

# Exploring jet quenching in expanding media

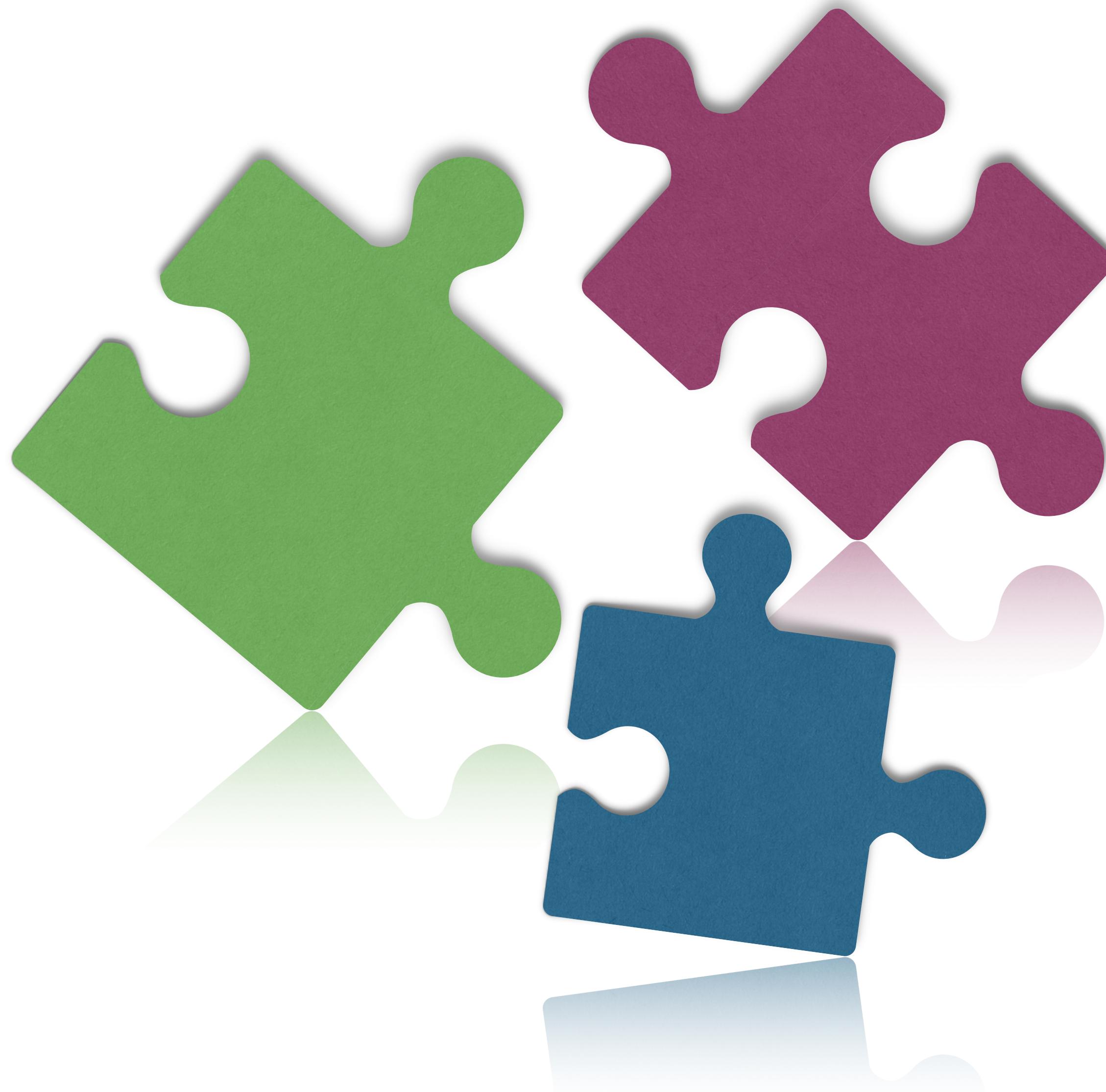
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# Collecting the pieces

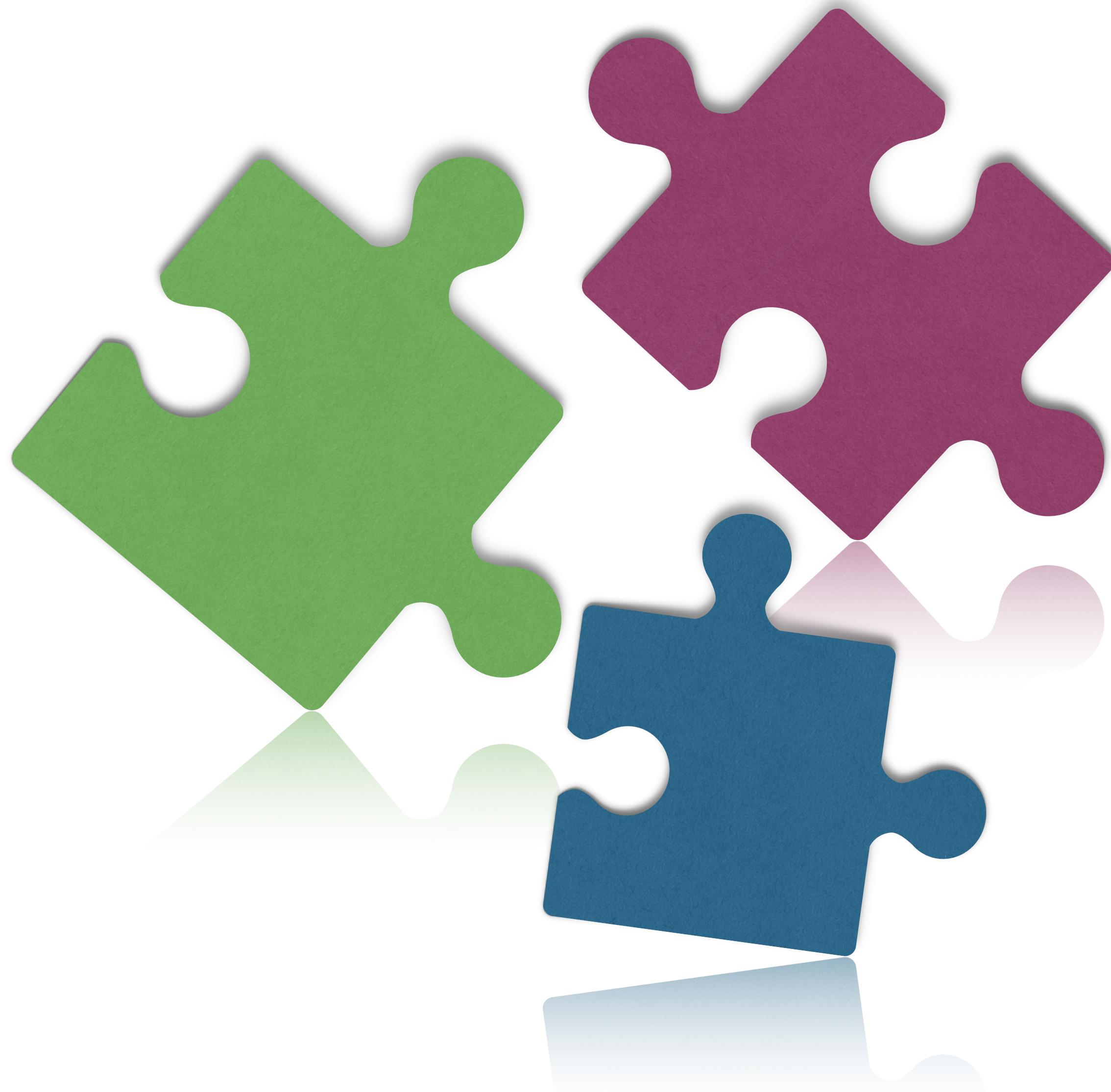


- **Gluonic cascades in expanding medium.**
- **Multi- partonic cascades in expanding medium.**
- **Transverse momentum broadening in cascades in expanding medium.**

*Complexity/ Completeness  
towards understanding*



# Collecting the pieces



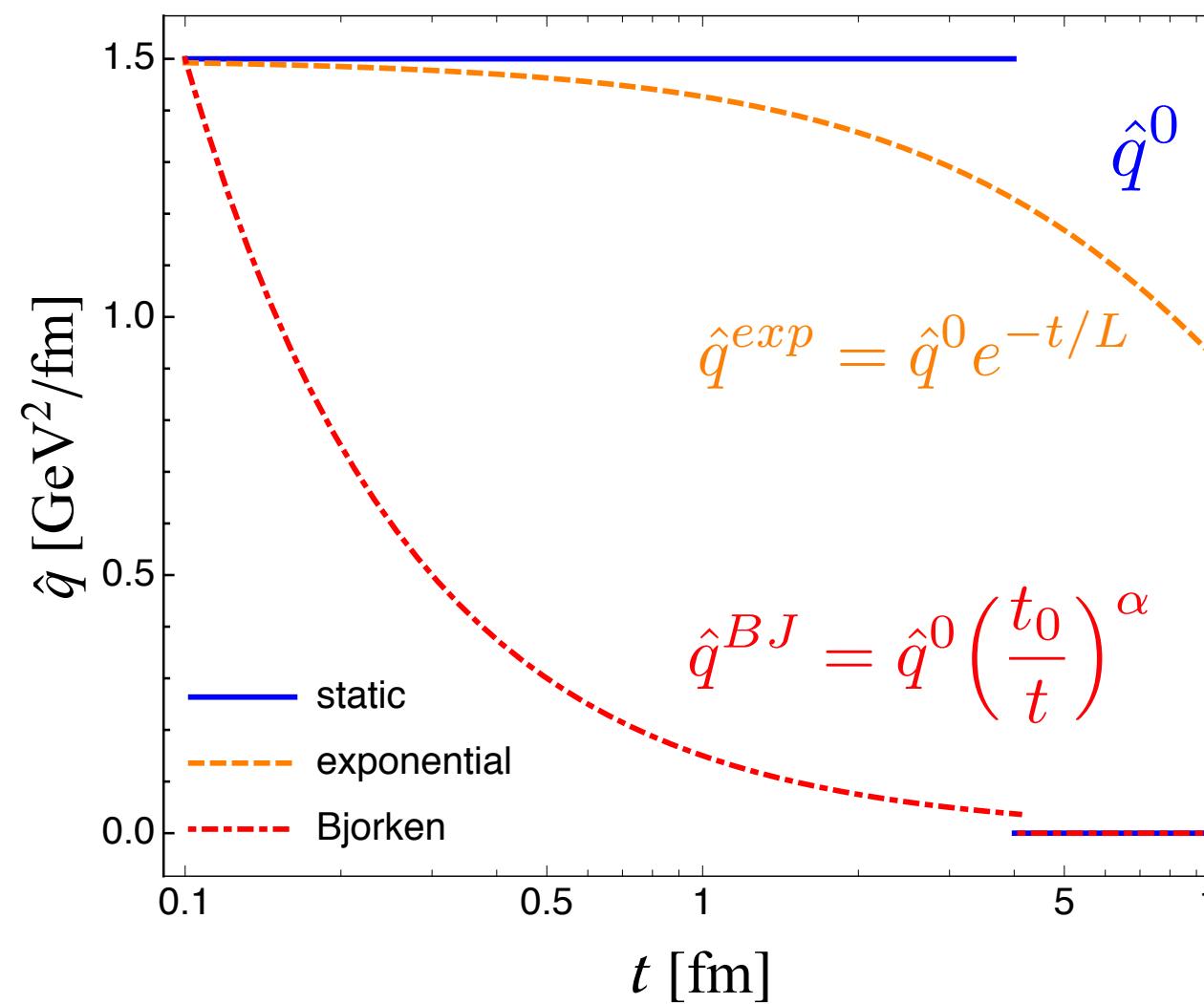
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- Multi- partonic cascades with expanding medium.
- Transverse momentum broadening in cascades in expanding medium.



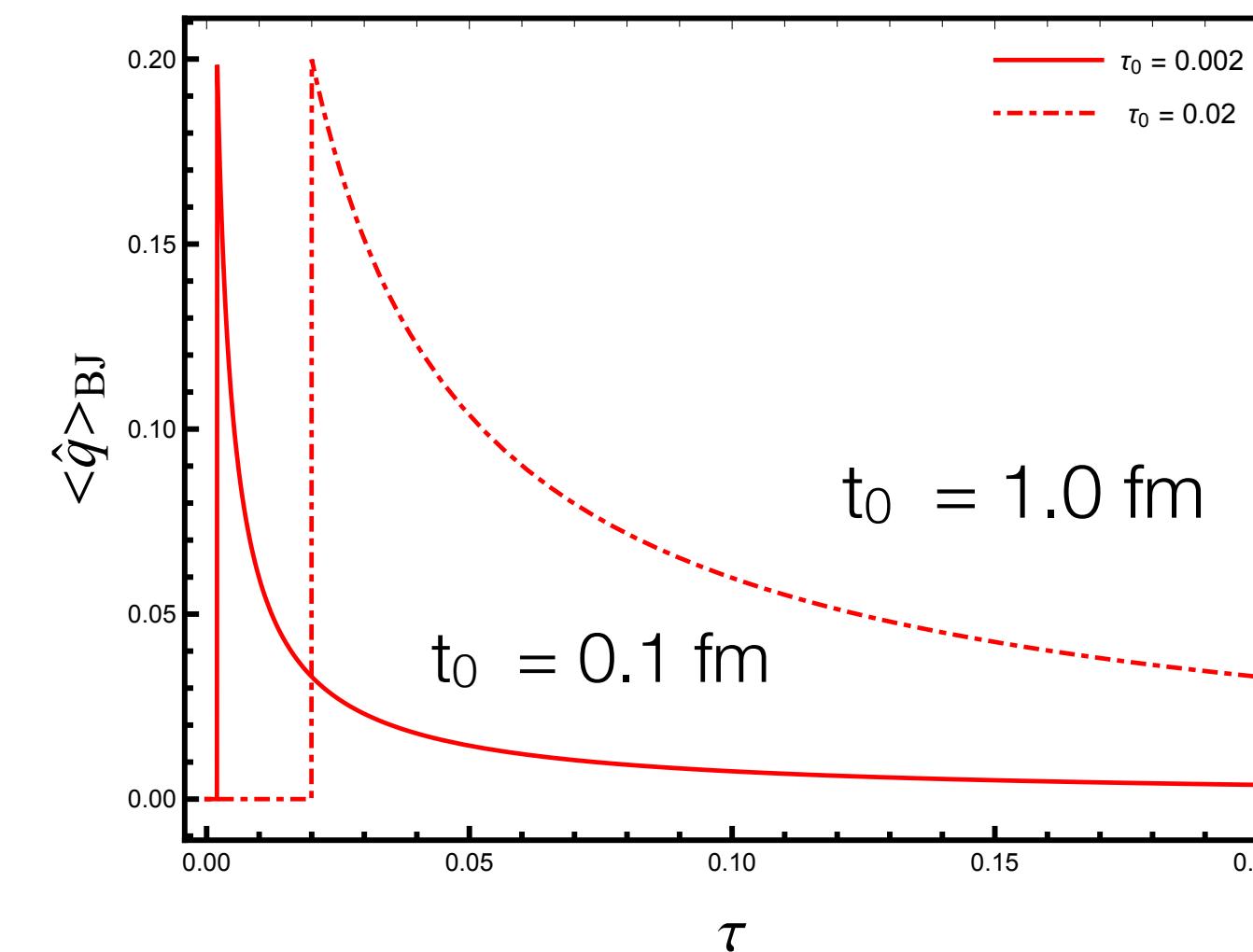
*Complexity/ Completeness  
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# Medium profiles and calculation workflow :

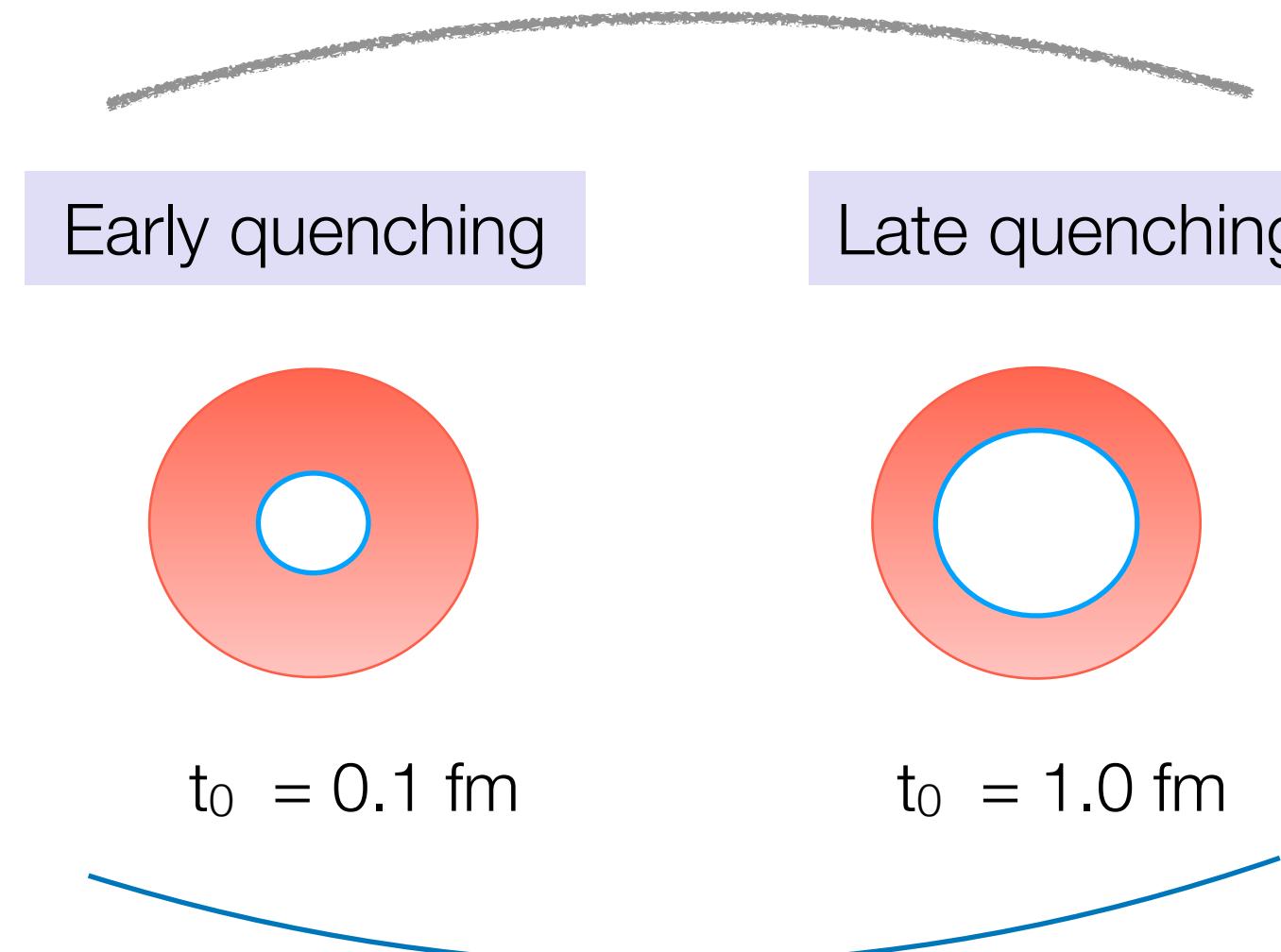


Quenching  
parameter



Early quenching

Late quenching



Bjorken initial conditions

**Evolution equations =>**

$$\star \quad \frac{\partial}{\partial \tau} D_g(x, \tau) = \int_0^1 dz K_{gg} \left[ \sqrt{\frac{z}{x}} D_g \left( \frac{x}{z} \right) - \frac{z}{\sqrt{x}} D_g(x) \right] - \int_0^1 z K_{qg}(z) \frac{z}{\sqrt{x}} D_g(x) + \int_0^1 z K_{gq}(z) \sqrt{\frac{z}{x}} D_S \left( \frac{x}{z} \right)$$

$$\star \quad \frac{\partial}{\partial \tau} D_S(x, \tau) = \int_0^1 dz K_{qq}(z) \left[ \sqrt{\frac{z}{x}} D_S \left( \frac{x}{z} \right) - \frac{1}{\sqrt{x}} D_S(x) \right] + \int_0^1 dz K_{qg}(z) \sqrt{\frac{z}{x}} D_g \left( \frac{x}{z} \right)$$

