

Forward quark jet-nucleus scattering in a light-front Hamiltonian approach

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DE GALICIA**

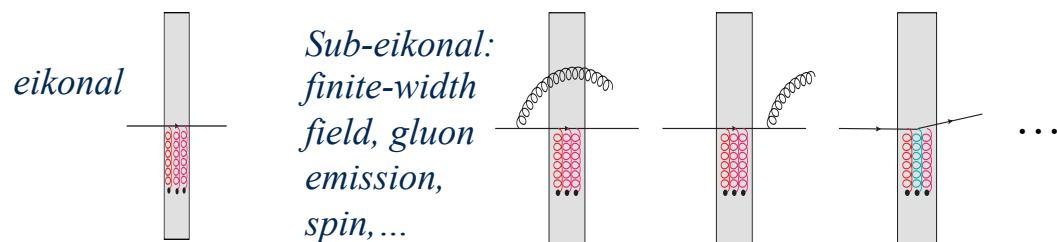
Motivation: Quark scattering off a color field

The general picture of a high energy quark scattering off a color field for two different processes:

- **High-energy quark nucleus scattering**

An infinitely energetic quark passes through the color glass condensate state of small x gluons insides a high-energy nucleus, and the field is treated as an infinitely thin shockwave, using the eikonal approximation.

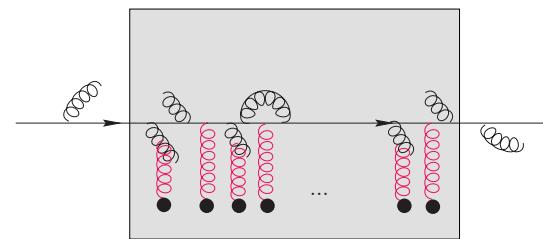
Interests: sub-eikonal effects



- **Jet quenching in colored medium**

The probe has infinitely large energy, the field is spatially extended, and the quark loses energy by gluon emission.

Interests: interplay between coherence time of emission and the timescales of the scattering centers of the medium



- Using a nonperturbative approach, we could
 - Relax the infinite-energy approximation
 - Explicitly solve the time evolution and therefore study the scattering and gluon emission/absorption at the same time

Outline

□ Methodology

- A light-front Hamiltonian approach for solving time-dependent problems
- Application to quark-nucleus scattering in the $|q\rangle + |qg\rangle$ Fock space

□ Results

1. Evolution of the quark system
 - a) Gluon emission and absorption
 - b) Interaction with a background color field
2. Momentum broadening

□ Summary and outlooks

Based on: M. Li, T. Lappi , and X. Zhao, Phys. Rev. D 104, 056014 (2021), and ongoing works.

Methodology: Time-dependent Basis Light-Front Quantization (tBLFQ)¹

➤ Light-front quantization

- The quantum field is quantized on the equal light-front time surface $x^+(\equiv x^0 + x^3)=0$

	instant form	front form	point form
time variable	$t = x^0$	$x^+ \triangleq x^0 + x^3$	$\tau \triangleq \sqrt{t^2 - \vec{x}^2 - a^2}$
quantization surface			
Hamiltonian	$H = P^0$	$P^- \triangleq P^0 - P^3$	P^μ
kinematical	\vec{P}, \vec{J}	$\vec{P}^\perp, P^+, \vec{E}^\perp, E^+, J^-$	\vec{J}, \vec{K}
dynamical	\vec{K}, P^0	\vec{F}^\perp, P^-	\vec{P}, P^0
dispersion relation	$p^0 = \sqrt{\vec{p}^2 + m^2}$	$p^- = (\vec{p}_\perp^2 + m^2)/p^+$	$p^\mu = mv^\mu \ (v^2 = 1)$

➤ Hamiltonian formalism

- The state obeys the time-evolution equation

$$\frac{1}{2} P^-(x^+) |\psi(x^+) \rangle = i \frac{\partial}{\partial x^+} |\psi(x^+) \rangle$$

A nonperturbative treatment: the time evolution is divided into many small timesteps, each timestep is evaluated by numerical method (e.g., direct exponentiation, finite difference methods)

$$\begin{aligned} |\psi(x^+) \rangle &= \mathcal{T}_+ \exp \left[-\frac{i}{2} \int_0^{x^+} dz^+ P^-(z^+) \right] |\psi(0) \rangle \\ &= \lim_{n \rightarrow \infty} \prod_{k=1}^n \mathcal{T}_+ \exp \left[-\frac{i}{2} \int_{x_{k-1}^+}^{x_k^+} dz^+ P^-(z^+) \right] |\psi(0) \rangle \\ \delta x^+ &= x^+ / n, x_k^+ = k \delta x^+ (k = 0, 1, 2, \dots, n) \end{aligned}$$

