Transport of hard probes through glasma

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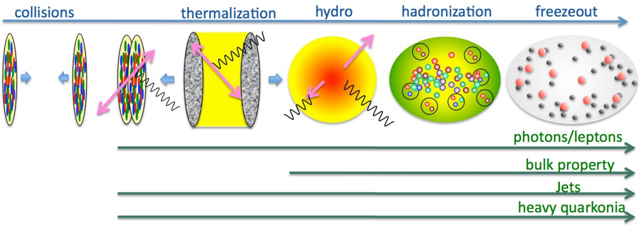
based on:
arXiv:2202.00357 (accepted in PRC)

Jet Quenching in the Quark-Gluon Plasma, Trento, 13-17 June, 2022

Hard probes in heavy-ion collisions



HARD PROBES (AND JETS) IN QGP - old and broad field - actively investigated

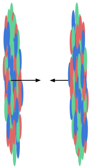
HARD PROBES IN GLASMA - can the effect of the early stage be important?

We propose a model to assess this effect:

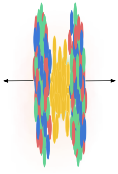
- expansion of glasma fields in the proper time
  $\rightarrow$ analytical approach to study the initial state
  $\rightarrow$ purely classical
- Fokker-Planck equation
  $\rightarrow$ allows to study the interaction of a probe with the medium

Nuclei before the collision

before the collision



after the collision



MV model - a specific realization of CGC:

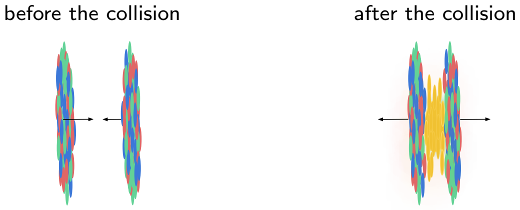
- large x partons represented by $J^{\mu}(x^-, \vec{x}_\perp) = \delta^{\mu +}\rho(x^-, \vec{x}_\perp)$
- small x partons represented by soft gluon fields $\beta^{\mu}(x)$: $F^{\mu\nu} = \frac{i}{g}[D^{\mu}, D^{\nu}]$ with $D^{\mu} = \partial^{\mu} - ig\beta^{\mu}$
- gluons are in the saturation regime controlled by the saturation scale Q_s
- separation scale between small-x and large-x partons is fixed
- alternatively: $\mathbf{E}(x)$ and $\mathbf{B}(x)$ fields

Yang-Mills equations: $[D_{\mu}, F^{\mu\nu}] = J^{\nu}$

solutions: $\beta^{-}(x^-, \vec{x}_\perp) = 0 \qquad \beta^{i}(x^-, \vec{x}_\perp) = \theta(x^-)\frac{i}{g}U(\vec{x}_\perp)\partial^{i}U^{\dagger}(\vec{x}_\perp)$

$$U(\vec{x}_\perp) - \text{Wilson line}$$

Glasma



Glasma:

* highly energetic and anisotropic medium made of mostly gluon fields
* glasma fields $\alpha(\tau,\vec{x}_\perp)$ and $\alpha^i_\perp(\tau,\vec{x}_\perp)$ develop in the forward light-cone region:

$$\alpha^+(x) = x^+\alpha(\tau,\vec{x}_\perp) \qquad \alpha^-(x) = -x^-\alpha(\tau,\vec{x}_\perp) \qquad \alpha^i(x) = \alpha^i_\perp(\tau,\vec{x}_\perp)$$

* evolve in time parametrized by $\tau = \sqrt{t^2 - z^2} = \sqrt{2x^+x^-}$
* are boost-independent
* gluon fields obtained as solutions to classical source-less Yang-Mills equations
* current dependence enters through boundary conditions, which connect different light-cone sectors

$$\alpha^i_\perp(\tau=0,\vec{x}_\perp) = \beta^i_1(\vec{x}_\perp) + \beta^i_2(\vec{x}_\perp) \qquad \alpha(\tau=0,\vec{x}_\perp) = -\frac{ig}{2}[\beta^i_1(\vec{x}_\perp),\beta^i_2(\vec{x}_\perp)]$$

* general solutions not known
* here: temporal evolution of glasma fields is obtained in the proper time expansion

Expansion in the proper time

An analytical approach to solve Yang-Mills equations proposed in:
Fries, Kapusta, Li, arXiv:0604054
Chen, Fries, Kapusta, Li, Phys. Rev. C 92, 064912 (2015)

