

Jet quenching in evolving QCD matter

Andrey Sadofyev

IGFAE (USC)



MARIE CURIE

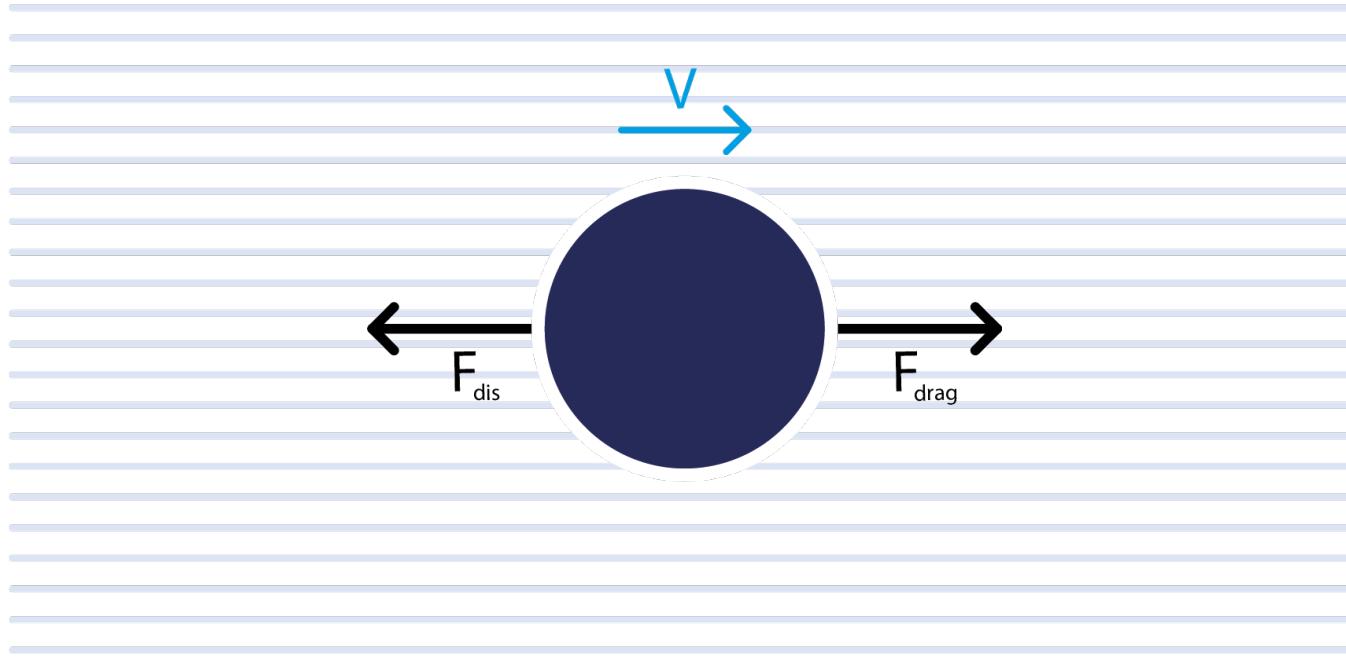
Jet Tomography

- The hot nuclear matter in HIC undergoes multi-phase evolution and its details are hard to access through the soft sector. In turn, jets see the matter at multiple scales, and essentially X-ray it;
- However, most approaches to the jet-medium interaction are either empirical or based on multiple simplifying assumptions – static matter, no fluctuations, etc;
- In what follows I will highlight our recent progress on the medium motion and structure effects in the QCD calculations for jet broadening and medium-induced radiation;
- The developed formalism can be also applied to include orbital motion of nucleons and some of in-medium fluctuations in the DIS context;





