Jet thermalization

Based on Y. Mehtar-Tani, S. Schlichting I. Soudi, in preparation



also see earlier work Y. Mehtar-Tani, S. Schlichting JHEP 09 (2018) 144 S. Schlichting, I. Soudi JHEP 07 (2021) 077



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Scope/Motivation

Jet quenching phenomena in HIC probe variety of different aspects of QCD

initial production vacuum shower energy loss medium response hadronization



Develop clear understanding of mechanism for degradation of energy, out-of-cone energy loss, medium response and thermalization of hard partons/jets

Especially the equilibration of soft large angle fragments in high-energy jets and the complete disappearance of low energy jets is interesting, as it may be one of the only ways to explore QCD thermalization in experiment

Methodology

Challenge: Description of jet thermalization requires theoretical description, that is valid at scales ~E (hard fragments) down to scales ~T (soft fragments & thermal medium)

Describe in-medium evolution jet fragments as collection of on-shell quarks/ gluons in QCD kinetic theory (HTL screened 2<->2, eff. 1<->2 processes)

$$\left(\partial_t + rac{oldsymbol{p}}{|oldsymbol{p}|}
abla
ight) f_a(oldsymbol{p}, \mathbf{x}, t) = -C_a^{2\leftrightarrow 2}[\{f_i\}] - C_a^{1\leftrightarrow 2}[\{f_i\}]$$
 .

with jet as linearized perturbation on top of static equilibrium background

Same setup as in studies of thermalization of QGP at early times

Characterize evolution in terms of energy distribution $D(t, x, \theta) = x \frac{dN}{dxd\cos(\theta)}$ starting from initial state of single primary quark/gluon

> Effectively calculating a Green's function for in-medium evolution of hard parton

Eff. kinetic theory

Elastic scattering processes treated with leading order HTL screened matrix elements

$$C_a^{2\leftrightarrow 2}[\{f_i\}] = \frac{1}{2|p_1|\nu_a} \sum_{bcd} \int d\Omega^{2\leftrightarrow 2} \left| \mathcal{M}_{cd}^{ab}(\boldsymbol{p}_1, \boldsymbol{p}_2; \boldsymbol{p}_3, \boldsymbol{p}_4) \right|^2 \delta \mathcal{F}(\boldsymbol{p}_1, \boldsymbol{p}_2; \boldsymbol{p}_3, \boldsymbol{p}_4) ,$$

including full statistical factors with Bose enhancement/Fermi suppression and inverse processes

$$\begin{split} \delta \mathcal{F}(\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3, \boldsymbol{p}_4) &= \delta f_a(\boldsymbol{p}_1) \left[\pm_a n_c(p_3) n_d(p_4) - n_b(p_2) (1 \pm n_c(p_3) \pm n_d(p_4)) \right] \\ &+ \delta f_b(\boldsymbol{p}_2) \left[\pm_b n_c(p_3) n_d(p_4) - n_b(p_1) (1 \pm n_c(p_3) \pm n_d(p_4)) \right] \\ &- \delta f_c(\boldsymbol{p}_3) \left[\pm_c n_a(p_1) n_b(p_2) - n_b(p_4) (1 \pm n_a(p_1) \pm n_b(p_2)) \right] \\ &- \delta f_d(\boldsymbol{p}_4) \left[\pm_d n_a(p_1) n_b(p_2) - n_b(p_3) (1 \pm n_a(p_1) \pm n_b(p_2)) \right] \end{split}$$

Exact conservation of energy, momentum and valence charge of the jet allows to study evolution from ~E to ~T including thermalization of the jet

Eff. kinetic theory

Numerical studies take into account full re-construction of in-medium rates in the AMY framework (incl. LPM & Bethe-Heitler regime) for an infinite medium

$$\begin{split} C_g^{g\leftrightarrow gg}[\{D_i\}] &= \int_0^1 dz \frac{d\Gamma_{gg}^g(\left(\frac{xE}{z}\right), z)}{dz} \bigg[D_g\left(\frac{x}{z}\right) \left(1 + n_B(xE) + n_B\left(\frac{\bar{z}xE}{z}\right)\right) \\ &+ \frac{D_g(x)}{z^3} \left(n_B\left(\frac{xE}{z}\right) - n_B\left(\frac{\bar{z}xE}{z}\right)\right) + \frac{D_g\left(\frac{\bar{z}xE}{z}\right)}{\bar{z}^3} \left(n_B\left(\frac{xE}{z}\right) - n_B(xE)\right)\bigg] \\ &- \frac{1}{2} \int_0^1 dz \frac{d\Gamma_{gg}^g(xE, z)}{dz} \bigg[D_g(x)(1 + n_B(zxE) + n_B(\bar{z}xE)) \\ &+ \frac{D_g(zx)}{z^3} (n_B(xE) - n_B(\bar{z}xE)) + \frac{D_g(\bar{z}x)}{\bar{z}^3} (n_B(xE) - n_B(zxE))\bigg] \,, \end{split}$$