- glasma is a short-lived phase and decays before the system reaches equilibrium ($\tau < 1$ fm/c)
- proper time of such a system is small and can be treated as an expansion parameter of glasma fields:

$$\alpha^i_\perp(\tau, \vec{x}_\perp) = \sum_{n=0}^{\infty} \tau^n \alpha^i_{\perp(n)}(\vec{x}_\perp), \qquad \alpha(\tau, \vec{x}_\perp) = \sum_{n=0}^{\infty} \tau^n \alpha_{(n)}(\vec{x}_\perp)$$

  and the chromodynamic fields:

$$\mathbf{E} = \mathbf{E}_{(0)} + \tau \mathbf{E}_{(1)} + \tau^2 \mathbf{E}_{(2)} + \dots \qquad \mathbf{B} = \mathbf{B}_{(0)} + \tau \mathbf{B}_{(1)} + \tau^2 \mathbf{B}_{(2)} + \dots$$

- the system of coupled Yang-Mills equations can be solved recursively to any order in τ
- 0th-rder coefficients are identified with boundary conditions
- solutions are written in terms of precollision potentials
- effective dimensionless parameter is $\tilde{\tau} = \tau Q_s$

Correlators of gauge potentials

- need for colour charge distributions which are not known
- average over colour sources assuming a Gaussian distribution of colour sources within each nucleus

$$\langle \rho_a(x^-, \vec{x}_\perp)\rho_b(y^-, \vec{y}_\perp)\rangle = g^2 \delta_{ab}\lambda(x^-, \vec{x}_\perp)\delta(x^- - y^-)\delta^2(\vec{x}_\perp - \vec{y}_\perp)$$

$\lambda(x^-, \vec{x}_\perp)$ - volume density of sources normalized as $\int dx^- \lambda(x^-, \vec{x}_\perp) = \mu(\vec{x}_\perp)$

- potentials of different nuclei are uncorrelated: $\langle \beta^i_{1a}\beta^j_{2b}\rangle = 0$

Basic building block - 2-point correlator

$$\delta_{ab} B^{ij}_n(\vec{x}_\perp, \vec{y}_\perp) \equiv \lim_{\mathrm{w}\to 0} \langle \beta^i_{n\,a}(x^\mp, \vec{x}_\perp)\beta^j_{n\,b}(y^\mp, \vec{y}_\perp)\rangle$$

$$B^{ij}_n(\vec{x}_\perp, \vec{y}_\perp) = \frac{2}{g^2 N_c \tilde{\Gamma}_n(\vec{x}_\perp, \vec{y}_\perp)} \left(\exp\left[\frac{g^4 N_c}{2} \tilde{\Gamma}_n(\vec{x}_\perp, \vec{y}_\perp) \right] - 1 \right) \partial^i_x \partial^j_y \tilde{\gamma}_n(\vec{x}_\perp, \vec{y}_\perp)$$

$\tilde{\Gamma}_n$ and $\tilde{\gamma}_n$ - given by Bessel functions and colour sources density

Correlators of gauge potentials

- **Wick's theorem:**
  - $\langle \beta_1^i \beta_1^j \, \beta_2^l \beta_2^m \beta_2^k \beta_2^r \rangle = \langle \beta_1^i \beta_1^j \rangle \left(\langle \beta_2^l \beta_2^m \rangle \langle \beta_2^k \beta_2^r \rangle + \langle \beta_2^l \beta_2^k \rangle \langle \beta_2^m \beta_2^r \rangle + \langle \beta_2^l \beta_2^r \rangle \langle \beta_2^k \beta_2^m \rangle \right)$
  - correlators of odd number of gauge fields vanish

- **charge density per unit transverse area**
  - $\bar{\mu} = g^{-4} Q_s^2$, where Q_s is the saturation scale (uniform nuclei)

- **IR regulator**

$m \sim \Lambda_{\mathrm{QCD}}$ - chosen so that because of confinement the effect of valence sources dies off at transverse length scales larger than $1/\Lambda_{\mathrm{QCD}}$

- **UV regulator**

Q_s - saturation scale

Summary of the method:

$$\rho(x^-, \vec{x}_\perp) \to \beta(x^-, \vec{x}_\perp) \to \alpha(0, \vec{x}_\perp) \to \alpha(\tau, \vec{x}_\perp) \to E(\tau, \vec{x}_\perp),\, B(\tau, \vec{x}_\perp)$$

