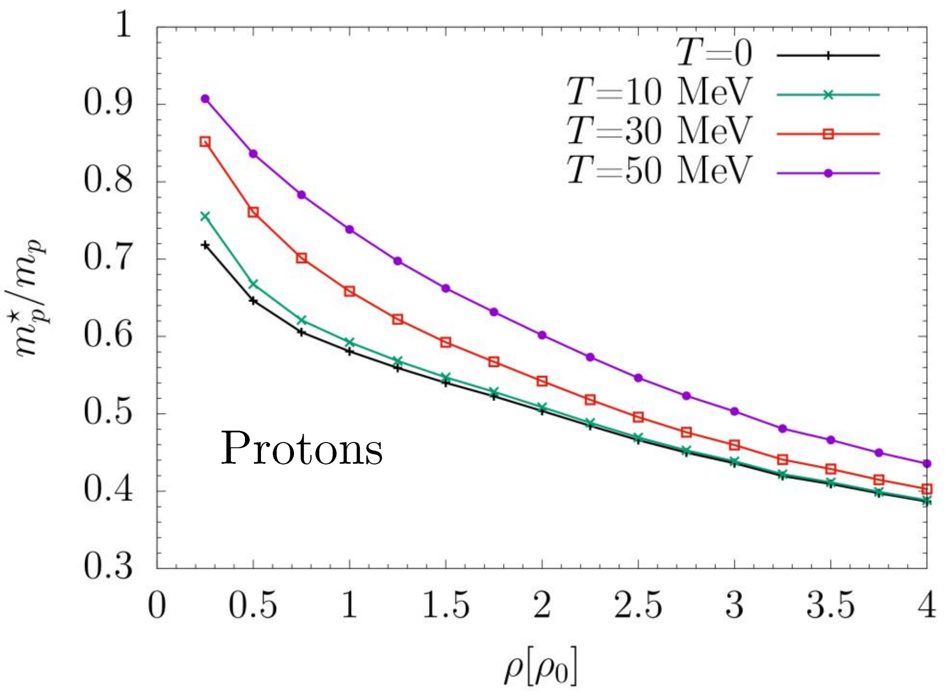
Single-particle energies (spectrum)

