# Time reclustering for jet quenching



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# AF7

### In collaboration with A. Cordeiro, P. Rodriguez and K. Zapp



## QGP: a fast expanding medium

What is the information that we get?

Integrated result of the whole medium (fast)  $\bigstar$ evolution

 However there is a strong timedependence of the medium properties (expansion and cooling of the system)

[talks: Carlota, Souvik, Andrey, João]

Can hard probes be able to probe different structures of the QGP?





## Sensitivity to QGP timescales

### Reconstructed hadronic W boson jet mass:



+







### Reconstructed hadronic W boson jet mass:





### @ FCC: Full QGP tomography!



Tops can be used as probes of the QGP time structure

Limited by statistics at current LHC energies

How about jets? +







### Jets

[C/A: Dokshitzer, et al (1997), Wobish, et al (1998)] [kT: Catani, et al (1993), Ellis et al (1993)] [Gen-kT (FastJet): Cacciari et al (2012)]







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What are jets?

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- Clustering of final state particles  $\bigstar$ 
  - User-defined hierarchical structure (sequential recombination algorithms)

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### Jets



What are jets?

+

Clustering of final state particles  $\bigstar$ 

User-defined hierarchical structure  $\mathbf{+}$ (sequential recombination algorithms)

Iterative distance between 2 pseudo-jets Generalized-k<sub>T</sub> family:

$$d_{ij} = \min(p_{t,i}^{2p}, p_{t,j}^{2p}) \frac{\Delta R_{ij}^2}{R^2} \quad d_{iB} = p_{t,i}^{2p}$$

p = 0: Cambridge/Aachen p = 1: k<sub>T</sub> p = -1: Anti- $k_T$ 

[C/A: Dokshitzer, et al (1997), Wobish, et al (1998)] [kT: Catani, et al (1993), Ellis et al (1993)] [Gen-kT (FastJet): Cacciari et al (2012)]

### Jets





Ordering in vacuum parton shower? To LL accuracy, all (virtuality,  $k_T$ ,...) equivalent

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### **Jets and Parton Showers**







**Expanding Medium** 

In-medium radiation will depend on local QGP parameters

## Medium-modified jets







































 $\theta_1 >> \theta_2 >> \theta_3 \dots \quad \mapsto \ t_1 >> t_2 >> t_3 \dots$ (Vacuum)  $\mapsto$  (QGP)



Generalised kt family of (reclustering) jet algorithms:

$$d_{ij} = \min(p_{t,i}^{2p}, p_{t,j}^{2p}) \frac{\Delta R_{ij}^2}{R^2} \quad d_{iB} = p_{t,i}^{2p}$$

0 (C/A)  $\vdots$ p = 0.5 ( $\tau$ )  $\vdots$ 1 (k<sub>T</sub>)

+





Generalised kt family of (reclustering) jet algorithms:

$$d_{ij} = \min(p_{t,i}^{2p}, p_{t,j}^{2p}) \frac{\Delta R_{ij}^2}{R^2} \quad d_{iB} = p_{t,i}^{2p}$$







Generalised kt family of (reclustering) jet algorithms:

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[PYTHIA 8: Sjostrand et al (2006)]



### Vacuum: PYTHIA 8 (k<sub>T</sub> ordered)





Generalised kt family of (reclustering) jet algorithms:

$$d_{ij} = \min(p_{t,i}^{2p}, p_{t,j}^{2p}) \frac{\Delta R_{ij}^2}{R^2} \quad d_{iB} = p_{t,i}^{2p}$$



### [PYTHIA 8: Sjostrand et al (2006)] [JEWEL: Zapp (2014)]



### Vacuum: PYTHIA 8 (k<sub>T</sub> ordered) Vacuum: JEWEL (~PYTHIA 6, Q<sup>2</sup> ordered)



Generalised kt family of (reclustering) jet algorithms:

$$d_{ij} = \min(p_{t,i}^{2p}, p_{t,j}^{2p}) \frac{\Delta R_{ij}^2}{R^2} \quad d_{iB} = p_{t,i}^{2p}$$



### [PYTHIA 8: Sjostrand et al (2006)] [JEWEL: Zapp (2014)]



Vacuum: PYTHIA 8 (k<sub>T</sub> ordered) Vacuum: JEWEL (~PYTHIA 6, Q<sup>2</sup> ordered) Medium: JEWEL (Q<sup>2</sup> ordered +  $\tau$  veto)