Energy loss of a probe: Fokker-Planck equation

Evolution equation on the distribution function of heavy quarks:
Mrówczyński, Eur. Phys. J, A54 no 3, 43 (2018)

$$\Big(D - \nabla_p^\alpha X^{\alpha\beta}(\mathbf{v})\nabla_p^\beta - \nabla_p^\alpha Y^\alpha(\mathbf{v})\Big)n(t,\mathbf{x},\mathbf{p}) = 0$$

$n(t,\mathbf{x},\mathbf{p})$ - distribution of hard probes $\qquad D \equiv \frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla$

Collision terms:

$$X^{\alpha\beta}(\mathbf{v}) = \frac{1}{2N_c}\int_0^t dt' \langle F_a^\alpha(t,\mathbf{x}) F_a^\beta(t', \mathbf{x} - \mathbf{v}(t-t'))\rangle$$

$$Y^\alpha(\mathbf{v}) = X^{\alpha\beta}\frac{v^\beta}{T}$$

T - temperature of a plasma that has the same energy density as in equilibrium
$\mathbf{F}(t,\mathbf{r}) = g(\mathbf{E}(t,\mathbf{r}) + \mathbf{v}\times\mathbf{B}(t,\mathbf{r}))$ - color Lorentz force
g - constant coupling
$\mathbf{E}(t,\mathbf{r}), \mathbf{B}(t,\mathbf{r})$ - chromoelectric and chromomagnetic fields
$\mathbf{v} = \frac{\mathbf{p}}{E_\mathbf{p}}$ - velocity of the probe:

$\mathbf{v} \simeq 1$ - light quarks and gluons $\qquad \mathbf{v} \leq 1$ - heavy quarks

Energy losses

Physical meaning of the collision terms:

$$\frac{\langle \Delta p^\alpha \rangle}{\Delta t} = -Y^\alpha(\mathbf{v}) \qquad\qquad \frac{\langle \Delta p^\alpha \Delta p^\beta \rangle}{\Delta t} = X^{\alpha\beta}(\mathbf{v}) + X^{\beta\alpha}(\mathbf{v})$$

Energy losses are defined by:

$$\frac{dE}{dx} = \frac{v^\alpha}{v}\frac{\langle \Delta p^\alpha \rangle}{\Delta t} \qquad\qquad \hat{q} = \frac{1}{v}\left(\delta^{\alpha\beta} - \frac{v^\alpha v^\beta}{v^2}\right)\frac{\langle \Delta p^\alpha \Delta p^\beta \rangle}{\Delta t}$$

Collisional energy loss and transverse momentum broadening

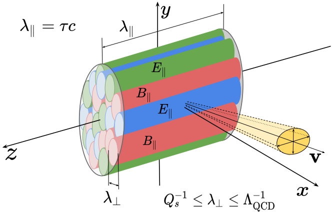
$$-\frac{dE}{dx} = \frac{v}{T}\frac{v^\alpha v^\beta}{v^2} X^{\alpha\beta}(\mathbf{v})$$

$$\hat{q} = \frac{2}{v}\left(\delta^{\alpha\beta} - \frac{v^\alpha v^\beta}{v^2}\right) X^{\alpha\beta}(\mathbf{v})$$

$$X^{\alpha\beta}(\mathbf{v}) = \frac{g^2}{2N_c}\int_0^t dt' \Big[\big\langle E_a^\alpha(t,\mathbf{x}) E_a^\beta(t-t',\mathbf{y}) \big\rangle + \epsilon^{\beta\gamma\gamma'} v^\gamma \big\langle E_a^\alpha(t,\mathbf{x}) B_a^{\gamma'}(t-t',\mathbf{y}) \big\rangle$$
$$+ \epsilon^{\alpha\gamma\gamma'} v^\gamma \big\langle B_a^{\gamma'}(t,\mathbf{x}) E_a^\beta(t-t',\mathbf{y}) \big\rangle + \epsilon^{\alpha\gamma\gamma'}\epsilon^{\beta\delta\delta'} v^\gamma v^\delta \big\langle B_a^{\gamma'}(t,\mathbf{x}) B_a^{\delta'}(t-t',\mathbf{y}) \big\rangle \Big]$$

Schematic picture

Hard probe traversing glasma at $\tau = 0$ ($\lambda_{\|}, \lambda_{\perp}$ - correlation lengths)