$D_S$  = q singlet spectra

$D_g$  = gluon spectra

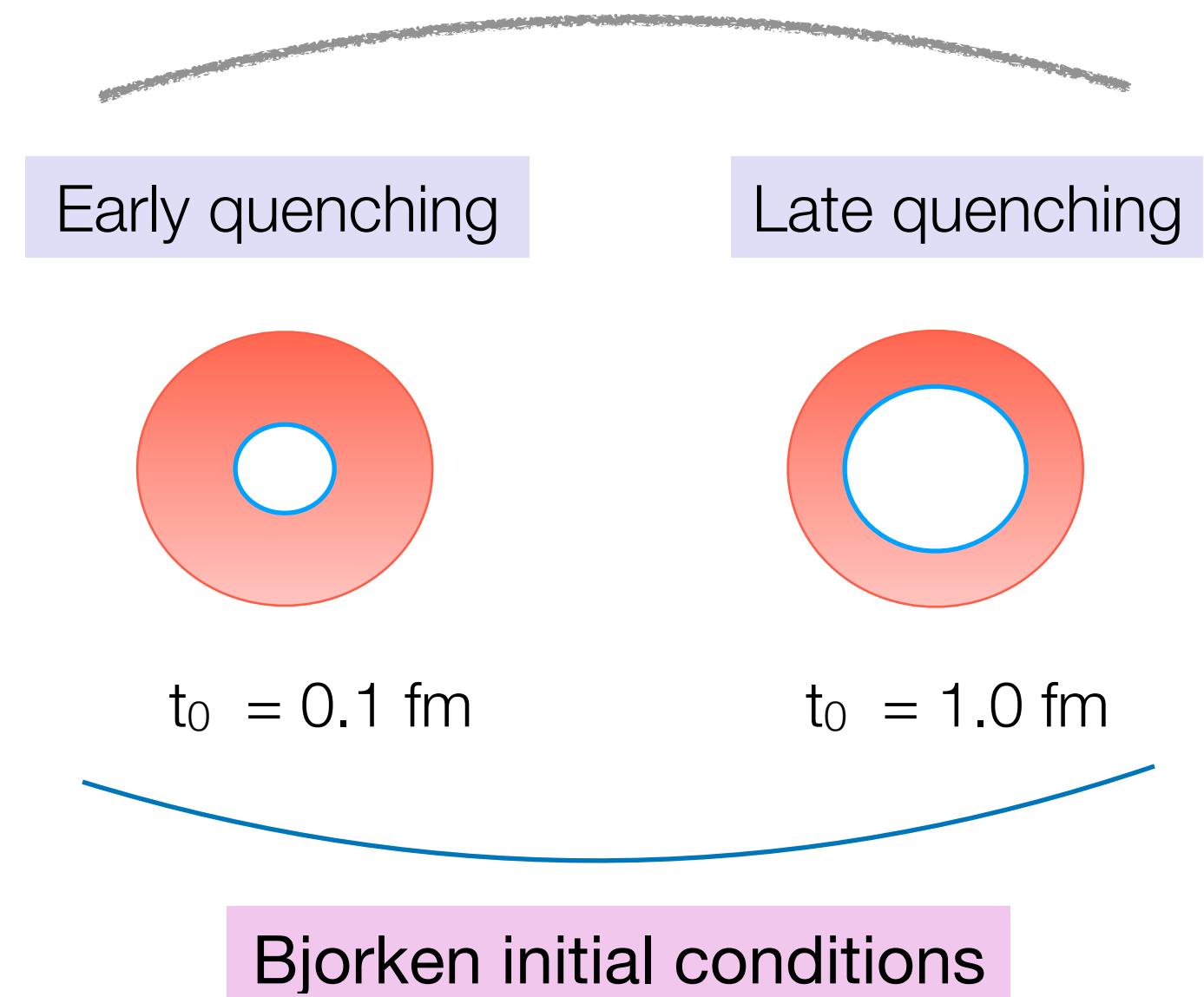
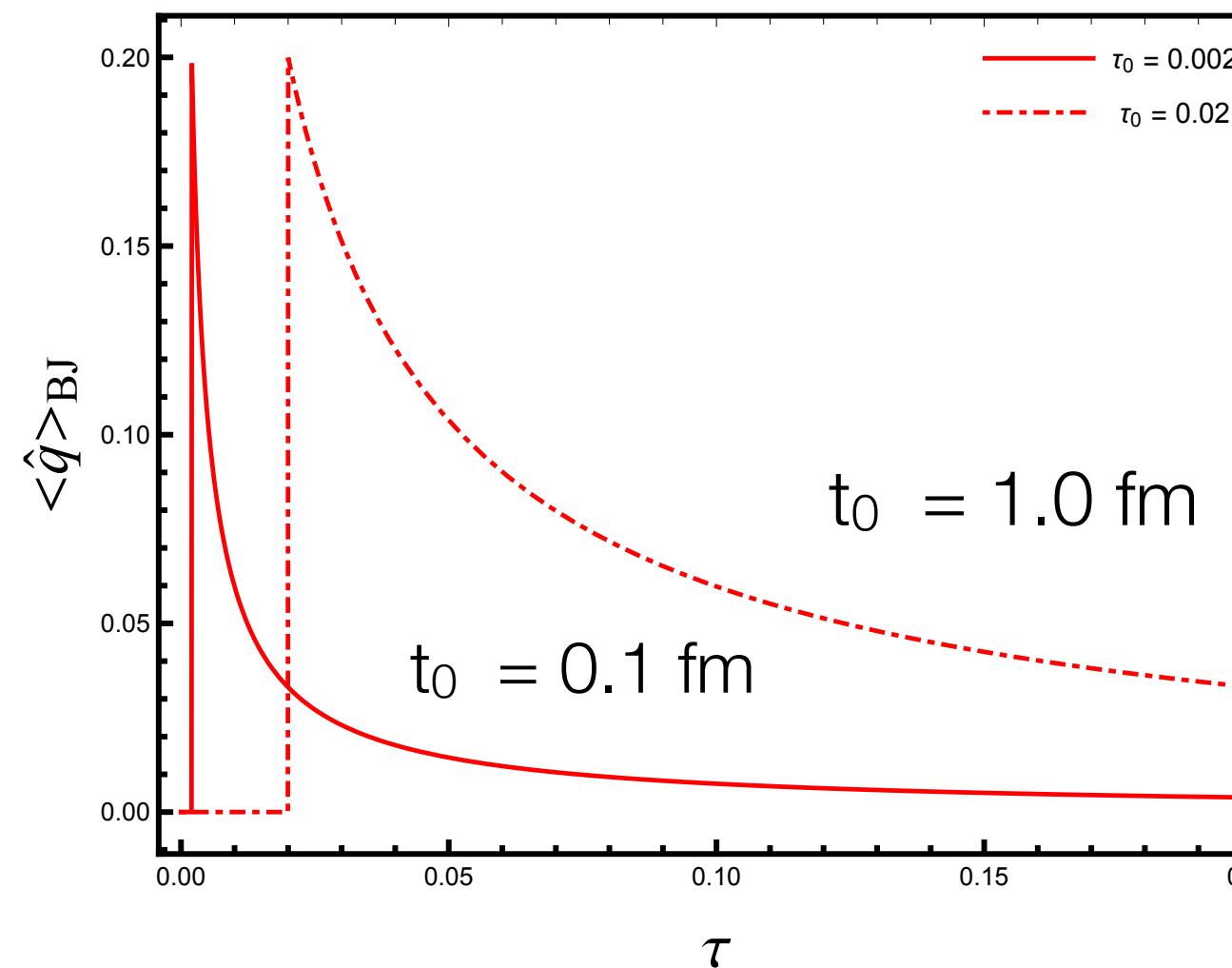
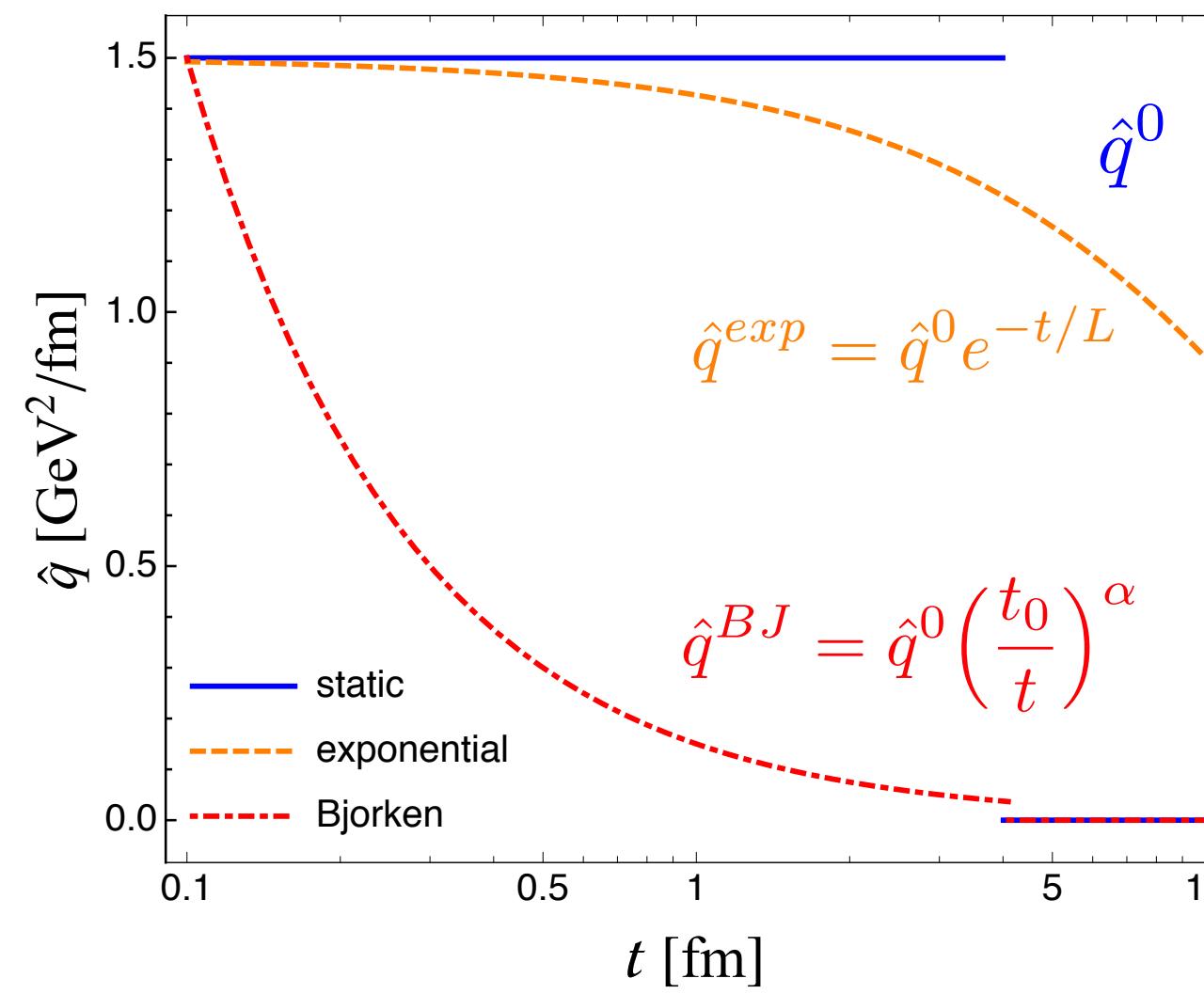
K = splitting rate

$\tau$  = evolution variable

S. S and Y. M-T ; JHEP 09 (2018) 144.

J-P. B., F. D., E. I, Y. M-T ; JHEP 06 (2014) 075.

# Medium profiles and calculation workflow :



Evolution equations =>

★  $\frac{\partial}{\partial \tau} D_g(x, \tau) = \int dz \mathcal{K}(z, \tau | p) \left[ \sqrt{\frac{z}{x}} D\left(\frac{x}{z}, \tau\right) - \frac{z}{\sqrt{x}} D(x, \tau) \right] - \int_0^1 z K_{qg}(z) \frac{z}{\sqrt{x}} D_g(x) + \int_0^1 z K_{gg}(z) \sqrt{\frac{z}{x}} D_S\left(\frac{x}{z}\right)$

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$D_s$  = q singlet spectra  
 $D_g$  = gluon spectra  
 $K$  = splitting rate  
 $\tau$  = evolution variable

S. S and Y. M-T ; JHEP 09 (2018) 144.

Single parton emission spectra ( $D$ ) in BDMPS-Z formalism for static, exponential and Bjorken expanding media

Splitting rates in static, exponential and Bjorken expanding media

Kinematic rate equation taking into account all the possible splittings for quark & gluon initiated jets

Optimisation in the Quenching factor for jets with combined q and g fractions through modified power law, nPDF and VLE

Study of rapidity dependence and estimation of elliptic flow

J-P. B., F. D., E. I., Y. M-T ; JHEP 06 (2014) 075.

# Scaling behaviour of the spectrum

The single gluon emission spectra are given as :

$$\frac{dI}{dz}^{static,soft} \simeq \frac{\alpha_s P(z)}{\pi} \sqrt{\frac{\omega_c}{2\omega}}$$

$$\frac{dI}{dz}^{static} = \frac{\alpha_s}{\pi} P(z) \operatorname{Re} \ln[\cos(\Omega_0 L)]$$

$$\frac{dI}{dz}^{expo} = \frac{\alpha_s}{\pi} P(z) \operatorname{Re} \ln J_0(2\Omega_0 L)$$

$$\frac{dI}{dz}^{BJ} = \frac{\alpha_s}{\pi} P(z) \operatorname{Re} \ln \left[ \left( \frac{t_0}{L+t_0} \right)^{1/2} \frac{J_1(z_0)Y_0(z_L) - Y_1(z_0)J_0(z_L)}{J_1(z_L)Y_0(z_L) - Y_1(z_L)J_0(z_L)} \right]$$

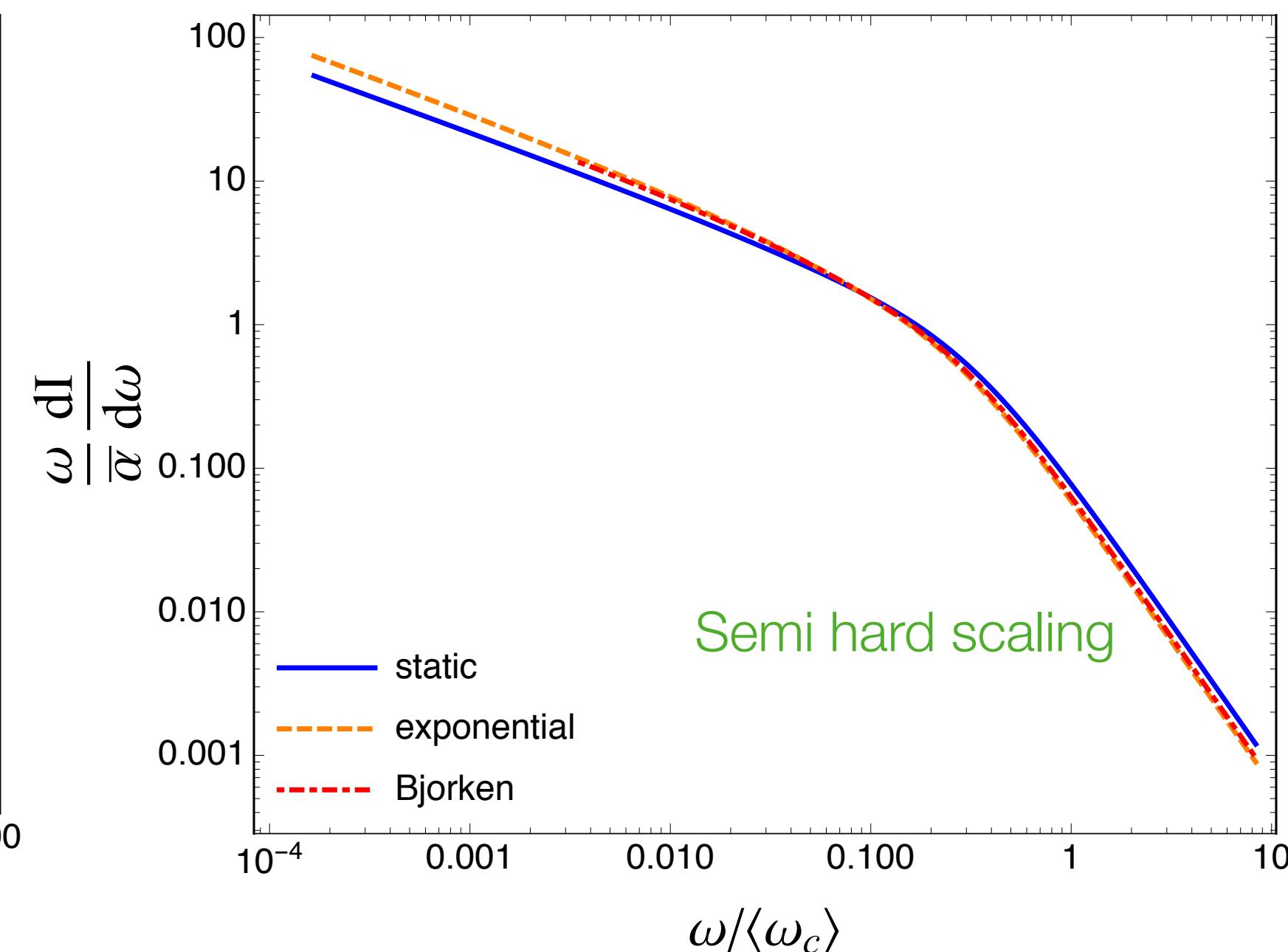
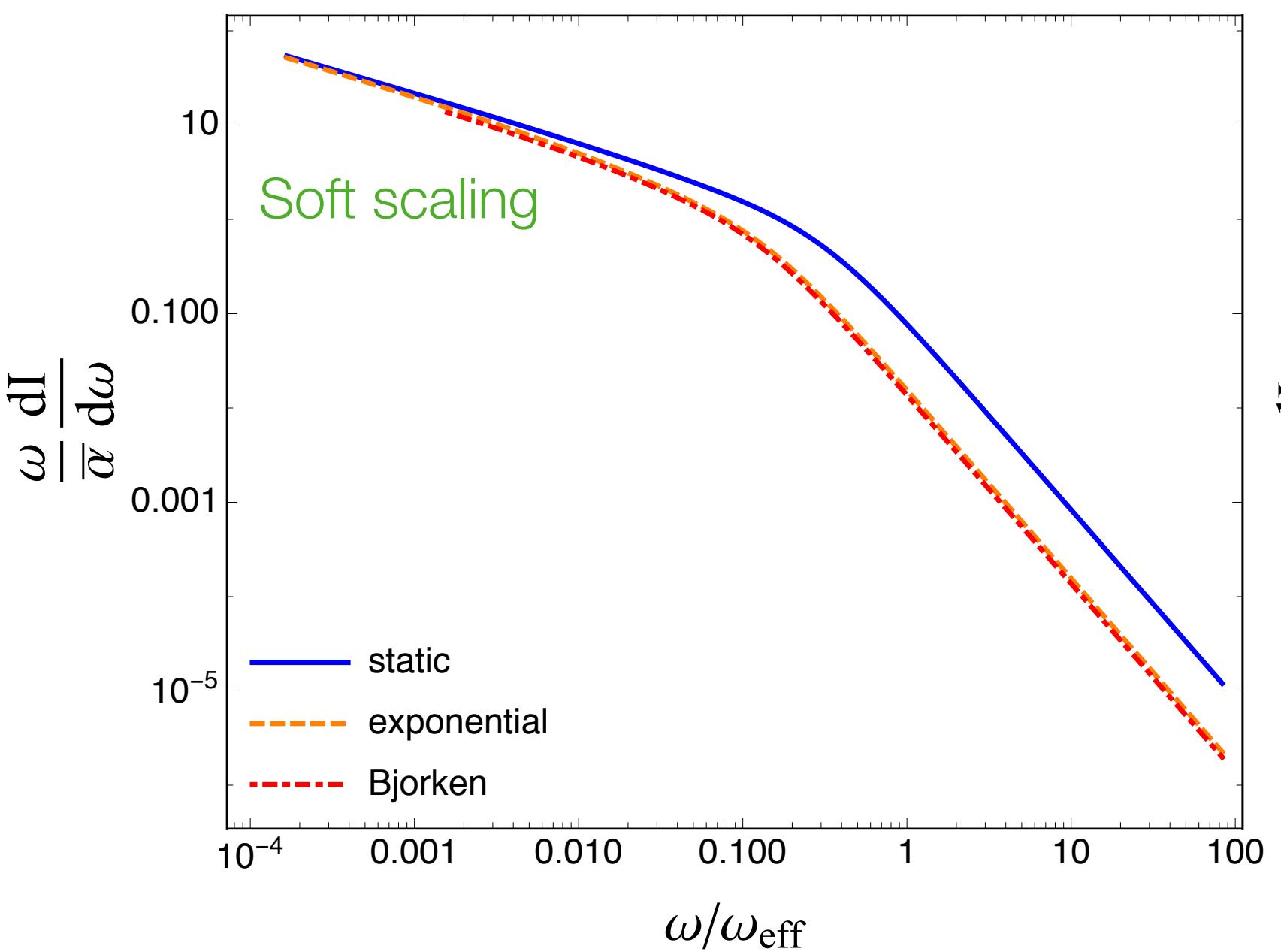
P. B. Arnold., PRD 79 (2009) 065025

$$\Omega_0 L = \sqrt{\frac{-i \hat{q}_0}{2p}} \kappa(z) L \quad \tau \equiv \sqrt{\frac{\hat{q}_0}{p}} L$$

$$z_0 \equiv (1-i)\kappa(z)\tau_0 \\ z_L \equiv (1-i)\kappa(z)\sqrt{\tau_0(\tau+\tau_0)},$$

Numerical approaches to emissions : C. Andres (Mon, Morning)

Can we interpret the scalings in different kinematical limits ?



Effective parameter

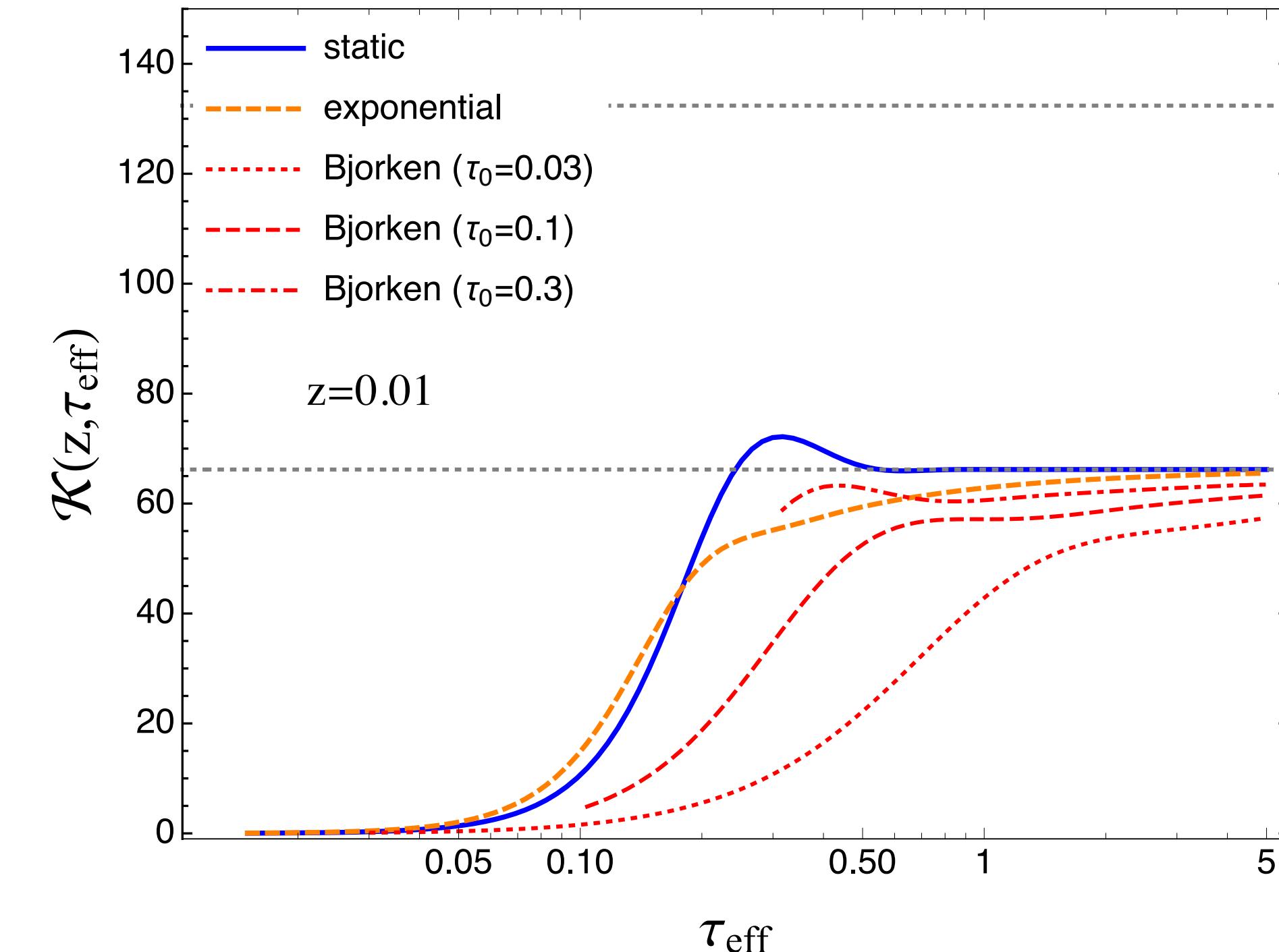
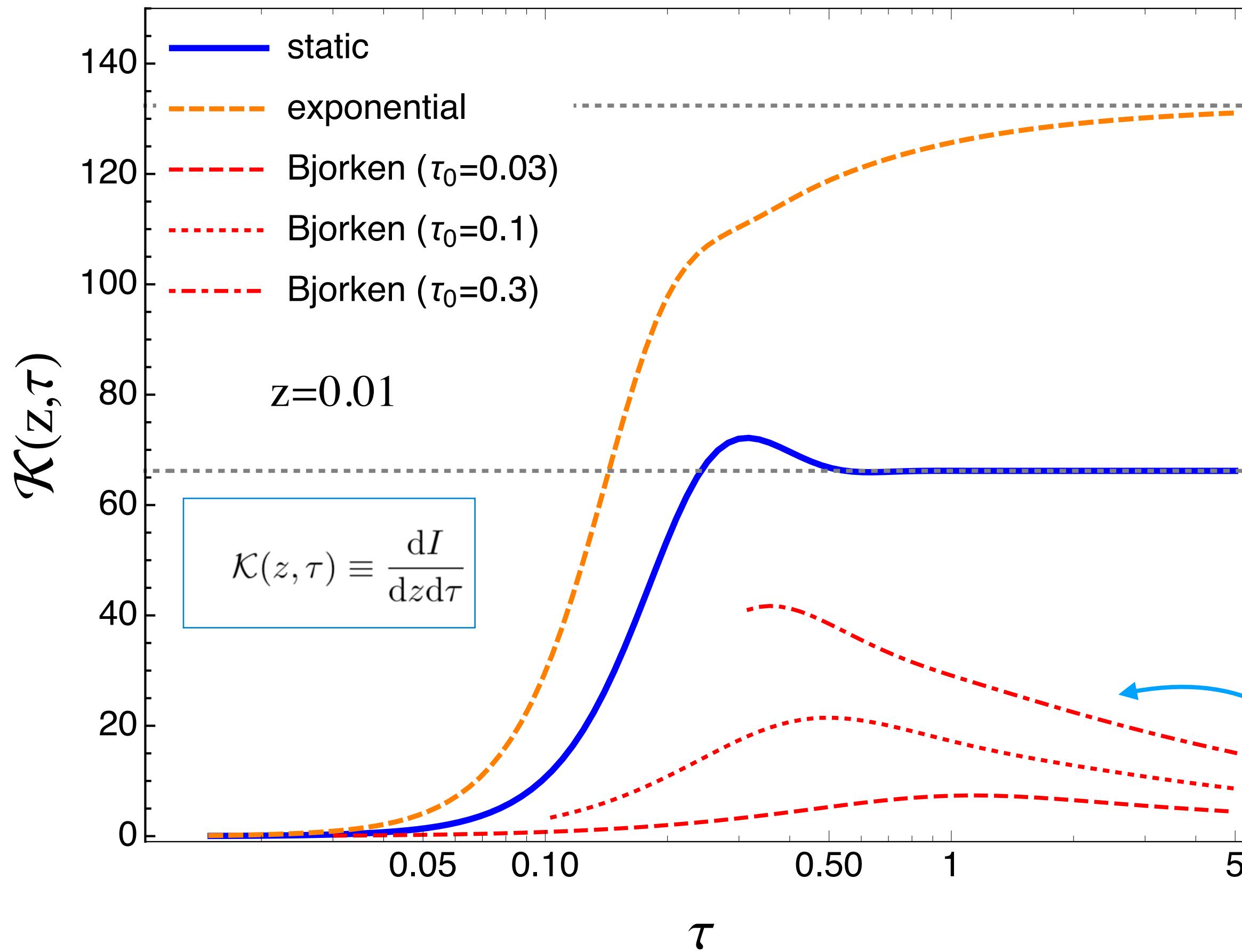
$$\frac{dI}{dz}^{static,sing} \simeq \frac{dI}{dz}^{expo,sing} \simeq \frac{dI}{dz}^{BJ,sing}$$

$$\omega_{\text{eff}} = \begin{cases} \frac{1}{2}\hat{q}_0 L^2 & \text{static medium} \\ 2\hat{q}_0 L^2 & \text{exponentially expansion} \\ 2\hat{q}_0 t_0 L & \text{Bjorken expansion} \end{cases} .$$