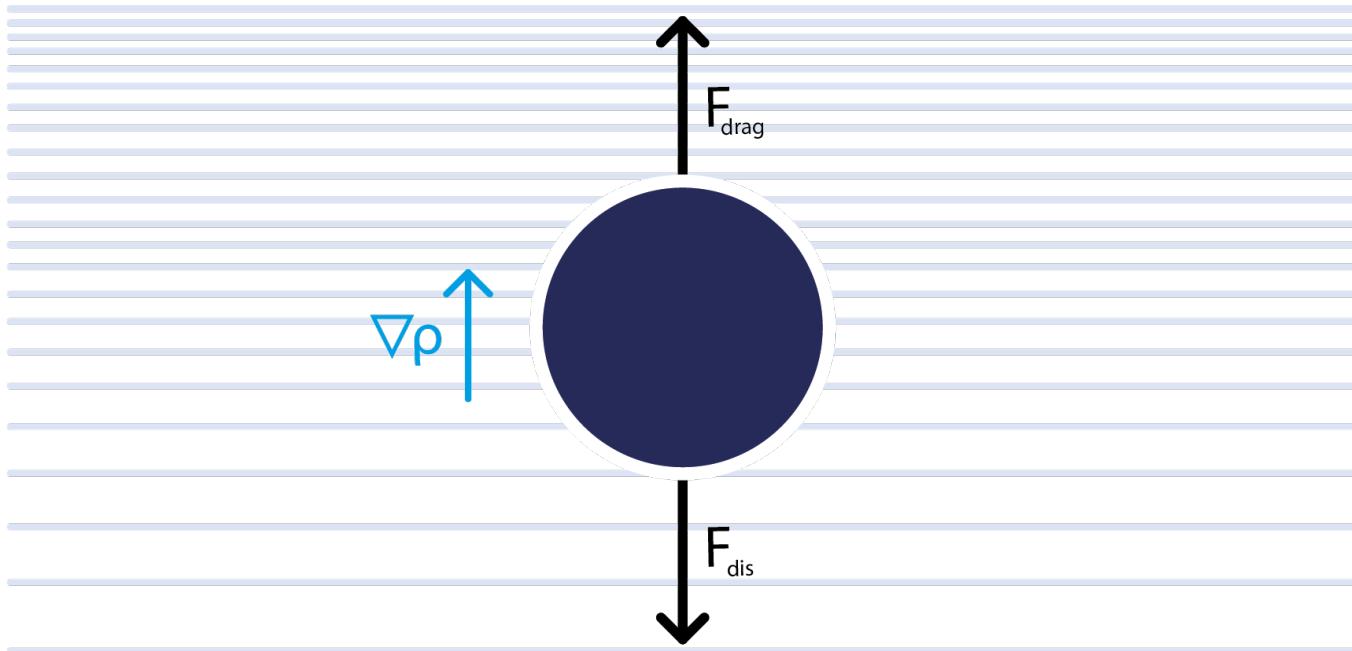
Drag Force



$$\vec{f} \sim T^2 \vec{v}$$



Drag Force

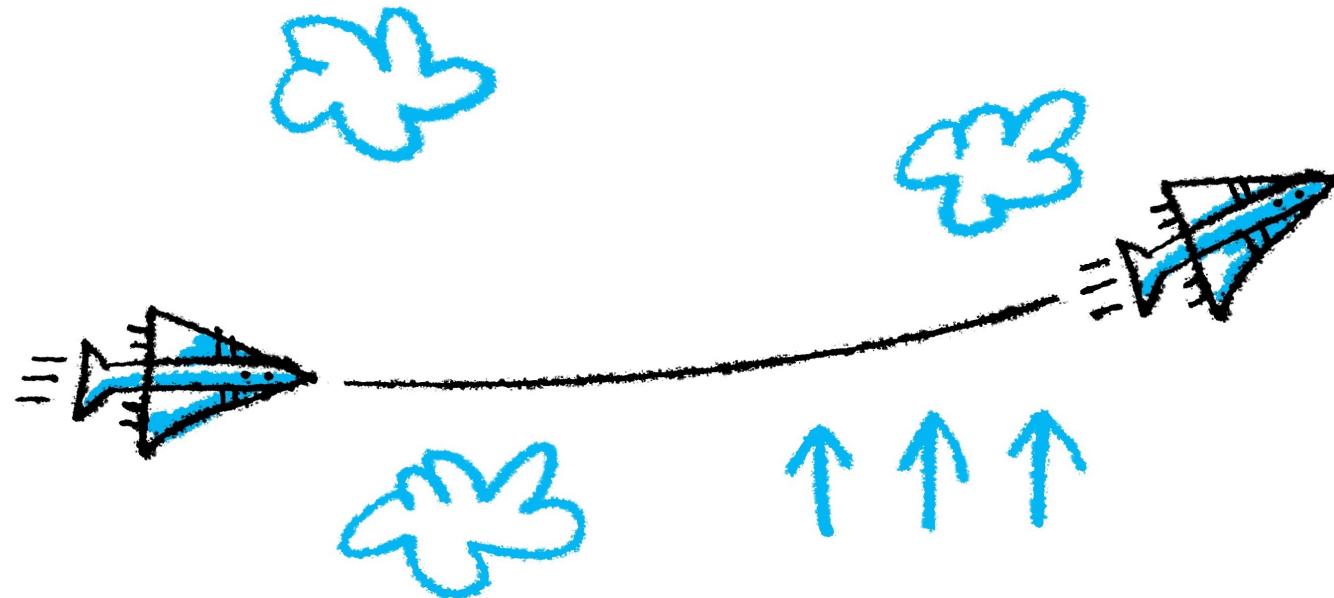


$$\vec{f} \sim \vec{\nabla} \rho$$



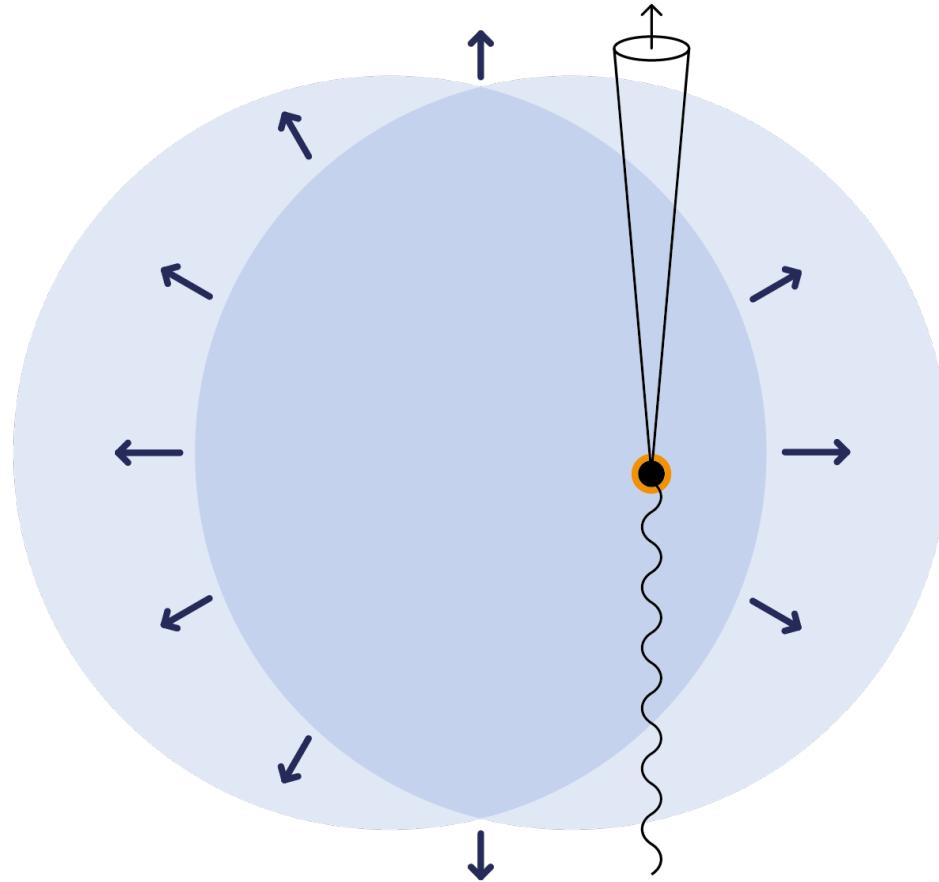
Jets

Does a jet feel the flow?



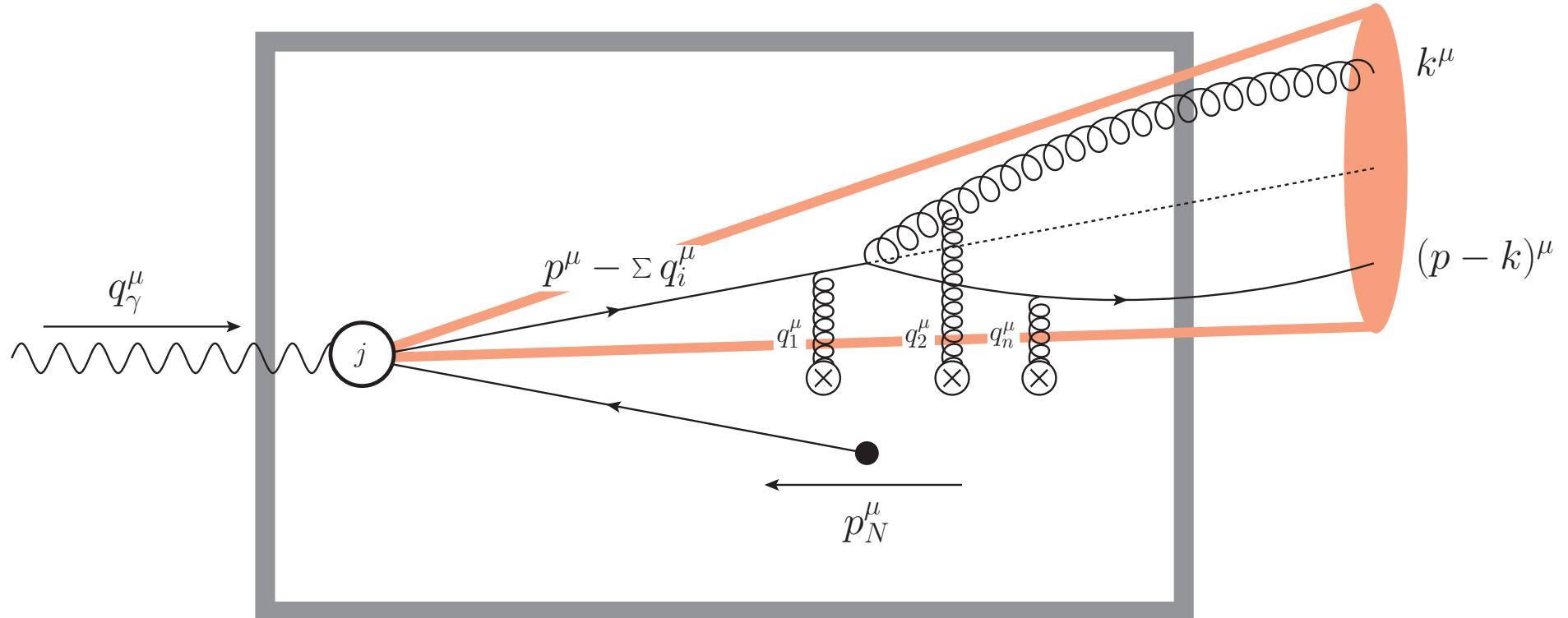


Jets



Jets

QCD broadening and gluon emission
(GLV/BDMPS-Z) with flow

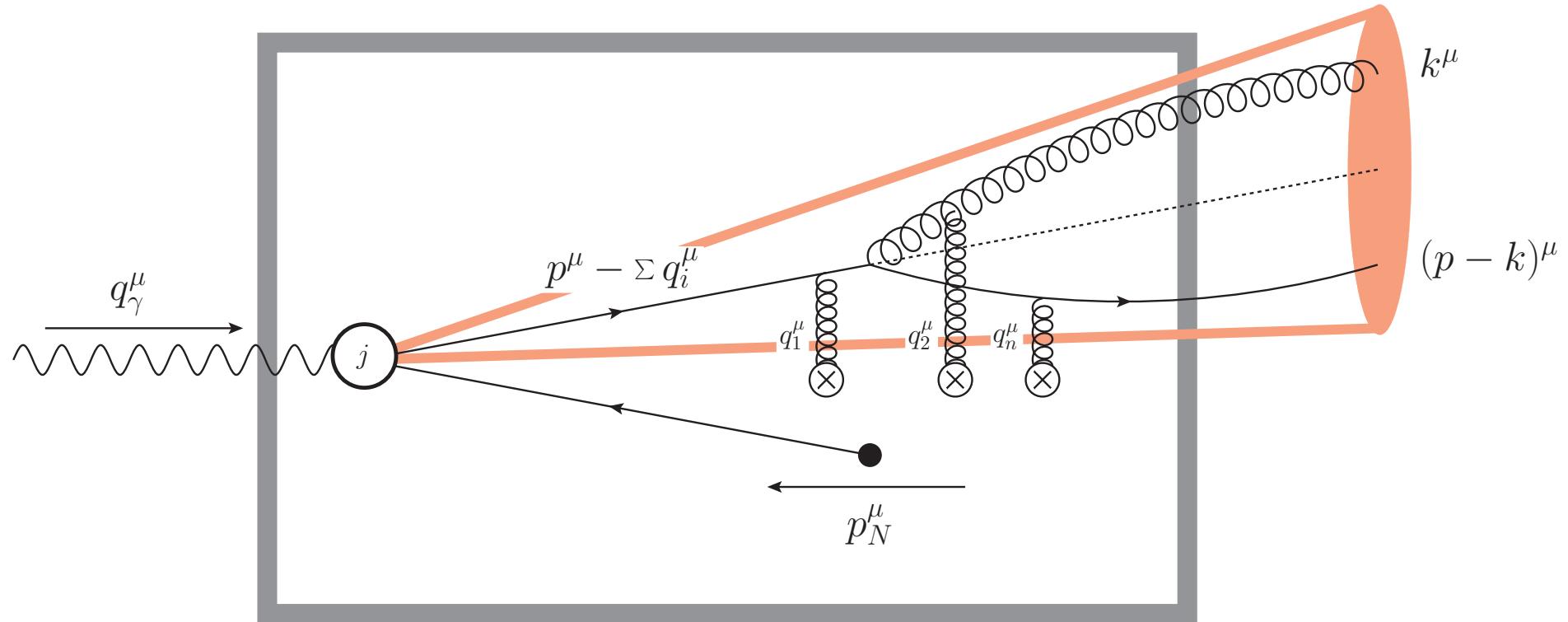


R. Baier et al, NPB, 1997
B. G. Zakharov, JETP, 1997
R. Baier et al, NPB, 1998
M. Gyulassy et al, NPB, 2000
M. Gyulassy et al, NPB, 2001
...

