

**ECT\***

EUROPEAN CENTRE FOR THEORETICAL STUDIES  
IN NUCLEAR PHYSICS AND RELATED AREAS



# Quantum simulation of jet quenching

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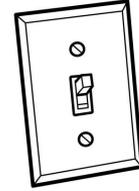
Jet quenching in the quark-gluon plasma, Trento, 17 June 2022

# Outline

1. Introduction to quantum computing (QC)
2. Applications in HEP/NP
3. Our recent work in jet quenching

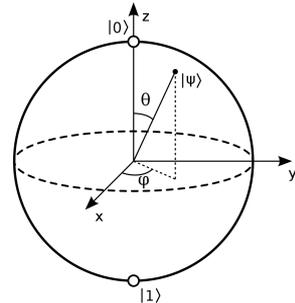
# Introduction to QC

- Classical computers:
  - classical bit: 0, 1
  - classical gates: AND, OR, NOT, Bitwise logic gates
  - deterministic nature



- Quantum computers:
  - quantum bit (qubit):  $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ,  $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
  - implementation: two-level quantum systems

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle, |\alpha|^2 + |\beta|^2 = 1$$



- quantum gates: **unitary operators**
- features: **superposition** & **entanglement**
- states only collapse when measured

# Developments in QC

Quantum computing has come a long way in past 40 years

Feynman (1981)

Toffoli Gate  
(1980)

Shor's Algo  
(1994)

Error Correction  
(1995)

Variational Eigensolver  
(2014)

Quantum Machine Learning  
(2017)

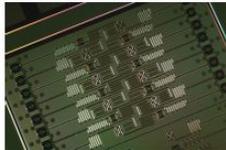
Currently, we are in the **Noisy Intermediate-Scale Quantum (NISQ)**: qubits and quantum operations are substantially imperfect. Nonetheless,

Preskill (2018)

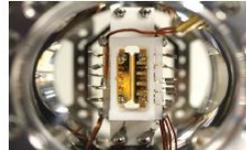
Image from Jong's talk (2022)



Google



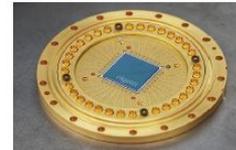
IBM



Rigetti



Intel



IonQ



Recently, “Quantum Simulation for High Energy Physics”, 2204.03381 (2022)

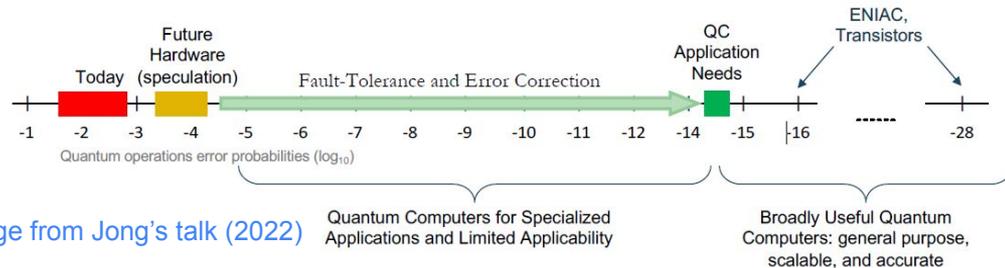
# Why are we interested in QC?

*“Nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical” (Richard Feynman)*



- Many problems are inherently quantum mechanical.
- Vast amount (exponential) of encoded information in a many-qubit state:  
Impossible to compete on classical computers; Nature does it automatically
- High scalability in quantum applications (compact encoding)
- Many-body problems and quantum computing are similar by nature
- Rapid progress in hardware, software, algorithms, benchmark

~ 100 qubits,  
improved volume,  
speed



# Applications in HEP/NP

Two main directions:

- **Variational Quantum Eigensolver (VQE)**, a hybrid optimization algorithm, widely-used in quantum chemistry, many variants
- **Quantum Phase Estimation (QPE)**, prepare, evolve, fourier transform, measure to find quantum state of the system (eigenvector)

# VQE paradigm, hybrid optimization approach

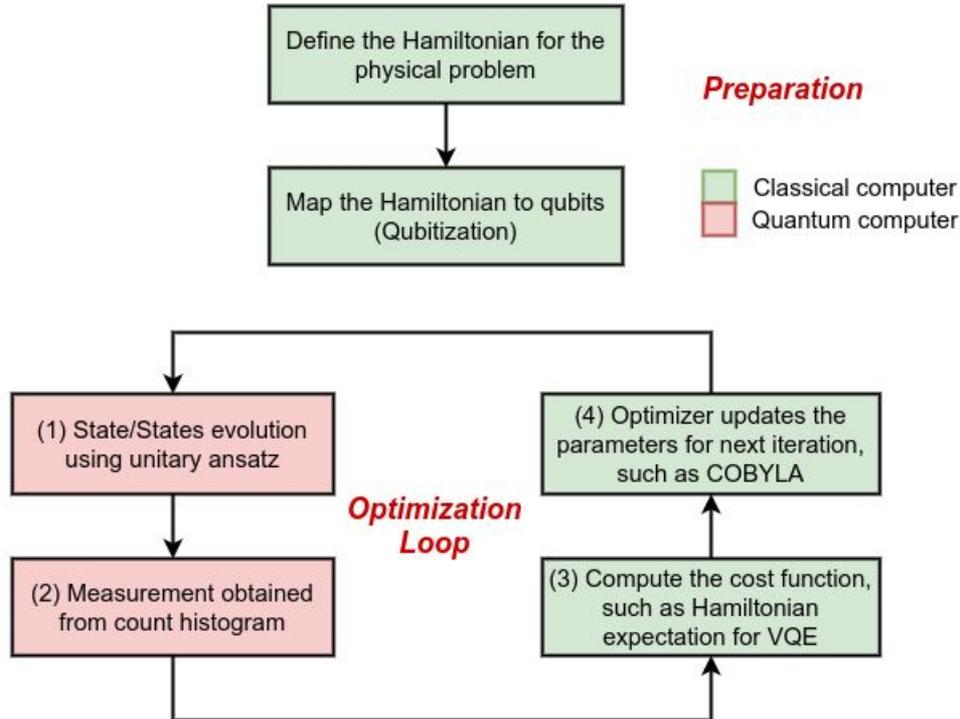
[Peruzzo et al., 1304.3061 \(2013\)](#)