The singular spectra can be re-scaled

$$\hat{q}_{eff}^{expo} = 4\hat{q}_0 \\ \hat{q}_{eff}^{BJ} = 4\hat{q}_0 t_0 / L$$

# Medium modified splitting rates



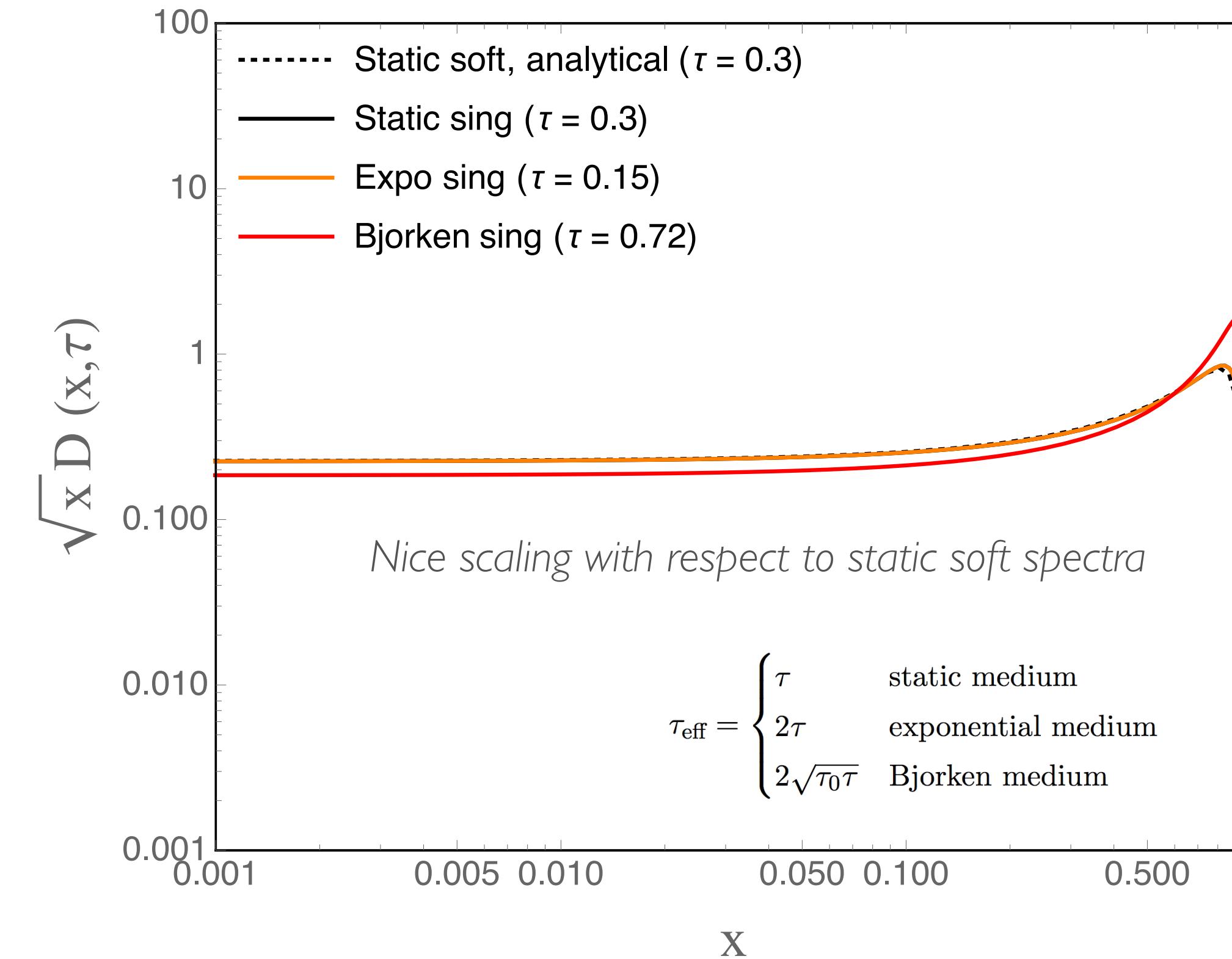
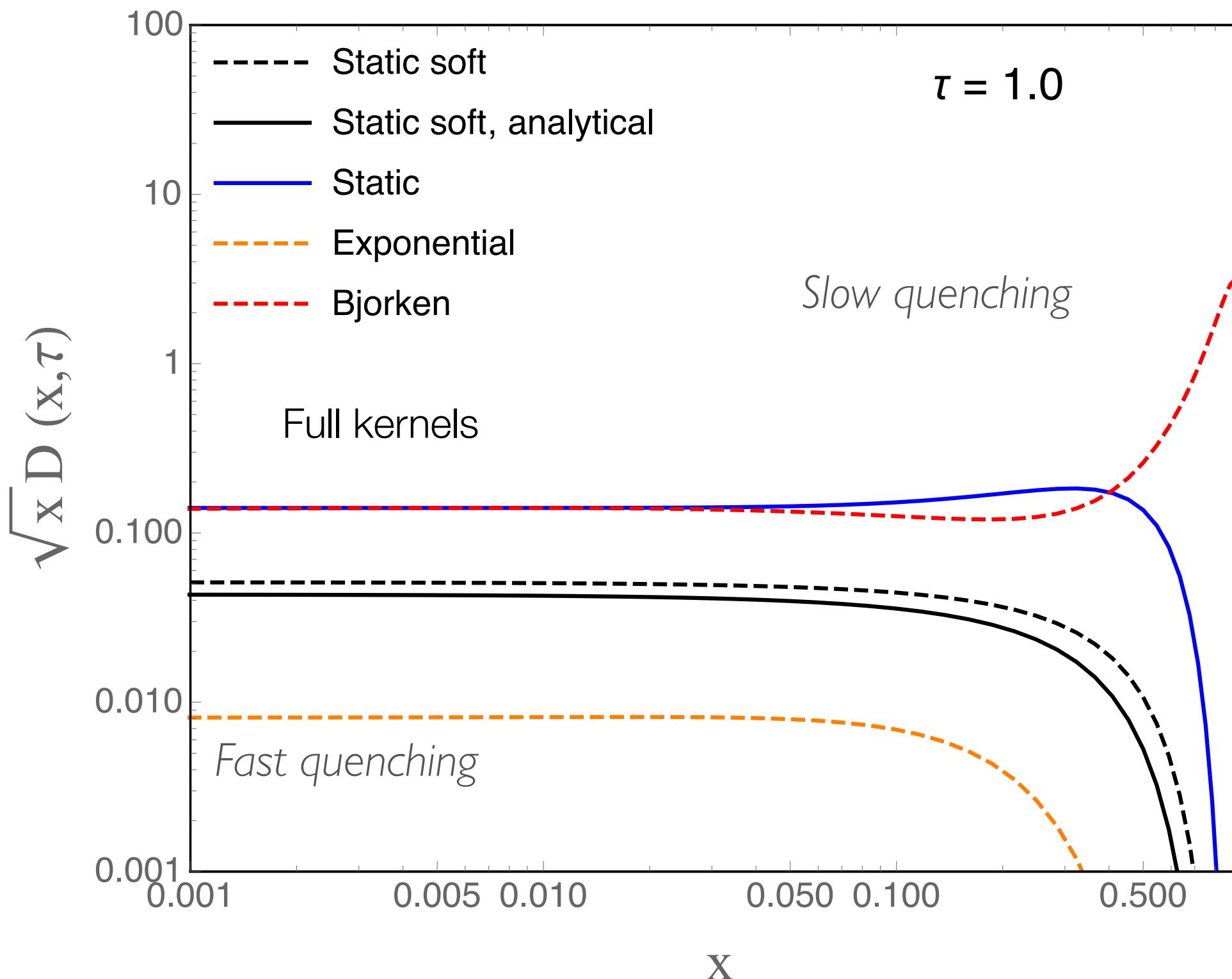
$$\mathcal{K}(z, \tau) = \frac{\alpha_s}{\pi} P(z) \kappa(z) \sqrt{\frac{\tau_0}{\tau + \tau_0}} \operatorname{Re} \left[ (1 - i) \frac{J_1(z_L) Y_1(z_0) - J_1(z_0) Y_1(z_L)}{J_1(z_0) Y_0(z_L) - J_0(z_L) Y_1(z_0)} \right]$$

- The BDMPS soft ( $w < w_c$ ) has a constant splitting rate independent of the time of evolution of the plasma.
- The rates for all the profiles except the BDMPS soft are similar at very low evolution time or length of the medium.
- In the Bjorken, the presence of the factor  $\sqrt{\tau_0/(\tau_0 + \tau)}$  leads to the dumping of the splitting rate for  $\tau > \tau_0$ .

# Medium evolved gluon spectra

- The kinematic evolution equation (**GAIN** + **LOSS** terms) in terms of gluon spectra :

$$\frac{\partial D(x, t)}{\partial \tau} = \int dz \mathcal{K}(z, \tau | p) \left[ \sqrt{\frac{z}{x}} D\left(\frac{x}{z}, \tau\right) - \frac{z}{\sqrt{x}} D(x, \tau) \right]$$



Static, soft gluon spectra (analytical)

$$D(x, \tau) = \frac{\tau}{\sqrt{x}(1-x)^{3/2}} e^{-\pi \frac{\tau^2}{1-x}}$$

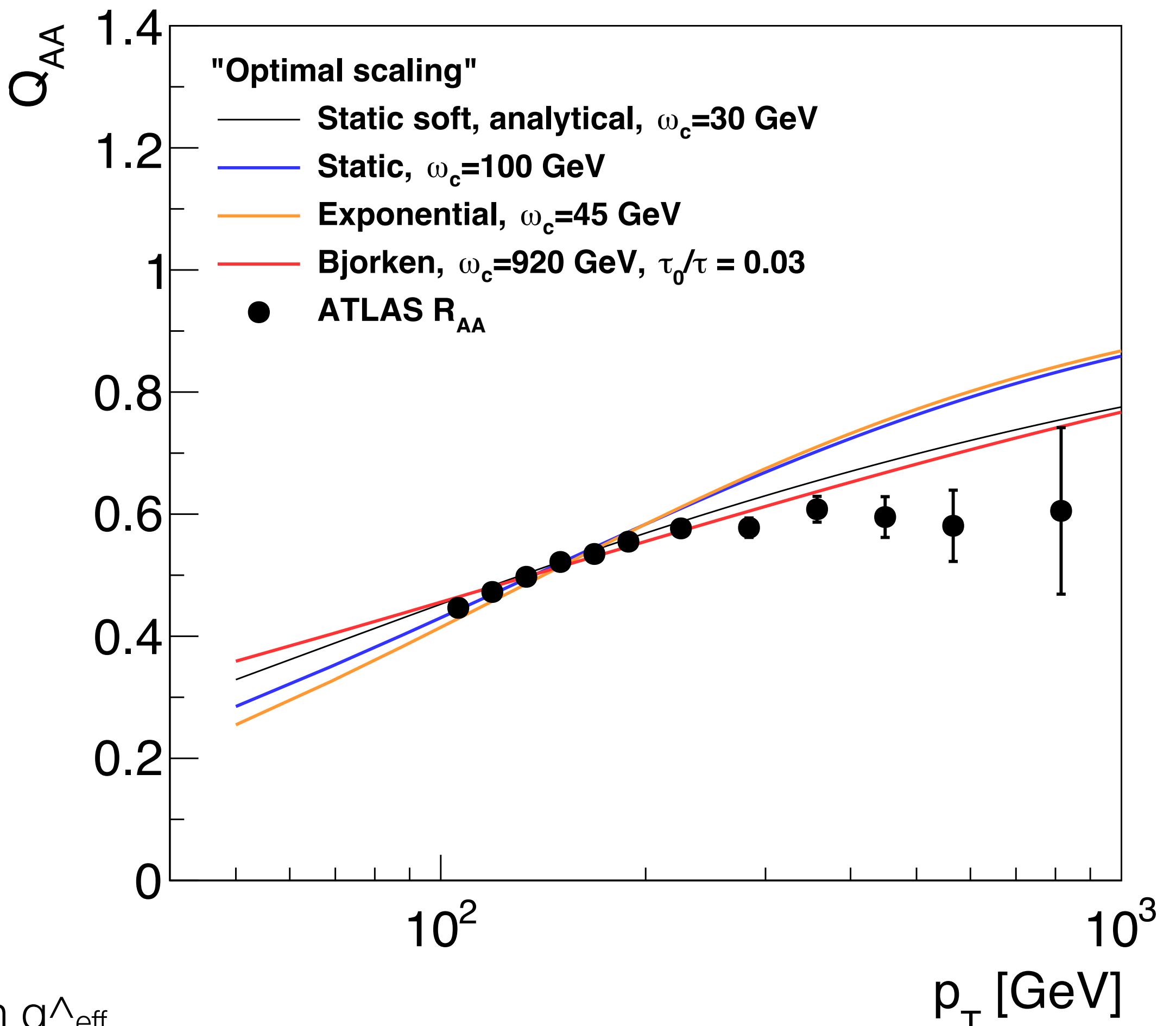
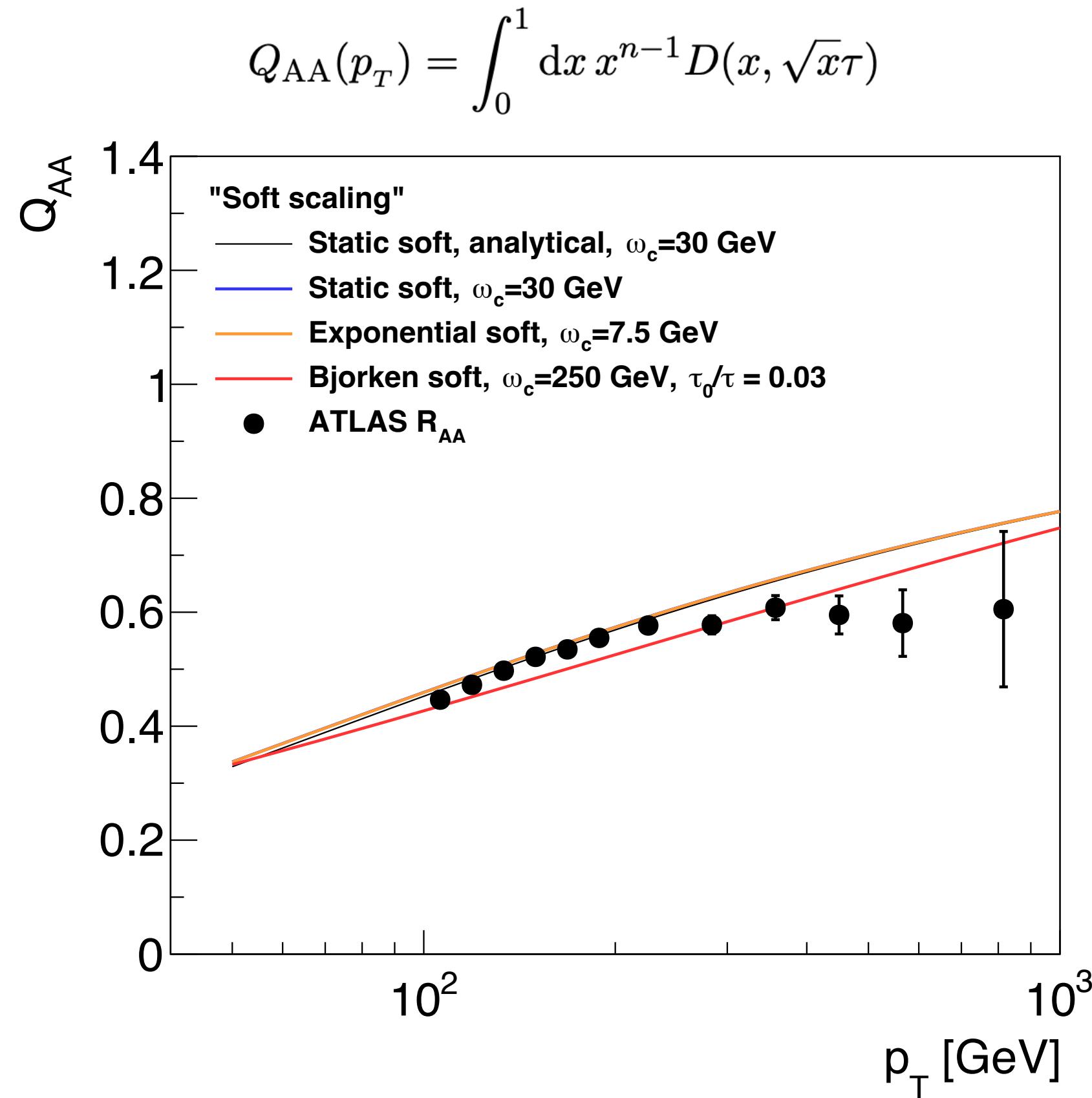
J. P. B, E. I., Y. M-T, PRL. 111 (2013) 052001.

- A. **Singular** spectra  
==> **Nice**  
scaling in  $\tau_{\text{eff}}$ .
- B. **Full** spectra  
==> **No** scaling  
in  $\tau_{\text{eff}}$ .

$$\mathcal{K}(z, \tau) \equiv \frac{dI}{dz d\tau}$$

- At low  $x$ , we see a  $1/(\sqrt{x})$  behaviour of all the profiles >> recovered from the similar gluon splitting at low  $x$ .

S. P. Adhya, C. Salgado, M. Spousta, K. Tywoniuk; JHEP 07 (2020) 150.



- **Soft scaling** => Scaling for singular spectra in  $q^\wedge_{\text{eff}}$ .
- The Bjorken profile depends on additional choice of  $(\tau_0/\tau)$  : **No universal scaling**.
- **Good, but not perfect scaling** is achieved by optimisation.
- Scaling for exponential medium ~ **average scaling**.

No

$\hat{q}_0$ [GeV $^3$ ]	static	exponential	Bjorken
no scaling	0.2	0.2	0.2
soft scaling	0.2	0.05	1.66
optimal scaling	0.2	0.09	1.84
scaling by $\langle \omega_c \rangle$	0.2	0.1	3.33

- **Significant differences** in values of  $q^\wedge$  for different types of medium and kinematical ranges point to the importance of **precise modelling of jet quenching phenomenon**.

