An effective theory of medium induced radiation

Jet Quenching In The Quark-Gluon Plasma, Trento Italy

13.06.2022

Johannes Hamre Isaksen

PhD student in Theoretical Physics

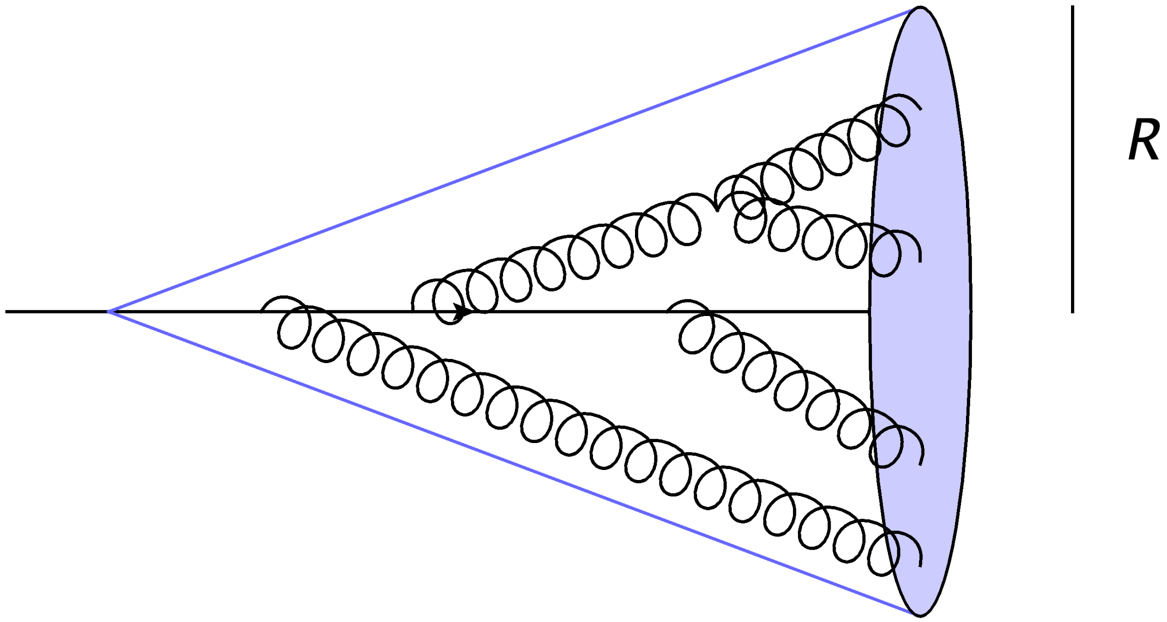
University of Bergen

In collaboration with Adam Takacs and Konrad Tywoniuk

Based on 2206.02811

Jet quenching

- Hard collision makes highly virtual particle
- Radiates and creates jet
- Medium interacts with jet and modifies it
  - This is called jet quenching



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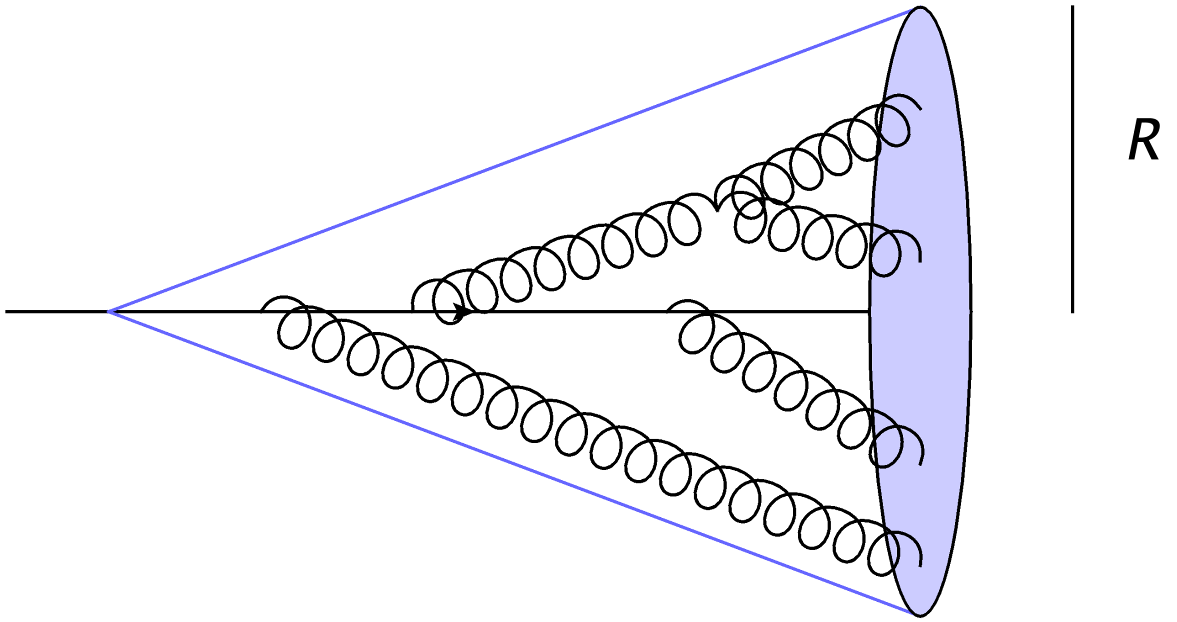

Jet quenching

- Partons go through the quark-gluon plasma
- Transverse momentum broadening
- Elastic collisions with medium constituents
- Radiation
  - Vacuum-like
  - Medium induced

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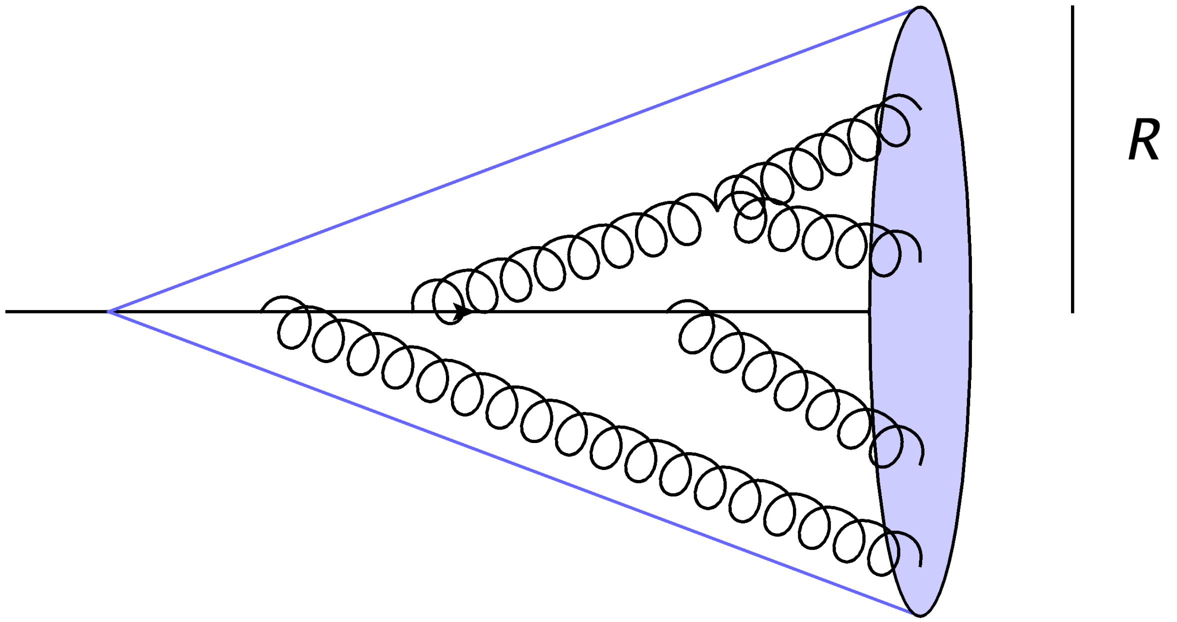
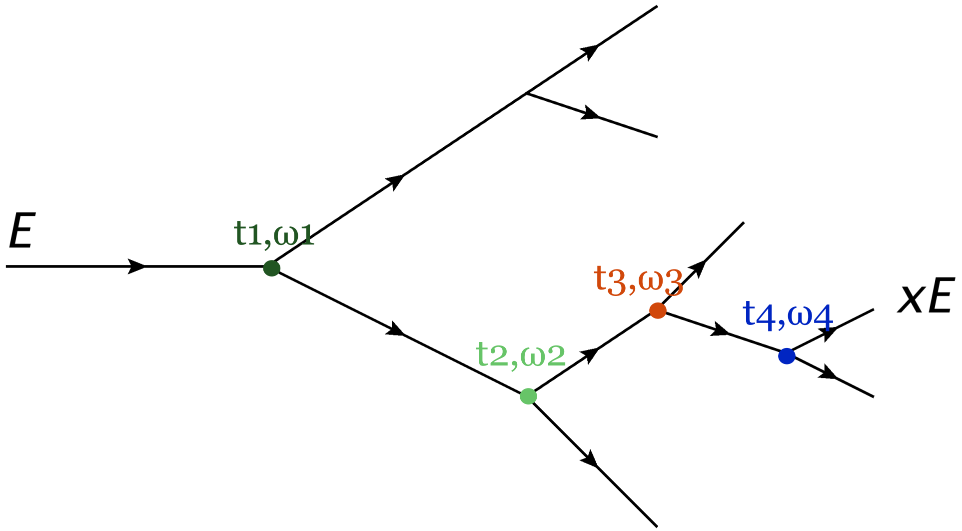
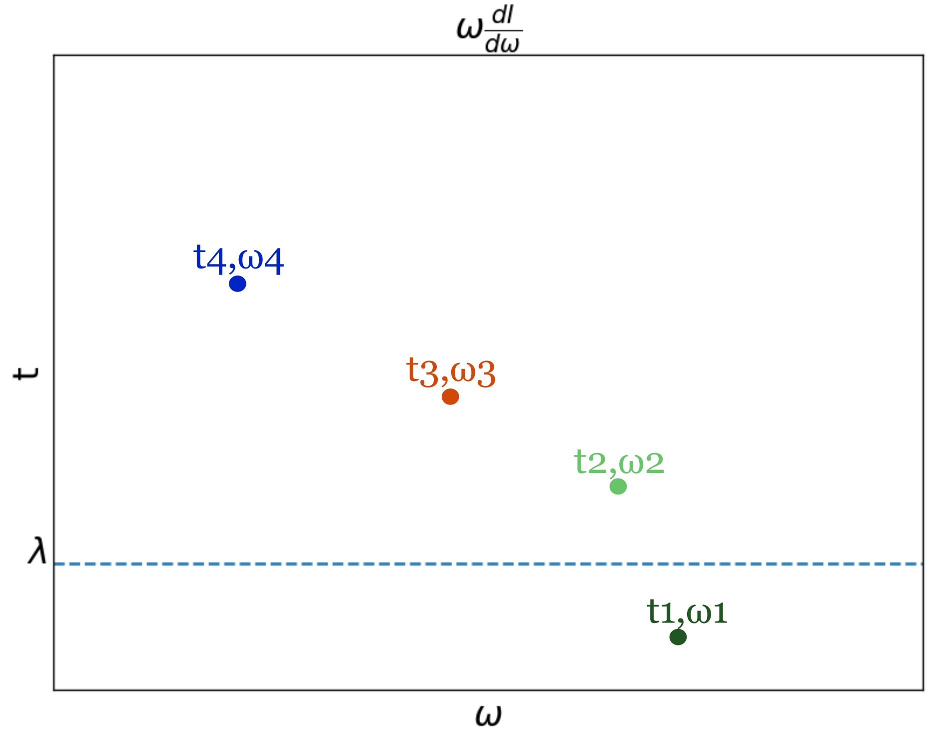
- Radiation
  - Vacuum-like
  - Medium induced → This is what this talk is about



Energy loss in the QGP

- Partons going through the medium scatter with medium constituents
- Scatterings induce emissions
- Emissions lead to radiative energy loss
  - Dominant contribution to energy loss for light quarks and gluons