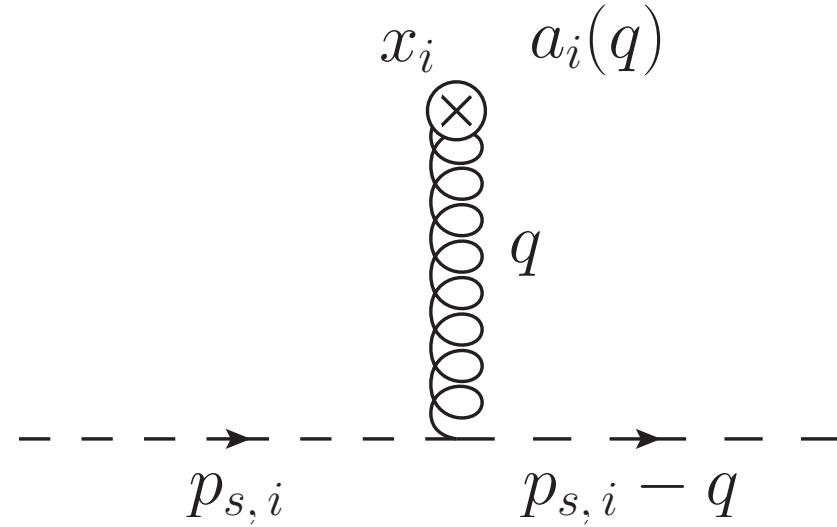


Jets

QCD broadening and gluon emission
(GLV/BDMPS-Z) with flow



Color Potential

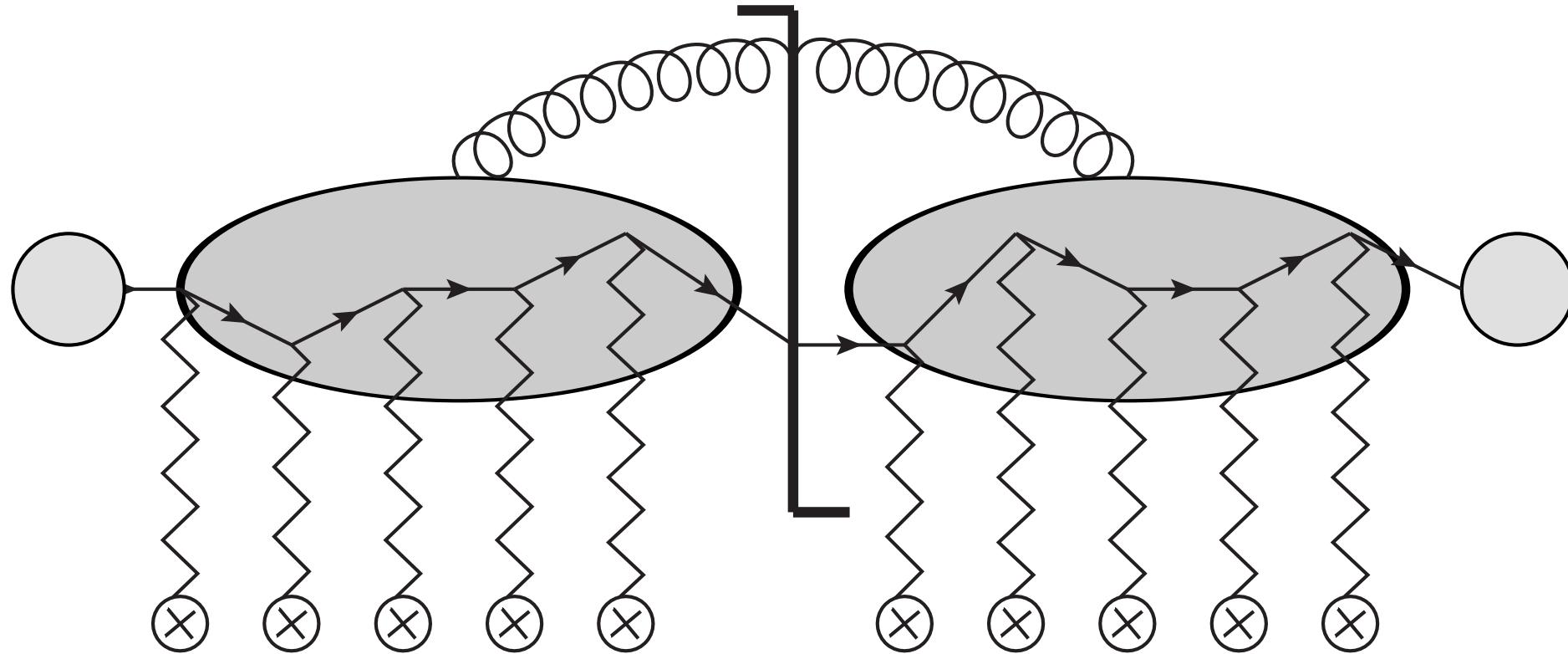


$$A^{\mu a}(q) = \sum_i (ig t_i^a) e^{iq \cdot x_i} (2p_{s,i} - q)_\nu \frac{-ig^{\mu\nu}}{q^2 - \mu_i^2 + i\epsilon} (2\pi) \delta((p_{s,i} - q)^2 - M^2).$$

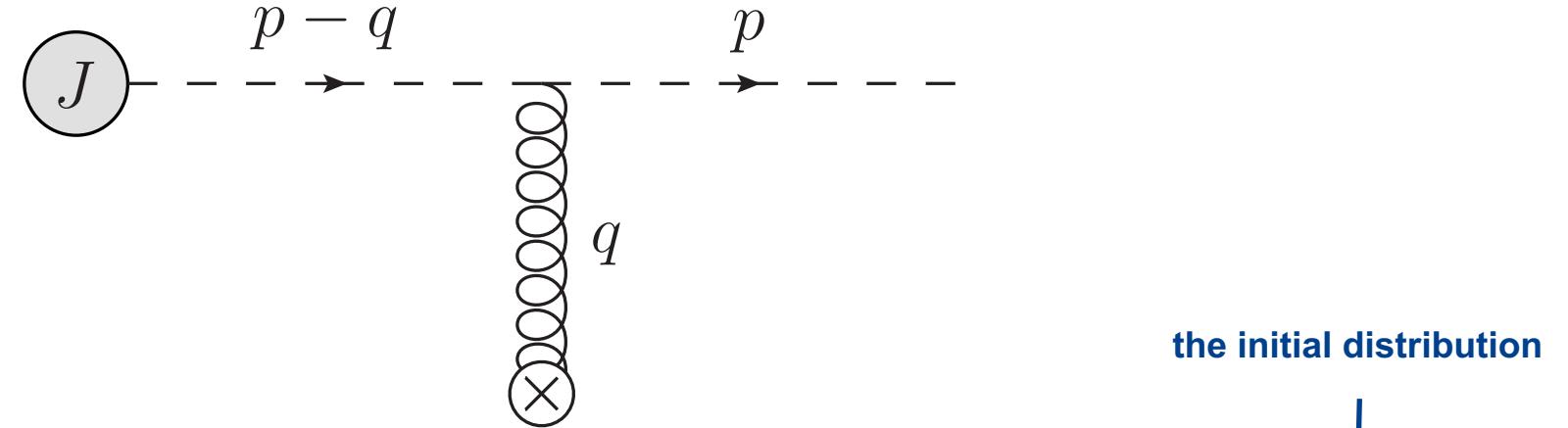
v(q²) -- the Gyulassy-Wang potential

large ↓





Jet Broadening



$$iM_1(p) = \int \frac{d^4q}{(2\pi)^4} \left[ig t_{\text{proj}}^a A_{\text{ext}}^{\mu a}(q) (2p - q)_\mu \right] \left[\frac{i}{(p - q)^2 + i\epsilon} \right] J(p - q)$$

$$gA_{\text{ext}}^{\mu a}(q) = \sum_i e^{iq \cdot x_i} t_i^a u_i^\mu v_i(q) (2\pi) \delta(q^0 - \vec{u}_i \cdot \vec{q})$$

the fluid velocity



Jet Broadening

Eikonal approximation -- $E \rightarrow \infty$

$$\frac{(2p - q)_\mu A_{ext}^\mu(q)}{(p - q)^2 + i\epsilon} \xrightarrow{\quad} \frac{2u_\mu p^\mu}{(1 - u_z^2)(Q^+ - Q^-)} = 1 - \frac{\vec{u}_\perp \cdot (\vec{p} - \vec{q})_\perp}{E(1 - u_z)} + \mathcal{O}\left(\frac{p_\perp^2}{E^2}\right)$$

$$v(q)$$

$$\mu \ll E \quad \mu z \gg 1$$

$$Q_{p-q}^+ = \frac{2E}{1 + u_{iz}} \left[1 - \frac{\vec{u}_{i\perp} \cdot \vec{q}_\perp}{2E} + \mathcal{O}\left(\frac{p_\perp^2}{E^2}\right) \right],$$

