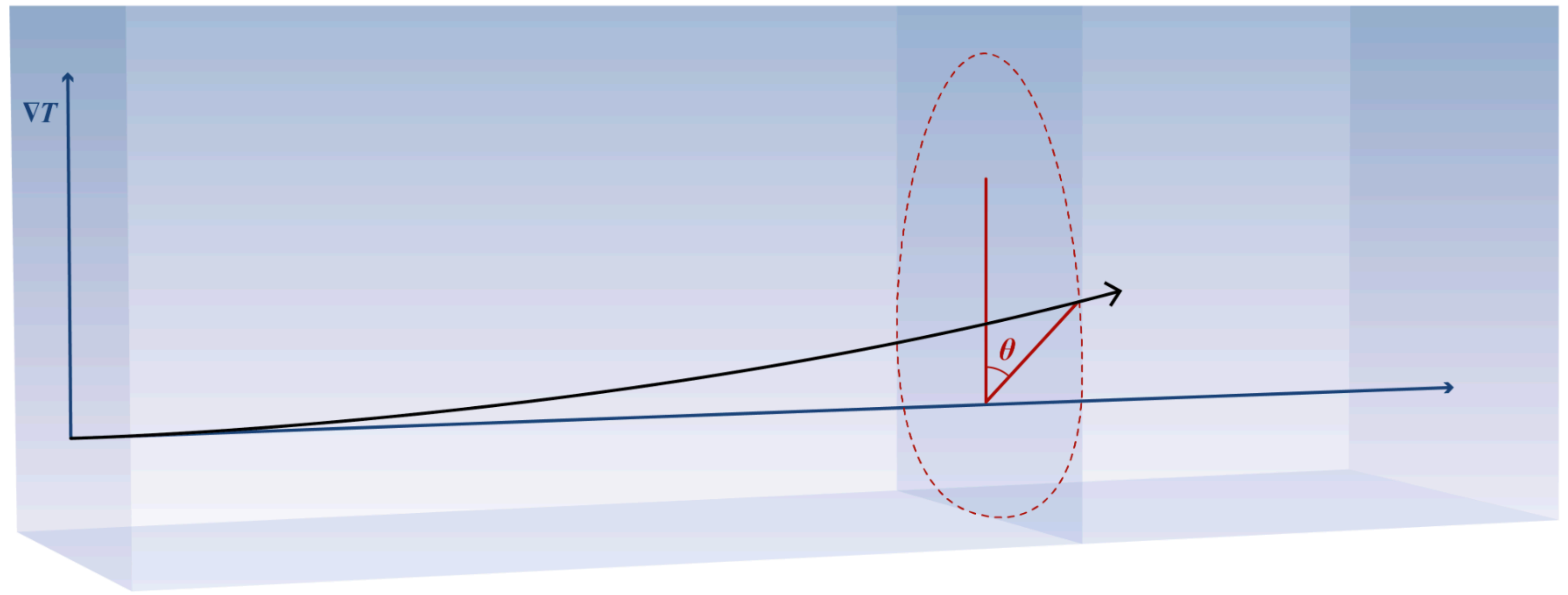


Jet evolution in inhomogeneous media

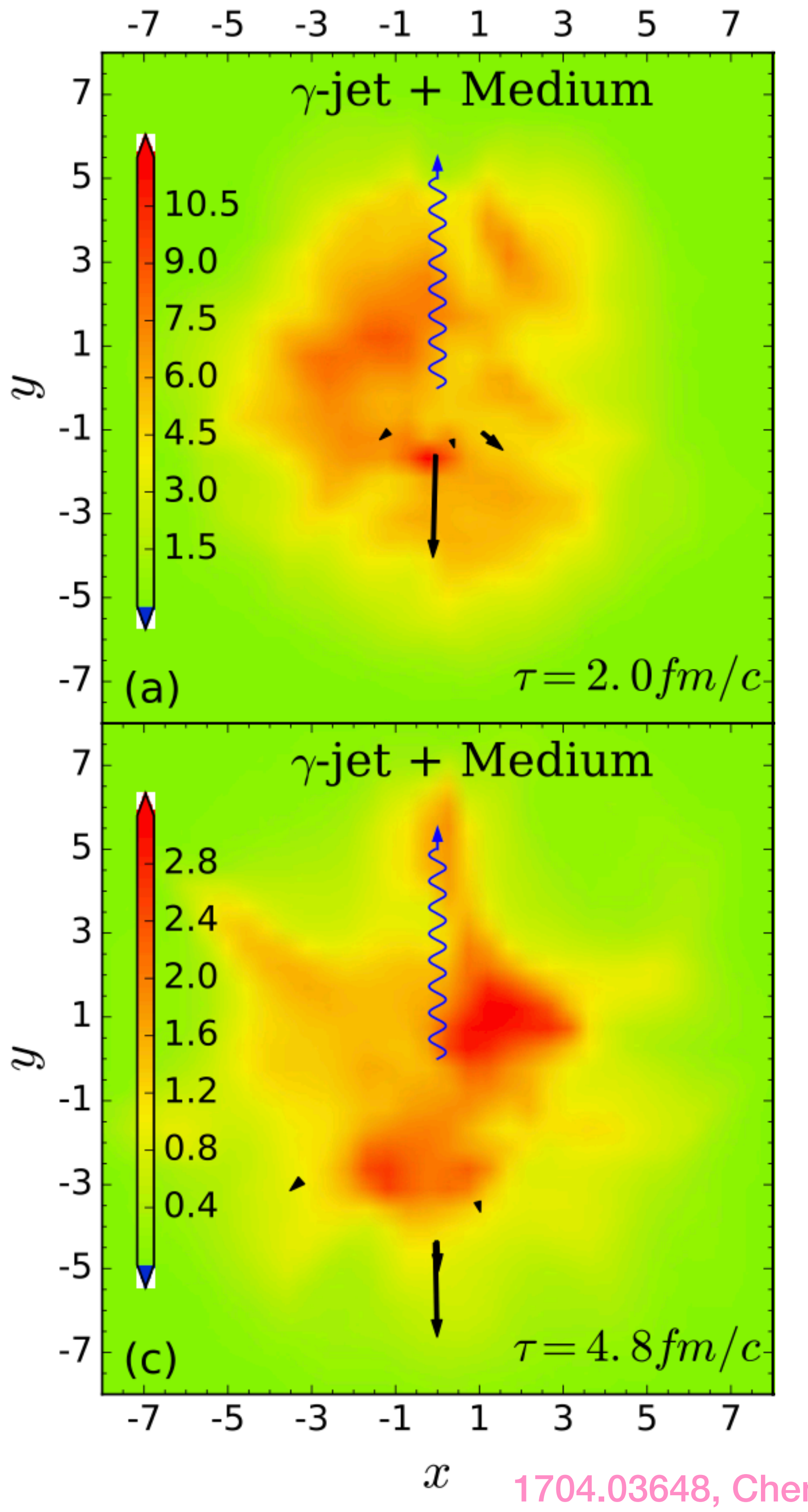
15th June 2022, Trento

João Barata, BNL

Mainly based on work done with Xoan Mayo, Andrey Sadofyev and Carlos Salgado

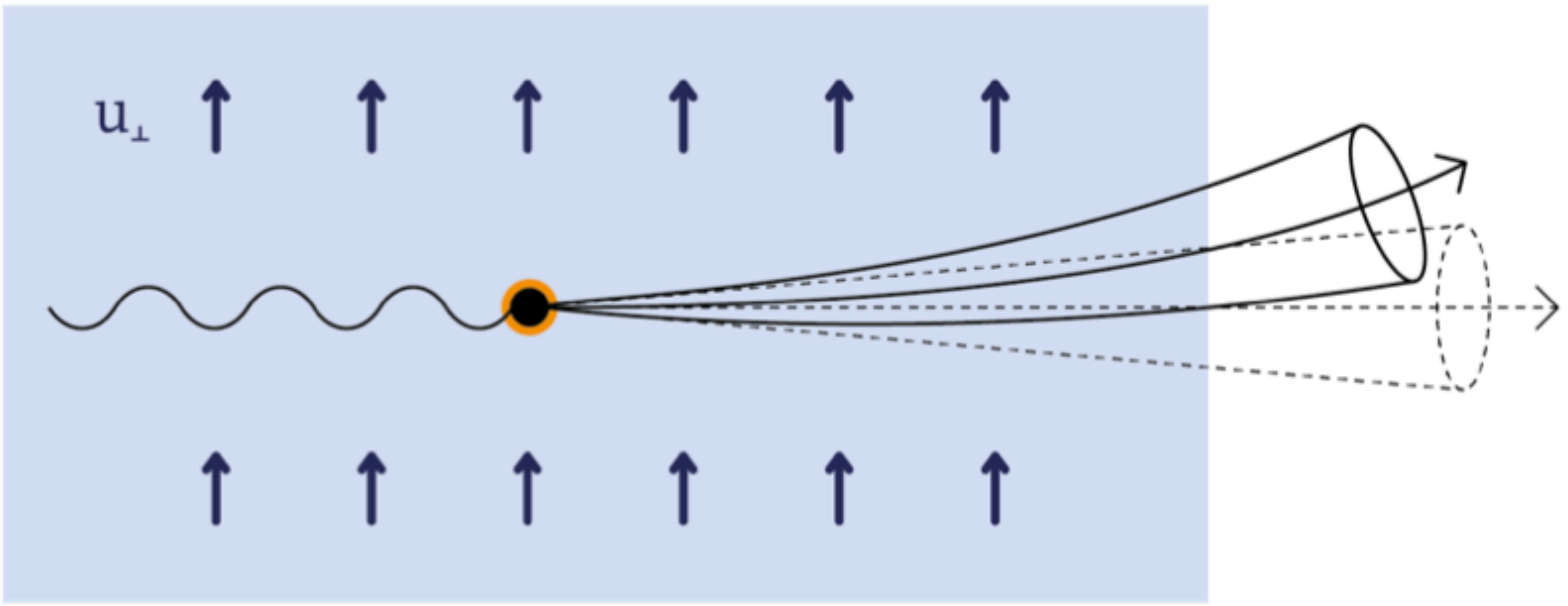
online

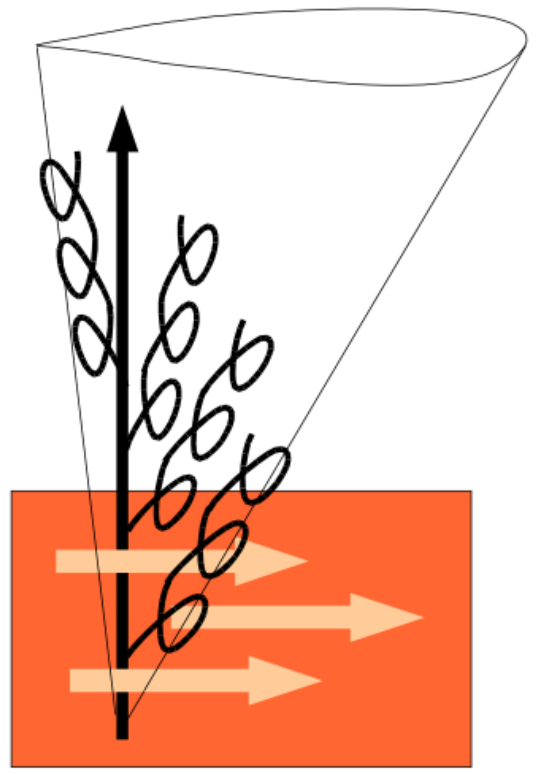
Jets in hot plasmas



1704.03648, Chen et al

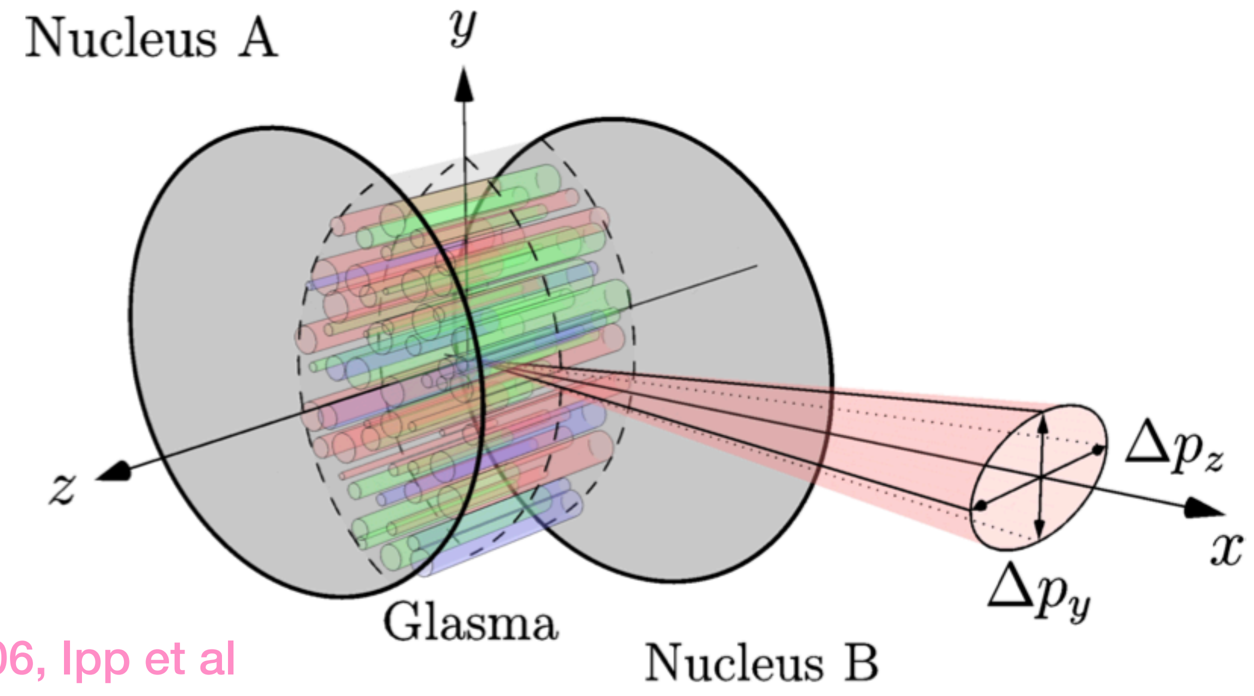
2104.09513, A. Sadofyev et al





hep-ph/0405301, Armesto et al

See talks on Friday (S. Hauksson, A. Czajka)



