

# Cone-size dependence of jet suppression in heavy-ion collisions

Daniel Pablos - INFN Torino

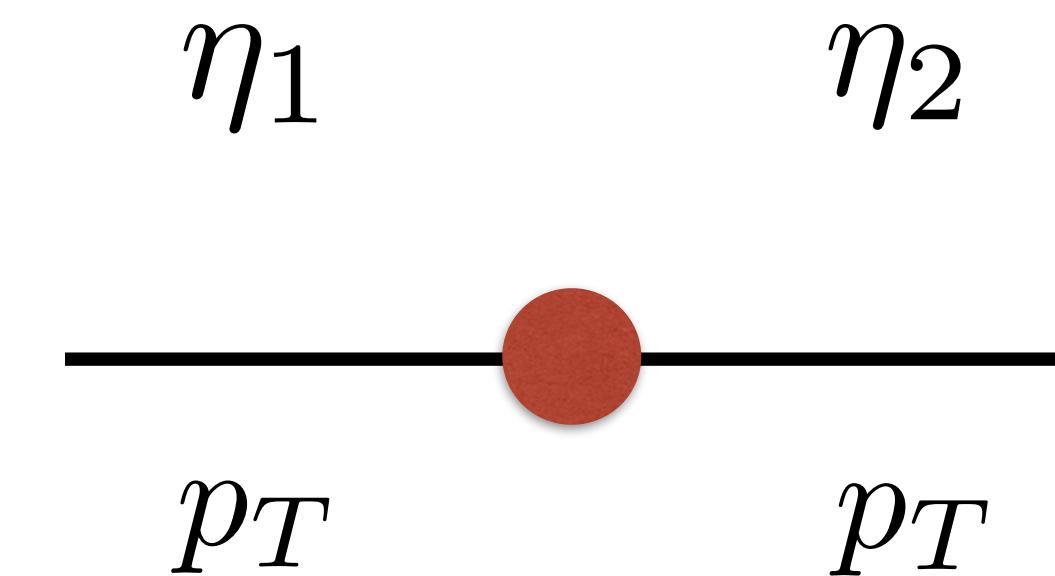


Istituto Nazionale di Fisica Nucleare

ECT\* - Jet Quenching in the Quark-Gluon Plasma  
15th June 2022

# Jets in pp

- Hard parton pairs produced back-to-back in transverse plane, misaligned in rapidity.

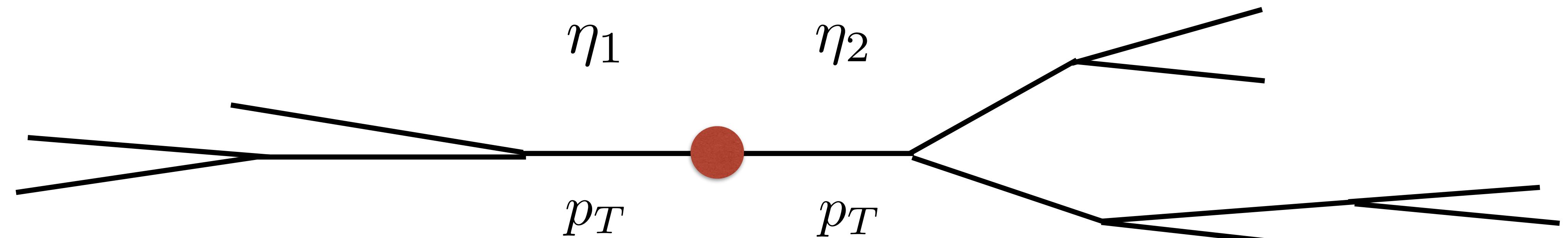


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$$t \frac{\partial}{\partial t} f(x, t) = \int \frac{dz}{z} \frac{\alpha_s}{2\pi} P_+(z) f\left(\frac{x}{z}, t\right)$$



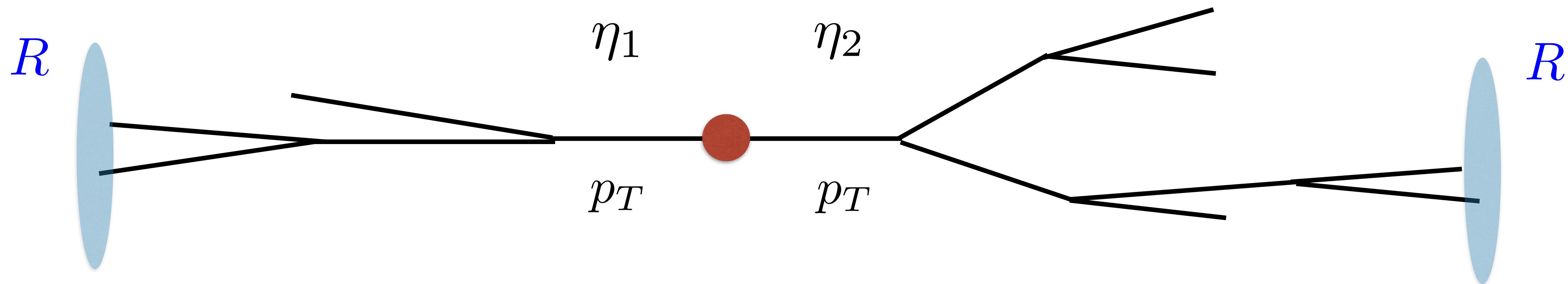
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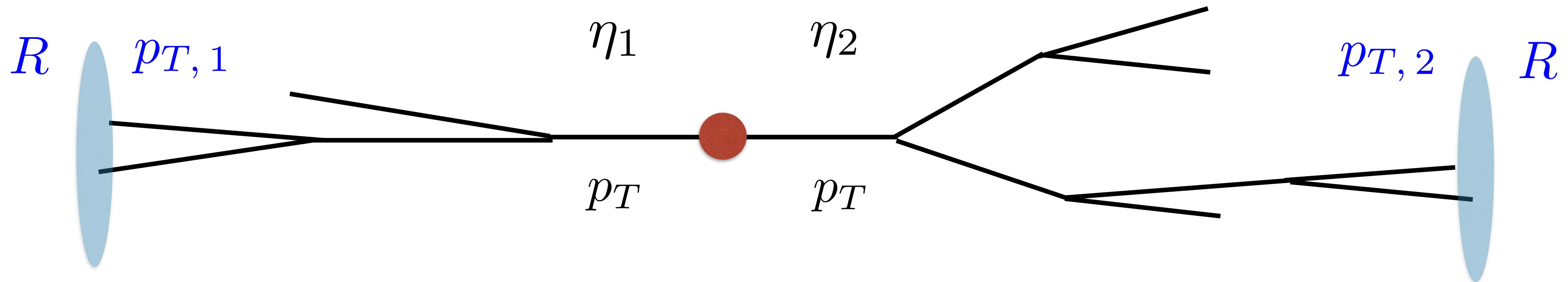
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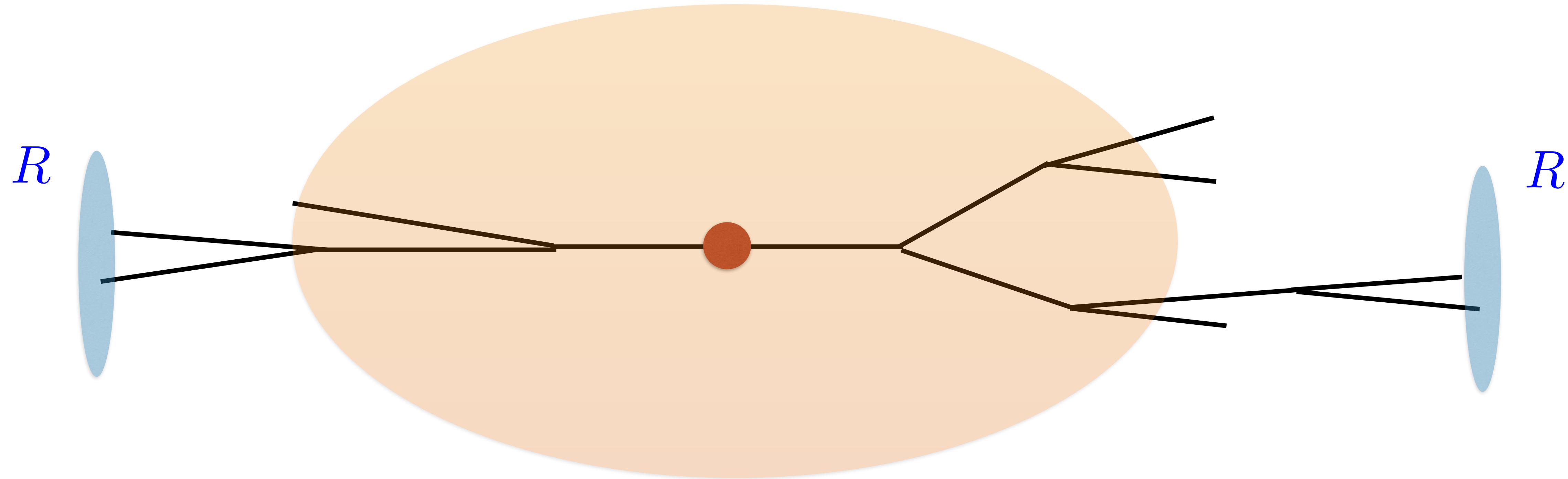
→ Degree of jet activity determines, e.g., out-of-cone radiation (causes dijet asymmetry in pp).

$$p_{T, 1} > p_{T, 2}$$

# Jets in AA

- Jet partons interact with QGP and experience energy loss.

- Crucial insights obtained with jet quenching MCs.



- Lesson example: total energy loss proportional to jet activity (more energy loss sources):

→ Causes dijet asymmetry in AA (for same traversed length).

Milhano & Zapp - EPJ '16

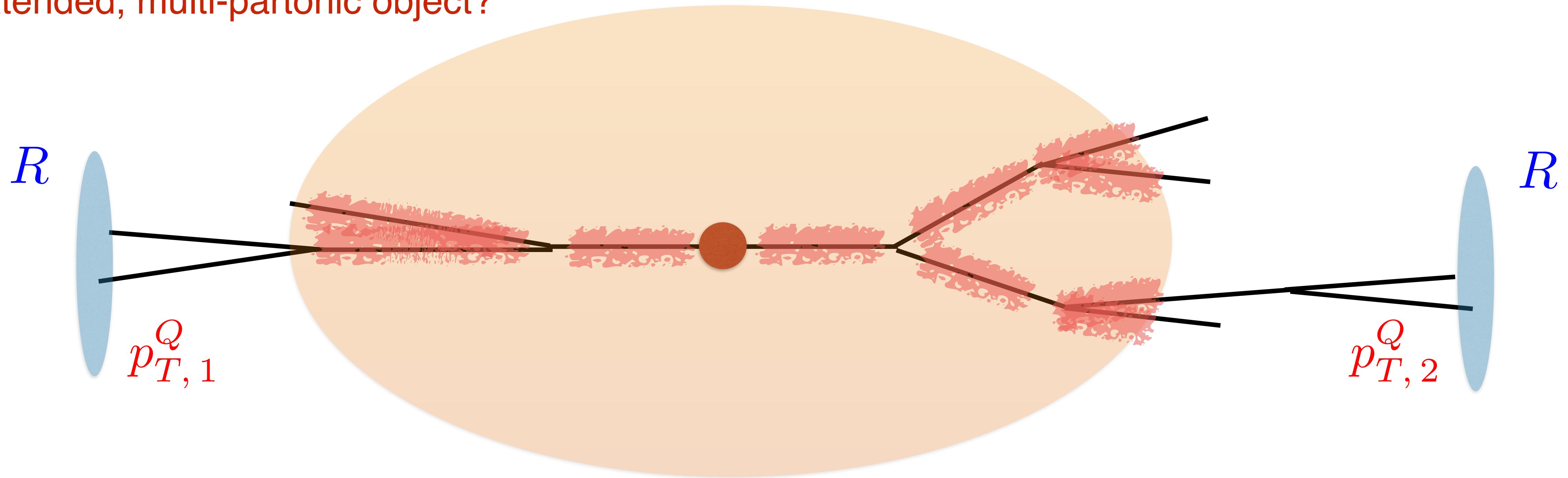
→ Causes selection bias towards narrower jets.

Hadron vs. Jet Suppression

Casalderrey, Hulcher, Milhano, DP, Rajagopal - PRC '19

- Jet suppression beyond MC:

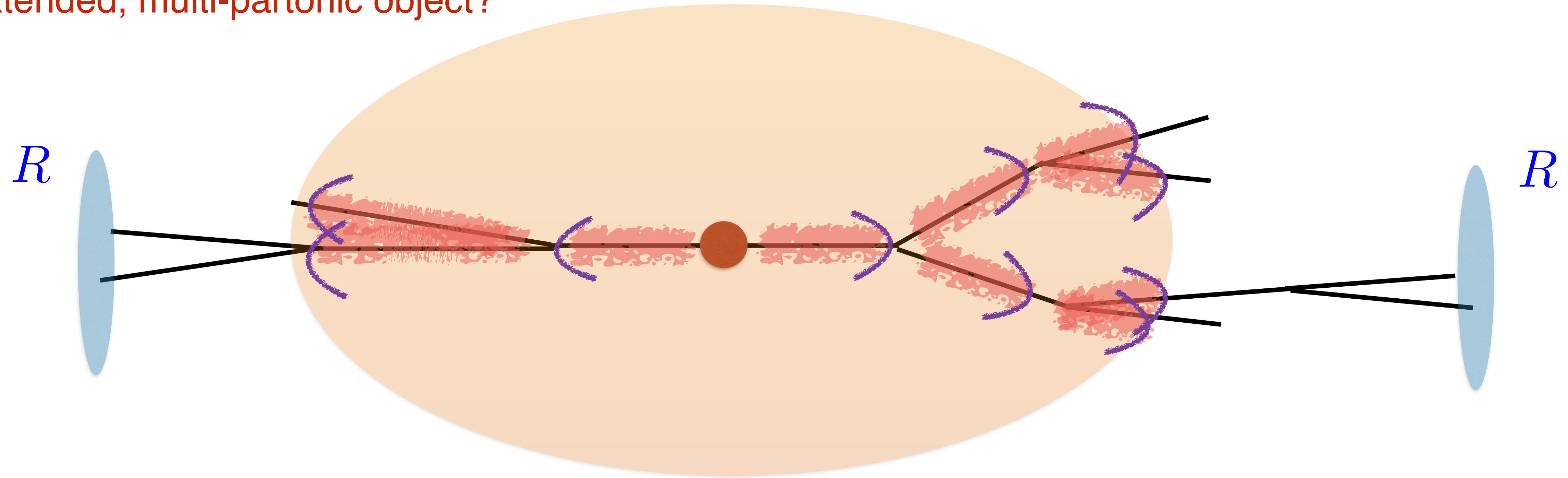
How to describe energy loss off  
an extended, multi-partonic object?



## Outline

- Jet suppression beyond MC:

How to describe energy loss off  
an extended, multi-partonic object?



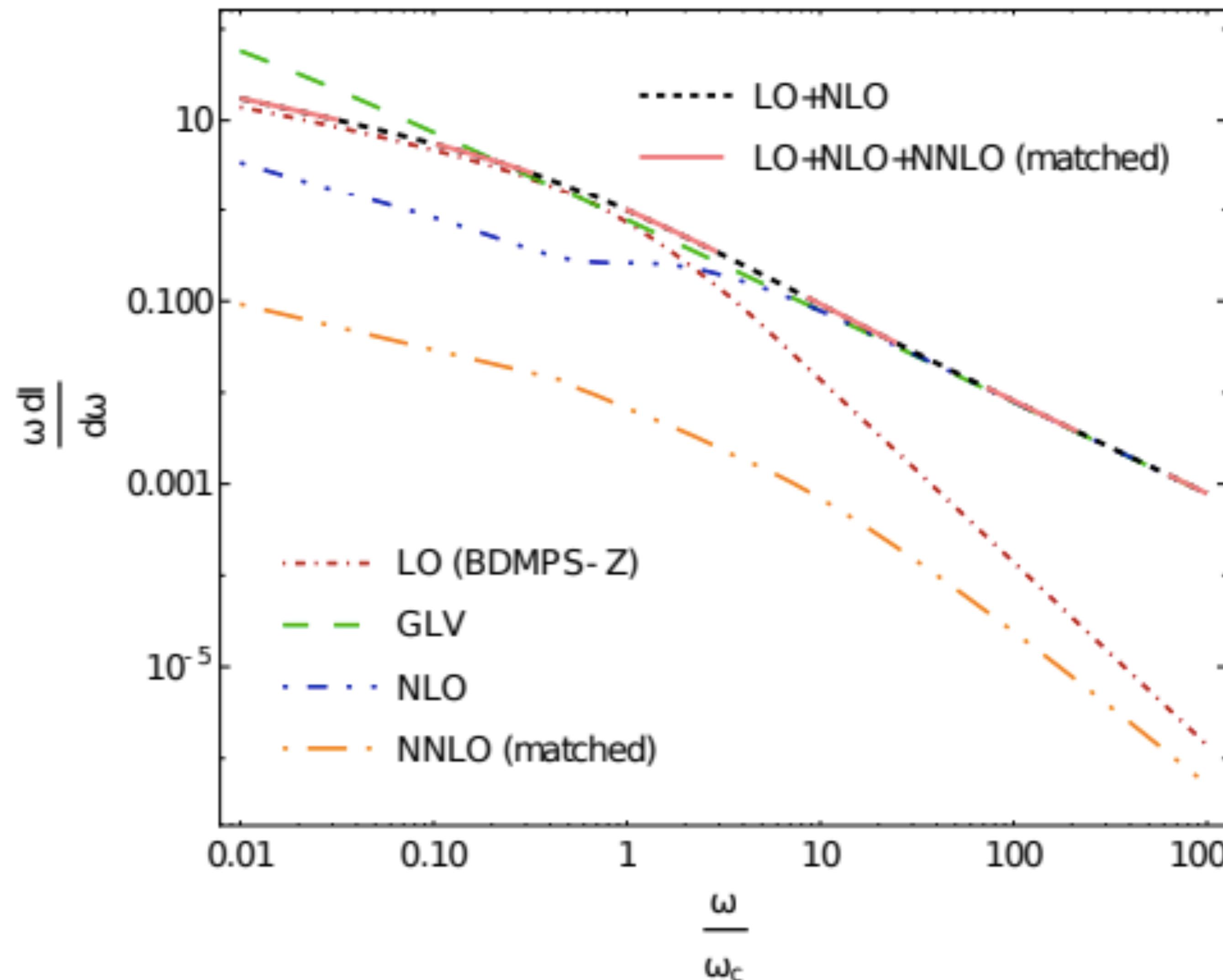
## Outline

- Non-perturbative modelling of long wavelength jet modes:  
*Where does “lost” energy go to?*

# Improved Opacity Expansion (IOE)

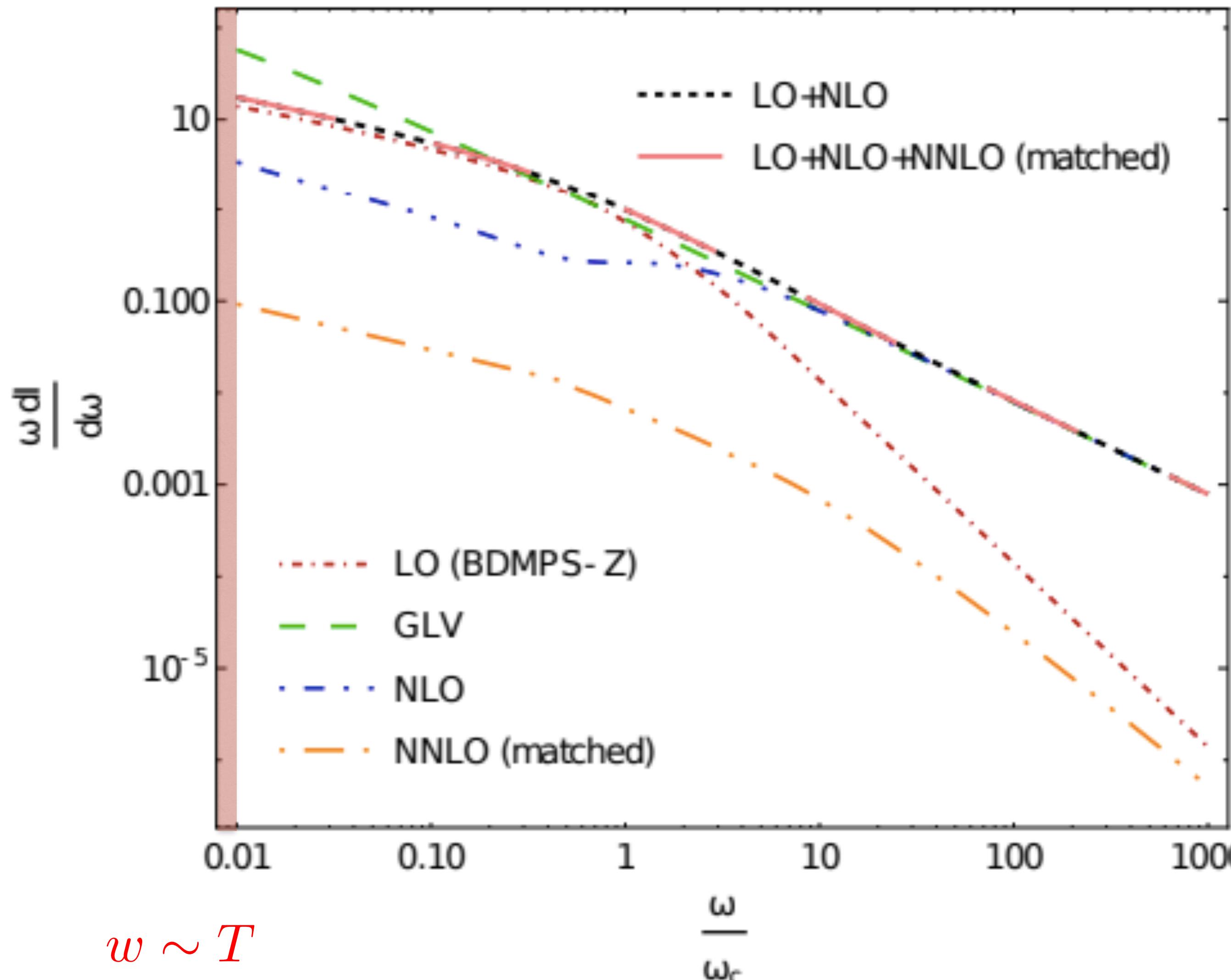
Barata, Mehtar-Tani - JHEP '20

$$t_{\text{coh}} = \omega/k_{\perp}^2 \quad k_{\perp}^2 \sim \hat{q} t_{\text{coh}} \quad t_{\text{coh}} \equiv \sqrt{\frac{\omega}{\hat{q}}}$$



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Barata, Mehtar-Tani - JHEP '20