# Collecting the pieces



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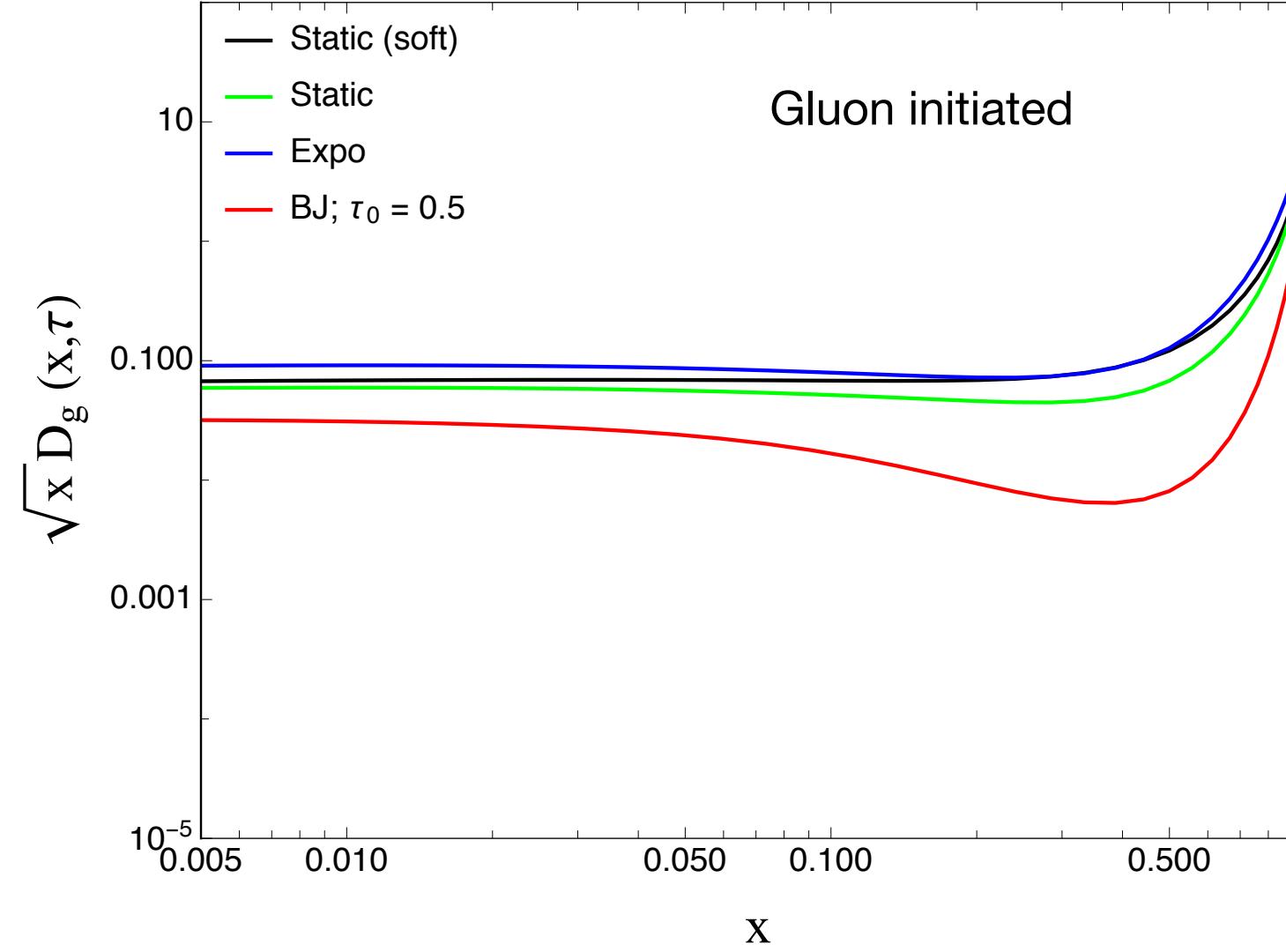


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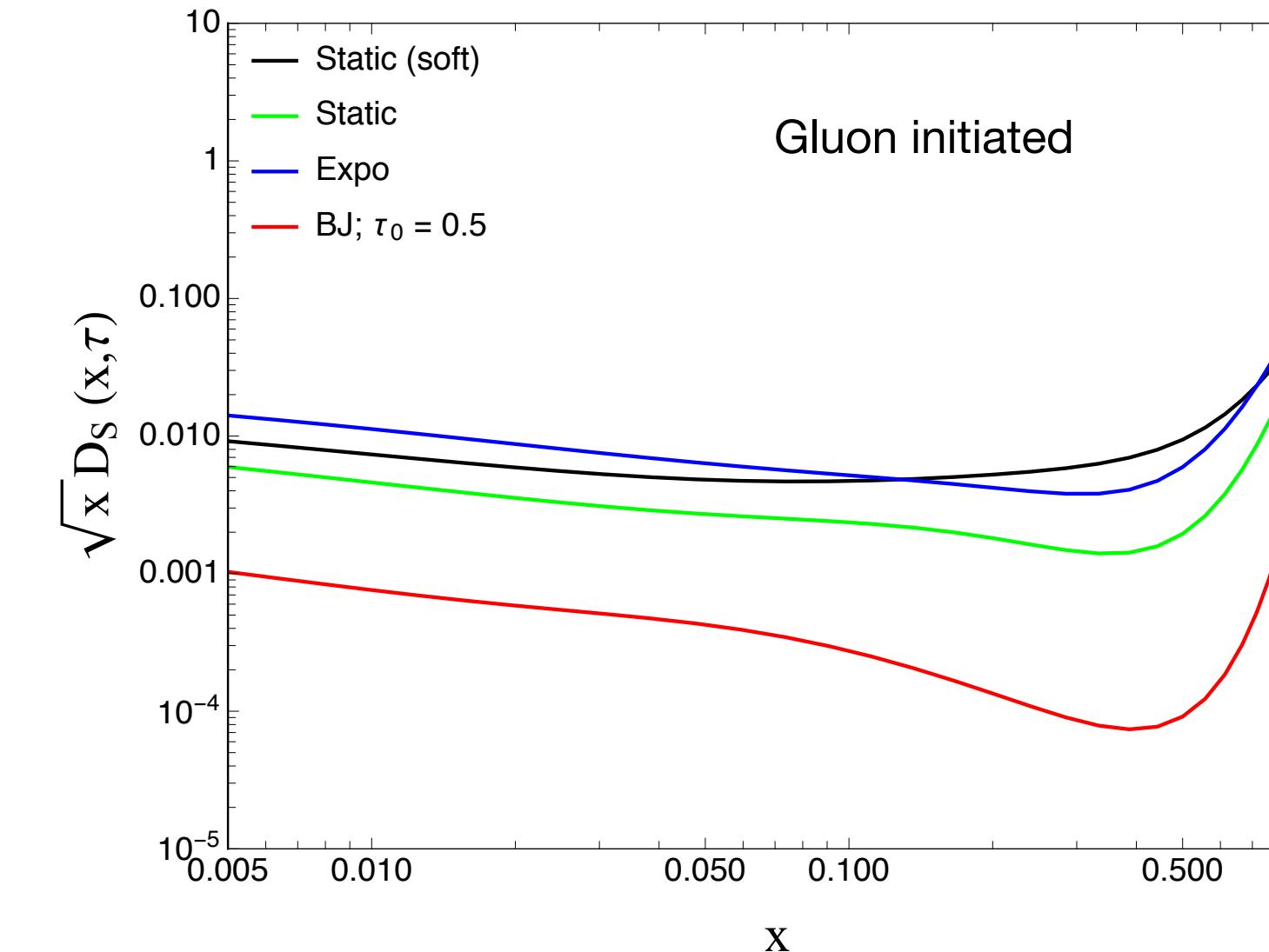
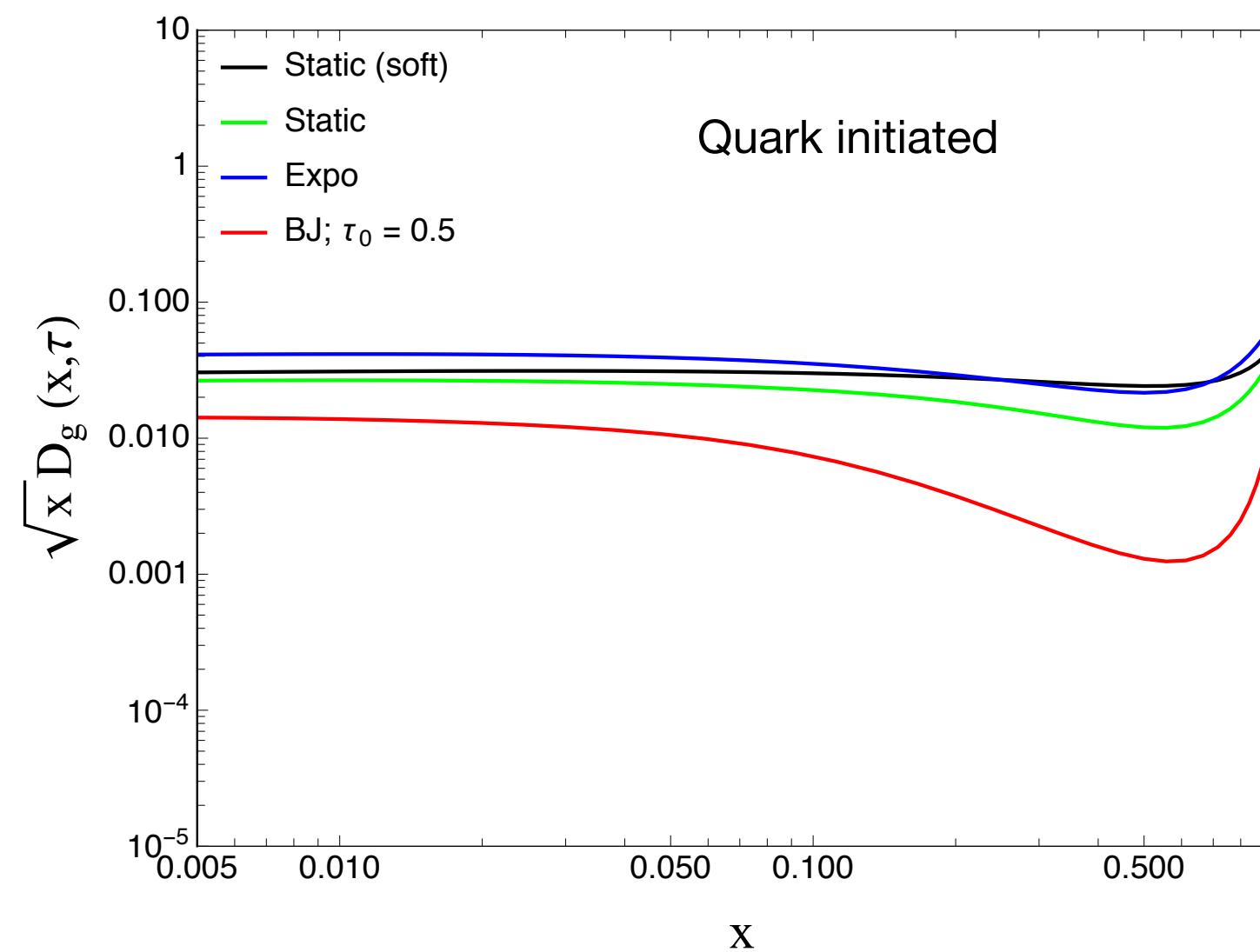
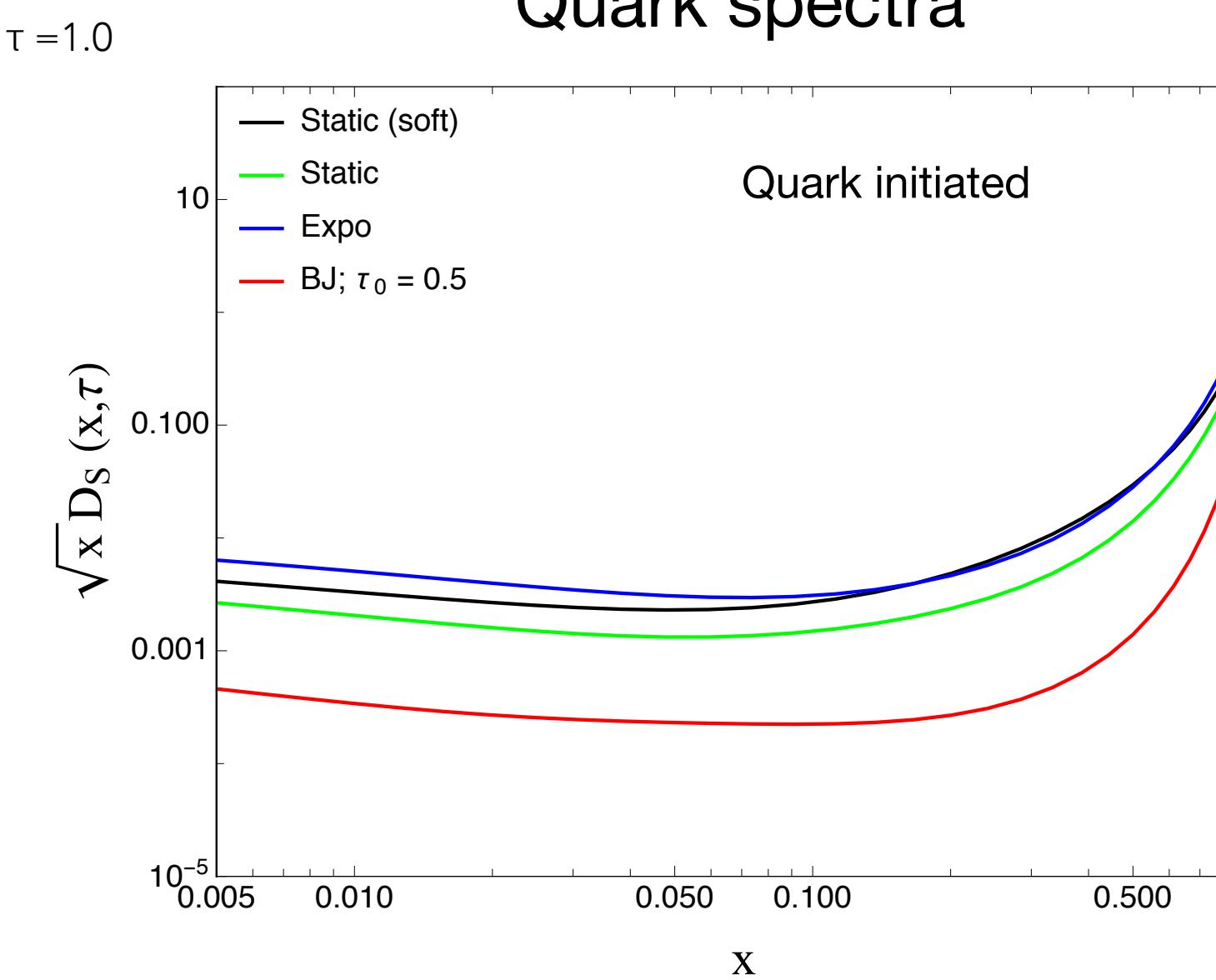


# Multi-partonic cascades for expanding media

Gluon spectra



Quark spectra



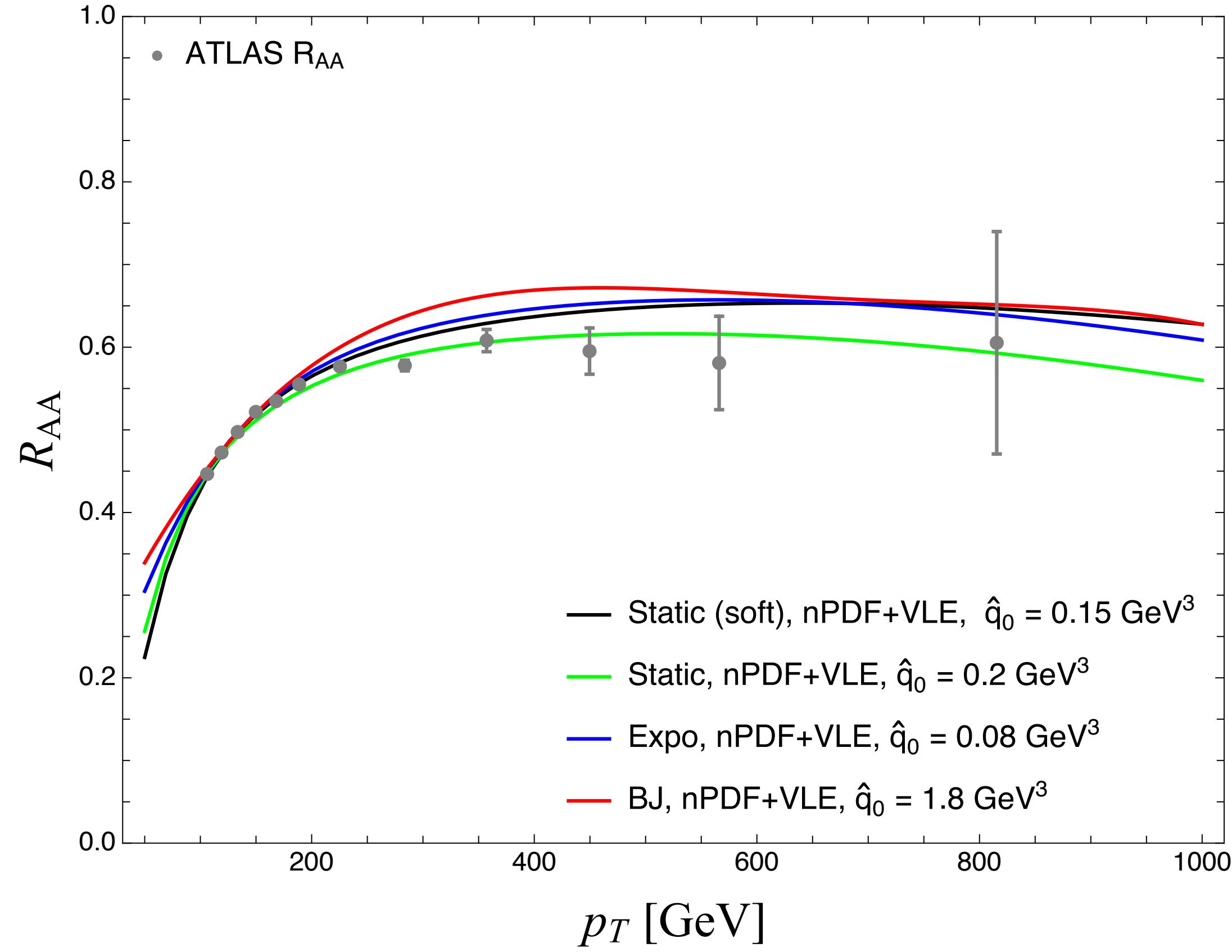
- One finds that at late times, quarks begin to dominate the large  $x$  part of the distribution.
- It is in fact more likely that such a large  $x$  fragment is a quark/anti-quark rather than a gluon.
- At late times, fragmentation pattern of quark and gluon initiated jets become similar  
==> effective memory loss of initial conditions.

# Does the media behave differently for rapidity ?

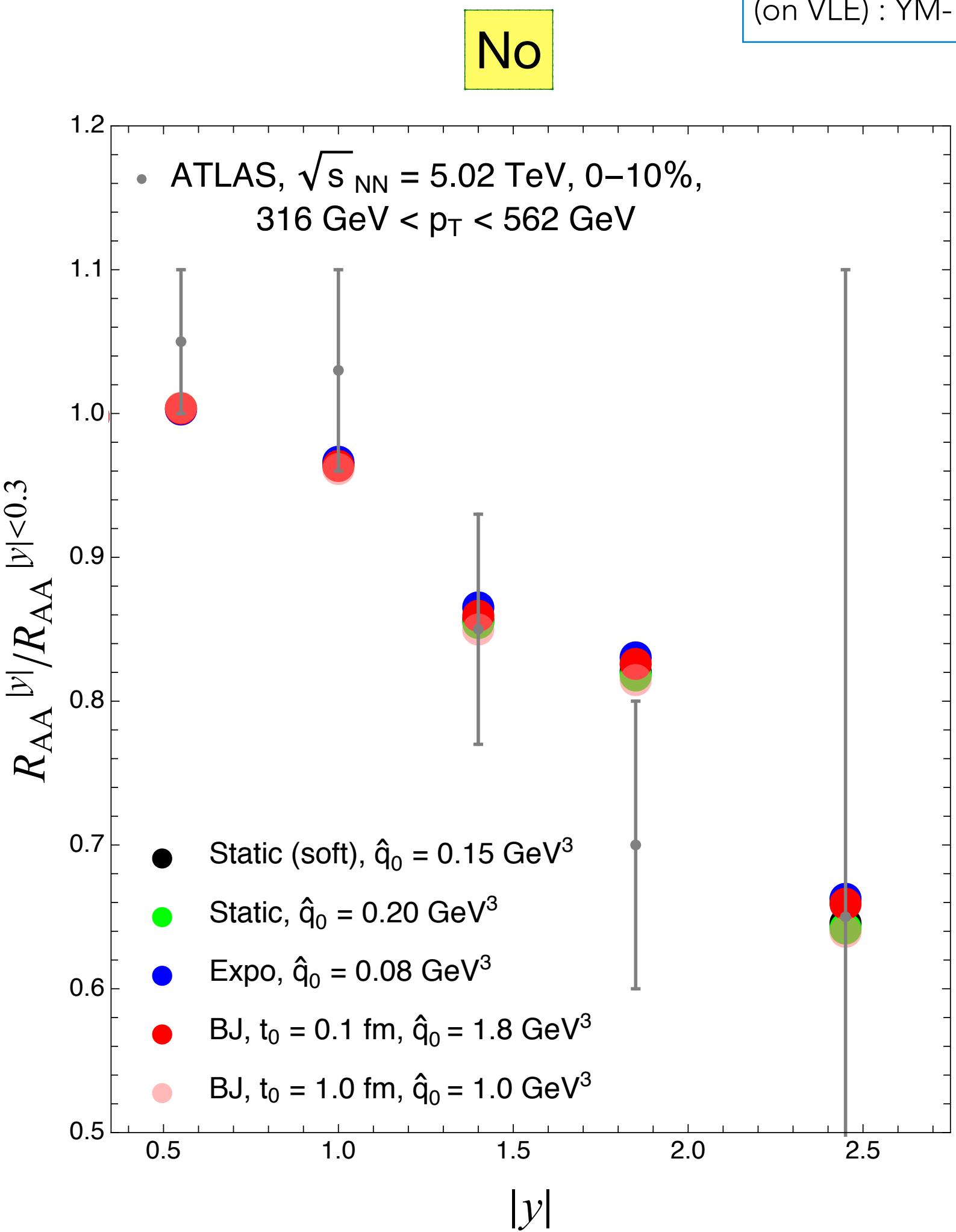
S. P. Adhya, C. Salgado, M. Spousta, K. Tywoniuk,  
EPJC 82 (2022) 1.