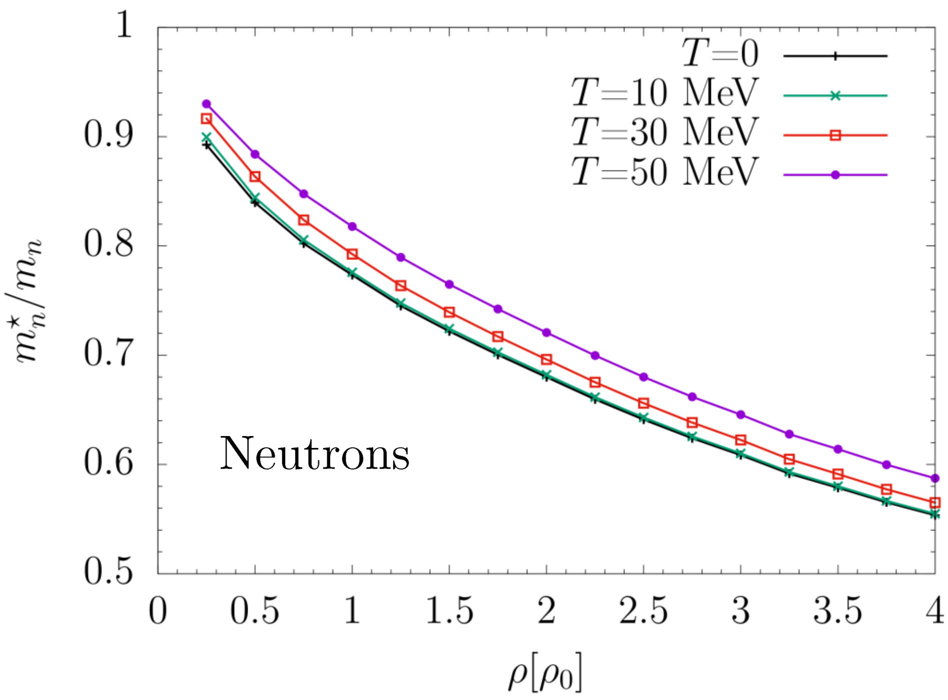


Beta-stable matter at $\rho = 2\rho_0$

Effective mass

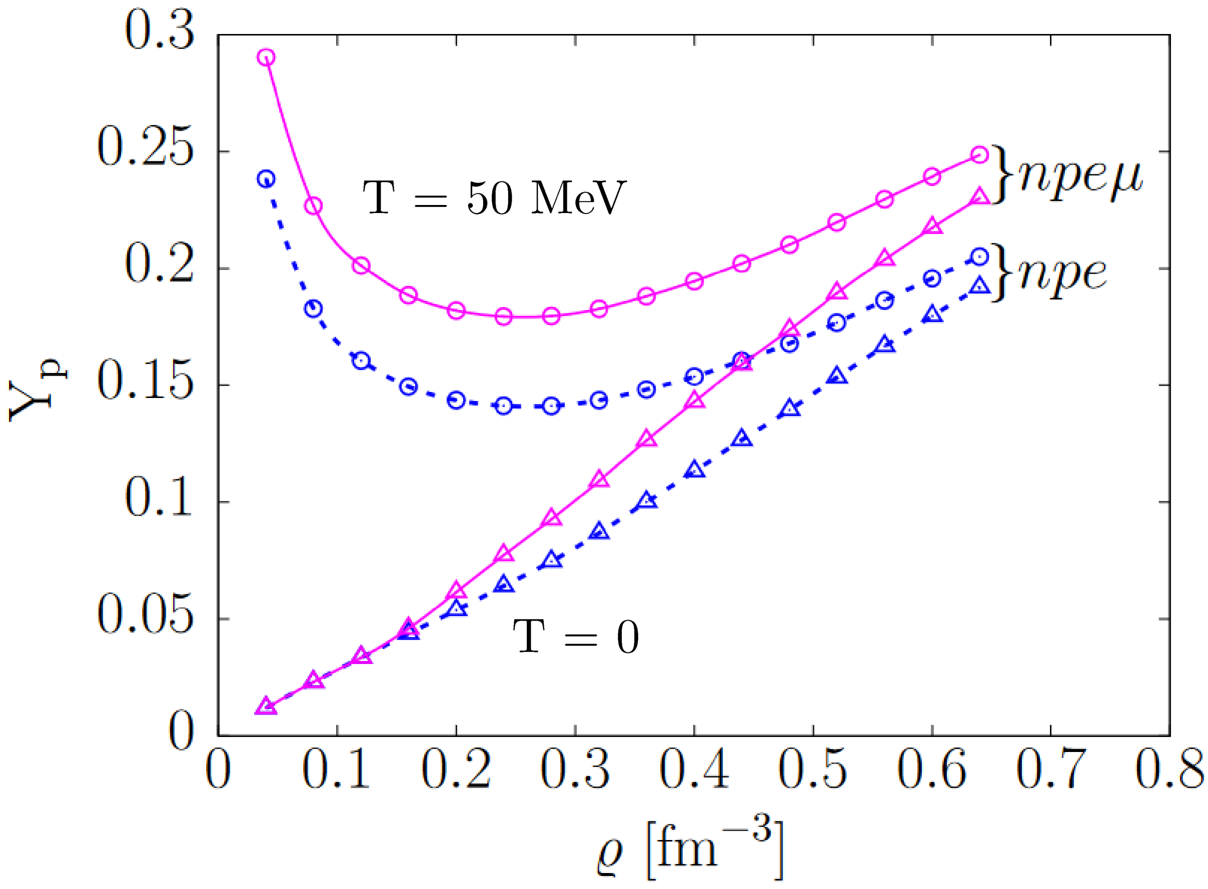
→ Determines the nucleon dispersion relation → $\frac{1}{m^\star(k,T)} = \left(\frac{1}{k}\frac{de(k,T)}{dk}\right)_{k=k_F}$