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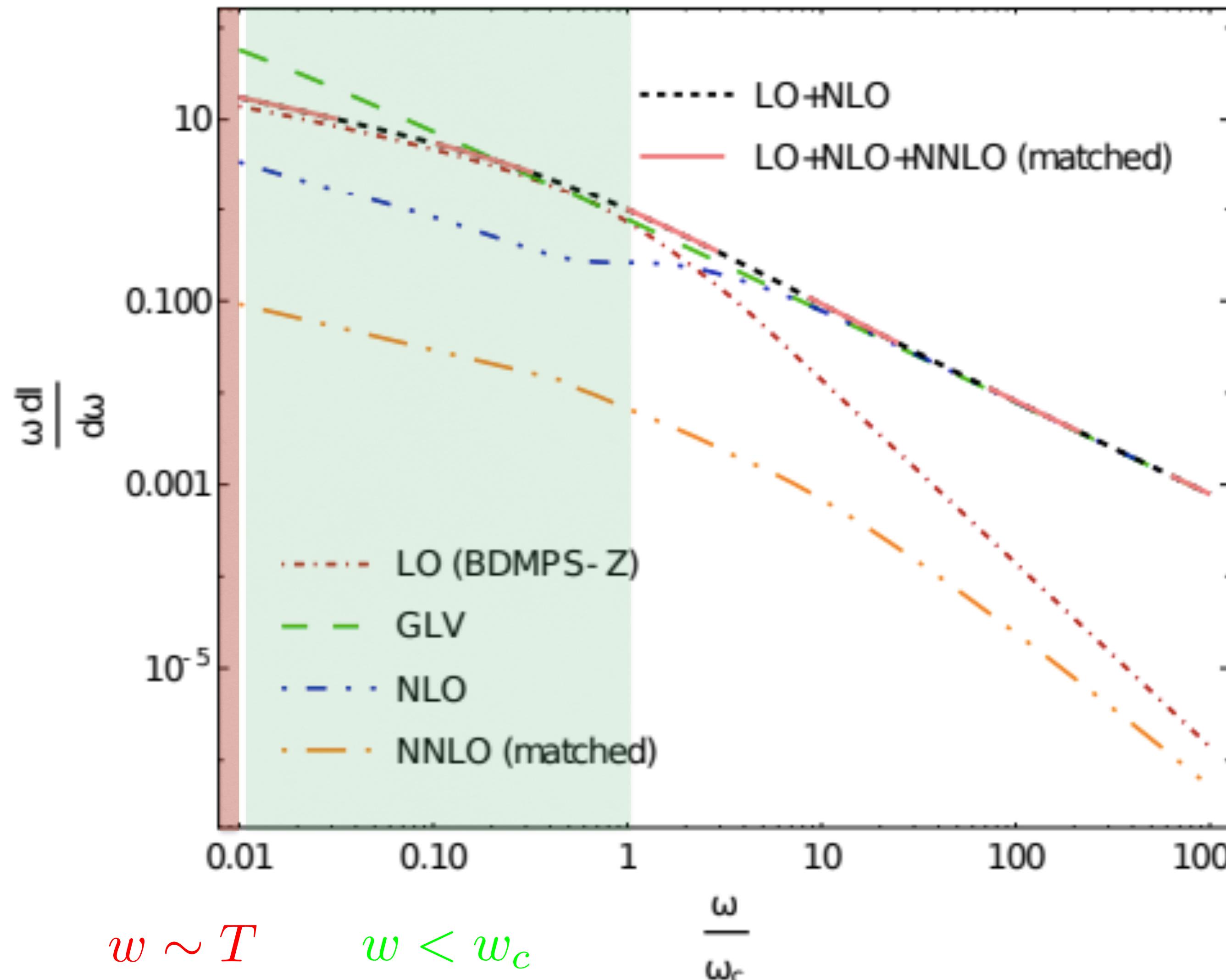
$$t_{\text{coh}} \equiv \sqrt{\frac{\omega}{\hat{q}}}$$

- Bethe-Heitler regime  $t_{\text{coh}} \sim \ell_{\text{mfp}}$

$$\omega \frac{dI_{\text{BH}}}{d\omega} \simeq \alpha_s \frac{L}{\ell_{\text{mfp}}} = \alpha_s N_{\text{scatt}}$$

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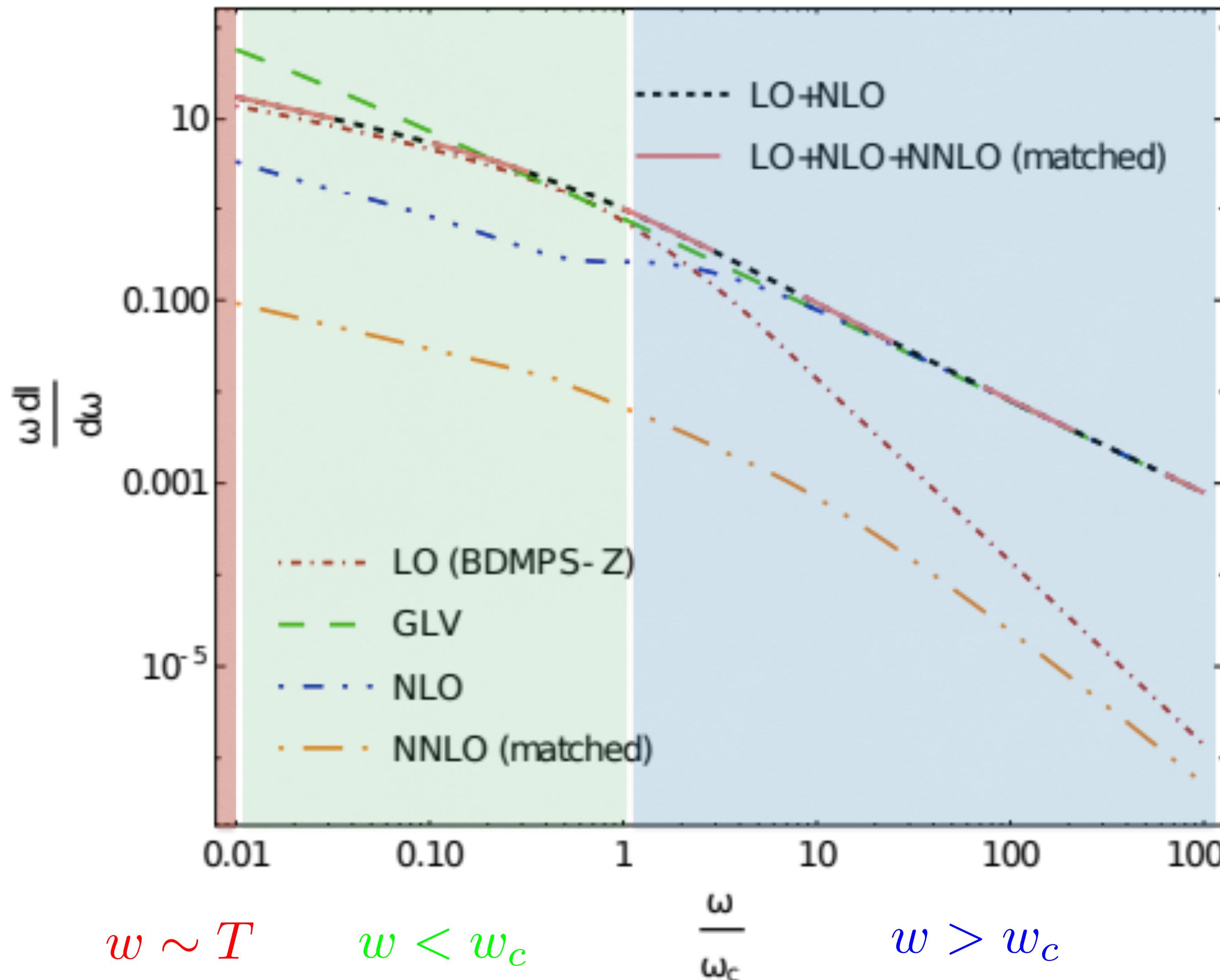
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- GLV regime  $k_{\perp}^2 \gg Q_s^2 \equiv \hat{q}L$

$$\omega \frac{dI}{d\omega} \sim \alpha_s^3 n L \int_{\omega/L}^{\infty} \frac{dk_{\perp}^2}{k_{\perp}^4} \simeq \alpha_s \frac{\omega_c}{\omega}$$

# Out of Cone Radiation

- Only emissions that end up out of the cone R should be accounted for:

Multiplicative Ansatz:  $\omega \frac{dI_>}{d\omega} = \int_{(\omega R)^2}^{\infty} dk_{\perp}^2 \omega \frac{dI}{d\omega dk_{\perp}^2} \simeq B(\omega R; Q_{\text{broad}}^2) \times \omega \frac{dI}{d\omega}$

Mehtar-Tani, DP, Tywoniuk - PRL '21

$$B(\omega R; Q_{\text{broad}}^2) = \frac{Q_{\text{broad}}^2}{4\pi} \int_y^{\infty} dx \boxed{\mathcal{P}(x)}$$

Broadening dist.

$$\mathcal{P}(\mathbf{k}) \simeq \begin{cases} \frac{4\pi}{Q_s^2} e^{-\mathbf{k}^2/Q_s^2} & k_{\perp}^2 \ll Q_{\text{med}}^2 \\ \frac{4\pi Q_s^2}{\mathbf{k}^4} & k_{\perp}^2 \gg Q_{\text{med}}^2 \end{cases}$$

$$\frac{\partial}{\partial L} \mathcal{P}(\mathbf{k}, L) = C_R \int_{\mathbf{q}} \gamma(\mathbf{q}) [\mathcal{P}(\mathbf{k} - \mathbf{q}, L) - \mathcal{P}(\mathbf{k}, L)]$$

$y = (\omega R)^2 / Q_{\text{broad}}^2$

Characteristic broadening scale

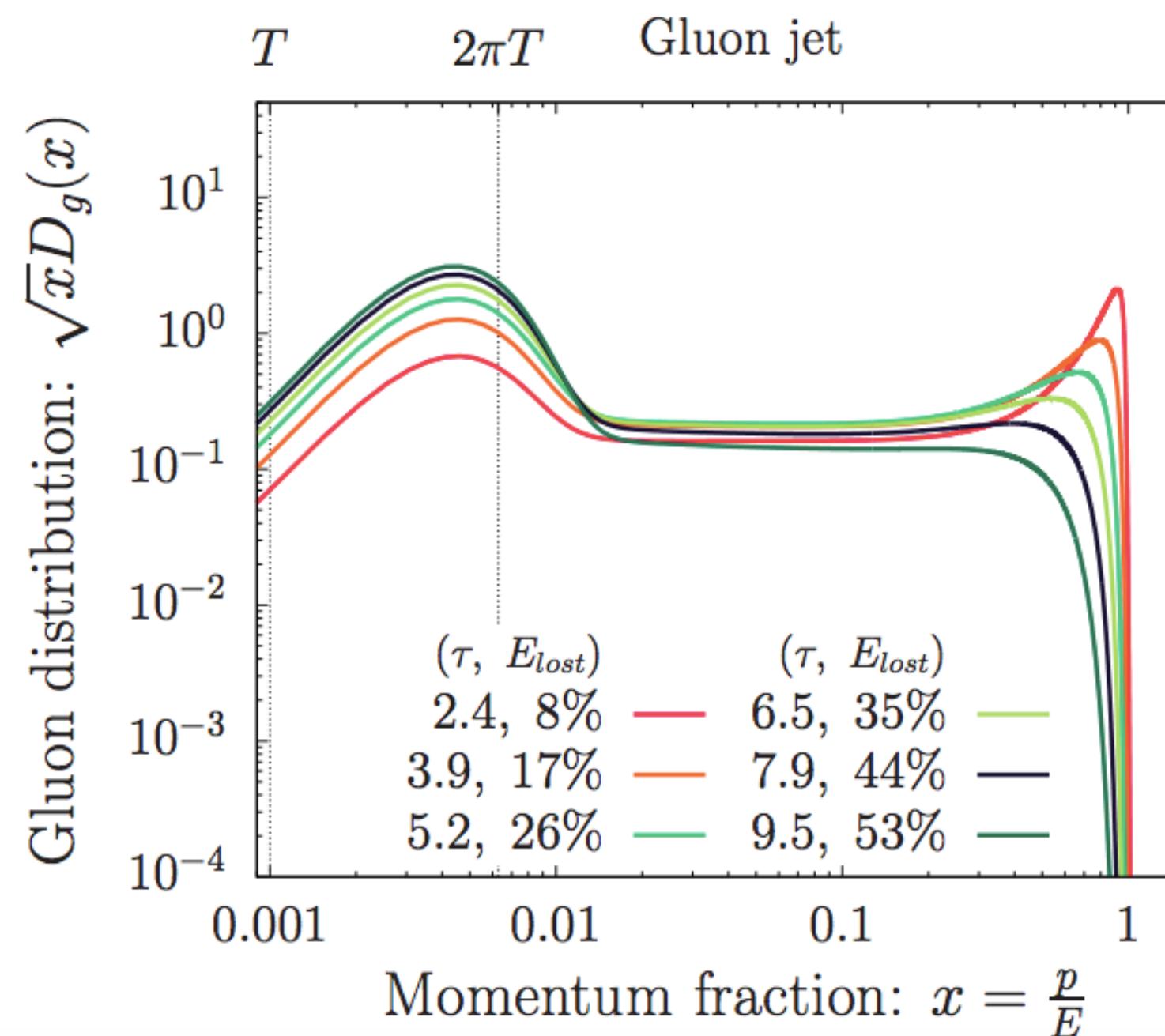
- Use Molière expansion around multiple soft scatterings (a.k.a. IOE). Barata et al. - PRD '21
- Can be improved with fully differential spectrum. Barata et al. - JHEP '21

# Further Improvements on Single Charge Energy Loss

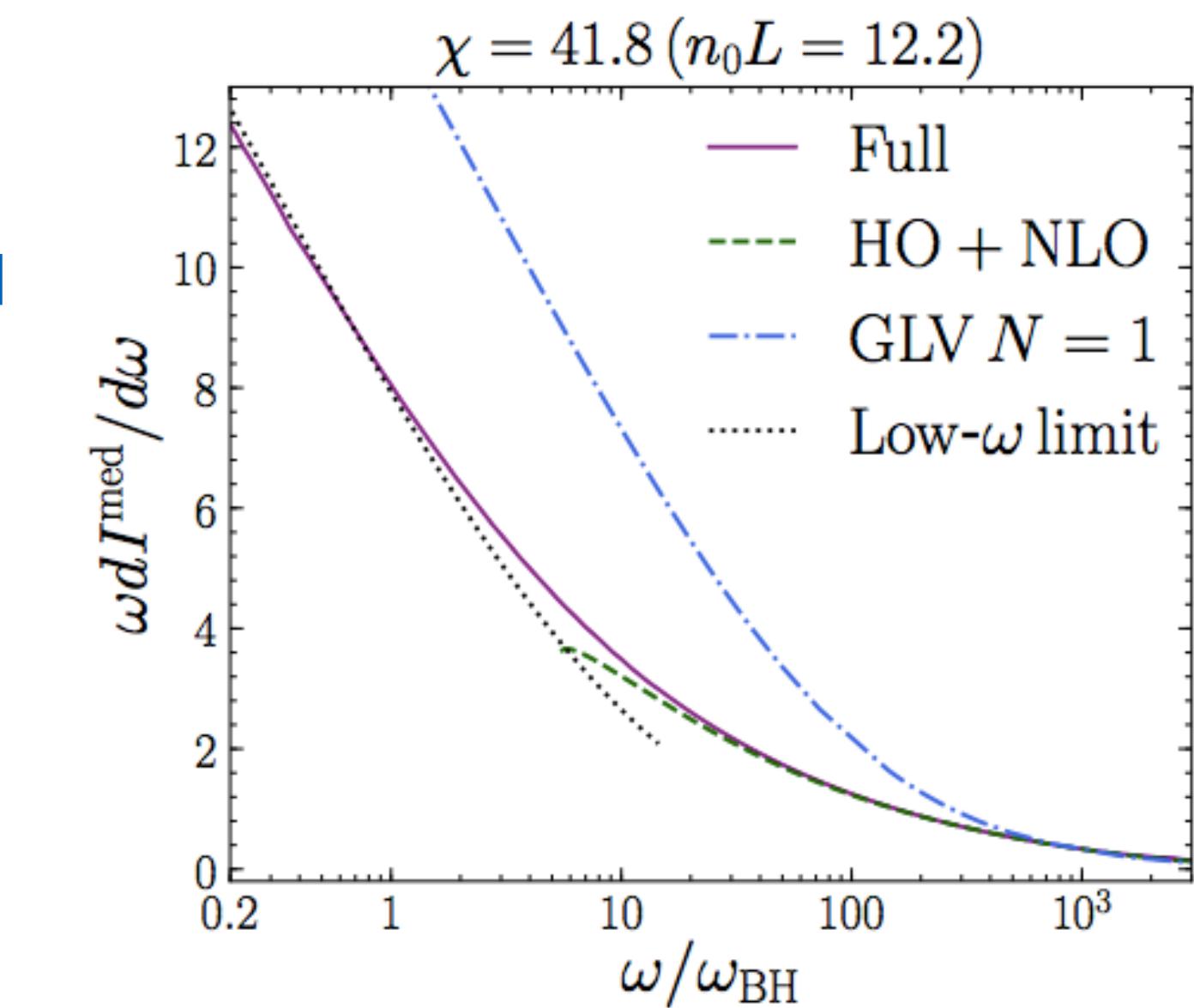
- All order resummation of medium induced radiation spectrum.
- Resummed Opacity Expansion (ROE) to cover Bethe-Heitler regime.

Feal et al. - PRD '18 & '19  
Andrés et al. - JHEP '20 & '21

Isaksen et al. - arXiv:2206.02811



Schlichting & Soudi - JHEP '20



- In-medium fragmentation of hard parton in QGP through effective kinetic theory.
  - Includes  $1 \leftrightarrow 2$  and  $2 \leftrightarrow 2$  processes.
  - Features turbulent cascade, modified chemistry around the jet.

Detailed analysis of dynamics, can account for medium response.

# Bare Quenching Factor

- For steeply falling spectrum and small energy loss:

Baier, Dokshitzer, Mueller - JHEP '01  
 Salgado, Wiedemann - PRD '03

$$\frac{d\sigma_{\text{med}}}{dp_T} = \int_0^\infty d\epsilon \mathcal{P}(\epsilon) \left. \frac{d\sigma_{\text{vac}}}{dp'_T} \right|_{p'_T=p_T+\epsilon} \approx \underbrace{\frac{d\sigma_{\text{vac}}}{dp_T} \int_0^\infty d\epsilon \mathcal{P}(\epsilon) e^{-\epsilon \frac{n}{p_T}}}_{Q(p_T)}$$

Mehtar-Tani, DP, Tywoniuk - PRL '21

- Quenching factor of a single parton for multiple independent emissions (R dependent):

$$Q_{\text{rad}}^{(0)}(p_T) = \exp \left[ - \int_{\omega_s}^\infty d\omega \frac{dI_{>}}{d\omega} (1 - e^{-\nu\omega}) - \int_T^{\omega_s} d\omega \frac{dI^{(0)}}{d\omega} \left( 1 - e^{-\nu\omega(1 - (\frac{R}{R_{\text{rec}}})^2)} \right) \right]$$

$$\nu \equiv \frac{n}{p_T}$$

Full out-of-cone spectrum  
for semi-hard emissions

$$\omega_s \equiv (g_{\text{med}}^2 N_c / (2\pi)^2)^2 \pi \hat{q}_0 L^2$$

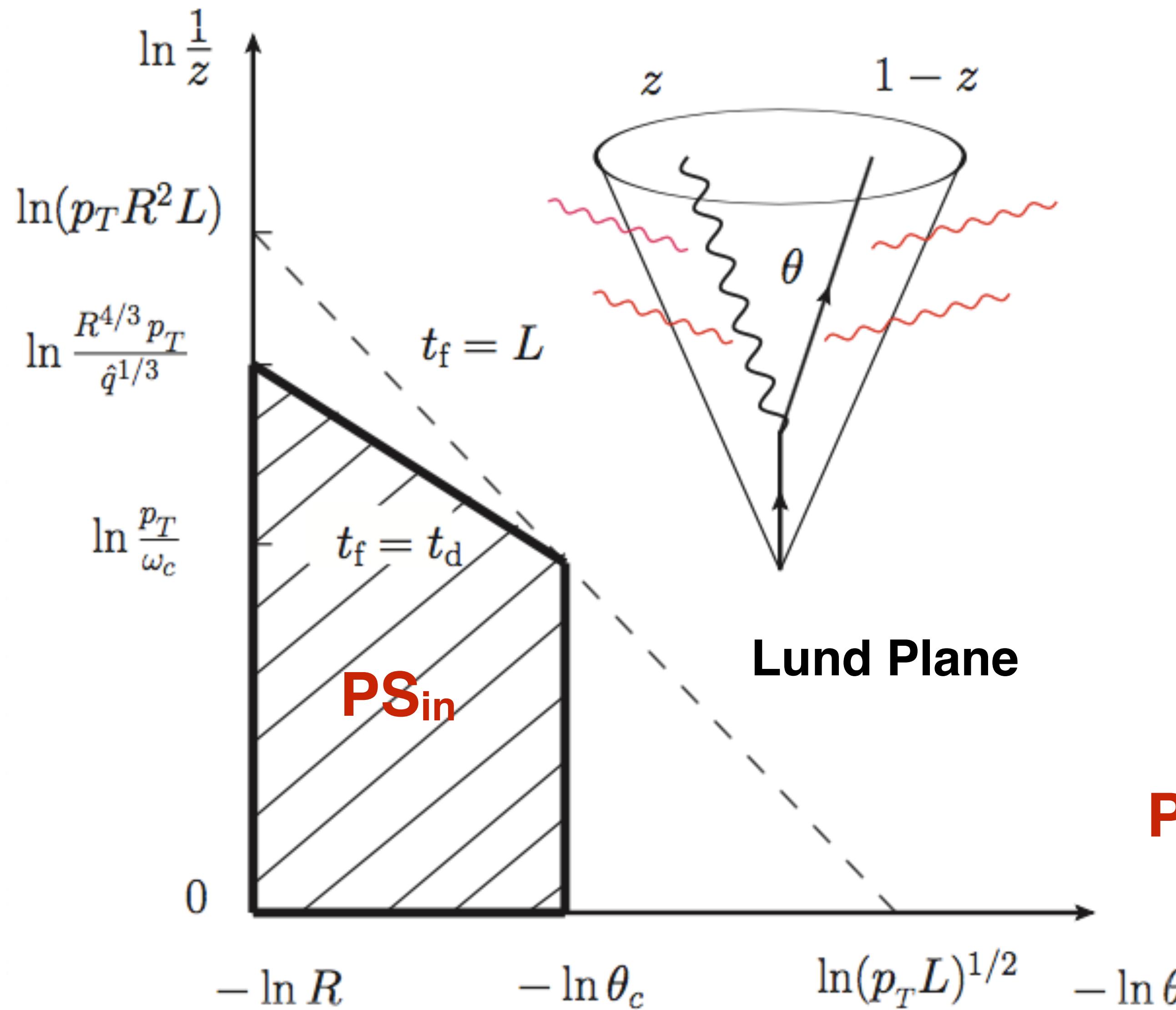
- O(1) emission probability
- undergo turbulent cascade, thermalise
- if uniformly distributed in jet hemisphere

$$R_{\text{rec}} \sim \pi$$

Note that:

$$\Delta E = (1 - (\frac{R}{R_{\text{rec}}})^2) \int_T^{\omega_s} dw w \frac{dI^{(0)}}{dw} = -\frac{d}{d\nu} Q_{\text{rad}}^{(0),\text{turb}}(p_T) \Big|_{\nu=0}$$

