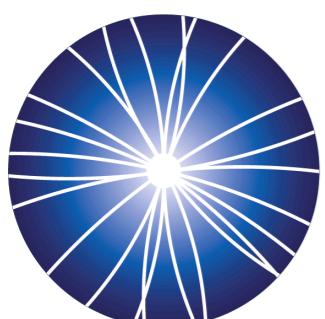


Full evaluation of in-medium splittings beyond the soft approximation

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Universidade de Santiago de Compostela

Jet Quenching in the Quark-Gluon Plasma
Trento
June 13th, 2022

**Ongoing collaboration with C. Andres, L. Apolinario,
M. Gonzalez Martinez, and C. Salgado**



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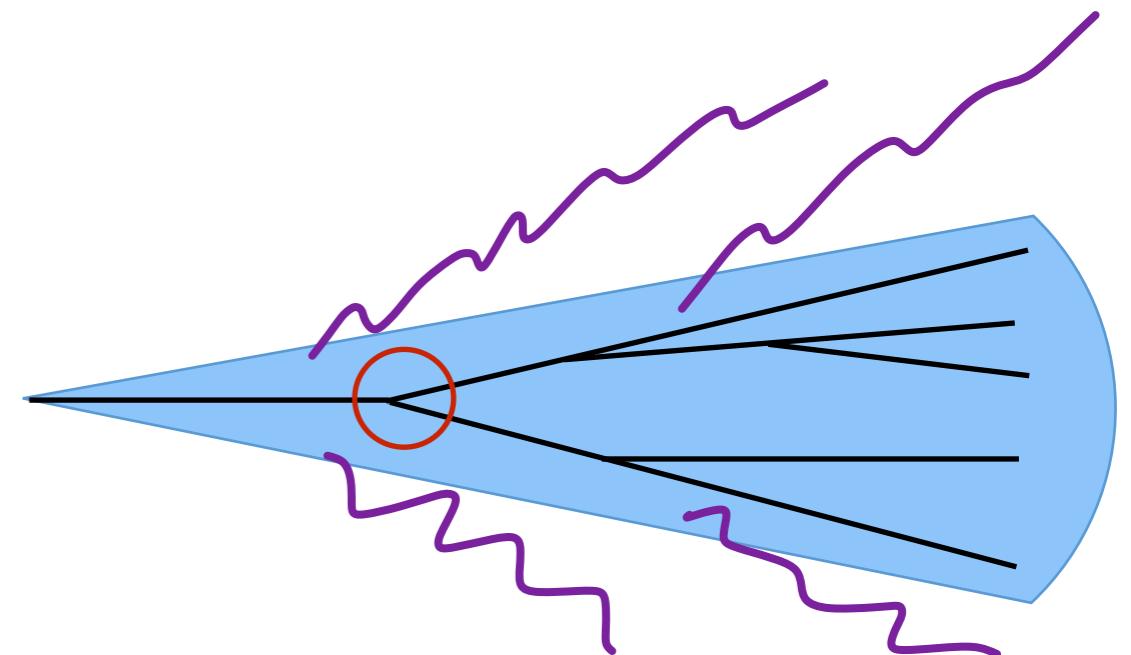
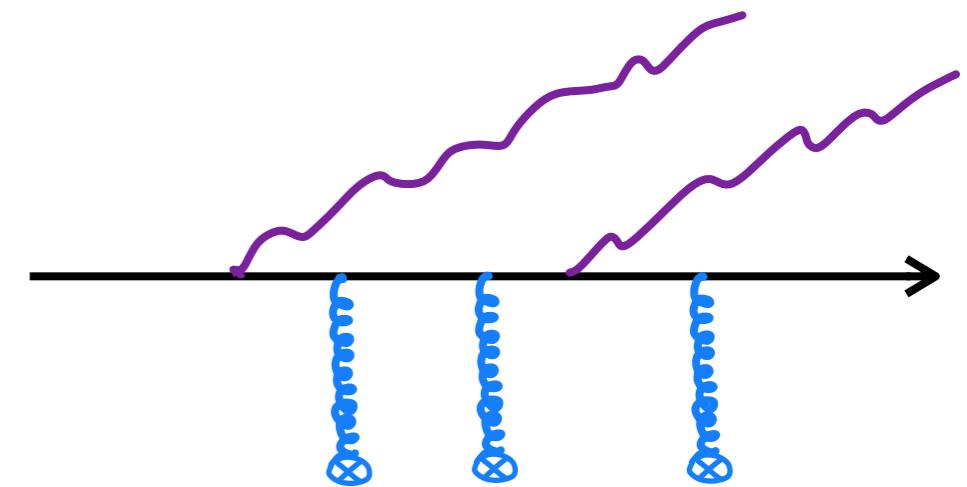
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**XUNTA
DE GALICIA**

From energy loss to jet substructure

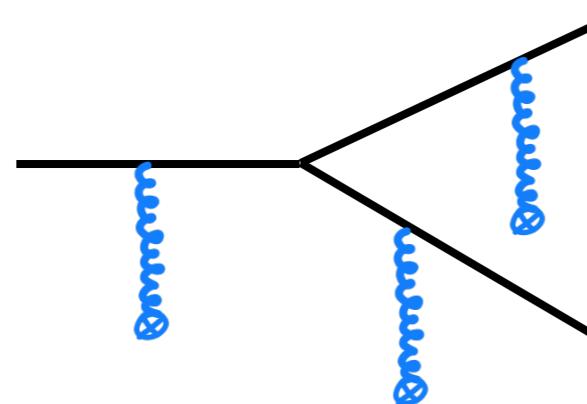
- For energy loss calculation we only need the soft limit $z \ll 1$
 - ◆ Soft divergence of the vacuum vertex
- What about jets?
 - ◆ Emissions from multiple sources
 - ◆ Harder vertices