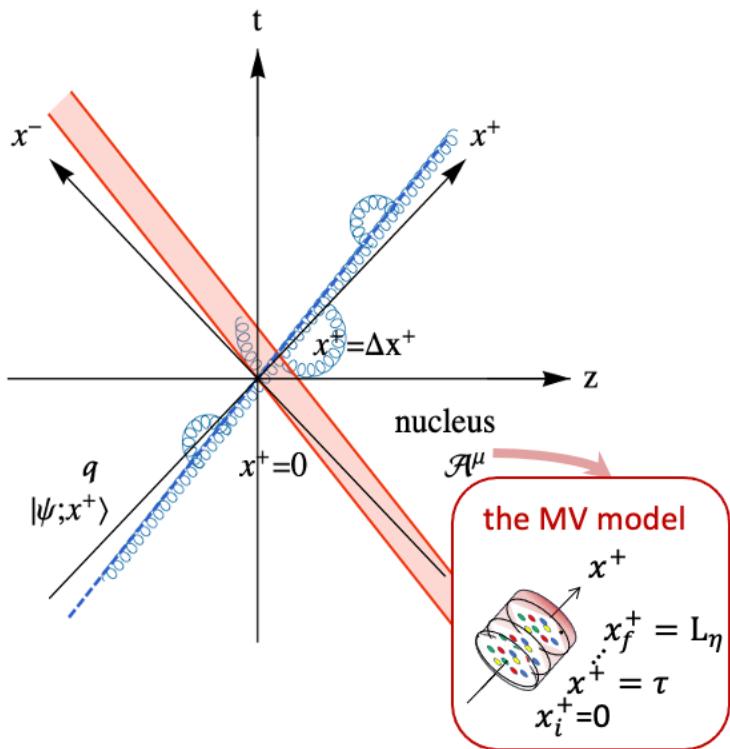
➤ Basis representation

- Optimal basis has the same symmetries of the system, and it is the key to numerical efficiency

1. J. P. Vary, H. Honkanen, Jun Li, P. Maris, S. J. Brodsky, A. Harindranath, G. F. de Teramond, P. Sternberg, E. G. Ng, C. Yang., Phys. Rev. C81, 035205 (2010); X. Zhao, A. Ilderton, P. Maris, and J. P. Vary, Phys. Rev. D88, 065014 (2013).

Methodology for quark nucleus scattering in the $|q\rangle + |qg\rangle$ space: A. The light-front Hamiltonian $P^-(x^+)$

We consider scattering of a high-energy quark moving in the positive z direction, on a high-energy nucleus moving in the negative z direction.



- **The light-front Hamiltonian** is derived from the QCD Lagrangian with a background color field \mathcal{A}_μ ,

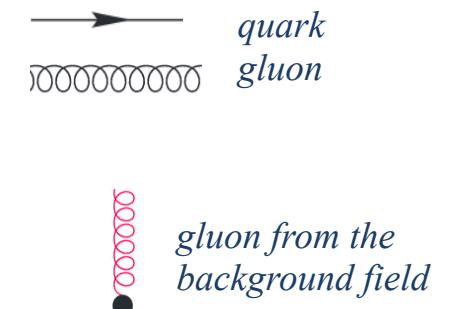
$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu}{}_a F^a_{\mu\nu} + \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi$$

where $D_\mu = \partial_\mu + ig(A_\mu + \mathcal{A}_\mu)$. In the $|q\rangle + |qg\rangle$ space, it includes the kinetic energy, the interaction with the background field, and gluon emission/absorption:

$$P^-(x^+) = P_{KE}^- + V(x^+)$$

The interaction matrix $V(x^+) = V_{qg} + V_A(x^+)$

Fock sector	$ q\rangle$	$ qg\rangle$
$\langle q $		
$\langle qg $		



Methodology: A.1 The background color field \mathcal{A}_μ

- The background field from the nucleus, $\mathcal{A}(x^+, \vec{x}_\perp)$, is a classical gluon field described by the color glass condensate¹

Color charges are stochastic variables with correlations

$$\langle \rho_a(x^+, \vec{x}_\perp) \rho_b(y^+, \vec{y}_\perp) \rangle = g^2 \tilde{\mu}^2 \delta_{ab} \delta^2(\vec{x}_\perp - \vec{y}_\perp) \delta(x^+ - y^+)$$

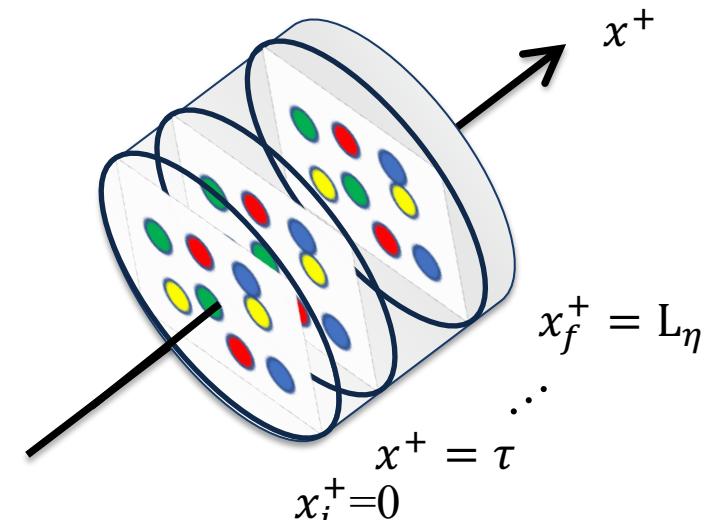
The color field is solved from

$$(m_g^2 - \nabla_\perp^2) \mathcal{A}_a^-(x^+, \vec{x}_\perp) = \rho_a(x^+, \vec{x}_\perp)$$

where m_g is a chosen infrared (IR) regulator.

The color sources of different layers are independent of each other, simulating the quarks from different nucleons of the heavy ion

- The duration of the interaction: $x^+ = [0, L_\eta]$
- Number of layers: N_η
- The duration of each layer: $\tau = L_\eta / N_\eta$



¹L. D. McLerran and R. Venugopalan, Phys. Rev. D49, 2233 (1994); L. D. McLerran and R. Venugopalan, Phys. Rev. D49, 3352 (1994); L. D. McLerran and R. Venugopalan, Phys. Rev. D50, 2225 (1994).