# Quenched Phase Space of a Jet



- Only those jet modes that:
    - are formed inside the medium, and,  $t_f < L$
    - are resolved by the medium,  $t_f < t_d$
- contribute to double-logarithmic enhancement of quenched phase space:

$$\text{PS}_{\text{in}} = \bar{\alpha} \int_{t_f < t_d < L} \frac{d\theta}{\theta} \int \frac{dz}{z} \equiv \bar{\alpha} \ln \frac{R}{\theta_c} \left( \ln \frac{p_T}{\omega_c} + \frac{2}{3} \ln \frac{R}{\theta_c} \right)$$

Mehtar-Tani, Tywoniuk - PRD '18  
see also Caucal, Iancu, Mueller, Soyez - PRL '18

# Jet Suppression: Framework

- Use microjet distributions derived using Generating Functional (GF) framework:

Vacuum evol.  
obeys DGLAP:

$$\frac{df_{j/i}^{\text{incl}}(z, t)}{dt} = \sum_k \int_z^1 \frac{dz'}{z'} P_{jk}(z') f_{k/i}^{\text{incl}}(z/z', t)$$

Dasgupta et al. - JHEP '14

- Extend GF in the medium to resum energy loss effects due to multi-particle nature of jet:

$$\begin{aligned} \frac{\partial Q_i(p, \theta)}{\partial \ln \theta} &= \int_0^1 dz \frac{\alpha_s(k_\perp)}{2\pi} p_{ji}^{(k)}(z) \Theta_{\text{res}}(z, \theta) \\ &\times [Q_j(zp, \theta)Q_k((1-z)p, \theta) - Q_i(p, \theta)] \end{aligned}$$

**PS<sub>in</sub> constraint**

Initial condition at zero angle  
is single charge quenching factor:

$$Q_i(p, 0) = Q_{\text{rad}, i}^{(0)}(p_T) Q_{\text{el}, i}^{(0)}(p_T)$$

Radiative  
energy loss

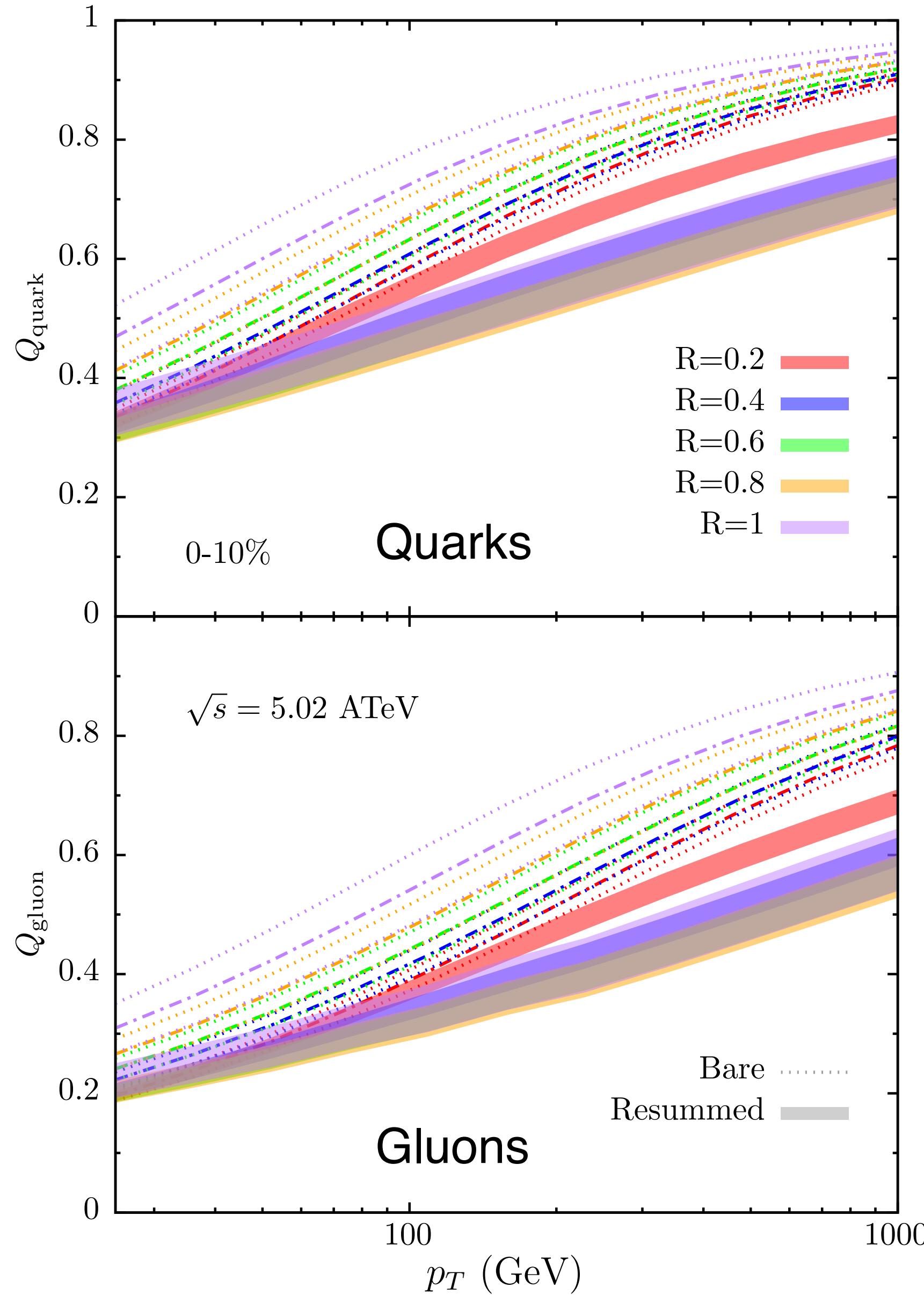
Elastic  
energy loss

- Energy loss versus R displays non-monotonic behaviour. Competing effects:

- Increasing R means more likely to retain emitted (or thermalised) quanta: **less quenching**.
- Increasing R means larger quenched phase space: **more quenching**.

Mehtar-Tani, DP, Tywoniuk - PRL '21

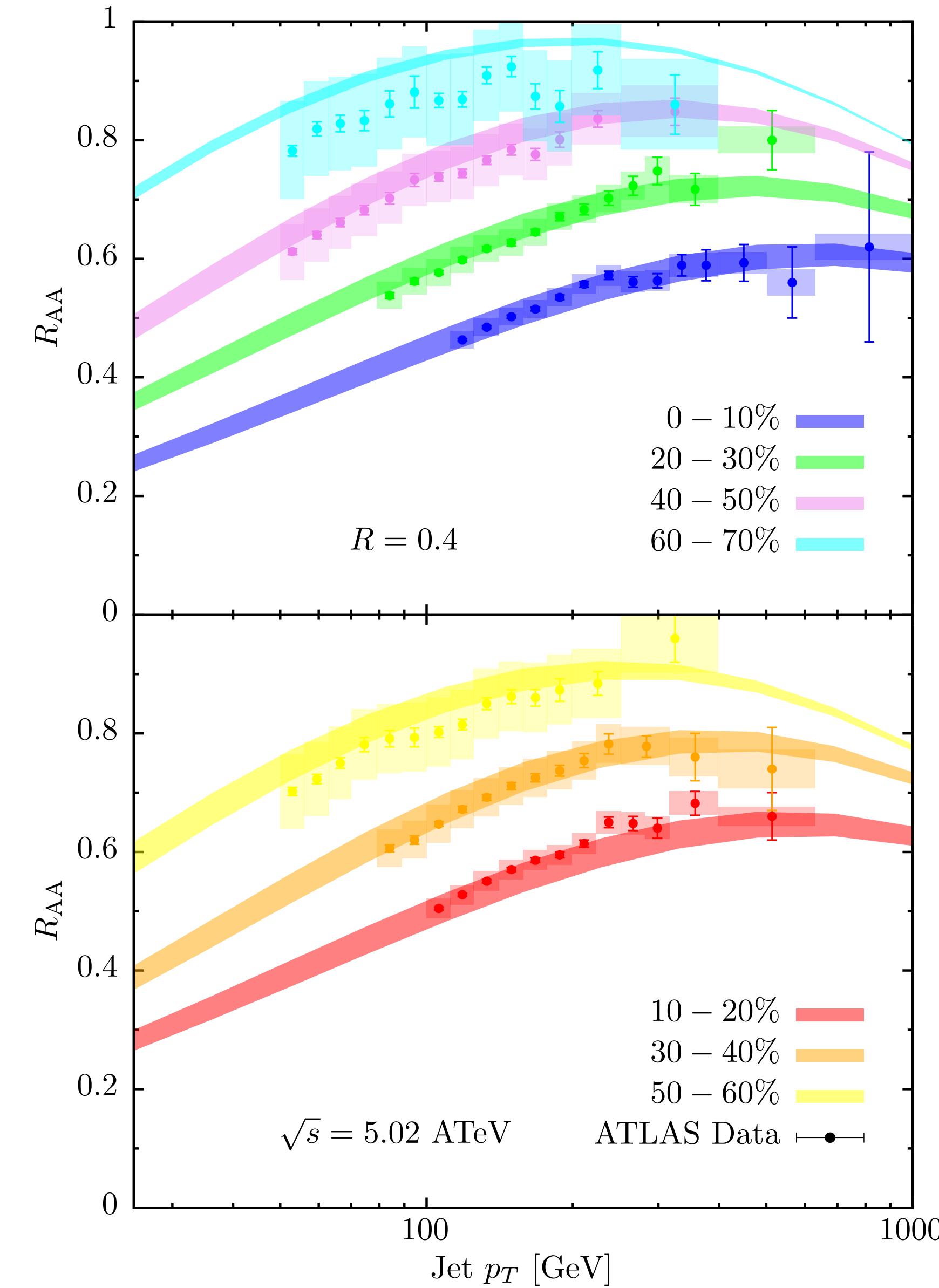
# Resummed Quenching Factor



- Bare quenching factors (dashed):
  - less quenching for larger  $R$ .
  - Easier to keep (recover) the emitted (thermalised) modes.
- Resummed quenching factors (solid):
  - larger  $R$  can lead to more quenching.
  - Interplay between energy recovery and size of quenched phase space.

# Jet Suppression

Mehtar-Tani, DP, Tywoniuk - PRL '21



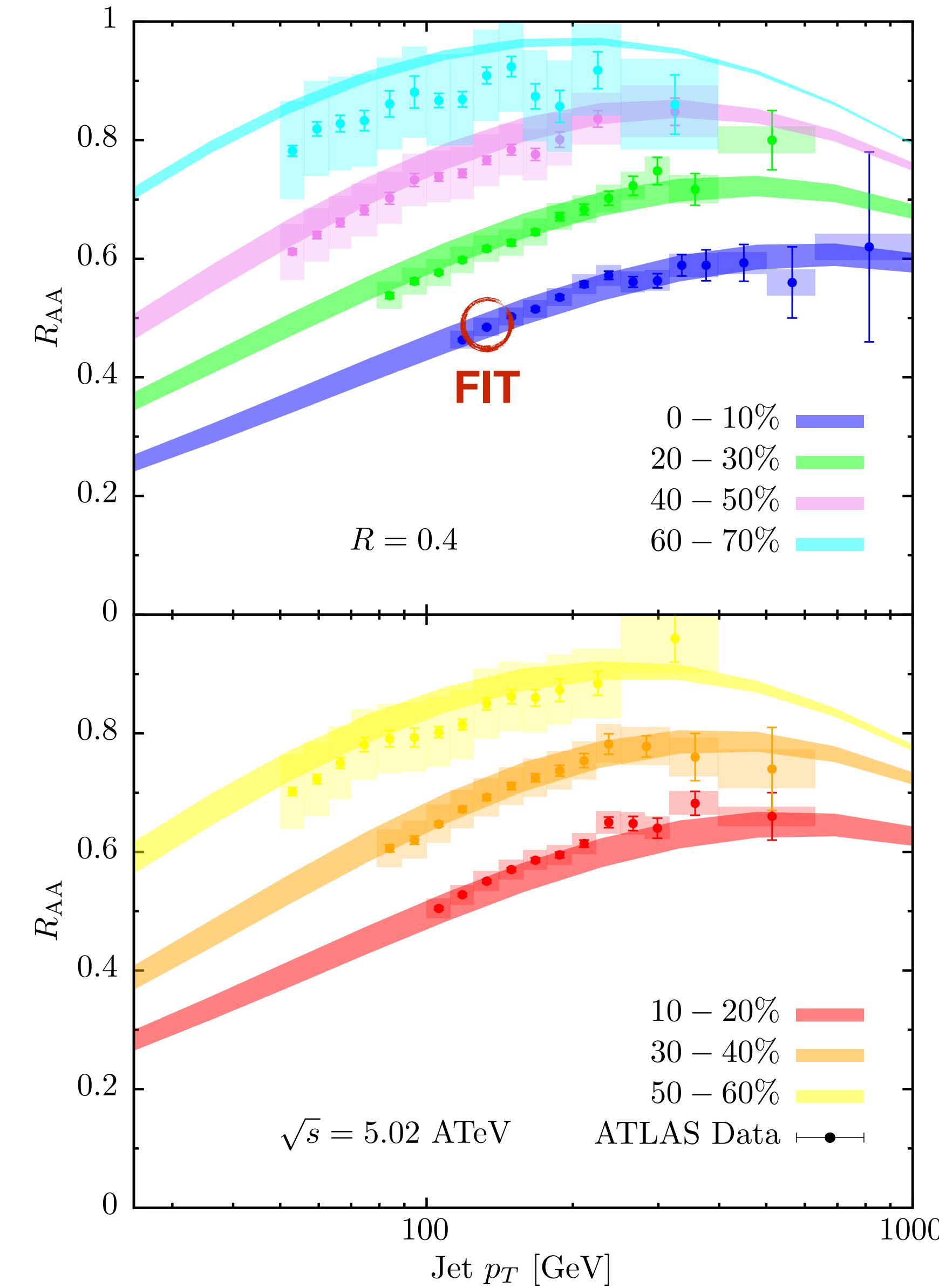
- Use PYTHIA8 to generate spectrum at initial angle  $R_0=1$  (with nuclear PDFs EPS09 LO in medium case)
- Evolve microjets using DGLAP down to jet  $R$ .
- Compute resummed quenching factors for each jet  $p_T$  and  $R$ :
  - Bare quenching factor requires knowledge of event-by-event, centrality dependent QGP properties:
  - Embed framework into realistic heavy-ion environment:
  - Glauber sampling, random azimuthal orientation.
  - Compute event-by-event relevant quantities, e.g.:
    - (in local fluid rest frame)

$$L = \int_{\Gamma(t)} dx_F \quad \hat{q}_0 \propto \frac{1}{L} \int_{\Gamma(t)} dx_F T^3(x) \left( \frac{p \cdot u(x)}{p^0} \right)$$

Path of jet through hydro. profile (VISHNU) down to  $T_c$

# Jet Suppression

Mehtar-Tani, DP, Tywoniuk - PRL '21



- Only two unconstrained parameters:

→  $R_{rec}$  varied between

$$R_{rec} = \pi/2$$

$$R_{rec} = (5/6)\pi/2$$

(isotropic)

(wake inspired)

Casalderrey, Milhano, DP, Rajagopal, Yao - JHEP '21

→  $g_{med}$  fit to ATLAS  $R=0.4$  around  $p_T \sim 120$  GeV at 0-10%

- $g_{med} \in \{2.2, 2.3\}$

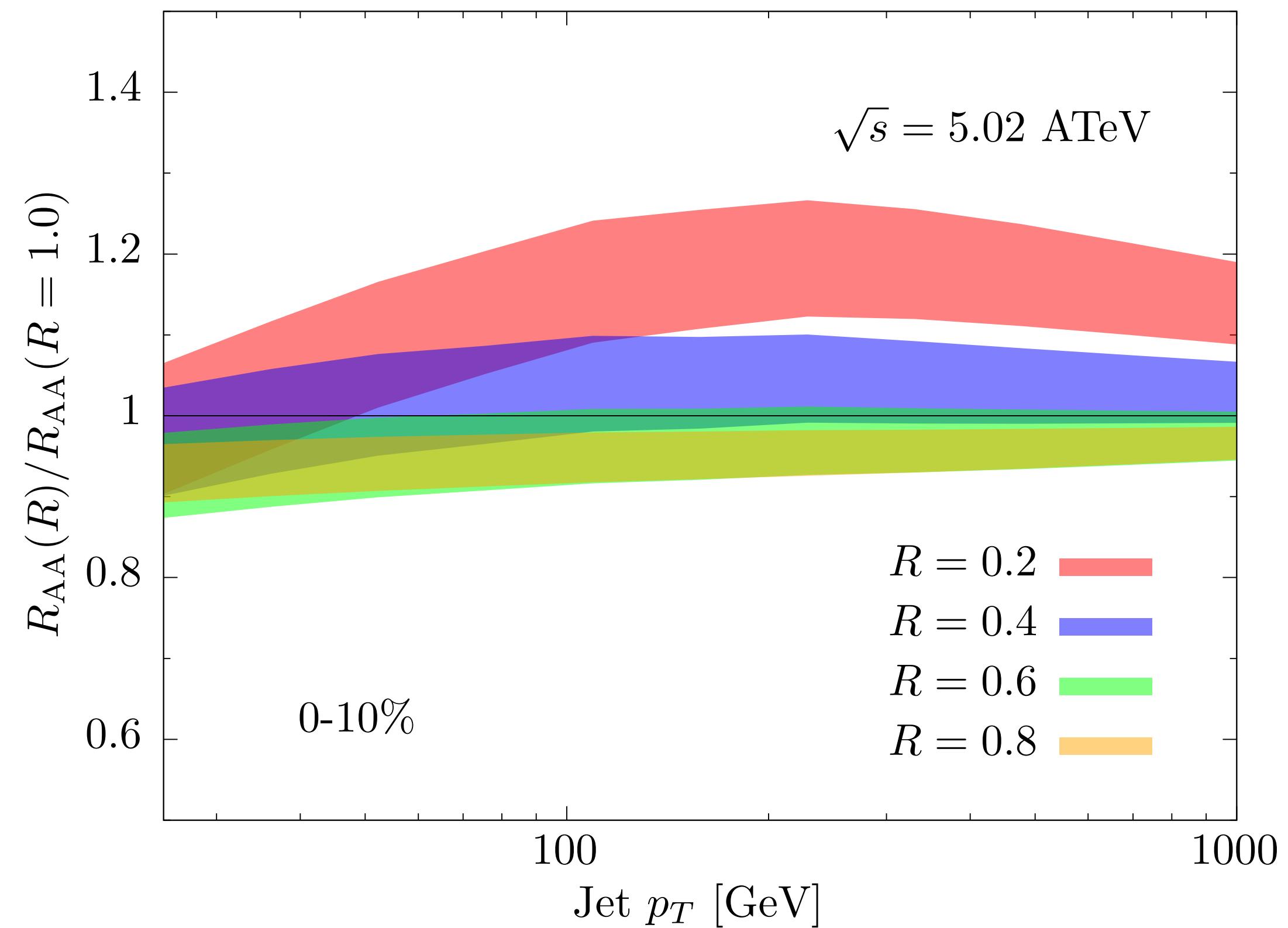
→  $\langle \hat{q}_0 \rangle \simeq 0.41 \text{ GeV}^2/\text{fm}$  in 0-10%

→  $\hat{q} = 2.46 \text{ GeV}^2/\text{fm}$   $\omega_c \approx 65 \text{ GeV}$

due to logarithmic corrections.

Good description of both centrality and jet  $p_T$  suppression.

# R Dependence & Modelling Uncertainties



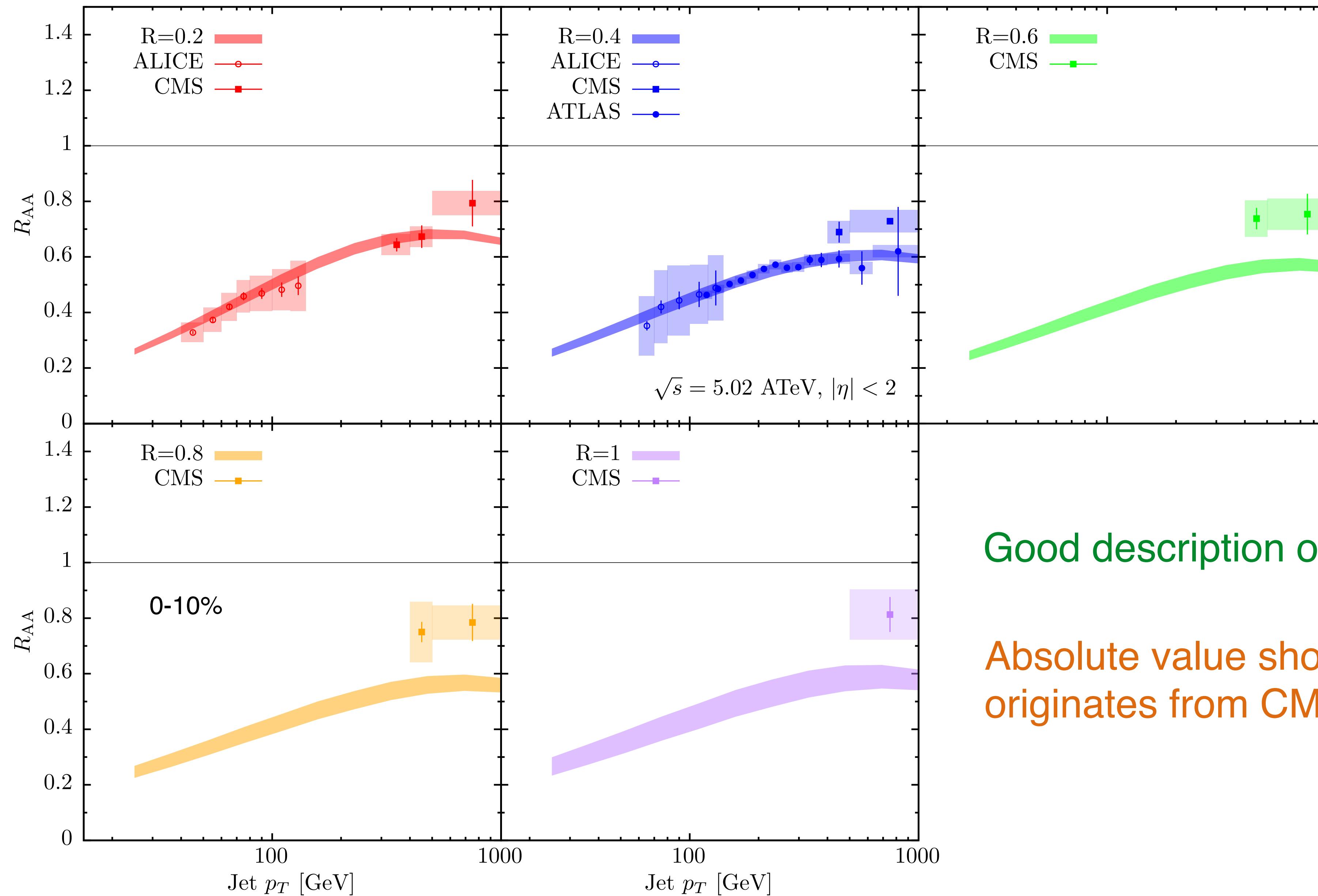
Mild R dependence,  
in agreement with CMS data.