2009.14206, Ipp et al

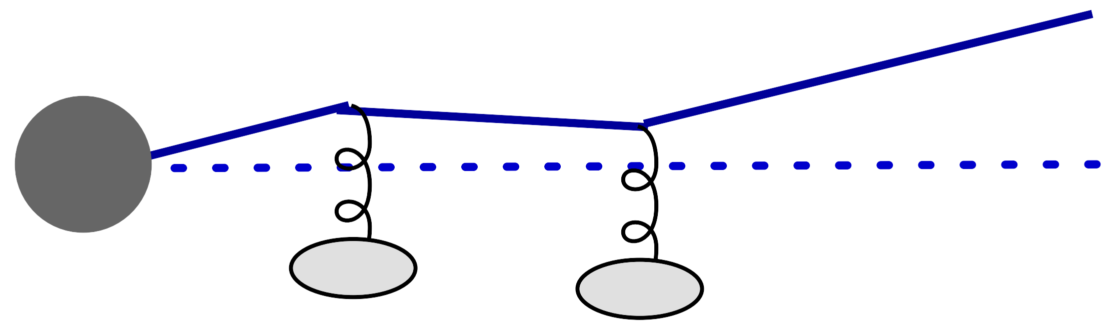
Medium response to jets

Jet response to medium structure

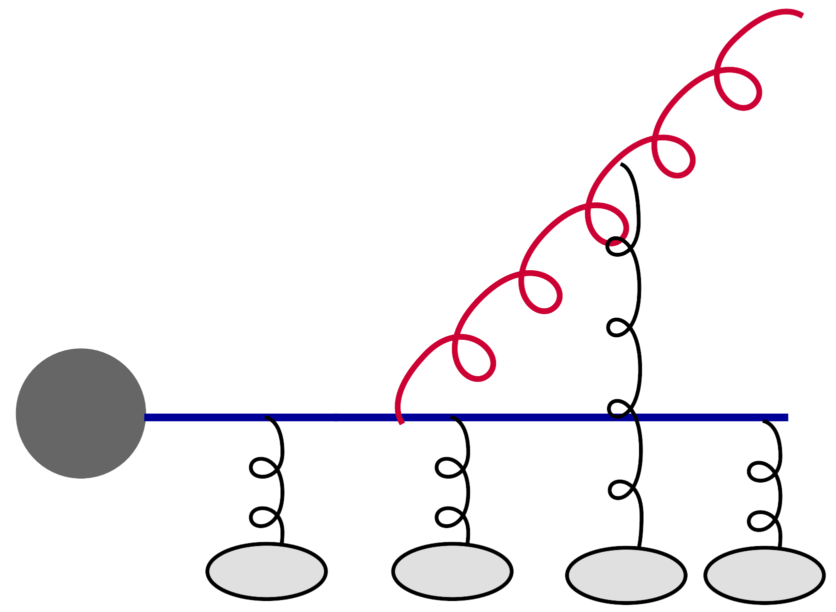
Jets in hot plasmas

Understanding jets' modifications is hard: **major focus on observables up to $\mathcal{O}(\alpha_s(Q_\perp \gg T))$**

Single particle broadening $\mathcal{O}(\alpha_s^0)$ today



Medium induced soft gluon radiation $\mathcal{O}(\alpha_s)$ today



All theoretical approaches to jet quenching require many strong assumptions. Some are:

- Eikonal expansion; keep only some sub-eikonal length enhanced terms
- Background is typically described by a model
- In the simplest scenario the medium is static, homogeneous (today) and infinitely long

✓ Problem becomes tractable

✗ Jets decouple from the plasma

Jets in hot plasmas

How are large can these effects be?

In the dilute regime, we can look at the leading moments 2104.09513, A. Sadofyev, M. Sievert, I. Vitev

sub-eikonal vs **enhancing medium factor**

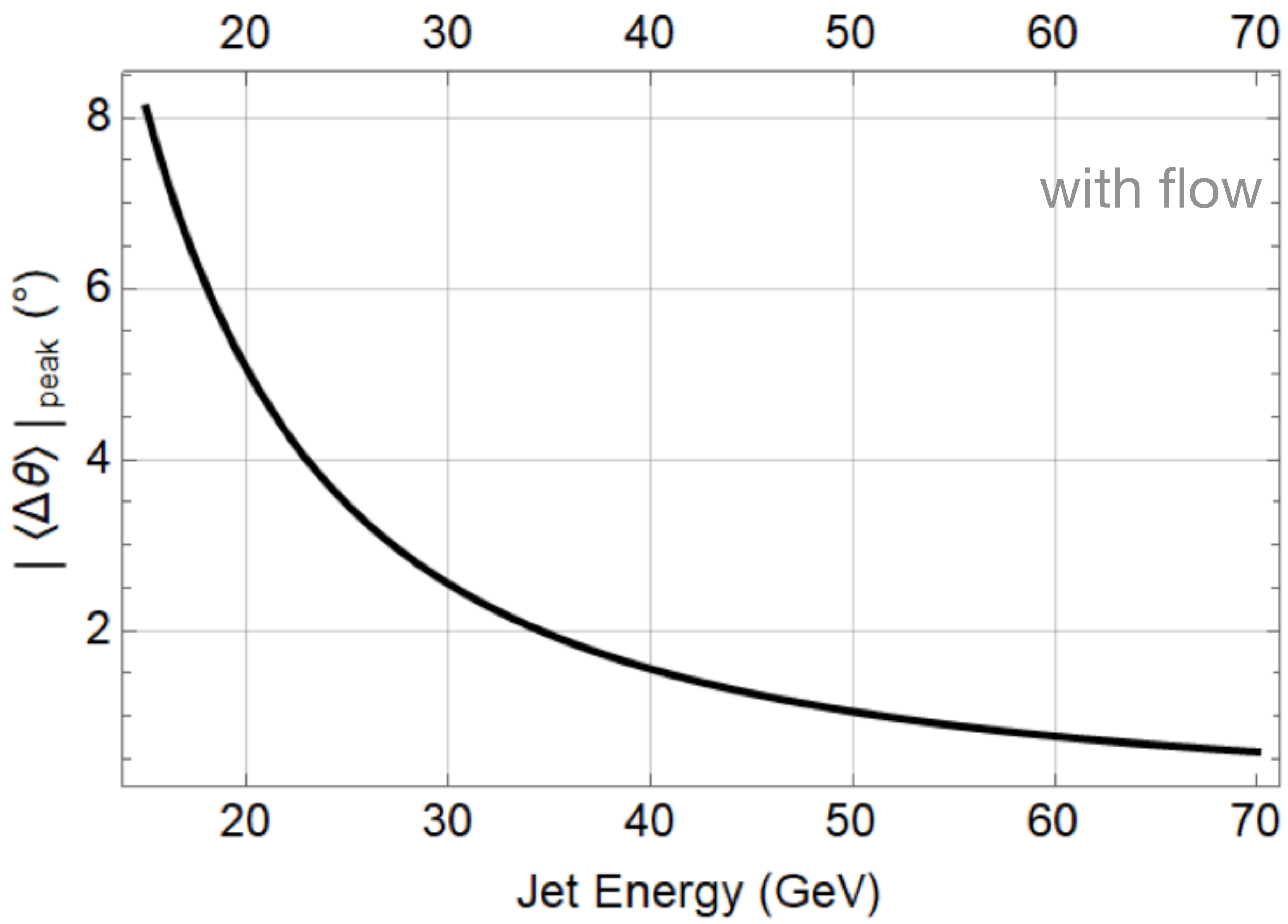
with flow

$$\langle \mathbf{p} \rangle_{\mathbf{u}\neq 0, \nabla T = 0} \propto \frac{u_\perp}{1 - u_z} \frac{\mu^2 L}{E\lambda}$$

with gradients

$$\langle \mathbf{p}\, \mathbf{p}^2 \rangle_{\mathbf{u}=0, \nabla T \neq 0} \propto \left(\frac{\nabla T}{T} L \right) \frac{\mu^2 L}{E\lambda}$$

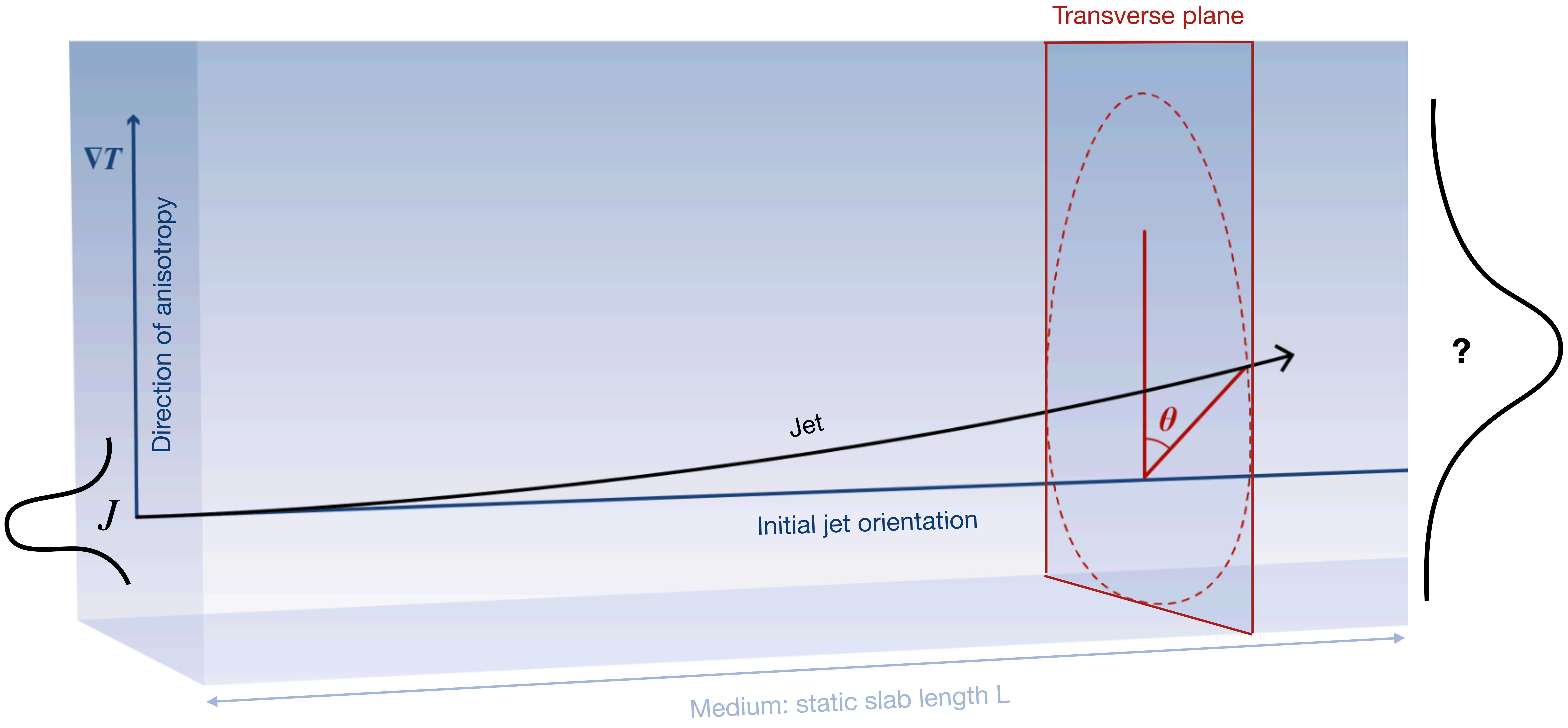
2110.03590, L. Antiporda et al



Outline

1. Momentum broadening in dense anisotropic media
2. Radiative energy loss in dense anisotropic media
3. Conclusion

Momentum broadening in inhomogeneous media



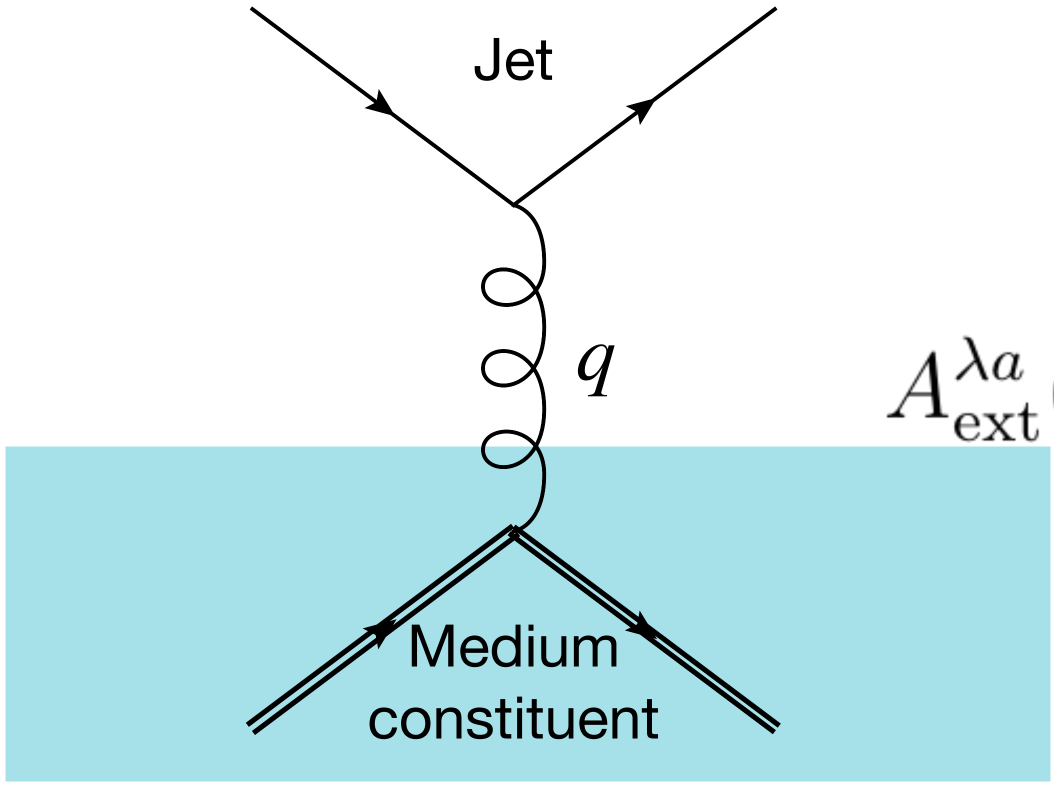
Momentum broadening in inhomogeneous media