Neutrons → larger density → weaker temperature effects

Chemical composition



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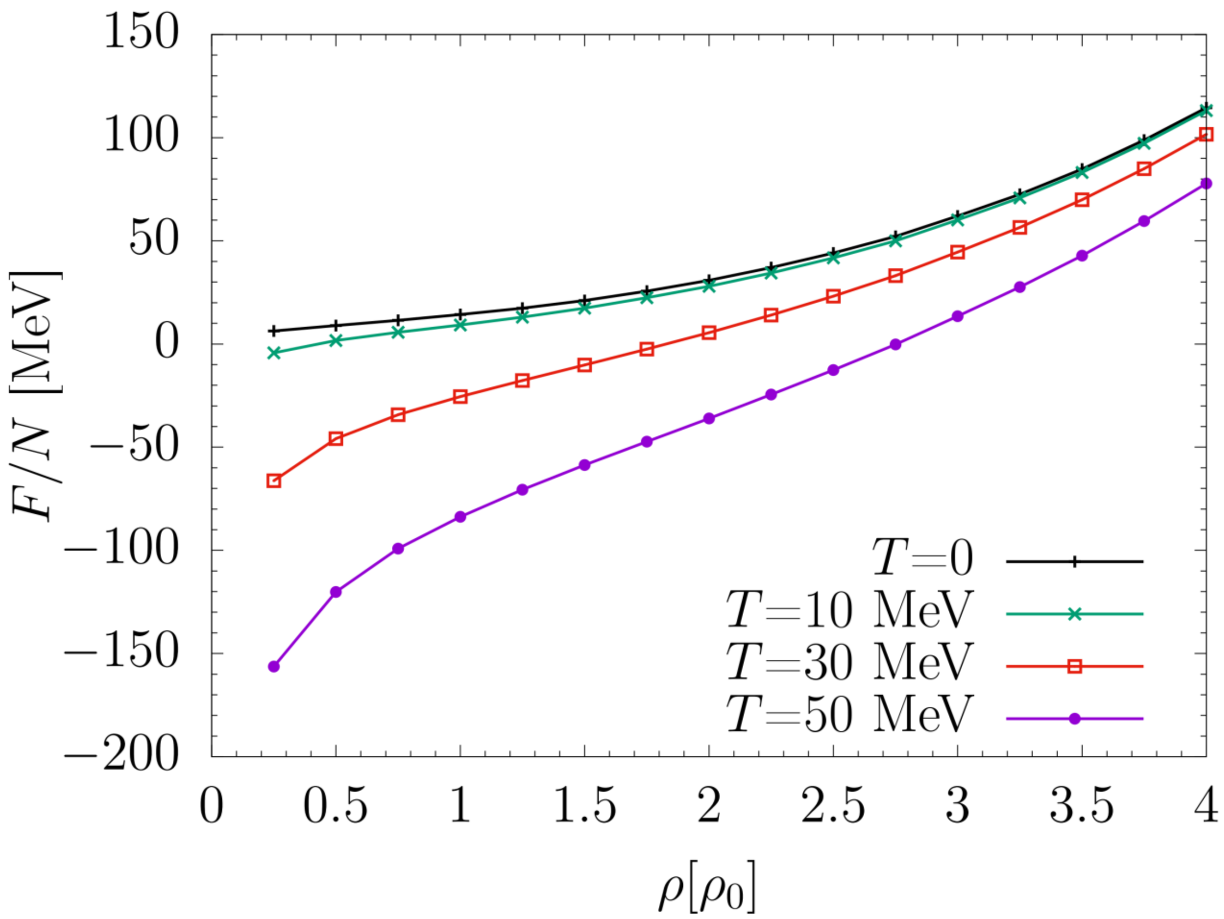
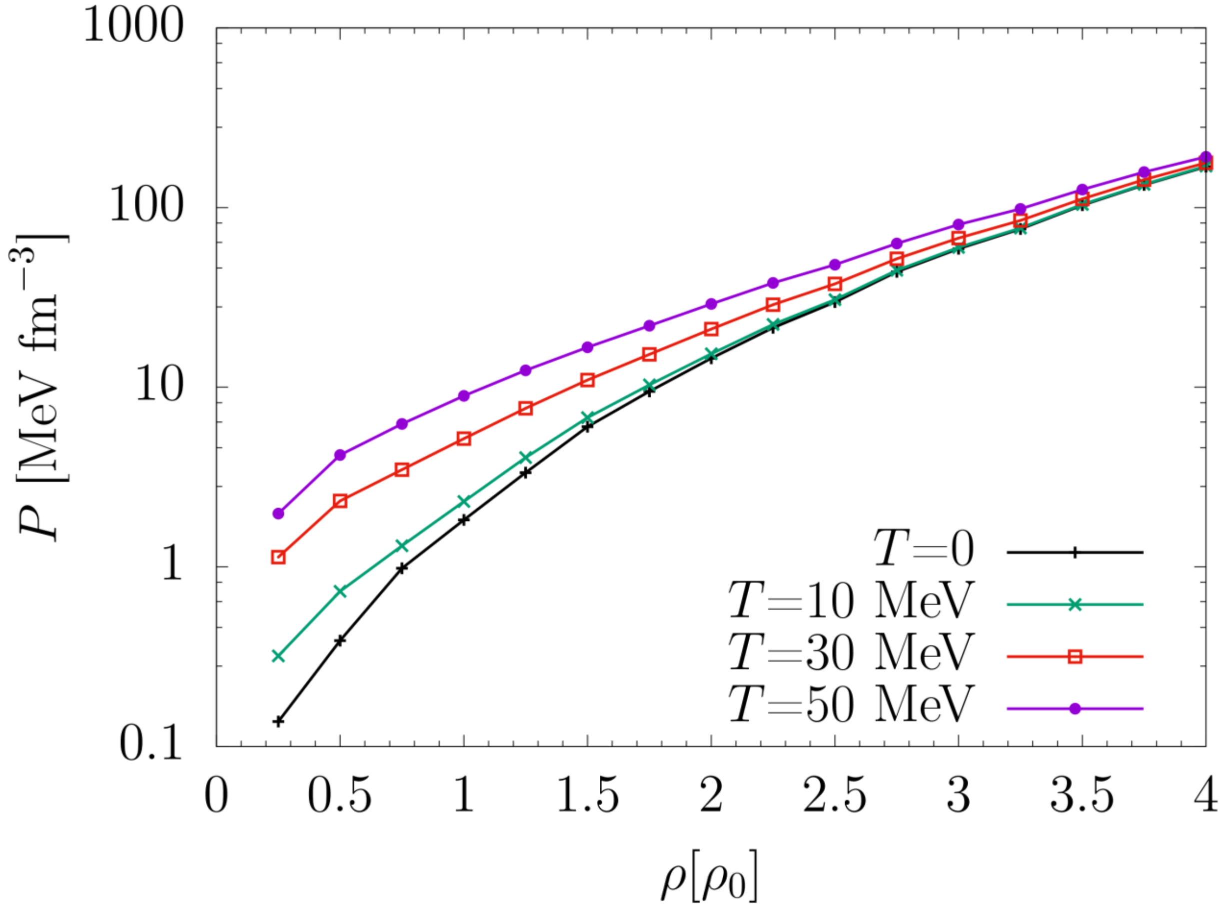
Free energy