### **Proxy for** $\tau$ form



$$au_{form}^{Uncluster}$$





### **Proxy for** $\tau$ form







### **Proxy for** $\tau$ form



$$\tau_{form}^{Uncluster}$$





### **Proxy for** $\tau$ form



$$au_{form}^{Uncluster}$$





### **Proxy for** $\tau$ form



$$au_{form}^{Uncluster}$$



Correlation MC-truth vs Unclustering:

### C/A



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+

## **Correlation (no grooming)**

ECT\* Jet Quenching In The Quark-Gluon Plasma

τ



+



Testing several types of grooming:

C/A + TimeDrop (a=2)



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[Larkoski, Marzani, Soyez, Thaler (14)] [Mehtar-Tani, Soto-Ontoso, Tywoniuk (20)]

### Grooming

### Cleaning soft fragments improves correlation

### $\tau$ + Soft-drop (z<sub>cut</sub>=0.1)









### Correlation

### +



Using the difference between the two formation times:  $\Delta \tau = \tau_{form}^{Parton \ Shower} - \tau_{form}^{Unclustering}$ 



### Correlation



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### Correlation



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### Correlation



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## Vacuum vs Medium

### $\Delta \tau$ distribution (vacuum quartiles):



 $\Delta \tau$  distribution (in-medium quartiles):



1st emission (with SD) has always median (Q2) centred at 0 (change in IQR)

## Vacuum vs Medium

### $\Delta \tau$ distribution (vacuum quartiles):

![](_page_39_Figure_2.jpeg)

 $\Delta \tau$  distribution (in-medium quartiles):

![](_page_39_Figure_6.jpeg)

### Shift in the median (Q2) of 2nd emission

# **Algorithms Comparison**

### 1st emission/unclustering step (IQR)

![](_page_40_Figure_2.jpeg)

### 2nd emission/unclustering step (Q2)

![](_page_40_Figure_6.jpeg)

### Absolute differences w.r.t to minimum value

![](_page_40_Picture_11.jpeg)

![](_page_40_Picture_12.jpeg)

## **Available timescales**

Focus on first emission:

What are the  $\tau_{form}$  available?  $\bigstar$ 

Is it modified by the medium?

Harder fragmentation  $\Leftrightarrow$  Longer  $\tau$ 

 $\diamond$ 

 $\blacklozenge$ 

R = 0.5, p<sub>T,jet</sub> > 300 GeV

![](_page_41_Figure_7.jpeg)

![](_page_41_Picture_10.jpeg)

![](_page_41_Picture_11.jpeg)

## **Available timescales**

Focus on first emission:

What are the  $\tau_{form}$  available?  $\bigstar$ 

Is it modified by the medium?

Harder fragmentation  $\Leftrightarrow$  Longer  $\tau$ 

 $\Leftrightarrow$  Survive more to in-medium jets

+

 $\blacklozenge$ 

R = 0.5, p<sub>T,jet</sub> > 300 GeV

![](_page_42_Figure_8.jpeg)

![](_page_42_Picture_11.jpeg)

![](_page_42_Picture_12.jpeg)

## Example application: RAA lead jet

Easily select two classes of jets:

![](_page_43_Figure_2.jpeg)

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 $\blacklozenge$ 

![](_page_43_Picture_6.jpeg)

## Example application: RAA lead jet

Easily select two classes of jets:

• "early" jets:  $\tau_1 < 1$  fm/c (strongly modified)

![](_page_44_Figure_3.jpeg)

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![](_page_44_Picture_7.jpeg)

## **Example application:** RAA<sup>lead jet</sup>

Easily select two classes of jets:

+ "early" jets:  $\tau_1 < 1$  fm/c (strongly modified)

"late" jets:  $\tau_1 > 3$  fm/c (weakly modified)

![](_page_45_Figure_4.jpeg)

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![](_page_45_Figure_7.jpeg)

# Example application: RAA lead jet

Easily select two classes of jets:

+ "early" jets:  $\tau_1 < 1$  fm/c (strongly modified)

"late" jets:  $\tau_1 > 3$  fm/c (weakly modified)

![](_page_46_Figure_4.jpeg)

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[talks on jet quenching selection: Laura, Raymond, Mateusz]

### Previous results

![](_page_46_Figure_8.jpeg)

![](_page_46_Figure_11.jpeg)

![](_page_46_Picture_12.jpeg)

# 1st unclustering: V

R S S L

and a diversity of the

![](_page_47_Figure_1.jpeg)

Time (fm/c)

![](_page_47_Picture_3.jpeg)

# 1st unclustering: V

# Can we go further? (2nd unclustering)

Expanding QGP

Time (fm/c)

![](_page_48_Picture_4.jpeg)

## **1st vs 2nd Unclustering Step**

Uncertainty on 2nd unclustering step is larger...