More details in for example:

2104.09513, A. Sadofyev, M. Sievert, I. Vitev

1807.03799, M. Sievert, I. Vitev

The medium is described by a classical field



Model dependent elastic scattering potential for source j

No energy transfer in each scattering: transverse t-channel gluon exchanges only

$$gA^{\lambda a}_{\text{ext}}(q) = -(2\pi)\, g^{\lambda 0} \sum_j e^{-i(\boldsymbol{q}\cdot\boldsymbol{x}_j + q_z z_j)}\, t^a_j\, v_j(q)\, \delta\left(q^0\right)$$

where we use the GW model

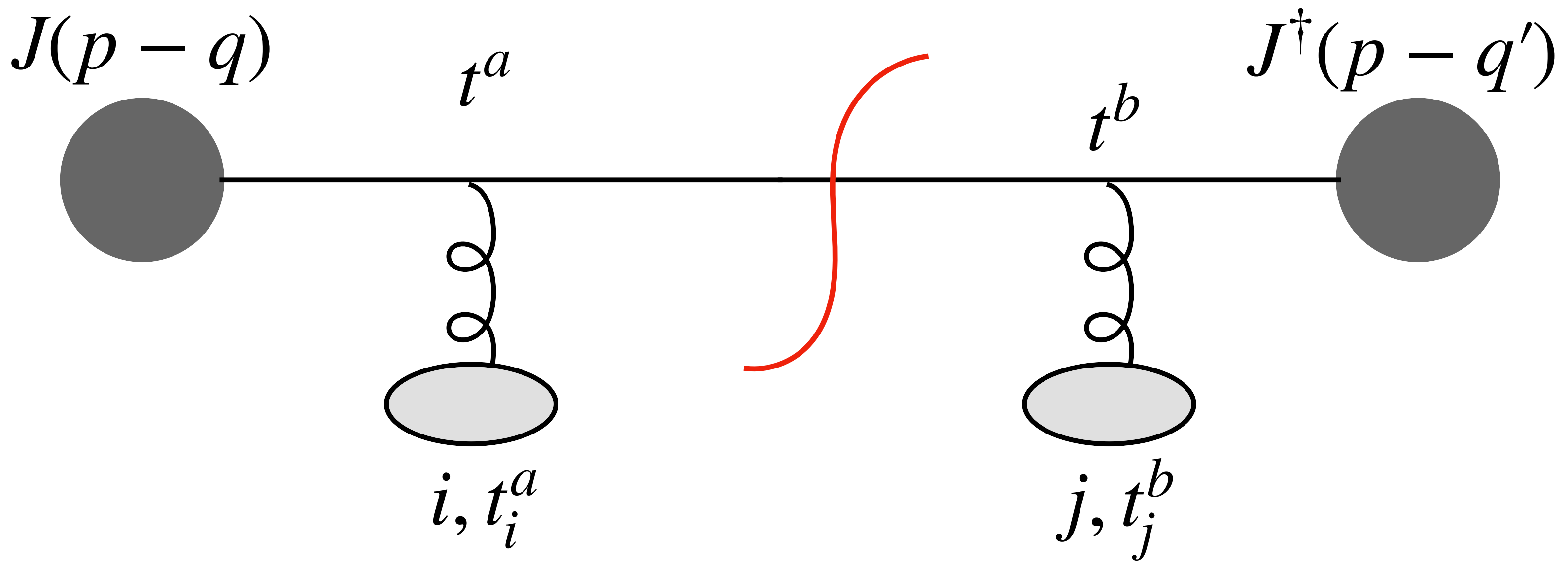
$$v_i(q) \equiv \frac{-g^2}{-q_0^2 + \boldsymbol{q}^2 + q_z^2 + \mu_i^2 - i\epsilon}$$

Medium statistics follow from 2-gluon approximation

$$\langle t^a_i t^b_j \rangle = \frac{1}{d_{\text{tgt}}} \text{tr}\left(t^a_i t^b_j\right) = \frac{1}{2C_{\bar{R}}} \delta_{ij}\delta^{ab}$$

Only non-trivial correlator

Probe interacts with the same scattering center in amplitude and conjugate amplitude



Momentum broadening in homogeneous media

More details in for example:

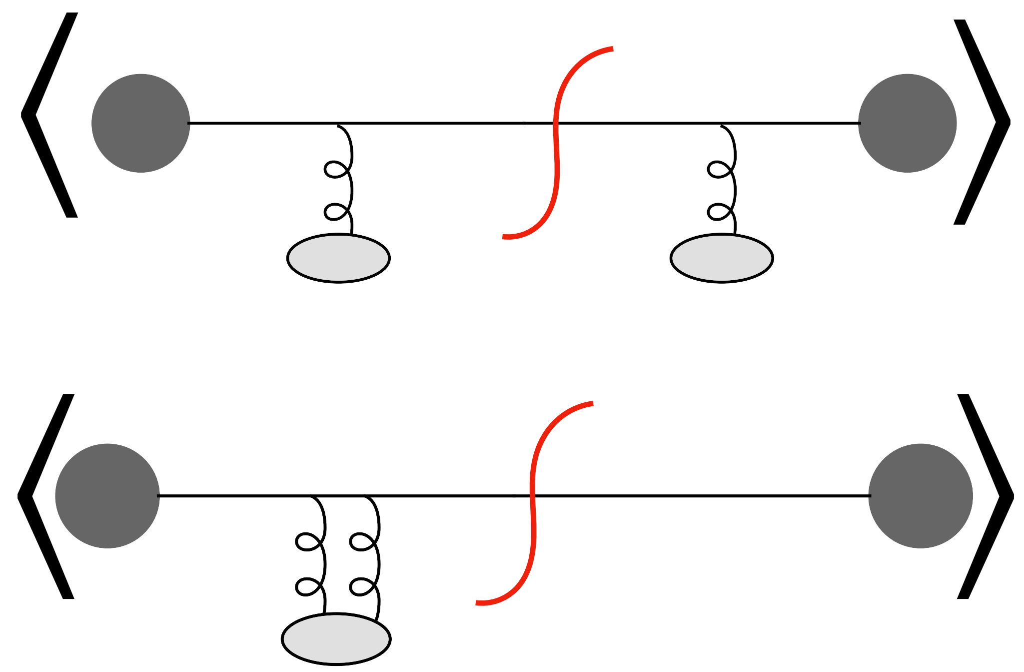
nucl-th/9306003, M. Gyulassy, X.-N. Wang

$\boxed{1}$ Compute all diagrams up to $2N$ field insertions

$$Q_n \equiv \frac{\boldsymbol{p}_n^2 - \boldsymbol{p}_f^2}{2E} \qquad p_n = p_f - \sum_{m=n}^{N} q_m ,\; p_{in} = p_1$$

For example, the diagram with $r = N$ insertions at distinct i_n reads

$$iM_r = \prod_{n=1}^{r} \left[\sum_{i_n} \int \frac{d^2 \boldsymbol{q}_n}{(2\pi)^2} \, i t^a_{\text{proj}} t^a_{i_n} \, \theta_{i_n, i_{n-1}} \, e^{-i \boldsymbol{q}_n \cdot \boldsymbol{x}_{i_n}} \underbrace{e^{-i Q_n \left(z_{i_n} - z_{i_{n-1}} \right)}}_{\text{LPM phase factor}} v_{i_n}(q_n) \right] J\left(p_{in}\right)$$



$\boxed{2}$ For each N, square and average the respective diagrams

$$\langle |M|^2 \rangle = \underbrace{\langle |M_0|^2 \rangle}_{N=0} + \underbrace{\langle |M_1|^2 \rangle + \langle M_2 M_0^* \rangle + \langle M_0 M_2^* \rangle}_{N=1} + \underbrace{\langle |M_2|^2 \rangle + \langle M_3 M_1^* \rangle + \langle M_1 M_3^* \rangle + \langle M_4 M_0^* \rangle + \langle M_0 M_4^* \rangle}_{N=2} + \ldots$$

The averaging is performed by taking the limit of continuous distribution in the medium

$$\sum_i f_i = \int d^2 \boldsymbol{x} \, dz \, \underbrace{\rho(\boldsymbol{x}, z)}_{\text{Density of scattering centers}} f(\boldsymbol{x}, z) \xrightarrow[\nabla T = 0]{\rho(\cancel{\boldsymbol{x}}, z) \qquad \mu^2(\cancel{\boldsymbol{x}}, z)} \int d^2 \boldsymbol{x}_n \, e^{-i(\boldsymbol{q}_n \pm \bar{\boldsymbol{q}}_n) \cdot \boldsymbol{x}_n} = (2\pi)^2 \, \delta^{(2)}(\boldsymbol{q}_n \pm \bar{\boldsymbol{q}}_n)$$

Momentum broadening in homogeneous media

More details in for example:

nucl-th/9306003, M. Gyulassy, X.-N. Wang

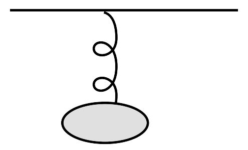
3 Resum the Opacity Series

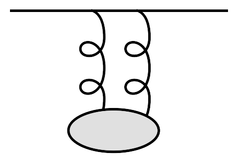
A detailed derivation shows that the square amplitude for $2N$ insertions has the form

$$\left\langle |M|^2 \right\rangle^{(N)} = \prod_{n=1}^{N} \left[(-1) \int_0^{z_{n+1}} dz_n \int \frac{d^2 \boldsymbol{q}_n}{(2\pi)^2} \mathcal{V}(\boldsymbol{q}_n, z_n) \right] \left| J\left(E, \boldsymbol{p}_{in}\right) \right|^2$$

where we identify the effective scattering potential

$$\mathcal{V}(\boldsymbol{q}, z) \equiv -\mathcal{C}\, \rho(z) \left(\left| v(\boldsymbol{q}^2) \right|^2 - \delta^{(2)}(\boldsymbol{q}) \int d^2\boldsymbol{l} \left| v(\boldsymbol{l}^2) \right|^2 \right)$$





The resummation in this case is:

$$\frac{d\mathcal{N}}{d^2\boldsymbol{x}dE} = \sum_{N=0}^{\infty} \int \frac{d^2\boldsymbol{p}\, d^2\boldsymbol{r}}{(2\pi)^2} e^{i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{r})} \frac{(-1)^N \left[\mathcal{V}(\boldsymbol{r})L\right]^N}{N!} \frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{r}dE} = e^{-\mathcal{V}(\boldsymbol{x})L} \frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}$$

Momentum broadening in inhomogeneous media

J.B., A. Sadofyev, C. Salgado

Brookhaven National Laboratory

When averaging in $\boxed{2}$ **we used**

$$\sum_i f_i = \int d^2\boldsymbol{x}\, dz\, \rho(\boldsymbol{x}, z)\, f(\boldsymbol{x}, z) \xrightarrow[\nabla T = 0]{} \int d^2\boldsymbol{x}_n\, e^{-i(\boldsymbol{q}_n \pm \bar{\boldsymbol{q}}_n)\cdot \boldsymbol{x}_n} = (2\pi)^2\, \delta^{(2)}(\boldsymbol{q}_n \pm \bar{\boldsymbol{q}}_n)$$

For anisotropic media this no longer holds; hard to tackle in general

We perform a gradient expansion for the 2 relevant parameters: ρ and μ 2104.09513, A. Sadofyev, M. Sievert, I. Vitev

$$\rho(\boldsymbol{x}, z) \approx \rho(z) + \boldsymbol{\nabla}\rho(z)\cdot \boldsymbol{x} \qquad \mu^2(\boldsymbol{x}, z) \approx \mu^2(z) + \boldsymbol{\nabla}\mu^2(z)\cdot \boldsymbol{x}$$

So that when averaging instead of a momentum space Dirac delta one obtains

$$\int d^2\boldsymbol{x}_n\, x_n^\alpha\, e^{-i(\boldsymbol{q}_n \pm \bar{\boldsymbol{q}}_n)\cdot \boldsymbol{x}_n} = i\,(2\pi)^2 \frac{\partial}{\partial (q_n \pm \bar{q}_n)_\alpha} \delta^{(2)}(\boldsymbol{q}_n \pm \bar{\boldsymbol{q}}_n)$$