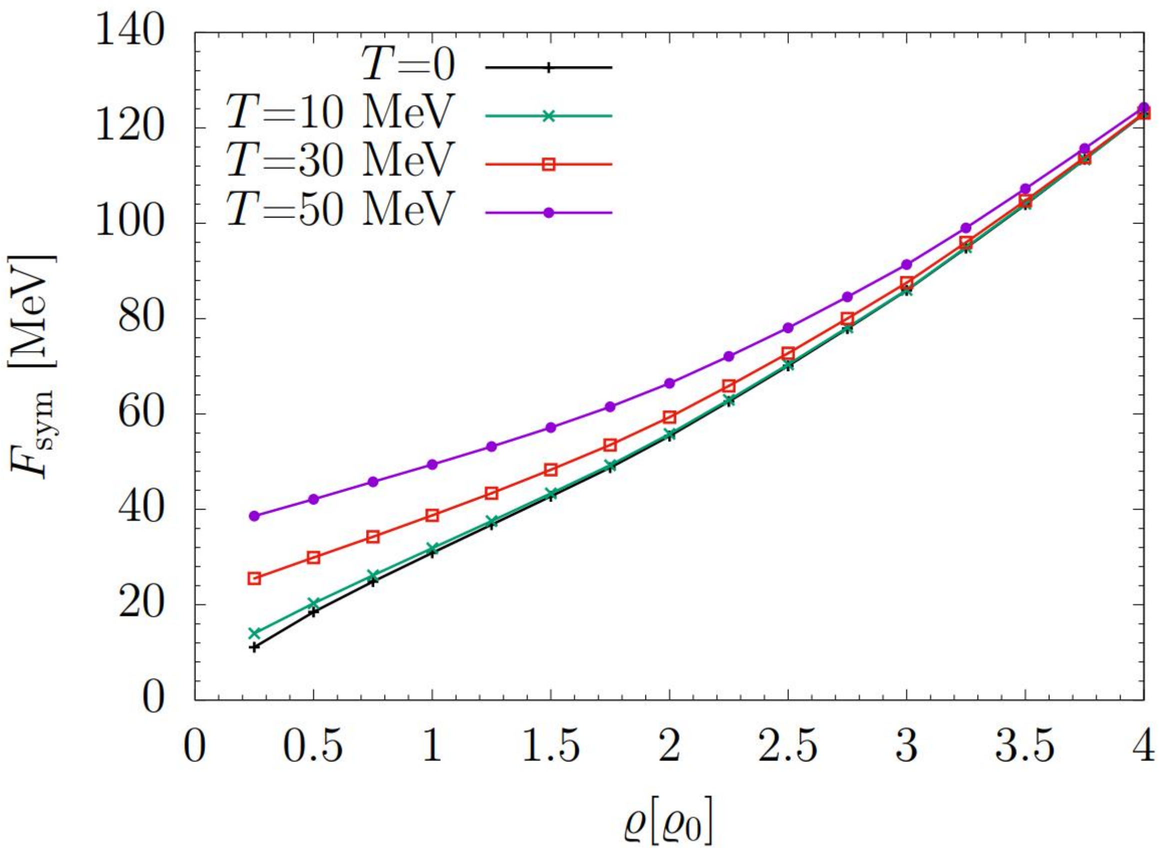
150
100
50
0
−50
−100
−150
−200
F/N [MeV]
0
0.5
1
1.5
2
2.5
3
3.5
4
$\rho[\rho_0]$
T=0
T=10 MeV
T=30 MeV
T=50 MeV

Pressure



Clearly at high densities the thermal contribution becomes less relevant

Symmetry free energy



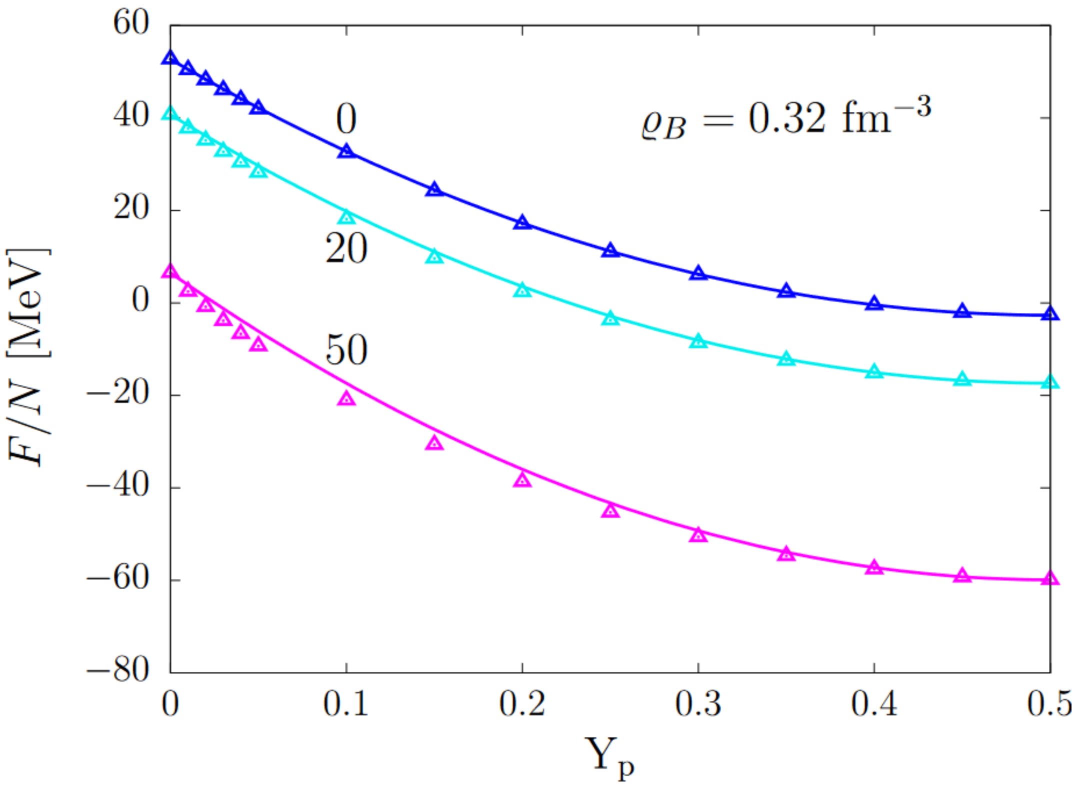
At T = 0:

$E_{sym} = 30.9$ MeV

$L = 67.9$ MeV

Symmetry free energy: quadratic approximation

$$\frac{F}{N}(\rho, T, Y_p) = \frac{F}{N}(\rho, T, 1/2) + F_{\text{sym}}(\rho, T)(1 - 2Y_p)^2$$