Inspired by **Variational Principle**

Large group of QC approaches (UCC, QAOA, etc); very successful in quantum chemistry

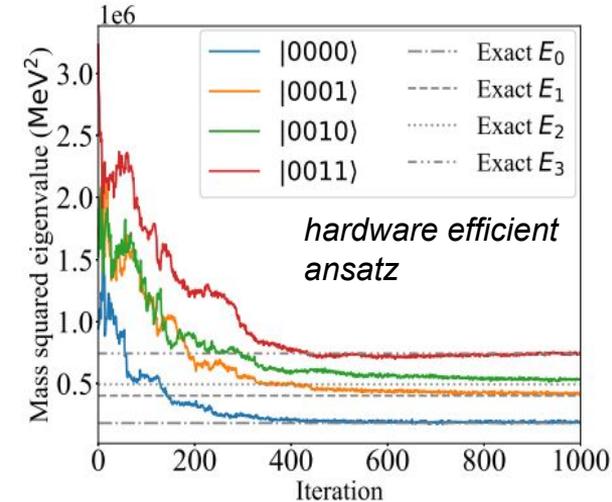
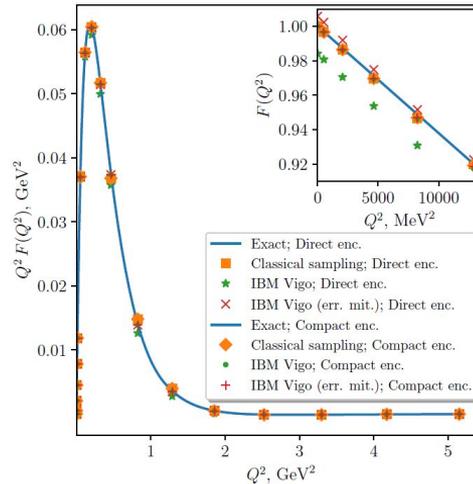
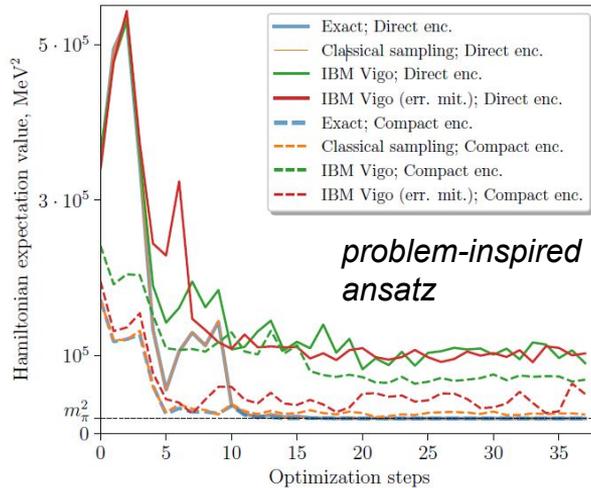
[Review at Bharti et al., 2101.08448 \(2021\)](#)

With optimized/finalized quantum state, we compute additional observables directly or indirectly on the quantum circuit



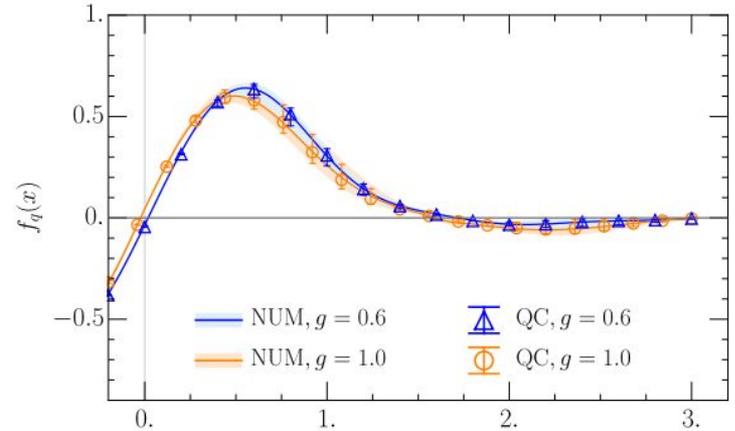
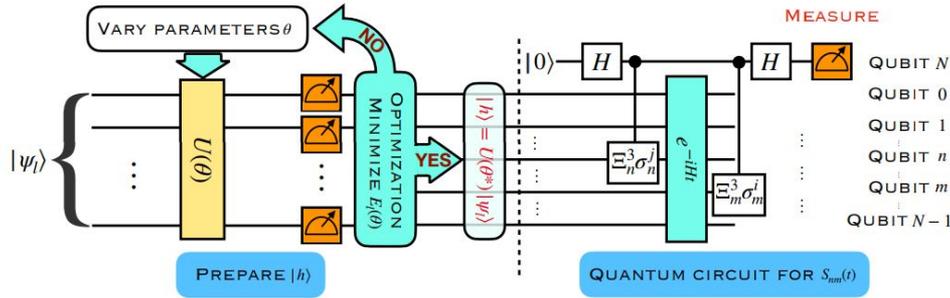
# Quantum simulation of quantum field theory in the light-front formulation

Kreshchuk, Jia, Kirby, Goldstein, Vary, Love, 2011.13443 (2020) & 2009.07885 (2020)  
 WQ, Basili, Pal, Luecke, Vary, 2112.01927 (2021)



Direct/Compact qubit encoding and careful design of ansatz in VQE calculation for bound state properties.