- Modelling sensitivity for **R** between **0.2 and 0.6**:

Parameter	Variation	Effect
$\theta_c$	$[\theta_c/2, 2\theta_c]$	$\lesssim 20\%$
IOE	LO/NLO	$\sim 2\%$
$n$	$\pm 1$	$\sim 10\%$
$R_{rec}$	$[1, \infty]$	$\lesssim 10\%$
$\omega_s$	$[\omega_s/2, 2\omega_s]$	$\lesssim 8\%$

- NLO contribution very small  
(hard emissions tend to be collinear).
- Modelling of fate of lost energy relatively small.
- Determination of quenched phase space  
relatively large. Improvable in pQCD.  
  
Need to improve perturbative sector before  
non-perturbative becomes relevant (for  $R < 0.6$ !).

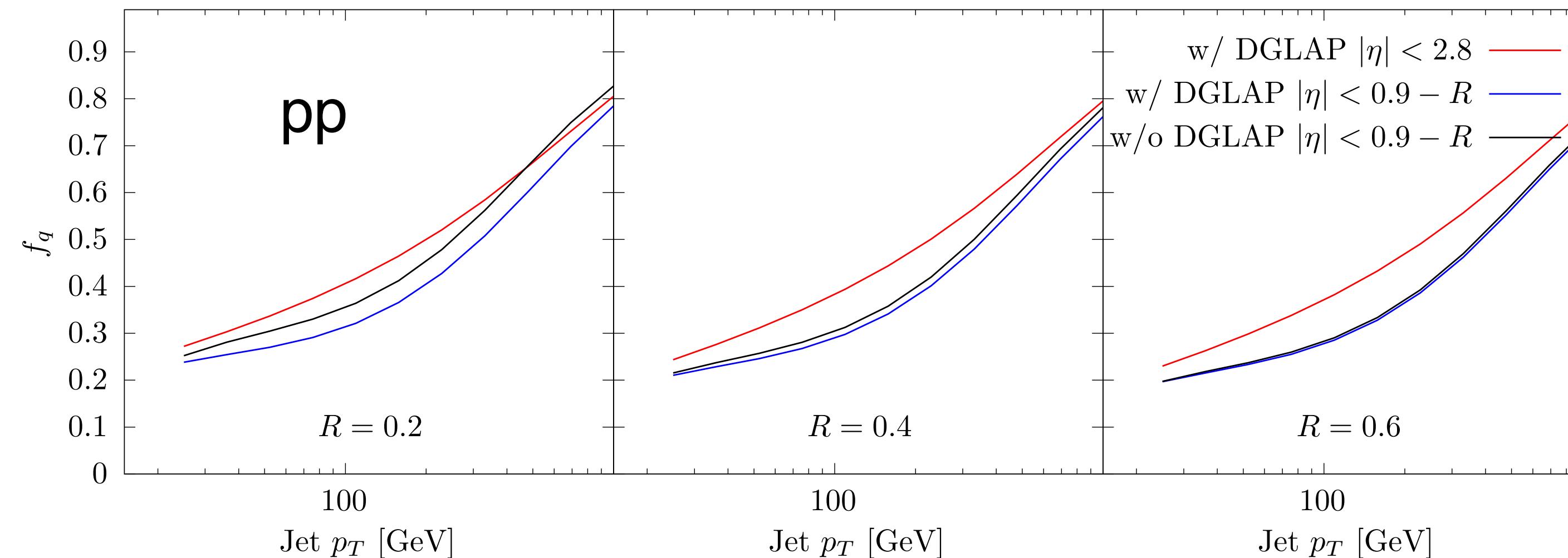
# More Data Comparison



Good description of ALICE at lower  $p_T$ .

Absolute value shows some tension with CMS,  
originates from CMS/ATLAS tension.

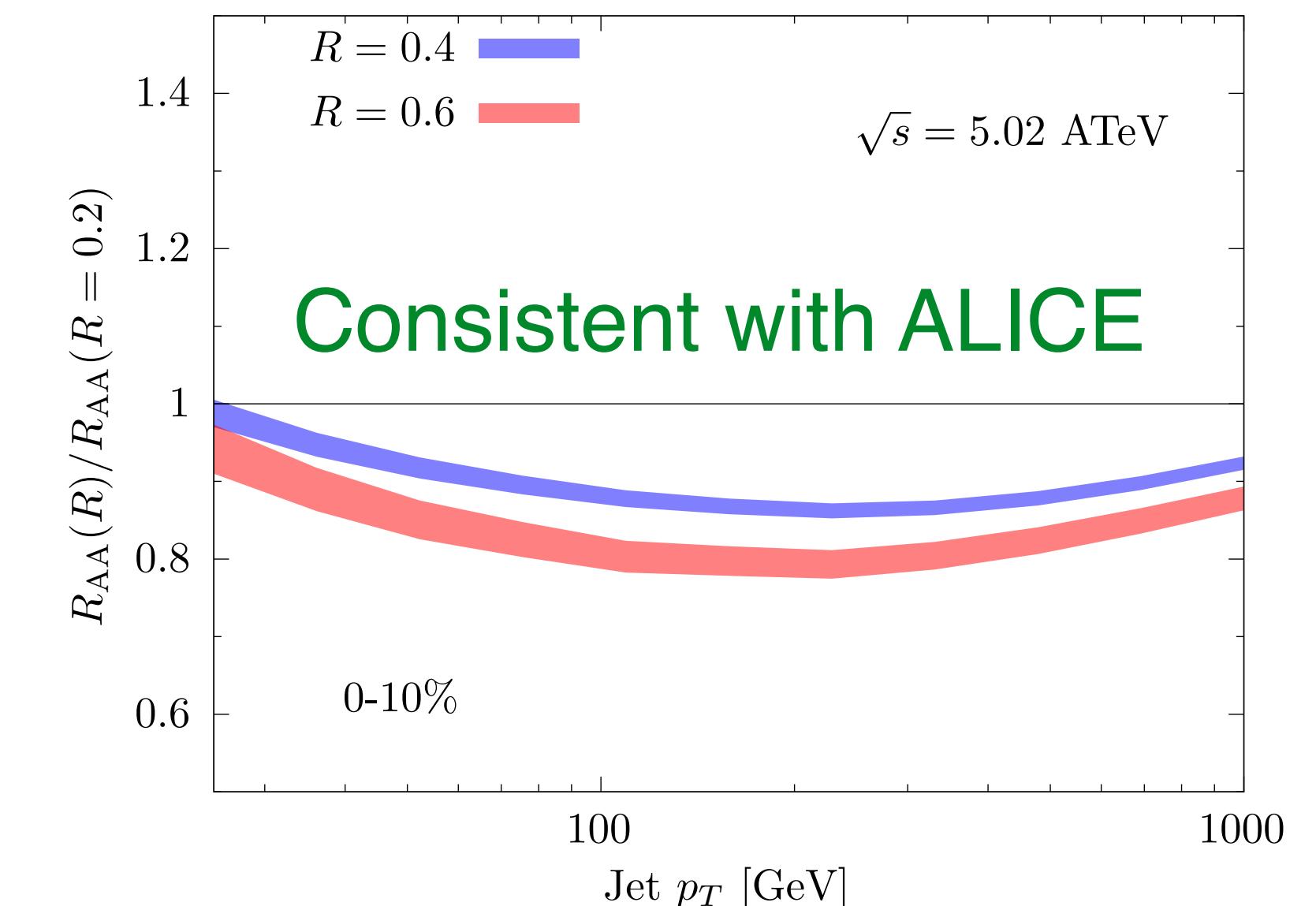
# Accounting for rapidity cuts



**w/ DGLAP:** fit pp spectrum at  $R_0=1$ , evolve down to  $R$ .

**w/o DGLAP:** fit pp spectrum directly at  $\eta = 0.9 - R$ .

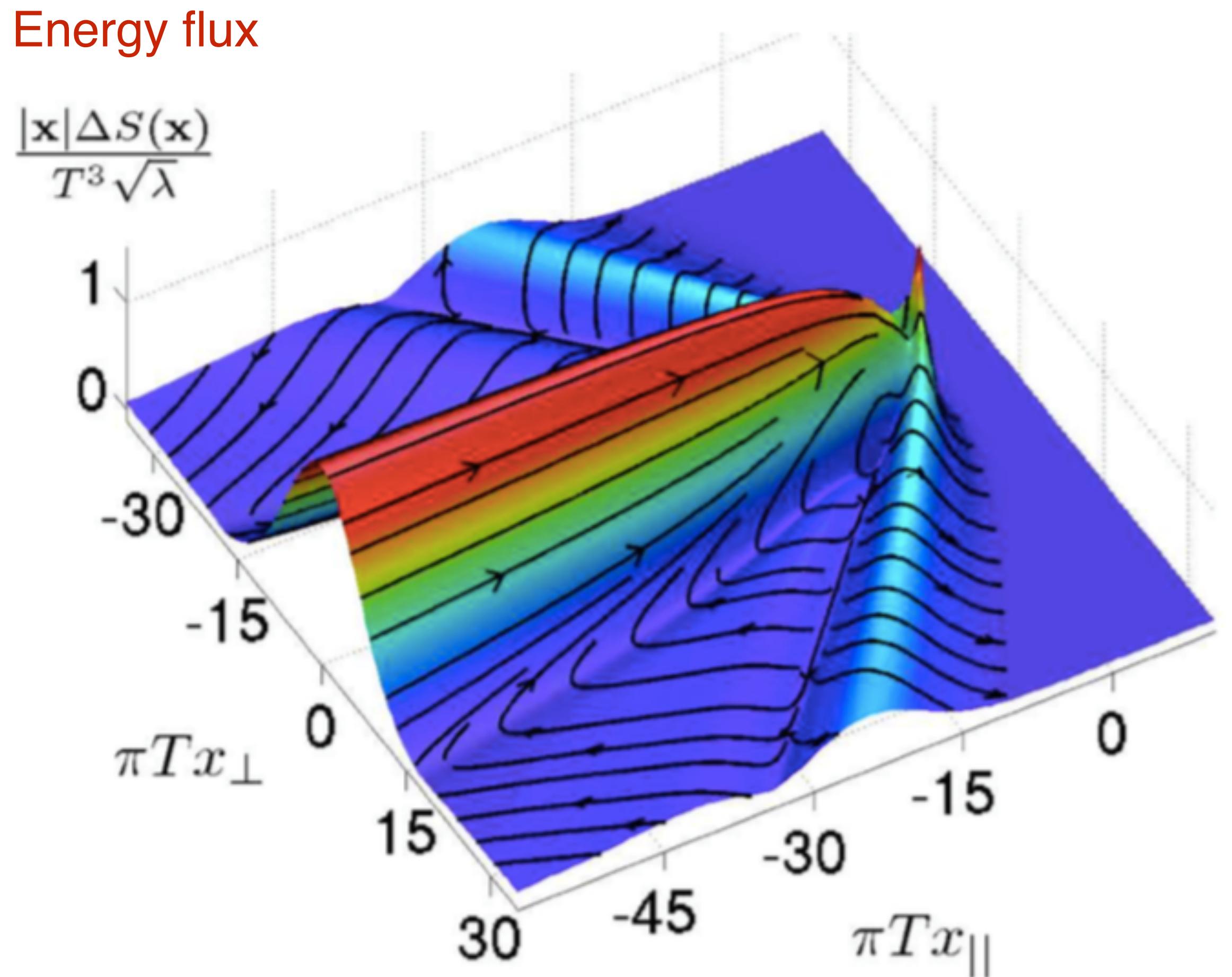
- Quark fraction depends on:
  - Rapidity, via PDF.
  - Cone  $R$ , via microjet evolution.



- Redoing calculation with ALICE rapidity cuts makes no difference.
  - Quark fraction change small in ratio.
  - Effect of spectral index change negligible.

# The Wake of a Quark

- At strong coupling:
  - Modification of stress-energy tensor due to supersonic quark contains sound and diffusive modes.
  - Effective source for hydro corresponds to drag force on the quark.
  - Agreement between hydrodynamics & wake of a quark even for small distances  $\sim 1/T$ .



*Fulfils Energy-Momentum Conservation  
in the Jet+Plasma Interplay.*

Chesler & Yaffe - PRD '07

# Estimation of the Hadrons from the Wake

- Assuming:

- small perturbations on top of Bjorken flow.
- perturbation stays localised near jet's rapidity.

Expand Cooper-Frye spectrum to first order in perturbations:

$$E \frac{d\Delta N}{d^3 p} = \frac{1}{32\pi} \frac{m_T}{T^5} \cosh(y - y_j) \exp \left[ -\frac{m_T}{T} \cosh(y - y_j) \right]$$

$$\left\{ p_T \Delta P_T \cos(\phi - \phi_j) + \frac{1}{3} m_T \Delta M_T \cosh(y - y_j) \right\}$$

$$\Delta P_{\perp}^i = w \tau \int d^2 x_{\perp} d\eta \delta u_{\perp}^i$$

Velocity pert.

$$\Delta S = \frac{s \tau}{c_s^2} \int d\eta d^2 x_{\perp} \frac{\delta T}{T}$$

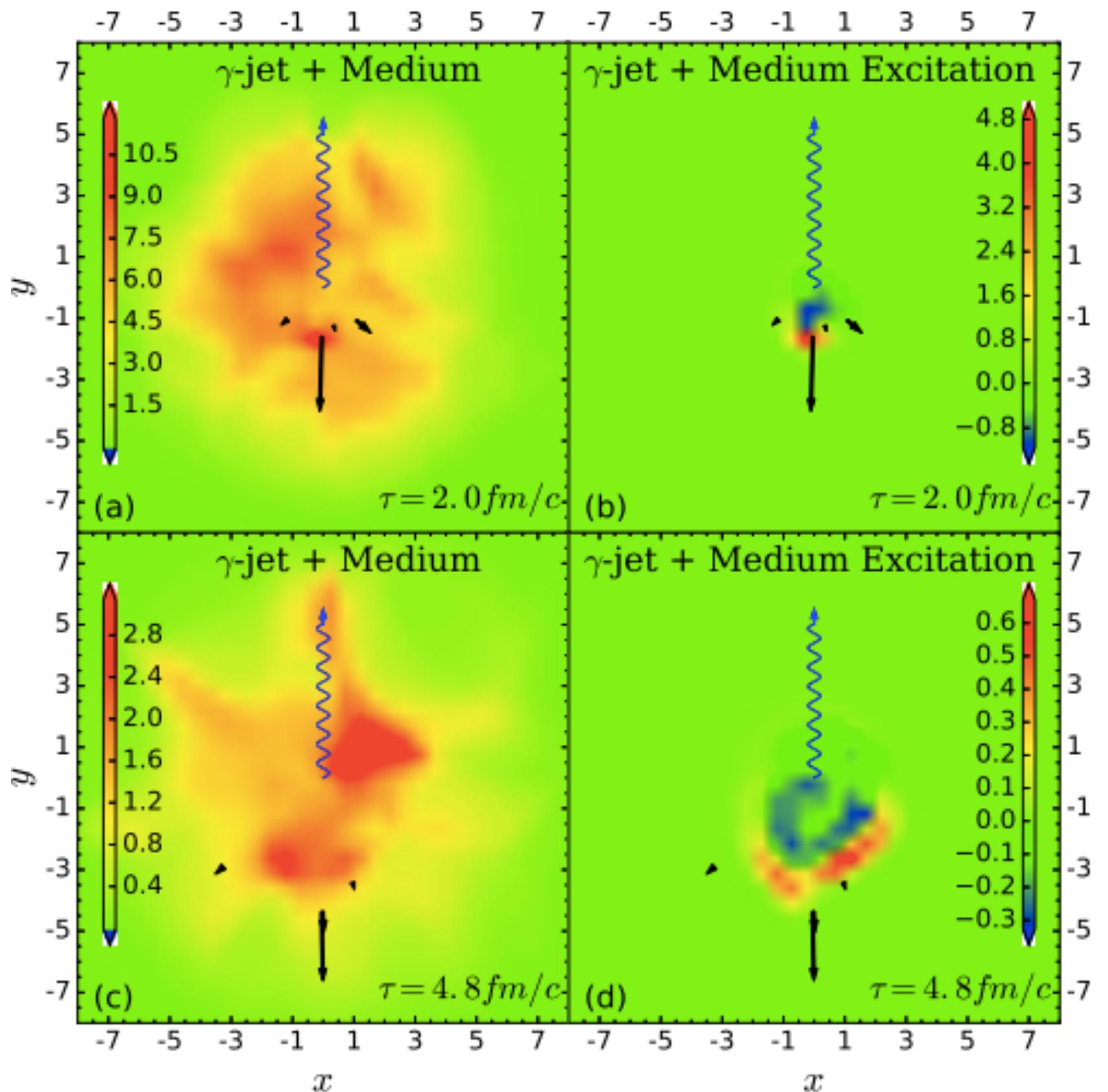
Temperature pert.

✓ Fully constrained by energy-momentum conservation.

✓ Computationally efficient.

✗ Neglects important effects from local flow.

# The Diffusion Wake

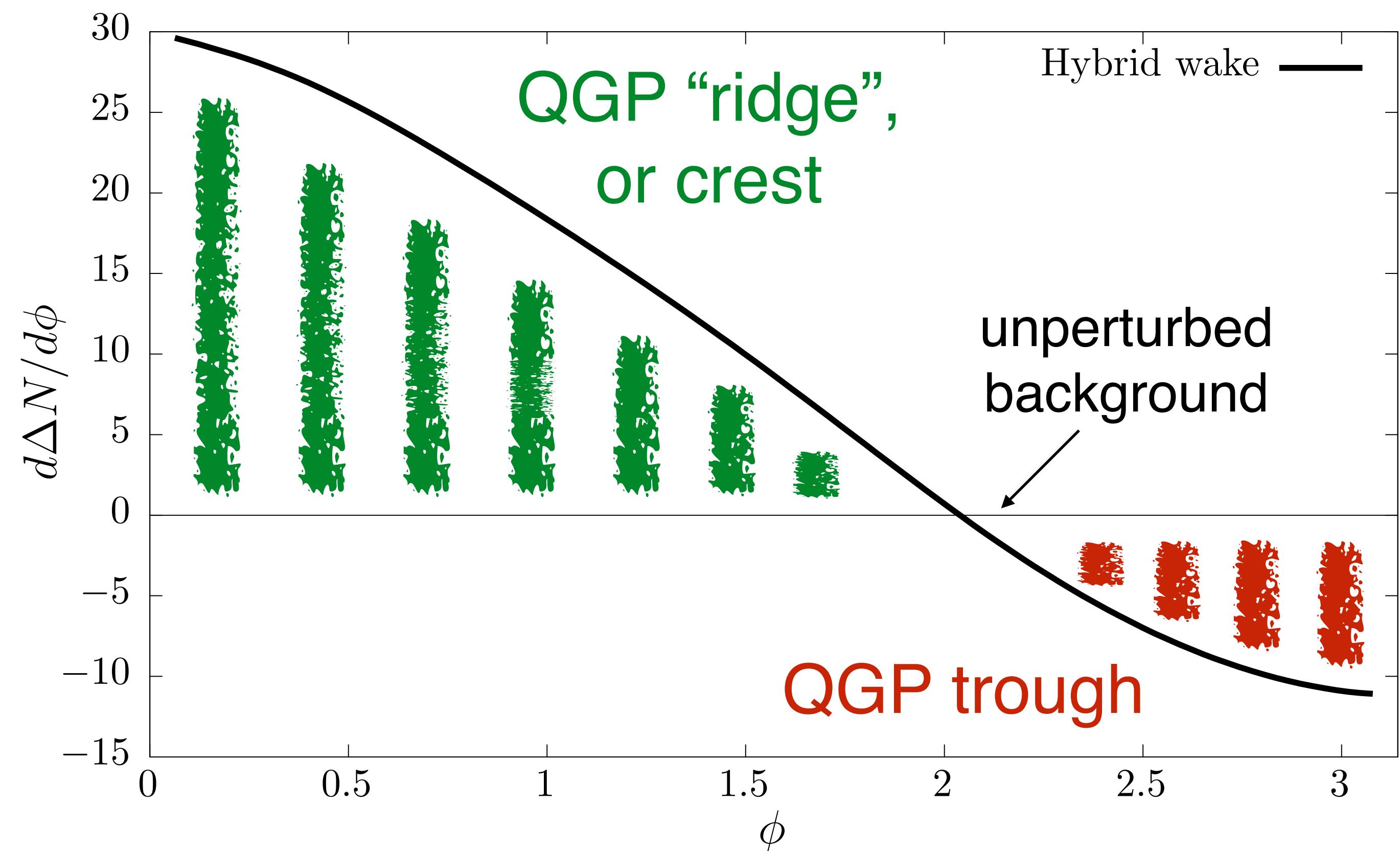


Chen et al. - PLB '18

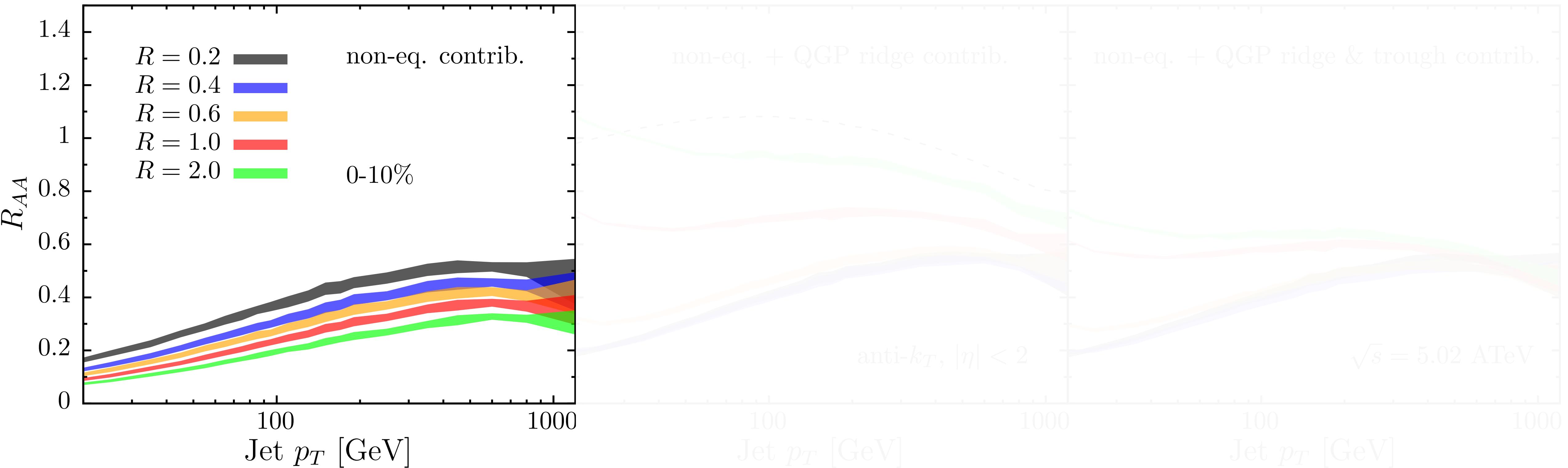
Effect observed also in non-linear  
hydro. + source from jet.