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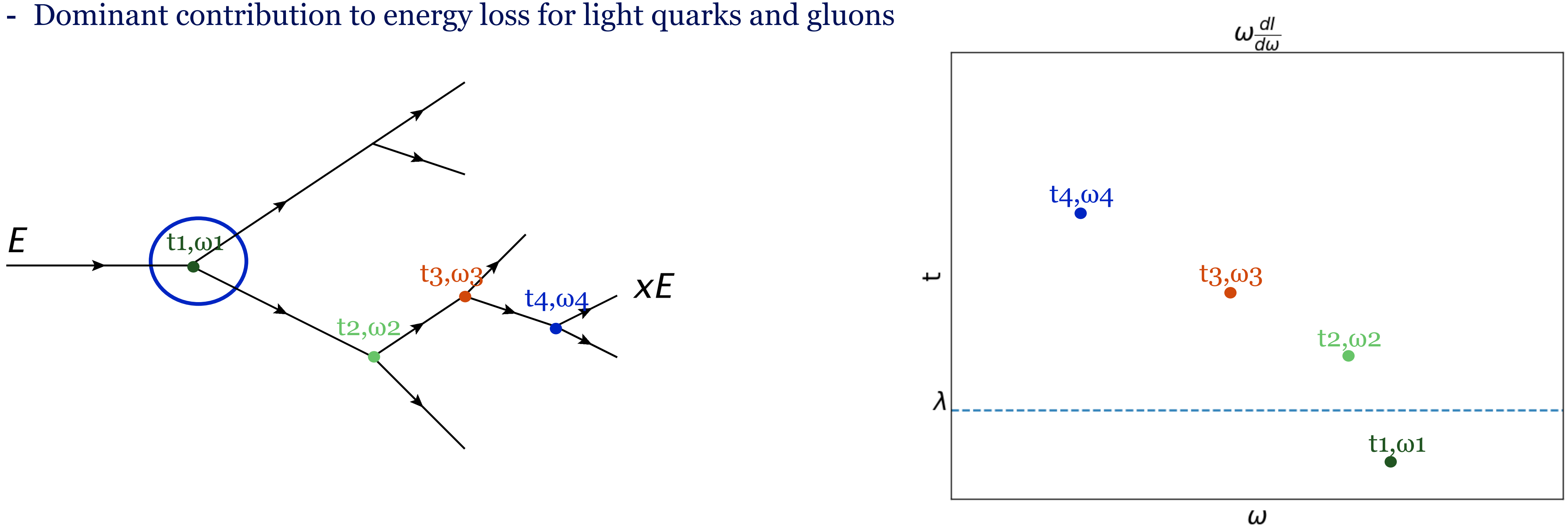


- To understand the process we need to zoom in and calculate the emission spectrum for each splitting

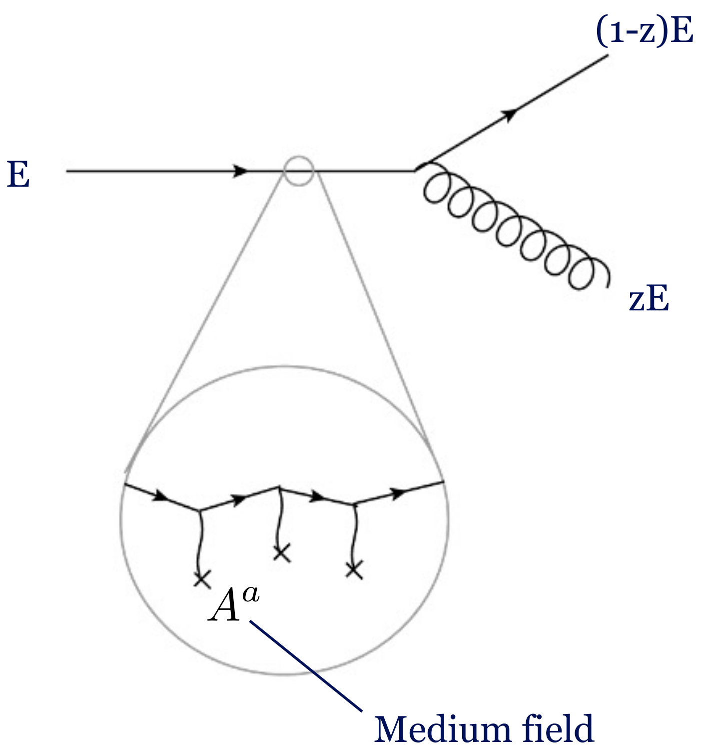
Medium induced emissions

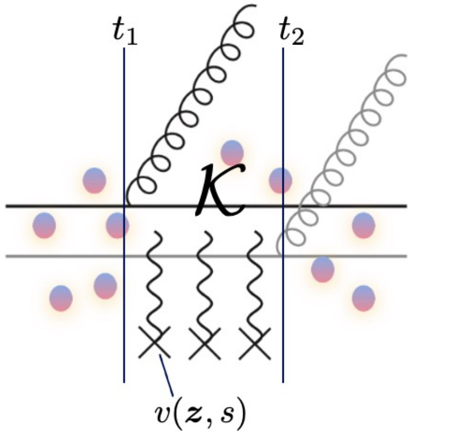
- The emission spectrum is given by

$$\omega \frac{\mathrm{d}I}{\mathrm{d}\omega} = \frac{2\alpha_s C_R}{\omega^2} \mathrm{Re} \int_0^\infty \mathrm{d}t_2 \int_0^{t_2} \mathrm{d}t_1\, \partial_{\boldsymbol{x}} \cdot \partial_{\boldsymbol{y}} \left[\mathcal{K}(\boldsymbol{x}, t_2; \boldsymbol{y}, t_1) - \mathcal{K}_0(\boldsymbol{x}, t_2; \boldsymbol{y}, t_1) \right]_{\boldsymbol{x}=\boldsymbol{y}=0}$$

- The three-point correlator $\mathcal{K}$ solves the Schrödinger equation

$$\left[i\partial_t + \frac{\partial_{\boldsymbol{x}}^2}{2\omega} + iv(\boldsymbol{x}, t) \right] \mathcal{K}(\boldsymbol{x}, t; \boldsymbol{y}, t_0) = i\delta(t - t_0)\delta(\boldsymbol{x} - \boldsymbol{y})$$





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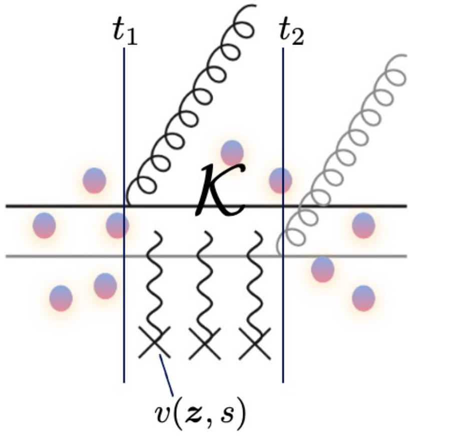
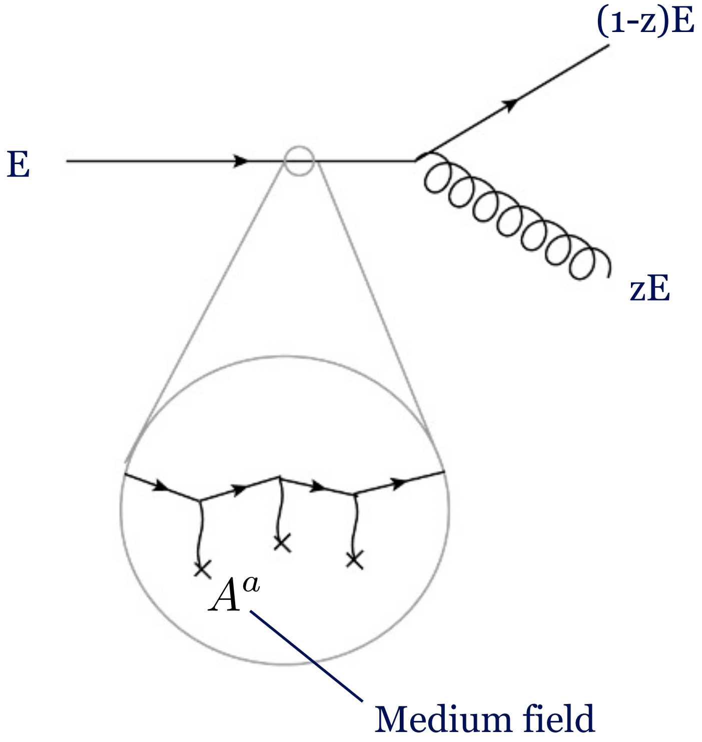
- Can in general only be evaluated numerically
- Analytical solutions of the spectrum are based on approximations





Medium induced emissions

Two well-known analytical solutions of the spectrum

- Opacity expansion
  - Expand $\mathcal{K}$ in the number of scatterings with the medium

$$\mathcal{K} \sim \mathcal{K}_0 + \mathcal{K}_0 v \mathcal{K}_0 + \mathcal{K}_0 v \mathcal{K}_0 v \mathcal{K}_0 + \ldots$$

  - Truncate at a finite order

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- None of these methods gives satisfying results in the whole phase space
- Combining three expansions gives a very good approximation
  - Opacity expansion (OE)*
  - Resummed opacity expansion (ROE)*
  - Improved opacity expansion (IOE)*

*Gyulassy et al. 9907461
Wiedemann 0005129

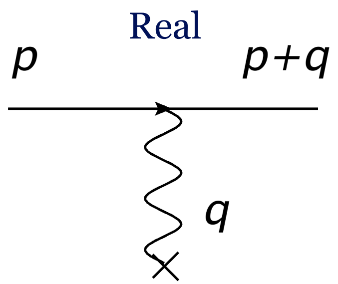
*Isaksen, Takacs, Tywoniuk 2206.02811
Schlicting, Soudi 2111.13731, Andres et al. 2011.06522

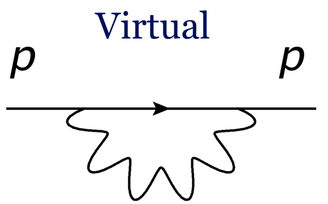
*Mehtar-Tani, Tywoniuk, Barata, Soto-Ontoso
1903.00506, 2106.07402

The opacity expansion

- Expansion in scatterings around the vacuum solution $\mathcal{K}_0$

$$\mathcal{K}(\boldsymbol{p},t_2;\boldsymbol{p}_0,t_1) = (2\pi)^2\delta(\boldsymbol{p}-\boldsymbol{p}_0)\mathcal{K}_0(\boldsymbol{p};t_2-t_1) - \int_{t_1}^{t_2}\mathrm{d}s\int_{\boldsymbol{q}}\mathcal{K}_0(\boldsymbol{p};t_2-s)v(\boldsymbol{q},s)\mathcal{K}(\boldsymbol{p}-\boldsymbol{q},s;\boldsymbol{p}_0,t_1)$$





- The scattering potential contains both a real and virtual part

$$v(\boldsymbol{q},s) = (2\pi)^2\delta(\boldsymbol{q})\Sigma(s) - \sigma(\boldsymbol{q},s)$$

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$$\omega\frac{\mathrm{d}I^{N=1}}{\mathrm{d}\omega} \simeq \begin{cases} 2\bar{\alpha}\frac{L}{\lambda}\left(\ln\frac{\bar{\omega}_c}{\omega}-1+\gamma_E\right), & \text{for } \omega \ll \bar{\omega}_c\,, \\ \frac{\pi}{2}\bar{\alpha}\frac{L}{\lambda}\frac{\bar{\omega}_c}{\omega}, & \text{for } \omega \gg \bar{\omega}_c\,. \end{cases}$$

$$\omega\frac{\mathrm{d}I^{N=2}}{\mathrm{d}\omega} \simeq \begin{cases} -\bar{\alpha}\left(\frac{L}{\lambda}\right)^2, & \text{for } \omega \ll \bar{\omega}_c\,, \\ \sim\bar{\alpha}\left(\frac{L}{\lambda}\right)^2\left(\frac{\bar{\omega}_c}{\omega}\right)^2, & \text{for } \omega \gg \bar{\omega}_c\,. \end{cases}$$

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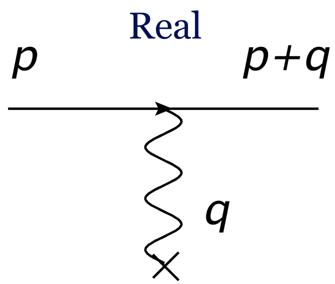