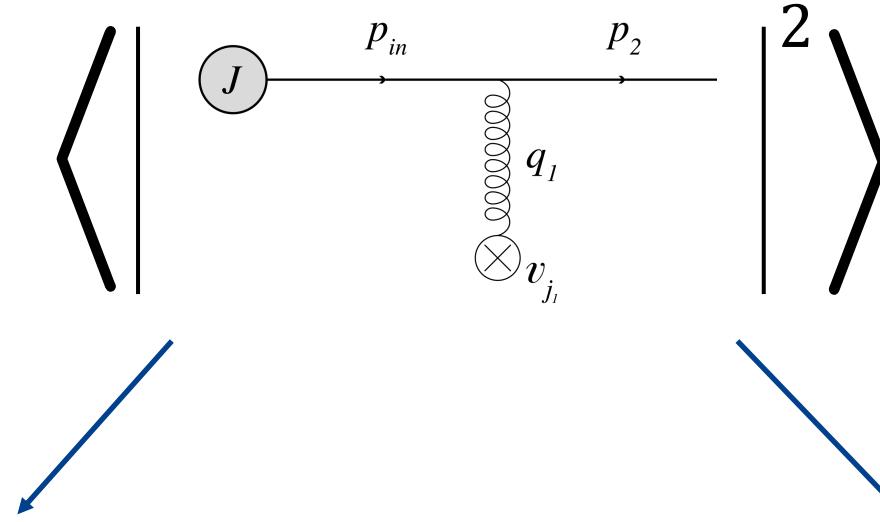
$$Q_{p-q}^- = \frac{\vec{u}_{i\perp} \cdot \vec{q}_\perp}{1 - u_{iz}} + \frac{(p - q)_\perp^2 - p_\perp^2}{2E(1 - u_{iz})} + \mathcal{O}\left(\frac{p_\perp^2}{E^2}\right)$$

↑
modified LPM phase





Medium Averaging



$$\langle t_i^a t_j^b \rangle = C \delta_{ij} \delta^{ab}$$

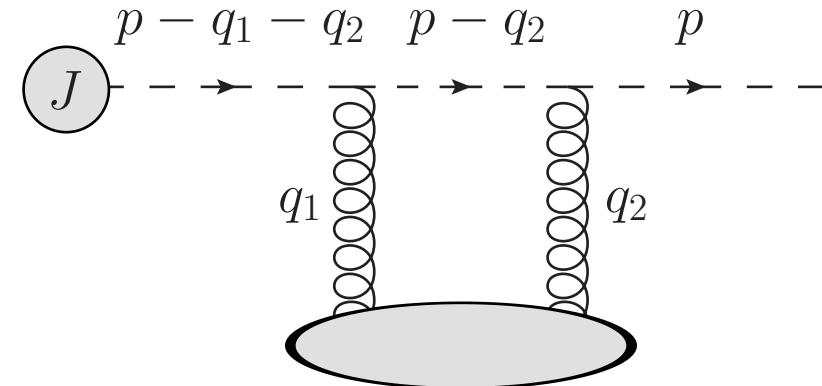
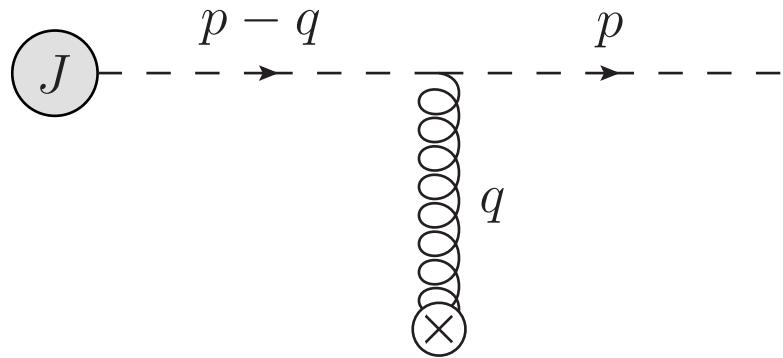
color neutrality
(one sum out of the two)

$$\sum_i = \int d^3x \rho(\vec{x})$$

source averaging
(if only the phase depends on x_\perp there
is only one momentum integration)

Jet Broadening

static case:



$$E \frac{dN^{(1)}}{d^3p} = - \int dz \int d^2\mathbf{q} \ \mathcal{V}(\mathbf{q}, z) E \frac{dN^0}{d^2(\mathbf{p} - \mathbf{q})dE}$$



$$\mathcal{V}(\mathbf{q}, z) \equiv -\mathcal{C} \rho(z) \left(|v(q_\perp^2)|^2 - \delta^{(2)}(\mathbf{q}) \int d^2\mathbf{l} \ |v(l_\perp^2)|^2 \right)$$

Jet Broadening

with uniform flow:

$$E \frac{dN^{(1)}}{d^3 p} = - \int dz \int d^2 \mathbf{q} \ \mathcal{V}(\mathbf{q}, z) \left[1 + \mathbf{u} \cdot \boldsymbol{\Omega}(\mathbf{q}) \right] \left(1 - \mathbf{u} \cdot \mathbf{q} \frac{\partial}{\partial E} \right) E \frac{dN^{(0)}}{d^2(\mathbf{p} - \mathbf{q}) dE}$$

$\boldsymbol{\Omega} = -\frac{\mathbf{p} - \mathbf{q}}{E} + \frac{\mathbf{q}}{E} \frac{(\mathbf{p} - \mathbf{q})^2 - \mathbf{p}^2}{v(\mathbf{q}^2)} \frac{\partial v}{\partial \mathbf{q}^2}$

- The finite collisional energy transfer q^0 to the jet results in a small shift in the energy of the initial jet distribution and in a shift of the transverse momentum spectrum;
- The first term in Ω , in turn, appears due to a sub-eikonal correction to the vertex, a penalty for bending the jet, and the modification of the propagator due to the energy transfer, which can increase the scattering amplitude;





Jet Broadening

with uniform flow:

$$E \frac{dN^{(0)}}{d^3 p} = \frac{1}{2(2\pi)^3} |J(p)|^2 = f(E) \delta^{(2)}(\vec{p}_\perp)$$

$$\langle \mathbf{p}_\perp (p_\perp^2)^k \rangle = -\frac{\mathbf{u}_\perp}{(1-u_z)} \frac{L}{2\lambda} (\mu^2)^{k+1} \left(-\frac{2}{E} \int_0^\infty d\xi \frac{\xi^{k+2}}{(1+\xi)^3} + \frac{1}{f(E)} \frac{\partial f}{\partial E} \int_0^\infty d\xi \frac{\xi^{k+1}}{(1+\xi)^2} \right)$$

\downarrow \uparrow

1/ $\rho\sigma_0$ **the mean free path**

the distribution in energies

depend on the elastic scattering cross section (GW here)

$$\left\langle \frac{\vec{p}_\perp}{p_\perp^2} \right\rangle = \frac{5}{2} \frac{\vec{u}_\perp}{(1-u_z)} \frac{L}{\lambda} \frac{1}{E}$$