QGP trough arises due to the diffusion wake:

- Depletion of energy density behind the jet (the jet drags the fluid along its direction of propagation, reduces yield of particles in the opposite direction).



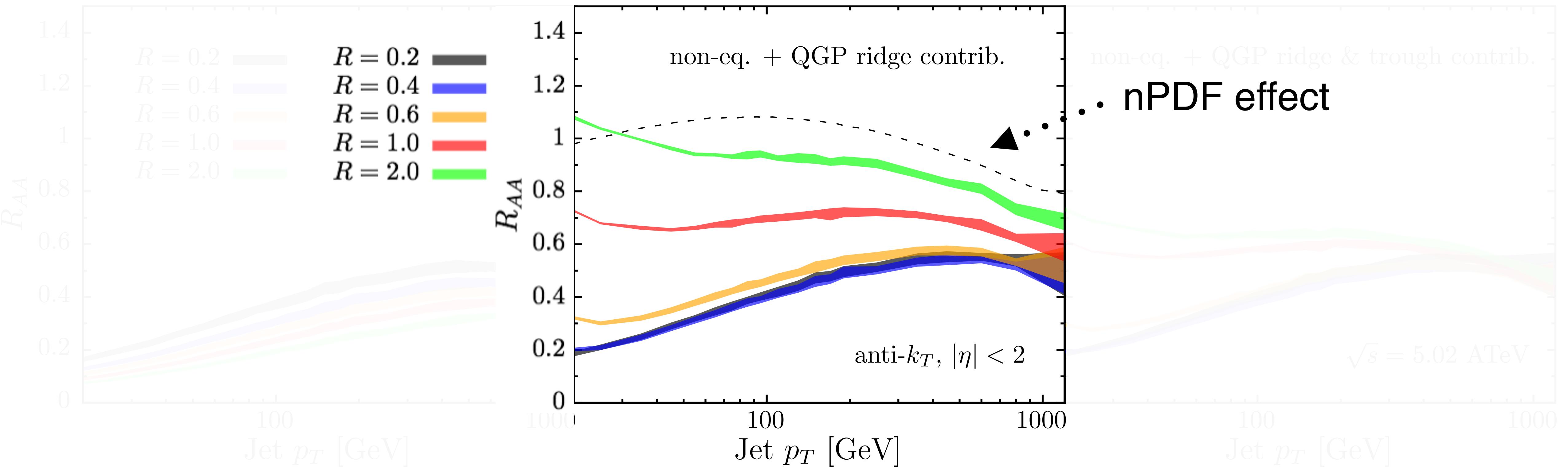
# Jet R<sub>AA</sub> at LHC with Hybrid Model



Include **non-eq. contribution** only, i.e. jet particles that did not hydrodynamize:

- Jet suppression increases with increasing  $R$ .
- Wider jets “lose” more energy, more energy loss sources.

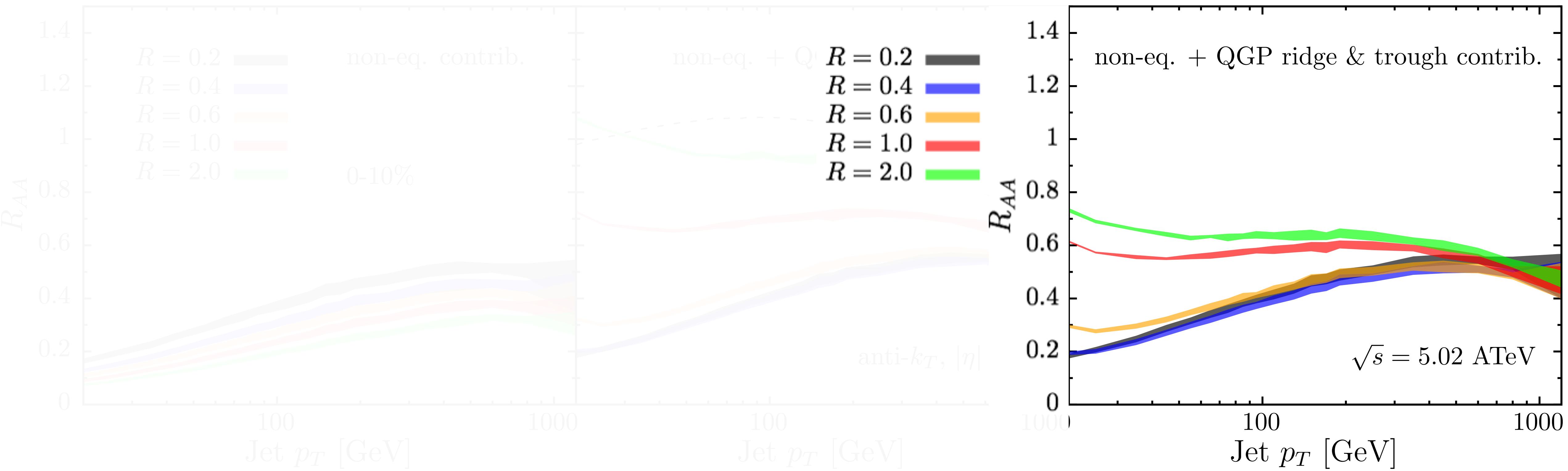
# Jet $R_{AA}$ at LHC with Hybrid Model



Include both **non-eq.** and **QGP “ridge” contributions:**

- Energy is progressively recovered with increasing  $R$ .
- ! nPDF effect sets an upper limit on  $R_{AA}$  at very high  $p_T$ .

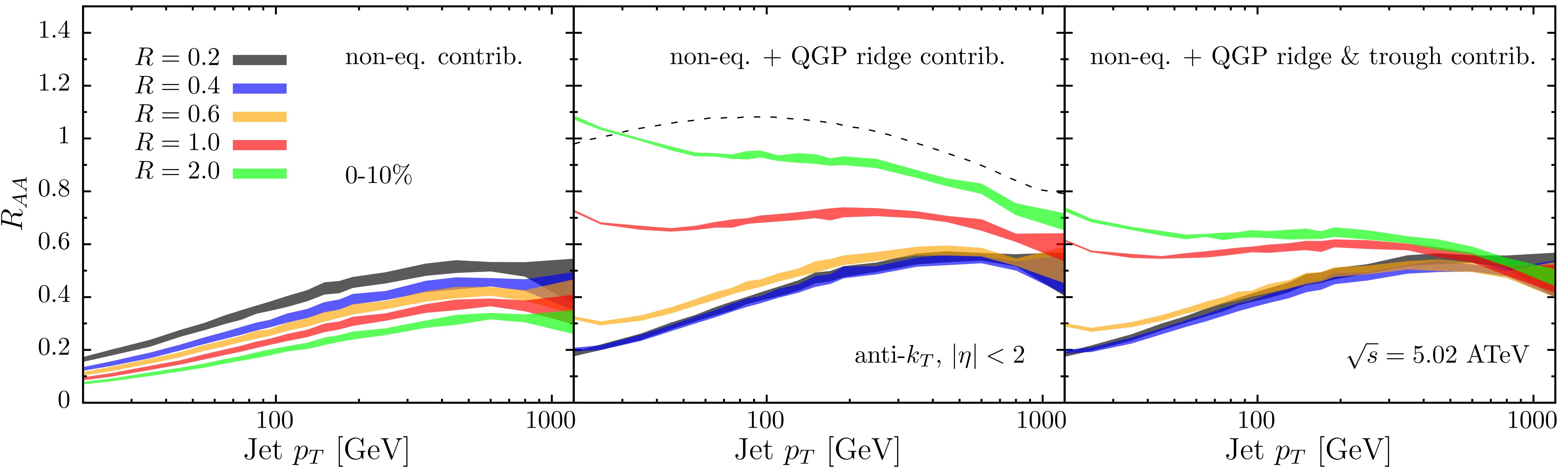
# Jet R<sub>AA</sub> at LHC with Hybrid Model



Include **non-eq.**, **QGP “ridge”** and **QGP trough** contribution:

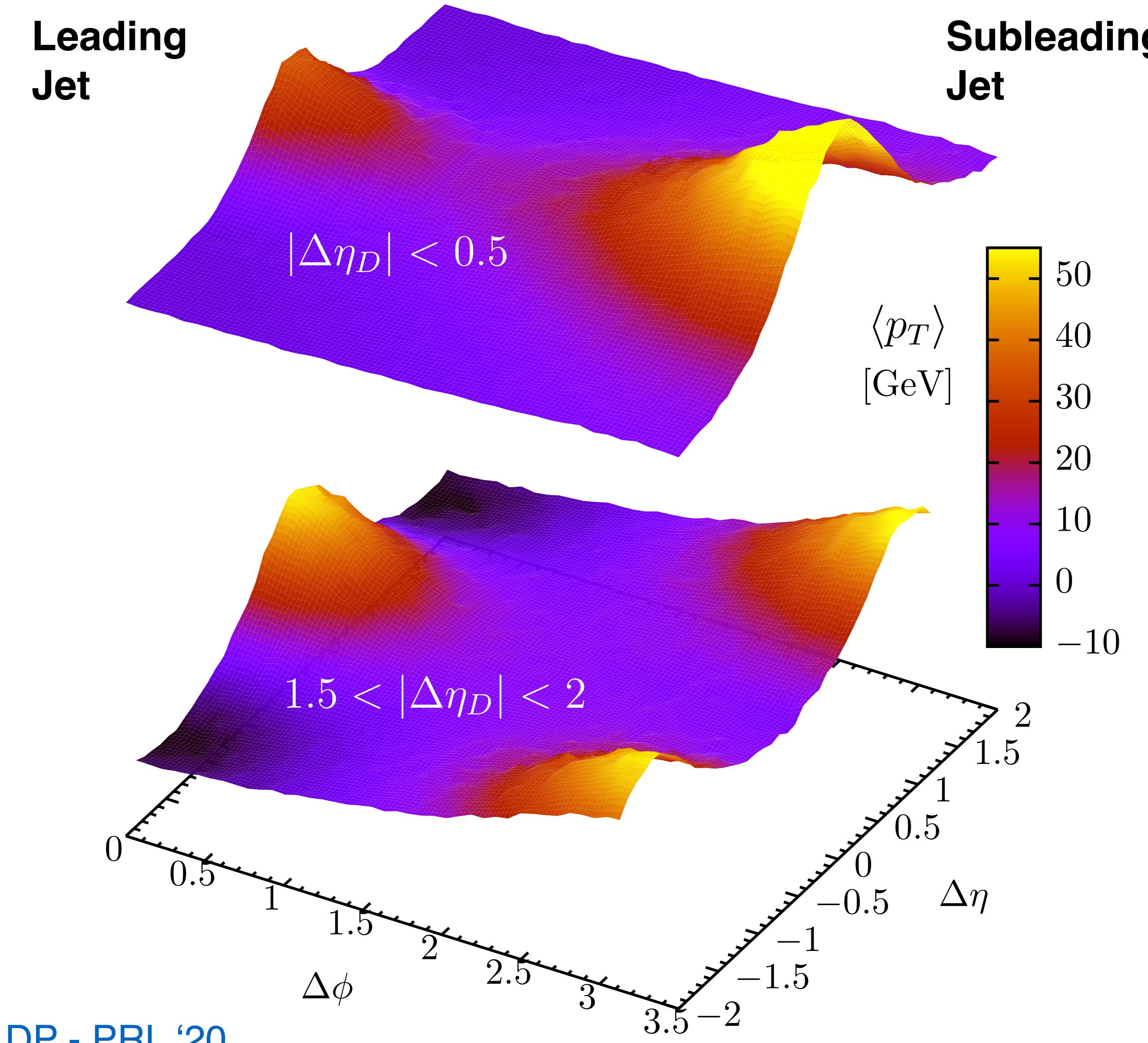
- QGP trough amounts to jet suppression; over-subtraction effect.
- Effect increases with increasing  $R$ .

# Jet R<sub>AA</sub> at LHC with Hybrid Model



Competition of effects that yield, overall,  
a very mild evolution from small to large  $R$ .

# The Effect of the Recoiling Jet



$\langle p_T \rangle$  density of wake hadrons w.r.t leading jet axis.

*Aligned in rapidity*

Subleading jet's **QGP trough hits leading jet.**

*Separated in rapidity*

Subleading jet's **QGP trough misses leading jet.**

$$\begin{aligned} p_T^L &> 250 \text{ GeV} \\ p_T^S &> 80 \text{ GeV} \\ \Delta\phi_D &> 2\pi/3 \end{aligned}$$

differential in  
 $|\eta_D| \equiv |\eta_L - \eta_S|$

# Leading Jet Suppression vs. $|\eta_{DL}|$

DP - PRL '20

*A new observable.*

**R = 0.4**

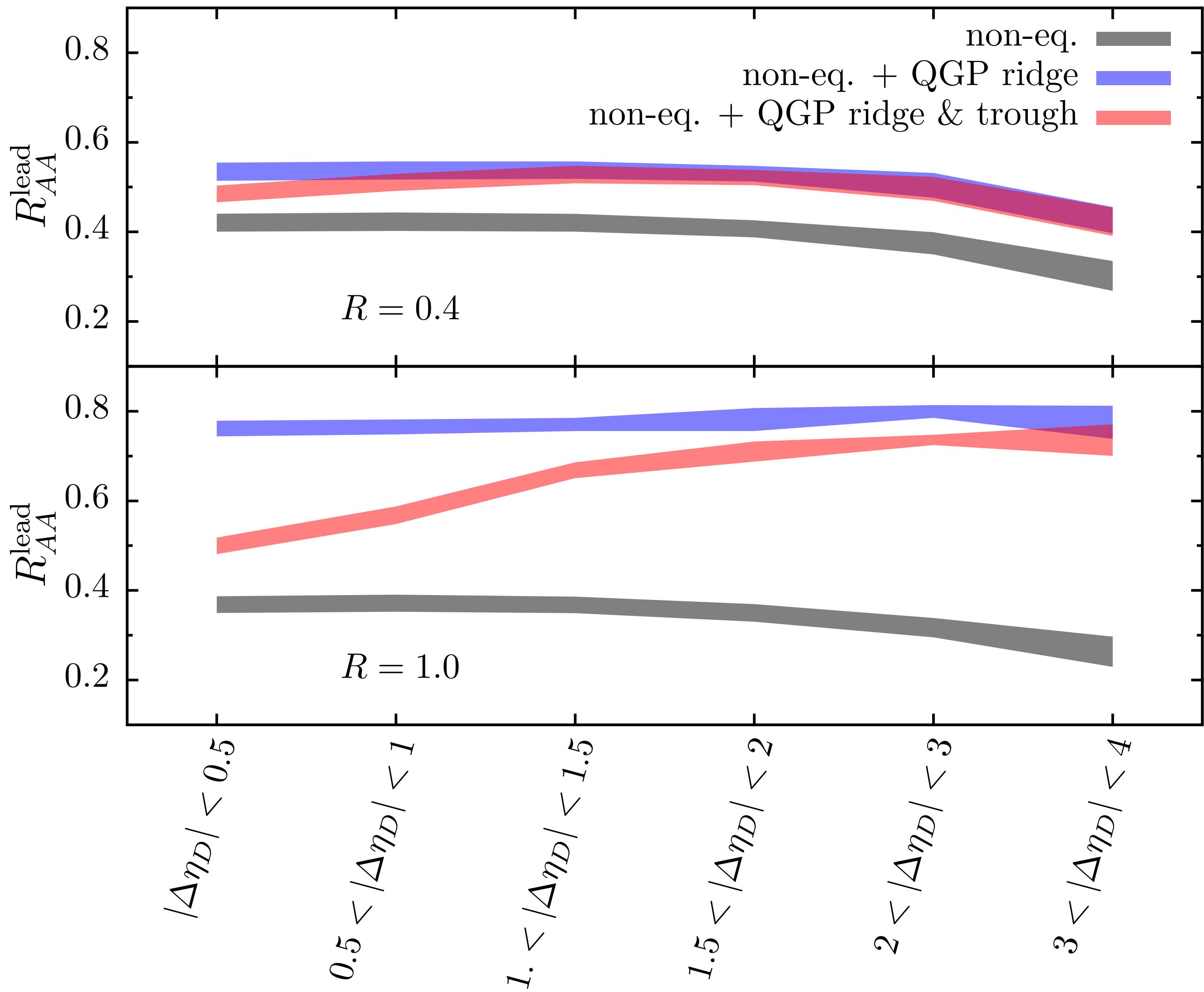
leading jet area easy to miss;  
small effect from QGP trough.

**R = 1.0**

strong dependence on  $|\eta_{DL}|$ ;  
knee visible when  $|\eta_{DL}| \sim R$ .

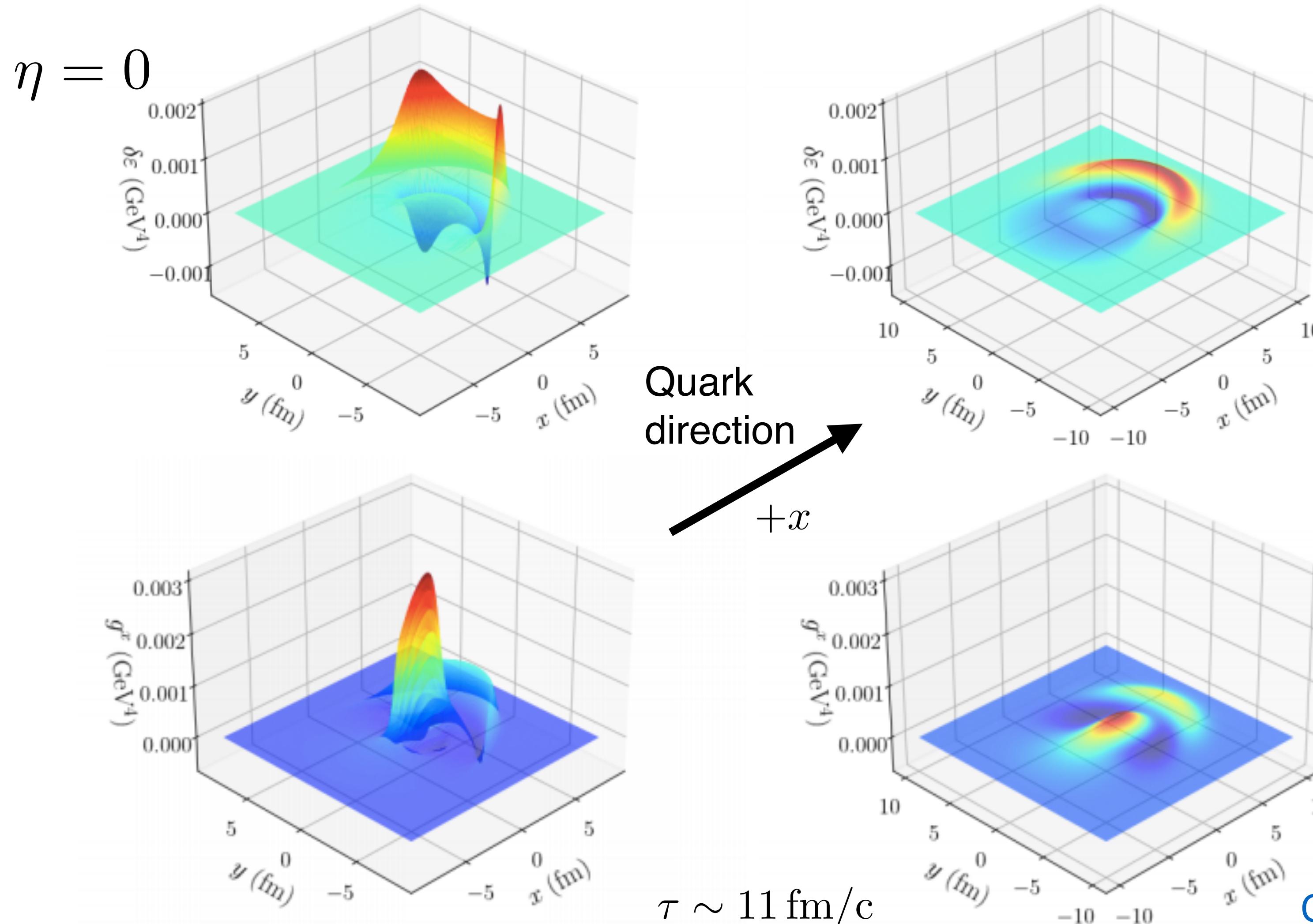
$p_T^L > 250$  GeV  
 $p_T^S > 80$  GeV  
 $\Delta\phi_D > 2\pi/3$