Methodology: B. Basis representation

- We choose the momentum states of the quark as **the basis states**: $P_{\text{KE}}^- |\beta\rangle = P_\beta^- |\beta\rangle$

The quark state is expanded in the basis space: $|\psi; x^+\rangle = \sum_{\beta} c_{\beta}(x^+) |\beta\rangle$
 $|q\rangle: |\beta_q\rangle; \quad |qg\rangle: |\beta_{qg}\rangle = |\beta_q\rangle \otimes |\beta_g\rangle$

Each single particle state carries five quantum numbers: $\beta_l = \{k_l^x, k_l^y, k_l^+, \lambda_l, c_l\}, (l = q, g)$
 the transverse momenta, the longitudinal momentum, helicity, and color

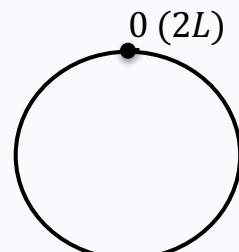
- The longitudinal space is circle of $2L$ with (anti)periodic boundary condition for the gluon (quark)

- $x^- = [0, 2L]$

- $p_l^+ = \frac{2\pi}{L} k_l^+$

$$k_q^+ = \frac{1}{2}, \frac{3}{2}, \dots, K + \frac{1}{2}$$

$$k_g^+ = 1, 2, \dots, K$$

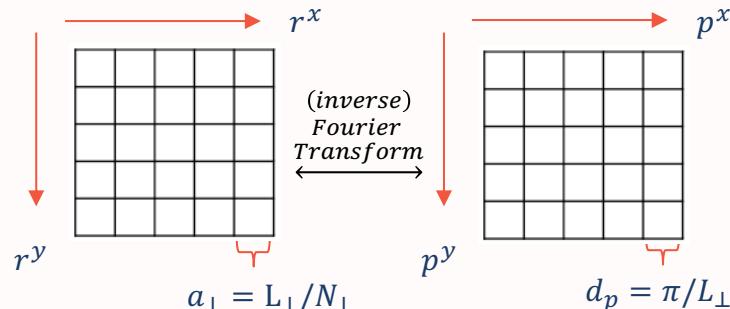


Basis size: $N_{tot} = (2N_{\perp})^2 \times 2 \times 3 + K \times (2N_{\perp})^4 \times 4 \times 24$

- The transverse coordinate space is discretized on a lattice $[-L_{\perp}, L_{\perp}]$ with periodic boundary conditions

- $r_l^{\perp} = [-N_{\perp}, \dots, N_{\perp} - 1] L_{\perp}/N_{\perp}$

- $p_l^{\perp} = \frac{2\pi}{2L_{\perp}} k_l^{\perp}, k_l^{\perp} = -N_{\perp}, \dots, N_{\perp} - 1$



Methodology: C. Time evolution

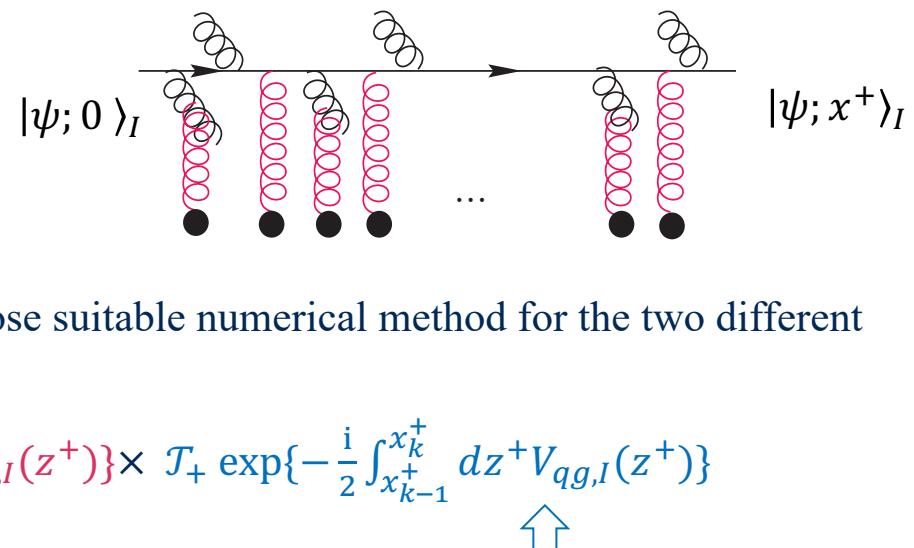
- **Solve the time-evolution equation** in the interaction picture

$$\frac{1}{2}V_I(x^+)|\psi; x^+\rangle_I = i\frac{\partial}{\partial x^+}|\psi; x^+\rangle_I$$

- P_{KE}^- as a phase factor: $|\psi; x^+\rangle_I = e^{\frac{i}{2}P_{KE}^- x^+} |\psi; x^+\rangle$, $V_I(x^+) = e^{\frac{i}{2}P_{KE}^- x^+} V(x^+) e^{-\frac{i}{2}P_{KE}^- x^+}$
- Time evolution as a product of many small timesteps

$$|\psi; x^+\rangle_I = \mathcal{T}_+ \exp\left\{-\frac{i}{2} \int_0^{x^+} dz^+ V_I(z^+)\right\} |\psi; 0\rangle_I$$

$$= \lim_{n \rightarrow \infty} \prod_{k=1}^n \mathcal{T}_+ \exp\left\{-\frac{i}{2} \int_{x_{k-1}^+}^{x_k^+} dz^+ V_I(z^+)\right\} |\psi; 0\rangle_I$$



Each timestep contains two successive operations, and we choose suitable numerical method for the two different interactions:

$$\mathcal{T}_+ \exp\left\{-\frac{i}{2} \int_{x_{k-1}^+}^{x_k^+} dz^+ V_I(z^+)\right\} = \mathcal{T}_+ \exp\left\{-\frac{i}{2} \int_{x_{k-1}^+}^{x_k^+} dz^+ V_{\mathcal{A},I}(z^+)\right\} \times \mathcal{T}_+ \exp\left\{-\frac{i}{2} \int_{x_{k-1}^+}^{x_k^+} dz^+ V_{qg,I}(z^+)\right\}$$

↑ ↑
matrix exponential in coordinate space +
Fast Fourier Transform, $\sim O(N_{tot} \log N_{tot})$ *4th-order Runge-Kutta method,*
 $\sim O(N_{tot})$

The total computational complexity of each timestep is $O(N_{tot} \log N_{tot})$.