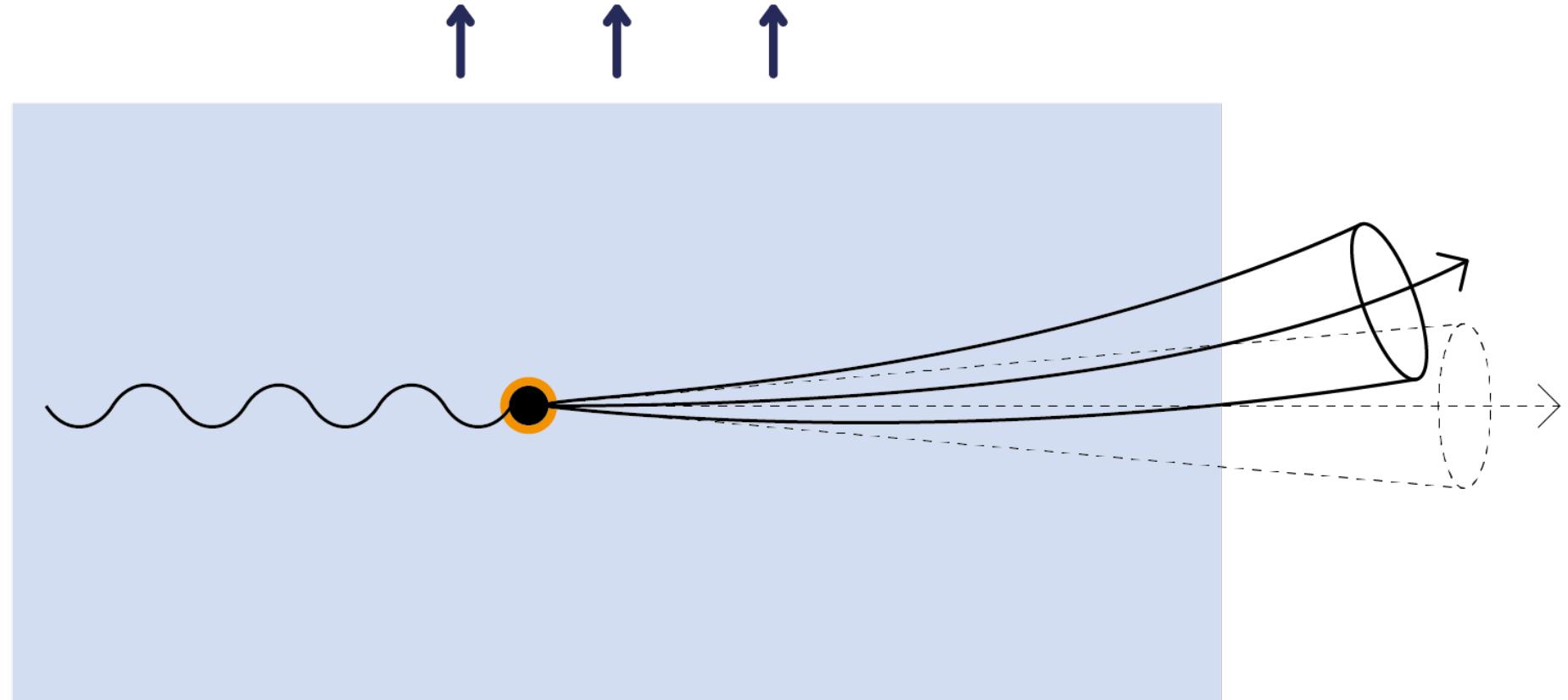




IGFAE
Instituto Galego de Física de Altas Enerxías

USC
UNIVERSITY
OF SANTIAGO
DE COMPOSTELA

XUNTA
DE GALICIA

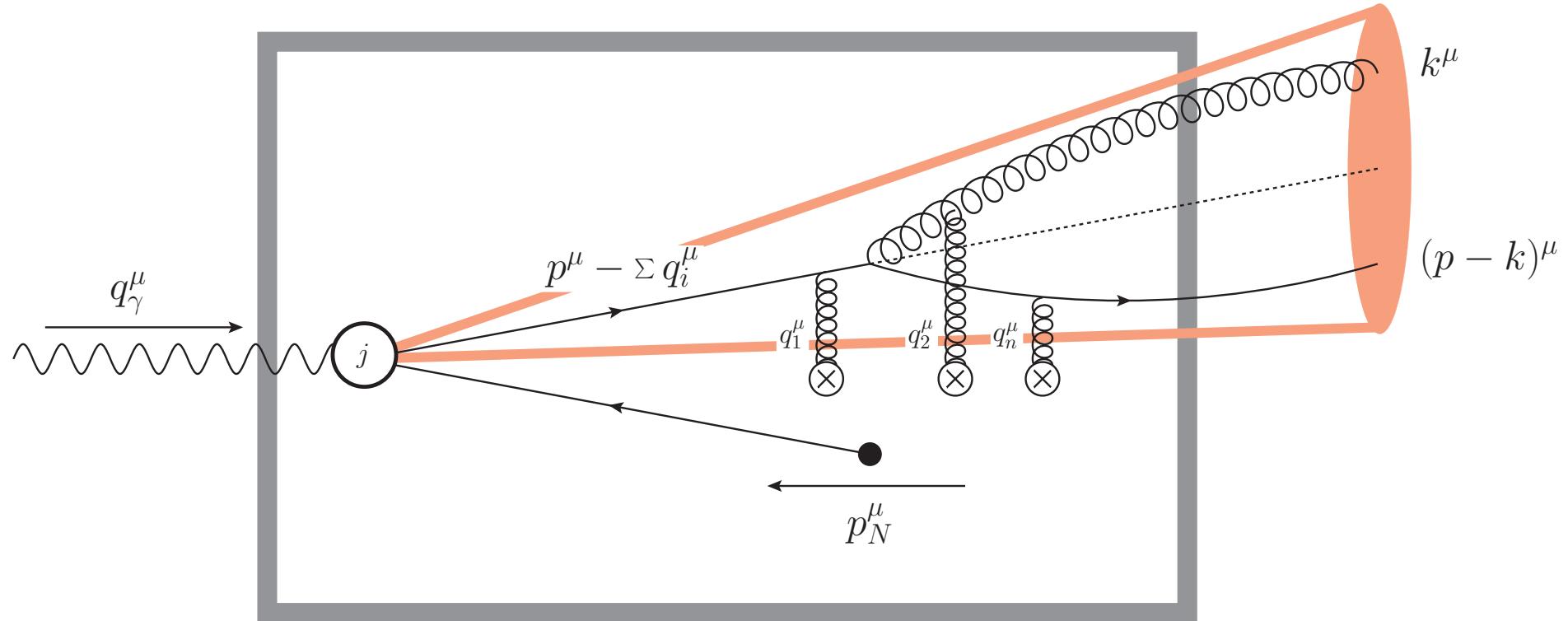


Instituto Galego de Física de Altas Enerxías (IGFAE)

igfae.usc.es

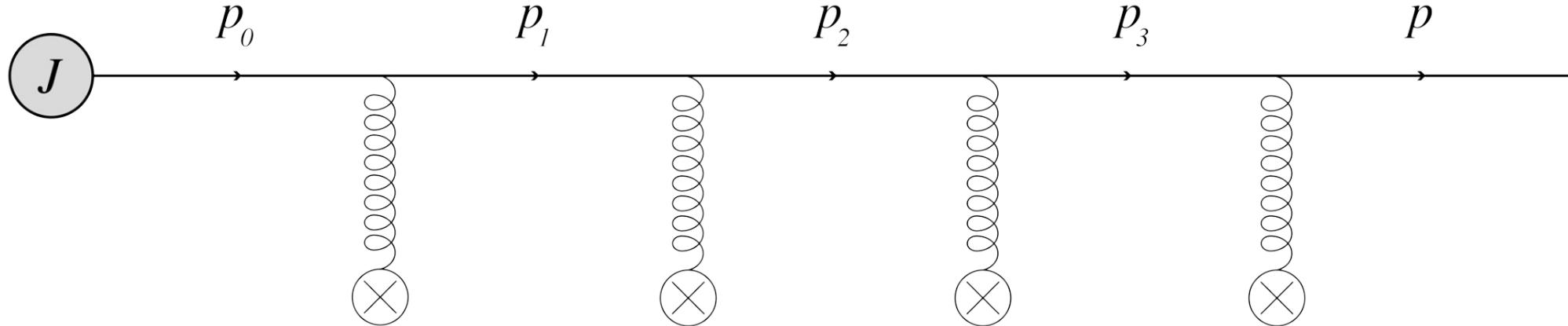
Jets

QCD broadening and gluon emission
(GLV/BDMPS-Z) with flow





The Propagator



$$i\mathcal{M}_n(p) = e^{-i\left(\mathbf{u}\cdot\mathbf{p} - \frac{p_\perp^2}{2E}\right)L} \int \frac{d^2\mathbf{p}_0}{(2\pi)^2} \mathcal{G}_n(\mathbf{p}, L; \mathbf{p}_0, 0) J(E - \mathbf{u} \cdot (\mathbf{p} - \mathbf{p}_0), \mathbf{p}_0),$$





$$\Omega = -\frac{\mathbf{p} - \mathbf{q}}{E} + \frac{\mathbf{q}}{E} \frac{(p - q)^2 - p^2}{v(\mathbf{q}^2)} \frac{\partial v}{\partial \mathbf{q}^2}$$

The Propagator

$$\begin{aligned} \frac{\partial}{\partial L} \mathcal{G}(\mathbf{p}, L; \mathbf{p}_0, z_0) &= i \left(\mathbf{u} \cdot \mathbf{p} - \frac{p_\perp^2}{2E} \right) \mathcal{G}(\mathbf{p}, L; \mathbf{p}_0, z_0) \\ &+ i \int \frac{d^2 \mathbf{q}}{(2\pi)^2} [1 + \mathbf{u} \cdot \Omega(\mathbf{p}, \mathbf{q})] v(q_\perp^2) \hat{\rho}^a(\mathbf{q}, L) t^a \mathcal{G}(\mathbf{p} - \mathbf{q}, L; \mathbf{p}_0, z_0), \end{aligned}$$

↑

not a convolution anymore

$$\mathcal{G}(\mathbf{p}, L; \mathbf{p}_0, z_0) = \mathcal{G}^{(0)}(\mathbf{p}, L; \mathbf{p}_0, z_0) + \mathcal{G}^{(1)}(\mathbf{p}, L; \mathbf{p}_0, z_0) + \mathcal{O}\left(\frac{p_\perp^2}{E^2}\right)$$

**still can be solved
analytically**



$$\boldsymbol{\Omega} = -\frac{\mathbf{p} - \mathbf{q}}{E} + \frac{\mathbf{q}}{E} \frac{(\mathbf{p} - \mathbf{q})^2 - \mathbf{p}^2}{v(\mathbf{q}^2)} \frac{\partial v}{\partial \mathbf{q}^2}$$

The Propagator

$$\begin{aligned} \mathcal{G}^{(0)}(\mathbf{x}, L; \mathbf{x}_0, z_0) &= \int_{\mathbf{x}_0}^{\mathbf{x} + (L-z_0)\mathbf{u}} \mathcal{D}\mathbf{r} \exp \left(\frac{iE}{2} \int_{z_0}^L d\xi \dot{\mathbf{r}}^2(\xi) \right) \\ &\quad \times \mathcal{P} \exp \left(i \int_{z_0}^L d\xi \mathcal{A}(\mathbf{r}(\xi) - (\xi - z_0)\mathbf{u}, \xi) \right) \end{aligned}$$



$$\begin{aligned} \mathcal{G}^{(1)}(\mathbf{p}, L; \mathbf{p}_0, z_0) &= i \int_{z_0}^L dz \int \frac{d^2\mathbf{q}}{(2\pi)^2} \frac{d^2\mathbf{l}}{(2\pi)^2} \mathcal{G}^{(0)}(\mathbf{p}, L; \mathbf{l}, z) (\mathbf{u} \cdot \boldsymbol{\Omega}(\mathbf{l}, \mathbf{q})) v(q_\perp^2) \\ &\quad \times \hat{\rho}^a(\mathbf{q}, z) t^a \mathcal{G}^{(0)}(\mathbf{l} - \mathbf{q}, z; \mathbf{p}_0, z_0) \end{aligned}$$