differential in  
 $|\eta_D| \equiv |\eta_L - \eta_S|$



# Linearized Hydrodynamics

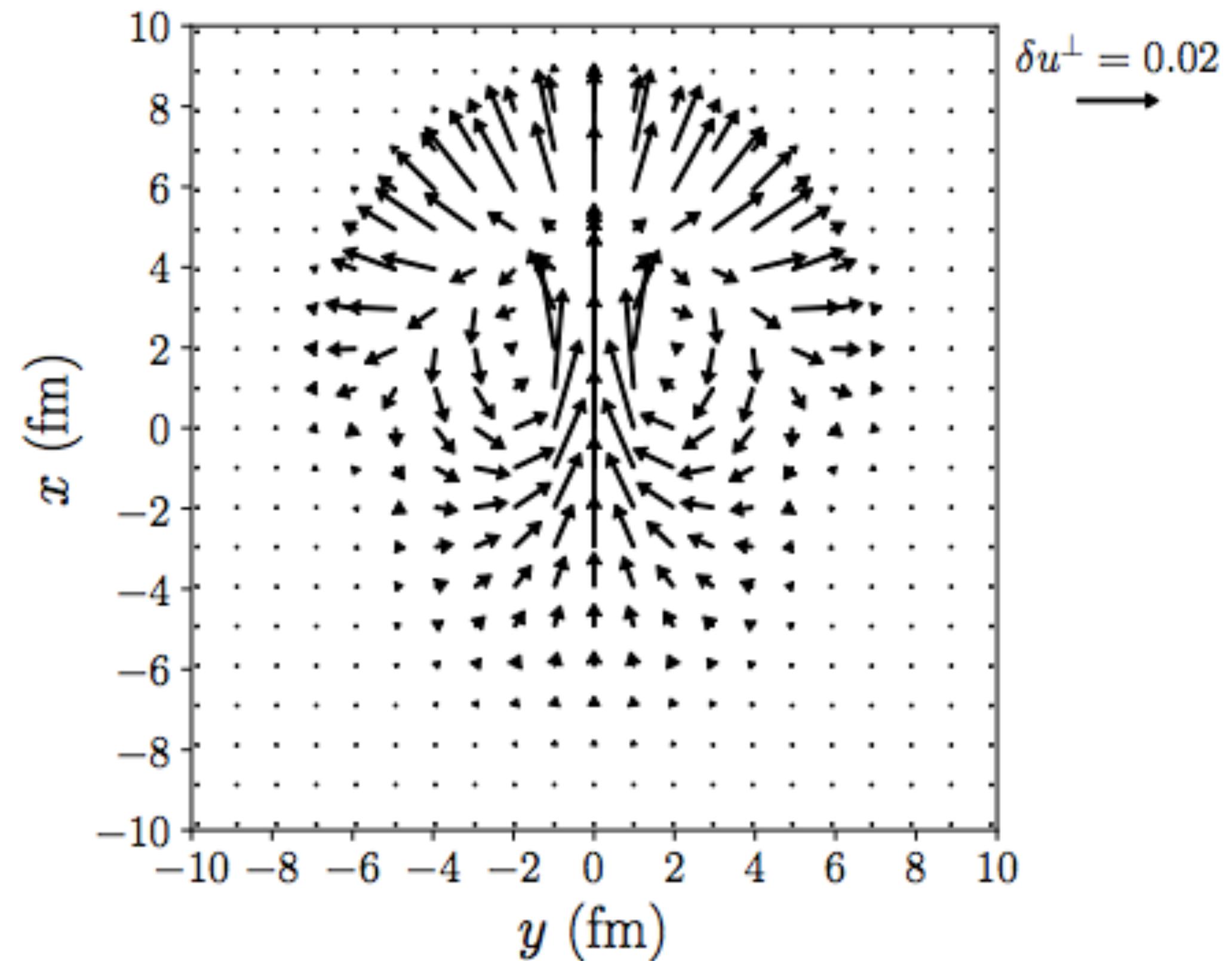
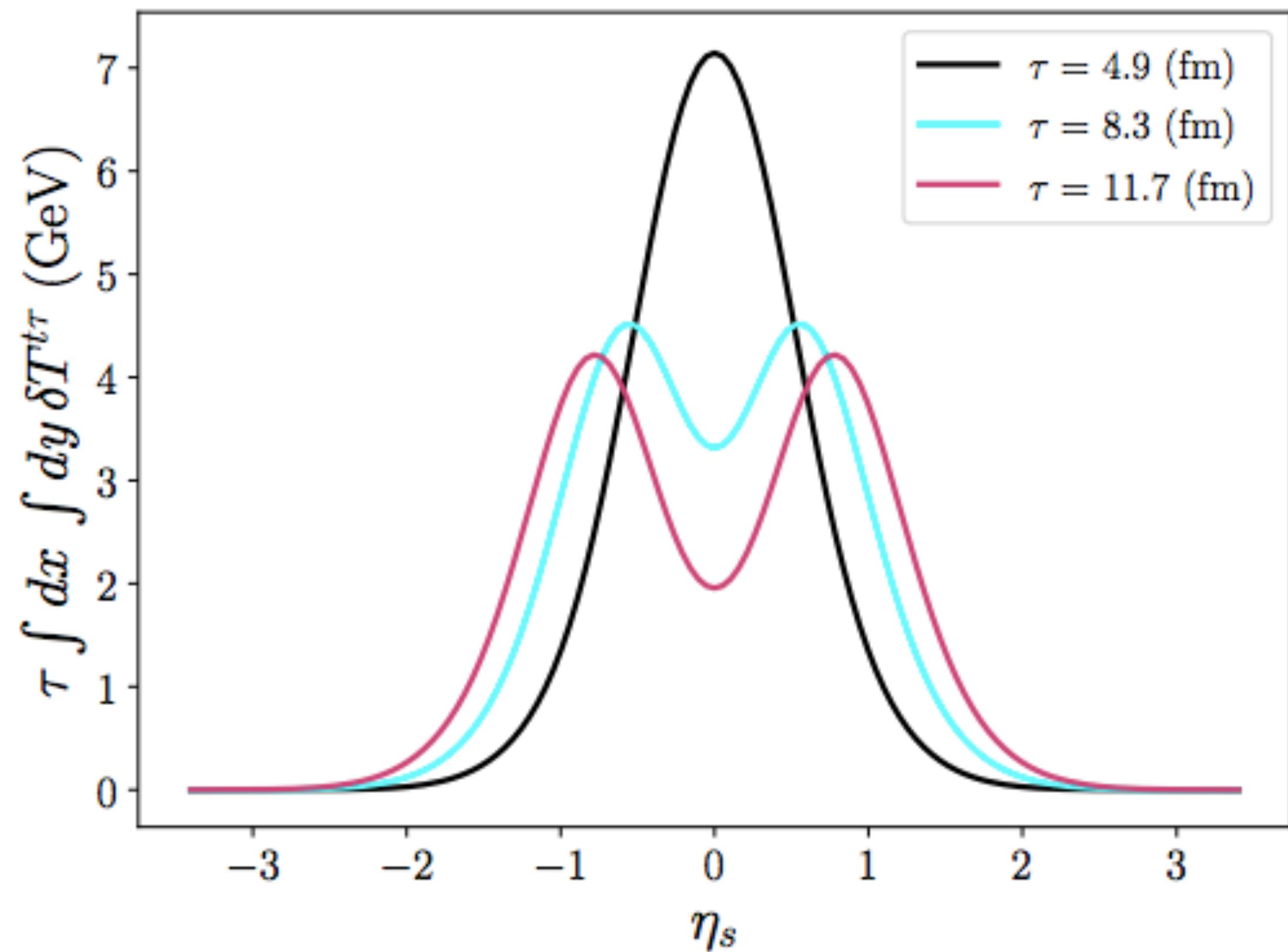
- Analytic, but over-simplified medium response needs to be improved:  
→ Starting point: linearised hydro eqs. for perturbations on top of viscous Bjorken flow.



$$\eta = \frac{s}{4\pi}$$

- $\delta\epsilon$  : energy density pert.  
Wavefront structure (Mach cone)  
diffuses due to viscosity.
- $g_x$  : momentum pert.  
Wing shaped structure diffuses  
due to viscosity (diffusion wake).

# Linearized Hydrodynamics



- Sound modes make wake energy spread in rapidity with time.
- Jet breaks long. boost invariance.

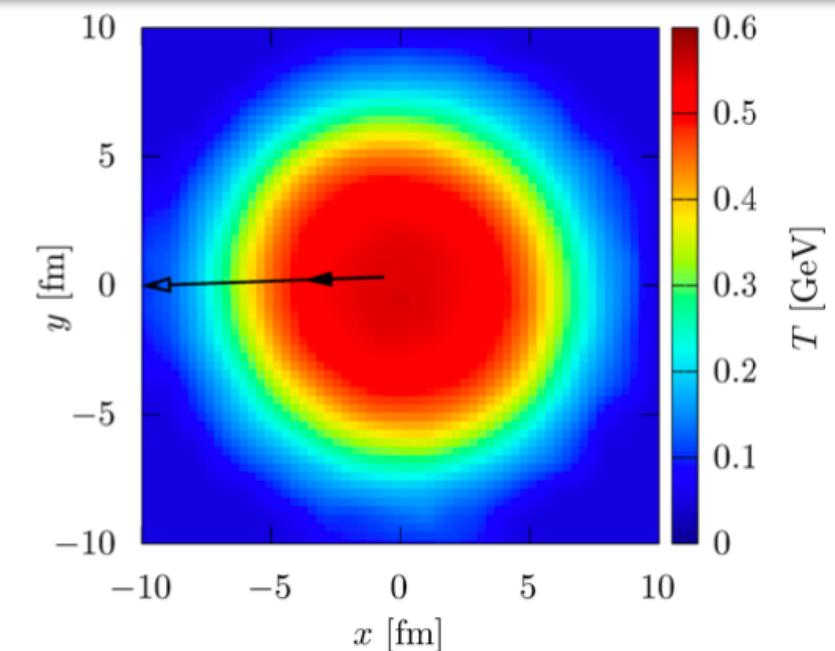
- Vortex ring around jet direction.
  - Imprints on  $\Lambda$  polarisation?

Serenone et al. - PLB '21

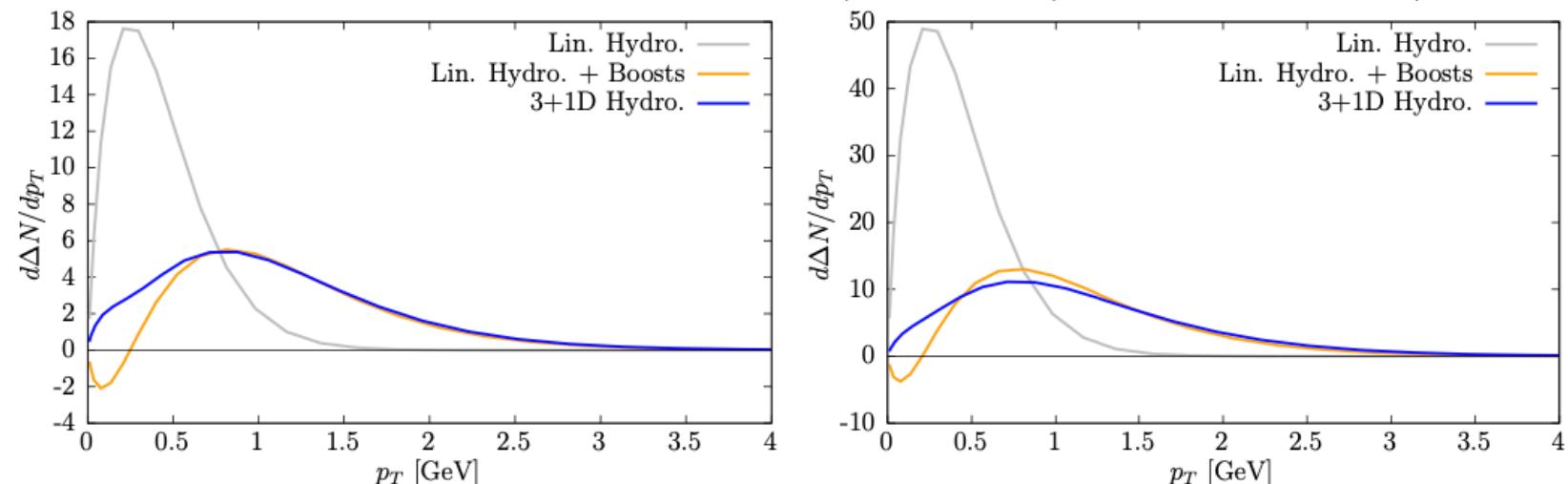
# Efficient Computations of the Wake

Casalderrey, Milhano, DP, Rajagopal, Yao - in preparation

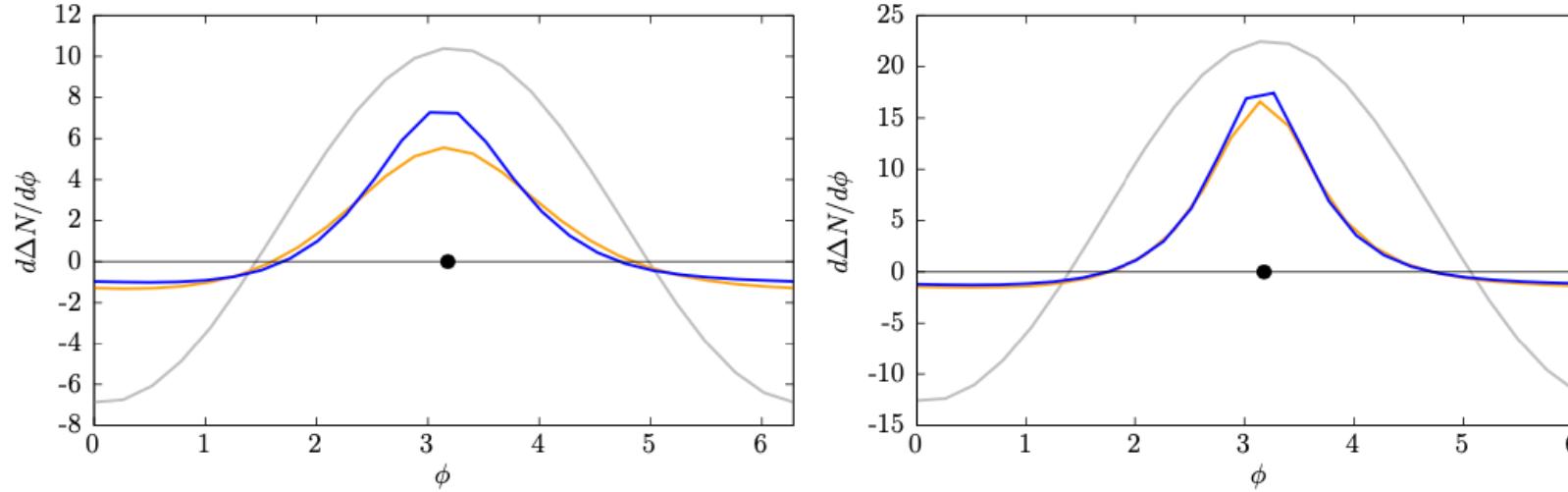
Hadrons from the wake  
depend on evolution time, local flow...  
thousands of possibilities.



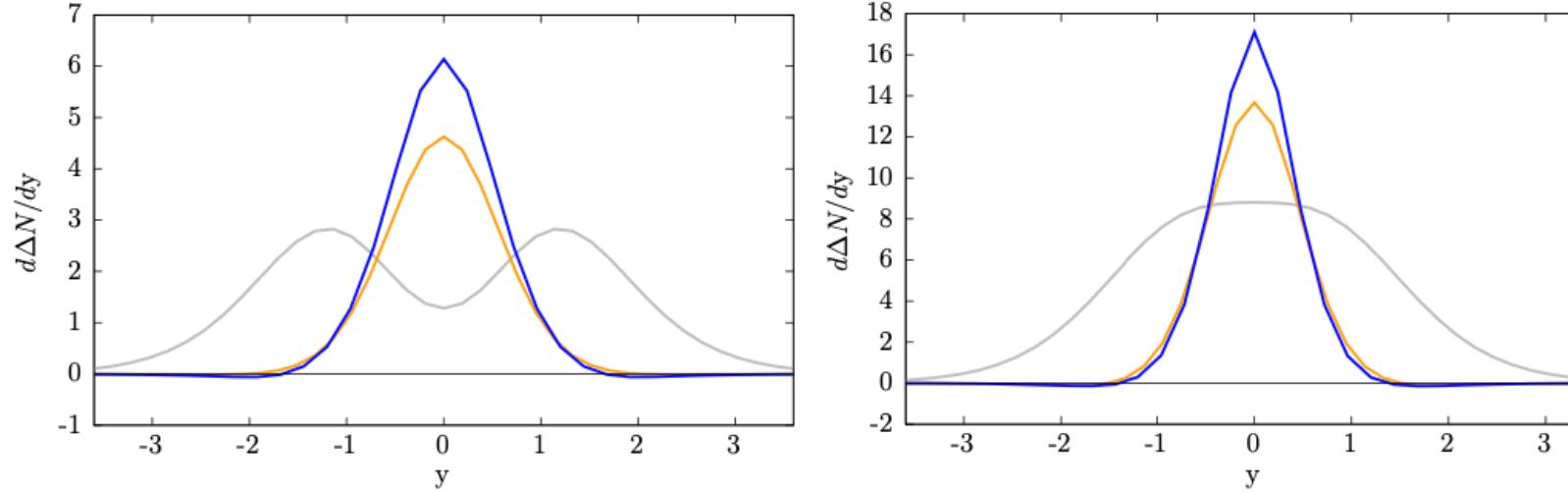
(a) Energy-momentum deposition for  $E_i = 10$  GeV (filled arrow) and  $E_i = 50$  GeV (empty arrow).



(b)  $p_T$  distribution for  $E_i = 10$  GeV.

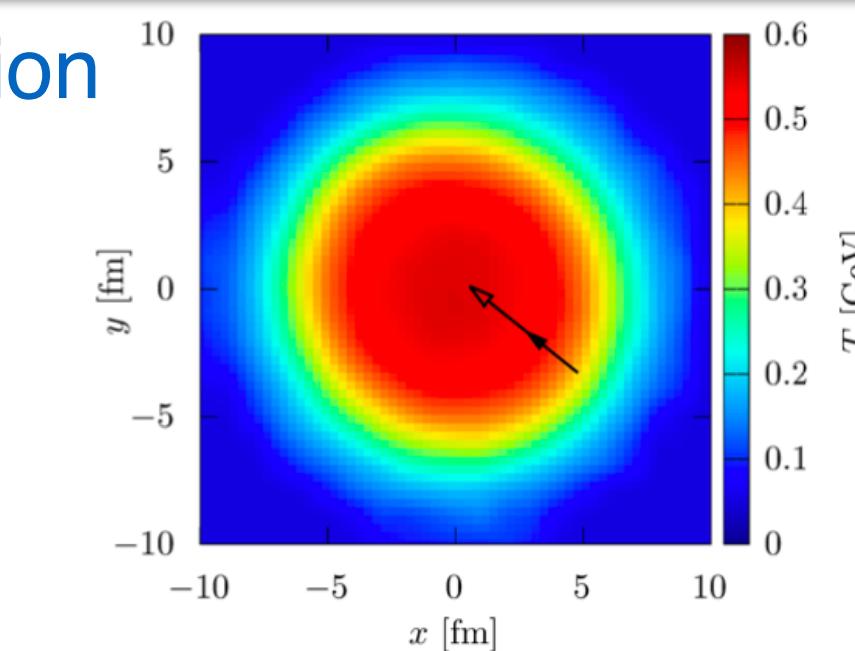


(d)  $\phi$  distribution for  $E_i = 10$  GeV.

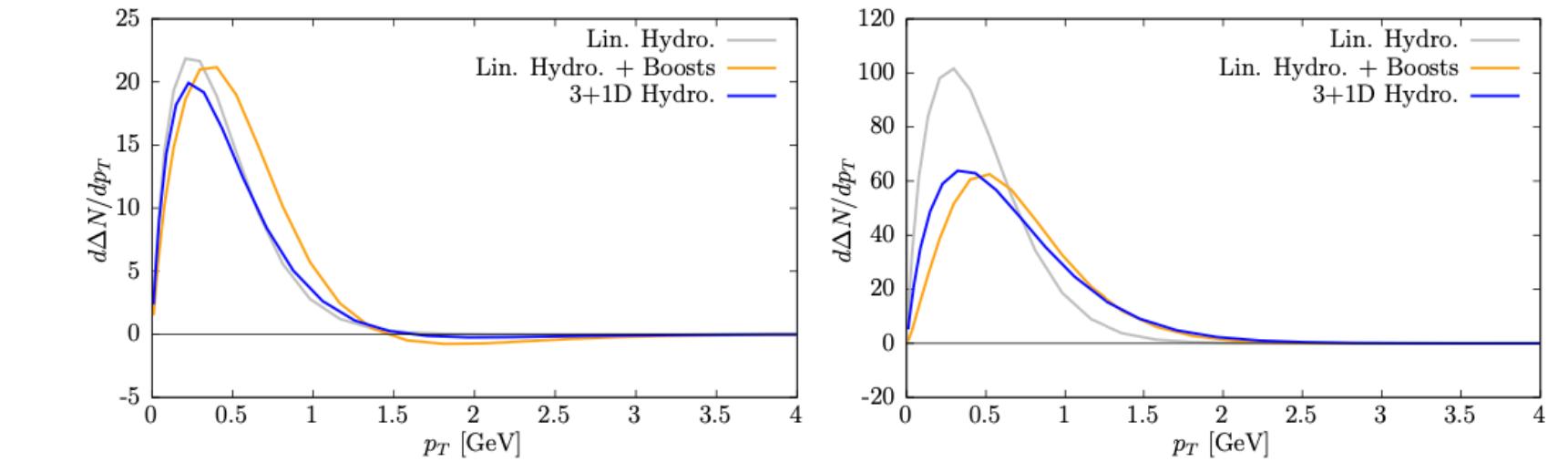


(f)  $y$  distribution for  $E_i = 10$  GeV.

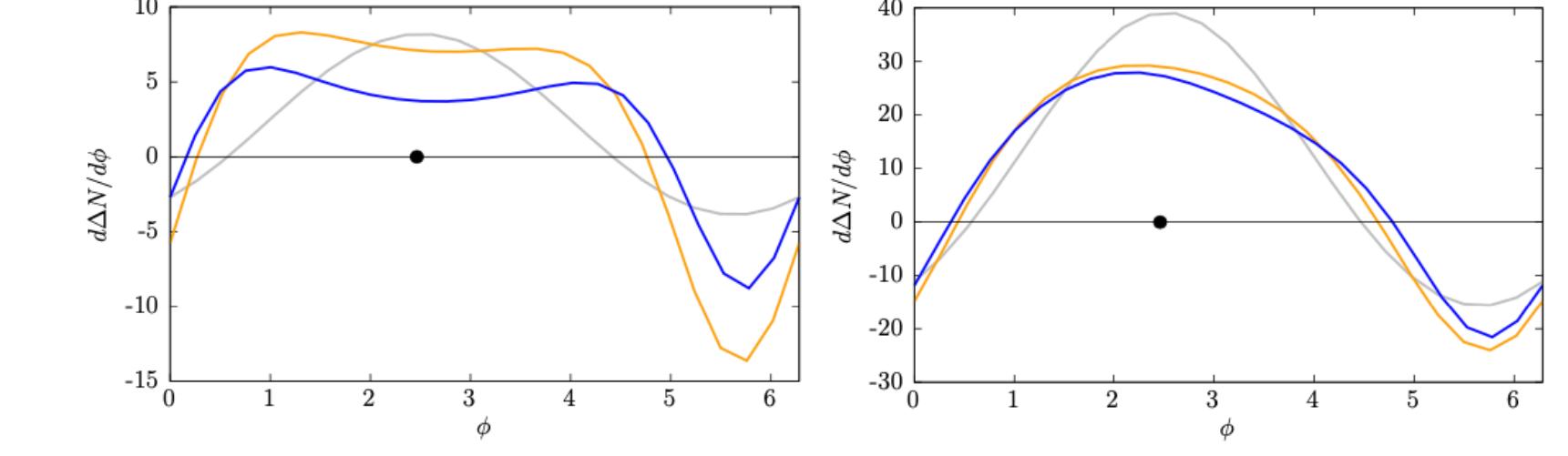
(g)  $y$  distribution for  $E_i = 50$  GeV.