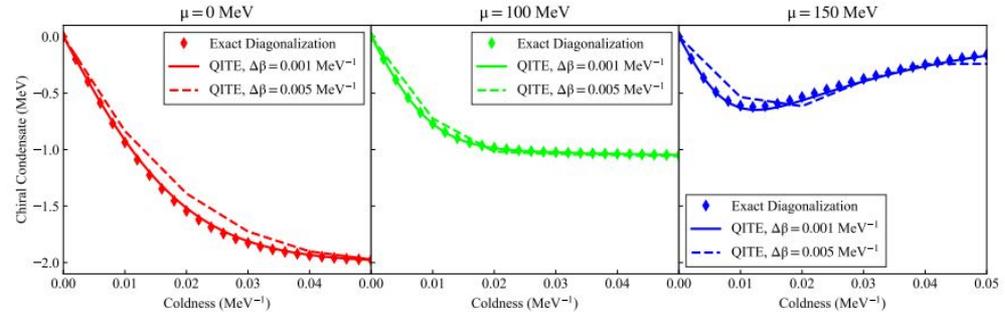
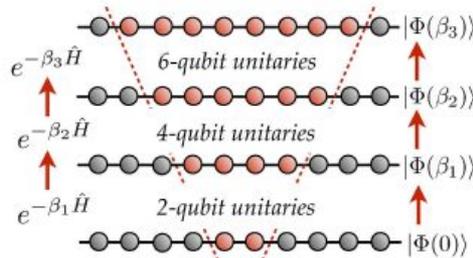
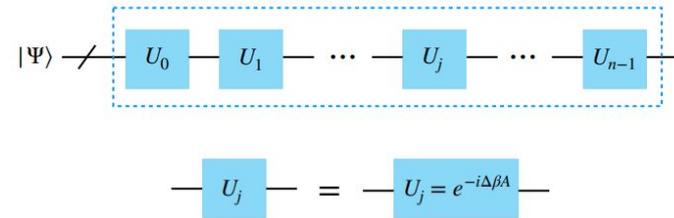
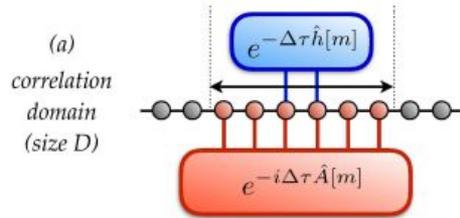
# Partonic Structure by Quantum Computing



Evaluate PDF for 1+1 Nambu-Jona-Lasinio (NJL) model

Li et al., 2106.03865 (2021)

# Quantum Imaginary Time Evolution (QITE) & Quantum Simulation of Chiral Phase Transitions



Competitors of VQE & application to 1+1 NJL model

# Quantum Simulation Algorithm

1. Define problem Hamiltonian
2. Encode Hamiltonian onto basis
3. Prepare initial states
4. Evolution
5. Measurement protocol

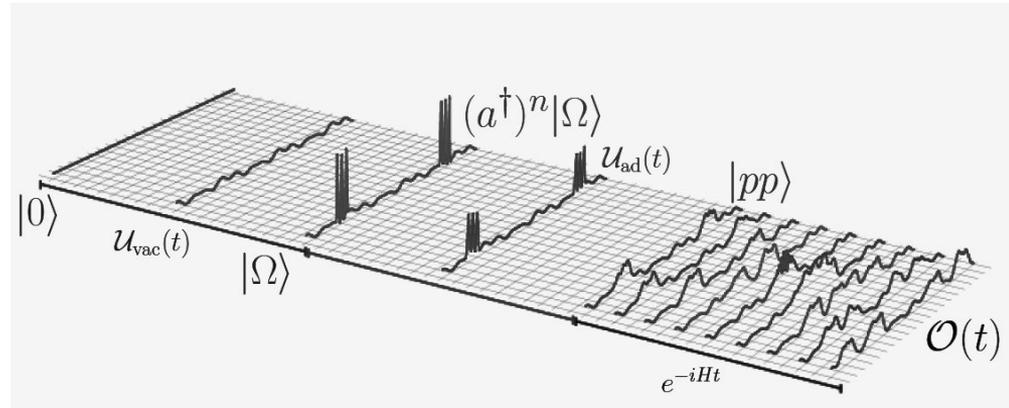
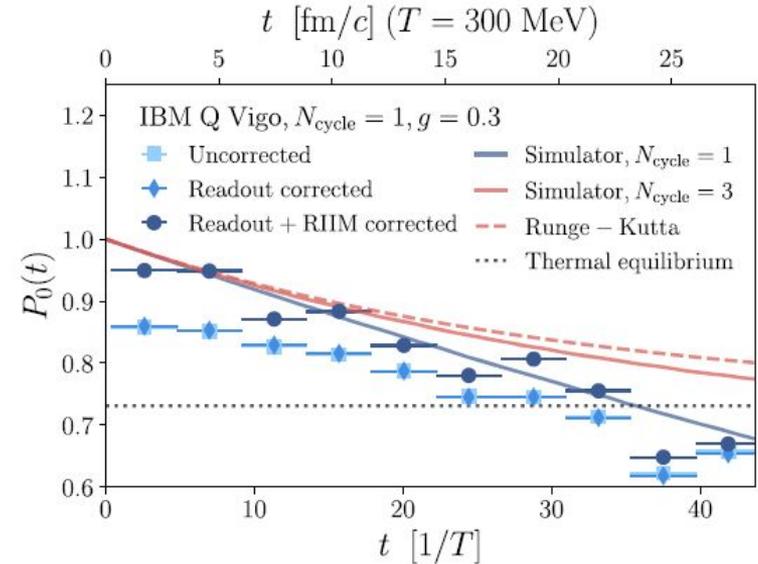
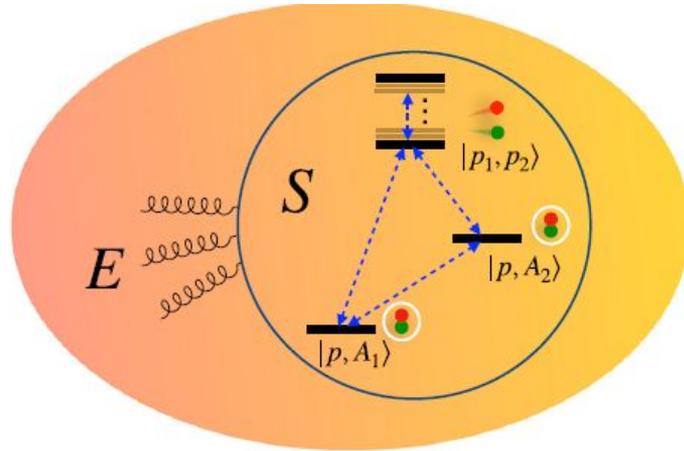