Jet Broadening

dense matter

$$\frac{1}{d_{proj}} \left\langle \text{Tr} \left[\mathcal{G}^\dagger(\mathbf{p}'_0, 0; \mathbf{p}, L) \mathcal{G}(\mathbf{p}, L; \mathbf{p}_0, 0) \right] \right\rangle \equiv (2\pi)^2 \delta^{(2)}(\mathbf{p}_0 - \mathbf{p}'_0) \mathcal{P}(\mathbf{p}, L; \mathbf{p}_0, 0)$$

$$\frac{\partial}{\partial L} \mathcal{P}(\mathbf{p}, L; \mathbf{p}_0, 0) = - \int \frac{d^2 \mathbf{q}}{(2\pi)^2} \sigma(\mathbf{p}, \mathbf{q}; L) \mathcal{P}(\mathbf{p} - \mathbf{q}, L; \mathbf{p}_0, 0)$$

$$\mathcal{P}(\mathbf{p}, 0; \mathbf{p}_0, 0) = (2\pi)^2 \delta^{(2)}(\mathbf{p} - \mathbf{p}_0)$$



Jet Broadening

dense matter

$$\mathcal{P}^{(0)}(\mathbf{r}, L; \mathbf{r}_0, 0) = e^{-\mathcal{V}(\mathbf{r})L} \delta^{(2)}(\mathbf{r} - \mathbf{r}_0)$$

$$\mathcal{V}(\mathbf{q}, z) \equiv -\mathcal{C} \rho(z) \left(|v(q_\perp^2)|^2 - \delta^{(2)}(\mathbf{q}) \int d^2\mathbf{l} |v(l_\perp^2)|^2 \right)$$



Jet Broadening

dense matter

$$\begin{aligned} \mathcal{P}^{(1)}(\mathbf{p}, L; \mathbf{p}_0, 0) = & \int d^2\mathbf{r} e^{-i(\mathbf{p}-\mathbf{p}_0)\cdot\mathbf{r}} e^{-\mathcal{V}(\mathbf{r})L} \frac{u_\alpha}{E} \left[2L \mathbf{p}_{0\beta} \left(\mathcal{V}(\mathbf{r})\delta_{\alpha\beta} + \mathcal{V}_{\alpha\beta}(\mathbf{r}) \right) \right. \\ & \left. - iL \nabla_\beta \mathcal{V}_{\alpha\beta}(\mathbf{r}) + iL^2 \left(\mathcal{V}(\mathbf{r})\delta_{\alpha\beta} + \mathcal{V}_{\alpha\beta}(\mathbf{r}) \right) \nabla_\beta \mathcal{V}(\mathbf{r}) \right] \end{aligned}$$

$$\mathcal{V}_{\alpha\beta} = \mathcal{C} \rho \left[-\mathbf{q}_\alpha \mathbf{q}_\beta \frac{\partial v^2}{\partial q_\perp^2} - (2\pi)^2 \delta^{(2)}(\mathbf{q}) \frac{\delta_{\alpha\beta}}{2} \int \frac{d^2\mathbf{l}}{(2\pi)^2} v(l_\perp^2)^2 \right]$$



Jet Broadening

dense matter

Eikonal approximation -- $E \rightarrow \infty$

$$\langle p_{\perp}^{2k} \mathbf{p} \rangle = \int \frac{d^2 \mathbf{p} d^2 \mathbf{r}}{(2\pi)^2} p_{\perp}^{2k} \mathbf{p} e^{-i \mathbf{p} \cdot \mathbf{r}} e^{-\mathcal{V}(\mathbf{r})L} = 0 + \mathcal{O}\left(\frac{\perp}{E}\right)$$

Opacity expansion -- $\chi \ll 1$

$$\begin{aligned} \langle p_{\perp}^{2k} \mathbf{p} \rangle|_{N=1} &= -\frac{\mathbf{u}}{2} \frac{f'(E)}{f(E)} \langle p_{\perp}^{2k+2} \rangle|_{N=1} - \frac{\mathbf{u}}{2E} L \int \frac{d^2 \mathbf{p} d^2 \mathbf{r}}{(2\pi)^2} p_{\perp}^{2k} e^{-i \mathbf{p} \cdot \mathbf{r}} \nabla_{\beta} \nabla_{\gamma} \mathcal{V}_{\beta\gamma}(\mathbf{r}) \\ &= -\frac{\mathbf{u}}{2E} \mathcal{C} \rho L \int \frac{d^2 \mathbf{p}}{(2\pi)^2} p_{\perp}^{2k+2} \left[E \frac{f'(E)}{f(E)} v(p_{\perp})^2 + p_{\perp}^2 \frac{\partial v^2}{\partial p_{\perp}^2} \right] \end{aligned}$$



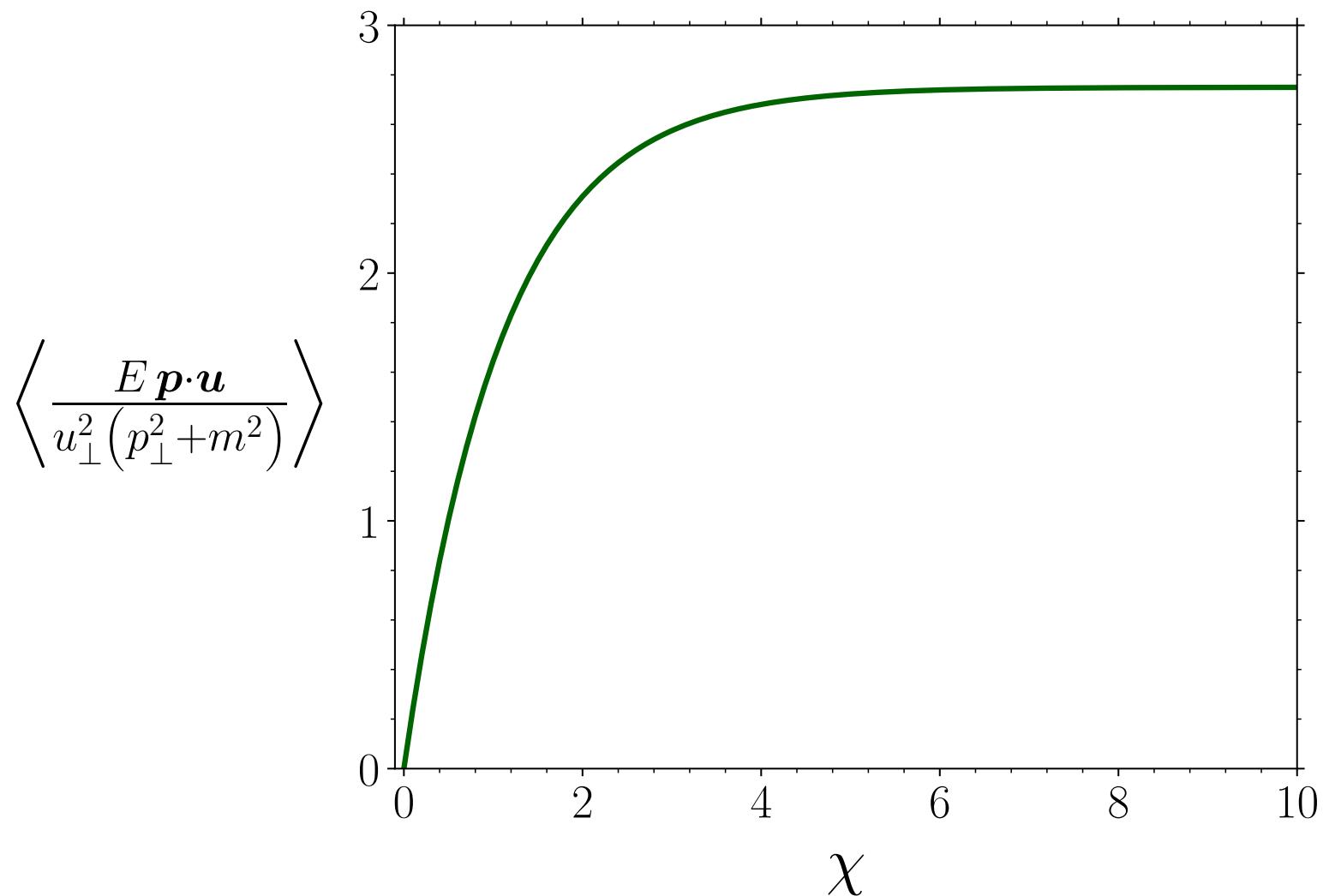
Jet Broadening

dense matter

$$\begin{aligned}
 \langle \mathbf{p} \rangle &= -\frac{\mathbf{u}}{2} \frac{f'(E)}{f(E)} \langle p_\perp^2 \rangle - i \frac{\mathbf{u}}{2E} \nabla_\beta \left\{ e^{-\mathcal{V}(\mathbf{r})L} \left[iL^2 \left(\mathcal{V}(\mathbf{r})\delta_{\beta\gamma} + \mathcal{V}_{\beta\gamma}(\mathbf{r}) \right) \nabla_\gamma \mathcal{V}(\mathbf{r}) - iL \nabla_\gamma \mathcal{V}_{\beta\gamma}(\mathbf{r}) \right] \right\} \Big|_{\mathbf{r}=\mathbf{0}} \\
 &= -\frac{\mathbf{u}}{2E} \mathcal{C} \rho L \int \frac{d^2 \mathbf{p}}{(2\pi)^2} p_\perp^2 \left[E \frac{f'(E)}{f(E)} v(p_\perp)^2 + p_\perp^2 \frac{\partial v^2}{\partial p_\perp^2} \right] + \mathcal{O}\left(\frac{\perp^2}{E^2}\right)
 \end{aligned}$$