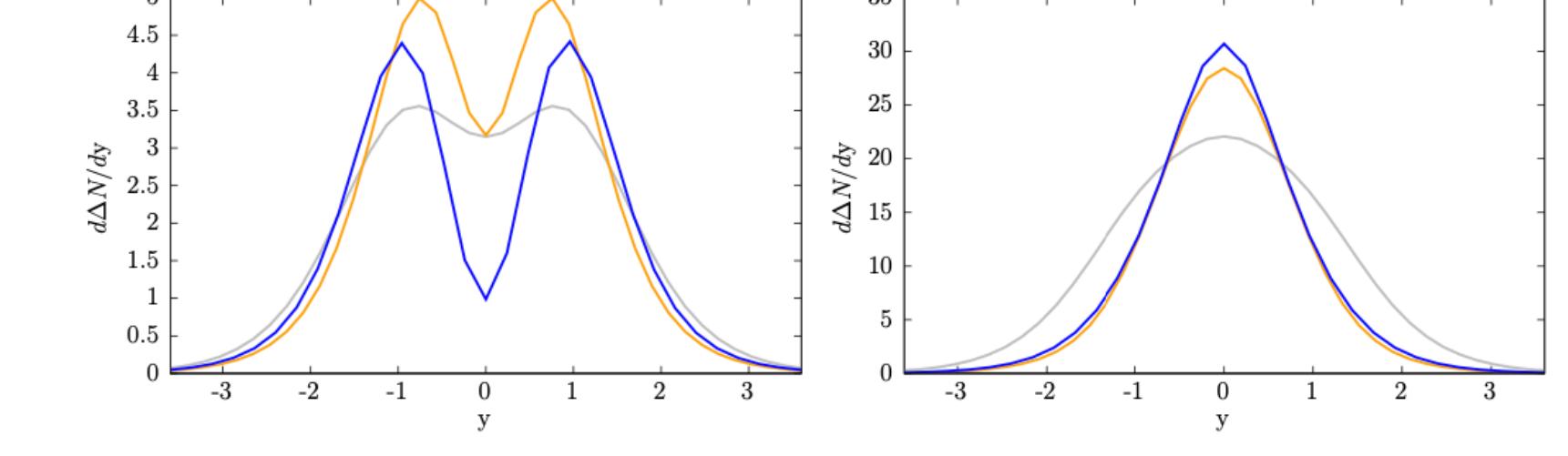
(a) Energy-momentum deposition for  $E_i = 10$  GeV (filled arrow) and  $E_i = 50$  GeV (empty arrow)



(b)  $p_T$  distribution for  $E_i = 10$  GeV.



(d)  $\phi$  distribution for  $E_i = 10$  GeV.



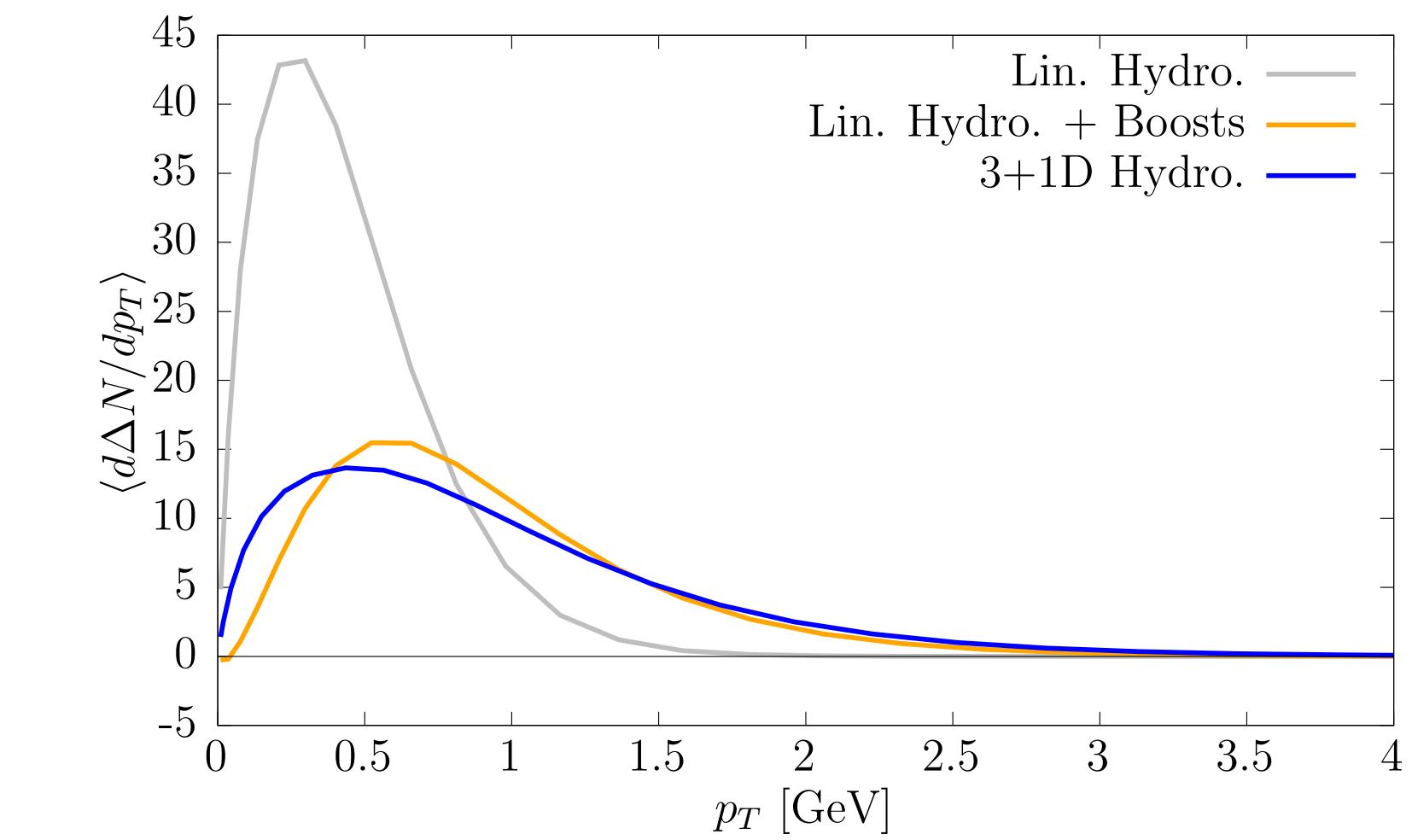
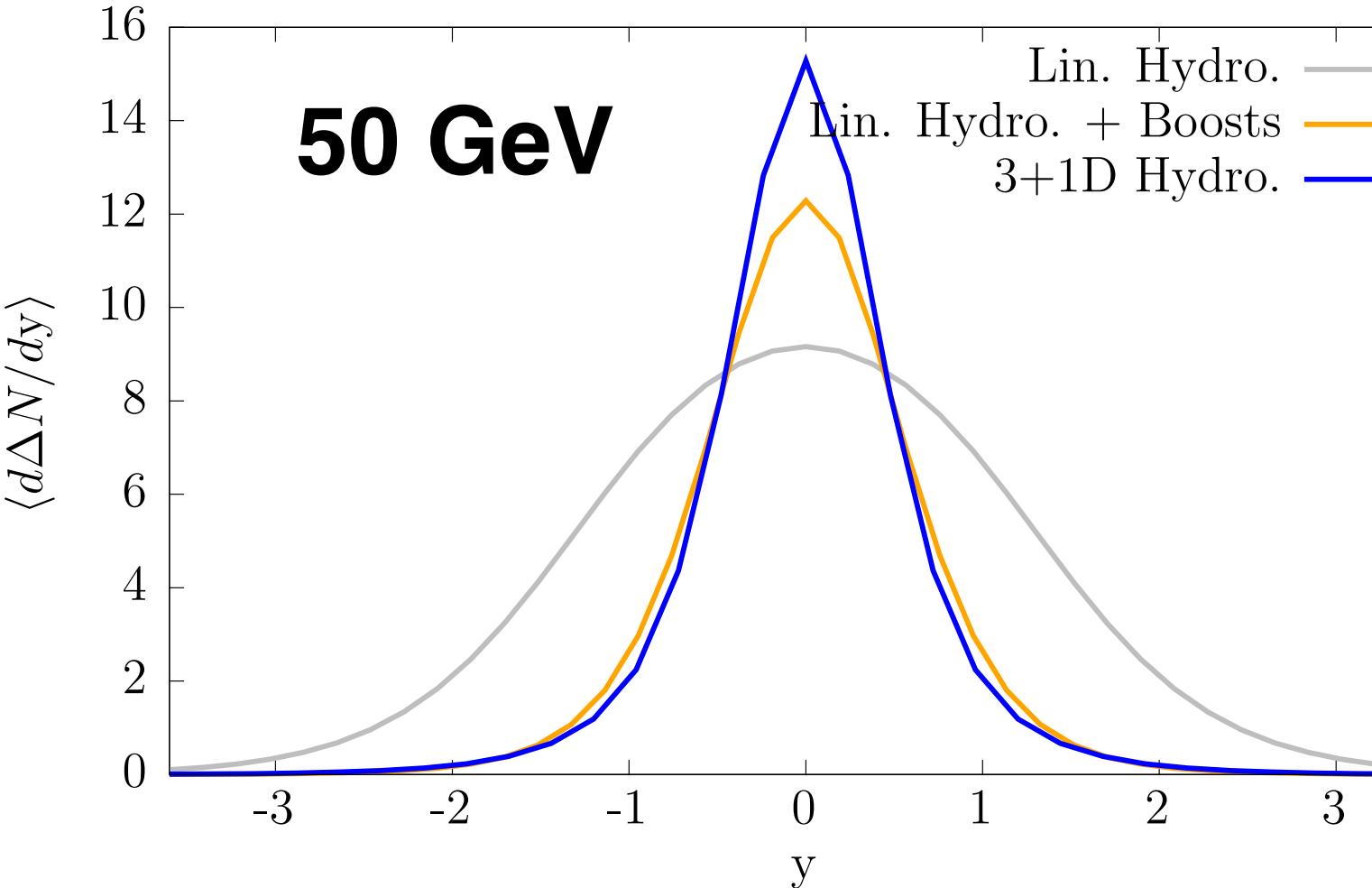
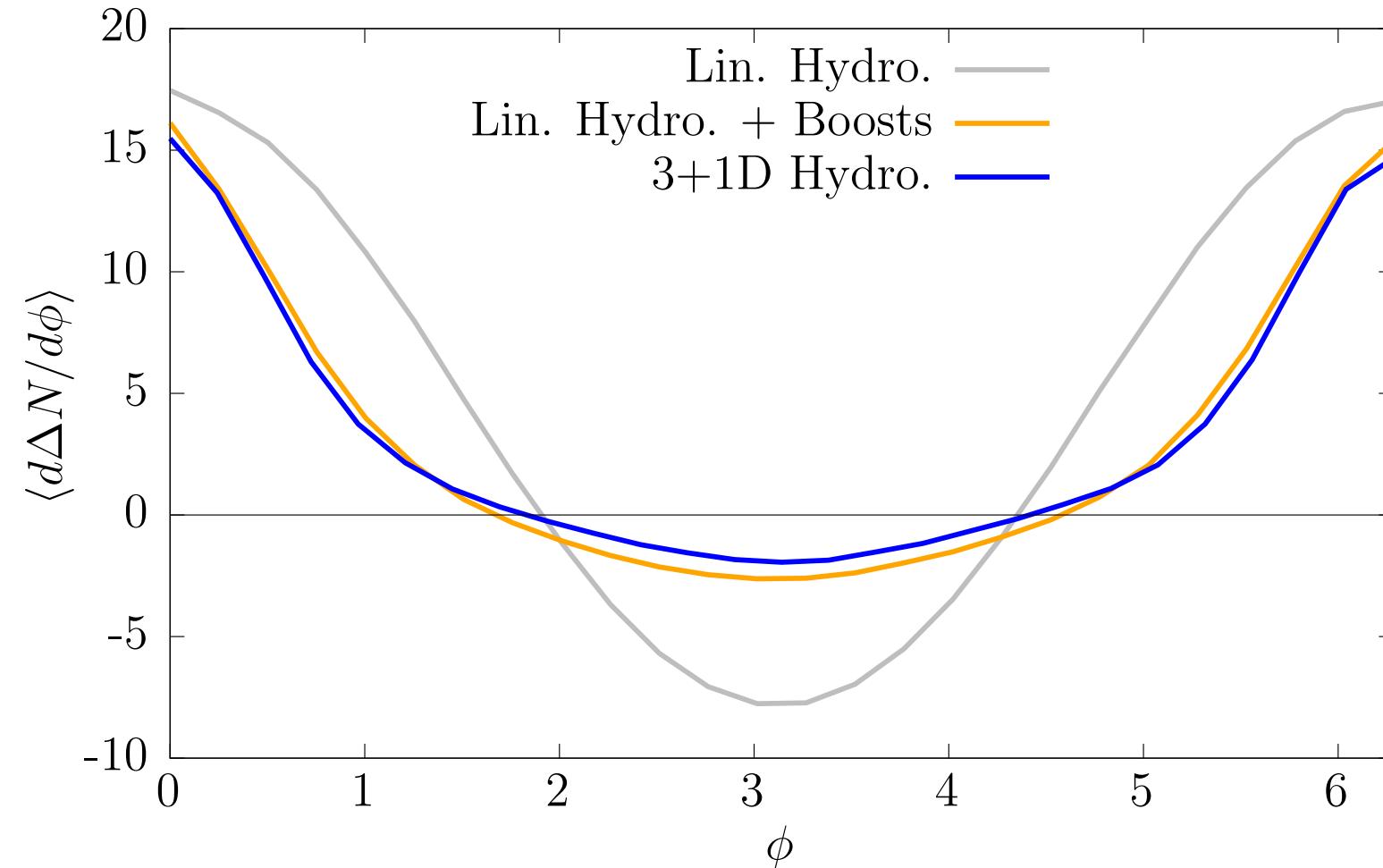
(f)  $y$  distribution for  $E_i = 10$  GeV.

(g)  $y$  distribution for  $E_i = 50$  GeV.

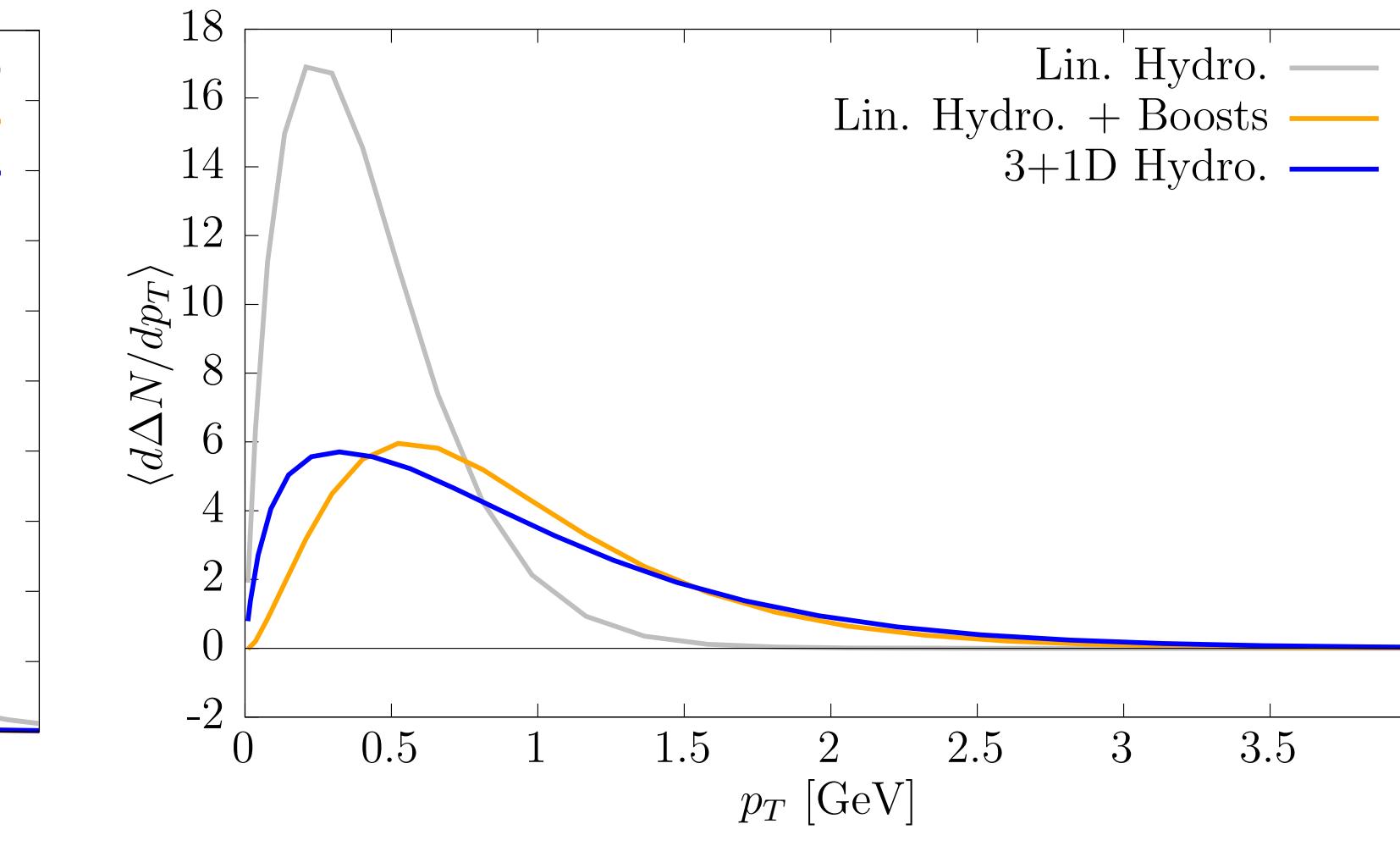
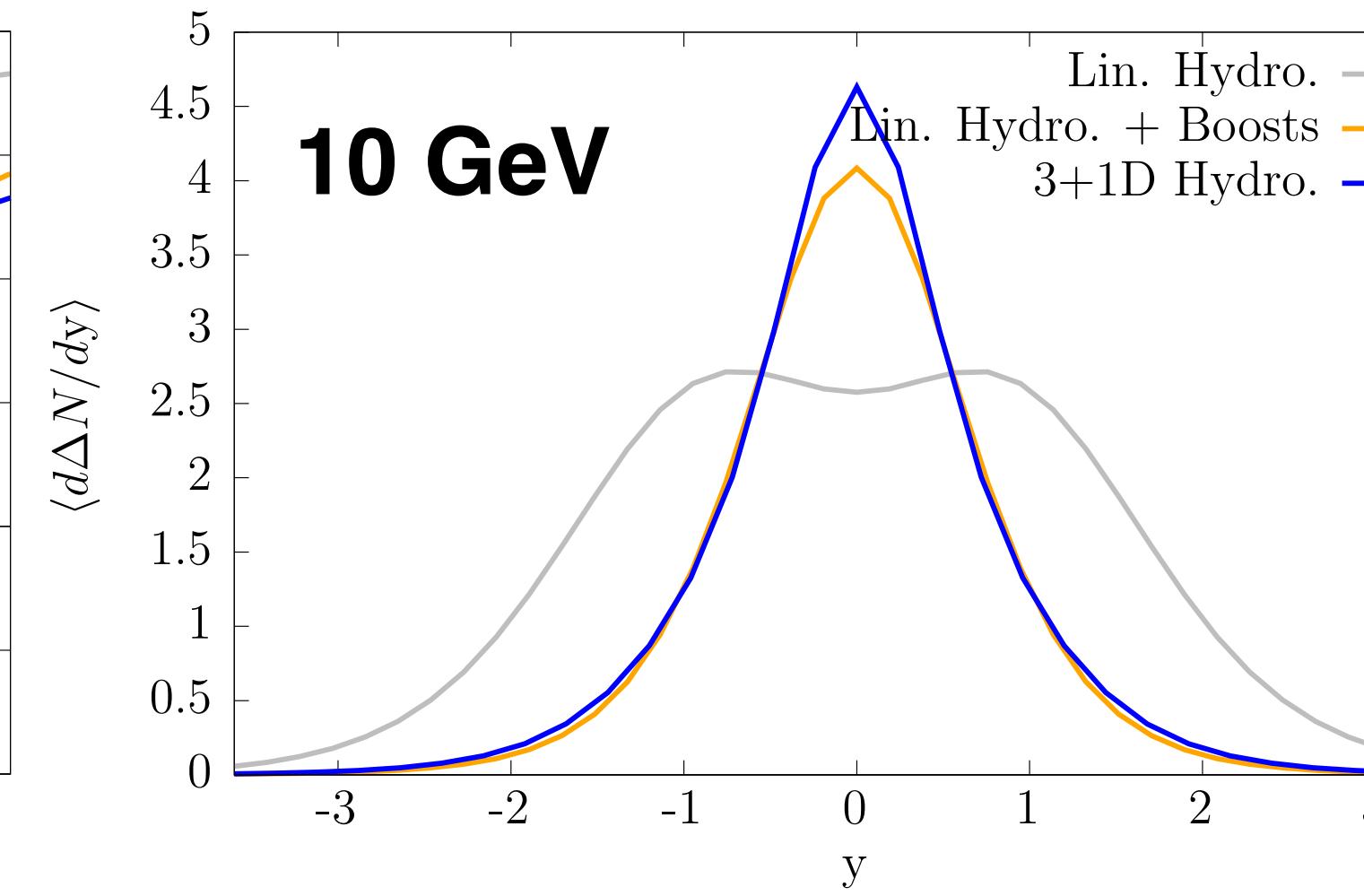
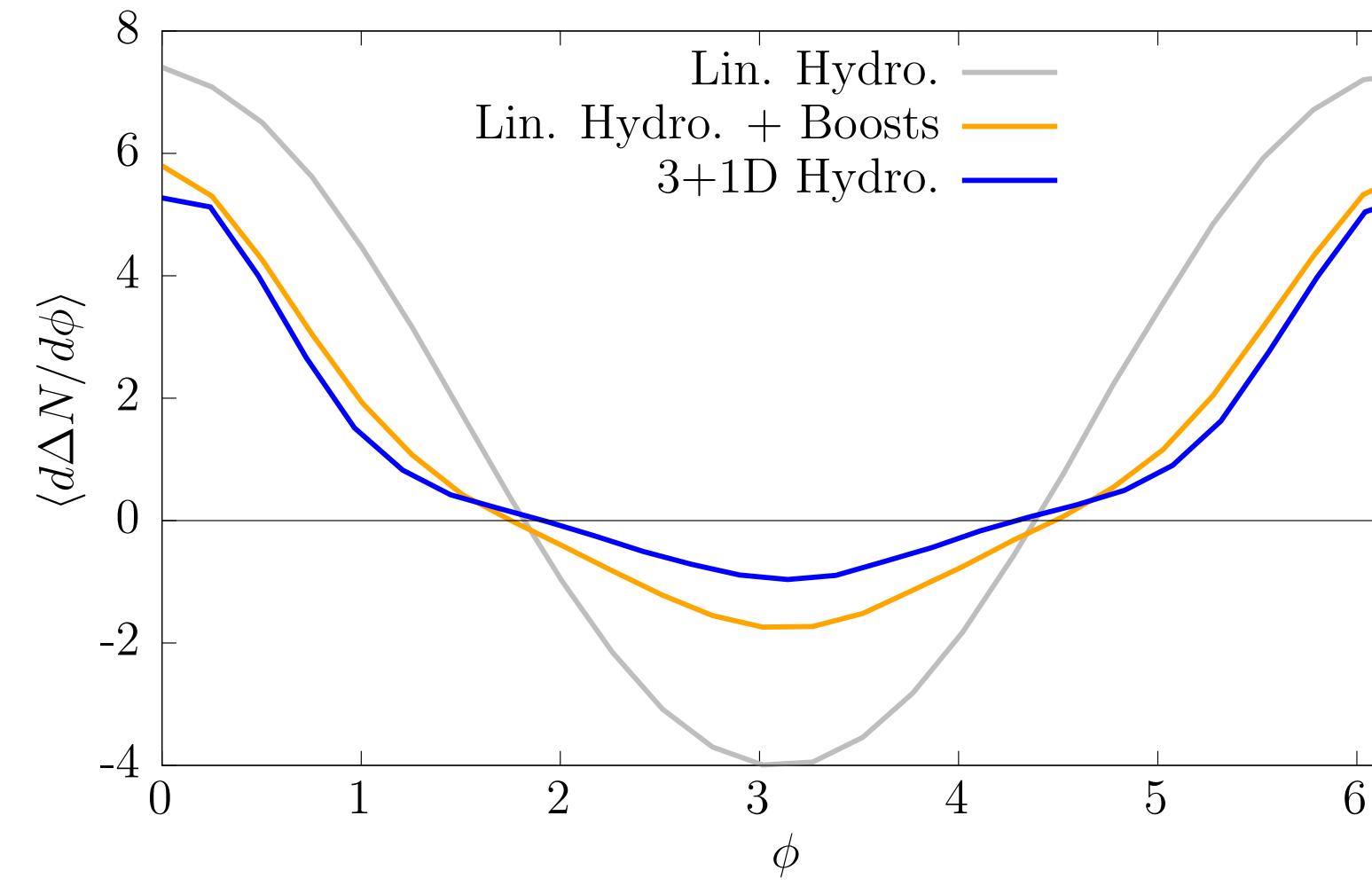
3+1D  
Viscous Hydrodynamics  
~ 2 hours

Superposition of  
Bjorken flow solutions  
+  
Trans., Rot., & Boosts  
~ 3 seconds

# Efficient Computations of the Wake



Average distribution ( $\sim 50$  events) becomes *harder* and more *collimated* than without transverse flow.