Statistical factors include Bose enhancement/Fermi suppression as well as inverse (2->1) processes important at energies ~T

Exact conservation of energy, momentum and valence charge of the jet allows to study evolution from ~E to ~T including thermalization of the jet

Eff. kinetic theory

Breakdown of advantages & disadvantages

Unified treatment of jet and medium response within same QCD kinetic theory framework

Infinite medium rates higher than more realistic finite medium rates, resulting in too much quenching

Not including any vacuum like effects, as they effectively enter initial condition/ source terms for kinetic equation

Not straightforward to study/ include jet-by-jet fluctuations as this is not actually a Monte-Carlo simulation





Out-of-cone energy loss:

1) Energy deposition into soft sector via nearly collinear cascade of radiative break-up confined to narrow cone θ <0.3

Stationary turbulent solution (T/E << x << 1)

In-elastic processes dominate at intermediate T/E << x << 1, can approximate kinetic equations by only considering radiative emissions

$$\begin{aligned} \frac{\partial}{\partial \tau} D_{\rm g}\left(x,\tau\right) &= \int_{0}^{1} dz \,\mathcal{K}_{\rm gg}(z) \left[\sqrt{\frac{z}{x}} D_{\rm g}\left(\frac{x}{z}\right) - \frac{z}{\sqrt{x}} D_{\rm g}(x)\right] - \int_{0}^{1} dz \,\mathcal{K}_{\rm qg}(z) \frac{z}{\sqrt{x}} \,D_{\rm g}\left(x\right) \\ &+ \int_{0}^{1} dz \mathcal{K}_{\rm gq}(z) \sqrt{\frac{z}{x}} \,D_{\rm S}\left(\frac{x}{z}\right), \quad \cdots \\ \frac{\partial}{\partial \tau} D_{\rm S}\left(x,\tau\right) &= \int_{0}^{1} dz \,\mathcal{K}_{\rm qq}(z) \left[\sqrt{\frac{z}{x}} D_{\rm S}\left(\frac{x}{z}\right) - \frac{1}{\sqrt{x}} D_{\rm S}(x)\right] + \int_{0}^{1} dz \,\mathcal{K}_{\rm qg}(z) \sqrt{\frac{z}{x}} D_{\rm g}\left(\frac{x}{z}\right) \end{aligned}$$

where $\sqrt{z/x}$ and $1/\sqrt{x}$ factors follow from shorter formation time for successive branchings

(c.f. Blaizot, Mehtar-Tani; Blaizot, Mehtar-Tani, lancu)

Stationary solution (T/E << x << 1)

Stationary non-equilibrium solution:

$$\begin{aligned} \frac{\partial}{\partial \tau} D_{\mathrm{g}}\left(x,\tau\right) &= \int_{0}^{1} dz \, \mathcal{K}_{\mathrm{gg}}(z) \left[\sqrt{\frac{z}{x}} D_{\mathrm{g}}\left(\frac{x}{z}\right) - \frac{z}{\sqrt{x}} D_{\mathrm{g}}(x) \right] - \int_{0}^{1} \mathrm{d}z \, K_{\mathrm{qg}}(z) \frac{z}{\sqrt{x}} \, D_{\mathrm{g}}\left(x\right) \\ &+ \int_{0}^{1} \mathrm{d}z K_{\mathrm{gq}}(z) \sqrt{\frac{z}{x}} \, D_{\mathrm{S}}\left(\frac{x}{z}\right), \end{aligned}$$

$$\frac{\partial}{\partial \tau} D_{\rm S}\left(x,\tau\right) = \int_0^1 dz \, \mathcal{K}_{\rm qq}(z) \left[\sqrt{\frac{z}{x}} D_{\rm S}\left(\frac{x}{z}\right) - \frac{1}{\sqrt{x}} D_{\rm S}(x)\right] + \int_0^1 dz \, \mathcal{K}_{\rm qg}(z) \, \sqrt{\frac{z}{x}} D_{\rm g}\left(\frac{x}{z}\right)$$

Chemistry of fragments fixed by balance of g->qq and q->gq processes

$$\frac{S}{G} = \frac{\int_0^1 \mathrm{d}z \ z \ \mathcal{K}_{\mathrm{qg}}(z)}{\int_0^1 \mathrm{d}z \ z \ K_{\mathrm{gq}}(z)} \approx 0.07 \ \times 2N_f$$

 $D_g(x) = rac{G}{\sqrt{x}} , \quad D_S = rac{S}{\sqrt{x}} ,$

Existence of solution does not rely on detailed form of K(z) but only on the fact that emission rates behave as $1/\sqrt{xE}$ due to formation time