Image from Lamm's talk (2021)

[Wiesner, 9603028 \(1996\)](#); [Zalka, 9603026 \(1996\)](#)

# Quantum simulation of open quantum systems in heavy-ion collisions

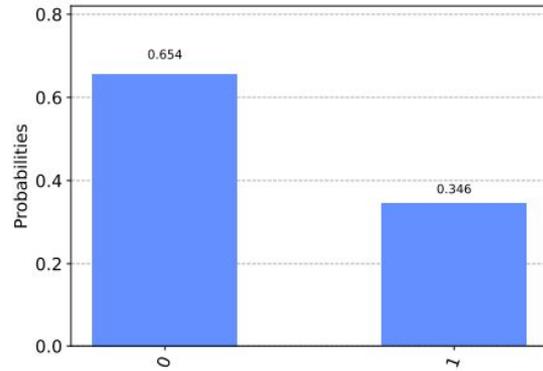
Jong, Metcalf, Mulligan, Ploskon, Ringer, Yao, 2010.03571 (2021)



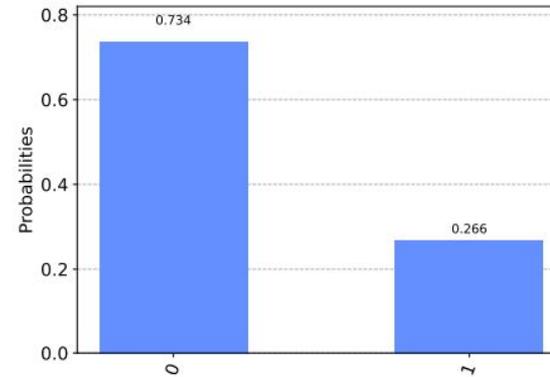
Simulating Lindblad equation for hard probes in thermal bath

# Quantum Simulation of Light-Front QCD for Jet Quenching in Nuclear Environments

Yao, 2205.07902 (2022)



(a) Vacuum case.

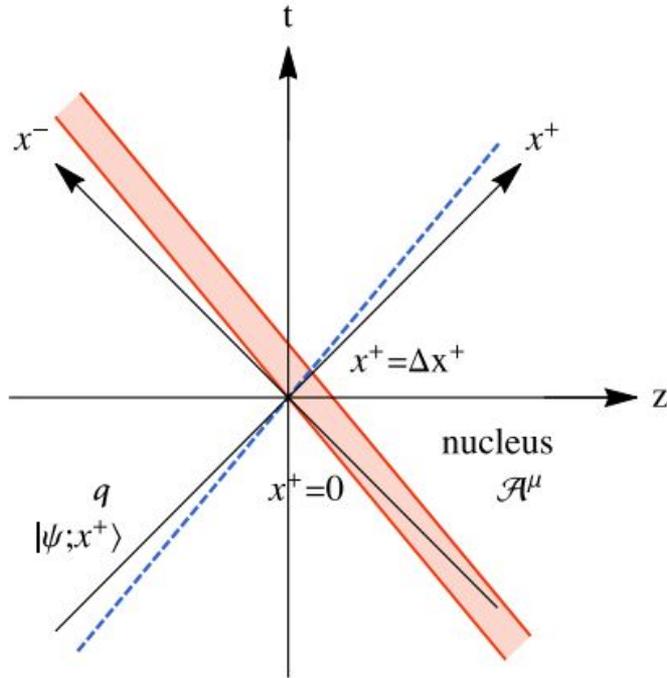


(b) Medium case.

Demo of Landau-Pomeranchuk-Migdal (LPM) effect observed in the quantum simulator (2+1 scalar fields, allowing 1-particle and 2-particle state splitting)

# Ongoing work: Medium induced jet broadening in a quantum computer

Barata, Salgado, 2104.04661 (2021)  
(In preparation) Barata, Du, Li, Salgado, WQ



High-energy quark moving close to the light cone scattering on a dense nucleus medium

$$\mathcal{L}_q = \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi$$

$$D^\mu \equiv \partial_\mu \mathbf{I} + ig\mathcal{A}^\mu$$

Light-front Hamiltonian becomes:

$$P^-(x^+) = P_{\text{KE}}^- + V_A(x^+) = \frac{p_\perp^2}{p^+} + gA(x^+) \cdot T$$

# Background field

The background field uses the MV model

$$\langle\langle \rho_a(\vec{x}_\perp, x^+) \rho_b(\vec{y}_\perp, y^+) \rangle\rangle = g^2 \tilde{\mu}^2 \delta_{ab} \delta^2(\vec{x}_\perp - \vec{y}_\perp) \delta(x^+ - y^+)$$