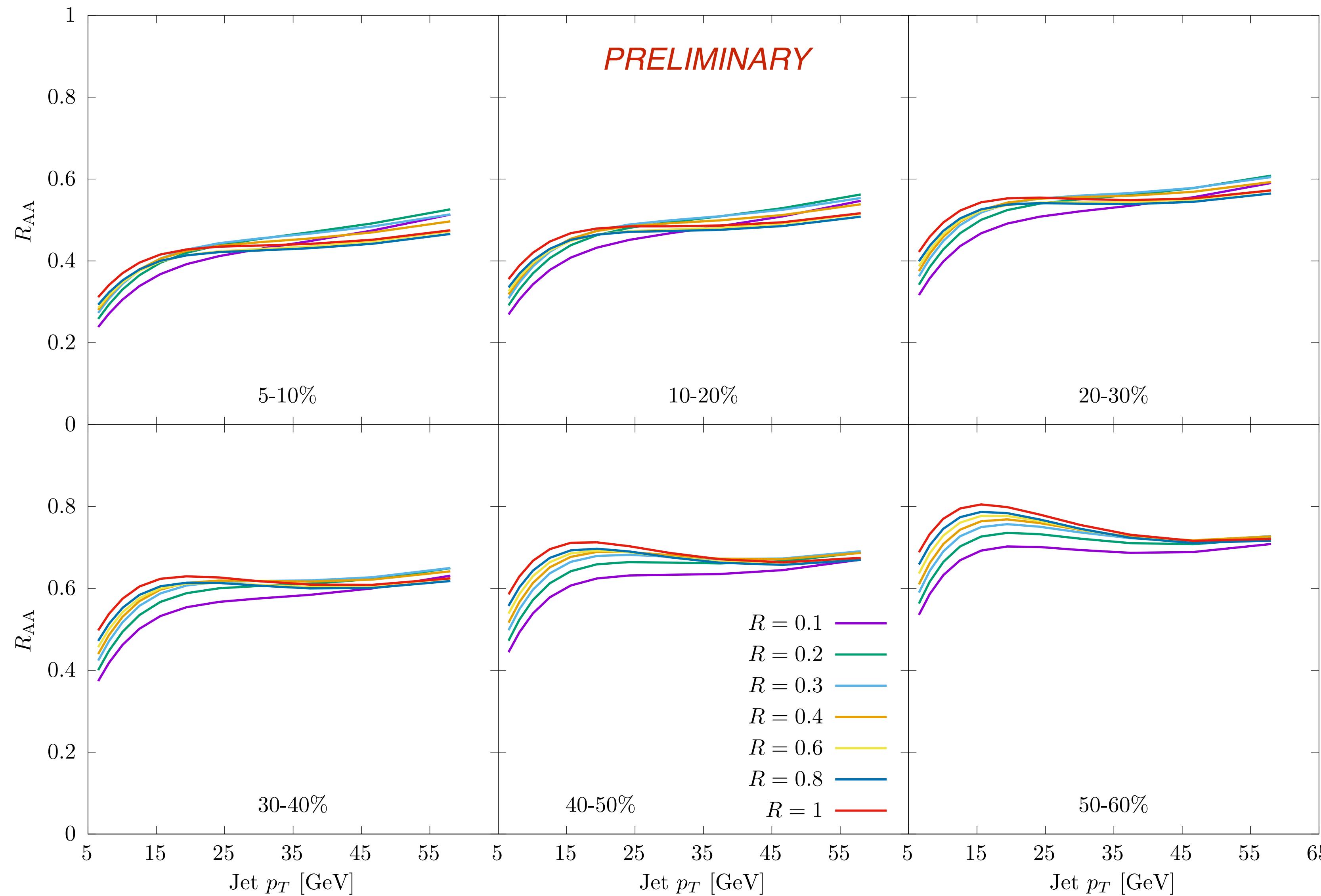
# Conclusions & Outlook

- Jet energy loss requires accounting for several effects:
  - Resummation of multi-particle energy loss inside R from high-virtuality shower.
  - Determination of the resolved phase-space. Improvable in pQCD.
  - Modelling of hydrodynamized component. Important for larger R.
- Cone-size dependence of jet suppression:
  - Competing effects between larger phase-space and energy recovery.
- Computationally efficient jet-by-jet, flow-dependent wakes:
  - Insights into devising tailored observables to reveal wake signatures.
  - Useful for massive medium response Monte Carlo simulations.
  - Can be supplemented as non-perturbative component to analytical computations.
- Recent impressive theory advances in many crucial ingredients:
  - Reliable predictions to confront experiments and test very exciting physics (color coherence, jet thermalization...).

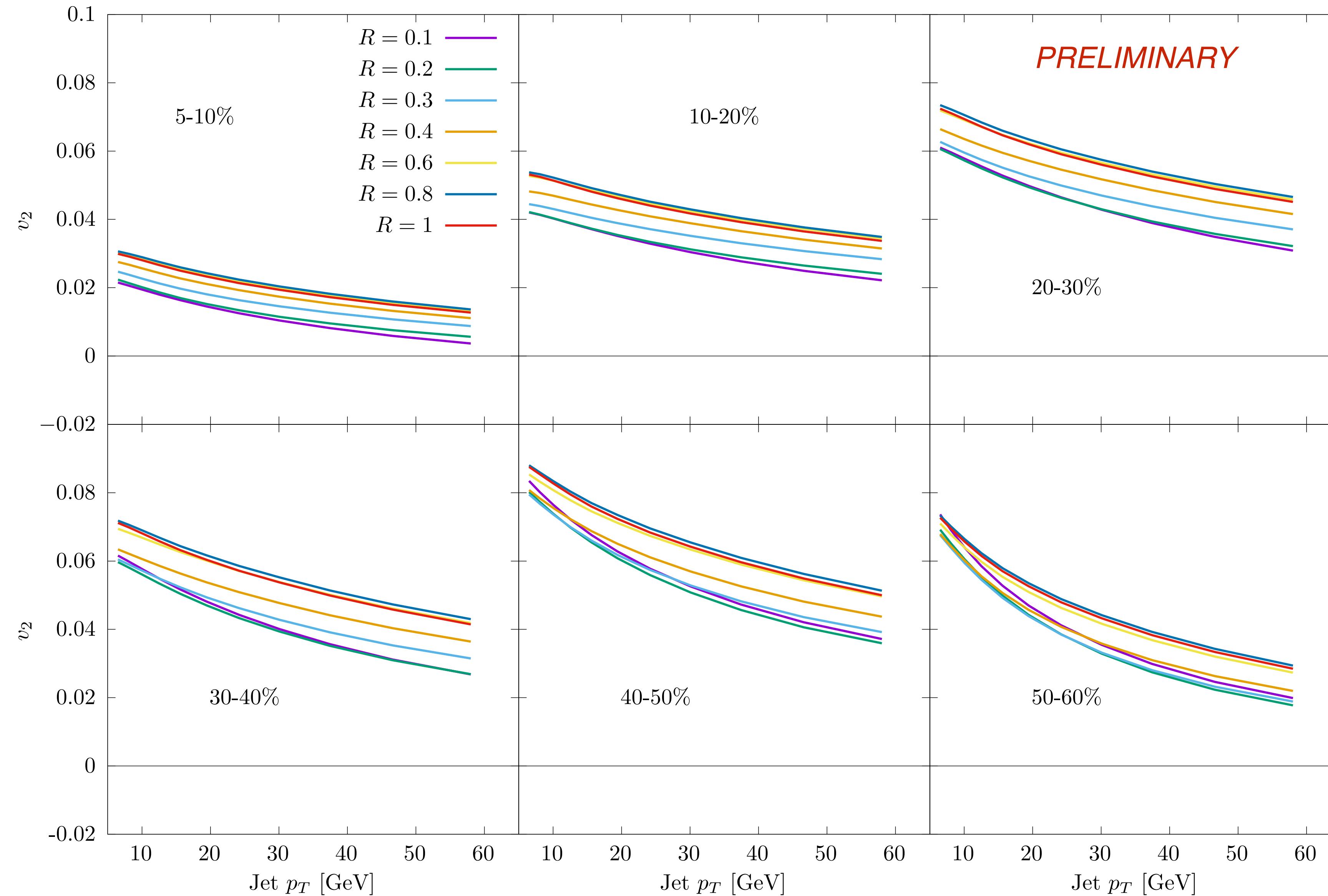
*Thanks for your  
attention!*

# Backup Slides

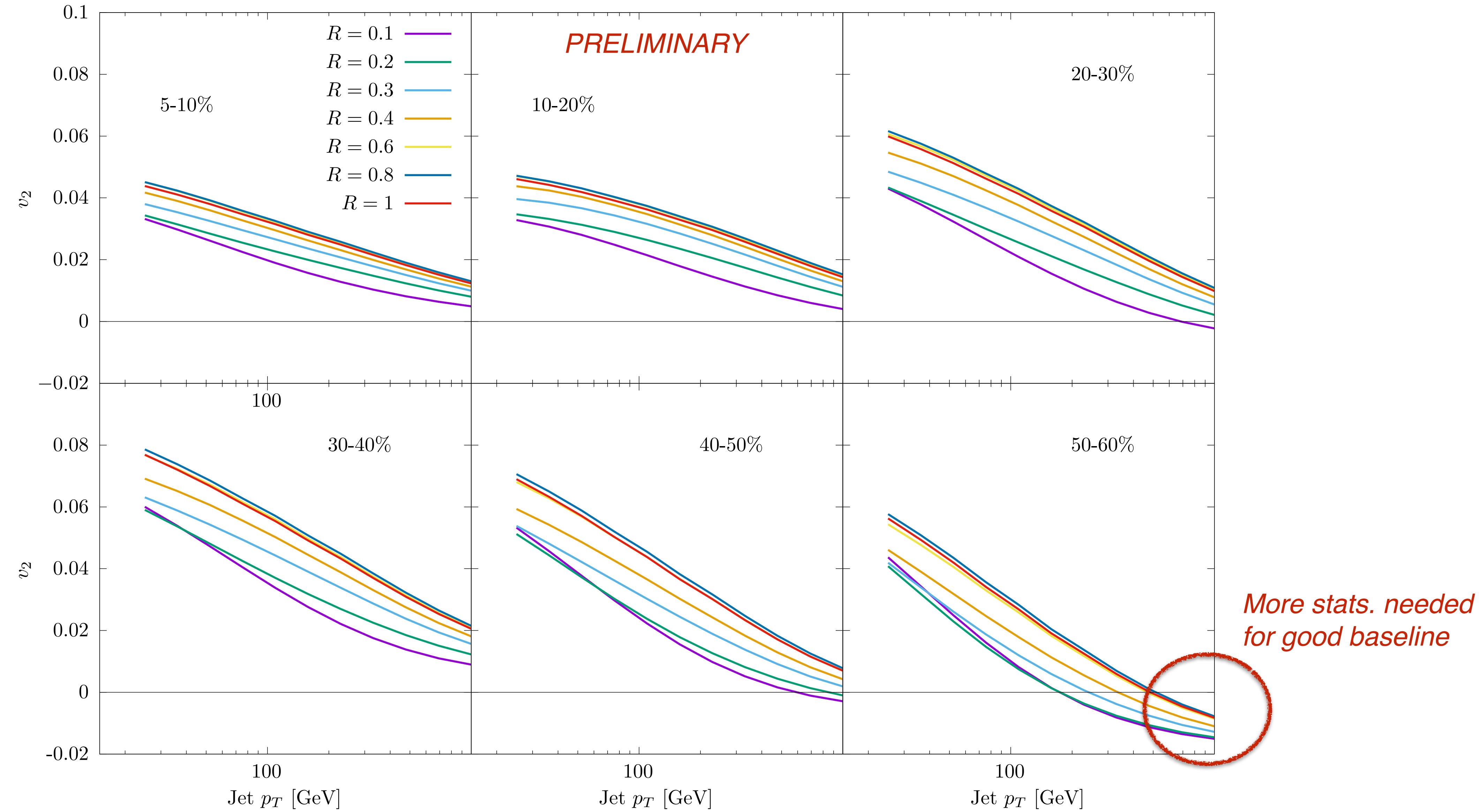
# Jet R<sub>AA</sub> at RHIC



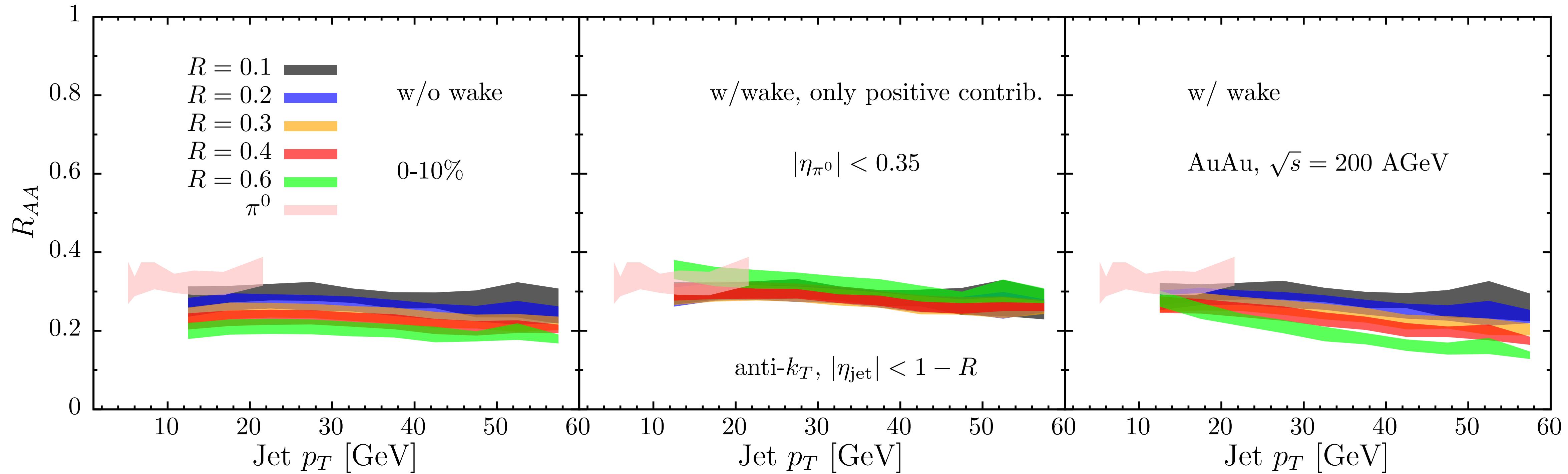
# Jet $v_2$ at RHIC



# Jet $v_2$ at LHC



# Jet R<sub>AA</sub> at RHIC with Hybrid Model



# Hybrid Strong/Weak Coupling Model

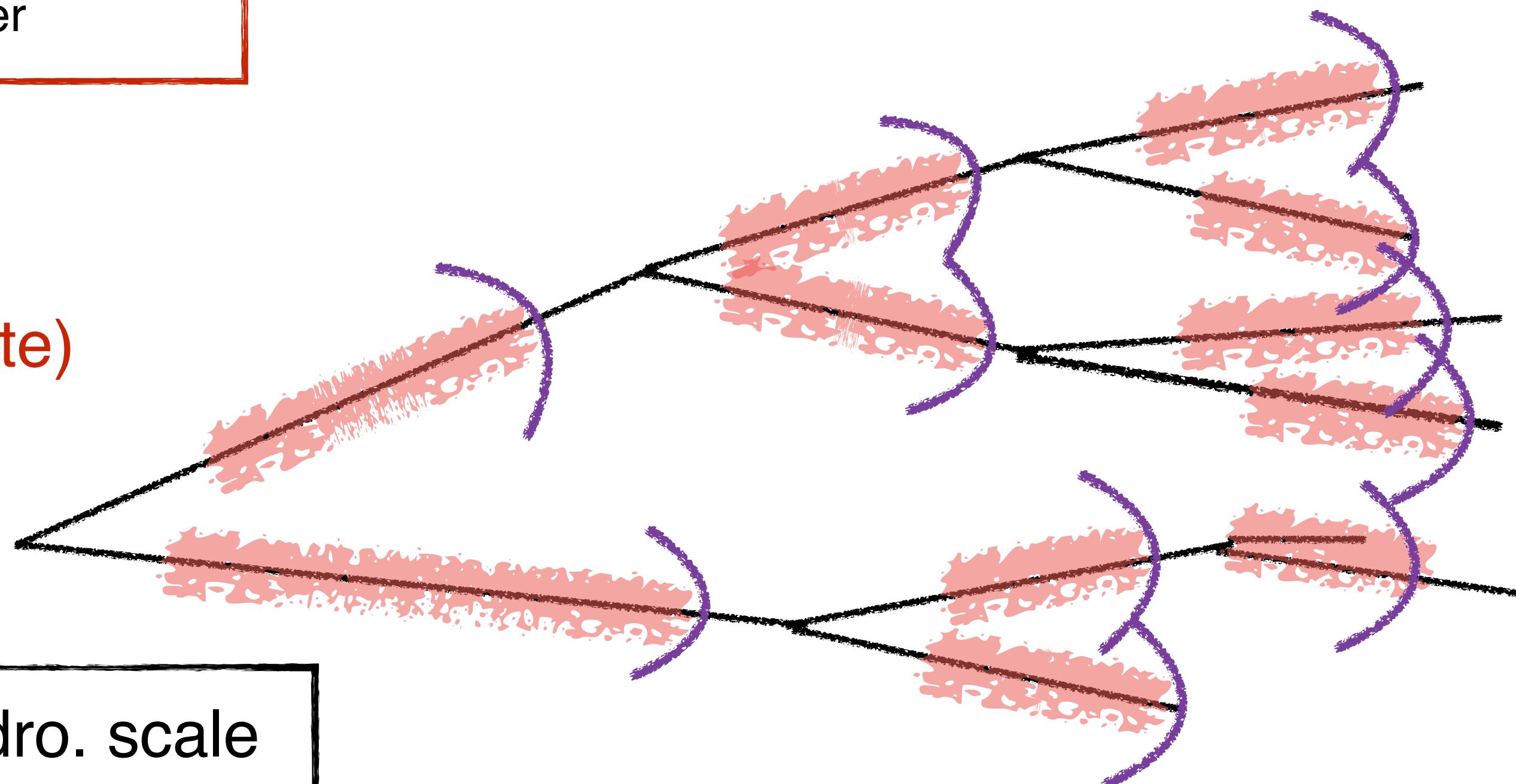
$$\frac{1}{E_{\text{in}}} \frac{dE}{dx} = -\frac{4}{\pi} \frac{x^2}{x_{\text{stop}}^2} \frac{1}{\sqrt{x_{\text{stop}}^2 - x^2}}$$
$$x_{\text{stop}} = \frac{1}{2} \frac{E_{\text{in}}^{1/3}}{\kappa_{\text{sc}} T^{4/3}}$$

$\mathcal{O}(1)$  free parameter

Strongly coupled  
energy loss  
(hydrodynamization rate)

$$E \frac{d\Delta N}{d^3p} = \frac{1}{32\pi} \frac{m_T}{T^5} \cosh(y - y_j) \exp \left[ -\frac{m_T}{T} \cosh(y - y_j) \right]$$
$$\left\{ p_T \Delta P_T \cos(\phi - \phi_j) + \frac{1}{3} m_T \Delta M_T \cosh(y - y_j) \right\}$$

Hadrons from the hydro.  
wake (medium response)



PYTHIA8 down to hadro. scale  
(formation time argument  
for spacetime picture)

Casalderrey-Solana, Gulhan,  
Milhano, DP,  
Rajagopal JHEP '15, '16, '17

# Accounting for Radial Flow

Casalderrey, Milhano, DP, Rajagopal, Yao - JHEP '21

- Modified spectra after Cooper-Frye:

$$\frac{d\Delta N}{p_T dp_T d\phi dy} = \frac{1}{(2\pi)^3} \int \tau dx dy d\eta_s m_T \cosh(y - \eta_s) \left[ f\left(\frac{u^\mu p_\mu}{T_f + \delta T}\right) - f\left(\frac{u_0^\mu p_\mu}{T_f}\right) \right]$$

With wake       No wake 

- Ansatz to account for radial flow:

→ Boost particle momentum to frame in which fluid cell moves with  $u^\mu = \gamma_\perp (\cosh \eta, v_x, v_y, \sinh \eta)$ .

$$p_{\text{cell}}^\mu = \Lambda^\mu_\nu(\mathbf{v}_{\text{cell}}) p_{\text{lab}}^\nu$$

Use flow profile at freeze out  
from 2+1D hydro. simulations  
such as VISHNU.