![](_page_49_Figure_2.jpeg)

+

![](_page_49_Picture_6.jpeg)

## **1st vs 2nd Unclustering Step**

Uncertainty on 2nd unclustering step is larger...

But  $\tau_{form}$ -distribution is also displaced towards larger values

![](_page_50_Figure_3.jpeg)

+

 $\bigstar$ 

![](_page_50_Picture_8.jpeg)

# Normalised IQR

Formation time gets logarithmic larger at each unclustering step

Normalising by the average  $<\tau_{form}>$ :  $\bullet$ 

L.

![](_page_51_Figure_3.jpeg)

![](_page_51_Figure_4.jpeg)

![](_page_51_Picture_7.jpeg)

# Normalised IQR

2nd step (+2SD)

Formation time gets logarithmic larger at each unclustering step

Normalising by the average  $<\tau_{form}>$ :  $\blacklozenge$ 

![](_page_52_Figure_3.jpeg)

![](_page_52_Figure_4.jpeg)

Relative IQR for 2nd unclustering still looks manageable

![](_page_52_Figure_6.jpeg)

![](_page_52_Picture_8.jpeg)

![](_page_53_Picture_0.jpeg)

There are several biases one needs to consider...

Phase space restrictions...  $\bigstar$ 

+

![](_page_53_Figure_5.jpeg)

![](_page_53_Picture_8.jpeg)

![](_page_54_Picture_0.jpeg)

![](_page_54_Figure_4.jpeg)

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![](_page_54_Picture_8.jpeg)

![](_page_55_Picture_0.jpeg)

![](_page_55_Figure_4.jpeg)

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![](_page_55_Picture_8.jpeg)

# 2nd unclustering: ?

# Need a more calibrated setup:

Expanding QGP

Time (fm/c)

## Dijets $\Rightarrow$ Z+jet

![](_page_56_Picture_5.jpeg)

![](_page_56_Picture_6.jpeg)

## From di-jets to Z+jet events

More calibrated results: boson + (quark) jet

Recoiling jet R = 0.5,  $p_{T,Z} > 60$  GeV,  $p_{Tjet} > 30$  GeV  $\bigstar$ 

![](_page_57_Figure_3.jpeg)

+

![](_page_57_Figure_6.jpeg)

![](_page_57_Picture_9.jpeg)

![](_page_57_Picture_10.jpeg)

![](_page_58_Figure_3.jpeg)

![](_page_58_Picture_7.jpeg)

# $\tau_{\text{form}} = \tau_{\text{form}}(\Delta \mathbf{R}, \mathbf{z})$

an a same a s

4

10 B

25 Ta 25

1111

# Kinematic cut?

![](_page_59_Picture_2.jpeg)

### Selecting on $\Delta R$ vs selecting on $\tau_{form}$ :

![](_page_60_Figure_2.jpeg)

+

### $\Delta \mathbf{R} \mathbf{VS} \tau_{form}$

![](_page_60_Figure_5.jpeg)

![](_page_60_Figure_6.jpeg)

![](_page_60_Picture_9.jpeg)

### Selecting on $\Delta R$ vs selecting on $\tau_{form}$ :

![](_page_61_Figure_2.jpeg)

Harder fragmentation  $\Leftrightarrow$  Longer  $\tau \Leftrightarrow$  Smaller  $\Delta R$ 

+

### $\Delta \mathbf{R} \mathbf{VS} \tau_{form}$

![](_page_61_Picture_8.jpeg)

### Selecting on $\Delta R$ vs selecting on $\tau_{form}$ :

![](_page_62_Figure_2.jpeg)

+

### $\Delta \mathbf{R} \mathbf{VS} \tau_{form}$

### Softer fragmentation $\Leftrightarrow$ Smaller $\tau \Leftrightarrow$ Larger $\Delta R$

![](_page_62_Picture_9.jpeg)

![](_page_63_Figure_2.jpeg)

+

### $\Delta \mathbf{R} \mathbf{VS} \tau_{form}$

![](_page_63_Picture_8.jpeg)

![](_page_63_Picture_9.jpeg)

![](_page_64_Figure_2.jpeg)

### $\Delta \mathbf{R} \mathbf{VS} \tau_{form}$

![](_page_64_Picture_8.jpeg)

![](_page_65_Figure_2.jpeg)

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### $\Delta \mathbf{R} \mathbf{VS} \tau_{form}$