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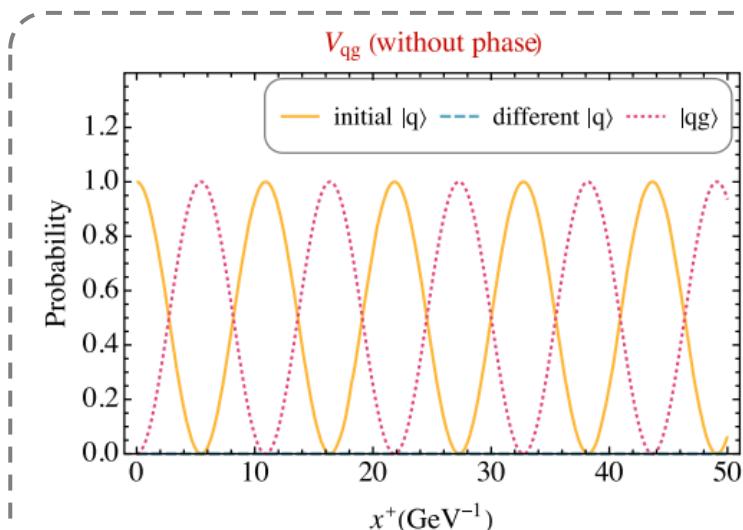
Based on: [M. Li, X. Zhao, P. Maris, G. Chen, Y. Li, K. Tuchin, and J. P. Vary, Phys. Rev. D 101, 076016 \(2020\)](#); [M. Li, T. Lappi , and X. Zhao, Phys. Rev. D 104, 056014 \(2021\)](#), and ongoing works.

Results: Evolution of the quark system

a) Gluon emission and absorption

The interaction contains the gluon emission/absorption term: $V_I(x^+) = e^{\frac{i}{2}P_{KE}^- x^+} V_{qg} e^{-\frac{i}{2}P_{KE}^- x^+}$

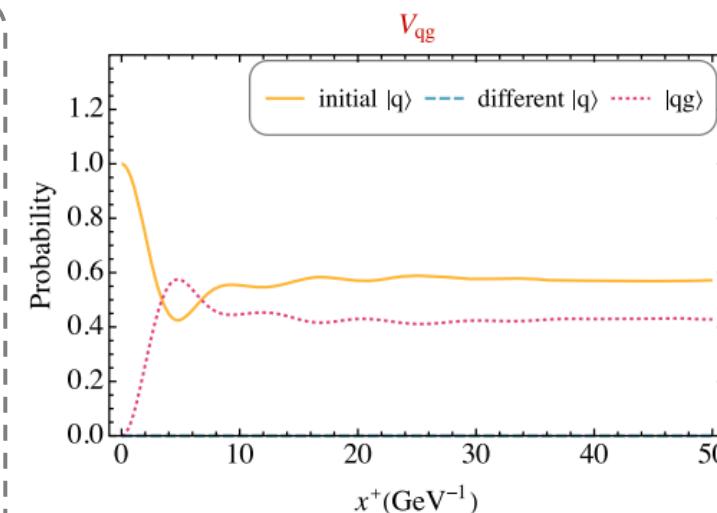
- Transition of the state (*Initial state is a single quark state $|q\rangle$ with $p^+ = 8.5 \text{ GeV}$, $\vec{p}_\perp = \vec{0}_\perp$, $\lambda_q = \uparrow$, $c_q = 1$*)



(1) Without the phase factor:

$$V_I = V_{qg}$$

Coherence: the probability oscillates between the initial $|q\rangle$ state and the $|qg\rangle$ sector



(2) With the phase factor:

$$V_I(x^+) = e^{\frac{i}{2}P_{KE}^- x^+} V_{qg} e^{-\frac{i}{2}P_{KE}^- x^+}$$

Decoherence: the probability of the initial $|q\rangle$ state and the $|qg\rangle$ sector each approaches some asymptotic value