Benhar, Lovato & Camelio, arXiv: 2205.11183 (2012)

Exact results VS fit

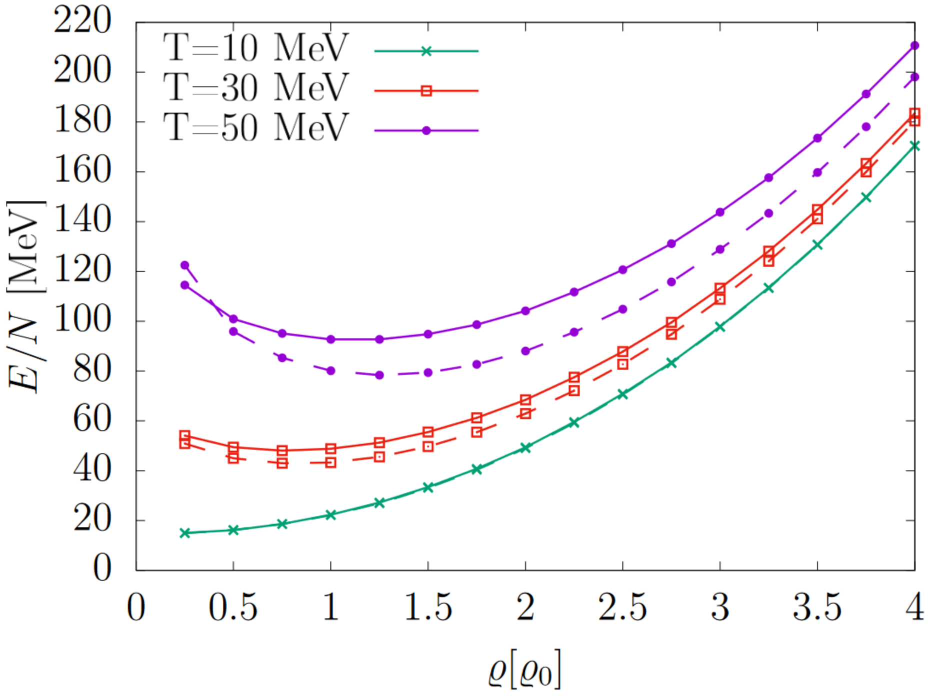
- Lack of hot EOSs: resort to parametrisations that generalise the thermal contribution to an arbitrary $T = 0$ EOS

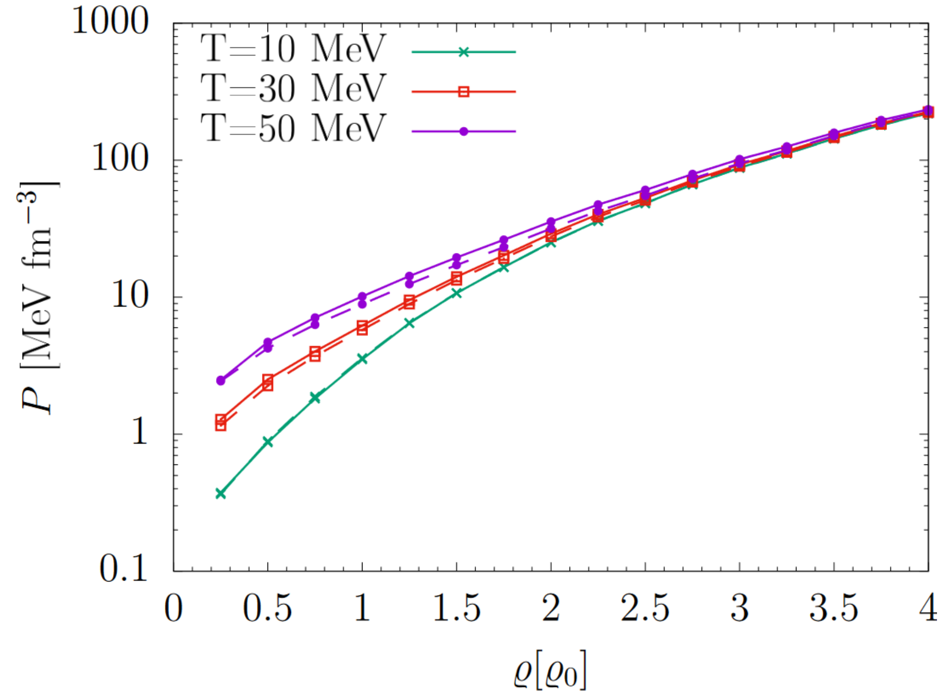
- Widely used model: thermal contribution added as an ideal fluid at T

$$\begin{aligned} p &= p_{\rm cold} + p_{\rm th} \\ e &= e_{\rm cold} + e_{\rm th} \end{aligned} \longrightarrow p_{\rm th}(\varrho, T) = \varrho\, e_{\rm th}(\Gamma_{\rm th} - 1)$$

- Recently Raithel et al. (2019) have proposed a different parametrisation
  - Intermediate densities: ideal fluid
  - High densities: degenerate Fermi gas (Sommerfeld expansion)

Exact results VS Raithel+ (2019)