With this modification, we find that the N order squared contribution now reads

$$\left\langle |M|^2 \right\rangle^{(N)} = \prod_{n=1}^{N} \left[\int_0^{z_{n+1}} dz_n \int \frac{d^2\boldsymbol{q}_n}{(2\pi)^2} \right] \left(1 + \frac{1}{E} \sum_{m=1}^{N} (z_m - z_{m-1}) \boldsymbol{p}_m \cdot \sum_{k=m}^{N} \left(\boldsymbol{\nabla}\rho \frac{\delta}{\delta \rho_k} + \boldsymbol{\nabla}\mu^2 \frac{\delta}{\delta \mu_k^2} \right) \right) (-1)^N \mathcal{V}_1(\boldsymbol{q}_1) \ldots \mathcal{V}_N(\boldsymbol{q}_N) |J(E, \boldsymbol{p}_{in})|^2$$

$$p_n = p_f - \sum_{m=n}^{N} q_m,\; p_{in} = p_1$$

Momentum broadening in inhomogeneous media

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Proceeding as in 3 **we find that**

$$\mathcal{V}'(\boldsymbol{x}) \equiv \frac{\partial}{\partial \mu^2}\mathcal{V}(\boldsymbol{x})$$

$$\frac{d\mathcal{N}^{(N)}}{d^2\boldsymbol{x}dE} = \int \frac{d^2\boldsymbol{p}\, d^2\boldsymbol{r}}{(2\pi)^2}\, e^{i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{r})}(-1)^N\left[\mathcal{V}(\boldsymbol{r})L\right]^N\left\{\frac{1}{N!} + \frac{L}{E(N+1)!} \times \sum_{m=1}^{N}\left[(N+1-m)\boldsymbol{p}\cdot\left(\frac{\mathcal{V}'(\boldsymbol{r})}{\mathcal{V}(\boldsymbol{r})}\boldsymbol{\nabla}\mu^2 + \frac{1}{\rho}\boldsymbol{\nabla}\rho\right) + i(N+1-m)^2\frac{\boldsymbol{\nabla}\mathcal{V}(\boldsymbol{r})}{\rho\,\mathcal{V}(\boldsymbol{r})}\cdot\boldsymbol{\nabla}\rho\right.\right.$$
$$\left.\left. + i(N+1-m)\left(\frac{\boldsymbol{\nabla}\mathcal{V}'(\boldsymbol{r})}{\mathcal{V}(\boldsymbol{r})} + (N-m)\frac{\mathcal{V}'(\boldsymbol{r})}{\mathcal{V}(\boldsymbol{r})}\frac{\boldsymbol{\nabla}\mathcal{V}(\boldsymbol{r})}{\mathcal{V}(\boldsymbol{r})}\right)\cdot\boldsymbol{\nabla}\mu^2\right]\right\}\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{r}dE}$$

Resumming the opacity series then leads to the compact expression

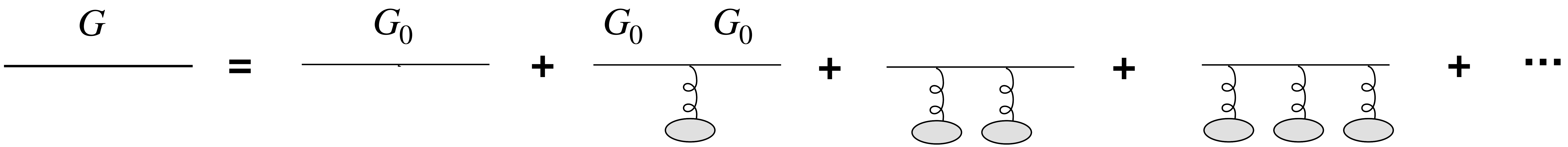
$$\boxed{\frac{d\mathcal{N}}{d^2\boldsymbol{x}dE} = e^{-\mathcal{V}(\boldsymbol{x})L}\left\{\left[1 - i\frac{\mathcal{V}(\boldsymbol{x})L^3}{6E}\left(\frac{\mathcal{V}'(\boldsymbol{x})}{\mathcal{V}(\boldsymbol{x})}\boldsymbol{\nabla}\mu^2 + \frac{1}{\rho}\boldsymbol{\nabla}\rho\right)\cdot\boldsymbol{\nabla}\mathcal{V}(\boldsymbol{x})\right]\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE} + i\frac{\mathcal{V}(\boldsymbol{x})L^2}{2E}\left(\frac{\mathcal{V}'(\boldsymbol{x})}{\mathcal{V}(\boldsymbol{x})}\boldsymbol{\nabla}\mu^2 + \frac{1}{\rho}\boldsymbol{\nabla}\rho\right)\cdot\boldsymbol{\nabla}\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}\right\}}$$

Momentum broadening in homogeneous media

More details in for example:

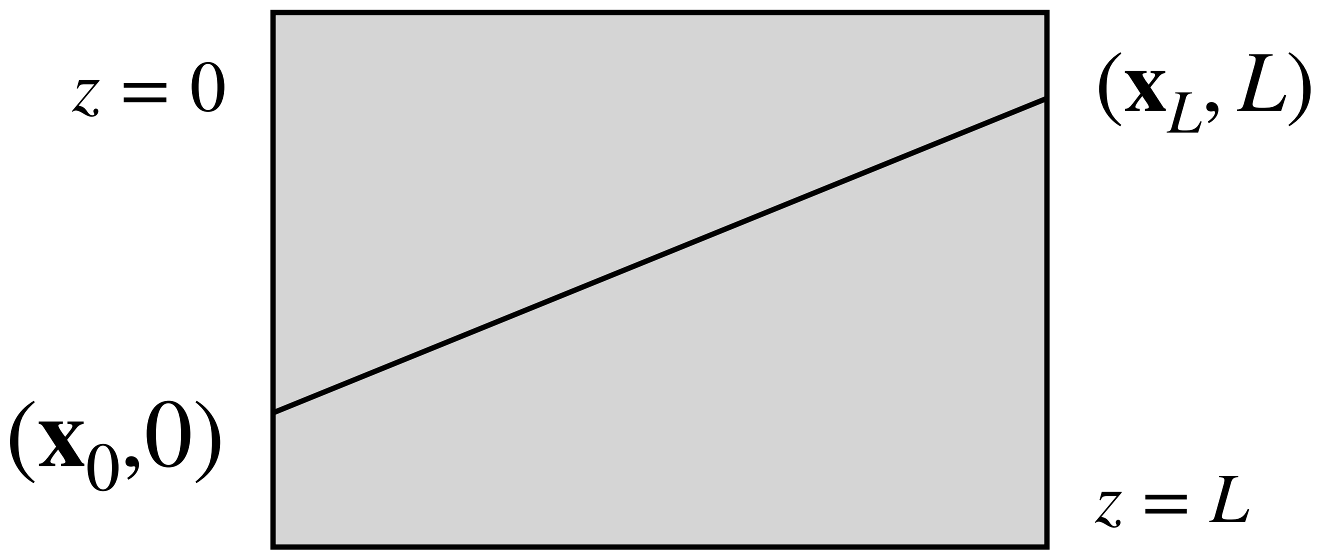
1302.2579, Y. Mehtar-Tani, J. Milhano, K. Tywoniuk

1 Compute an effective in-medium propagator



This results in an effective propagator G

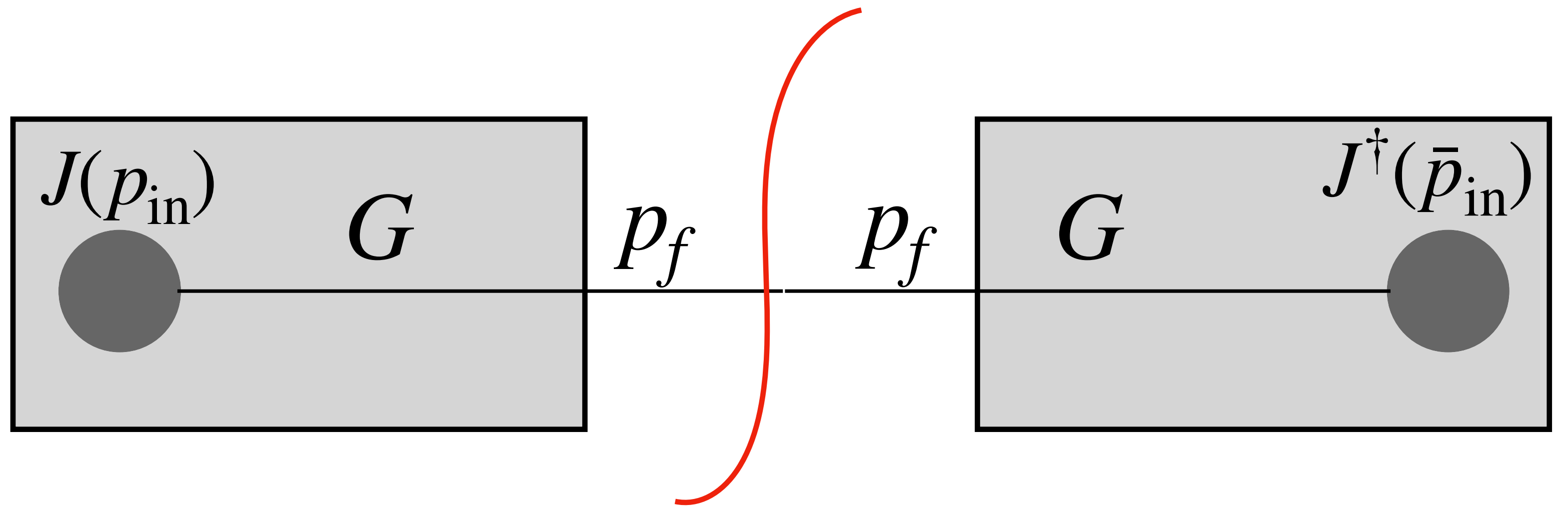
$$G(\boldsymbol{x}_L, L; \boldsymbol{x}_0, 0) = \int_{\boldsymbol{x}_0}^{\boldsymbol{x}_L} \mathcal{D}\boldsymbol{r} \exp\left(\frac{iE}{2}\int_0^L d\tau\, \dot{\boldsymbol{r}}^2\right) \mathcal{P}\exp\left(-i\int_0^L d\tau\, t^a_{\mathrm{proj}} v^a(\boldsymbol{r}(\tau), \tau)\right)$$



2 Compute the relevant Feynman diagrams

$$\langle |M|^2 \rangle = \int \frac{d^2\boldsymbol{p}_{in} d^2\overline{\boldsymbol{p}}_{in}}{(2\pi)^4} \langle G(\boldsymbol{p}_f, L; \boldsymbol{p}_{in}, 0) G^\dagger(\boldsymbol{p}_f, L; \overline{\boldsymbol{p}}_{in}, 0)\rangle J(\boldsymbol{p}_{in}) J^*(\overline{\boldsymbol{p}}_{in})$$

$$gA^{\mu a}_{\mathrm{ext}}(q) = -(2\pi)\, g^{\mu 0}\, v^a(q)\, \delta\left(q^0\right) \qquad v^a(q) = \sum_i e^{-i\vec{q}\cdot\vec{x}_i}\, t^a_i\, v_i(q)$$



Momentum broadening in homogeneous media

More details in for example:

1302.2579, Y. Mehtar-Tani, J. Milhano, K. Tywoniuk

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3 Solve the remaining average of dressed propagators

Option 1) Solve first the path integrals and then average

Equivalent to Opacity Series approach

Option 2) Perform the average before integration

In practice, by solving the remaining integrals one performs the resummation of averaged quantities directly

The **key step** is to use the fact that the color average of potential at different positions

$$\langle t^a_{\text{proj}} v^a(\boldsymbol{r},\tau) t^b_{\text{proj}} v^{\dagger b}(\overline{\boldsymbol{r}},\overline{\tau})\rangle = \mathcal{C}\, g^4 \int dz\, d^2\boldsymbol{x}\, \rho(\boldsymbol{x},\, z) \int \frac{d^2\boldsymbol{q}\, dq_z\, d^2\overline{\boldsymbol{q}}\, d\overline{q}_z}{(2\pi)^6} \frac{e^{i\boldsymbol{q}\cdot(\boldsymbol{r}-\boldsymbol{x})} e^{-i\overline{\boldsymbol{q}}\cdot(\overline{\boldsymbol{r}}-\boldsymbol{x})} e^{iq_z(\tau-z)} e^{-i\overline{q}_z(\overline{\tau}-z)}}{(\boldsymbol{q}^2 + q_z^2 + \mu^2(\boldsymbol{x},z))(\overline{\boldsymbol{q}}^2 + \overline{q}_z^2 + \mu^2(\boldsymbol{x},z))}$$