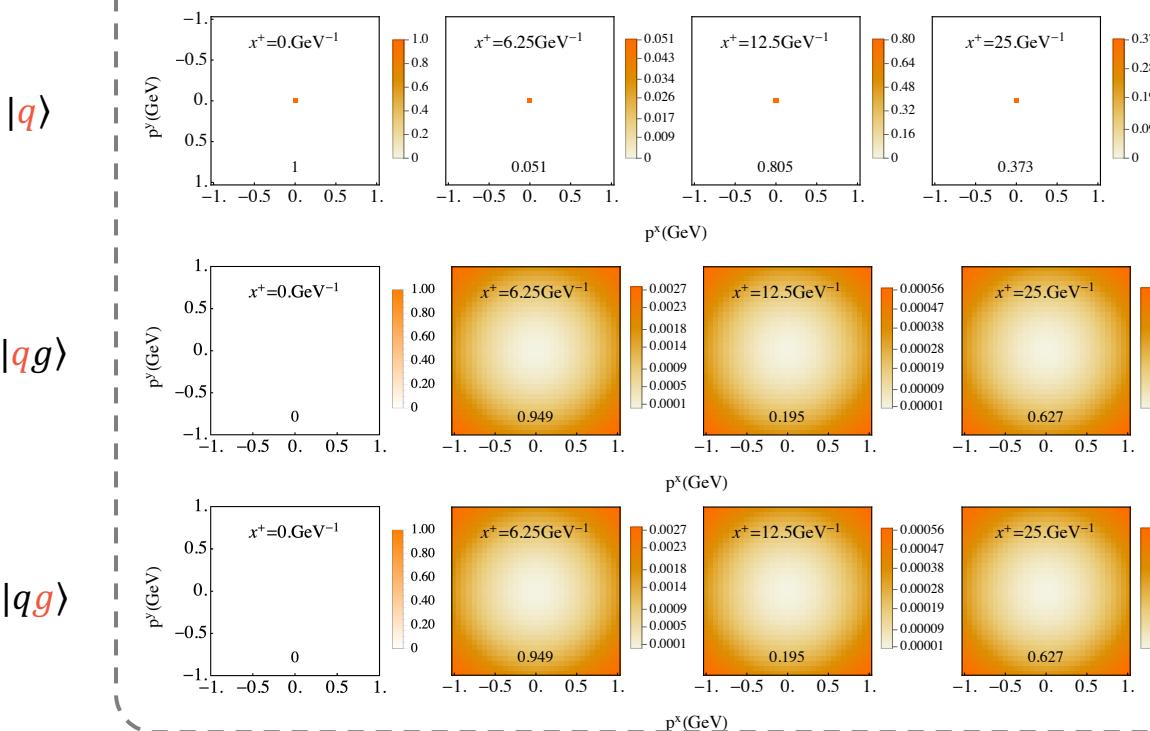
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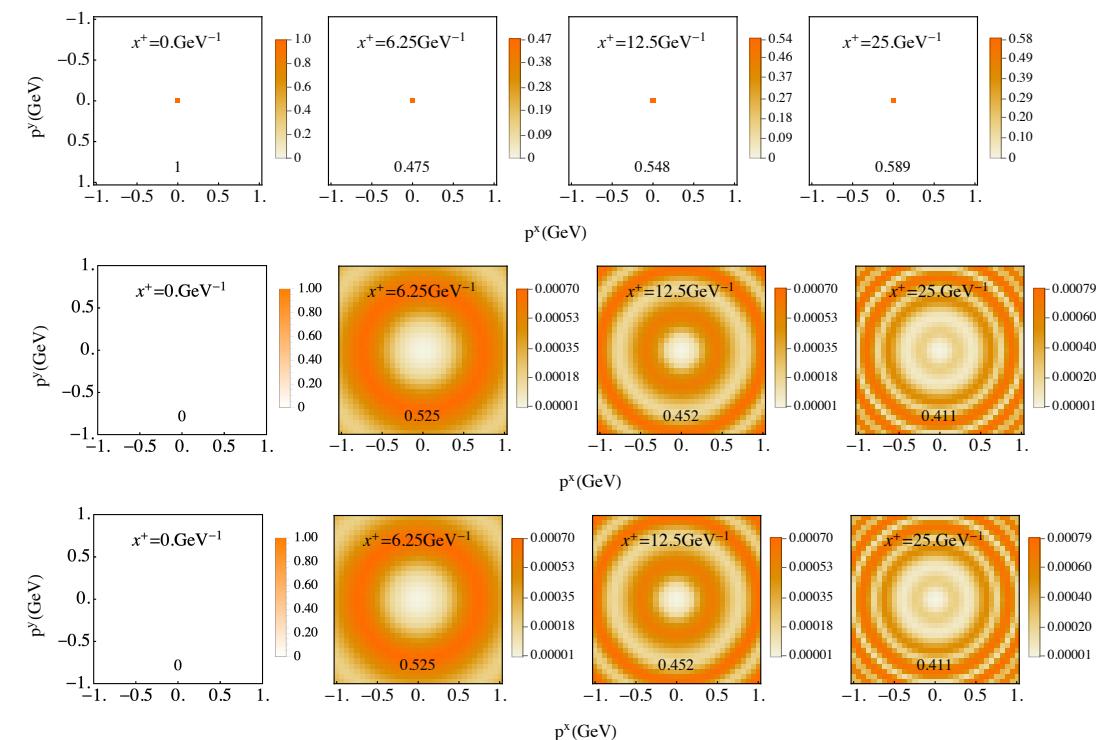
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- Evolution in the \vec{p}_\perp space