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## Multi- partonic cascades



Jet  $R_{AA}$  for different medium profiles

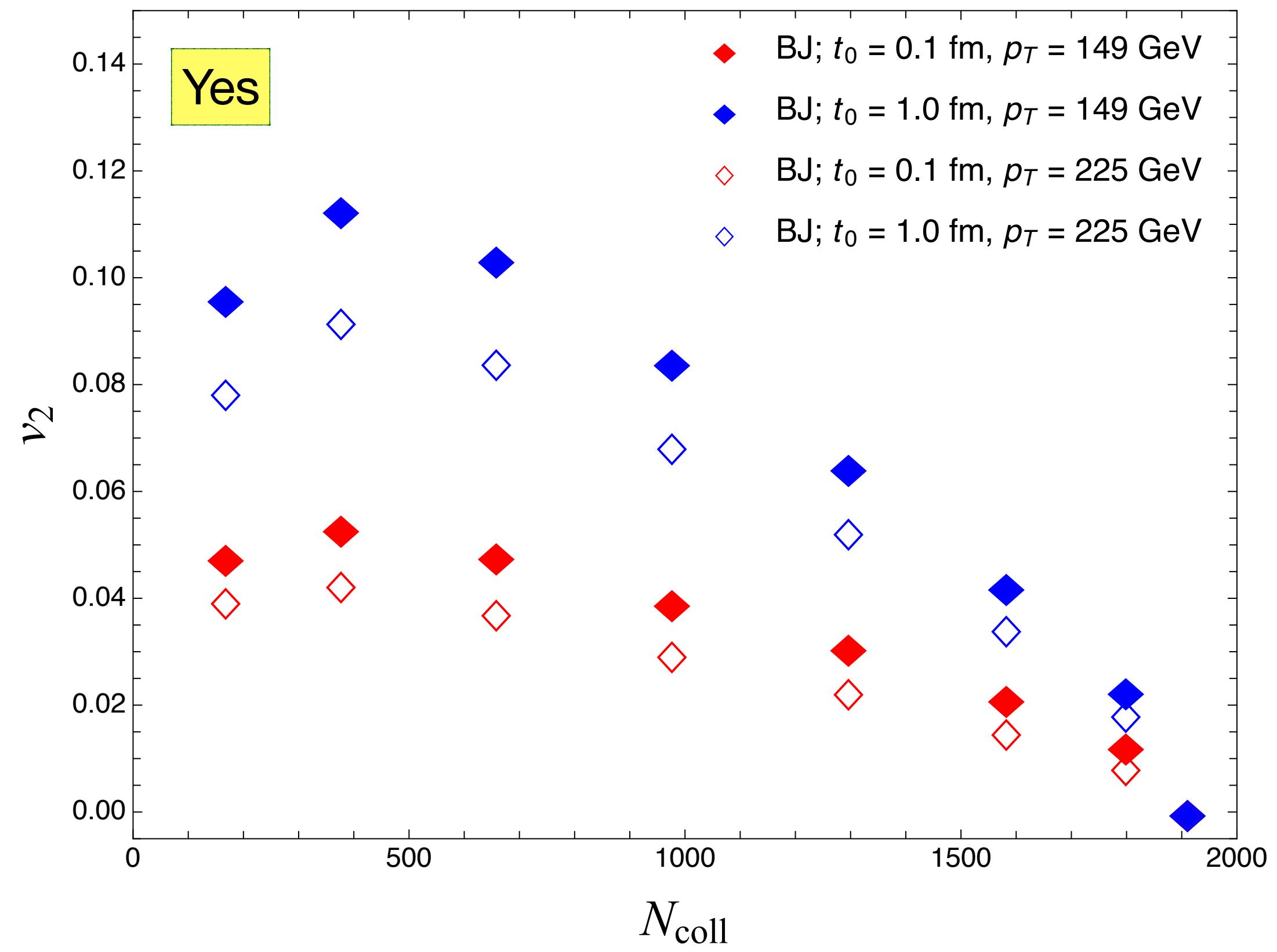
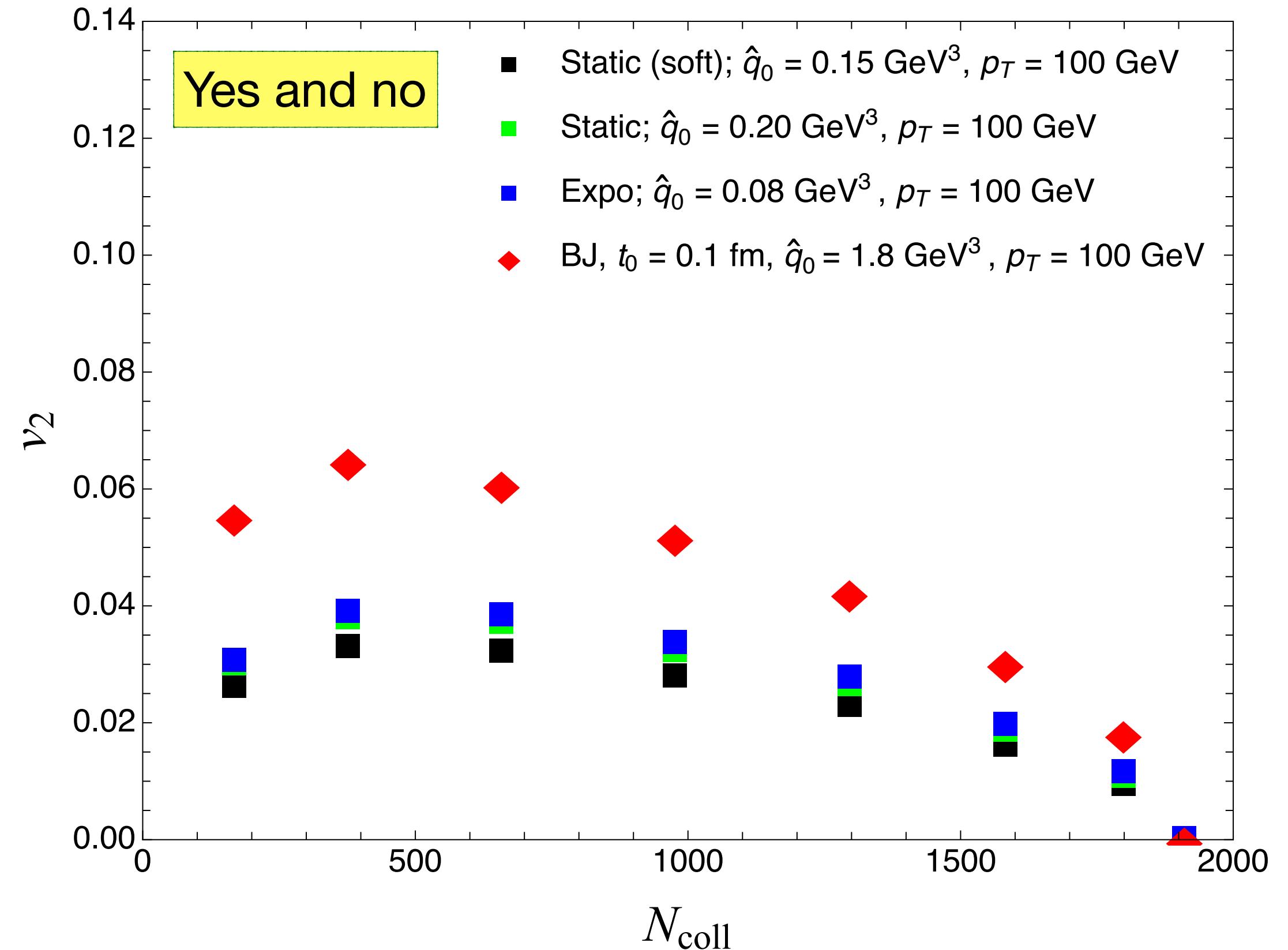


Rapidity ratio with respect to  $|y|$  for different medium profiles

# Does the media behave differently for $v_2$ ?

S. P. Adhya, C. Salgado,  
M. Spousta, K. Tywoniuk,  
EPJC 82 (2022) 1.

13/23



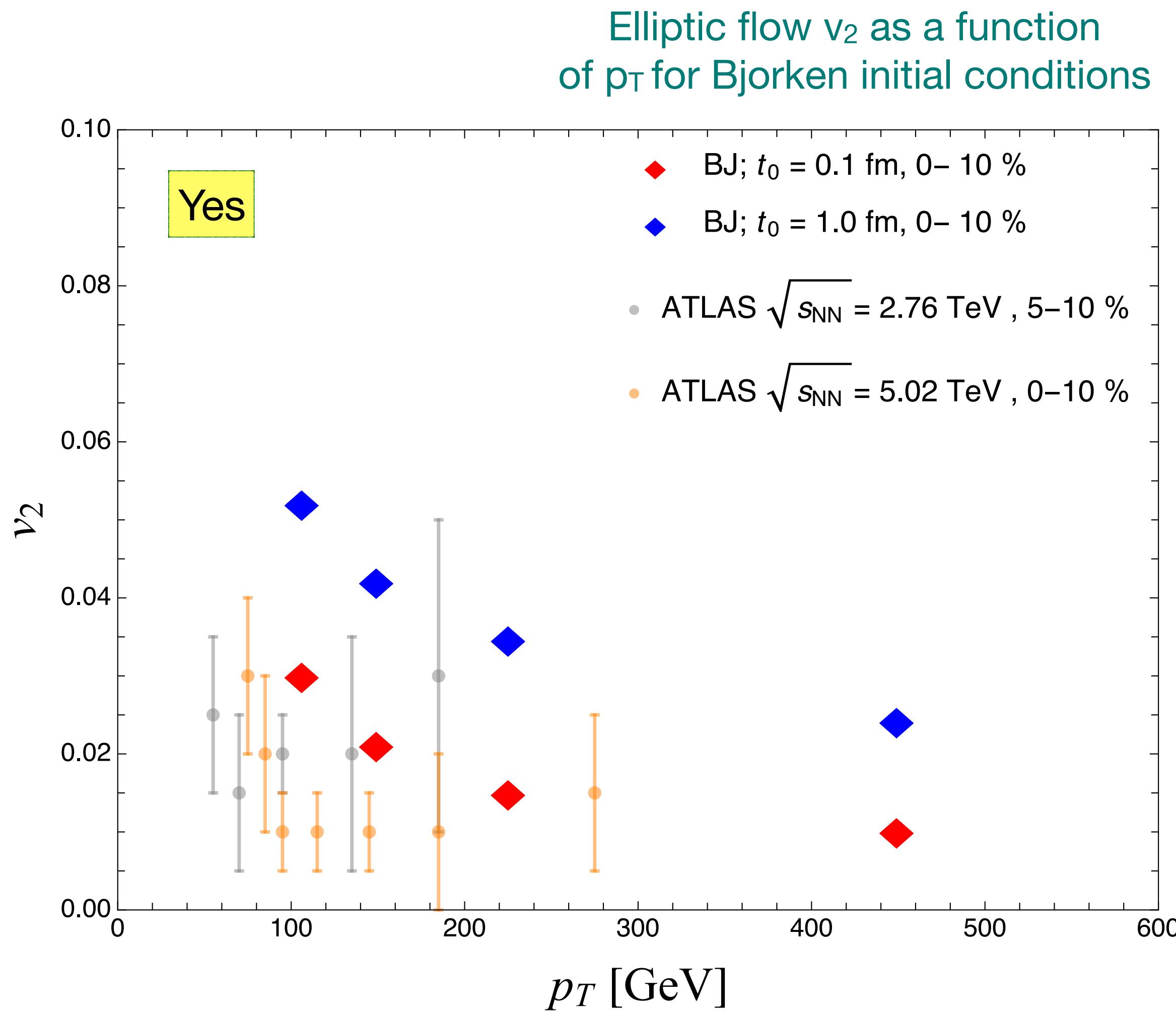
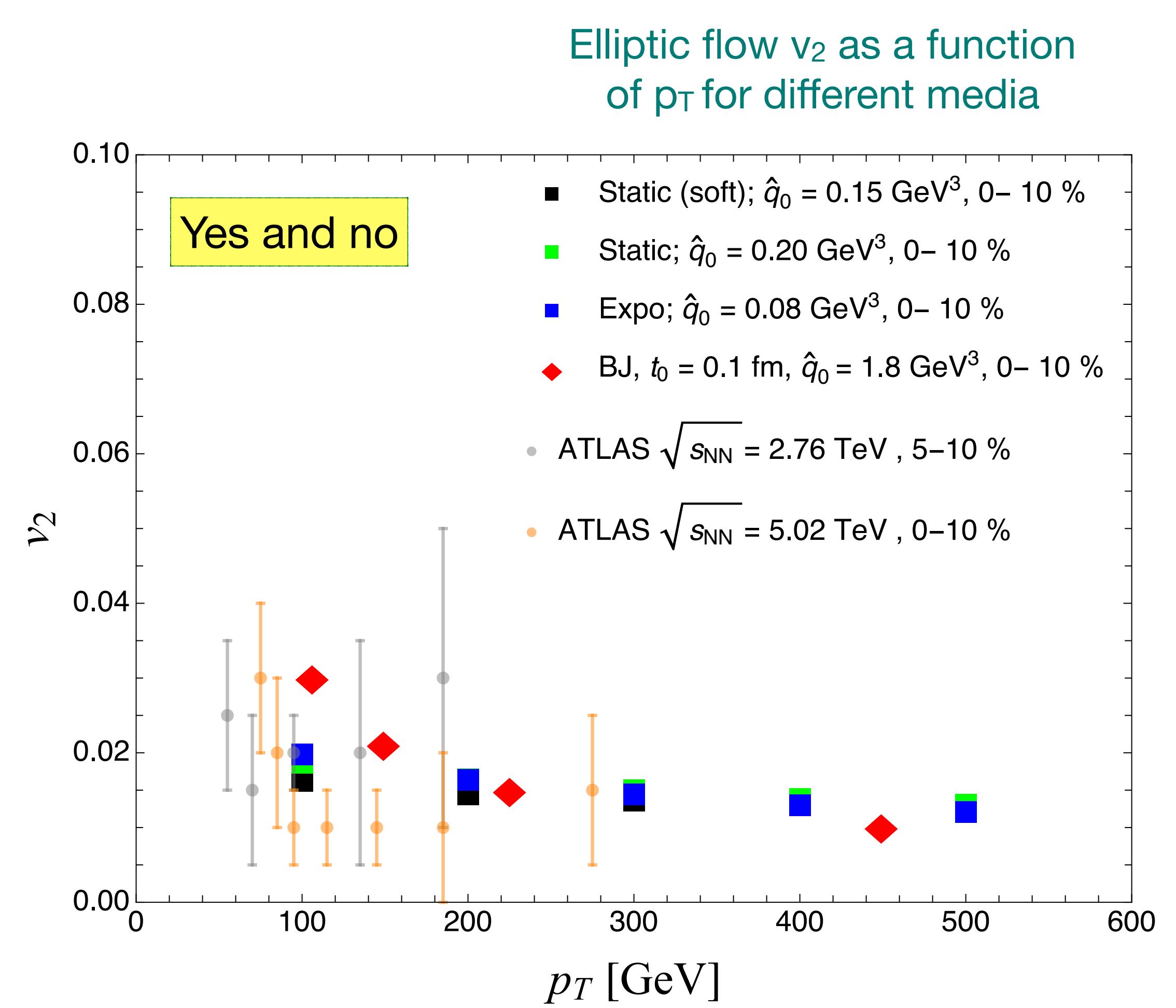
- The impact of the medium expansion can be largely scaled out by a suitable choice of  $\hat{q}$  [confirming Adhya et. al., 2020].
- The jet  $v_2$  remains sensitive to choice of starting time of Bjorken quenching to.

$$v_2 = \frac{1}{2} \frac{R_{\text{AA}}(L^{in}) - R_{\text{AA}}(L^{out})}{R_{\text{AA}}(L^{in}) + R_{\text{AA}}(L^{out})}$$

# Does the media behave differently for $v_2$ ?

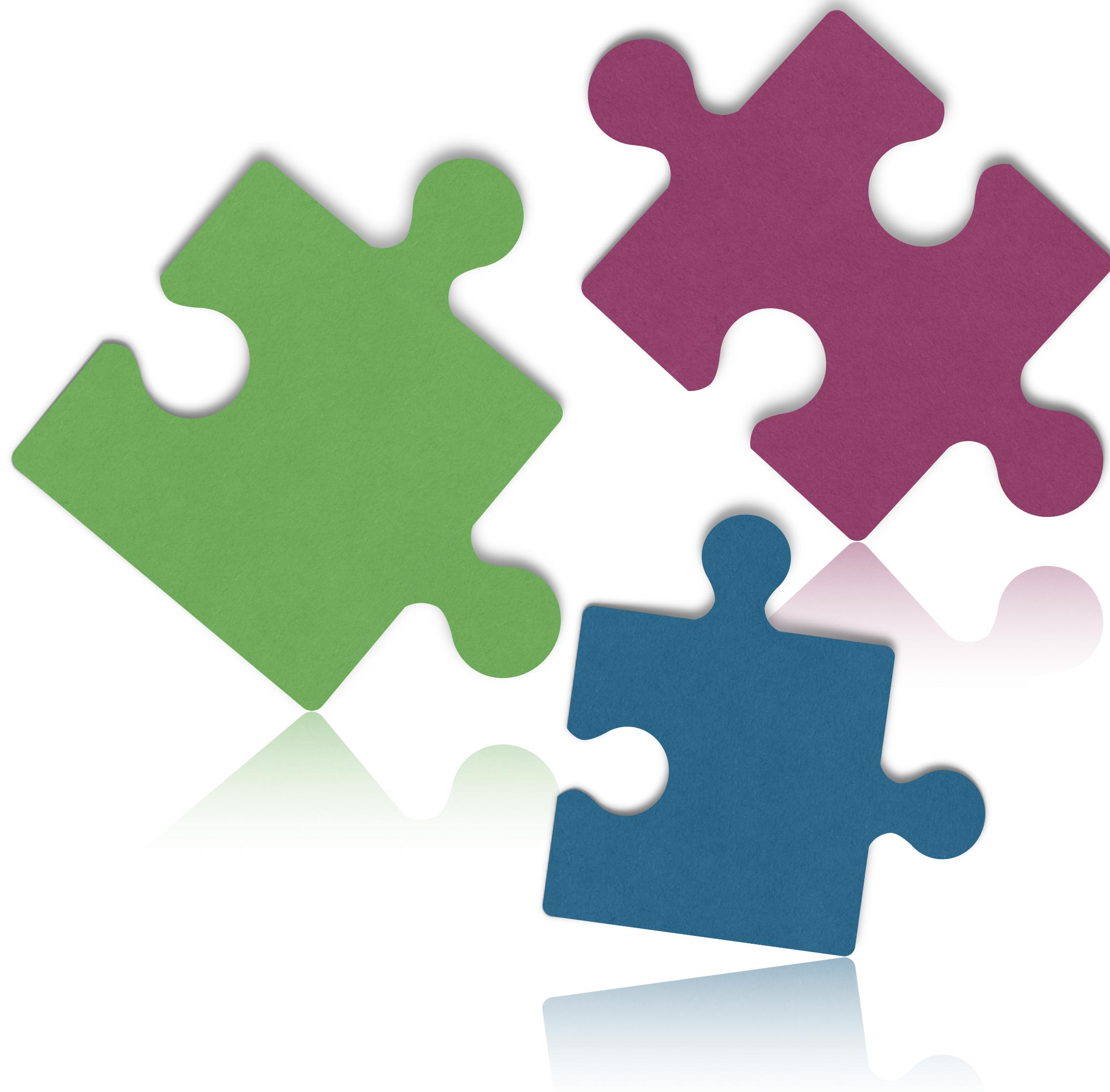
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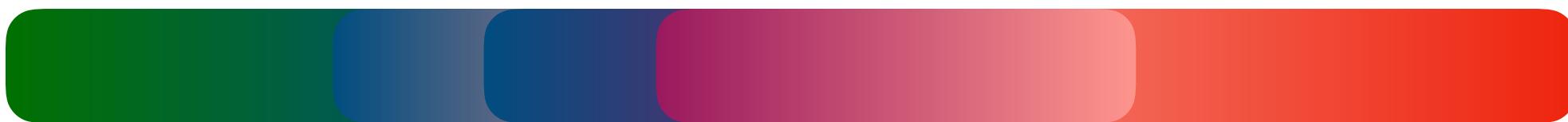


- **Agreement** with findings of the **sensitivity of  $v_2$  on  $t_0$**  [Carlota et. al., PLB, 2020] which was done in more complex modelling of the collision geometry, but less complex modelling of the medium induced showering.

# Collecting the pieces



- Gluonic cascades with expanding medium.
- Multi- partonic cascades with expanding medium.
- Transverse momentum broadening in cascades in expanding medium.



*Complexity/ Completeness  
towards understanding*



# The evolution equation (once more)

- Evolution equation for gluon transverse-momentum-dependent distribution  $D(x, \mathbf{k}, t)$  :

$$\frac{\partial}{\partial t} D(x, \mathbf{k}, t) = \frac{1}{t^*} \int_0^1 dz \mathcal{K}(z, \tau) \left[ \frac{1}{z^2} \sqrt{\frac{z}{x}} D\left(\frac{x}{z}, \frac{\mathbf{k}}{z}, t\right) \theta(z-x) - \frac{z}{\sqrt{x}} D(x, \mathbf{k}, t) \right] \\ + \int \frac{d^2 \mathbf{q}}{(2\pi)^2} C(\mathbf{q}) D(x, \mathbf{k} - \mathbf{q}, t),$$

HTL approx.

$$w(\mathbf{l}) = \frac{g^2 m_D^2 T}{\mathbf{l}^2 (\mathbf{l}^2 + m_D^2)}$$

- The elastic collision kernel  $\longrightarrow C(\mathbf{l}) = w(\mathbf{l}) - \delta(\mathbf{l}) \int d^2 \mathbf{l}' w(\mathbf{l}')$
- Re-writing the evolution equation in *integral form* :

$$D(x, \mathbf{k}, \tau) = e^{-\Psi(x)(\tau-\tau_0)} D(x, \mathbf{k}, \tau_0) \\ + \int_{\tau_0}^{\tau} d\tau' \int_0^1 dz \int_0^1 dy \int d^2 \mathbf{k}' \int d^2 \mathbf{q} \mathcal{G}(z, \mathbf{q}) \\ \times \delta(x - zy) \delta(\mathbf{k} - \mathbf{q} - z\mathbf{k}') e^{-\Psi(x)(\tau-\tau')} D(y, \mathbf{k}', \tau')$$

$$\Phi(x) = \frac{1}{\sqrt{x}} \int_0^{1-\epsilon} dz z \mathcal{K}(z), \\ W = t^* \int_{|\mathbf{q}| > q_{\min}} d^2 \mathbf{q} \frac{w(\mathbf{q})}{(2\pi)^2}, \\ \Psi(x) = \Phi(x) + W,$$

- One can integrate the evolution equation over  $k_T$  to obtain,

$$\frac{\partial}{\partial t} D(x, t) = \frac{1}{t^*} \int_0^1 dz \mathcal{K}(z, \tau) \left[ \sqrt{\frac{z}{x}} D\left(\frac{x}{z}, t\right) \theta(z-x) - \frac{z}{\sqrt{x}} D(x, t) \right]$$

$$\frac{1}{t^*} = \frac{\alpha_s N_c}{\pi} \sqrt{\frac{\hat{q}}{p_0^+}}$$

$$\tau = t/t^*$$

We used this evolution equation for previous results

J-P. B., F. D., E. I., Y. M-T ; JHEP 06 (2014) 075.

For further details and implementations of the integral form  $\rightarrow$  K. Kutak, W. Płaczek & R. Straka, EPJC 79 (2019), 317. Also solutions by MINCAS and TMDICE.

Sudakov form factor resums virtual and unresolved real emissions

# Comparison of momentum broadening probabilities

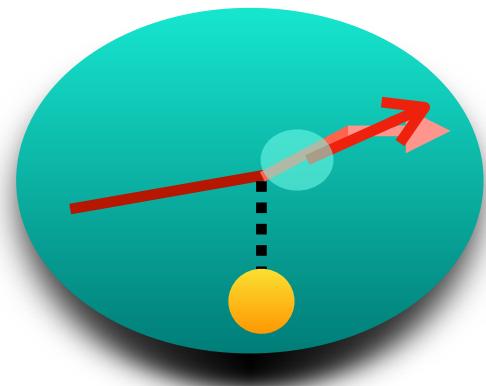
S. P. A. , K. Kutak, W. Placek, M. Rohrmorser, K. Tywoniuk (in preparation)

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'x' integrated  $D(x, k, t)$  integrated spectra =  $\rho(k) \rightarrow$  Broadening of jet

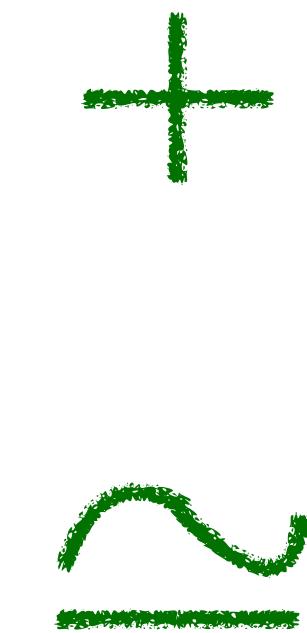
Single hard (SH) scattering

$$\mathcal{P}^{\text{SH}}(\mathbf{k}, L) \Big|^{eff} = 4\pi \frac{Q_{s0,eff}^2}{\mathbf{k}^4}$$



Multiple soft (MS) scattering

$$\mathcal{P}^{\text{MS}}(\mathbf{k}, L) \Big|^{eff} = \frac{4\pi}{Q_{s,eff}^2} e^{-\frac{\mathbf{k}^2}{Q_{s,eff}^2}}$$



Moli`ere's theory of multiple scattering

$$\mathcal{P}^{(0)+(1)}(\mathbf{k}, L) \Big|^{eff} = \frac{4\pi}{Q_s^2} e^{-x} \left\{ 1 - \lambda \left( e^x - 2 + (1-x) (\text{Ei}(x) - \log(4\pi a)) \right) \right\}$$

$$x \equiv \frac{\mathbf{k}^2}{Q_{s,eff}^2}$$

Static media  $Q_s^2$

$\longrightarrow$

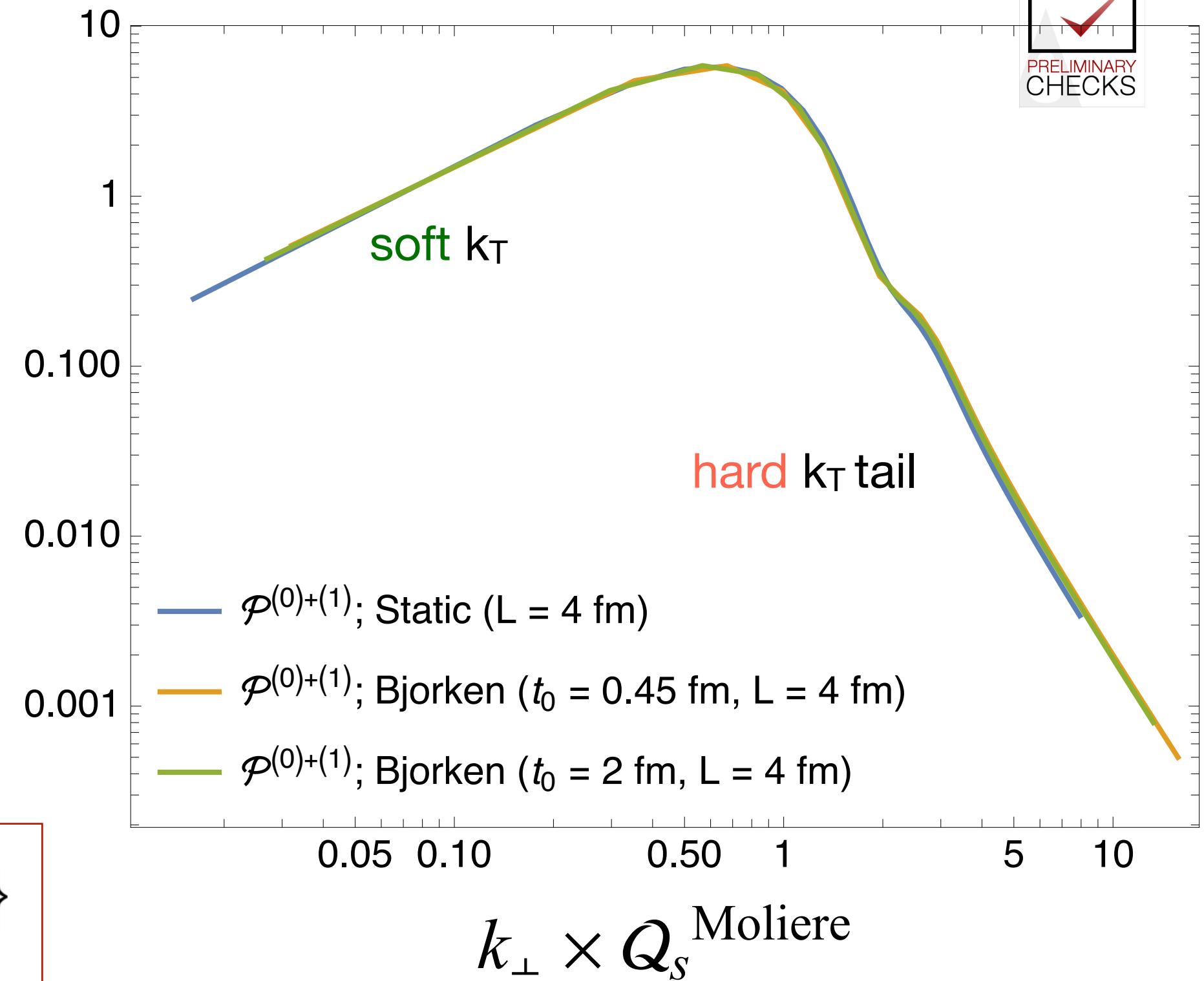
$$Q_s^2 \left( \frac{t_0}{L} \right) \log \left( \frac{L}{t_0} \right)$$

Bjorken expanding media

$$Q_s^2 = \hat{q}_0 L \log \frac{a Q_s^2}{\mu_*^2}$$

$$Q_{s0}^2 \equiv \hat{q}_0 L$$

Nice scaling of the Bjorken with static !