$$\chi \equiv \frac{\mathcal{C}g^4\rho}{4\pi\mu^2}L$$



igfae.usc.es

Jet Broadening

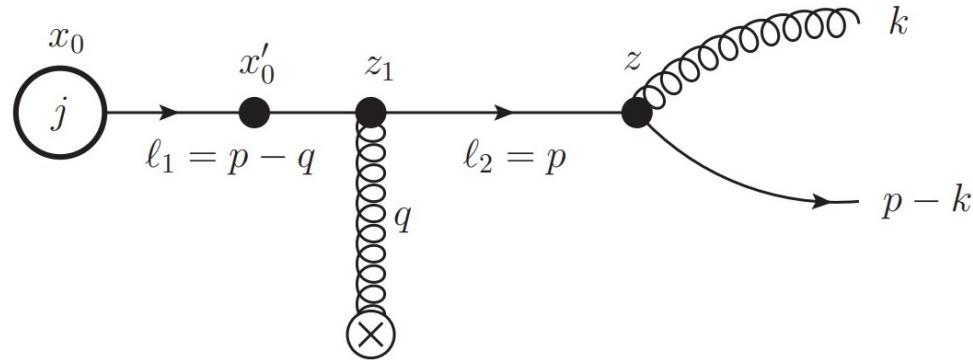
dense matter

$$E \frac{d\mathcal{N}}{d^2\mathbf{p} dE} = \int \frac{d^2\mathbf{p}_0}{(2\pi)^2} \mathcal{P}(\mathbf{p}, L; \mathbf{p}_0, 0) \left[1 - \mathbf{u} \cdot (\mathbf{p} - \mathbf{p}_0) \frac{\partial}{\partial E} \right] E \frac{d\mathcal{N}^{(0)}}{d^2\mathbf{p}_0 dE}$$

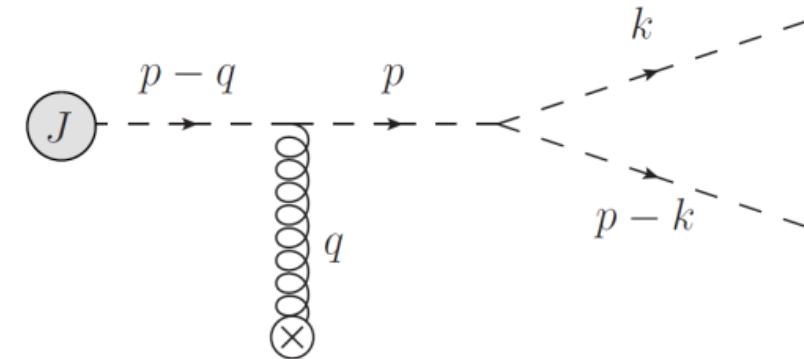
- The odd moments of this re-summed final distribution are proportional to the transverse flow velocity, while the even moments are unmodified;
- One can readily check that this distribution reproduces the results at the first order in opacity (expanding in powers of the potential);
- The initial and final distributions are not factorized anymore in coordinate space (due to the energy derivative);



“Gluon” Emission



$$\mathcal{L} = \mathcal{L}_{QCD} + g \bar{\psi} \gamma_\mu A_{ext}^{\mu a} t^a \psi + \dots$$



$$\mathcal{L} = \mathcal{L}_{\phi^3} - ig (\partial_\mu \phi) A_{ext}^{\mu a} t^a \phi^c$$

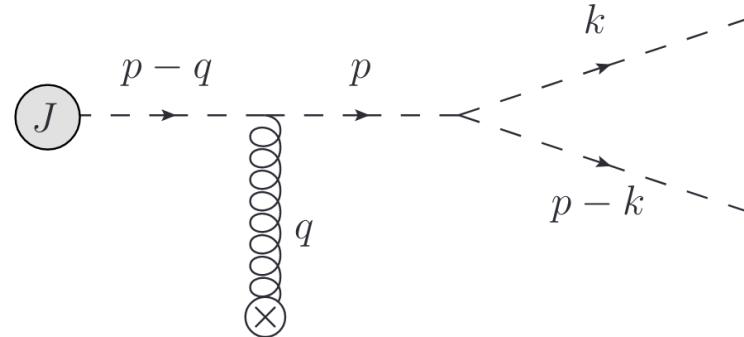
- Interchangeability of light-front wave functions;
- Universality of high-energy scattering;