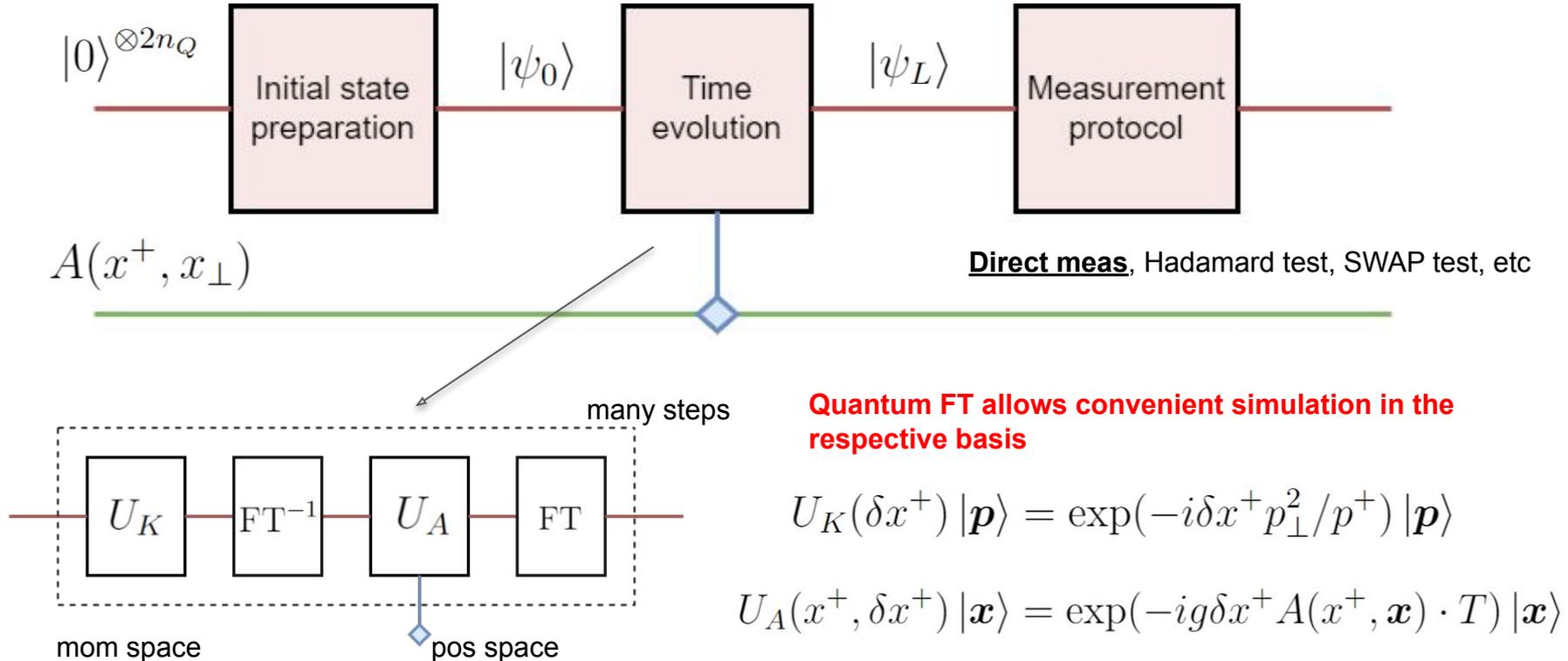
*static, non-dynamical*

$$(m_g^2 - \nabla_\perp^2) \mathcal{A}_a^-(\vec{x}_\perp, x^+) = \rho_a(\vec{x}_\perp, x^+)$$

The correlation is achieved by using a local Gaussian distribution

Here, we are interested in the jet evolution using quantum simulation.

# Quantum simulation



# Field evolution

$$A(x) \cdot T = \sum_{a=X,Y,Z} A^a \sigma^a / 2$$

*simplest colorful case,  
with two colors, SU(2)*

$$\exp(-ig\delta x^+ A(x) \cdot T) = \exp \left\{ -ig\delta x^+ (A^X \otimes \sigma^X + A^Y \otimes \sigma^Y + A^Z \otimes \sigma^Z) \right\}$$

Implementations:

$$e^{ia(\hat{n} \cdot \vec{\sigma})} = I \cos a + i(\hat{n} \cdot \vec{\sigma}) \sin a$$

- Exact method using property of the *exponential of a Pauli vector*
- Exponent matrix approximation
- **Linear-order, allows for separation of color and pos space** (circuit modularization)

- **higher orders**

$$\exp(-iA(x)\sigma^Z) |\mathbf{x}\rangle |c\rangle = \begin{cases} e^{-iA(x)} |\mathbf{x}\rangle |c\rangle, & \text{if } c = 0 \\ e^{+iA(x)} |\mathbf{x}\rangle |c\rangle, & \text{if } c = 1 \end{cases}$$

# Basis space encoding

## Basis space encoding

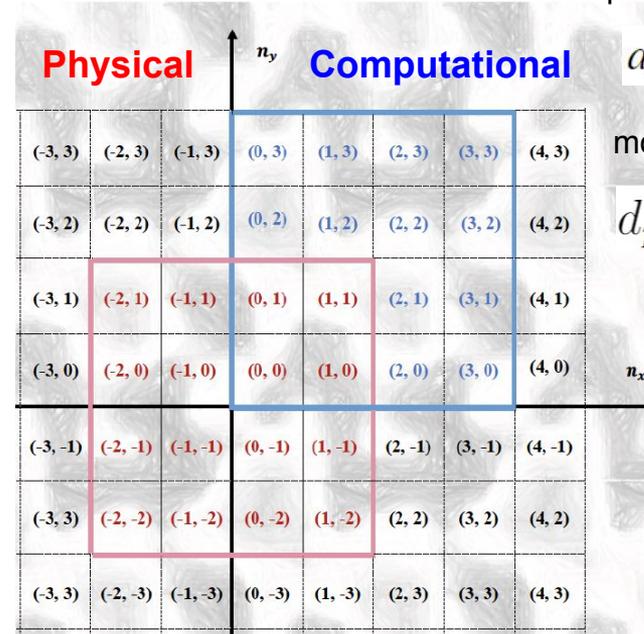
Here, we have 2D momentum/position space  
+ 1D color space

$$n_{tot} = (2N_{\perp})^2 N_c$$

For SU(2),  $N_c = 2$ ,

$$\begin{aligned} \text{state} &= |q_{2n_Q} \dots q_1\rangle |q_0\rangle \\ &= |n_x, n_y\rangle |c\rangle \\ &= |k_x, k_y\rangle |c\rangle \end{aligned} \quad c = 1, 2$$