Maximum errors of ~10%

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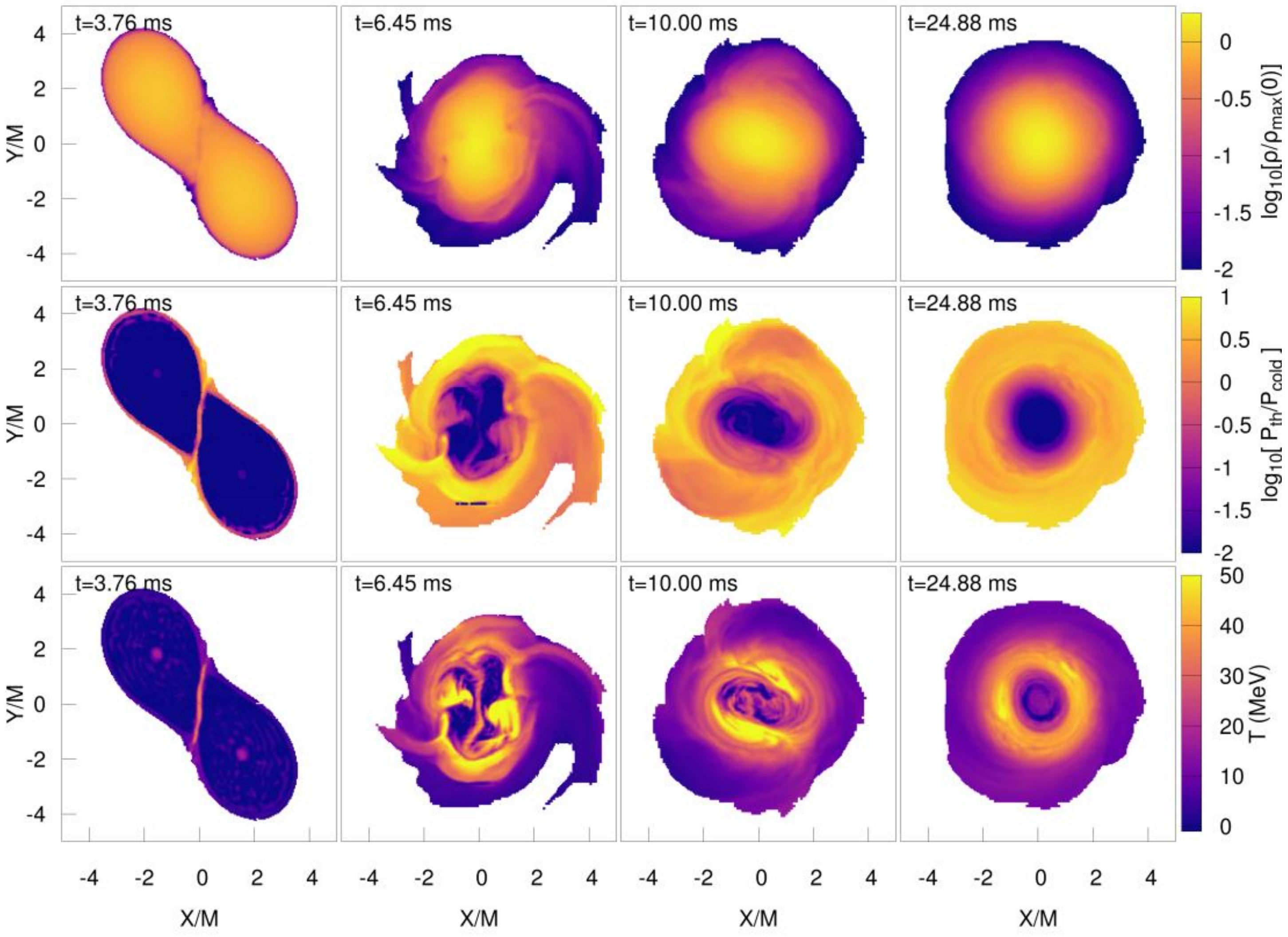
Conclusions and perspectives

- **Robust finite-temperature EOSs** are fundamental to multimessenger astrophysics
- **Thermal effects** are more important to **protons in beta-stable matter**, due to their **lower density**
- **Temperature** also affects the **chemical composition** of NS matter
- Changes in **single-particle energies** and effective masses → relevant to every **nucleon collision process**
- The application to the calculation of bulk viscosity is being carried out

Backup

Merger simulation

Simulations



Raithel+, **PRD** 104 063016 (2021)

CBF Effective Interaction

CBF Effective Interaction

- The effective interaction can be written as

$$v_{ij}^{\mathrm{eff}} = \sum_{p=1}^{6} v^{\mathrm{eff}\,,p}(r_{ij}) O_{ij}^{p}$$

- And the correlated ground state as

$$|\Psi_0\rangle \equiv \frac{F|\Phi_0\rangle}{\langle\Phi_0|F^\dagger F|\Phi_0\rangle^{1/2}}$$

- The correlation operator is written in the form

$$F(1, ..., N) = \mathcal{S} \prod_{j>i=1}^{N} f_{ij}$$

CBF Effective Interaction

- The two-body correlation functions reflect the complexity of the NN potential and can be conveniently expressed as

$$f_{ij} = \sum_{p=1}^{6} f^p(r_{ij}) O_{ij}^p$$

Finite-temperature perturbation theory

Finite-temperature perturbation theory

- Our basic assumption: at $T \neq 0$ and $T \ll m_\pi \approx$140 MeV, the Hamiltonian is largely unaffected by thermal effects
- All thermodynamic functions of a system in equilibrium at temperature T can be obtained from the grand canonical potential

$$\Omega = -\frac{1}{\beta} \ln Z$$

where the partition function Z is

$$Z = \mathrm{Tr}\ \Phi$$

with

$$\Phi = e^{-\beta(H - \mu N)}$$

Finite-temperature perturbation theory

- The basis for the derivation of finite-temperature perturbation theory is the Bloch equation

$$-\frac{\partial \Phi}{\partial \beta} = (H - \mu N)\Phi$$

- To find the perturbative expansion of Z we can explore the similarity between this equation and Schroedinger's