![](_page_65_Picture_7.jpeg)

![](_page_65_Picture_8.jpeg)

# 2nd unclustering:

# Selection on Tform: allows to

**Expanding QGP** 

![](_page_66_Picture_3.jpeg)

# evaluate $\Delta E$ in a $\Delta \tau$

# Z+jet

![](_page_66_Picture_6.jpeg)

![](_page_66_Picture_7.jpeg)

## Summary

New jet scale  $\tau_{form}$  allows to: +

Select quenched jets without biasing initial  $p_T$  (accurate evaluation of  $\blacklozenge$ energy loss)

+ 1st and 2nd unclustering steps with identical relative  $\tau_{form}$  resolution

+  $\Delta E = \Delta E (\tau_{form})$ ? (On-going...)

![](_page_67_Figure_7.jpeg)

![](_page_67_Picture_10.jpeg)

![](_page_67_Picture_11.jpeg)

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+  $\Delta E = \Delta E (\tau_{form})$ ? (On-going...)

- $\tau$ -algorithm:
  - + with SD: 1st unclustering C/A identical to  $\tau$
  - Without SD: better overall performance in evaluating  $\tau_{form}$ +

![](_page_68_Figure_10.jpeg)

![](_page_68_Figure_11.jpeg)

![](_page_68_Figure_12.jpeg)

![](_page_68_Picture_16.jpeg)

![](_page_68_Picture_17.jpeg)

![](_page_68_Figure_18.jpeg)

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New jet scale  $\tau_{form}$  allows to: +

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 $\tau$ -algorithm:

- + with SD: 1st unclustering C/A identical to  $\tau$
- Without SD: better overall performance in evaluating  $\tau_{form}$ +

### Thank you!

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![](_page_69_Figure_11.jpeg)

![](_page_69_Figure_12.jpeg)

![](_page_69_Figure_13.jpeg)

![](_page_69_Picture_17.jpeg)

![](_page_69_Picture_18.jpeg)

![](_page_69_Figure_19.jpeg)

![](_page_70_Picture_0.jpeg)

![](_page_70_Picture_1.jpeg)

# REPÚBLICA PORTUGUESA

![](_page_70_Picture_3.jpeg)

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## Acknowledgments

![](_page_70_Picture_6.jpeg)

![](_page_70_Picture_7.jpeg)

![](_page_70_Picture_10.jpeg)

![](_page_71_Picture_0.jpeg)

# **Backup slides**

.

![](_page_71_Picture_2.jpeg)
What happens when a high momentum particle travels through the QGP?



+







+

- parton shower



What happens when a high momentum particle travels through the QGP?

Medium-induced energy loss

 $\blacklozenge$ 

- parton shower





What happens when a high momentum particle travels through the QGP?



 $\blacklozenge$ 

- parton shower



What happens when a high momentum particle travels through the QGP?



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 $\blacklozenge$ 

- parton shower









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 $\blacklozenge$ 

### **Expanding Medium**









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+







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+







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+

Several unknowns!





## Sensitivity to early times?

A description of high-p<sub>T</sub> anisotropic flow needs both hard and soft sectors: +

Framework to change quenching during early stages  $\bigstar$ (based on quenching weights)



## Sensitivity to early times?

A description of high- $p_T$  anisotropic flow needs both hard and soft sectors: +

Framework to change quenching during early stages  $\bigstar$ (based on quenching weights)



## Sensitivity to early times?

A description of high- $p_T$  anisotropic flow needs both hard and soft sectors:

Framework to change quenching during early stages  $\bigstar$ (based on quenching weights)

Potential to constrain the dynamics of the initial stages of the evolution



+

## Sensitivity to later times?

### Reconstructed hadronic W boson jet mass:



+

[Citron et al (19)]



@ LHC: limited sensitivity (identify long vs short lived scenarios)







### Sensitivity to different timescales:

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### Sensitivity to different timescales: Early dynamics V -



# Sensitivity to different timescales: Early dynamics V - Late dynamics V



# Sensitivity to different timescales: Early dynamics $\sqrt{}$ - Late dynamics V - Anything else ... ?



# Sensitivity to different timescales: Early dynamics V Late dynamics $\checkmark$ - Anything else ... ?

Jets are multi-scale objects!





# Sensitivity to different timescales: Early dynamics $\sqrt{}$ Late dynamics $\checkmark$ - Anything else ... ?

High momentum particles (typically from vacuum-like parton shower)

Jets are multi-scale objects!