- Single particle momentum broadening distribution ( $\rho$ ) reproduce the Gaussian behavior at small- $k_\perp$  **together** with the power-law tail ==> *simple analytic expression* in Moli`ere prescription (effective).

# How does the $k_T$ dependent spectra look like ?

S. P. A. , K. Kutak, W. Placek, M. Rohrmorser, K. Tywoniuk (in preparation)

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- We consider different schemes for the transverse momentum broadening.

Numerical solution

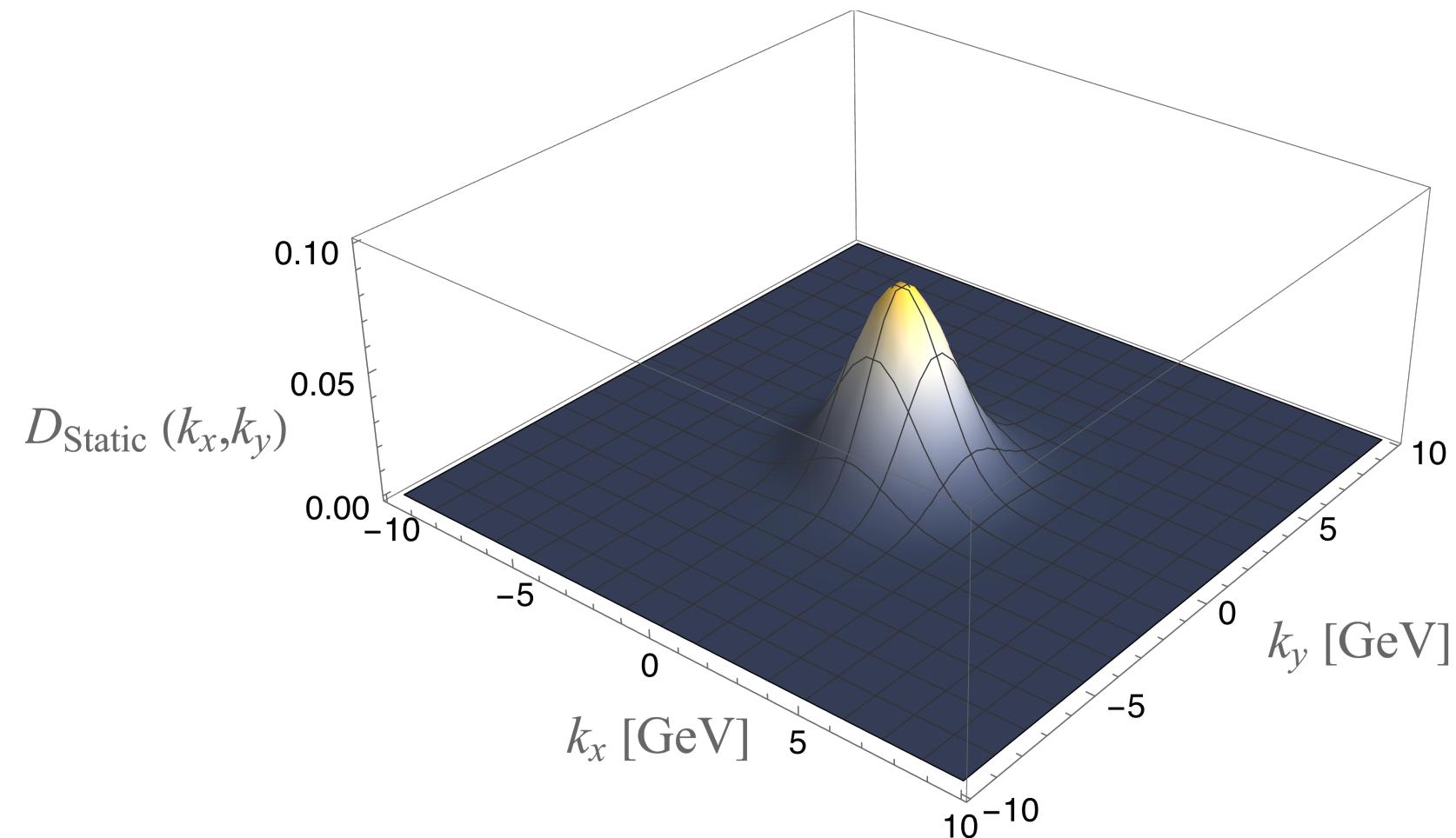
Ansatz

Schemes	Gaussian broadening only	General broadening	
Medium evolved spectra w/o broadening	$D(x, \tau)$	$D(x, \tau)$	–
Momentum broadening term	$\mathcal{P}^{GB}(\mathbf{k}, \tau)$	$\mathcal{P}^{(0)+(1)}(\mathbf{k}, \tau)$	–
Medium evolved spectra with broadening	$D^{GB}(x, \mathbf{k}_T, \tau) = D(x, \tau) \times \mathcal{P}^{GB}(\mathbf{k}, \tau)$	$D^{eGB}(x, \mathbf{k}_T, \tau) = D(x, \tau) \times \mathcal{P}^{(0)+(1)}(\mathbf{k}, \tau)$	$D^{nGB}(x, \mathbf{k}_T, \tau)$

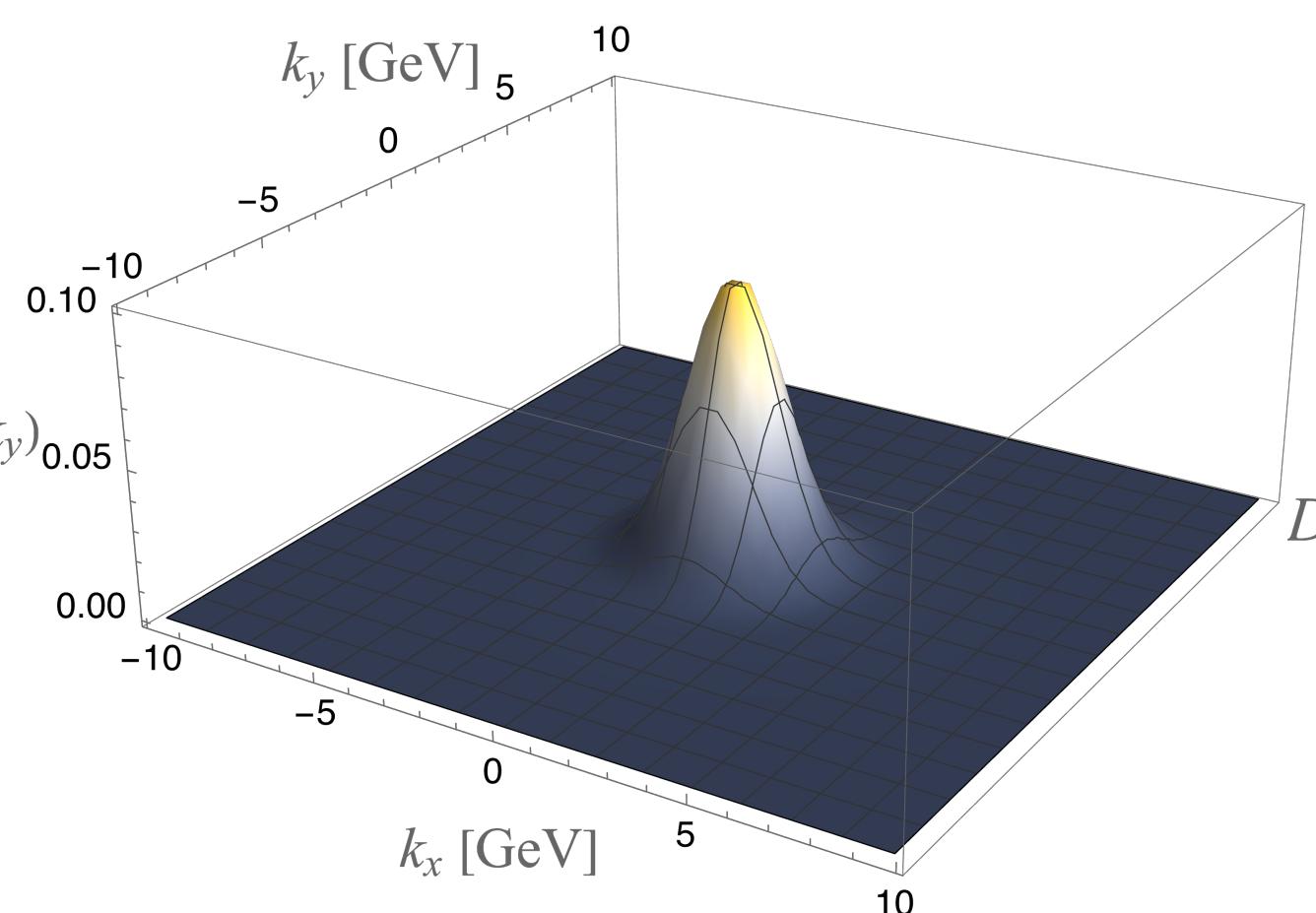
- $k_T$  independent kernel. This is an approximation. The whole broadening comes from rescattering.
- Non-Gaussianity**: Sum of many Gaussians of different widths; arbitrary number of the collisions with the medium.

(Static media) A. v-H , K. K, W. P. , M. R., K.T., PRC 102 (2020) 044910.

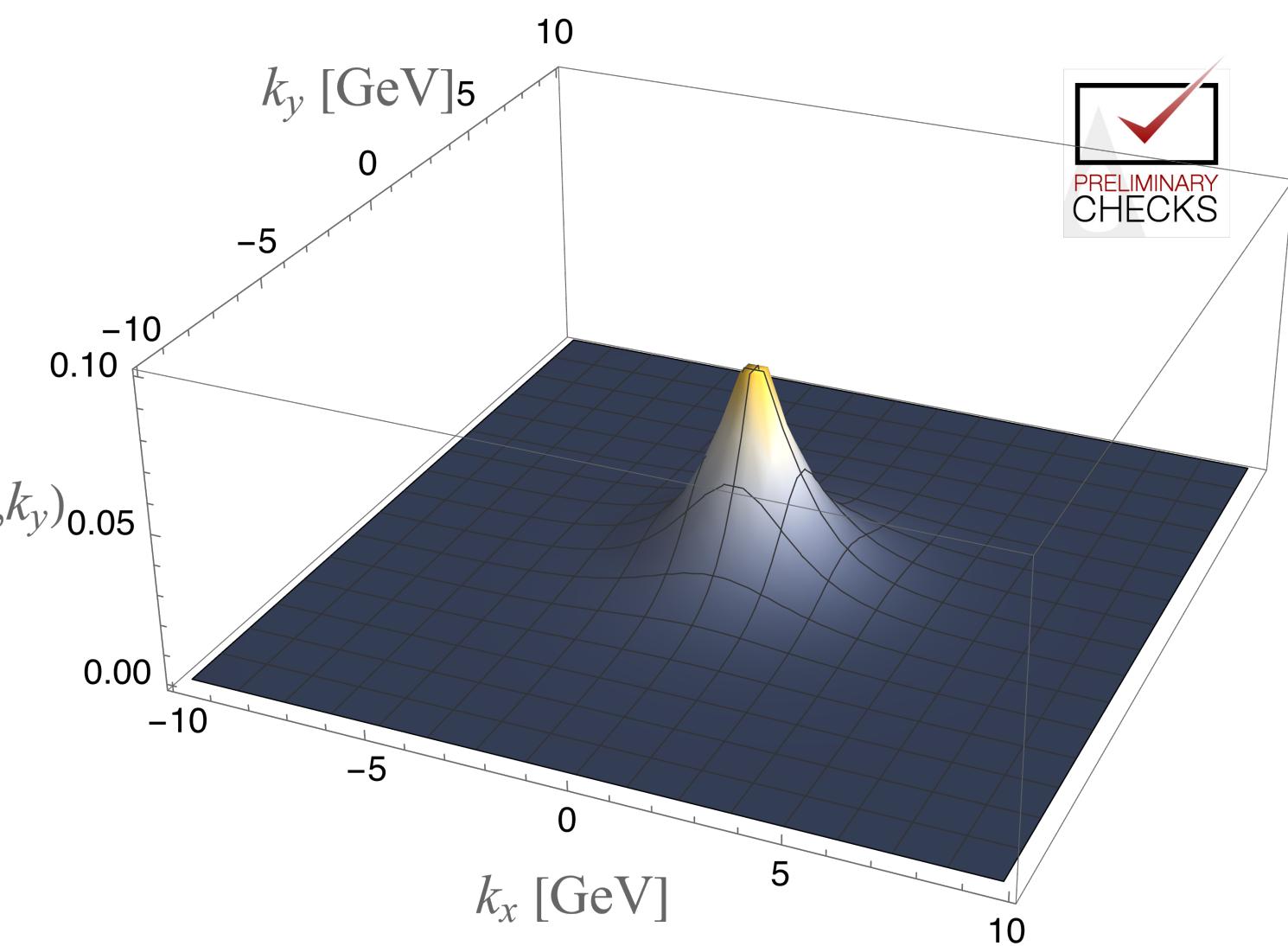
$D^{GB}$  (Gaussian broadening)



$D^{eGB}$  (effective broadening)



$D^{nGB}$  (non-Gaussian broadening)



STATIC medium

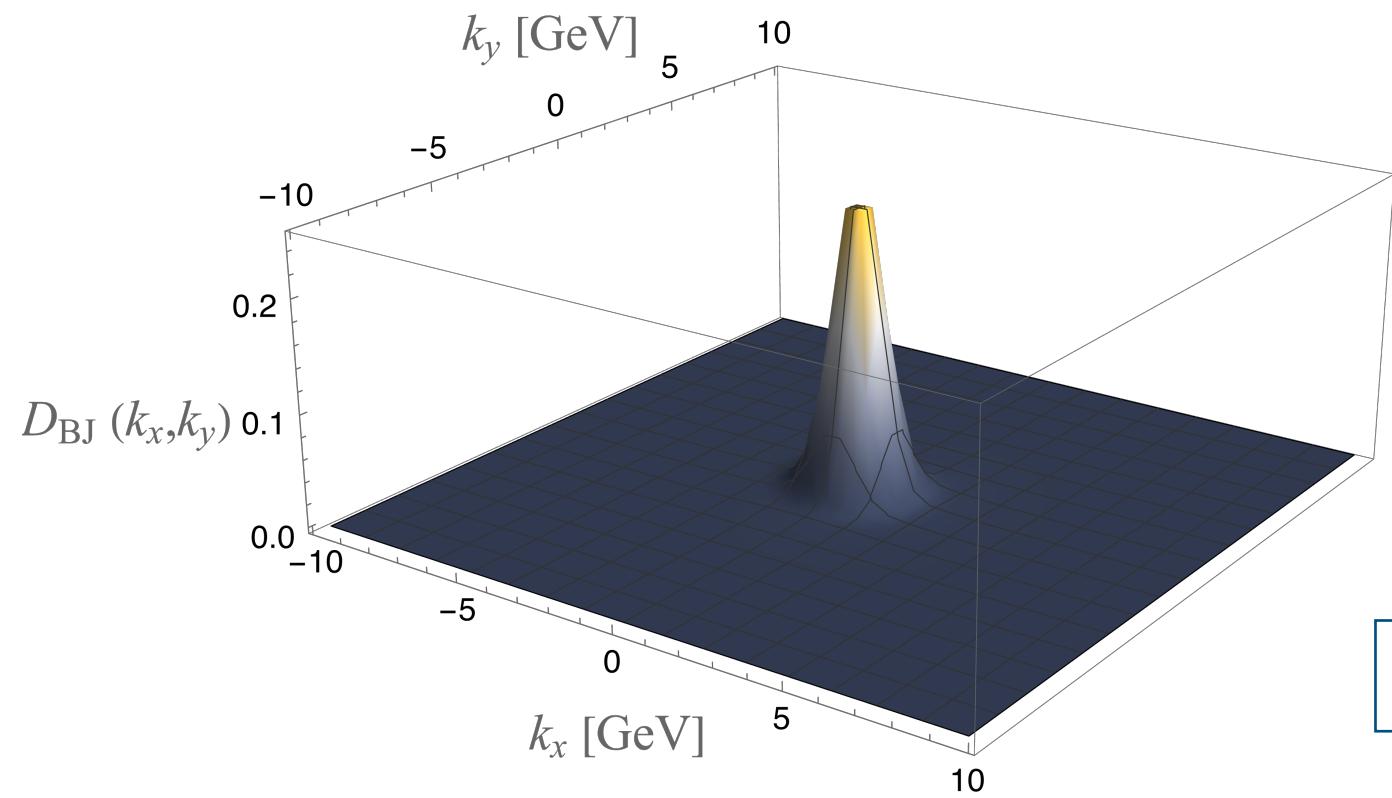
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Rohrmorser, K. Tywoniuk (in  
preparation)

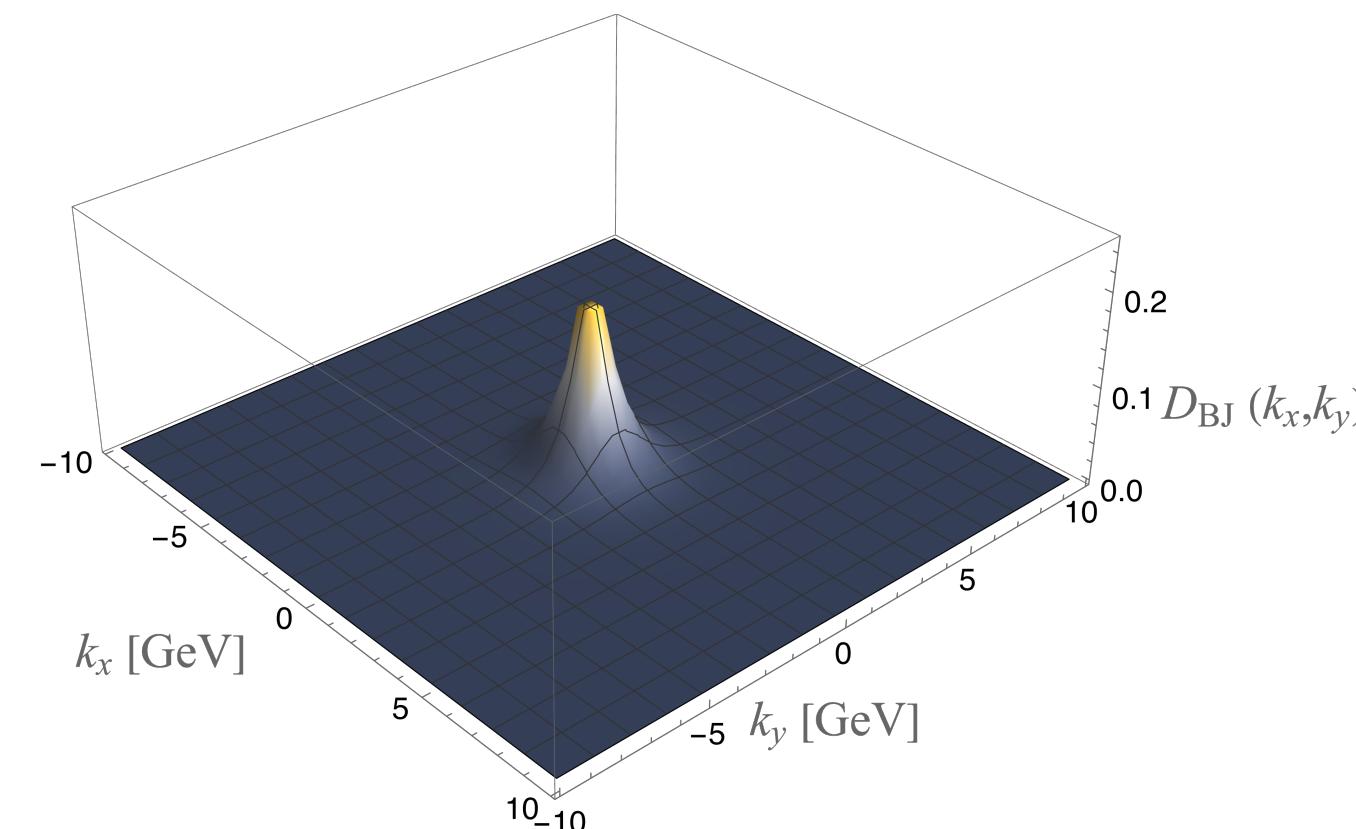
19/23

Bjorken medium  
(early quenching)

D<sup>e</sup>GB (effective broadening)