- Rewriting the Hamiltonian as $H = H_0 + H_I$,

$$-\frac{\partial \Phi}{\partial \beta} = [(H_0 - \mu N) + H_I]\Phi$$
$$= (H_0' + H_I)\Phi$$

Finite-temperature perturbation theory

- H_0 and H_I are written as

$$H_0 = \sum_k e_k a_k^\dagger a_k$$

$$H_I = \frac{1}{2} \sum_{k,k',q,q'} \langle k'q' \,|\, v \,|\, kq \rangle \, a_{k'}^\dagger a_{q'}^\dagger a_q a_k - \sum_k U_k a_k^\dagger a_k$$

where e_k is the single-particle energy, which plays an important role in determining thermal effects.

$$e_k = \frac{\mathbf{k}^2}{2m} + U_k = t_k + U_k$$

Finite-temperature perturbation theory

- It can be shown that, at first order, $\Omega = \Omega_0 + \Omega_1$ can be written as

$$\Omega_0 = -\frac{1}{\beta}\ln Z_0 = -\frac{1}{\beta}\sum_k \ln\left(1 + e^{-\beta(e_k-\mu)}\right)$$

$$= \sum_k (e_k - \mu) n_k + \frac{1}{\beta}\sum_k [n_k \ln n_k + (1-n_k)\ln(1-n_k)]$$

$$\Omega_1 = -\frac{1}{\beta}\ln Z_1 = \frac{1}{2}\sum_{kk'} \langle kk' \,|\, v \,|\, kk' \rangle_A n_k n_{k'} - \sum_k U_k n_k$$

Finite-temperature perturbation theory

- It can be shown that, at first order, $\Omega = \Omega_0 + \Omega_1$ can be written as

$$\Omega_0 = -\frac{1}{\beta}\ln Z_0 = -\frac{1}{\beta}\sum_k \ln\left(1+e^{-\beta(e_k-\mu)}\right)$$

$$= \sum_k (e_k-\mu)n_k + \frac{1}{\beta}\sum_k [n_k \ln n_k + (1-n_k)\ln(1-n_k)]$$

$E - \mu N$ $\qquad S$

$$n_k = \left[1+e^{\beta(e_k-\mu)}\right]^{-1}$$

Fermi distribution

$$\Omega_1 = -\frac{1}{\beta}\ln Z_1 = \frac{1}{2}\sum_{kk'}\langle kk' \,|\, v \,|\, kk'\rangle_A n_k n_{k'} - \sum_k U_k n_k$$