## Periodical Lattice



pos lattice spacing

$$a_{\perp} = L_{\perp} / N_{\perp}$$

mom lattice spacing

$$d_p = \pi / L_{\perp}$$

$$O(n_x, n_y) = O(n_x + i2N_{\perp}, n_y + j2N_{\perp})$$

## Simulation parameters:

- We study both evolutions with and without color, initial state:  $(p_x, p_y) = (0, 0)$
- Duration of static medium:  $L_\eta = 50 \text{ GeV}^{-1} \approx 10 \text{ fm}$
- 5 field configurations
- Two sets of lattice grids, 32 X 32 (10 qubits) and 64 X 64 (12 qubits)
- Fix  $g = 1$ , physical IR regulator  $m_g = 0.8 \text{ GeV} (\ll Q_s)$
- Selected values of saturation scales  $Q_s$

$$g^2 \tilde{\mu} = \sqrt{\frac{2\pi Q_s^2}{C_F L_\eta}}$$

$$\lambda_{IR} = \frac{\pi}{N_\perp a_\perp} \ll m_g \ll Q_s \ll \lambda_{UV} = \frac{\pi}{a_\perp}$$

range coverage cond

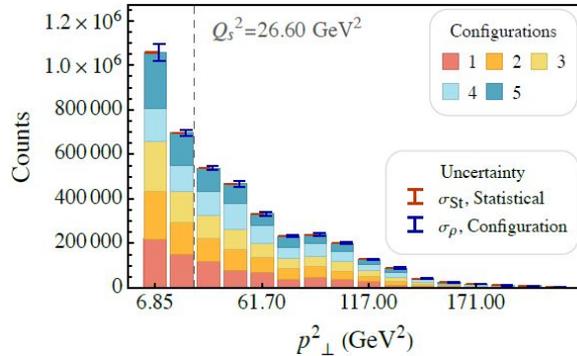
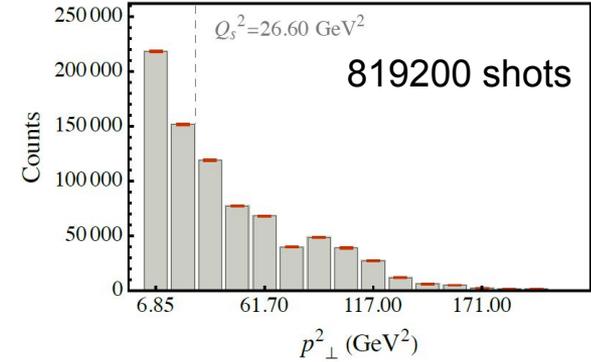
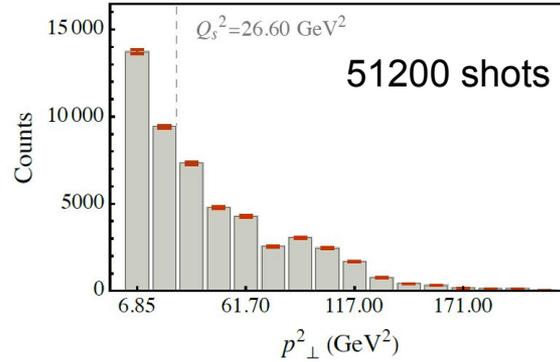
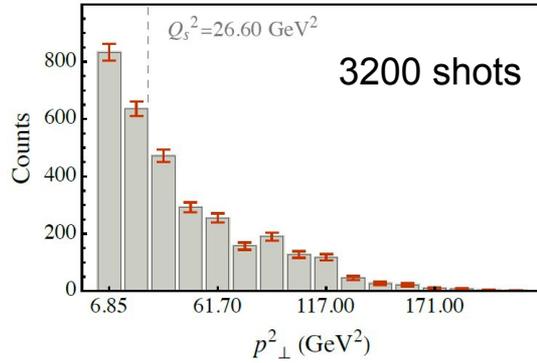
$$a_\perp^2 Q_s^2 < \frac{4\pi^2}{3} \left[ \log\left(\frac{1}{a_\perp^2 m_g^2 / \pi^2} + 1\right) - \frac{1}{1 + a_\perp^2 m_g^2 / \pi^2} \right]^{-1}$$