implies for $\nabla T = 0$

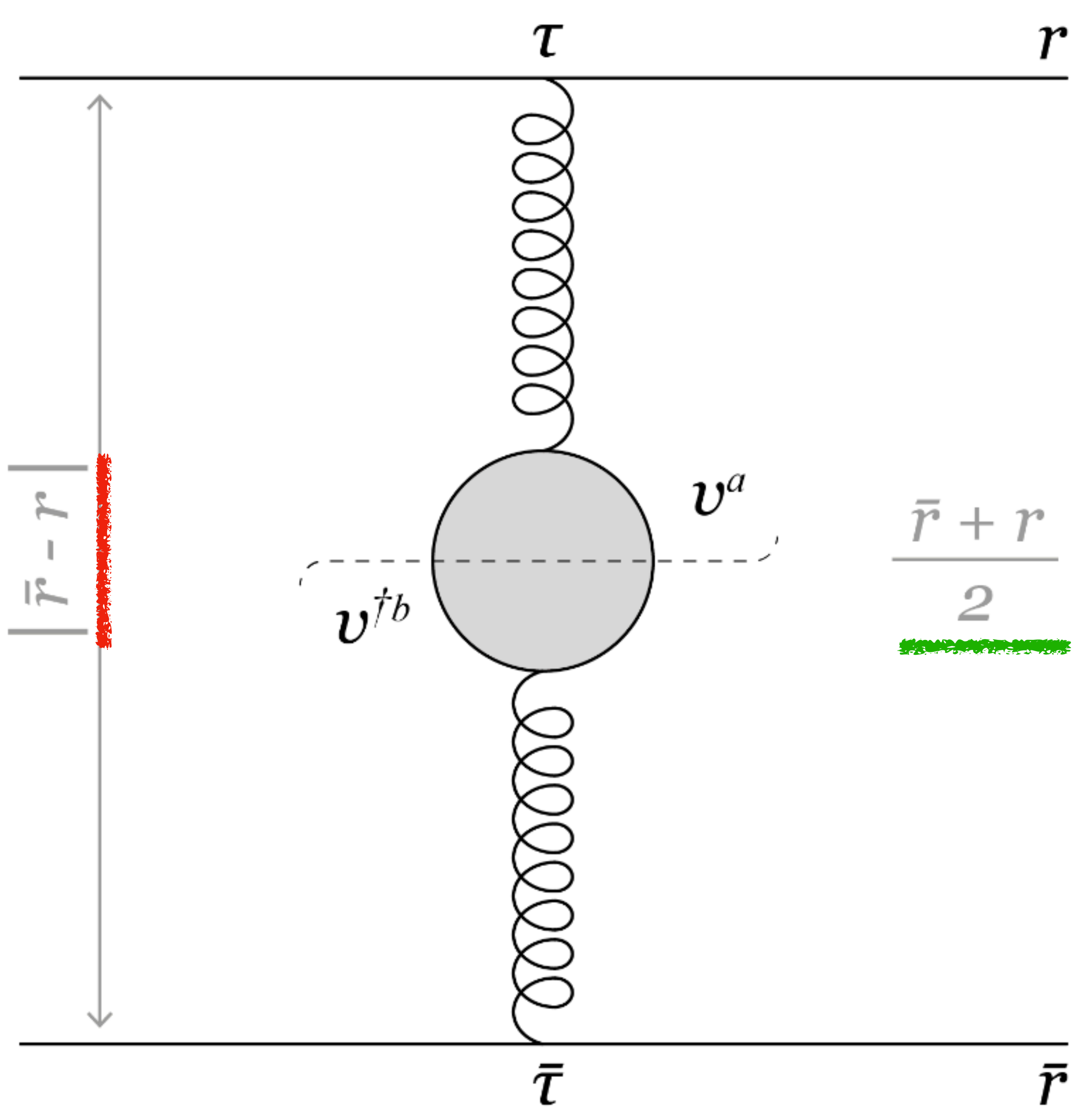
$$\left\langle \mathcal{P}\exp\left(-i\int_0^L d\tau\, t^a_{\text{proj}} v^a(\boldsymbol{r}(\tau),\tau)\right) \mathcal{P}\exp\left(i\int_0^L d\overline{\tau}\, t^b_{\text{proj}} v^b(\overline{\boldsymbol{r}}(\overline{\tau}),\overline{\tau})\right)\right\rangle = \exp\left\{-\int_0^L d\tau \quad \mathcal{V}\left(\boldsymbol{r}(\tau) - \overline{\boldsymbol{r}}(\tau)\right)\right\}$$

Momentum broadening in inhomogeneous media

J.B., A. Sadofyev, C. Salgado
2202.08847

To linear order in gradients from $\boxed{3}$ we find now

$$\langle t^a_{\rm proj} v^a(\boldsymbol{r},\tau) t^b_{\rm proj} v^{\dagger b}(\bar{\boldsymbol{r}},\bar\tau)\rangle \simeq \left(1 + \frac{\boldsymbol{r}(\tau)+\bar{\boldsymbol{r}}(\tau)}{2}\cdot\left(\boldsymbol{\nabla}\rho\frac{\delta}{\delta\rho} + \boldsymbol{\nabla}\mu^2\frac{\delta}{\delta\mu^2}\right)\right)\mathcal{C}\,\delta(\tau-\bar\tau)\,\rho\, g^4 \int \frac{d^2\boldsymbol{q}}{(2\pi)^2}\frac{e^{i\boldsymbol{q}\cdot(\boldsymbol{r}-\bar{\boldsymbol{r}})}}{(\boldsymbol{q}^2+\mu^2)^2}$$



One can still show that the 2-point correlator exponentiates

before

$$\left\langle \mathcal{P}\exp\left(-i\int_0^L d\tau\, t^a_{\rm proj} v^a(\boldsymbol{r}(\tau),\tau)\right)\mathcal{P}\exp\left(i\int_0^L d\bar\tau\, t^b_{\rm proj} v^b(\bar{\boldsymbol{r}}(\bar\tau),\bar\tau)\right)\right\rangle = \exp\left\{-\int_0^L d\tau \quad \mathcal{V}\left(\boldsymbol{r}(\tau)-\bar{\boldsymbol{r}}(\tau)\right)\right\}$$

$$\left\langle \mathcal{P}\exp\left(-i\int_0^L d\tau\, t^a_{\rm proj} v^a(\boldsymbol{r}(\tau),\tau)\right)\mathcal{P}\exp\left(i\int_0^L d\bar\tau\, t^b_{\rm proj} v^b(\bar{\boldsymbol{r}}(\bar\tau),\bar\tau)\right)\right\rangle = \exp\left\{-\int_0^L d\tau \left[1+\underbrace{\frac{\boldsymbol{r}(\tau)+\bar{\boldsymbol{r}}(\tau)}{2}}_{\text{Center of mass of dipole}}\cdot\left(\boldsymbol{\nabla}\rho\frac{\delta}{\delta\rho}+\boldsymbol{\nabla}\mu^2\frac{\delta}{\delta\mu^2}\right)\right]\mathcal{V}\left(\underbrace{\boldsymbol{r}(\tau)-\bar{\boldsymbol{r}}(\tau)}_{\text{Dipole size}}\right)\right\}$$

Momentum broadening in inhomogeneous media

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2202.08847

Combining all the results, computing $\langle |M|^2 \rangle = \int \frac{d^2\boldsymbol{p}_{in} d^2\overline{\boldsymbol{p}}_{in}}{(2\pi)^4} \langle G(\boldsymbol{p}_f, L; \boldsymbol{p}_{in}, 0) G^\dagger(\boldsymbol{p}_f, L; \overline{\boldsymbol{p}}_{in}, 0) \rangle J(\boldsymbol{p}_{in}) J^*(\overline{\boldsymbol{p}}_{in})$

is reduced to finding a close form for

$$\left\langle G(\boldsymbol{x}_L, L; \boldsymbol{x}_0, 0) G^\dagger(\overline{\boldsymbol{x}}_L, L; \overline{\boldsymbol{x}}_0, 0) \right\rangle = \int_{\boldsymbol{u}_0}^{\boldsymbol{u}_L} \mathcal{D}\boldsymbol{u} \int_{\boldsymbol{w}_0}^{\boldsymbol{w}_L} \mathcal{D}\boldsymbol{w} \, \exp\left\{ \int_0^L d\tau \, \left[iE\, \dot{\boldsymbol{u}} \cdot \dot{\boldsymbol{w}} - (1 + \boldsymbol{w} \cdot \hat{\boldsymbol{g}}) \, \mathcal{V}\left(\boldsymbol{u}(\tau)\right) \right] \right\}$$

$$\boldsymbol{u} \equiv \boldsymbol{r} - \overline{\boldsymbol{r}} \qquad \boldsymbol{w} \equiv \frac{\boldsymbol{r} + \overline{\boldsymbol{r}}}{2}$$

After some algebra, the particle distribution reduces to

$$\frac{d\mathcal{N}}{d^2\boldsymbol{x} dE} \simeq \frac{1}{L^2} \int d^2\boldsymbol{u}_0 \, d^2\boldsymbol{u}_L \, \delta^{(2)}(\boldsymbol{x} - \boldsymbol{u}_L) \; \delta^{(2)}\left(\dot{\boldsymbol{u}}_c(L)\right) \exp\left\{ - \int_0^L d\tau \, \mathcal{V}\left(\boldsymbol{u}_c(\tau)\right) \right\} \frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{u}_0 dE}$$

$$\boldsymbol{u}_c(\tau) = \boldsymbol{u}_L + \frac{i}{E} \hat{\boldsymbol{g}} \, \mathcal{V}\left(\boldsymbol{u}_L\right) \left\{ \frac{(\tau - L)^2}{2} \right\}$$

Momentum broadening in inhomogeneous media

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2202.08847

For real J this leads to

$$\frac{d\mathcal{N}}{d^2\boldsymbol{x}dE} \simeq \exp\left\{-\mathcal{V}\left(\boldsymbol{x}\right)L\right\}\left\{\left[1-\frac{iL^3}{6E}\boldsymbol{\nabla}\mathcal{V}\left(\boldsymbol{x}\right)\cdot\left(\boldsymbol{\nabla}\rho\frac{\delta}{\delta\rho}+\boldsymbol{\nabla}\mu^2\frac{\delta}{\delta\mu^2}\right)\mathcal{V}\left(\boldsymbol{x}\right)\right]\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}+\frac{iL^2}{2E}\left(\boldsymbol{\nabla}\rho\frac{\delta}{\delta\rho}+\boldsymbol{\nabla}\mu^2\frac{\delta}{\delta\mu^2}\right)\mathcal{V}\left(\boldsymbol{x}\right)\cdot\boldsymbol{\nabla}\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}\right\}$$

Same result as in Opacity Expansion approach

For time dependent medium profile

$$\hat{\boldsymbol{g}}\left(\tau\right)=\left(\boldsymbol{\nabla}\rho(\tau)\frac{\delta}{\delta\rho}+\boldsymbol{\nabla}\mu^2(\tau)\frac{\delta}{\delta\mu^2}\right)$$

$$\frac{d\mathcal{N}}{d^2\boldsymbol{x}dE} \simeq \exp\left\{-\int_0^L d\tau\,\mathcal{V}\left(\boldsymbol{x},\tau\right)\right\}\left\{\left[1-\frac{i}{E}\int_0^L d\tau\,\boldsymbol{\nabla}\mathcal{V}\left(\boldsymbol{x},\tau\right)\cdot\left(\int_L^\tau d\zeta\int_0^\zeta d\xi+(L-\tau)\int_0^L d\xi\right)\hat{\boldsymbol{g}}(\xi)\mathcal{V}\left(\boldsymbol{x},\xi\right)\right]+\frac{i}{E}\int_0^L d\zeta\int_0^\zeta d\xi\,\hat{\boldsymbol{g}}\left(\xi\right)\mathcal{V}\left(\boldsymbol{x},\xi\right)\cdot\boldsymbol{\nabla}\right\}\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}$$

Momentum broadening in inhomogeneous media

2204.05323
also 2001.08273

Brookhaven National Laboratory

Parallel work by Y. Fung, J. Casalderrey-Solana, X.-N. Wang

Use harmonic approximation and exponentiated form $\boldsymbol{R} = (\boldsymbol{r}_1 + \boldsymbol{r}_2)/2$ and $\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2$.

$$\frac{1}{N_c}\left\langle\!\!\left\langle tr\{W(\boldsymbol{r}_1)W^\dagger(\boldsymbol{r}_2)\}\right\rangle\!\!\right\rangle \approx \exp\{-\int_{x_0^+}^{x_f^+} \mathrm{d}x^+ \frac{1}{4\sqrt{2}}\hat{q}(\boldsymbol{R})\,\boldsymbol{r}^2\}$$

Then assume that

$$\hat{q}(\boldsymbol{R}) = \frac{\hat{q}_0}{1 - f(\boldsymbol{R})}$$

$$f(\boldsymbol{R}) = \delta \exp\left\{\frac{-\boldsymbol{R}^2}{\sigma^2}\right\}$$

To first order in f $\frac{d^2N}{d^2\boldsymbol{p}} \approx \frac{d^2N_0}{d^2\boldsymbol{p}} + \frac{d^2N_1}{d^2\boldsymbol{p}}$

$\mathbf{p_0}(\mathbf{x})$ is initial parton's transverse momentum (position)