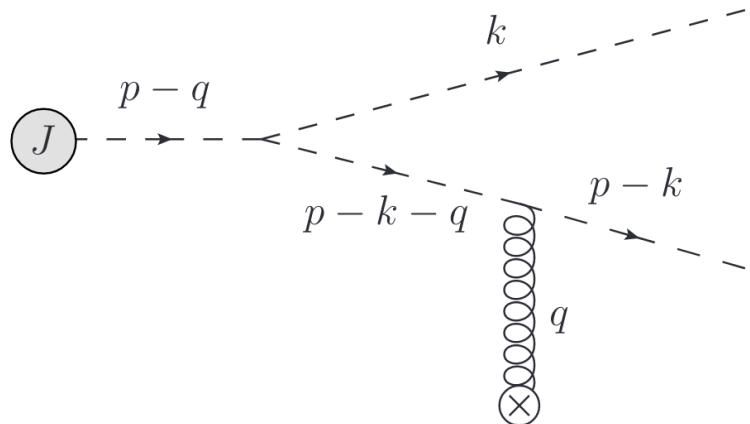


“Gluon” Emission

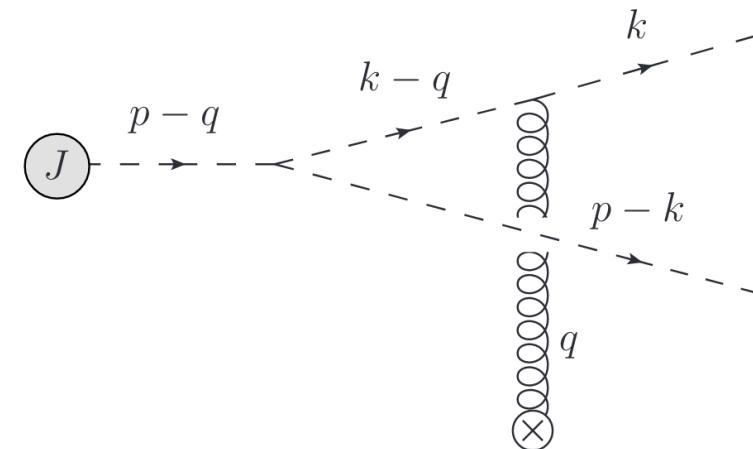
A



B

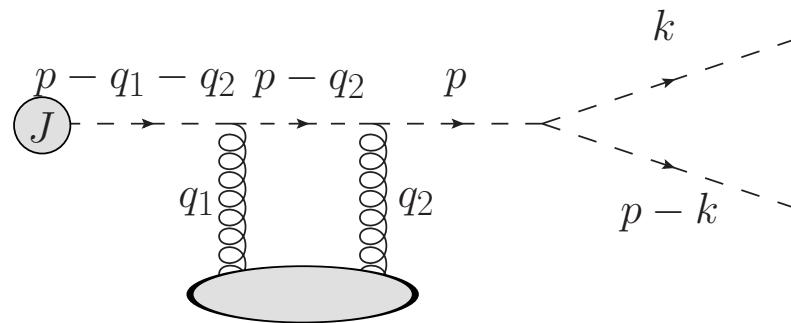
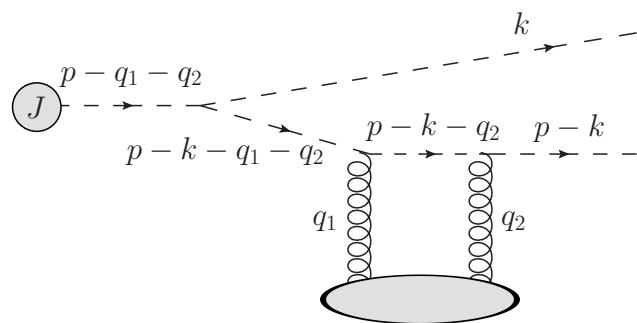
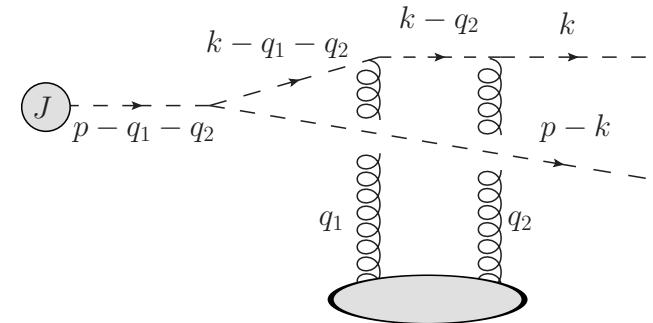
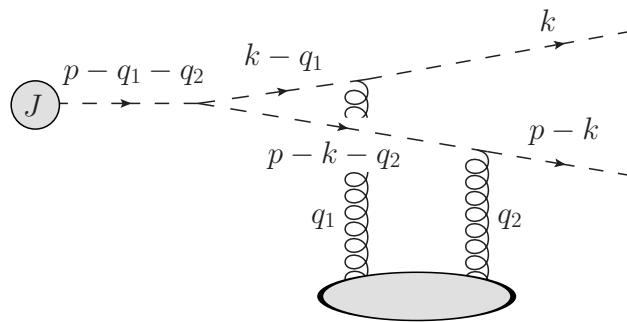
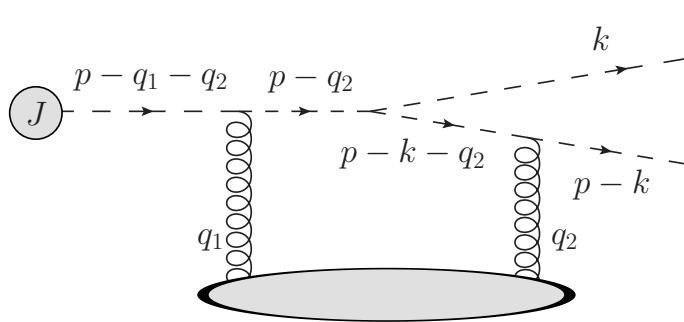
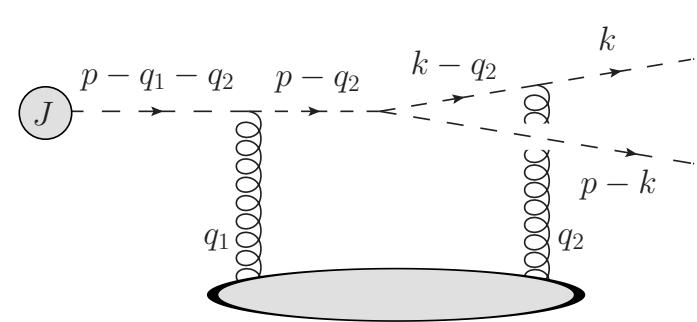


C





“Gluon” Emission

D**E****F****G****H****I**

“Gluon” Emission

In the small-x limit, and for a broad source, the gluon emission spectrum reduces to

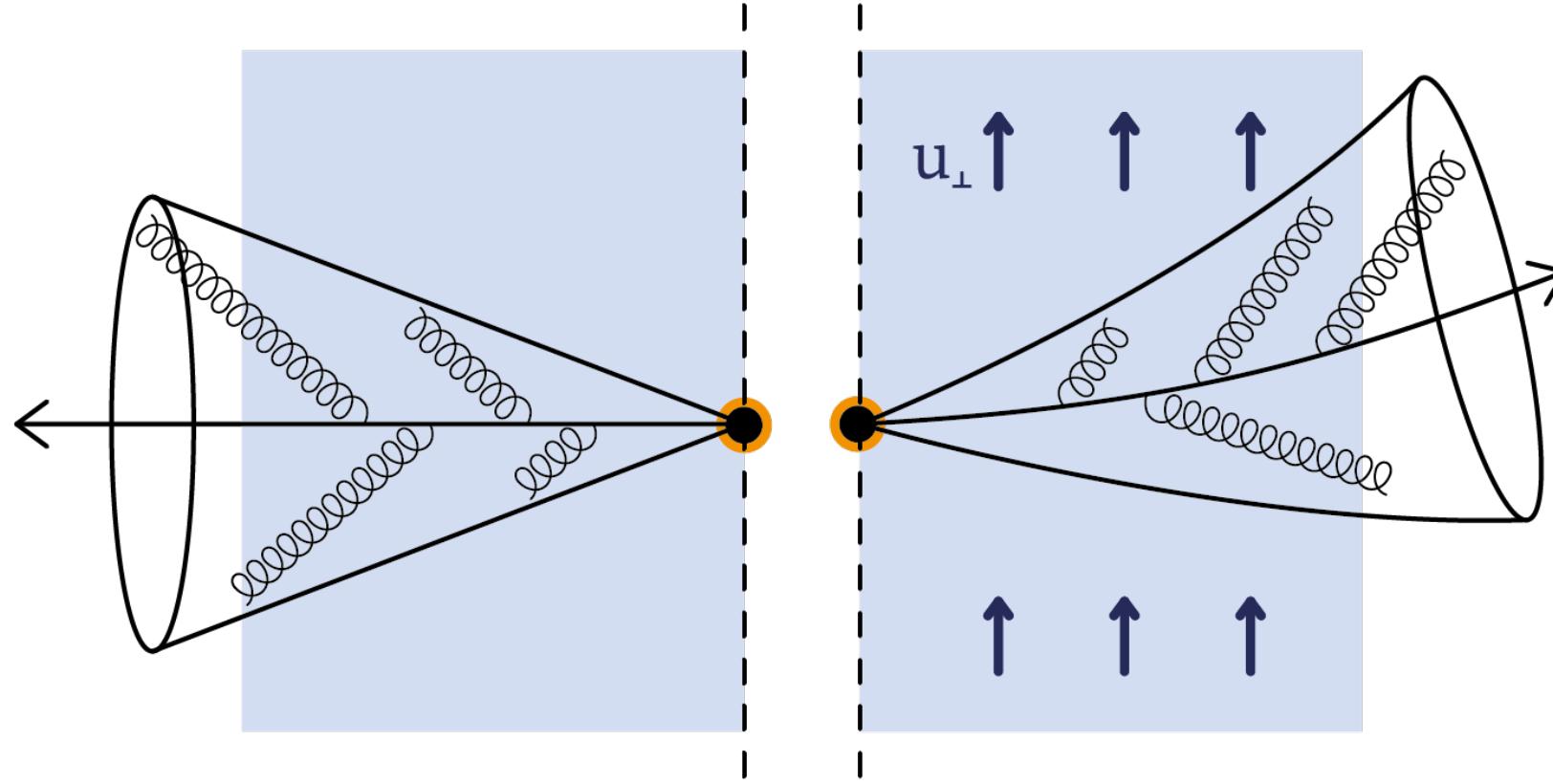
$$E \frac{dN^{(1)}}{d^2 k_\perp dx d^2 p_\perp dE} = \frac{\alpha_s N_c}{\pi^2 x} \left(E \frac{dN^{(0)}}{d^2 p_\perp dE} \right) \int_0^L dz \rho \int d^2 q_\perp \bar{\sigma}(q_\perp^2)$$

$$\times \left\{ \frac{2 \vec{k}_\perp \cdot \vec{q}_\perp}{k_\perp^2 (k - q)_\perp^2} \left(1 - \cos \left(\frac{(k - q)_\perp^2}{2xE(1 - u_z)} z \right) \right) + \frac{q_\perp^2}{k_\perp^2 (q_\perp^2 + \mu^2)} \frac{\vec{u}_\perp \cdot \vec{k}_\perp}{2(1 - u_z)xE} \right\},$$

where the QCD LFWFs were substituted



$$\left\langle \frac{\vec{k}_\perp}{k_\perp^2} \right\rangle = \frac{N_c L}{C_F \lambda} \frac{\vec{u}_\perp}{8(1 - u_z)xE}$$



$$\left\langle \frac{\vec{k}_\perp}{k_\perp^2} \right\rangle = \frac{N_c L}{C_F \lambda} \frac{\vec{u}_\perp}{8(1 - u_z) x E}$$

Summary

- We have re-summed the jet broadening distribution to all orders in opacity in flowing matter and constructed the corresponding single particle propagator;
- The transverse flow bends the leading parton trajectory, contributing to odd moments of the final momentum distribution (anisotropic broadening);
- The transverse flow also affects the medium-induced radiation (opacity expansion), leading to non-zero odd moments of the gluon momentum distribution;
- The initial and final state effects are not (fully) factorized anymore due to the energy shift in the initial distribution;
- These results open multiple opportunities to include the medium motion and in-medium fluctuation effects into studies of other probes of nuclear matter;



Outlook

- We have constructed a generalization of the GLV approach which includes the effects of the medium evolution (and structure). With this tool one can consider effects of the general flow, temperature, and source density profiles;
- We have extended the BDMPS-Z formalism, including the same effects to all orders in opacity (partially work in progress: radiation in dense matter);
- In the context of DIS our formalism can be also used to study nucleon orbital motion and spatial inhomogeneities in the system (a relation to GPDs and TMDs);
- The same formalism can be used to study the initial dynamics, and one may attempt coupling jets to non-equilibrium nuclear matter;
- The medium response naturally appears in our considerations, and we will return to that in the near future;