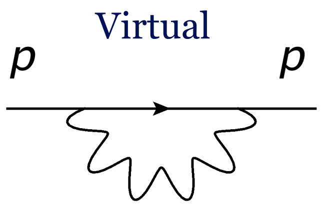

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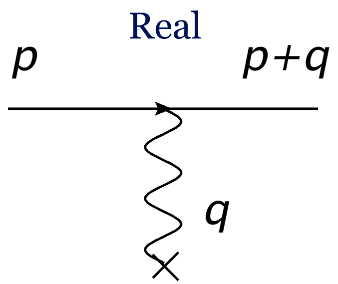

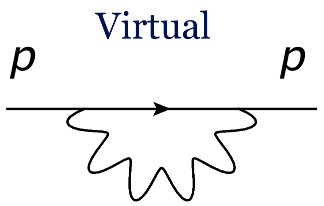

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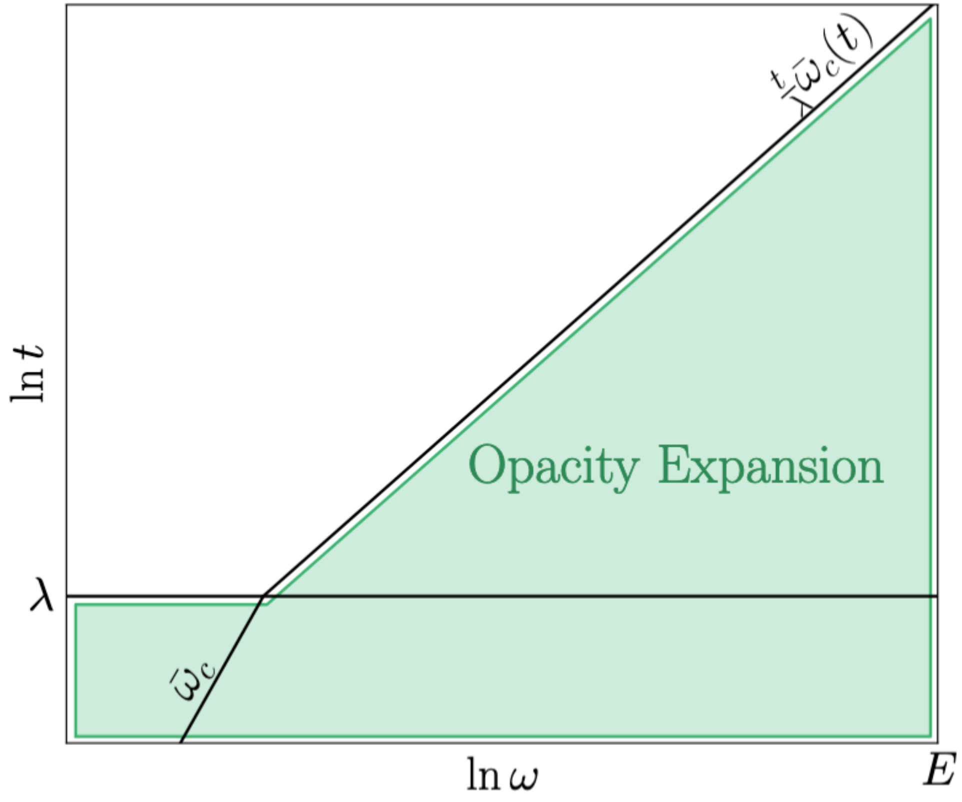
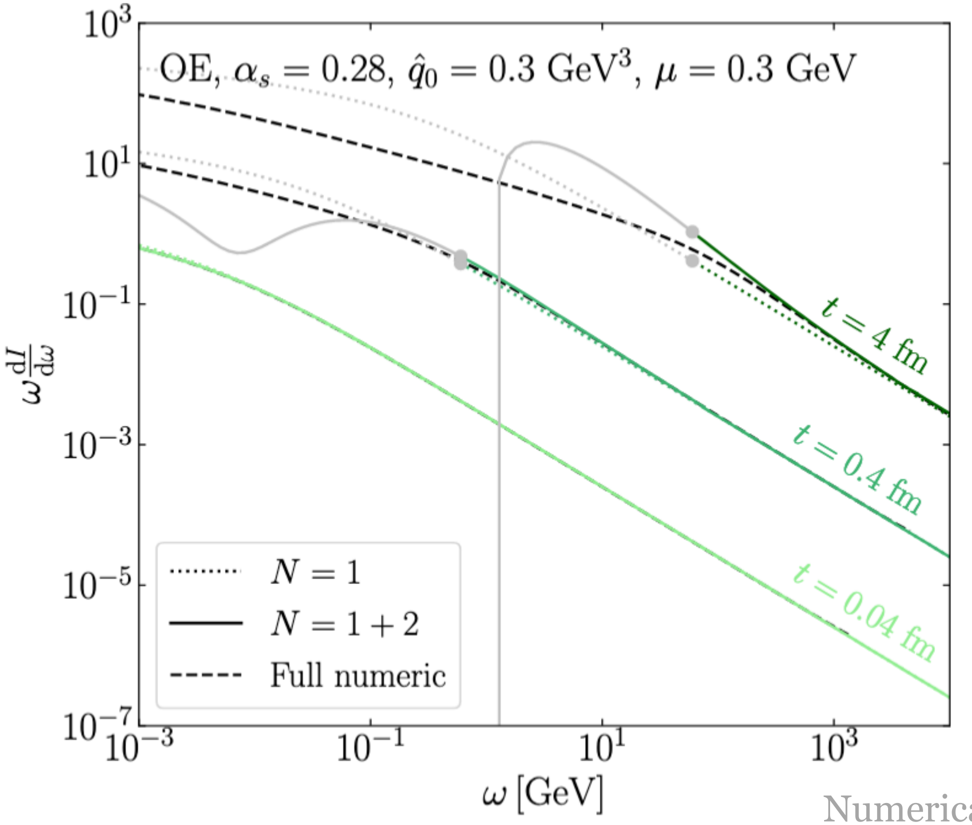
- At high energy the spectrum goes as $\sim \left(\frac{L}{\lambda}\frac{\bar{\omega}_c}{\omega}\right)^n = \left(\frac{\hat{q}_0 L^2}{2\omega}\right)^n$ → convergence when $\omega > \dfrac{\hat{q}_0 L^2}{2}$

The opacity expansion

- Valid for early times, but also late times if the energy is big
- Breaks down at later times for low energy





Numerical solution from
Andres, Dominguez, Gonzalez Martinez
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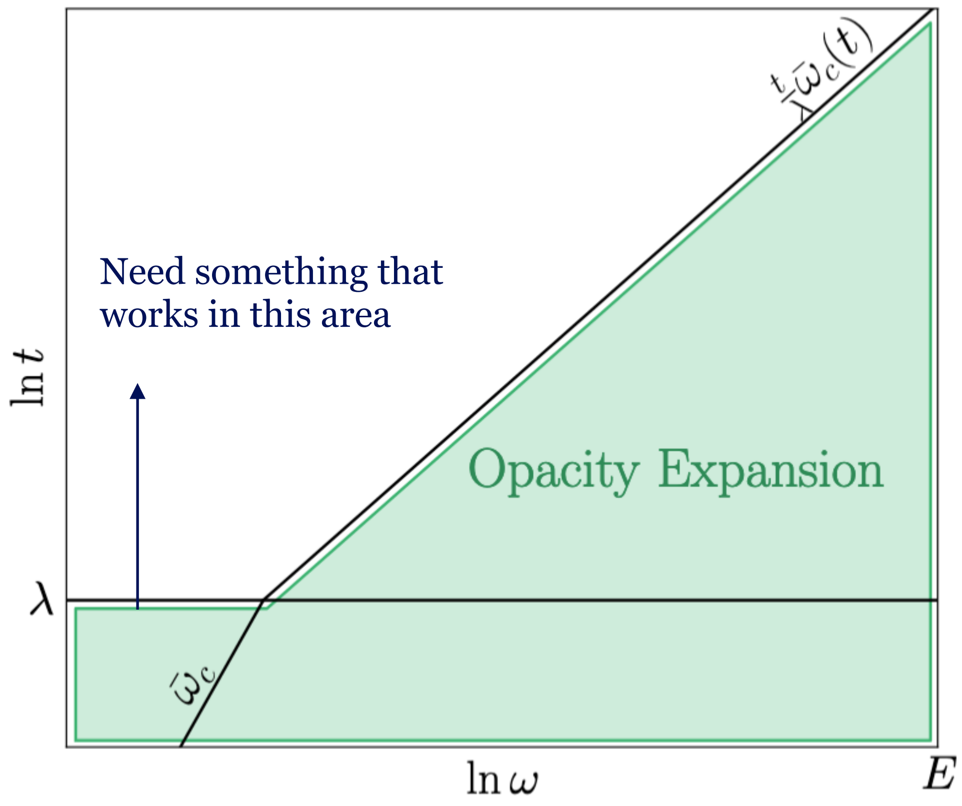

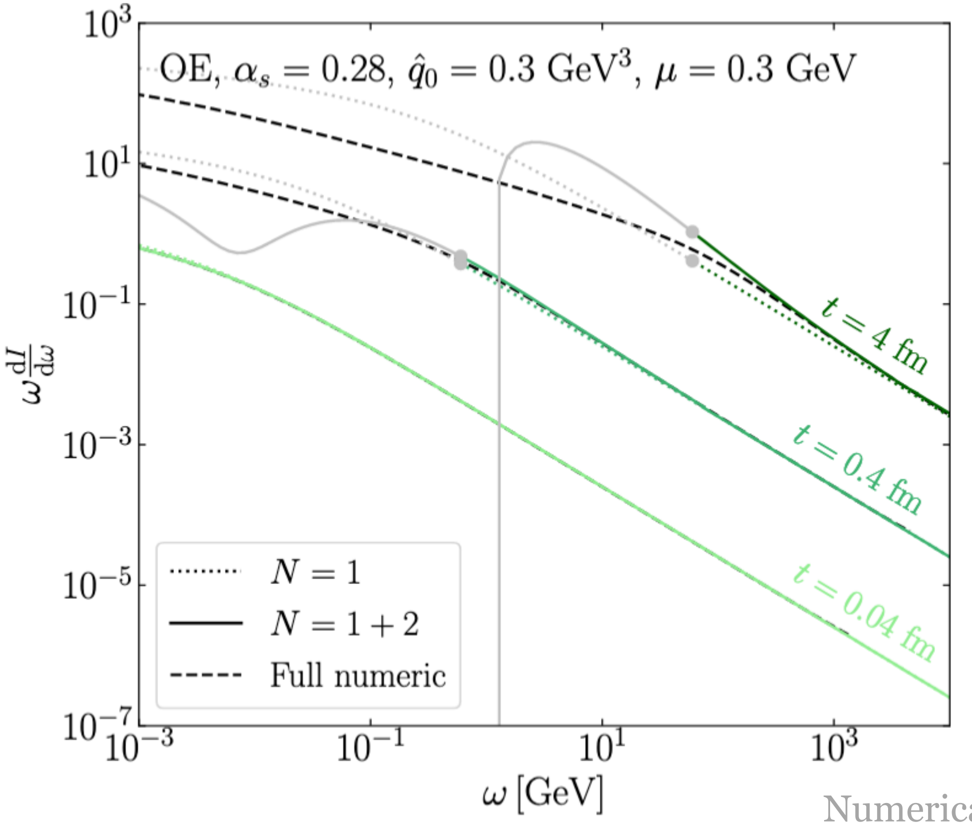

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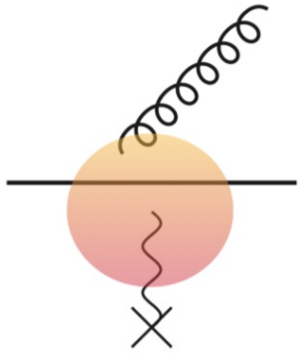
- To fill out more of the phase space another expansion is needed

The resummed opacity expansion

- Expand only in real scatterings
- All virtual scatterings are resummed in a Sudakov factor $\Delta(t, t_0) \equiv \mathrm{e}^{-\int_{t_0}^{t} \mathrm{d}s\, \Sigma(s)}$

$$\mathcal{K}(\boldsymbol{p}, t; \boldsymbol{p}_0, t_0) = (2\pi)^2 \delta(\boldsymbol{p} - \boldsymbol{p}_0) \Delta(t, t_0) \mathcal{K}_0(\boldsymbol{p}; t - t_0) \\ + \int_{t_0}^{t} \mathrm{d}s \frac{\Delta(t, t_0)}{\Delta(s, t_0)} \int_{\boldsymbol{q}} \mathcal{K}_0(\boldsymbol{p}; t - s) \sigma(\boldsymbol{q}, s) \mathcal{K}(\boldsymbol{p} - \boldsymbol{q}, s; \boldsymbol{p}_0, t_0)$$