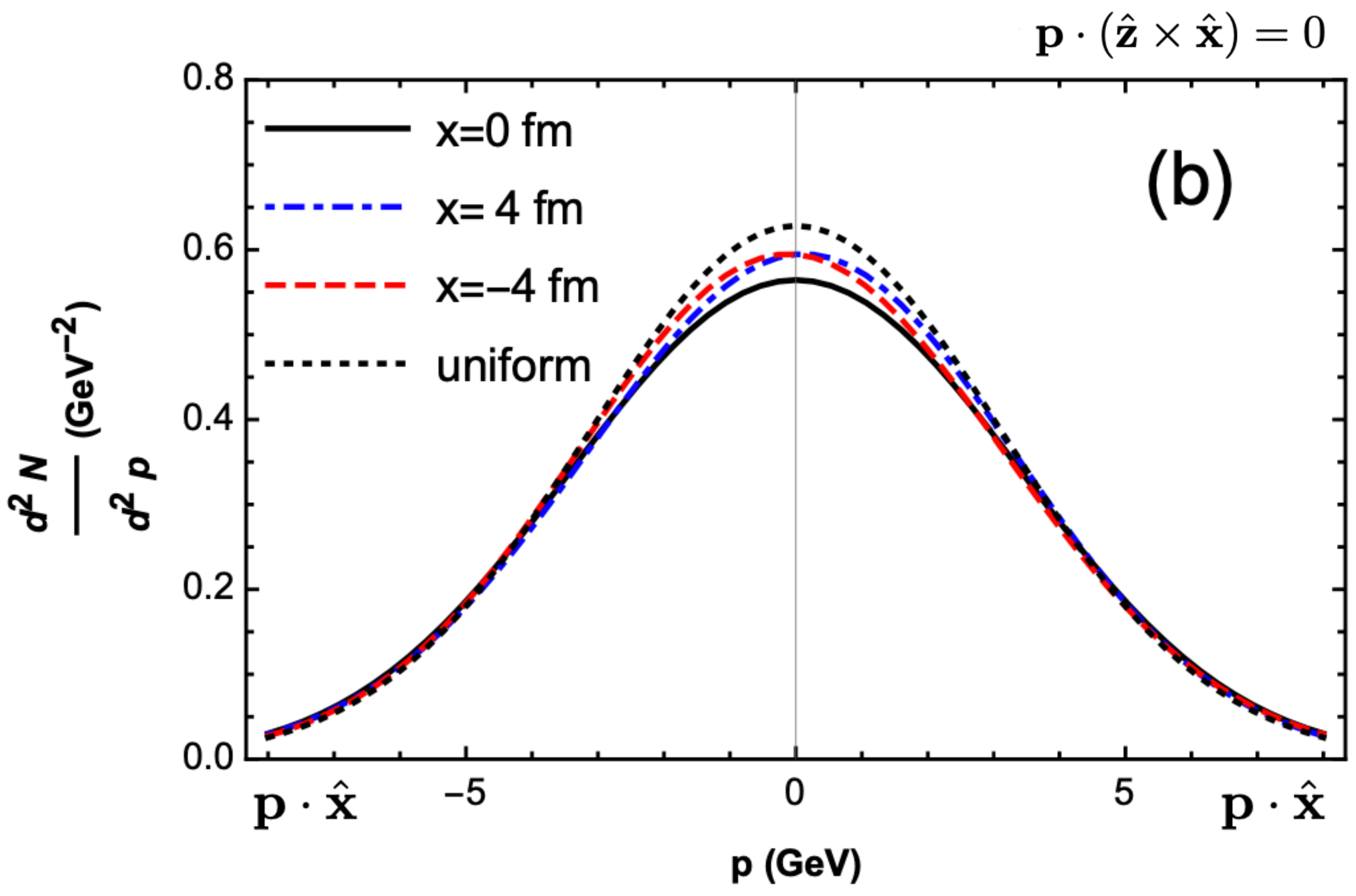
$$\frac{d^2N_0}{d^2\boldsymbol{p}} = \frac{4\pi}{\hat{q}_0(t_f - t_0)} \exp\{-\frac{(\boldsymbol{p} - \boldsymbol{p}_0)^2}{\hat{q}_0(t_f - t_0)}\}$$

$$\frac{d^2N_1}{d^2\boldsymbol{p}} = \int_{t_0}^{t_f} \mathrm{d}t \frac{4\pi\hat{q}_0\sigma^2\delta}{\Sigma(t)\Delta(t)} \exp\left\{-\frac{(\mathbf{x} + (t - t_0)\frac{\boldsymbol{p}_0}{\omega})^2}{\Sigma(t)} - \frac{\left[\boldsymbol{p} - \boldsymbol{p}_0 + \lambda(t)(\mathbf{x} + (t - t_0)\frac{\boldsymbol{p}_0}{\omega})\right]^2}{\Delta(t)}\right\}$$
$$\times \frac{1}{\Delta^2(t)}\left\{\left[\boldsymbol{p} - \boldsymbol{p}_0 + \lambda(t)(\mathbf{x} + (t - t_0)\frac{\boldsymbol{p}_0}{\omega})\right]^2 - \Delta(t)\right\}.$$

Momentum broadening in inhomogeneous media

2204.05323
also 2001.08273

Parallel work by Y. Fung, J. Casalderrey-Solana, X.-N. Wang



Broader and asymmetrical distribution

Vanishing leading moments

$$\int \mathrm{d}^2\mathbf{p} d^2N_1/d^2\mathbf{p} = \int \mathrm{d}^2\mathbf{p}\mathbf{p} d^2N_1/d^2\mathbf{p} = 0.$$

Quadratic moment is modified

$$\langle \mathbf{p}^2\rangle = (t_f - t_0)\hat{q}_0 + \langle \Delta\mathbf{p}^2(\mathbf{x}, t_f)\rangle,$$

$$\langle \Delta\mathbf{p}^2(\mathbf{x}, t_f)\rangle = \int_{t_0}^{t_f} \mathrm{d}t \frac{\hat{q}_0\sigma^2\delta}{\Sigma(t)} \exp\left[-\frac{\mathbf{x}^2}{\Sigma(t)}\right]$$

Momentum broadening distribution

J.B., A. Sadofyev, C. Salgado
2202.08847

The final distribution has the form

$$\frac{d\mathcal{N}}{d^2\boldsymbol{x}dE} \simeq \exp\left\{-\mathcal{V}(\boldsymbol{x})\,L\right\}\left\{\left[1-\frac{iL^3}{6E}\boldsymbol{\nabla}\mathcal{V}(\boldsymbol{x})\cdot\left(\boldsymbol{\nabla}\rho\frac{\delta}{\delta\rho}+\boldsymbol{\nabla}\mu^2\frac{\delta}{\delta\mu^2}\right)\mathcal{V}(\boldsymbol{x})\right]\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}+\underline{\frac{iL^2}{2E}\left(\boldsymbol{\nabla}\rho\frac{\delta}{\delta\rho}+\boldsymbol{\nabla}\mu^2\frac{\delta}{\delta\mu^2}\right)\mathcal{V}(\boldsymbol{x})\cdot\boldsymbol{\nabla}\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}}\right\}$$

$$\frac{d\mathcal{N}}{d^2\boldsymbol{x}dE} = \underline{\mathcal{P}(\boldsymbol{x})}\,\underline{\hat{\mathrm{S}}(\boldsymbol{x})}\,\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}$$

In the literature referred to as single particle broadening distribution (when Fourier transformed)

Usually a unit operator, but now it acts with ∇ on initial distribution

Effective factorization no longer holds in general due to operator nature

Still

$$\int d^2\boldsymbol{p}\,\frac{d\mathcal{N}}{d^2\boldsymbol{p}\,dE} = \left.\frac{d\mathcal{N}}{d^2\boldsymbol{x}dE}\right|_{\boldsymbol{x}=0} = \left.\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}\right|_{\boldsymbol{x}=0} = \int d^2\boldsymbol{p}\,\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{p}\,dE}$$

We consider first the case of a source with finite width

$$E\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{p}\,dE} = \frac{f(E)}{2\pi w^2}e^{-\frac{\boldsymbol{p}^2}{2w^2}}$$

It is possible to show that even though $\langle\boldsymbol{p}\rangle = 0$

higher odd moments can be generated, for example

$$\langle p^\alpha\,\boldsymbol{p}^2\rangle = \underbrace{\frac{w^2L^2\mu^2}{E\,\lambda}\frac{\nabla^\alpha\rho}{\rho}\ln\frac{E}{\mu}}_{N=1} + \underbrace{\frac{L^3\mu^4}{6E\,\lambda^2}\frac{\nabla^\alpha\rho}{\rho}\left(\ln\frac{E}{\mu}\right)^2}_{N=2}$$

Higher N terms dominate due to diverging potential at large momenta

Coulomb logarithm

Momentum broadening distribution

J.B., A. Sadofyev, C. Salgado
2202.08847

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If we neglect initial state effects, then we are left with $\chi = \frac{\mathcal{C} g^4 \rho}{4\pi\mu^2} L$ medium opacity

$$\mathcal{P}(\boldsymbol{p}) = \int d^2\boldsymbol{x}\, e^{-i\boldsymbol{p}\cdot\boldsymbol{x}} e^{-\mathcal{V}(\boldsymbol{x})L} \left[1 - \frac{iL^3}{6E} \boldsymbol{\nabla}\mathcal{V}(\boldsymbol{x}) \cdot \hat{\boldsymbol{g}}\mathcal{V}(\boldsymbol{x})\right]$$

where for GW model $\boldsymbol{p}^2 \gg \mu^2$

$$\frac{4L}{\chi}\mathcal{V}^{\text{GW}}(\boldsymbol{x}) = \mu^2\boldsymbol{x}^2 \log \frac{4e^{1-2\gamma_E}}{\mu^2\boldsymbol{x}^2} + \mathcal{O}(\mu^4\boldsymbol{x}^4)$$

In the hard region where $\boldsymbol{p}^2 \gg \chi\mu^2$ it can be written in a closed form

$$\mathcal{P}(\boldsymbol{p}) \simeq \underline{\frac{4\pi\mu^2\chi}{\boldsymbol{p}^4}} + \frac{16\pi\mu^4\chi^2}{\boldsymbol{p}^6}\left(\log\frac{\boldsymbol{p}^2}{\mu^2} - 2\right) + \frac{4\pi\mu^4\chi^2 L}{3E}\left[\frac{\boldsymbol{\nabla}\rho}{\rho}\left(\log\frac{\boldsymbol{p}^4}{\mu^4} - 4\right) - \frac{\boldsymbol{\nabla}\mu^2}{\mu^2}\right]\cdot\frac{\boldsymbol{p}}{\boldsymbol{p}^6}$$

Coulomb tail

In the complementary region where $\mu^2 \ll \boldsymbol{p}^2 \le \chi\mu^2$ one has $\frac{4L}{\chi}\mathcal{V}^{\text{GW}}(\boldsymbol{x}) \simeq \mu^2\boldsymbol{x}^2\left(\log\frac{Q^2}{\mu^2} + \log\frac{4e^{1-2\gamma_E}}{Q^2\boldsymbol{x}^2}\right)$

$$\mathcal{P}(\boldsymbol{p}) = \frac{4\pi}{\chi\mu^2\log\frac{Q^2}{\mu^2}}\left[1 + \frac{L}{6E}\underline{\frac{\boldsymbol{p}^2 - 2\chi\mu^2\log\frac{Q^2}{\mu^2}}{\chi\mu^2\log\frac{Q^2}{\mu^2}}}\left(\frac{\boldsymbol{\nabla}\rho}{\rho} - \frac{1}{\log\frac{Q^2}{\mu^2}}\frac{\boldsymbol{\nabla}\mu^2}{\mu^2}\right)\cdot\boldsymbol{p}\right]\underline{e^{-\frac{\boldsymbol{p}^2}{\chi\mu^2\log\frac{Q^2}{\mu^2}}}}$$