(1) Without the phase factor, different \vec{p}_\perp modes oscillate coherently



(2) With the phase factor, different \vec{p}_\perp modes oscillate out of phase



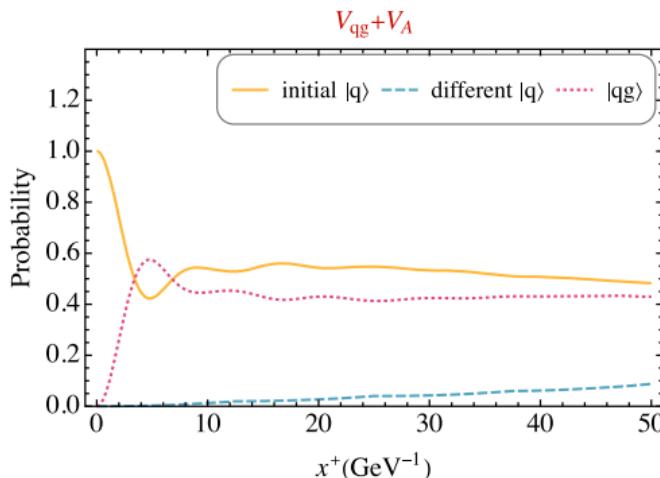
Results: Evolution of the quark system

b) Gluon emission and absorption inside a medium

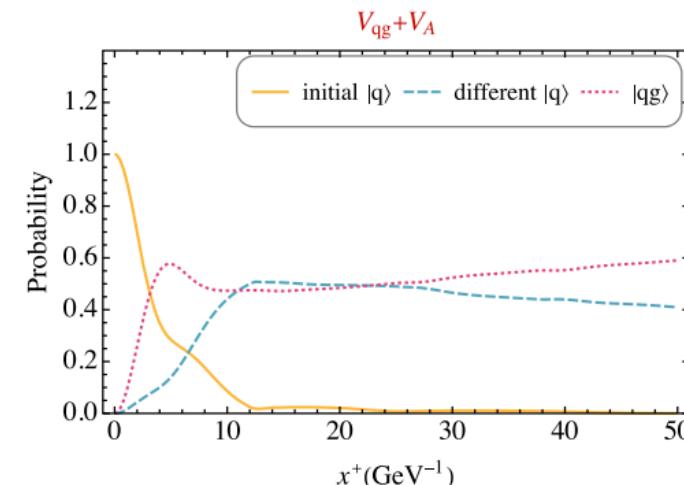
$$\text{The full interaction: } V_I(x^+) = e^{\frac{i}{2}P_{KE}^- x^+} [V_{qg} + V_A(x^+)] e^{-\frac{i}{2}P_{KE}^- x^+}$$

- Transition of the state (*Initial state is a single quark state $|q\rangle$ with $p^+ = 8.5 \text{ GeV}$, $\vec{p}_\perp = \vec{0}_\perp$, $\lambda_q = \uparrow$, $c_q = 1$*)

The probability of the initial $|q\rangle$ state decreases and that of the $|qg\rangle$ sector increases, and different momentum state appear.



with a relatively weaker background
 $g^2 \tilde{\mu} = 0.018 \text{ GeV}^{3/2}$



with a relatively stronger background
 $g^2 \tilde{\mu} = 0.144 \text{ GeV}^{3/2}$