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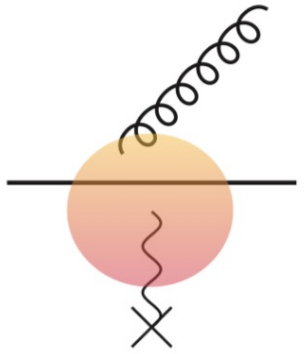

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- However, also works for longer media L > λ:
  - New constant scale emerges: $\omega_{\mathrm{BH}} = \dfrac{\mu^2\lambda}{2}$

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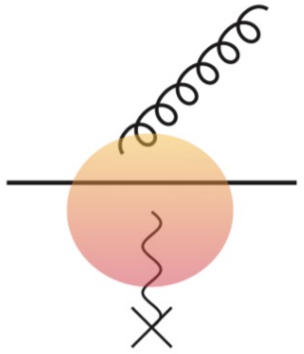

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- Nr=1 contribution leading at low energy: convergence

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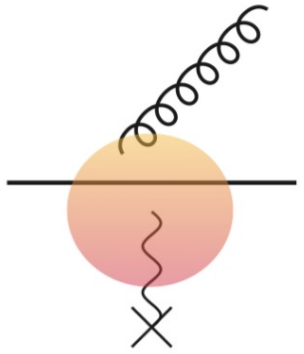

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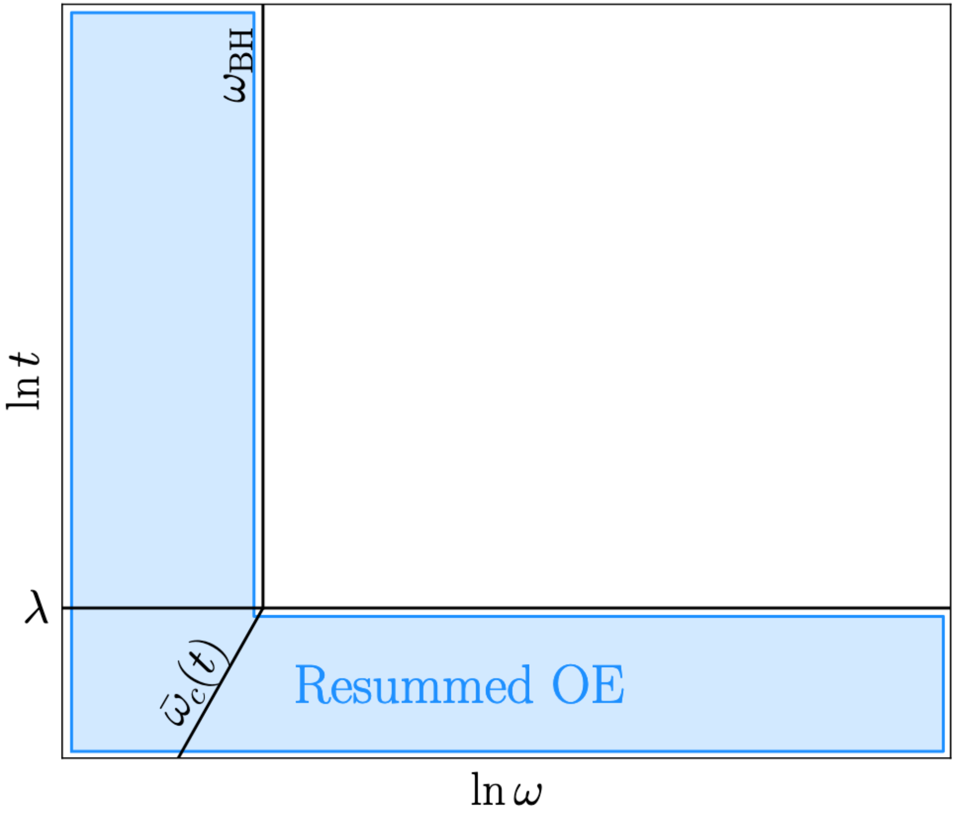
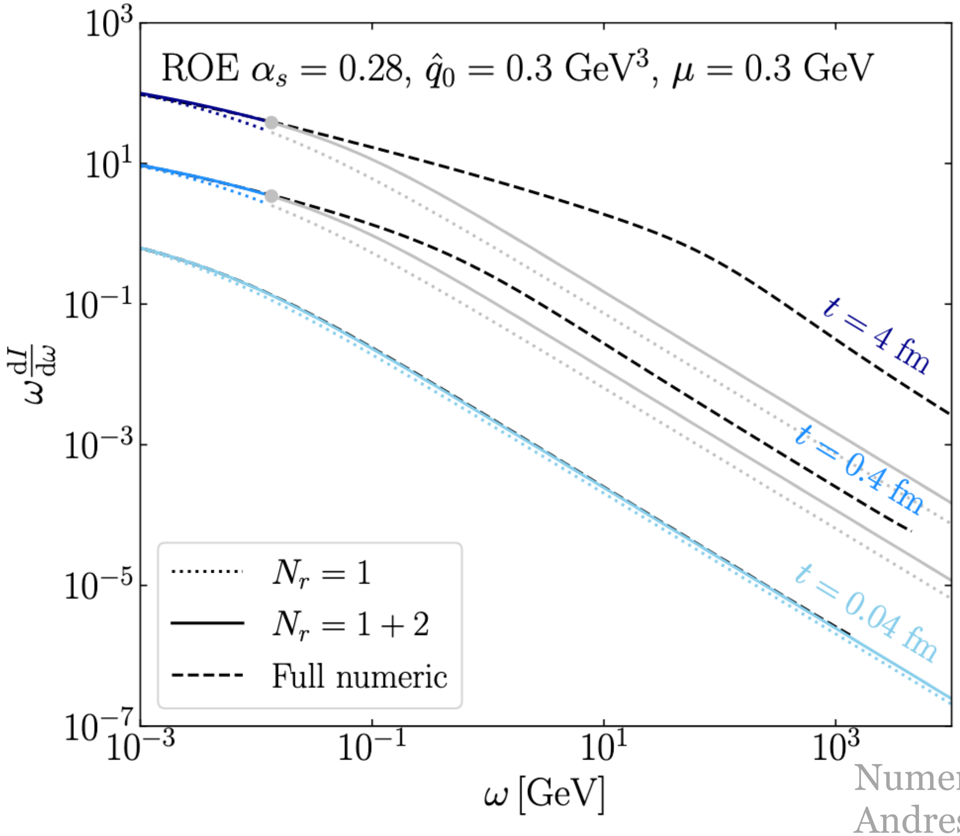
- Nr=1 contribution leading at low energy: convergence
- At high energy $\mathrm{d}I^{N_r=2} \sim \mathrm{d}I^{N_r=1}$: no sign of convergence

The resummed opacity expansion

- Valid for:
  - Early times (same as opacity expansion)
  - Late times at low energy (Bethe-Heitler region)





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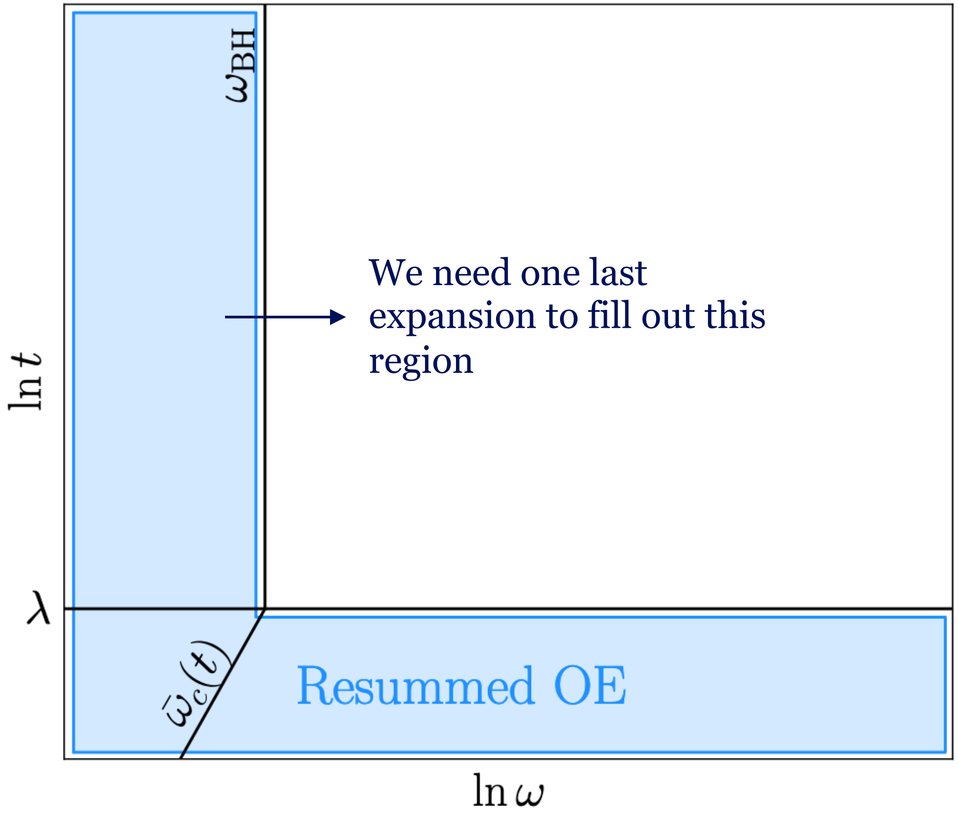
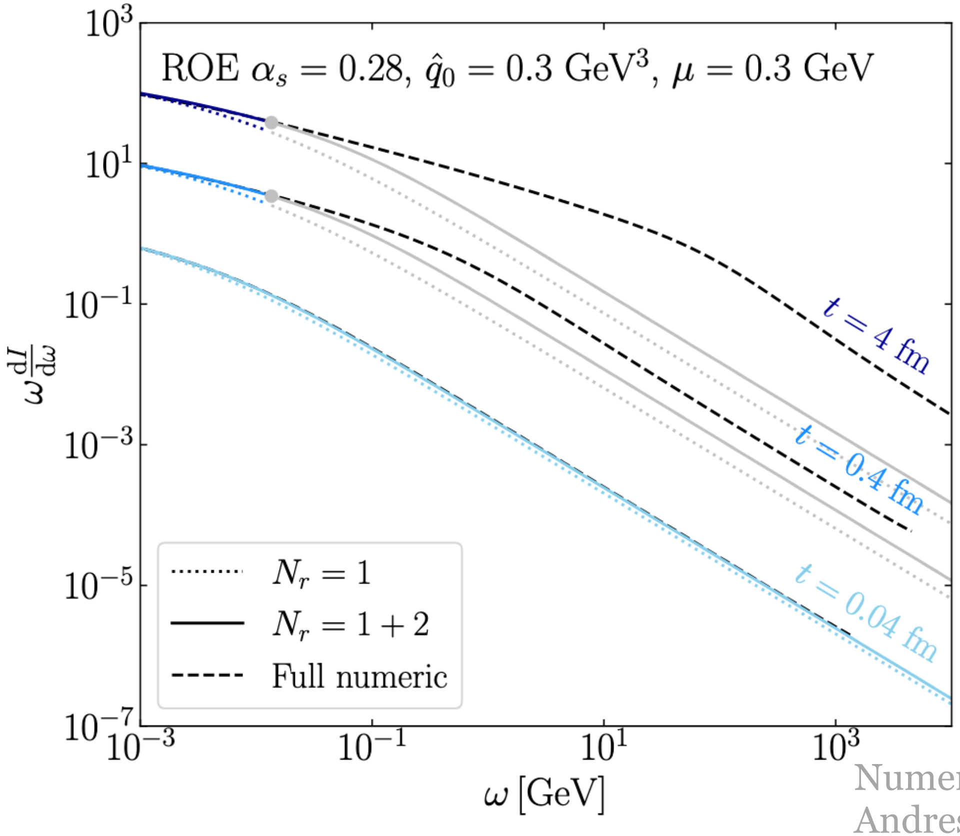
  - Late times at low energy (Bethe-Heitler region)