Usual Gaussian distribution

Gradient effects seem to be more relevant around the distribution average

Momentum broadening distribution

J.B., A. Sadofyev, C. Salgado
2202.08847

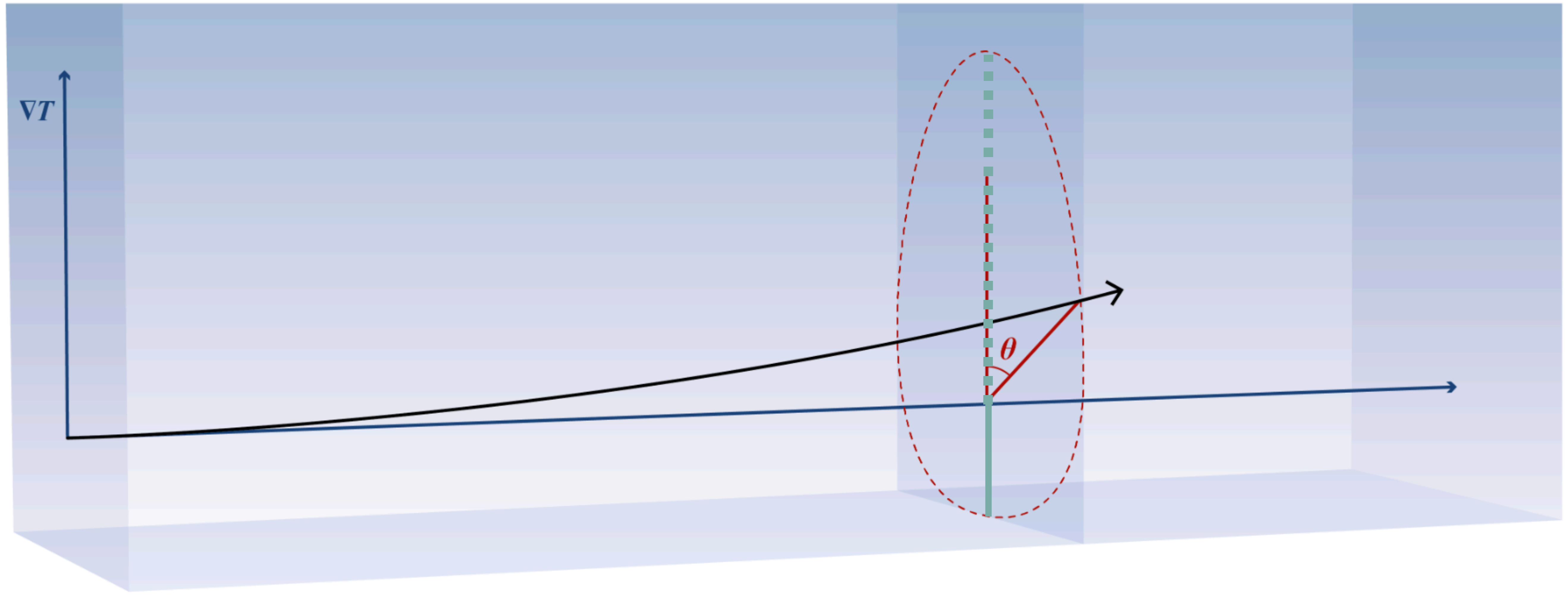
For the full GW model we have

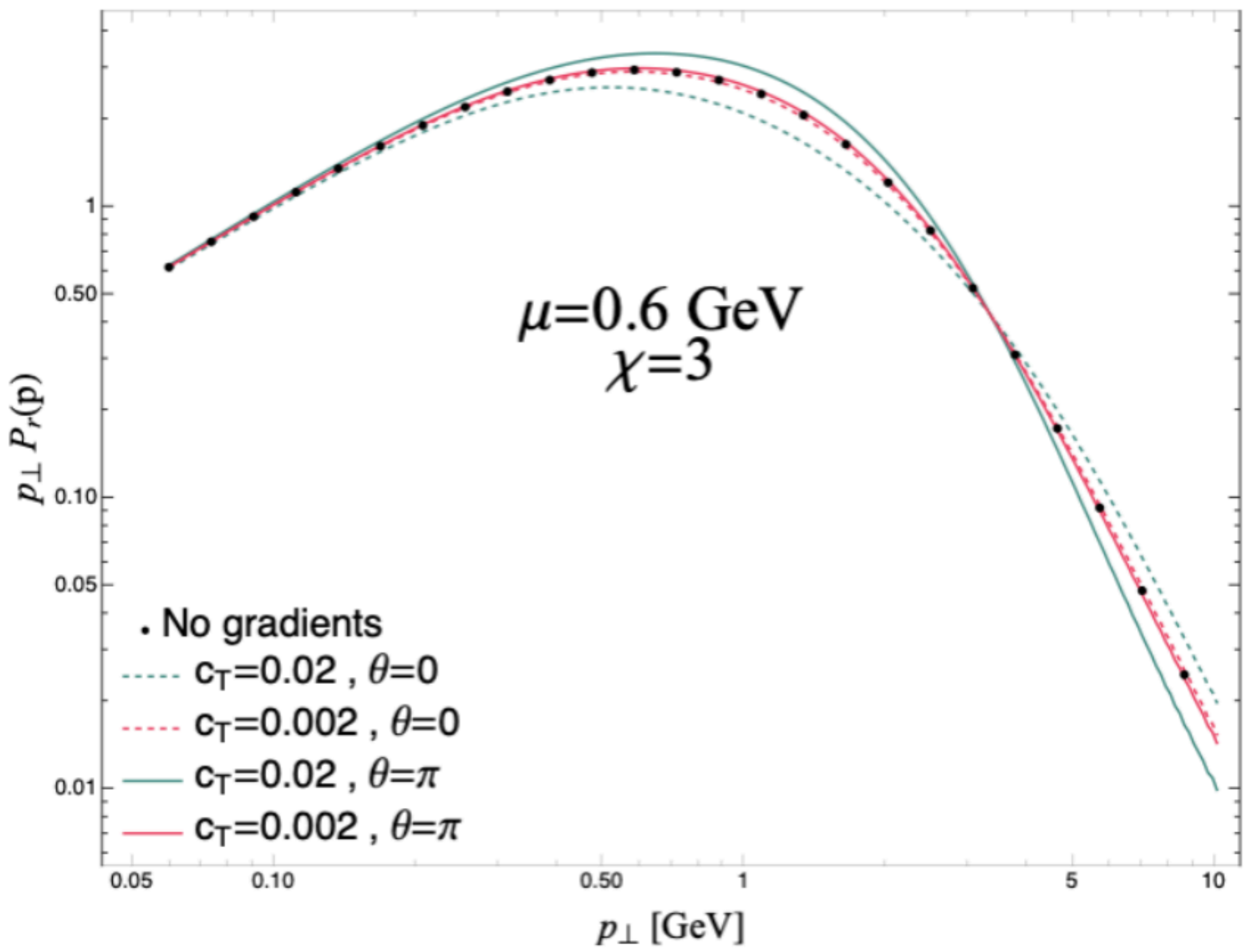
Using the parametric relation

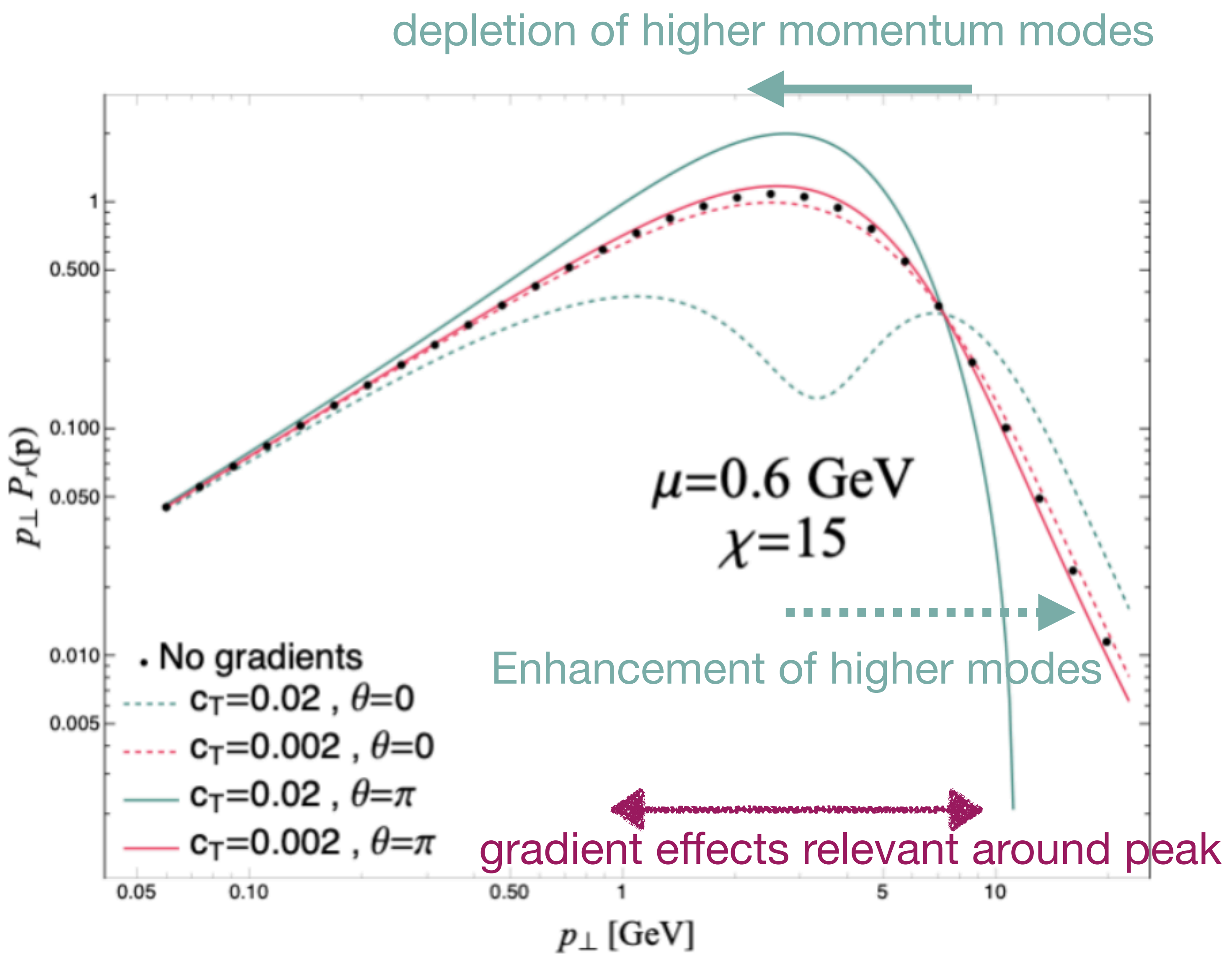
$$\frac{\boldsymbol{\nabla}\rho}{\rho} \sim 3\frac{\boldsymbol{\nabla}T}{T}\,, \qquad \frac{\boldsymbol{\nabla}\mu^2}{\mu^2} \sim 2\frac{\boldsymbol{\nabla}T}{T}$$

The full distribution is written in terms of the angle θ and parameter $c_T \equiv \left|\frac{\boldsymbol{\nabla}T}{ET}\right|$.

$$\mathcal{P}(\boldsymbol{p}) = 2\pi \int_0^\infty dx_\perp\, x_\perp\, e^{-\mathcal{V}^{\rm GW}(x_\perp)L} \Big\{ J_0(p_\perp x_\perp) - \frac{\chi^2\mu^2 L}{6}\, c_T\, x_\perp\, K_0(\mu\, x_\perp)\, J_1(p_\perp x_\perp) \times \left[1 - 3\mu\, x_\perp K_1(\mu\, x_\perp) + \mu^2 x_\perp^2 K_2(\mu\, x_\perp)\right] \cos\theta \Big\}$$

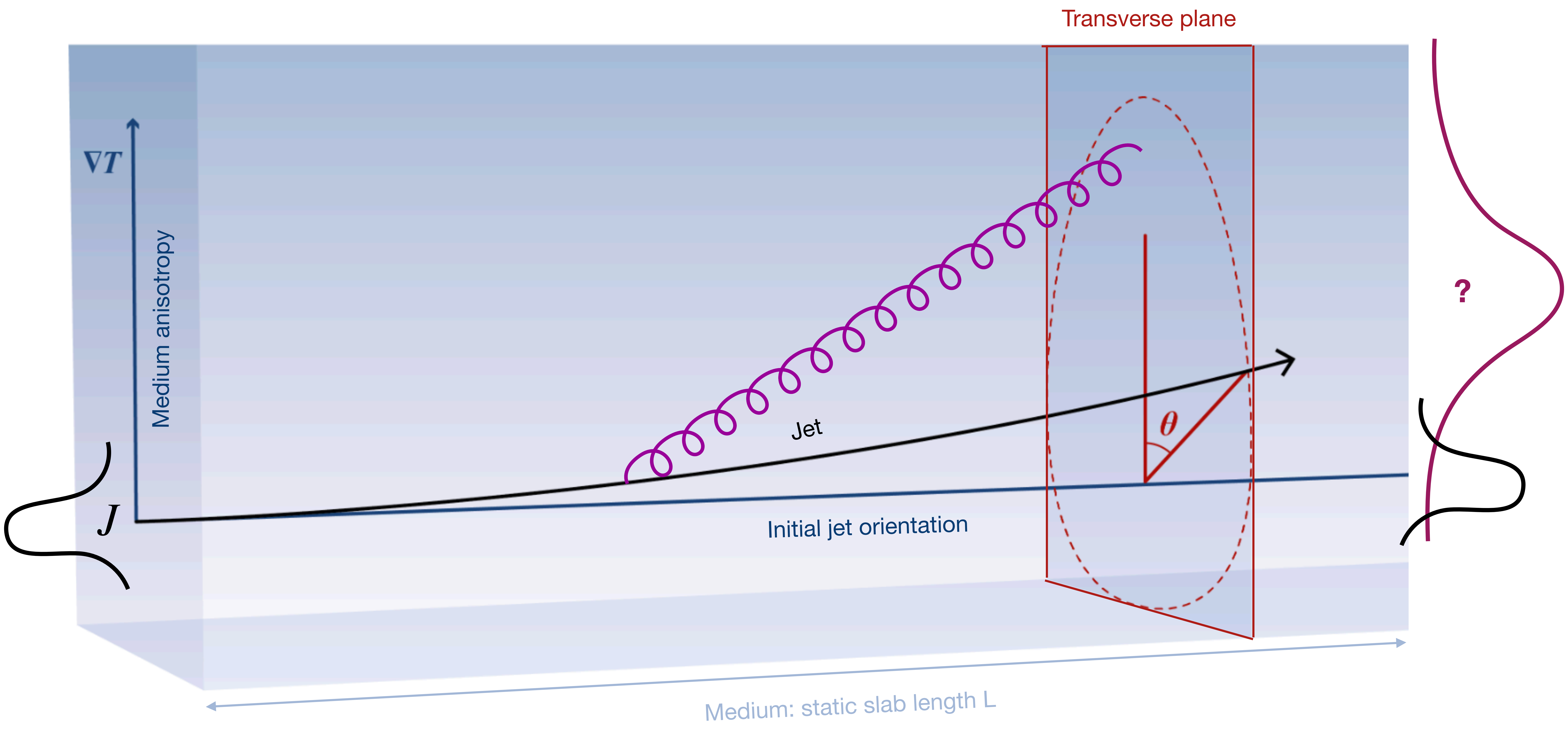






Induced soft gluon spectrum

J.B., X. Mayo, A. Sadofyev, C. Salgado



Induced soft gluon spectrum

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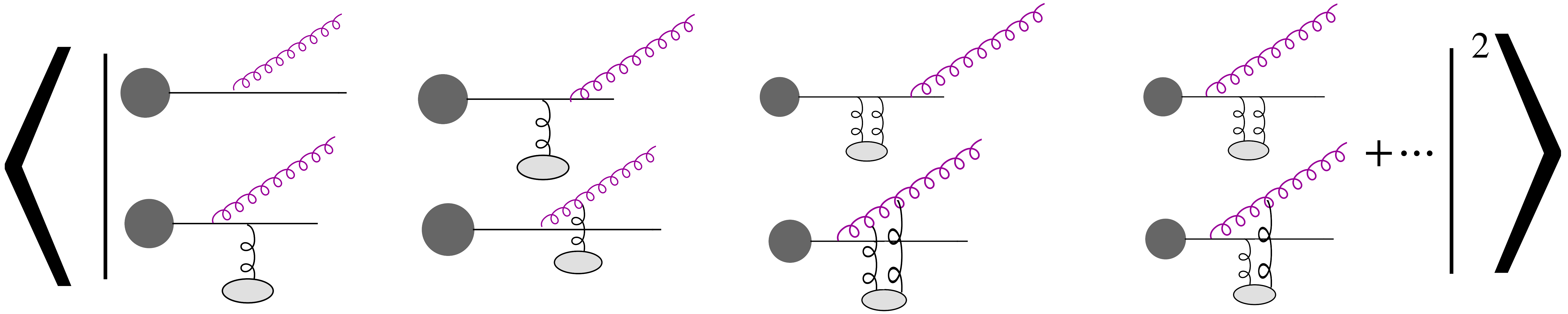
In Opacity Expansion style calculation one needs to compute