broadening coverage cond

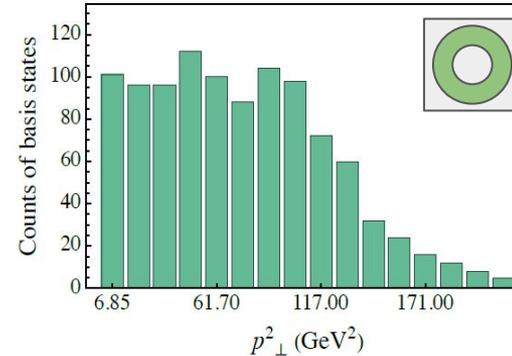
# Event simulations

## Measurement (collapse) of quantum state

sampling noise statistical uncertainty



configuration uncertainty

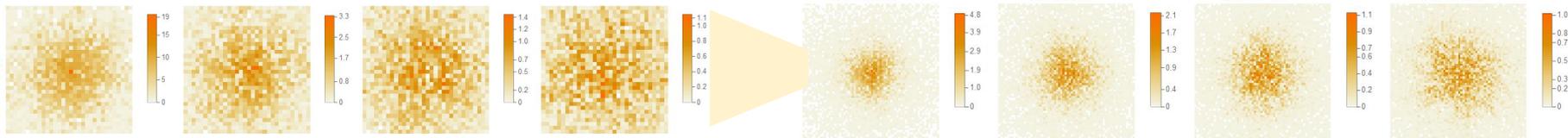
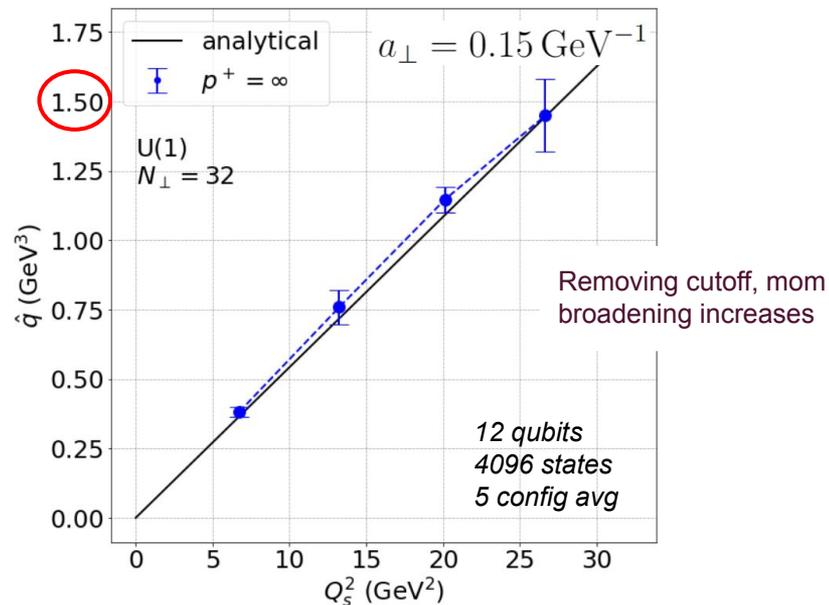
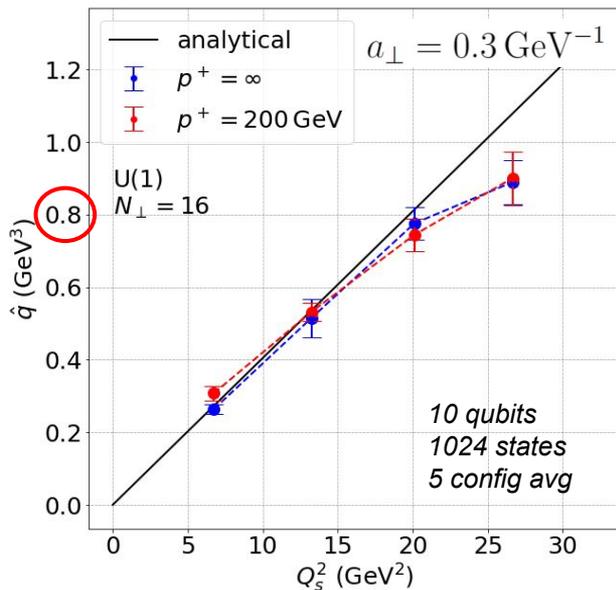


number of basis states

# Numerical results, colorless U(1) case

819200 shots  
quantum simulation

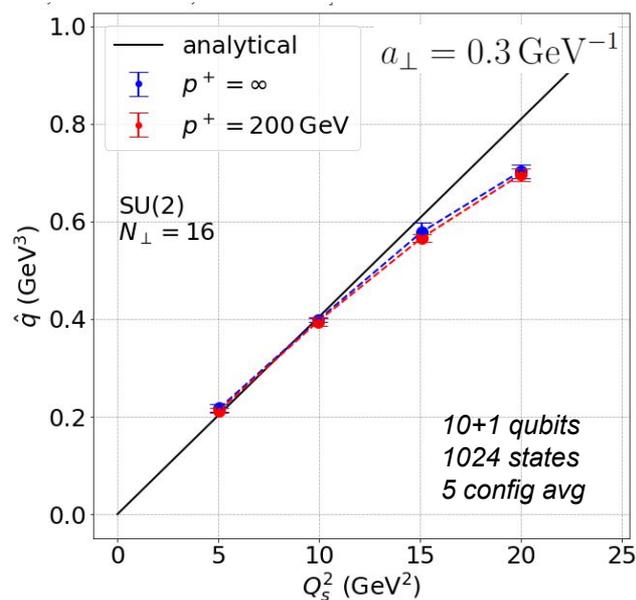
$$\hat{q} = \langle p_{\perp}^2 \rangle / L_{\eta}$$



$$p_{\perp} = [-\pi/a_{\perp}, \pi/a_{\perp}]$$

# Numerical results, colorful SU(2) case

819200 shots  
quantum simulation



Analytical

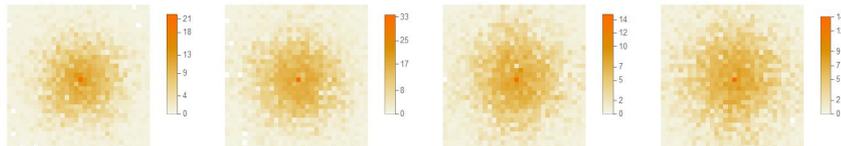
$$\hat{q} = \frac{g^4}{4\pi} C_F \tilde{\mu}^2 \left\{ \log \left( 1 + \frac{\frac{\pi^2}{a_{\perp}^2}}{m_g^2} \right) - \frac{1}{1 + \frac{a_{\perp}^2 m_g^2}{\pi^2}} \right\}$$

identical curve for U(1) and SU(2)

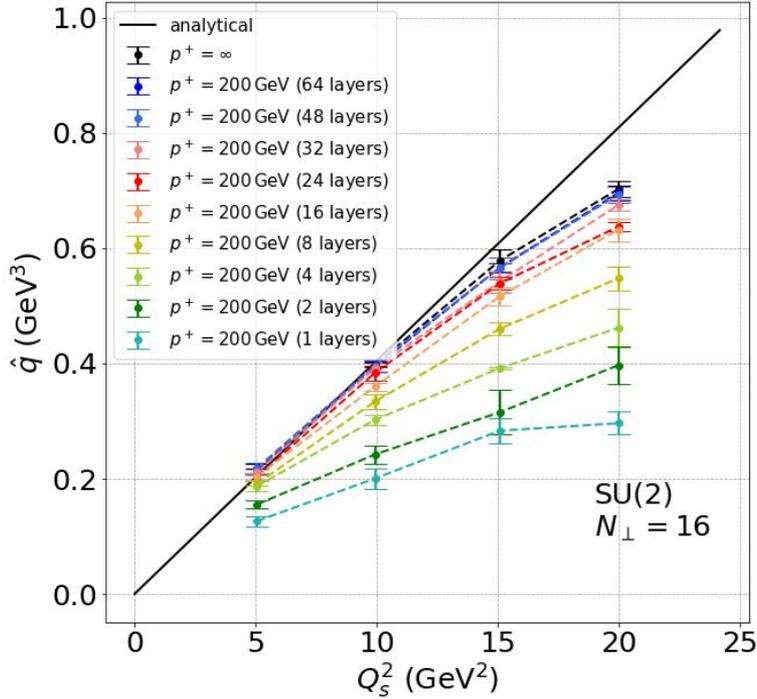
See Li's talk (06/14)