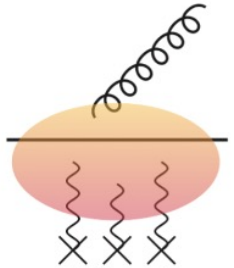


Numerical solution from
Andres, Dominguez, Gonzalez Martinez
2011.06522

The improved opacity expansion

- Comes from manipulating the scattering potential
$$v(\boldsymbol{x},t) \simeq \frac{\hat{q}_0}{4}\boldsymbol{x}^2 \ln\frac{1}{\mu_*^2\boldsymbol{x}^2} = \frac{\hat{q}}{4}\boldsymbol{x}^2 + \frac{\hat{q}_0}{4}\boldsymbol{x}^2\ln\frac{1}{Q^2\boldsymbol{x}^2} \equiv v_{\rm HO}(\boldsymbol{x},t) + \delta v(\boldsymbol{x},t)$$

Harmonic oscillator



- The harmonic oscillator problem is an expansion in many soft scatterings
  - Solved exactly, resums an arbitrary number of scatterings
  - Can only create emissions with energy up to the emergent scale $\omega_c = \dfrac{\hat{q}L^2}{2}$
  - Emissions above this scale must be created by harder scatterings, leading to

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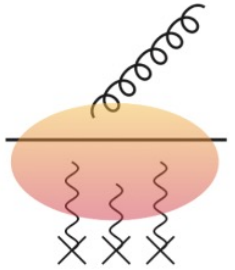

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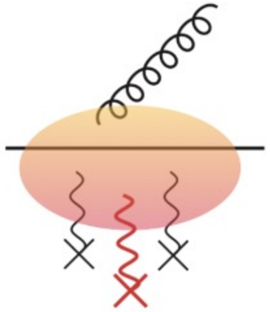
- The improved opacity expansion
  - Expansion in hard scatterings around the harmonic oscillator solution

$$\mathcal{K}(\boldsymbol{x},t_2;\boldsymbol{y},t_1) = \mathcal{K}_{\text{HO}}(\boldsymbol{x},t_2;\boldsymbol{y},t_1) - \int_{t_1}^{t_2} \mathrm{d}s \int_{\boldsymbol{z}} \mathcal{K}_{\text{HO}}(\boldsymbol{x},t_2;\boldsymbol{z},s)\delta v(\boldsymbol{z},s)\mathcal{K}(\boldsymbol{z},s;\boldsymbol{y},t_1)$$

- The improved opacity expansion makes it possible to go to higher energies than ω_c

Next-to harmonic oscillator



$\delta v(\boldsymbol{x},t)$

The improved opacity expansion

- The jet quenching parameter now depends on the matching scale Q

$$\hat{q}(t) = \hat{q}_0(t) \ln \frac{Q^2}{\mu_*^2}$$

  - It turns out* the expansion is only well-behaved when choosing the ω-dependent scale

$$Q_r^2(\omega) = \sqrt{\hat{q}\omega}$$

  - Together these two conditions provide an implicit equation for $\hat{q}$, which now runs with energy
  - Breaks down around the Bethe-Heitler energy $\omega_{\rm BH}$ → IOE only valid for $\omega > \omega_{\rm BH}$

*Discussed in detail by Barata, Mehtar-Tani 2004.02323

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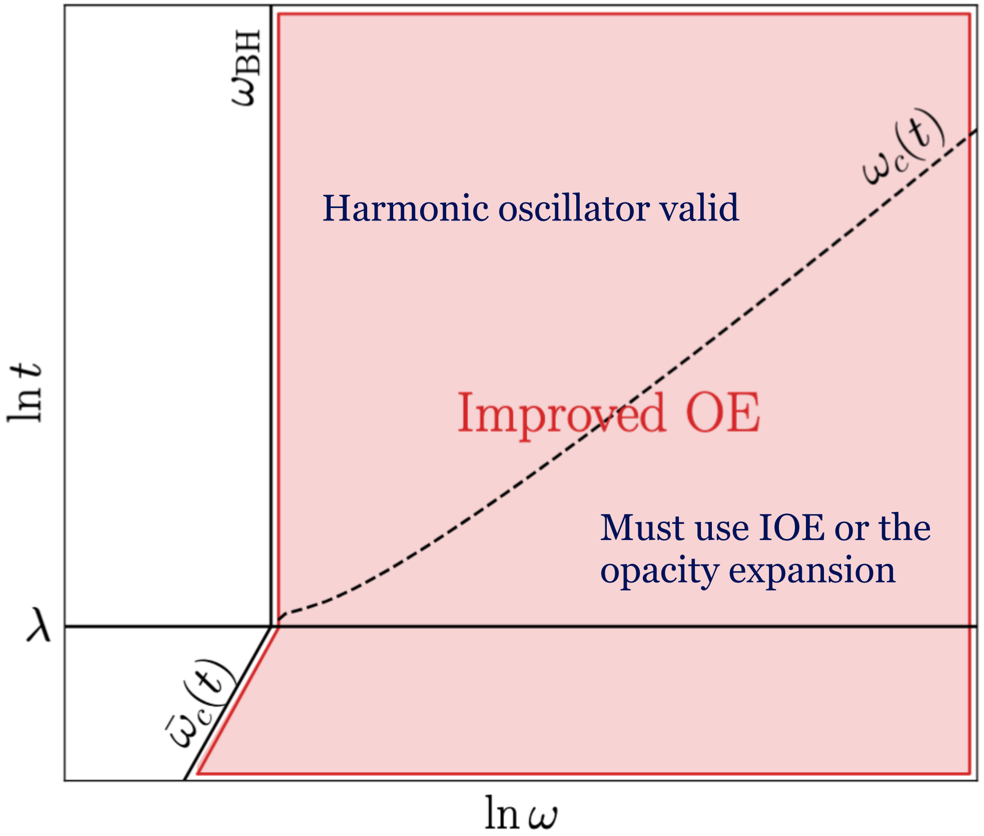
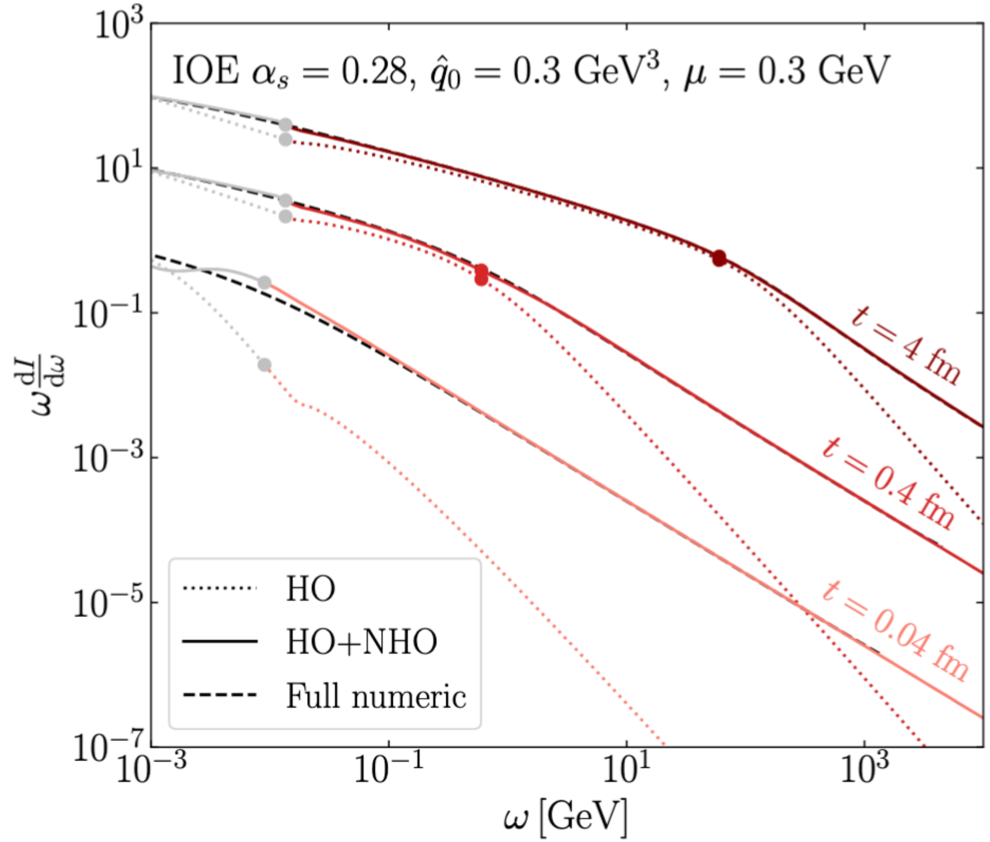
The improved opacity expansion

- Valid for energies over the Bethe-Heitler regime





Numerical solution from
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Summary of the expansions

- At early times $t < \lambda$ only few scatterings will induce emissions
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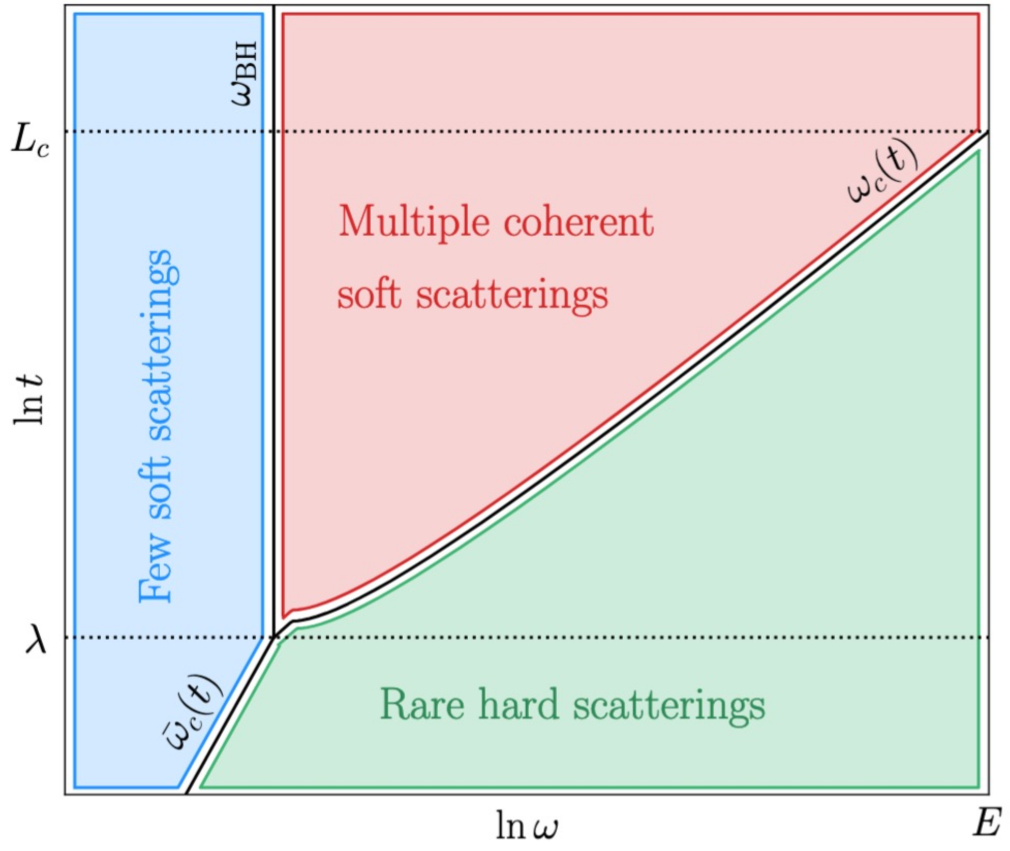