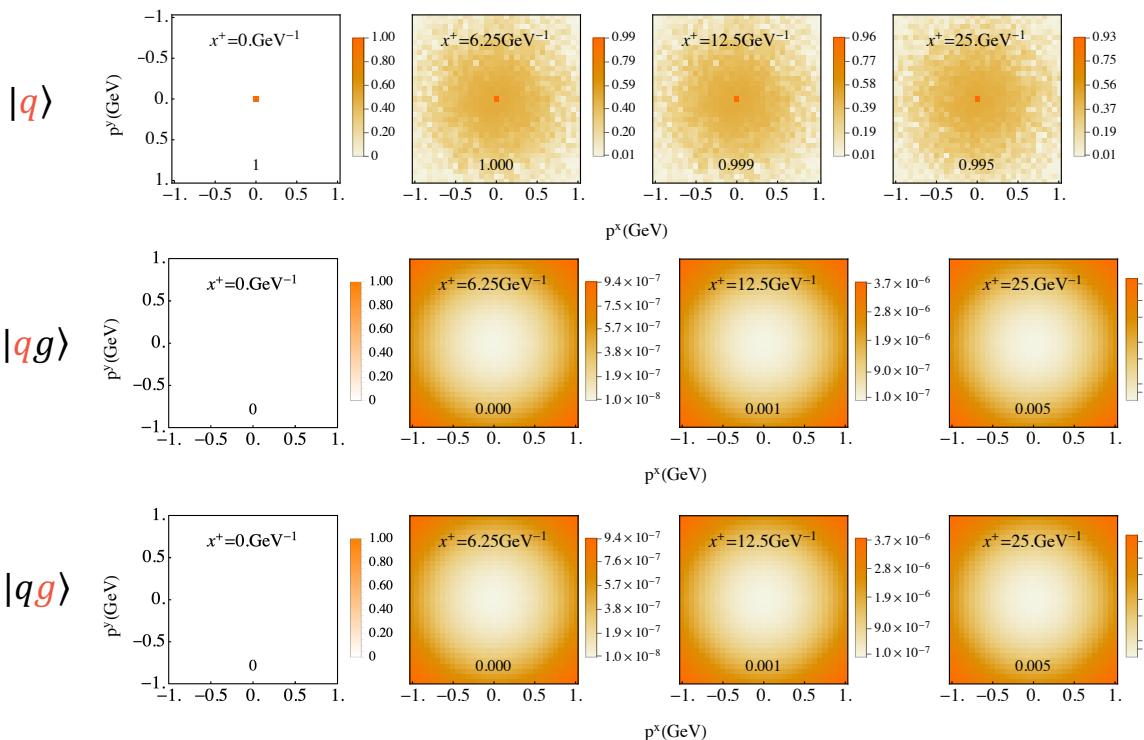
Results: Evolution of the quark system

b) Gluon emission and absorption inside a medium

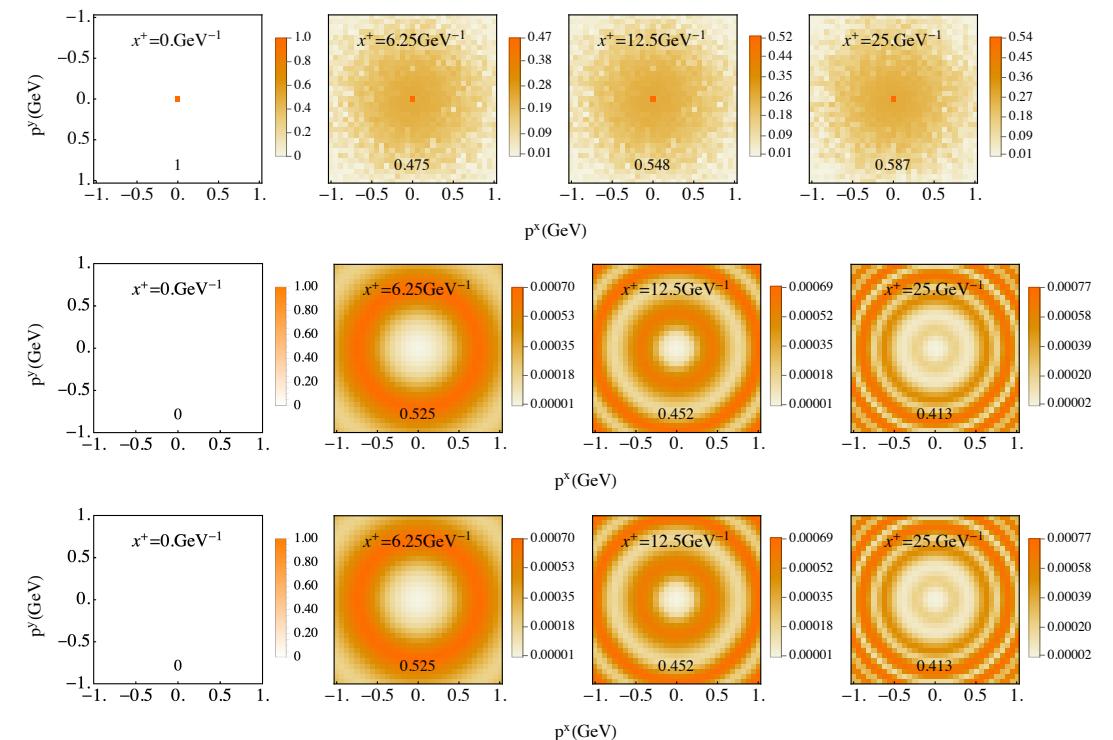
$$\text{The full interaction: } V_I(x^+) = e^{\frac{i}{2}P_{KE}^- x^+} [V_{qg} + V_A(x^+)] e^{-\frac{i}{2}P_{KE}^- x^+}$$

- Evolution in the \vec{p}_\perp space, $g^2 \tilde{\mu} = 0.018 \text{ GeV}^{3/2}$

$p^+ = 850 \text{ GeV}$, “fast” quark



$p^+ = 8.5 \text{ GeV}$, “slow” quark



Results: Momentum broadening

The quark state transfers to different momentum modes through evolution. Such momentum broadening can be characterized by the quenching parameter:

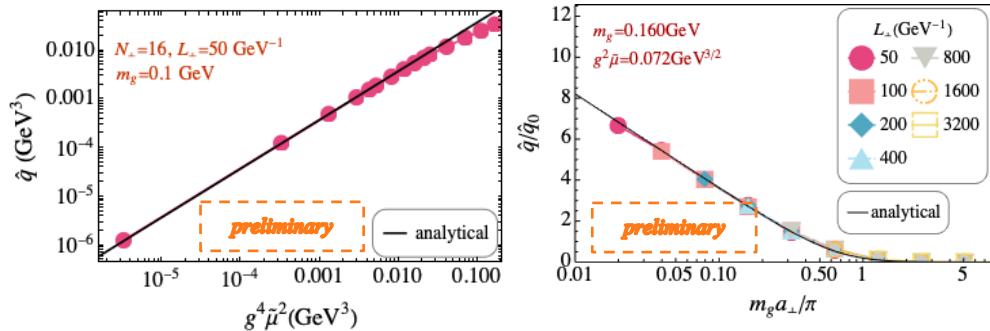
$$\hat{q} \equiv \frac{d\langle p_\perp^2(x^+) \rangle}{dx^+}$$

a) Bare quark, in the $|q\rangle$ space

In the $|q\rangle$ space, $\langle p_\perp^2(x^+) \rangle = \langle q | p_\perp^2(x^+) | q \rangle$, the quenching parameter can be calculated analytically in the eikonal limit:

$$\hat{q} = \frac{c_F g^4 \tilde{\mu}^2}{4\pi} \left[\log \frac{\lambda_{UV}^2 + m_g^2}{m_g^2} - \frac{\lambda_{UV}^2}{\lambda_{UV}^2 + m_g^2} \right], \quad \lambda_{UV} = \pi/a_\perp.$$

numerically verified:



b) With one gluon

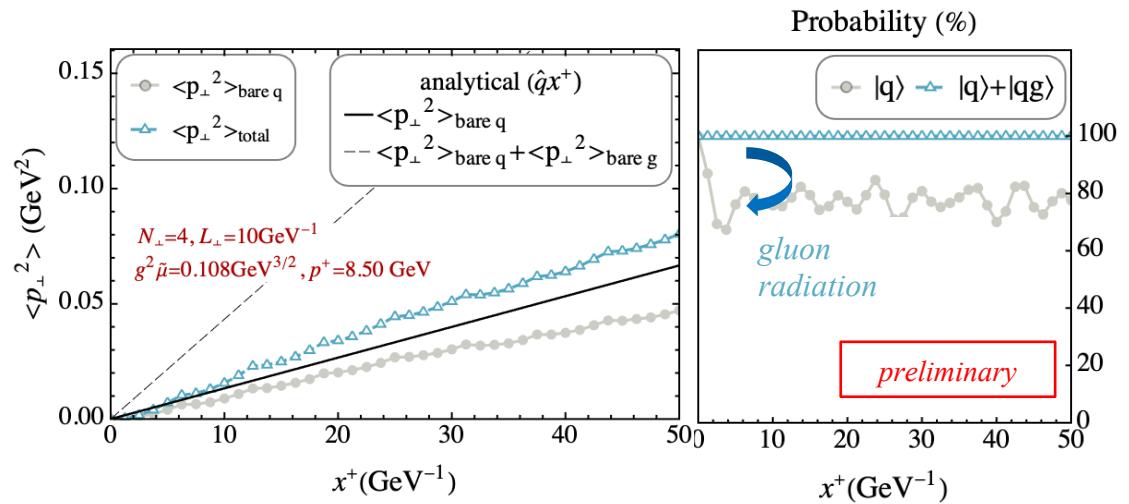
In the $|q\rangle + |qg\rangle$ space, consider the initial state as a single quark state,

$$|\psi; x^+ = 0\rangle = |q(\vec{0}_\perp)\rangle$$

The momentum broadening of the single quark component and of the total:

$$\langle p_\perp^2(x^+) \rangle_{\text{bare } q} = \langle q | p_\perp^2(x^+) | q \rangle$$