Energy cascade

Solution is analogous to Kolmogorov-Zhakarov spectrum in weak wave turbulence is associated with a finite scale invariant energy flux

$$\frac{dE}{d\tau} = \tilde{\gamma}_g G + \tilde{\gamma}_q S$$

with flux constants given by simple moments of the medium-induced splitting functions



$$\begin{split} \tilde{\gamma}_{g} &= \int_{0}^{1} dz \ z [\mathcal{K}_{gg}(z) + 2N_{f} \mathcal{K}_{qg}(z)] \ \log(z) = \frac{\alpha_{s}}{2\pi} \sqrt{\frac{\hat{\bar{q}}(\sqrt{TE})}{E}} (25.78 + 2N_{f} 0.177) \\ \tilde{\gamma}_{q} &= \int_{0}^{1} dz \ 2z [K_{gq}(z) + K_{qq}(z)] \log(z) \qquad = \frac{\alpha_{s}}{2\pi} \sqrt{\frac{\hat{\bar{q}}(\sqrt{TE})}{E}} (11.595) , \end{split}$$

dominated by quasi-democratic (z~1/2) branchings

(c.f. Baier, Mueller, Schiff, Son; Blaizot, Mehtar-Tani, Iancu)



Out-of-cone energy loss:

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2) Soft fragments (x~T/E)
spread out to large angles
(θ~1) via elastic interactions



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3) Eventually jet thermalizes, when all hard partons have decayed

$$D^{\rm eq}(x,\cos\theta) = x^3 \delta T \partial_T n_a(xE) + x^3 \frac{\delta P_z}{E} \partial_\beta n_a(xE(1-\beta\cos\theta)) \Big|_{\beta\to 0}$$

Energy loss

Single emission off the original hard parton create a $1/\sqrt{x}$ gluon spectrum at small x with amplitude

$$G(t) \simeq C_R C_A^{1/2} \frac{\alpha_s}{2\pi} \sqrt{\frac{\hat{q}(E)}{E}} t$$

Multiple emissions off soft fragments create turbulent energy flux

$$rac{dE}{d au} = ilde{\gamma}_g G + ilde{\gamma}_q S$$
 ,



all the way to T/E where energy is absorbed by the thermal QGP and goes out to large angles

$$\frac{dE}{d\tau} = \gamma^{\rm soft-radiation} + \gamma^{\rm recoil} + \left(\tilde{\gamma}_g + \frac{S}{G}\tilde{\gamma}_q\right)G(\tau)$$

(c.f Baier, Mueller, Schiff, Son; Blaizot, Iancu, Mehtar-Tani)

Out-of-cone energy loss for narrow cones (R~0.3) governed by radiative break-up of hard fragments + rapid broadening of soft fragments



Energy (E/T) dependence governed by radiative emission rates of the primary hard parton

Significant recovery of energy of soft thermal fragments beyond R~0.3, with strong sensitivity to soft sector for larger cone sizes



Can try to search for signatures of thermalization/ medium-response in outskirts of the jet

Jet Quenching

So far all calculations with QCD kinetic theory, not including finite length effects & fluctuations

Improve by calculating first splitting with BDMPS rate $d\Gamma/d\omega \sim 1/\omega^{3/2}$ including fluctuations and only sub-sequently use QCD Kinetic theory to calculate energy remaining inside jet cone



$$Q(p_T) = \exp\left[\int_0^L \mathrm{d}t \int \mathrm{d}\omega \frac{\mathrm{d}\Gamma}{\mathrm{d}\omega} \left(1 - \mathrm{e}^{-n\frac{\omega}{p_T} \left[1 - E\left(\omega, R, \tau = \frac{L-t}{t_{\mathrm{th}}}\right)\right]}\right)\right]$$

Employ QCD kinetic Green's functions to propagate the soft fragments through the medium and see how much of their energy remains inside cone of size R



Jet Quenching

Stochastic emission with BDMPS rate + QCD Kinetic theory to calculate amount of energy remaining inside jet cone



Bands highlight increased sensitivity to soft sector for R>0.3; this is where to search for medium response

Conclusions & Outlook

Out-of-cone energy loss and thermalization of highly energetic partons/jets governed by nearly collinear cascade + broadening of soft fragments



Dynamics accessible via variations of cone-size R and energy range pT_{min} for strongly quenched jets? Effects of modified q/g chemistry of jet? Jet photons as probe of equilibration of soft fragments?

Green's functions for energy loss of soft fragments can be used in semianalytic calculations of jet quenching observables

First step towards development of MC Generator for unified description of jet quenching & medium response within QCD Kinetic Theory

Backup

Angular structure



Gluon jet E/T = 500

Energy dependence of equilibration



Scale invariant energy flux confirms turbulent nature of energy loss mechanism

Eventually breaks down for low scale ~10T (mini-) partons, where direct energy loss is efficient



Jet chemistry

Balance of the g->qq and q->qg processes at the non-equilibrium steady state (x < <1) uniquely determines chemistry



Universal Kolmogorov ratio approximately realized at intermediate momentum fractions x and evolution times

Comparison to diffusion approximation