Variational calculation at finite T

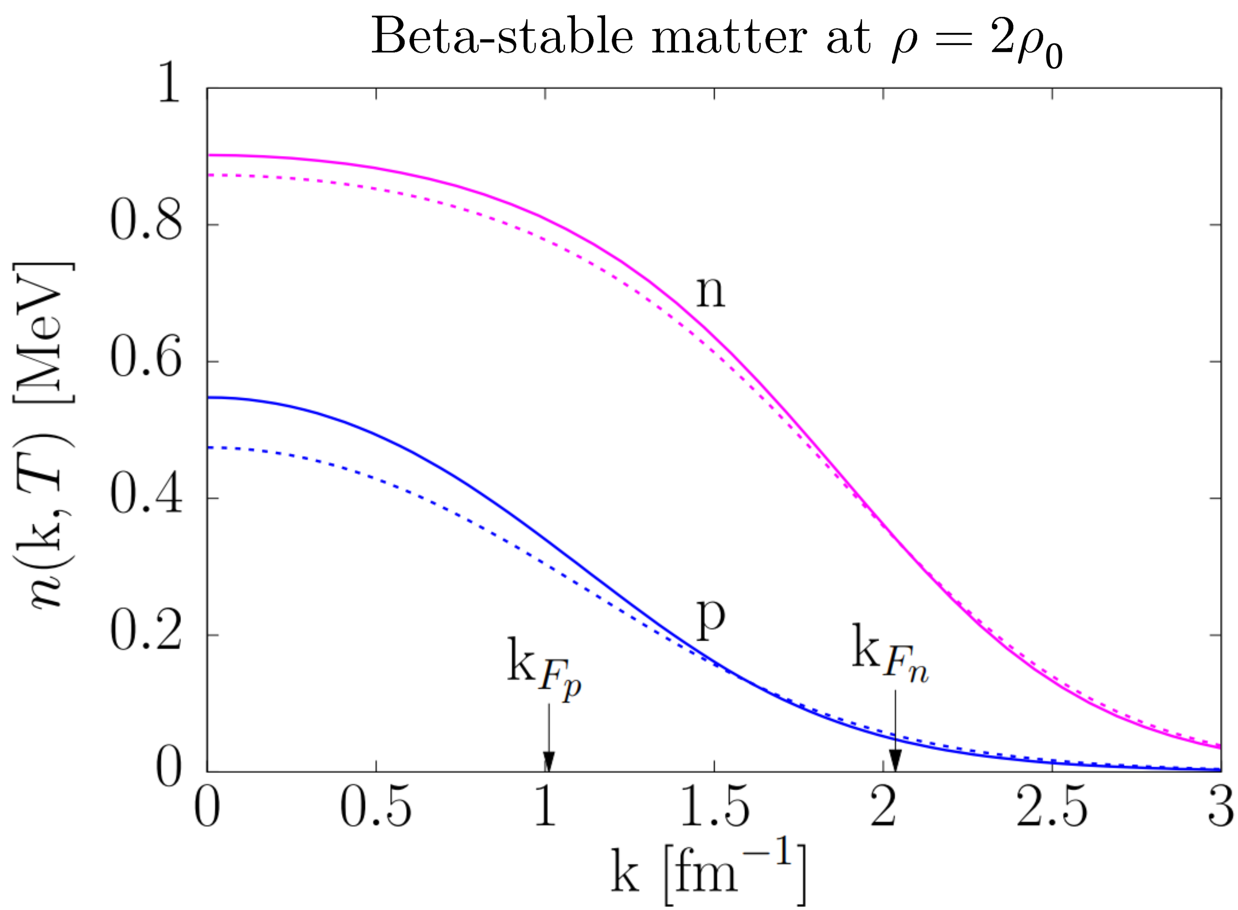
Hartree-Fock gran canonical potential

$$\widetilde{\Omega} = \sum_k t_k n_k + \frac{1}{2} \sum_{k,k'} \langle kk'|v|kk'\rangle_A \; n_k n_{k'} + \frac{1}{\beta} \sum_k \left[n_k \ln n_k + (1 - n_k) \ln(1 - n_k) \right]$$

- The given potential is totally valid in a variational context
- It recovers the HF approximation at T=0
- The formalism is thermodynamically consistent by construction

Frozen correlations

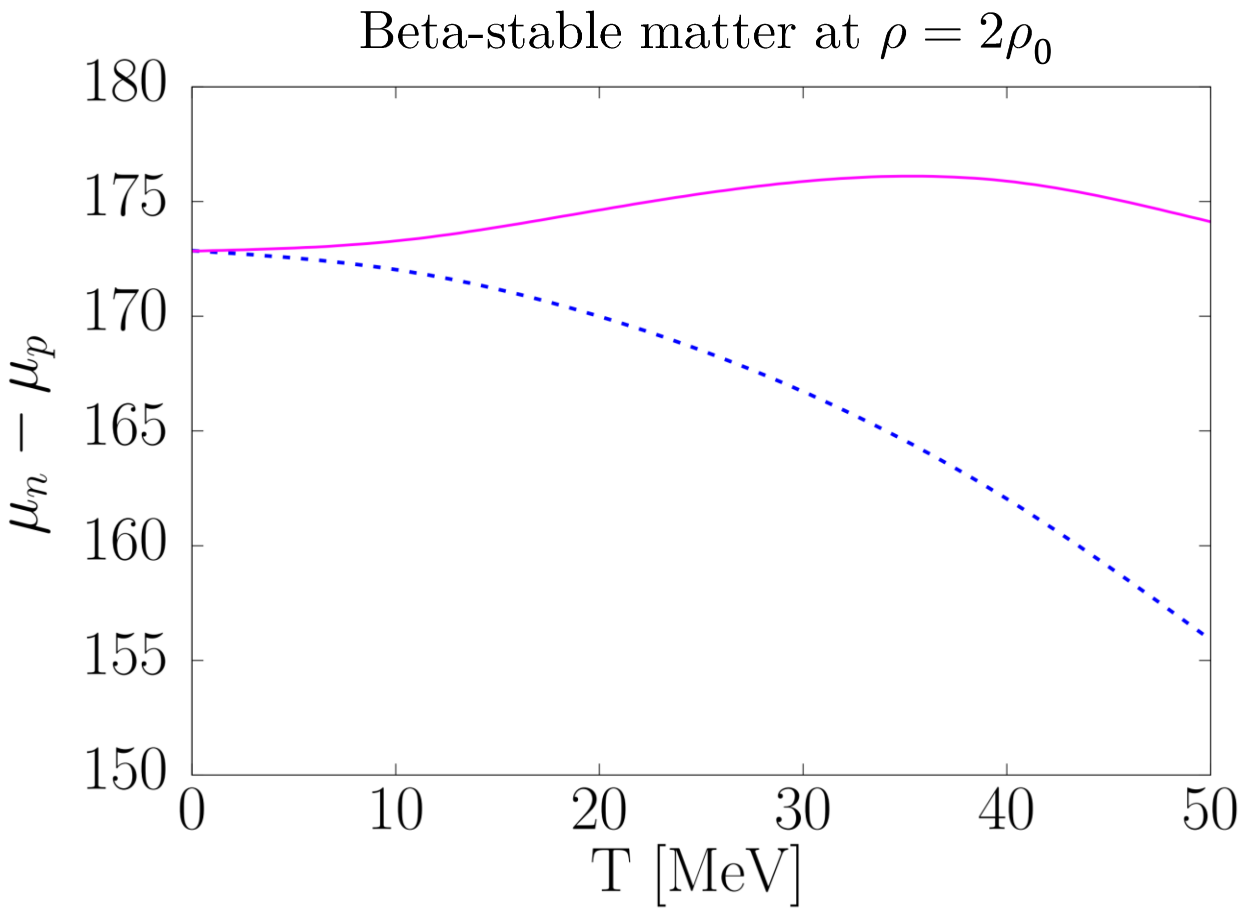
Spectrum → Fermi distribution



$T = 50$ MeV

Solid line: Fermi distribution assuming $e(k, T)$
Dashed line: Fermi distribution assuming $e(k, T = 0)$

Spectrum → Chemical potential



Changes of chemical composition in β-stable matter

Dashed line: assuming $e(k,T)$

Solid line: assuming $e(k,T=0)$