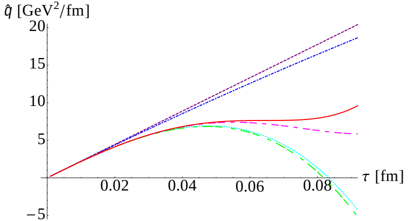


$\rightarrow$ momentum-space rapidity $y = \frac{1}{2} \ln \frac{1+v_{\|}}{1-v_{\|}}$
$*$ experiments focus on the region $y \in (-1, 1) \rightarrow v_{\|} \in (-0.76, 0.76)$
$\rightarrow$ transport coefficients built up during the time that the probe spends within the domain of correlated field
$*$ this time determined by $\lambda_{\perp}$ and $\mathbf{v}$
$*$ role of the velocity: $v_{\perp} = 1 \rightarrow dE/dx$ is minimal and $\hat{q}$ is maximal
$\rightarrow$ transport coefficients saturate when the probe leaves the region of correlated fields
$\rightarrow$ at higher order in τ $\rightarrow$ calculations needed

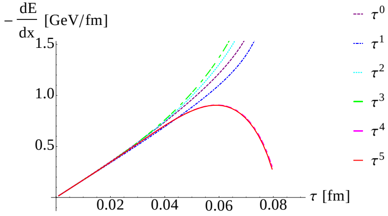
Consistency and reliability of the approach are fixed by convergence of the proper time expansion and saturation of the results.

Time dependence of $\hat{q}$ and dE/dx

- dE/dx and $\hat{q}$ calculated up to τ^5 order
- parameters $m = 0.2$ GeV, $Q_s = 2$ GeV, $N_c = 3$, $g = 1$
- in case of dE/dx we need temperature T:
  $\varepsilon_{\rm QGP} = \frac{\pi^2}{60}\big(4(N_c^2-1)+7N_fN_c\big)T^4$
  $\varepsilon_{\rm QGP} = 130.17\big(15.9773 - 29.6759\,\tilde{\tau}^2 + 42.6822\,\tilde{\tau}^4 - 49.2686\,\tilde{\tau}^6\big)$



$v = v_\perp = 1$

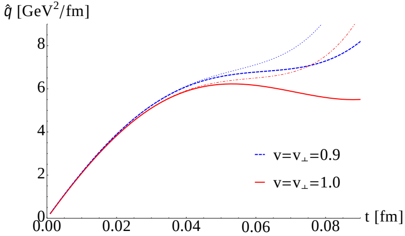


$v = 1, v_\parallel = v_\perp = 1/\sqrt{2}$

- $\hat{q}$: saturation observed before the τ expansion breaks down,
  $\hat{q} \simeq 6$ GeV^2/fm - maximal value,
  similar result was found using real-time QCD calculations
  Ipp, Müller, Schuh, Phys. Lett. B 810, 135810 (2020)
- dE/dx: reaches a maximal value 0.9 GeV/fm, no saturation $\rightarrow$ order of magnitude estimate only

Velocity dependence of $\hat{q}$

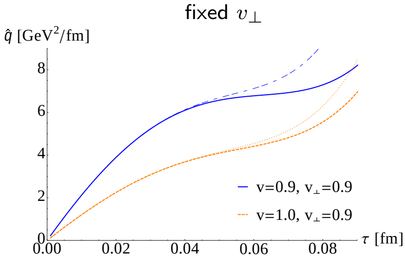
Purely transverse motion of hard probes through the glasma $(v_{\|} = 0)$

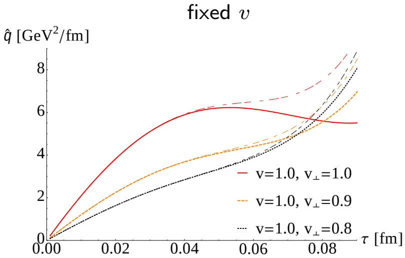


- the results at orders τ^4 and τ^5 agree quite well up to about $\tau \sim 0.07 - 0.08$ fm
- the probe spends less time in the region of correlated fields →reduction of the coefficient for ultra-relativistic quarks