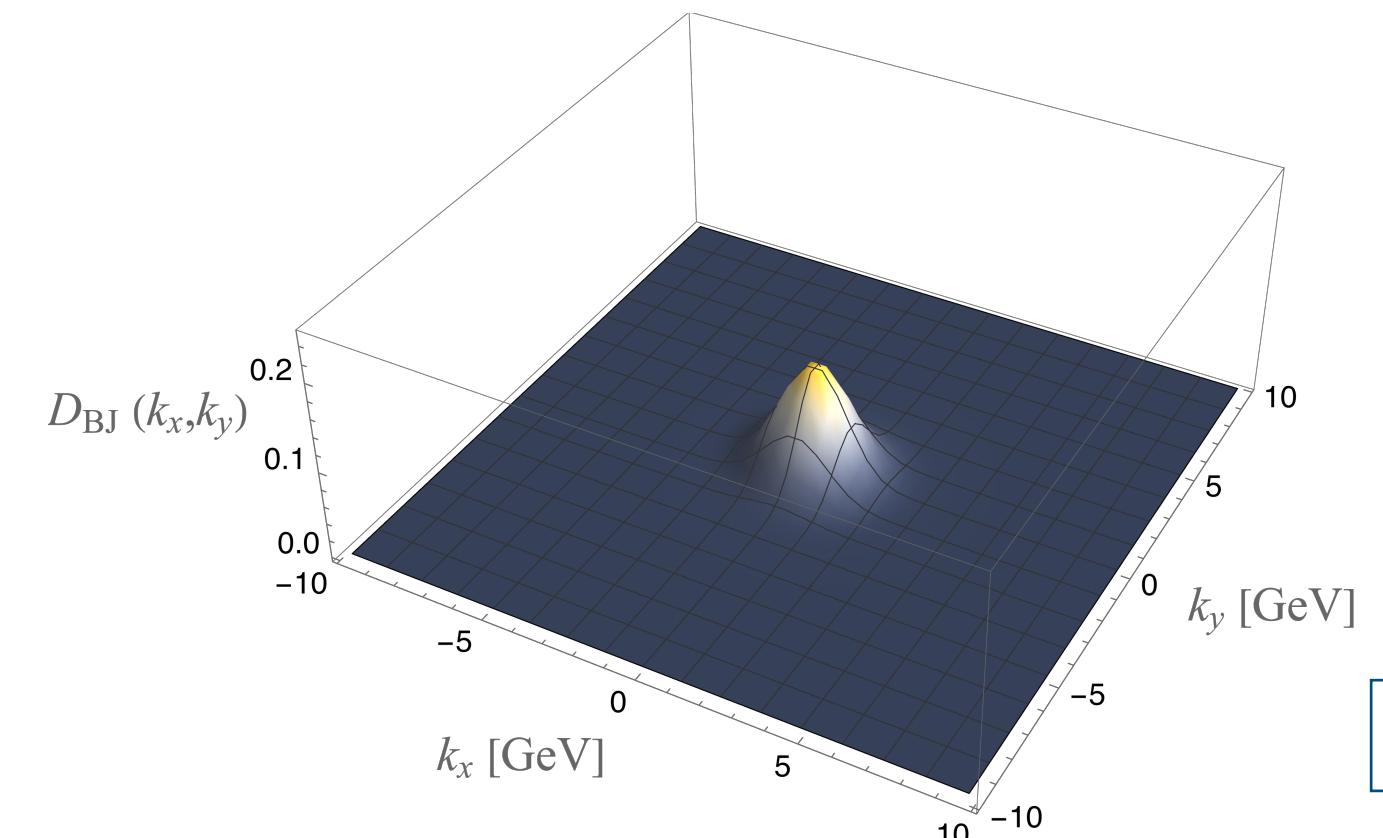
D<sup>n</sup>GB (non- Gaussian broadening)



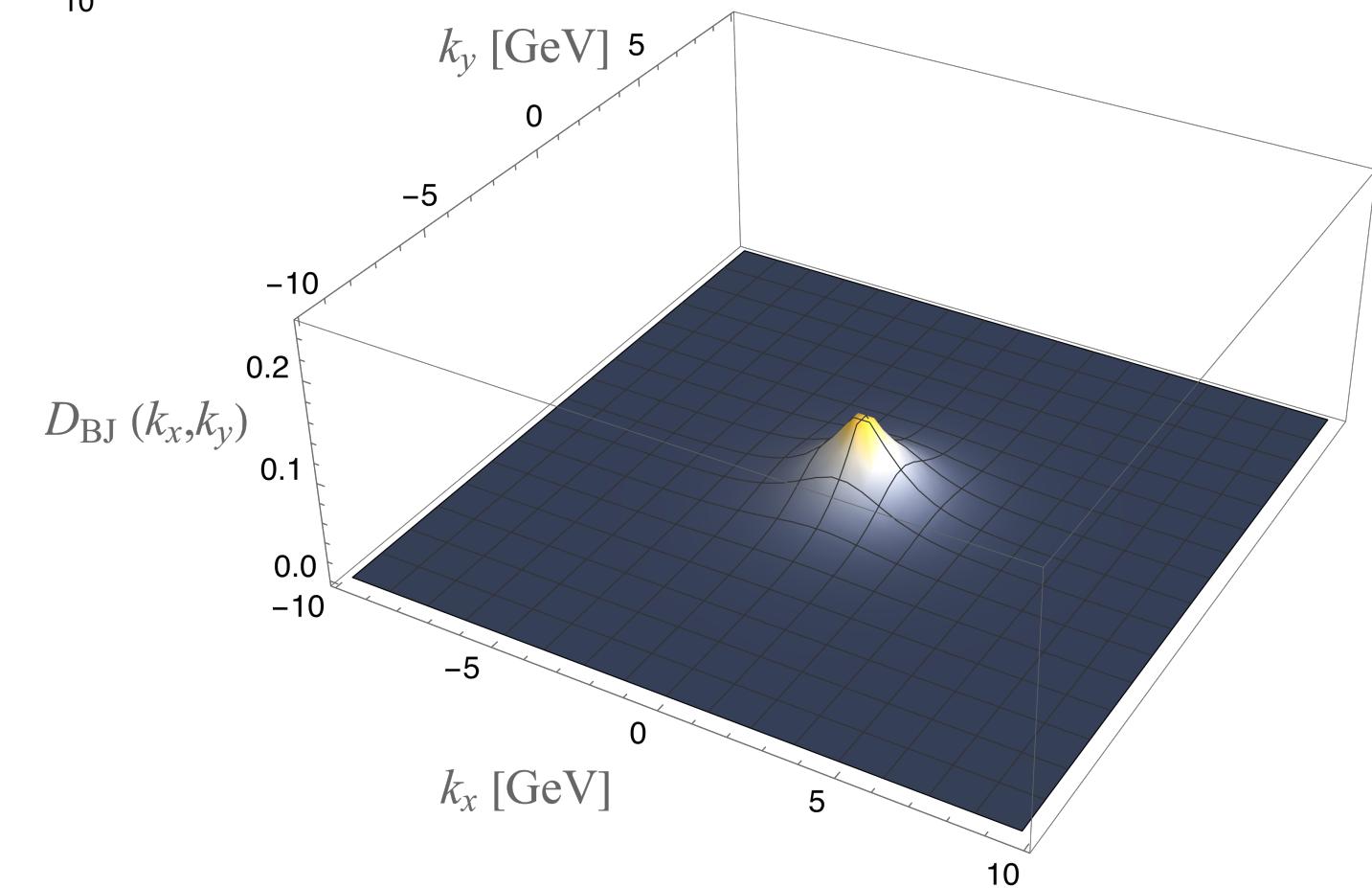
Total evolution = 4 fm,  $t_0$  (early) = 0.45 fm,  $t_0$  (late) = 2 fm

Bjorken medium  
(late quenching)

D<sup>e</sup>GB (effective broadening)



D<sup>n</sup>GB (non- Gaussian broadening)



# How does the $k_T$ dependent spectra look like ?

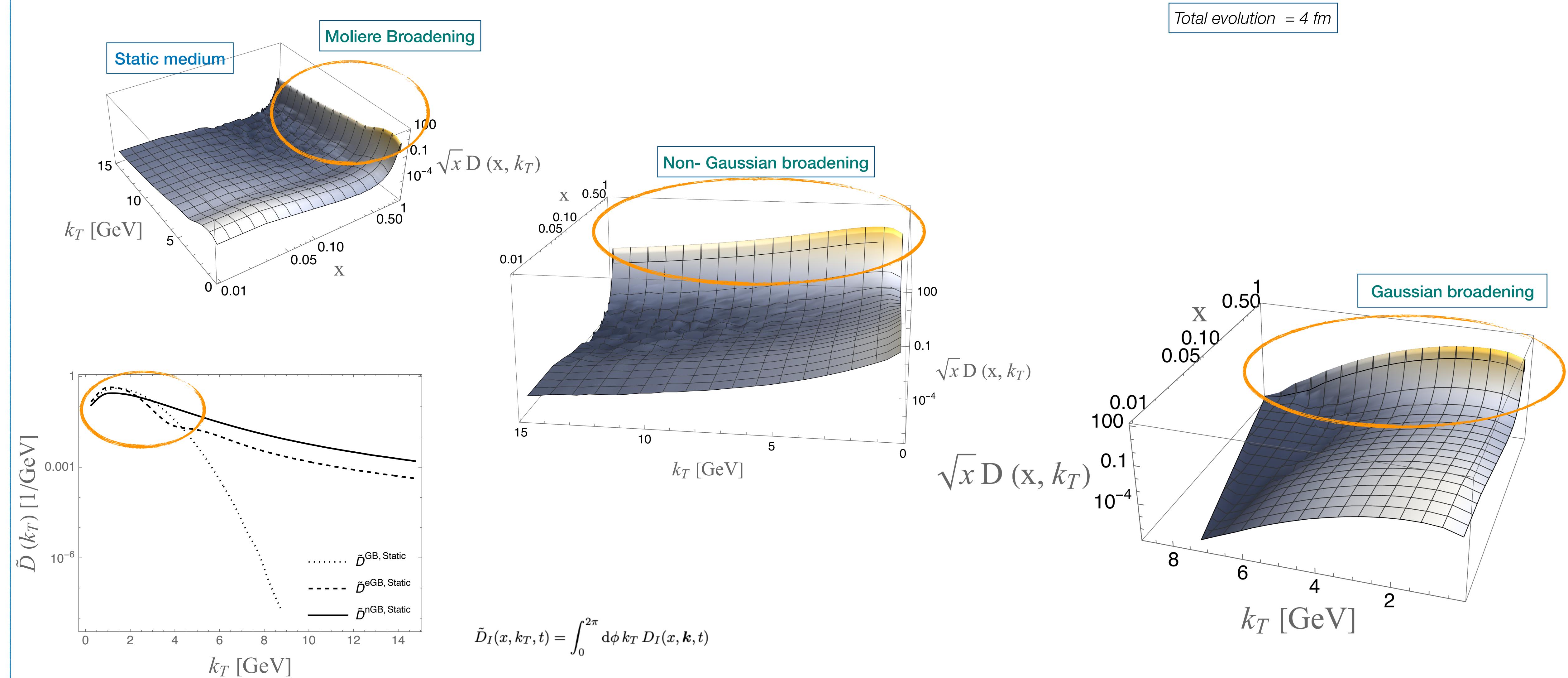
S. P. A. , K. Kutak, W. Placek, M. Rohrmorser, K. Tywoniuk (in preparation)

20/23

- Let's compare the broadening in the static spectra



PRELIMINARY  
CHECKS

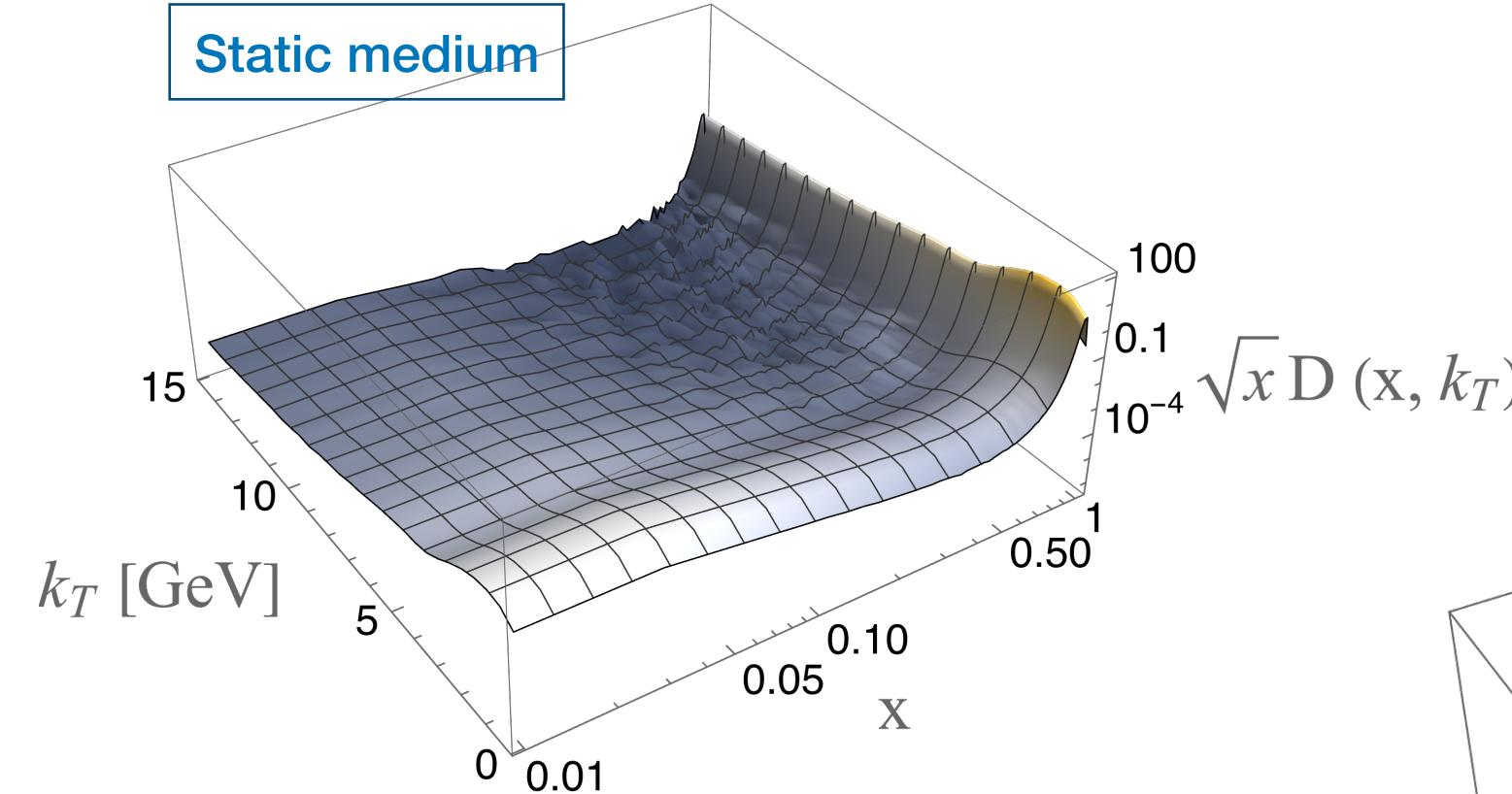


# How does the $k_T$ dependent spectra look like ?

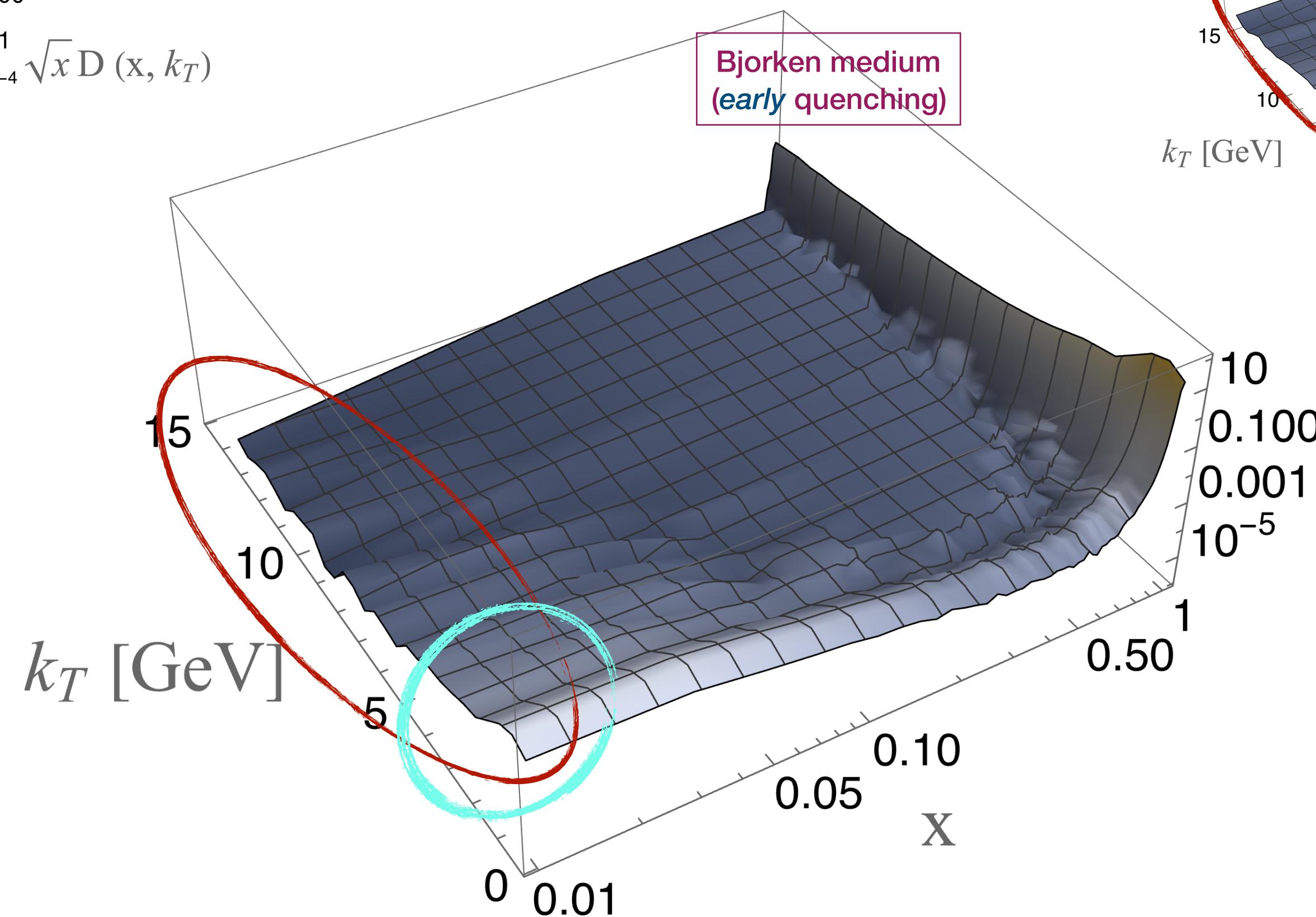
S. P. A. , K. Kutak, W. Placek, M.  
Rohrmorser, K. Tywoniuk (in  
preparation)

21/23

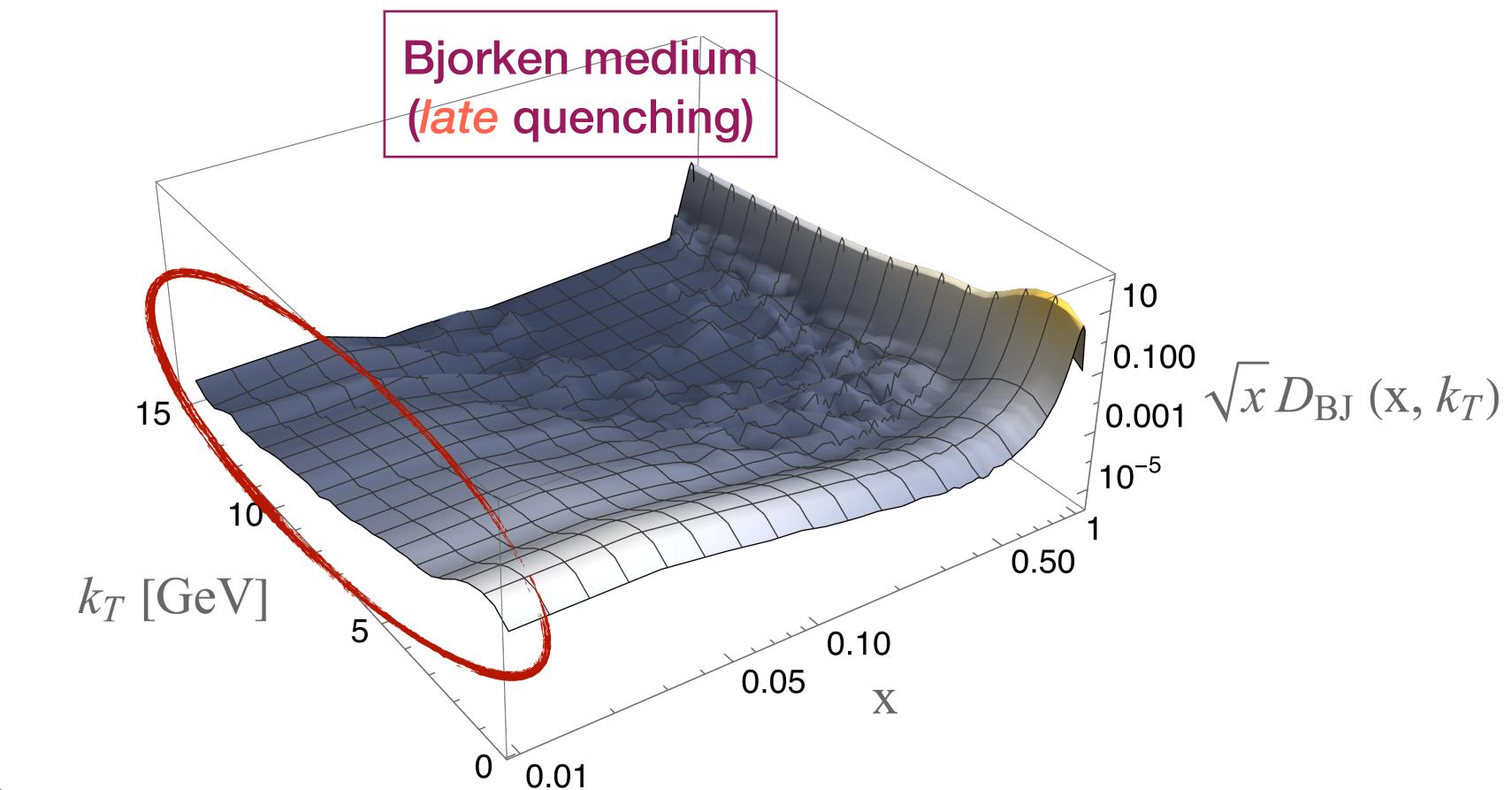
- Let's compare with the static spectra in  $x$  and  $k_T$  with the Bjorken media



Moliere Broadening



$\sqrt{x} D_{BJ}(x, k_T)$



Bjorken medium  
(late quenching)

We can see the behavior at large angle once we rescale  $k_T \rightarrow xE\Theta$  !

We expect different (and more) soft gluons at large angles among the medium profiles.