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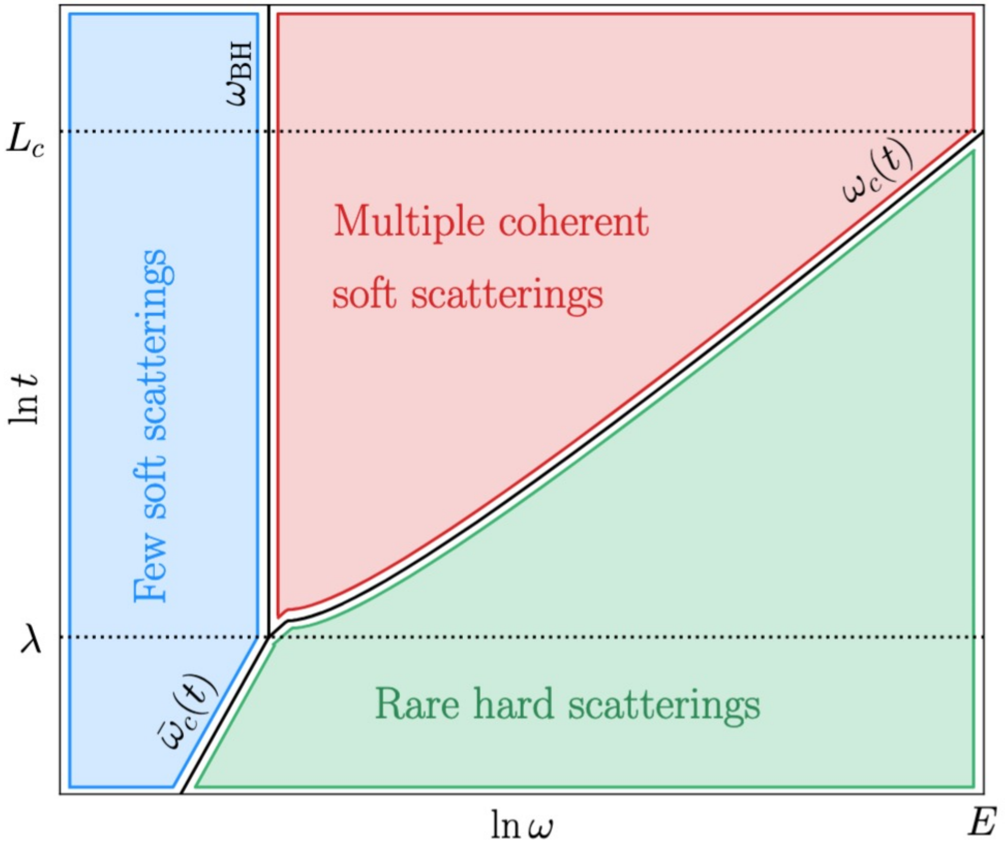

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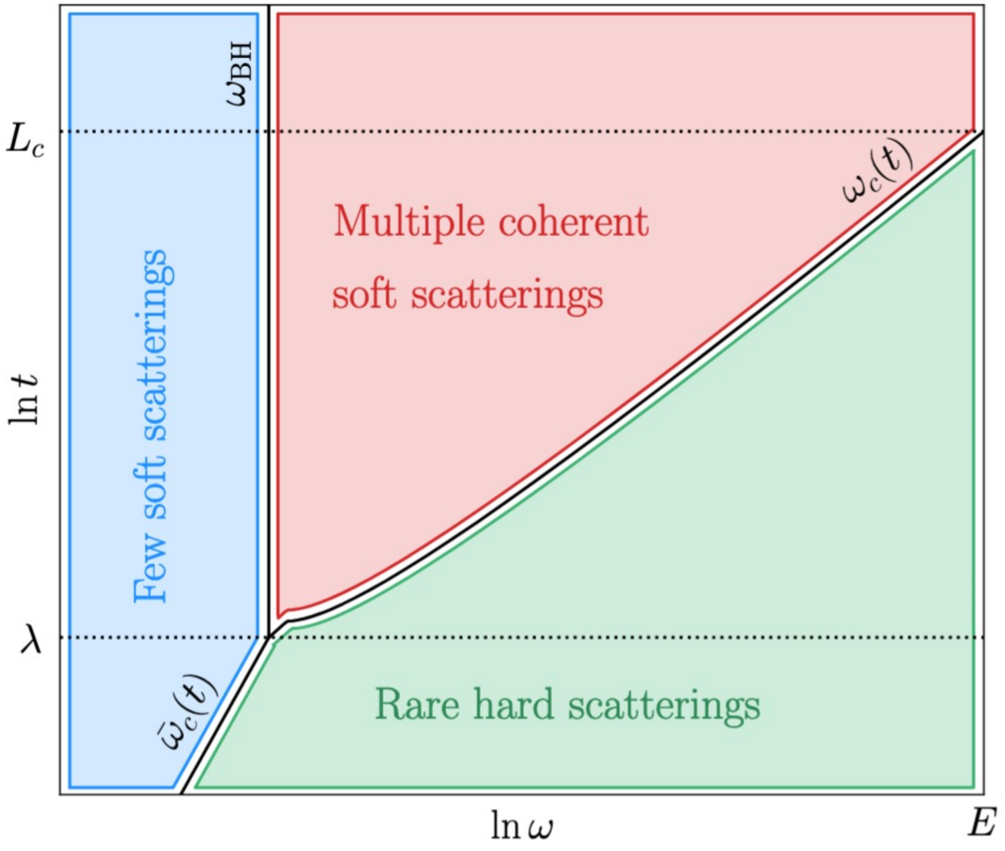

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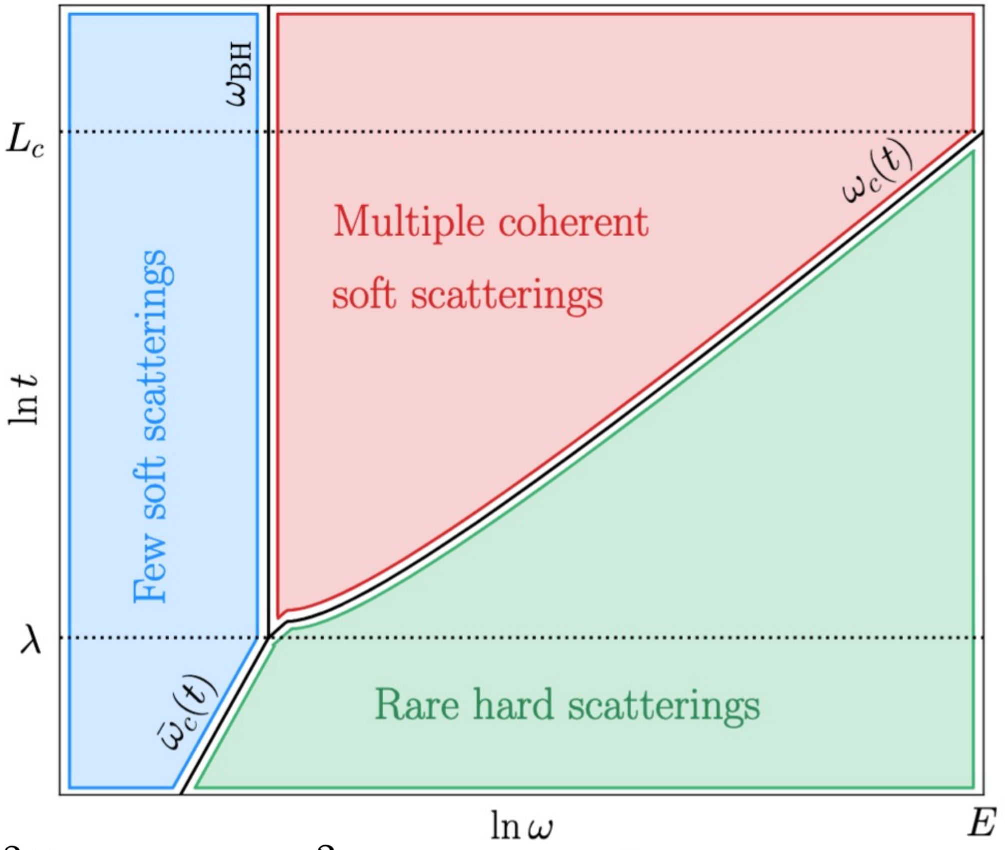

- Three energy scales naturally emerge from the expansions: $\bar{\omega}_c = \dfrac{\mu^2 L}{2}$ $\quad \omega_{\mathrm{BH}} = \dfrac{\mu^2 \lambda}{2}$ $\quad \omega_c = \dfrac{\hat{q} L^2}{2}$

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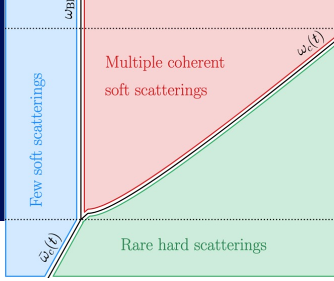
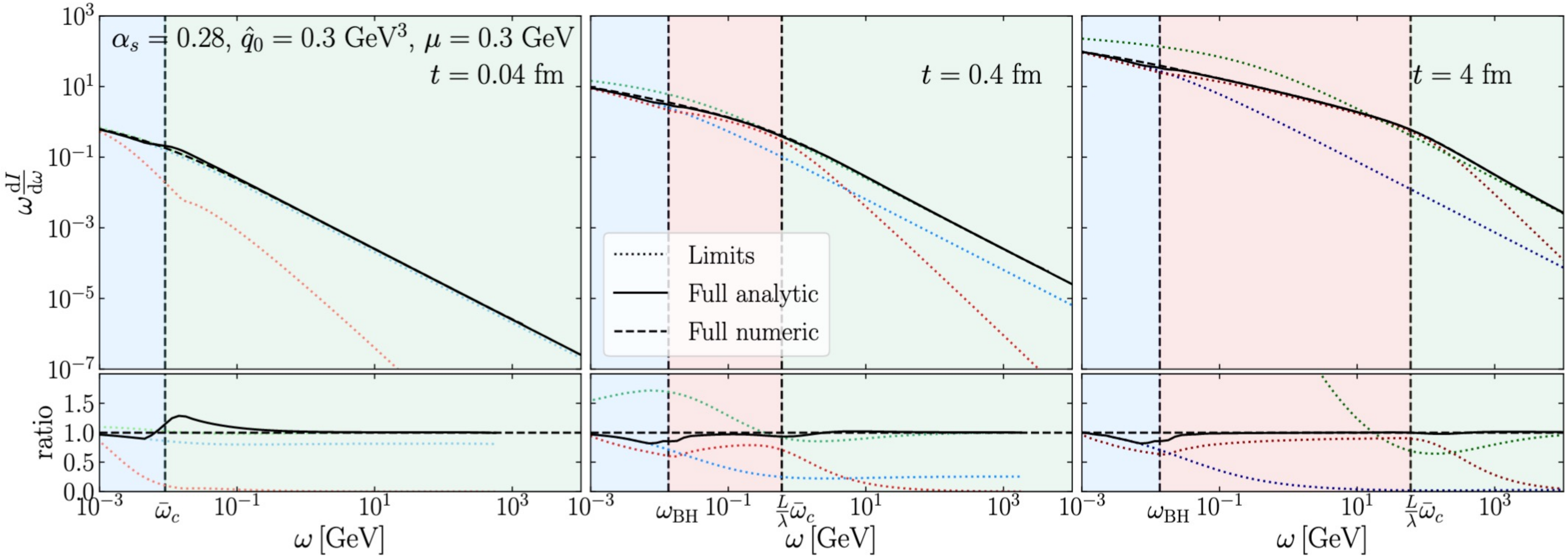


- For the full line we have used the ROE and IOE, with a smoothening transition function
- Error is biggest at the transitions between the areas, expect that higher orders make it smoother