Beyond N=1, this seems to be quite hard to do



Induced soft gluon spectrum

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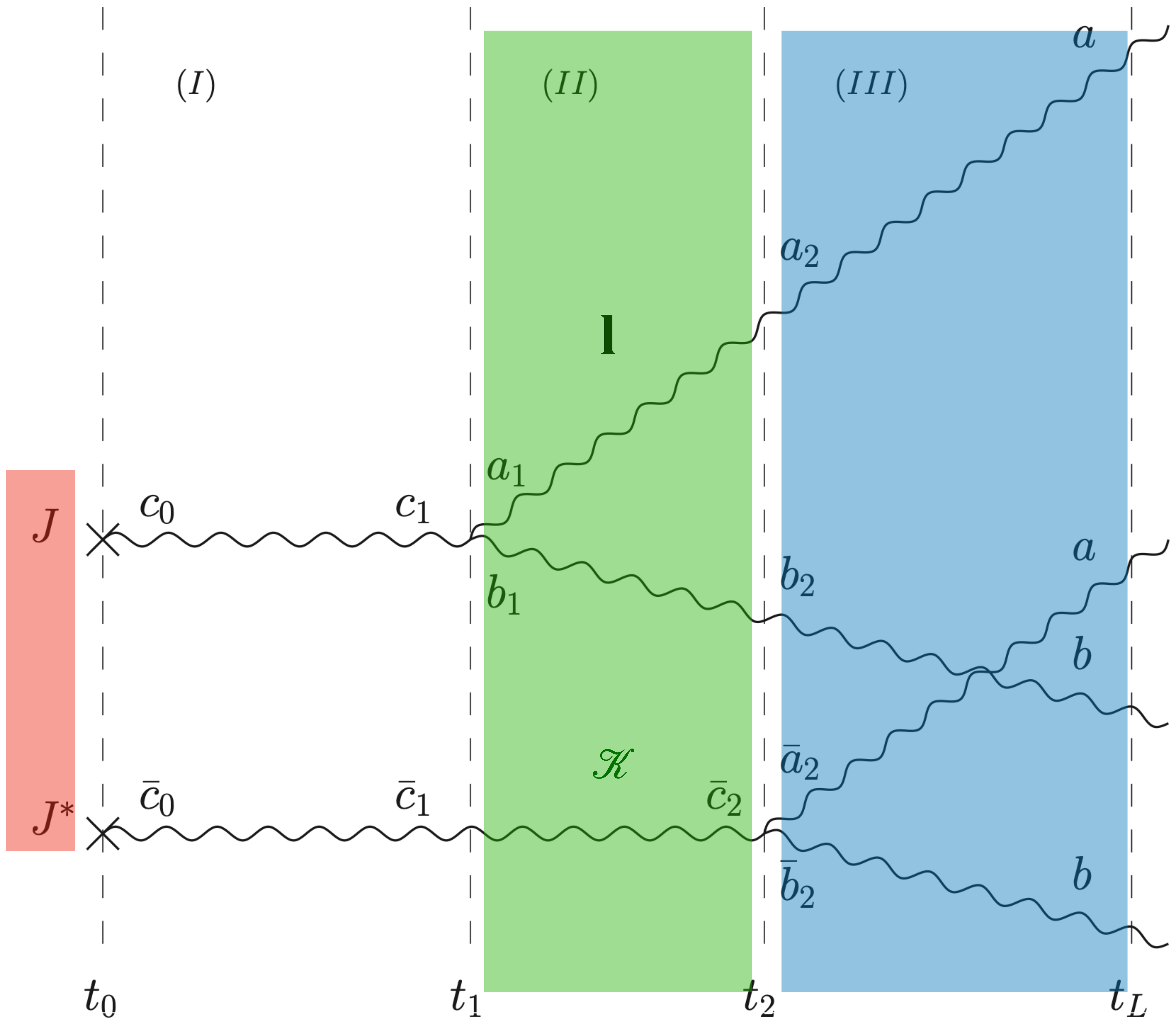
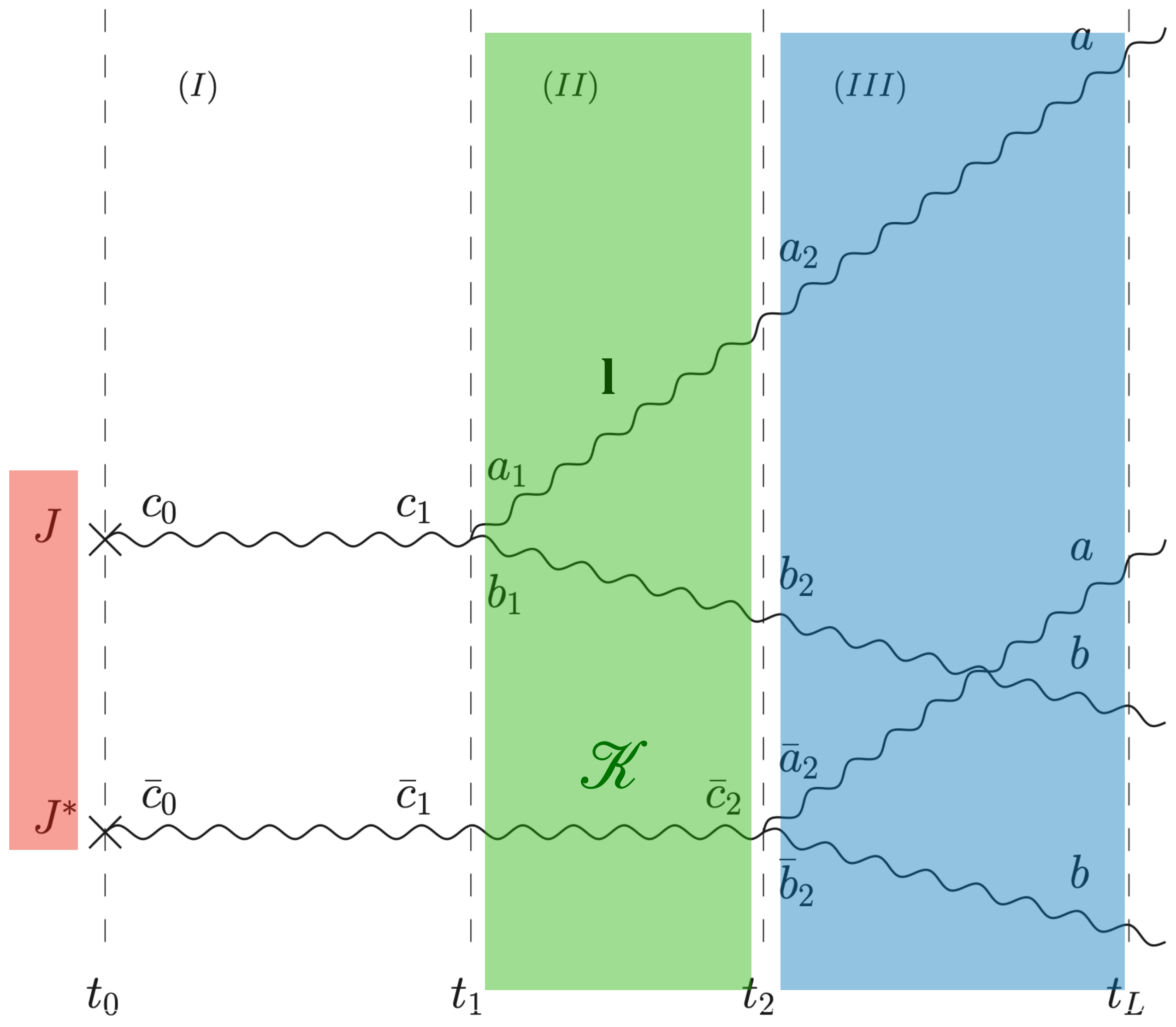
In the BDMPS-Z style calculation only averaging changes



see e.g. 1209.4585, J.-P. Blaizot, F. Dominguez, E. Iancu, Y. Mehtar-Tani

Induced soft gluon spectrum

J.B., X. Mayo, A. Sadofyev, C. Salgado



In this case, we can write the squared amplitude as

$$dN = \frac{\alpha_s C_F}{\omega^2} 2\Re \Big[\int_{\bar{s} s \boldsymbol{z} \boldsymbol{x}_0} \partial_{\boldsymbol{y}=\boldsymbol{x}_0} \mathcal{K}_\omega(\boldsymbol{z}, \bar{s}; \boldsymbol{y}, s|\boldsymbol{x}_0) \cdot \partial_{\bar{\boldsymbol{y}}=\boldsymbol{x}_0} \underbrace{\int_{\boldsymbol{q}_1 \boldsymbol{q}_2} e^{-i\boldsymbol{q}_1\cdot\boldsymbol{z}} e^{i\boldsymbol{q}_2\cdot\bar{\boldsymbol{y}}} S^{(2)}(\boldsymbol{k}, \boldsymbol{k}, \infty; \boldsymbol{q}_1, \boldsymbol{q}_2, \bar{s})}_{\text{Solved!}} |J(\boldsymbol{x}_0)|^2$$

$$\mathcal{K}_\omega(\boldsymbol{z}, \bar{s}; \boldsymbol{y}, s|\boldsymbol{x}_0) = \langle G_\omega(\boldsymbol{z}, \bar{s}; \boldsymbol{y}, s) W^\dagger(\boldsymbol{x}_0; \bar{s}, s)\rangle$$

For an arbitrary kernel we can always write $dN = dN^{(0)} + dN^{(1)}$

We take the splitting: $\mathcal{V}^{(0)}(\boldsymbol{x}, \boldsymbol{y}) = \frac{\hat{q}}{4}(\boldsymbol{x}-\boldsymbol{y})^2 \quad \mathcal{V}^{(1)}(\boldsymbol{x}, \boldsymbol{y}) = \boldsymbol{F} \cdot \frac{\boldsymbol{x}+\boldsymbol{y}}{2} \frac{\hat{q}}{4}(\boldsymbol{x}-\boldsymbol{y})^2 \quad \boldsymbol{F} = \nabla\rho\, \delta_\rho$

$$dN^{(0)} = \frac{\alpha_s C_F}{\omega^2} 2\Re \int_{\bar{s} s \boldsymbol{z}} e^{-i\boldsymbol{k}\cdot\boldsymbol{z}} \mathcal{P}(\boldsymbol{z}, L-\bar{s}) \times \left[-(L-\bar{s})\mathcal{V}(\boldsymbol{z})\boldsymbol{F} + \left(1-(L-\bar{s})\mathcal{V}(\boldsymbol{z})\boldsymbol{F}\cdot\frac{\boldsymbol{z}}{2} + \frac{i(L-\bar{s})^2}{2\omega}\mathcal{V}(\boldsymbol{z})\boldsymbol{F}\cdot\partial_{\boldsymbol{z}}\right)\partial_{\boldsymbol{z}}\right] \cdot \partial_{\boldsymbol{y}=\boldsymbol{0}} \mathcal{K}^{(0)}_\omega(\boldsymbol{z}, \bar{s}; \boldsymbol{y}, s|\boldsymbol{0})$$

$$dN^{(1)} = \frac{\alpha_s C_F}{\omega^2} 2\Re \int_{\bar{s} s \boldsymbol{z}} e^{-i\boldsymbol{k}\cdot\boldsymbol{z}} \mathcal{P}^{(0)}(\boldsymbol{z}, L-\bar{s})\, \partial_{\boldsymbol{z}} \cdot \partial_{\boldsymbol{y}=\boldsymbol{0}} \mathcal{K}^{(1)}_\omega(\boldsymbol{z}, \bar{s}; \boldsymbol{y}, s|\boldsymbol{0})$$

Induced soft gluon spectrum

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One can compute the spectrum at linear order in gradients with multiple soft insertions

The final result is not particularly illuminating. In summary:

→ The same result is (seemingly) harder to recover by resuming the Opacity Series

→ Gradient effects do not affect energy loss, as expected

For pheno it is important to have the soft gluon limit: still holds

$$dN^{(0)}_{\omega\ll\omega_c} \approx \int_0^L d\bar{s}\,\mathcal{P}(\boldsymbol{k}, L-\bar{s})\frac{\omega\, dI^{(0)}}{d\omega d\bar{s}}$$

Very schematically: In soft gluon limit $k_b^2 \gg k_f^2$

$$dN = \frac{\alpha_s C_F}{\omega^2} 2\Re\Big[\int_{\bar{s}s\boldsymbol{z}\boldsymbol{x}_0} \partial_{\boldsymbol{y}=\boldsymbol{x}_0}\mathcal{K}_\omega(\boldsymbol{z},\bar{s};\boldsymbol{y},s|\boldsymbol{x}_0)\cdot\partial_{\bar{\boldsymbol{y}}=\boldsymbol{x}_0}\int_{\boldsymbol{q}_1\boldsymbol{q}_2} e^{-i\boldsymbol{q}_1\cdot\boldsymbol{z}}e^{i\boldsymbol{q}_2\cdot\bar{\boldsymbol{y}}}S^{(2)}(\boldsymbol{k},\boldsymbol{k},\infty;\boldsymbol{q}_1,\boldsymbol{q}_2,\bar{s})|J(\boldsymbol{x}_0)|^2$$

Gluons typically acquire momentum $k_f^2 \sim \sqrt{\hat{q}\omega}$

Gluons typically acquire momentum $k_b^2 \sim \hat{q}L$

Conclusion

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1 Momentum broadening in dense anisotropic media

→ The **broadening distribution** can be resumed for **non-flowing anisotropic** media

→ Final distribution gives parametrically small corrections to the leading result. **However,** these contribute at leading order in the azimuthal distribution

2 Radiative energy loss in dense anisotropic media

→ **BDMPS-Z** style calculation is in principle possible in the multiple scattering regime

→ Pheno oriented **effective soft factorization is not broken**

→ Further observable oriented calculations are needed to gauge and extract these effects