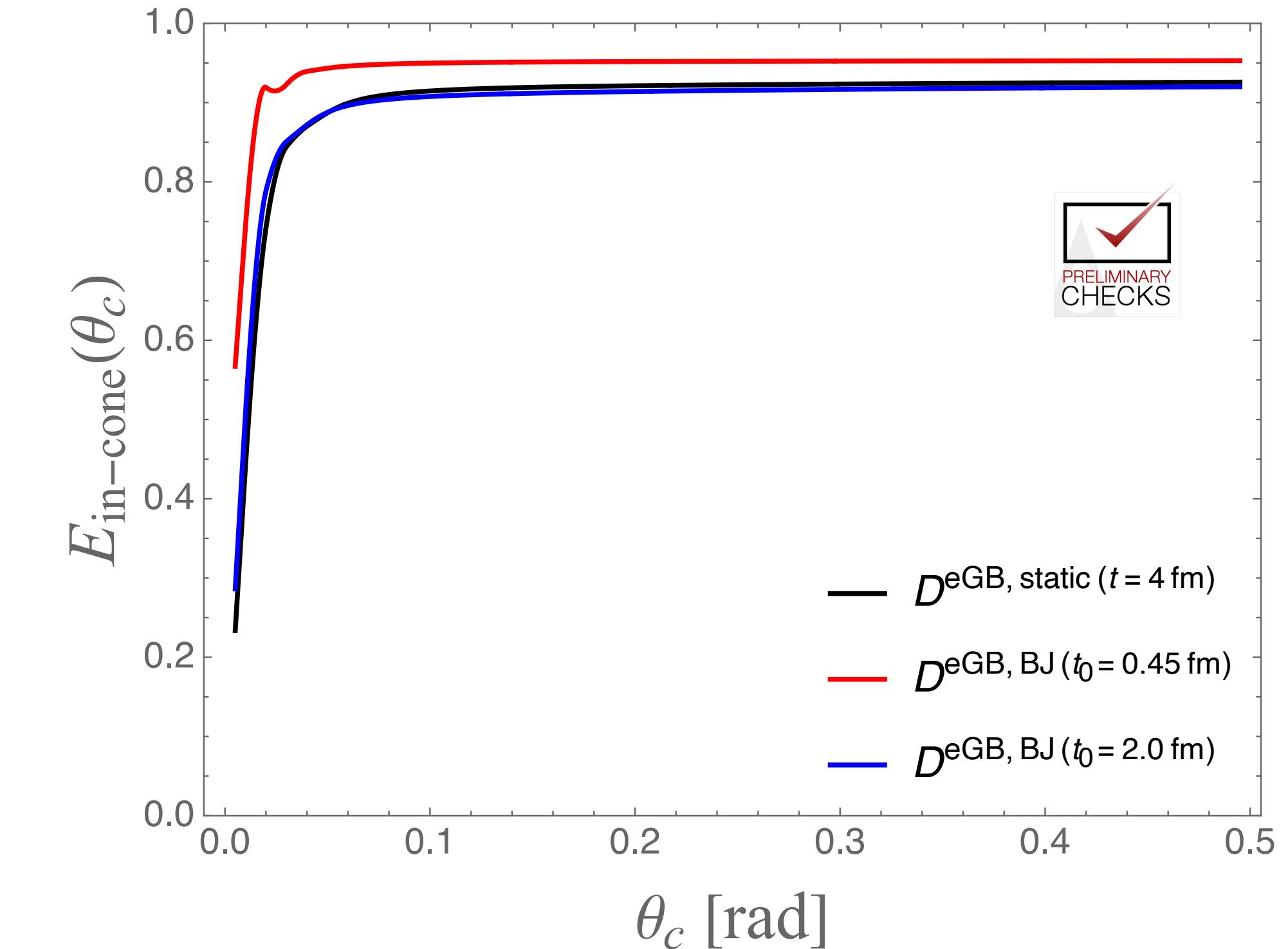
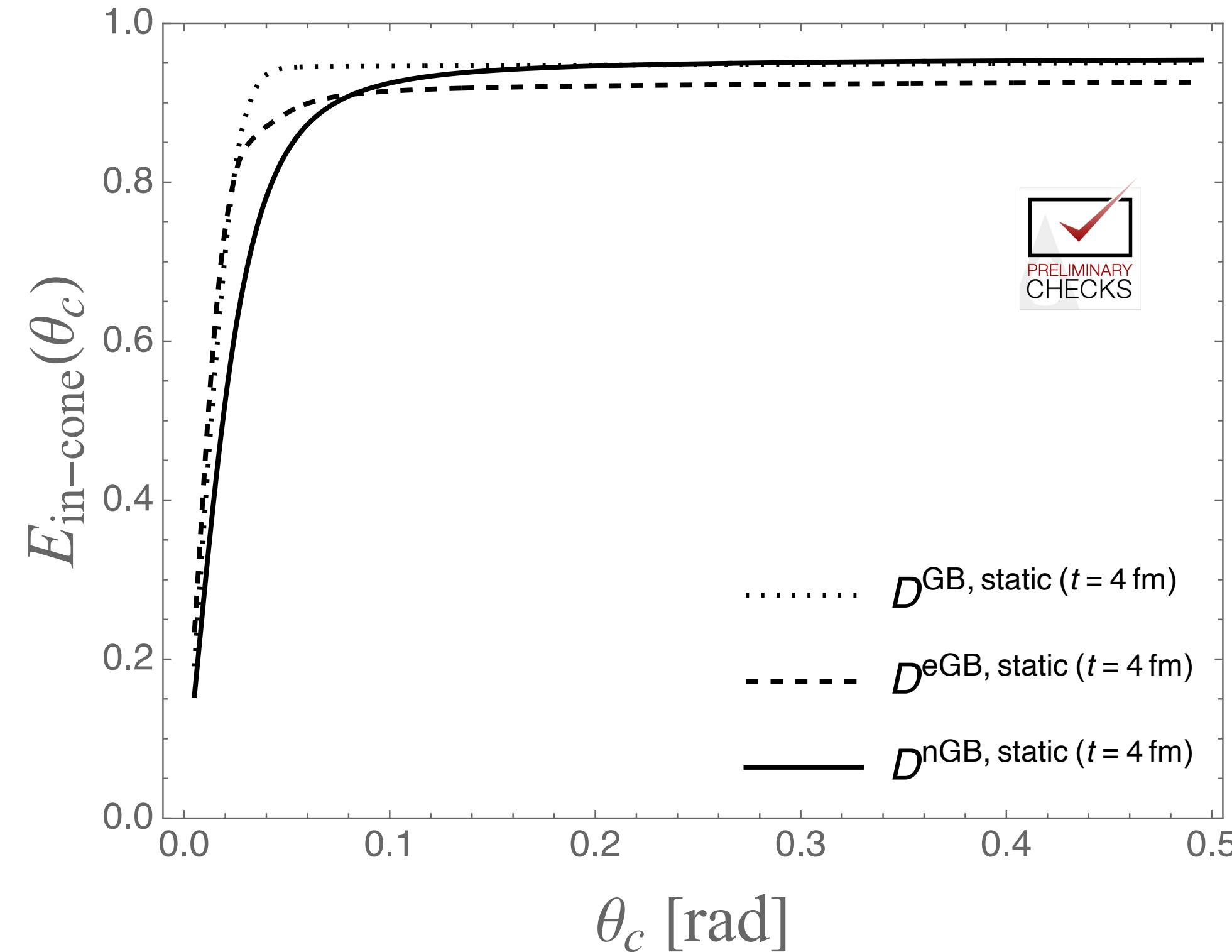
PRELIMINARY  
CHECKS

# Angular distributions for different medium profiles

S. P. A. , K. Kutak, W. Placek, M.  
Rohrmorser, K. Tywoniuk (in  
preparation)

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One can estimate the fraction of the parent gluon (jet) energy that is contained within a cone of size  $\Theta$ .

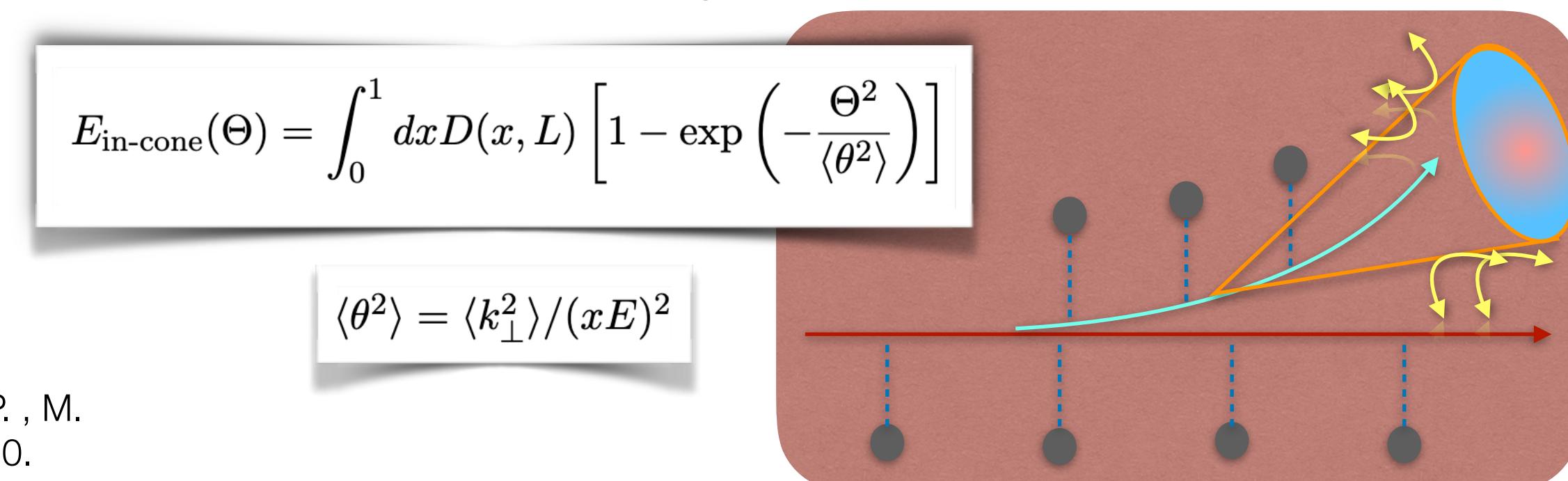


- Most of the energy is contained in a small cone.
- More in cone energy, more is the parton collimated.
- Need more checks and analysis !

(Static media) A. v-H , K. K, W. P. , M.  
R., K.T., PRC 102 (2020) 044910.

$$E_{\text{in-cone}}(\Theta) = \int_0^1 dx D(x, L) \left[ 1 - \exp \left( -\frac{\Theta^2}{\langle \theta^2 \rangle} \right) \right]$$

$$\langle \theta^2 \rangle = \langle k_\perp^2 \rangle / (xE)^2$$

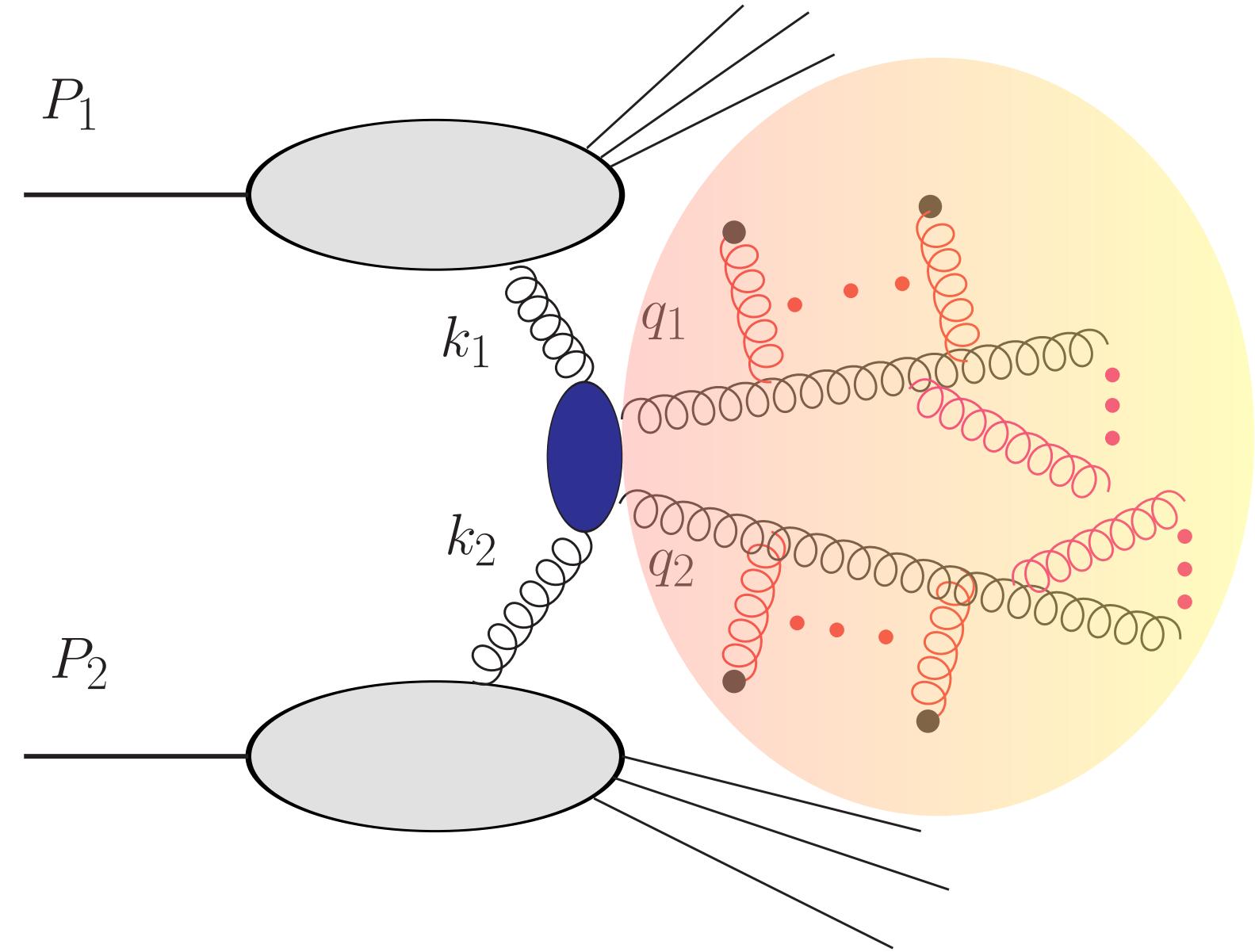


# Take home messages and Scopes ?

S. P. A. , K. Kutak, W. Placek, M.  
Rohrmorser, K. Tywoniuk (in  
preparation)

23/23

Pic courtesy : K. Kutak



- For the Bjorken expansion, the medium evolved spectra as well as quenching factors are still sensitive to the onset of quenching through the ratio  $(t_0/L)$ , which **spoils the universal scaling features**.
- Rapidity dependence of the inclusive jet suppression is **not very sensitive** to the way how the **medium expands**.
- **Jet  $v_2$  is sensitive** to the **medium expansion** with difference in initial starting time of expansion.
- Exploring the  **$k_T$  dependent cascades** and its role on observables for an **expanding medium**.

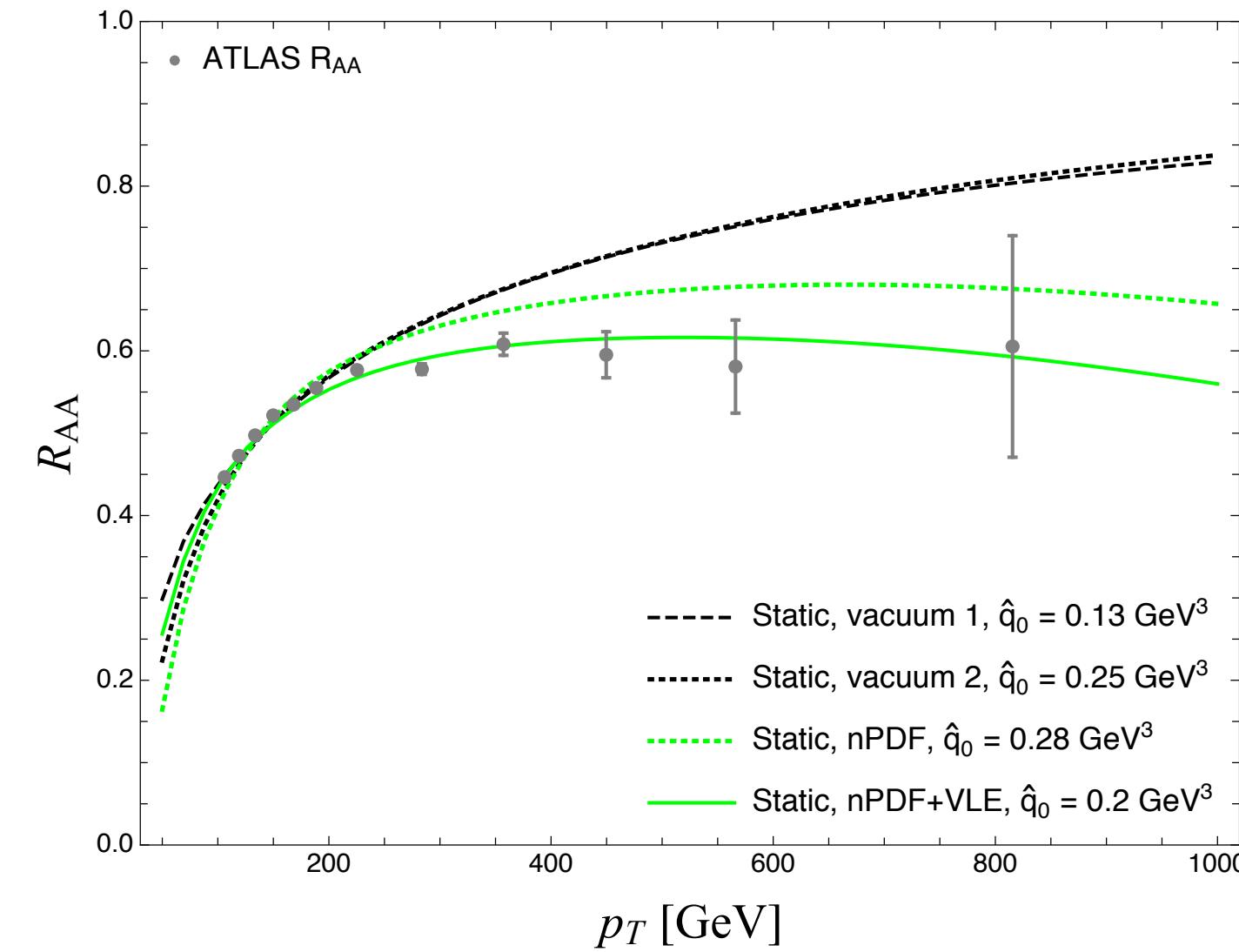
More on medium modelling impacts on jet quenching : A. Sadofyev and J. Barata  
(Wed, Morning)

Thanks !

Complexity/ Completeness  
towards understanding



# Effects of nPDF and VLE



- Vacuum like emissions (VLE) :

$$\text{Phase space for VLE : } \Pi_{\text{in}} = 2 \frac{\alpha_s C_i}{\pi} \int_{R_{\min}}^R \frac{d\theta}{\theta} \int_{k_{\perp,\min}}^{p_T \theta} \frac{dk_{\perp}}{k_{\perp}} = \frac{\alpha_s C_i}{\pi} \ln \left( \frac{R}{R_{\min}} \right) \ln \left( \frac{p_T^2 R R_{\min}}{k_{\perp,\min}^2} \right)$$

$$R_{\min} = \max[\theta_c, \theta_d] \text{ and } k_{\perp,\min} = \max[Q_s(L), Q_0]$$

$$\text{Collimator function : } Q_i(p_T, R) = Q_i^{(0)}(p_T) \exp \left[ \Pi_{\text{in}} \left( Q_g^{(0)}(p_T) - 1 \right) \right]$$

- Effect of VLE : Increase of the quenching.

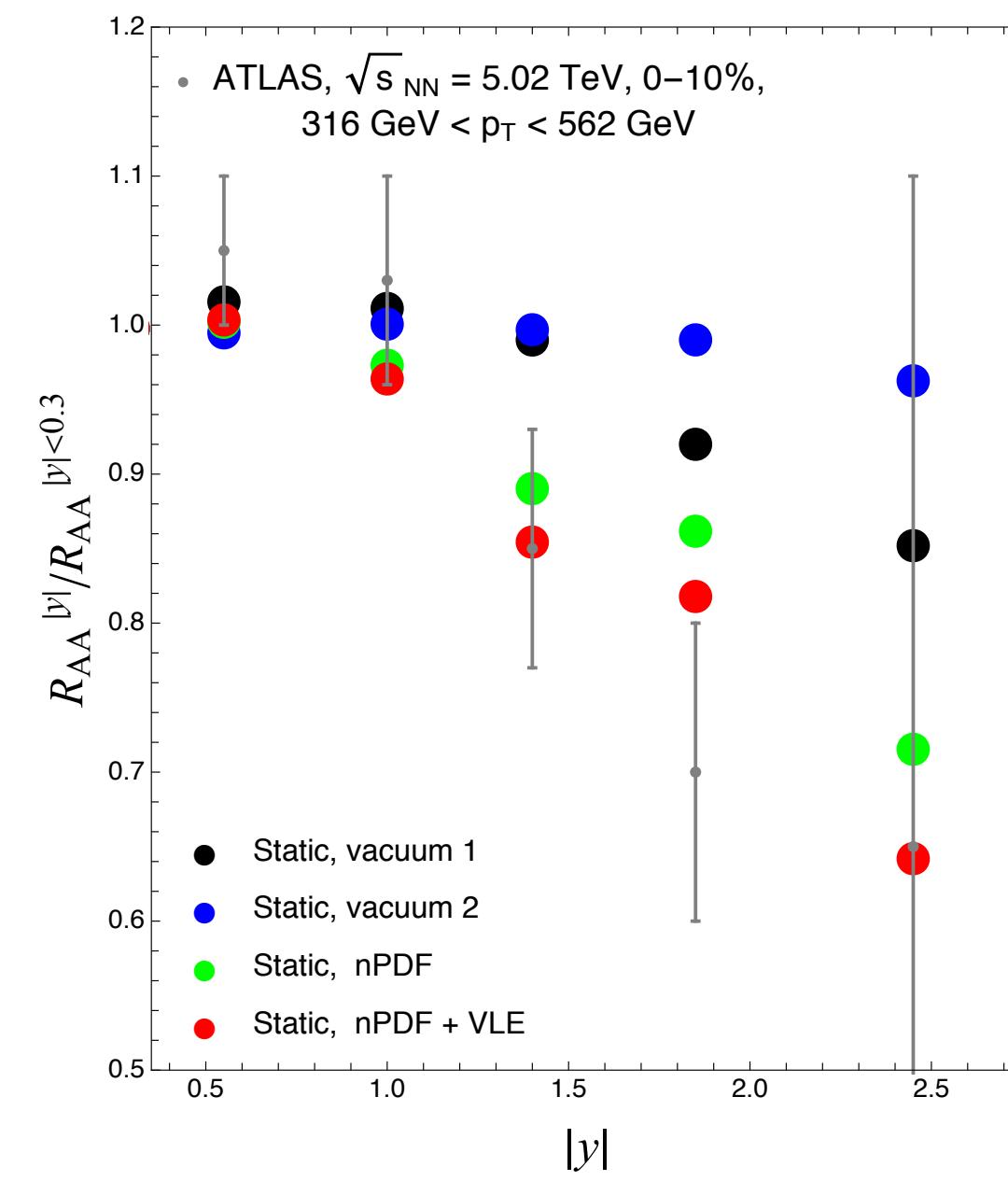
Configurations:

Vacuum 1 = Input parton spectra,  
PYTHIA8.185+AU2+CT10 PDF.

Vacuum 2 = PYTHIA 8.306  
(default).

nPDF = PYTHIA 8.306 (default),  
EPS09LO nPDF.

nPDF + VLE = PYTHIA 8.306  
(default), VLE.



- Effect of nPDF : Flattening at high  $p_T$ .

Quenching parameter ( $\hat{q}$ )	Static (soft)	Static	Expo	Bjorken $t_0 = 0.1 \text{ fm}$
$\hat{q}_0$ (nPDF+VLE) [ $\text{GeV}^3$ ]	0.15	0.2	0.08	1.8
$\hat{q}_0$ (gluon-only) [ $\text{GeV}^3$ ]	0.20	0.2	0.09	2.6

Comparison of “nPDF” and “nPDF+VLE” configuration implies that adding VLE to the calculation has an important impact on both the shape of  $R_{AA}$  and its overall normalization which has an impact on the extracted values of  $\hat{q}$ .

## ATLAS measurements