Numerical solution from Andres, Dominguez, Gonzalez Martinez 2011.06522

Multiple emissions

- Multiple emissions must be considered when the multiplicity $N(\omega) = \int_\omega^\infty \mathrm{d}\omega' \frac{\mathrm{d}I}{\mathrm{d}\omega'}$ is large
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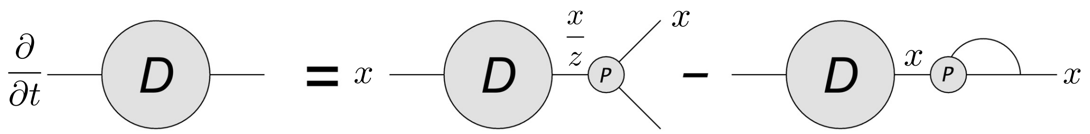
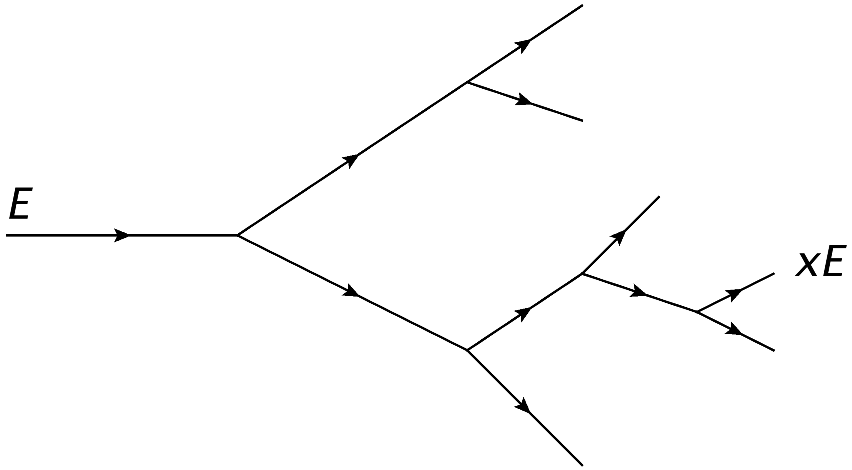
- Define the energy distribution of partons with energy xE after travelling time t in the medium:

$$D(x,t) \equiv x \frac{\mathrm{d}N}{\mathrm{d}x}$$

- Multiple emissions are resummed in a rate equation

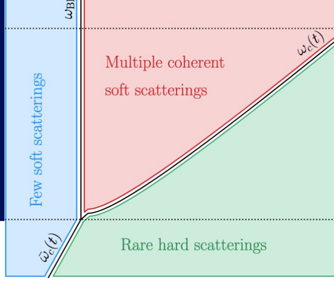
$$\frac{\partial}{\partial t} D(x,t) = \int_x^1 dz\, 2P\left(z, \frac{x}{z}E, t\right) D\left(\frac{x}{z}, t\right) - \int_0^1 dz\, P\left(z, xE, t\right) D\left(x, t\right)$$



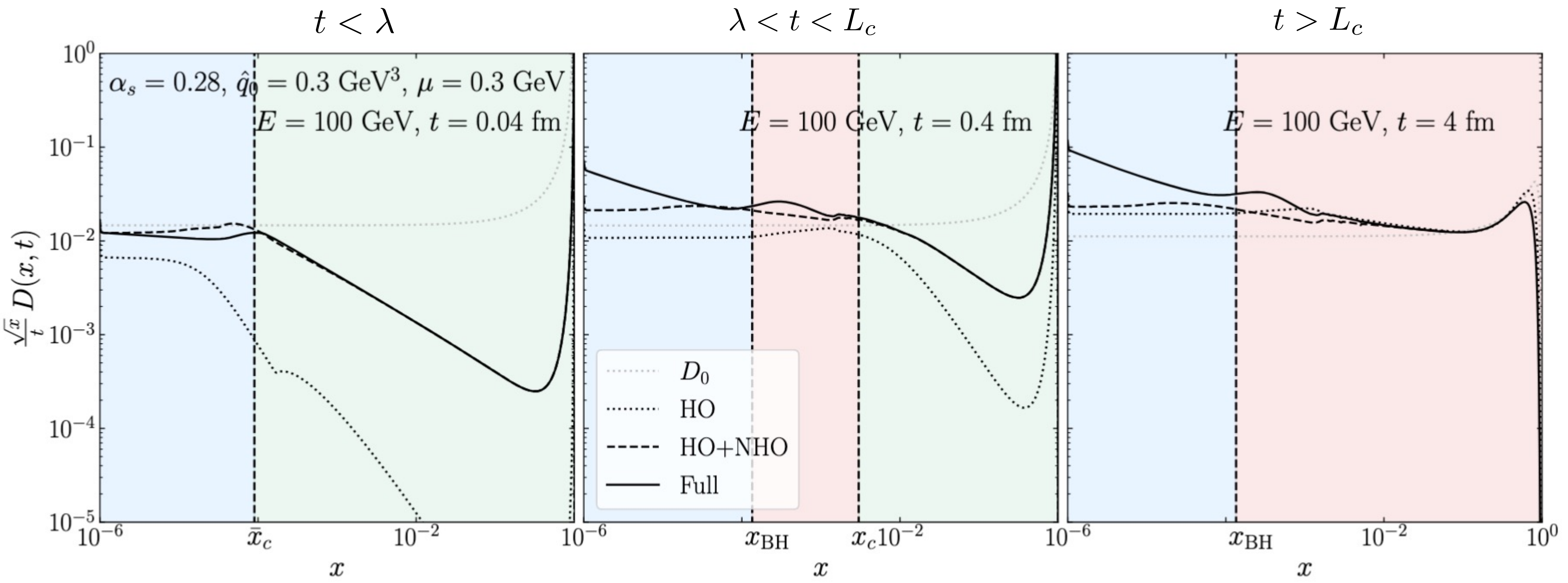


- The splitting rate is simply $P(z,E,t) = \left.\frac{\mathrm{d}I}{\mathrm{d}z\mathrm{d}t}\right|_E$
  - Follows directly from the emission spectrum

The energy distribution



- Numerical solution of the energy distribution



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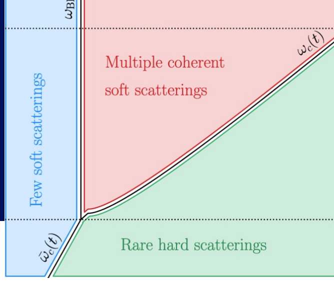
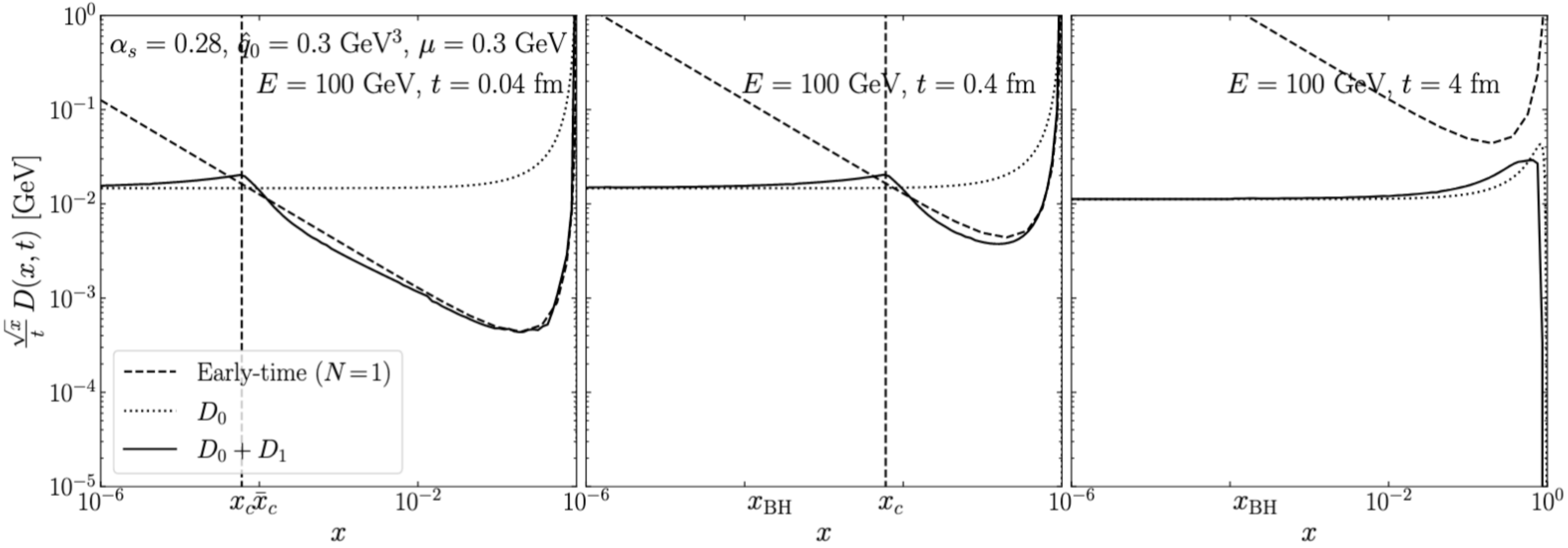


- The rate equation can be solved analytically for simplified systems
  - A single emission coming from one scattering $N = 1$ (good approximation at early times)
  - Pure harmonic oscillator solution D_0 (good approximation at late times)
  - Harmonic oscillator solution with one hard scattering $D_0 + D_1$ (good approximation at early to late times)



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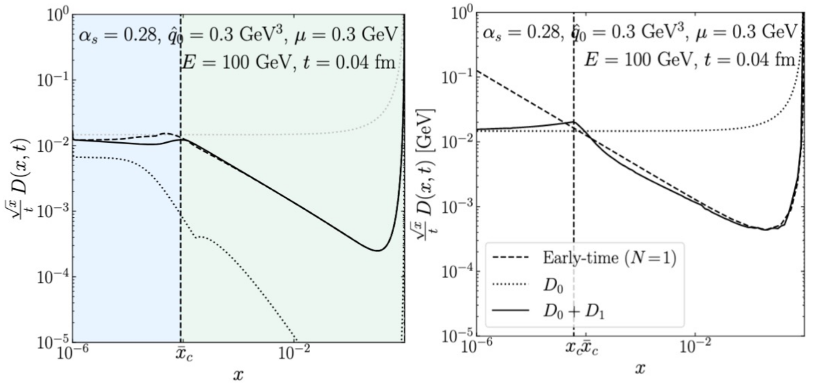

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  - Still residue of early time behavior
  - The analytical solution $D_0 + D_1$ works reasonably well

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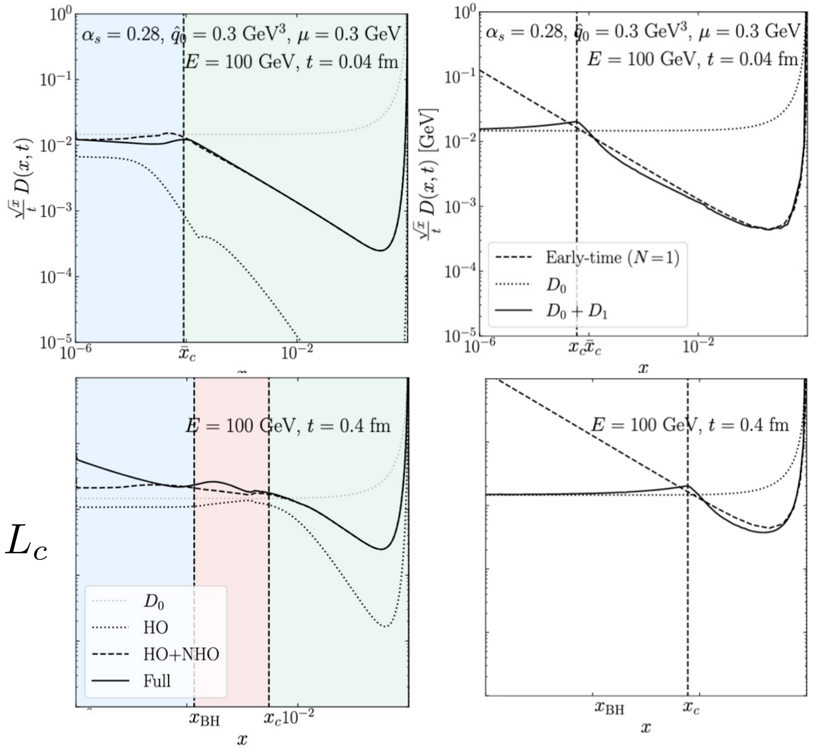