Simulation

$$\hat{q} = \langle \mathbf{p}_{\perp}^2 \rangle / L_{\eta}$$

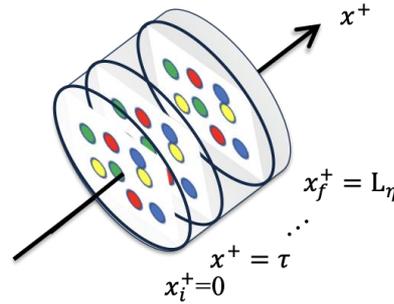


# Static field layers

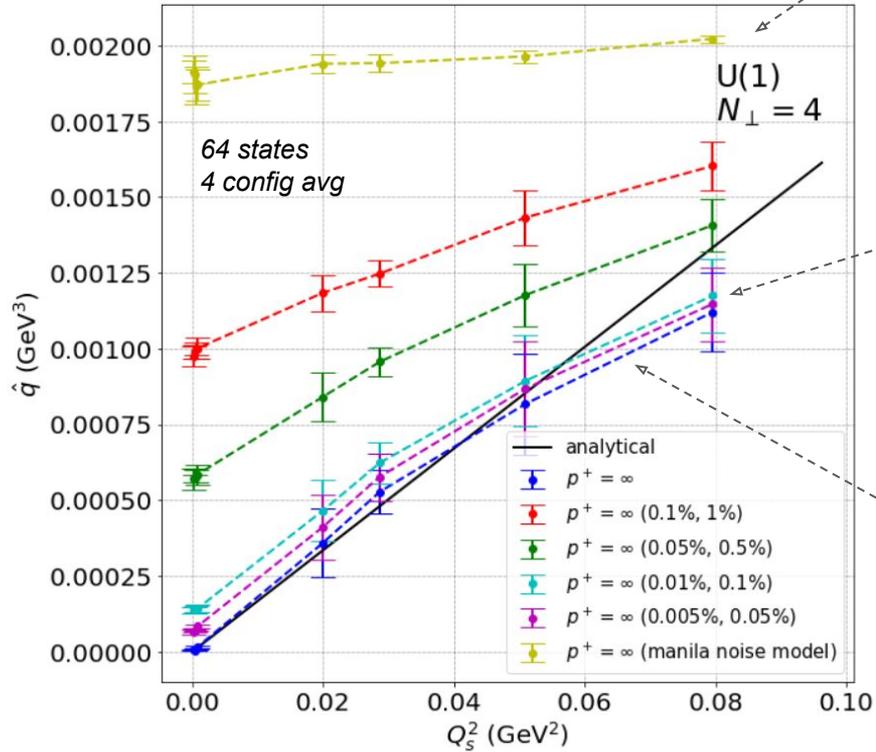


As layer increases, the correlation condition is approximately satisfied

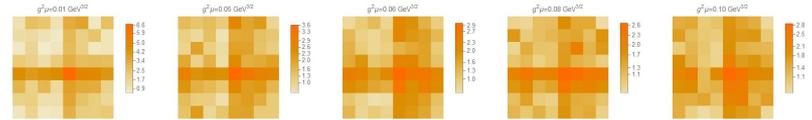
$$\langle\langle \rho_a(n^x, n^y, n_{\tau}) \rho_b(n'^x, n'^y, n'_{\tau}) \rangle\rangle = g^2 \tilde{\mu}^2 \delta_{ab} \frac{\delta_{n^x, n'^x} \delta_{n^y, n'^y} \delta_{n_{\tau}, n'_{\tau}}}{a_{\perp}^2 \tau}$$



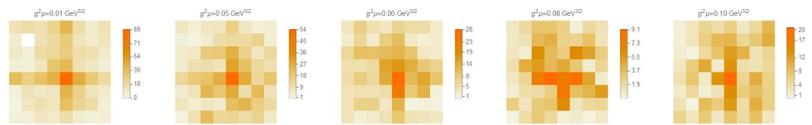
# In reality



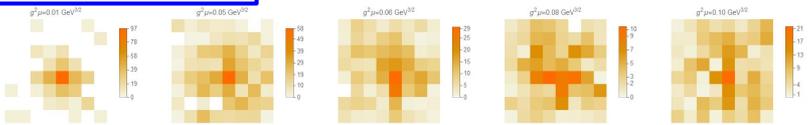
noise model using IBMQ manila



customized noise model with errors (0.01% for 1-qubit gates)



exact simulation



# Future plans

- Incorporate multiple Fock sectors, for example, to study gluon absorption and emission  $|\psi\rangle = c_1|q\rangle + c_2|qg\rangle + c_3|qgg\rangle + \dots$

Li, Lappi, Zhao, 2107.02225 (2021)

- Simulation-friendly implementation of the potential fields on the quantum circuit, for example, polynomial-time discretization method, approximation is well-controlled

Kassal, Jordan, Love, Mohseni, Aspuru-Guzik, 0801.2986 (2008)

- Investigate the effect of other medium and initial states, etc

Barenco, Ekerta, Suominenb, Torma, 9601018 (1996)

- Approximate QFT might help runtime on NISQ QCs

- Adding more colors, SU(3), to simulate a realistic QCD

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \equiv \tilde{\lambda}^1$$

*Thank you very much!*

Sainadh, 1309.2736 (2013)