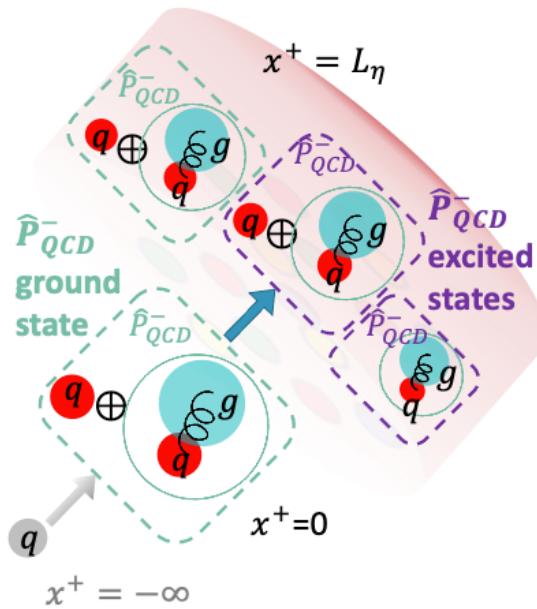
$$\langle p_\perp^2(x^+) \rangle_{\text{total}} = \langle q | p_\perp^2(x^+) | q \rangle + \langle qg | p_\perp^2(x^+) | qg \rangle$$



Results: Momentum broadening

c) Dressed quark

In a physical high-energy scattering process, the initial quark state has already developed a gluon cloud before the interaction.



We obtain the dressed quark state by solving the eigenvalue equation in the $|q\rangle + |qg\rangle$ space:

$$\hat{P}_{QCD}^- |\psi\rangle_{dressed} = P^- |\psi\rangle_{dressed}, \quad \hat{P}_{QCD}^- = P_{KE}^- + V_{qg}$$

Mass renormalization scheme^[1] is implemented to match the ground state mass with that of a physical quark.

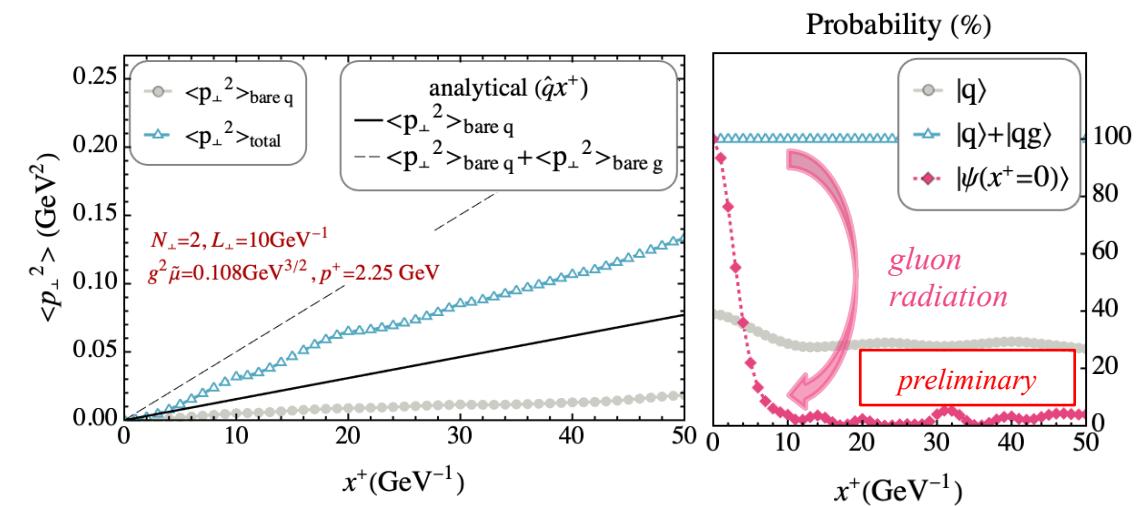
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$$\langle p_\perp^2(x^+) \rangle_{\text{total}} = \langle q | p_\perp^2(x^+) | q \rangle + \langle qg | p_\perp^2(x^+) | qg \rangle$$



^[1] V.A. Karmanov, J.-F. Mathiot, A.V. Smirnov, Phys. Rev. D 77 (2008), 085028; Phys. Rev. D 86 (2012) 085006.

Summary and outlooks

- We demonstrated a nonperturbative method to investigate time-evolution problems with light-front Hamiltonian formalism
 - We studied the quark-nucleus scattering in the $|q\rangle + |qg\rangle$ space, and analyzed the individual and combined effects from
 - the light-front energy
 - gluon emission/absorption
 - interaction with a background field
 - A main advantage of this method: one can smoothly vary the magnitudes of the above three effects separately
- Ongoing study:
 - Investigate the interplay between gluon emission medium interaction, in the momentum broadening
 - Investigate high-energy scattering with sub-eikonal effects
 - The initial quark state is an eigenstate of the QCD Hamiltonian in the $|q\rangle + |qg\rangle$ space

Thank you!