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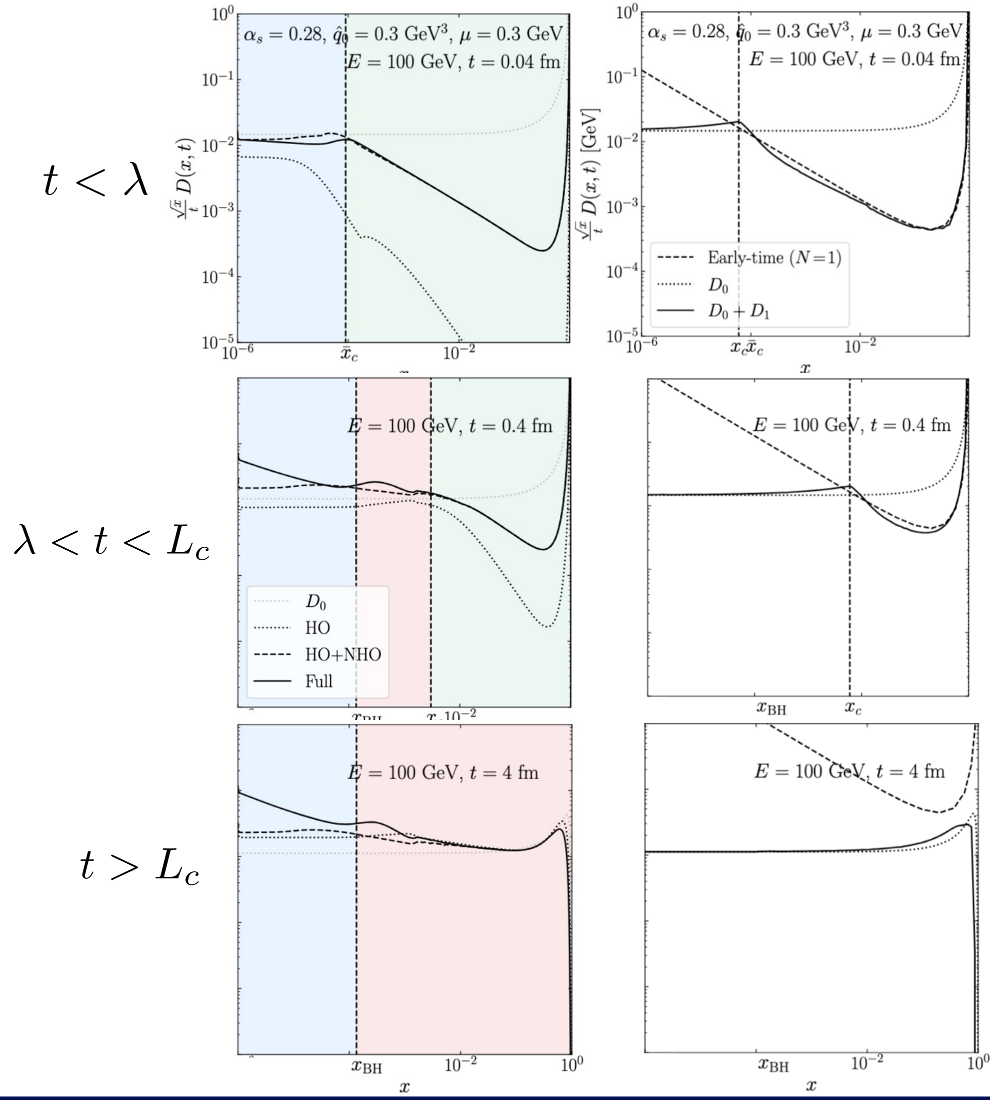

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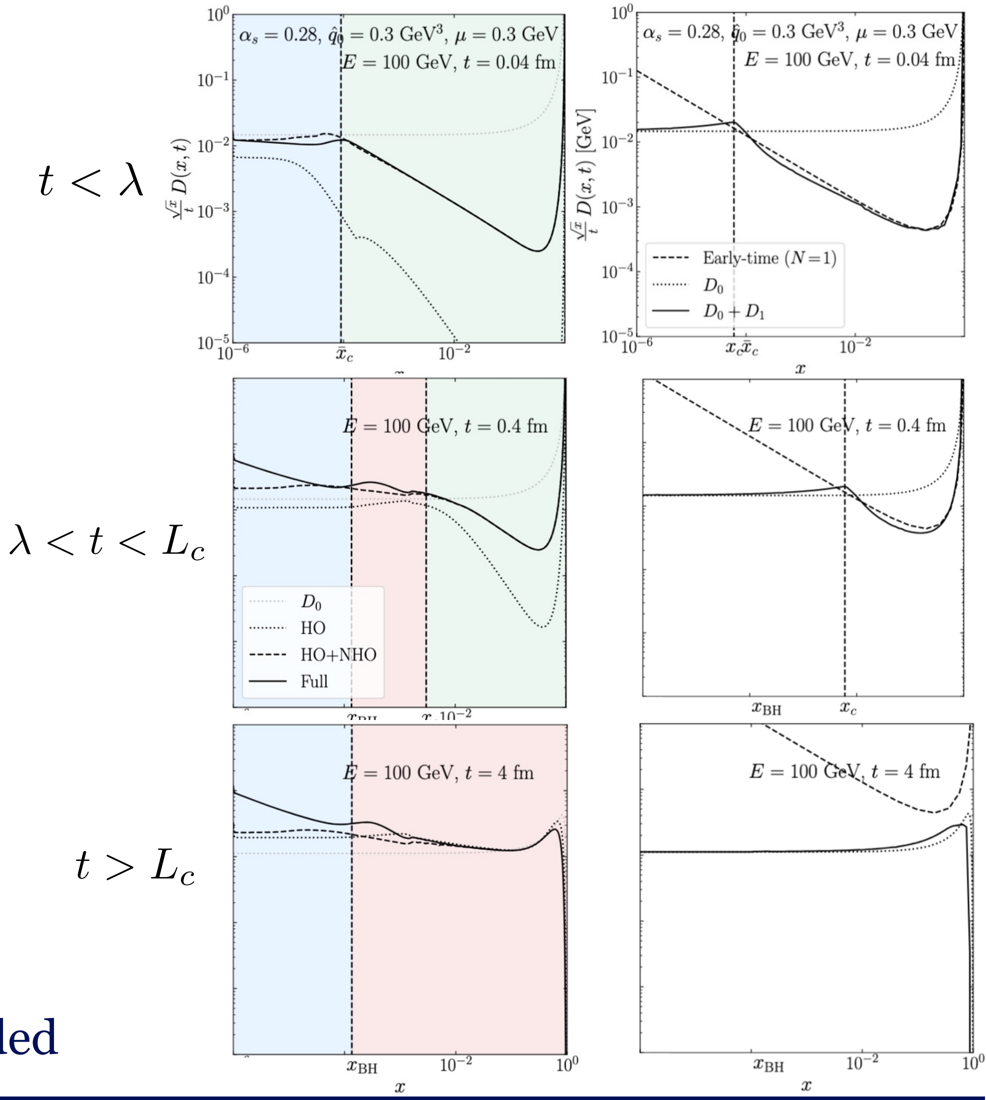
- The analytical solutions fail at small x, as the BH regime is not included



Conclusion and outlook

- Effective theory for the medium induced energy spectrum
  - Good theoretical understanding and control in the different regimes
  - Systematically improvable order by order
- This is achieved through three expansions
  - The opacity expansion, the resummed opacity expansion and the improved opacity expansion
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- Outlook
  - Get closer to phenomenology by incorporating the vacuum
  - Rigorously define the accuracy of medium induced emissions
  - Incorporating quark masses

Thank you for your time

Backup

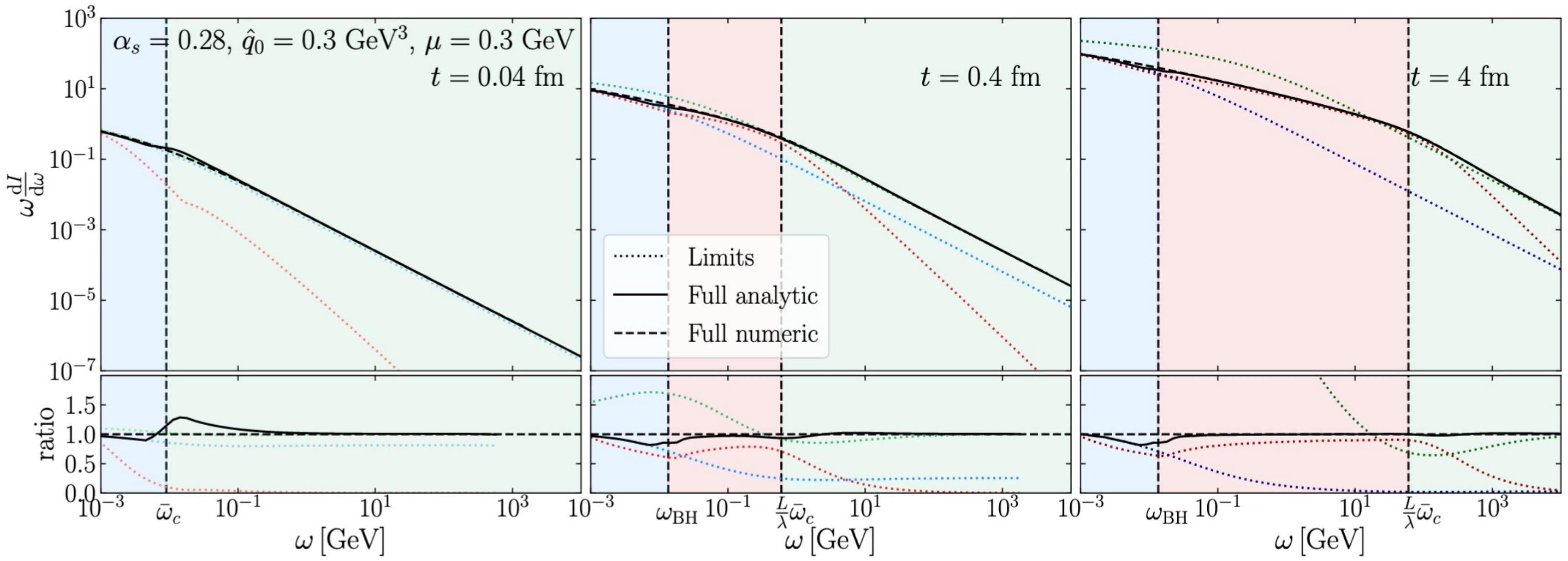
- The limits of the full emission spectrum are

$$\omega\frac{\mathrm{d}I}{\mathrm{d}\omega}\bigg|_{t\ll\lambda} = \begin{cases} 2\bar{\alpha}\frac{L}{\lambda}\left(\ln\frac{\bar{\omega}_c}{\omega} - 1 + \gamma_E\right), & \text{for } \omega \ll \bar{\omega}_c(t) \quad ** \\ \frac{\pi}{2}\bar{\alpha}\frac{L}{\lambda}\frac{\bar{\omega}_c}{\omega}, & \text{for } \bar{\omega}_c(t) \ll \omega \quad *** \end{cases}$$

$$\omega\frac{\mathrm{d}I}{\mathrm{d}\omega}\bigg|_{t\gg\lambda} = \begin{cases} 2\bar{\alpha}\frac{L}{\lambda}\ln\left(\frac{\omega_{\mathrm{BH}}}{\omega}\right), & \text{for } \omega \ll \omega_{\mathrm{BH}}, \quad * \\ \bar{\alpha}\sqrt{\frac{2\omega_c}{\omega}}, & \text{for } \omega_{\mathrm{BH}} \ll \omega \ll \omega_c(t) \quad * \\ \frac{\pi}{2}\bar{\alpha}\frac{L}{\lambda}\frac{\bar{\omega}_c}{\omega}, & \text{for } \omega_c(t) \ll \omega, \quad ** \end{cases}$$

*Opacity expansion
*Resummed opacity expansion
*Improved opacity expansion

The union of IOE and ROE covers the whole phase space!



Backup

- Analytic energy distributions
  - Early time

$$D(x,t) \simeq \begin{cases} 2\bar{\alpha}\frac{t}{\lambda}\frac{1}{1-x}\ln\left(\frac{\bar{\omega}_c(t)}{x(1-x)E}\right) & \text{for } x \ll \bar{x}_c\,, \\ \frac{\pi\bar{\alpha}}{4}\frac{\hat{q}_0}{E}\frac{t^2}{x(1-x)^2} & \text{for } \bar{x}_c \ll x \ll 1-\bar{x}_c \end{cases}$$

  - Late time

$$D_0(x,\tau) = \frac{\tau}{\sqrt{x}(1-x)^{3/2}}\mathrm{e}^{-\pi\frac{\tau^2}{1-x}}$$

  - Intermediate time

$$D_1(x,\tau) = \int_0^\tau \mathrm{d}\sigma \int_x^1 \mathrm{d}\xi\,\delta P(x,\xi,\sigma)D_0(\xi,\sigma) - \int_0^\tau \mathrm{d}\sigma D_0(x,\sigma)\int_0^x \mathrm{d}\xi\,\delta P(\xi,x,\sigma)$$

$$\delta P(x,\xi,\tau) = (P_{\text{hard}}(x,\xi,\tau) - P_{\text{coh}}(x,\xi,\tau))\Theta(x-x_c)\Theta(\xi-x_c-x)$$