Velocity dependence of $\hat{q}$

Dependence on the longitudinal component of the velocity $v_{\parallel}$

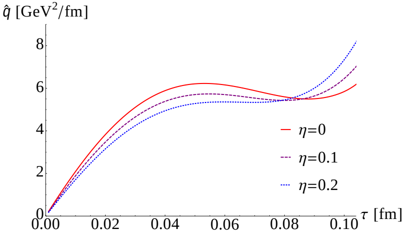


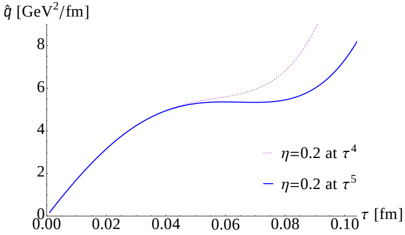


- for larger values of $v_{\parallel}$ saturation is less evident
- **fixed** $v_{\perp}$: the effect of the velocity dependence of the Lorentz force
  $\rightarrow$ the role of electric contribution decreases when $v_{\parallel}$ increases
- **fixed** v: the effect of changing the amount of time that the probe spends in the region of correlation
  $\rightarrow$ probes with larger $v_{\perp}$ escape from the region of correlated fields fast, before the fields become large
  $\rightarrow$ probes with smaller $v_{\perp}$ remain longer in the region and eventually interact with very large glasma fields

Space-time rapidity dependence of $\hat{q}$

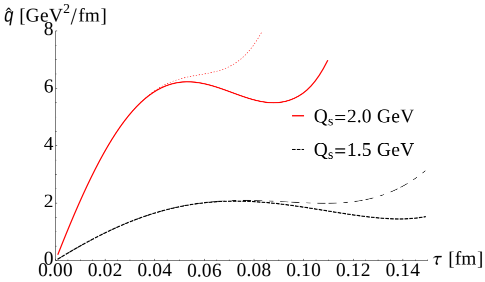
dependence on spatial rapidity $\eta \rightarrow$ dependence on the initial position of the probe in the glasma





- $\hat{q}$ at orders τ^4 and τ^5 agree well up to $\tau \simeq 0.07$ fm
- $\hat{q}$ is weakly dependent on η fo small values of η (CGC is expected to work best in the region of mid-spatial-rapidity region)

Dependence on Q_s

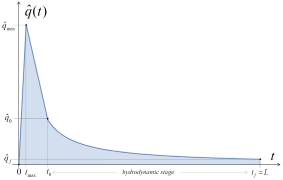


→ $\hat{q}$ sensitive to the choice of Q_s
→ decreasing Q_s decreases the maximal value of $\hat{q}$ but extends the validity region of τ
→ Q_s is smaller at smaller collision energies → $\hat{q}$ is smaller at smaller collision energies (RHIC vs LHC collision energies)

→ reduction in $\hat{q}$ at $\tau = 0.6$ fm for high-p_T hadron at the RHIC energies compared to LHC energies observed by the JET Collaboration
K. M. Burke et al (JET Collaboration), Phys. Rev. C 90, 014909 (2014)

Glasma impact on jet quenching

Total accumulated transverse momentum: $\Delta p_T^2 = \int_0^L dt\, \hat{q}(t)$



- non-equilibrium case: $\Delta p_T^2\Big|^{\text{non-eq}} = \frac{1}{2}\hat{q}_{\text{max}} t_0 + \frac{1}{2}\hat{q}_0(t_0 - t_{\text{max}})$
- equilibrium case: $\Delta p_T^2\Big|^{\text{eq}} = 3T_0^3\, t_0 \,\ln \frac{L}{t_0}$

  where we used $\hat{q}(t) = 3T^3$ and $T = T_0\left(\frac{t_0}{t}\right)^{1/3}$
- parameters:
  $\hat{q}_{\text{max}} \approx 6$ GeV^2/fm, $t_{\text{max}} \approx 0.06$ fm
  $L = 10$ fm, $\hat{q}_0 \approx 1.4$ GeV^2/fm, $t_0 \approx 0.6$ fm, $T_0 = 0.45$ GeV
  JETSCAPE, Phys. Rev. C 104, 024905 (2021), C. Shen et al, Phys. Rev. C 84, 044903 (2011)

$$\frac{\Delta p_T^2\Big|^{\text{non-eq}}}{\Delta p_T^2\Big|^{\text{eq}}} = 0.93$$

Non-equilibrium phase gives comparable contribution to the radiative energy loss as the equilibrium phase.

Summary and conclusions

- Transport of hard probes through glasma studied in the proper time expansion
- Impact of the glasma on hard probes quantified
- Convergence of the proper time expansion tested

- Both $\hat{q}$ and dE/dx are found to be relatively large
- Our approach is most reliable for probes moving transversally to the collision axis

Significant impact of glasma on hard probes