"Semi-hard" & Soft medium-induced radiation

Soft jet-induced medium response









### C/A + Soft drop

+

[Larkoski, Marzani, Soyez, Thaler (2014)] [Larkoski, Marzani, Thaler (2015)]

 $z_g = \frac{\min(p_{T,1}, p_{T,2})}{p_{T,1} + p_{T,2}} \quad \text{when} \quad \frac{\min(p_{T,1}, p_{T,2})}{p_{T,1} + p_{T,2}} > z_{cut} \left(\frac{R_{12}}{R_0}\right)^{\beta}$ 



### C/A + Soft drop = Jet splitting function +

[Larkoski, Marzani, Soyez, Thaler (2014)] [Larkoski, Marzani, Thaler (2015)]

 $\frac{\min(p_{T,1}, p_{T,2})}{p_{T,1} + p_{T,2}}$  $z_g =$ 





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## Grooming

### Testing several types of grooming:

### TimeDrop (a=2)





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+

[Larkoski, Marzani, Soyez, Thaler (14)] [Mehtar-Tani, Soto-Ontoso, Tywoniuk (20)]

### Soft-drop (z<sub>cut</sub>=0.1)







### Grooming

### Testing several types of grooming:

### TimeDrop (a=2)





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+

### [Larkoski, Marzani, Soyez, Thaler (14)] [Mehtar-Tani, Soto-Ontoso, Tywoniuk (20)]

### Cleaning soft fragments improves correlation

### Soft-drop (z<sub>cut</sub>=0.1)











## Testing ground

Z+Jet events

Leptonic decay of Z boson  $\bigstar$ 

Choose "cleanest" channel as a first setting

Recoiling jet

+ R = 0.5

◆ p<sub>T</sub> > 300 GeV

+ |η| < 1

 $\tau_{form} \approx \frac{1}{2Ez(1-z)(1-\cos\theta_{12})}$ 

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### All done at hadron level





### $\Delta \mathbf{R} \mathbf{VS} \tau_{form}$





### Vacuum vs Medium

 $\Delta \tau$  distribution ( $\tau$  algorithm):



In-medium scatterings deteriorate resolution...



Medium recoil

Shift in transverse momentum due to elastic scatterings (Shift in reconstructed  $\tau_{form}$ )





## **Example application:** RAA<sup>lead jet</sup>

Easily select two classes of jets:

+ "early" jets:  $\tau_1 < 1$  fm/c (strongly modified)

"late" jets:  $\tau_1 > 3$  fm/c (weakly modified)



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Even with soft-drop, emissions/unclustering steps might not be ordered in  $\tau_{form}$ +



Even with soft-drop, emissions/unclustering steps might not be ordered in  $\tau_{form}$ +



Even with soft-drop, emissions/unclustering steps might not be ordered in  $\tau_{form}$ +

From leading branch, select the one with the shortest  $\tau_{form}$ +



Even with soft-drop, emissions/unclustering steps might not be ordered in  $\tau_{form}$ 

From leading branch, select the one with the shortest  $\tau_{form}$ C/A (p = 0.0)



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Even  $\tau$  can yield a fraction unordered emissions...

 $\tau$  (p = 0.5)





## Time-drop vs Soft-drop

How about other grooming settings?

+

### Soft-drop

[Larkoski, Marzani, Soyez, Thaler (2014)] [Larkoski, Marzani, Thaler (2015)]

C/A re-clustered jet

$$z_g = \frac{\min(p_{T,1}, p_{T,2})}{p_{T,1} + p_{T,2}}$$

when 
$$\frac{\min(p_{T,1}, p_{T,2})}{p_{T,1} + p_{T,2}} > z_{cut} \left(\frac{R_{12}}{R_0}\right)^{\beta}$$



## Time-drop vs Soft-drop

### How about other grooming settings?

### Time-drop

[Metar-Tani, Soto-Ontoso, Tywoniuk (2020)]

### C/A re-clustered jet

$$\kappa^{(a)} = \frac{1}{p_{\mathrm{T}}} \max_{i \in \mathrm{C/A \ seq.}} \left[ z_i (1 - z_i) \, p_{\mathrm{T},i} \left( \frac{\theta_i}{R} \right)^a \right]$$

a = 2: 
$$t_{\rm f}^{-1} \sim \kappa^{(2)} p_{\rm T}$$
.

+

### Soft-drop

[Larkoski, Marzani, Soyez, Thaler (2014)] [Larkoski, Marzani, Thaler (2015)]

C/A re-clustered jet

$$z_g = \frac{\min(p_{T,1}, p_{T,2})}{p_{T,1} + p_{T,2}}$$

when 
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