From energy loss to jet substructure

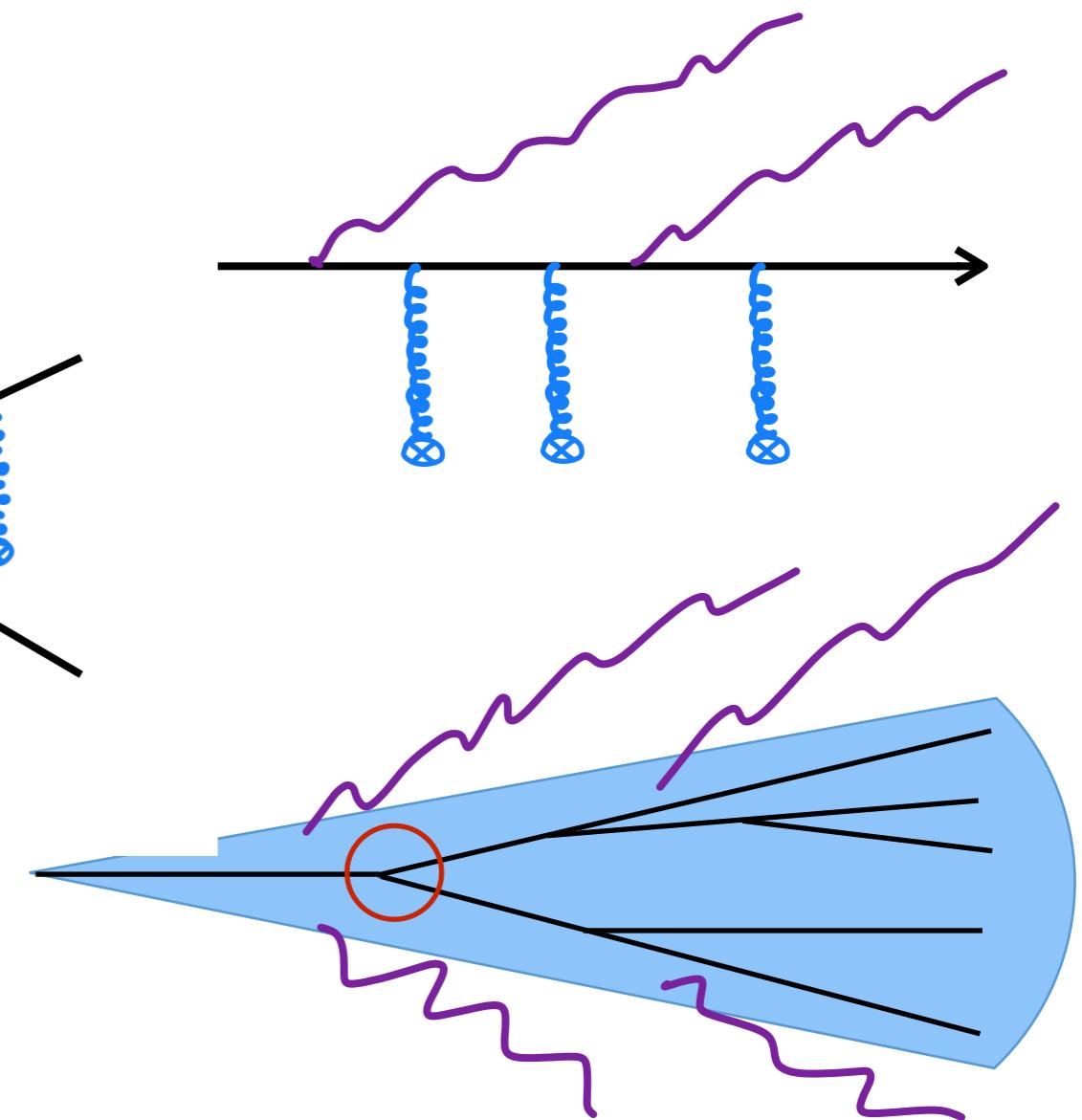
- For energy loss calculation we only need the soft limit $\tau \ll 1$

- ◆ Soft divergence c



- What about jets?

- ◆ Emissions from τ
 - ◆ Harder vertices



Formalism

- All particles have a large longitudinal momentum compared to their transverse momenta and therefore there is a decoupling between transverse and longitudinal dynamics
[Going beyond this limit, see talks on Wednesday by A. Sadofyev and J. Barata](#)
- We work in a mixed representation (\mathbf{p}, t) with momentum coordinates in the transverse direction and “time” (+ coordinate) in the longitudinal direction.
- Multiple scatterings resummed through propagators in a background field

$$= \mathcal{G}_R(\mathbf{p}_2, t_2; \mathbf{p}_1, t_1; \omega)$$

- Vacuum vertices

$$= V(\mathbf{k} - z \mathbf{p}, z) T^{\alpha\beta\gamma} (2\pi)^2 \delta^{(2)}(\mathbf{p} - \mathbf{k} - \mathbf{q})$$

- Background field averaged at the level of the cross section

$$\langle A^{a-}(\mathbf{q}_1, t_1) A^{b-\dagger}(\mathbf{q}_2, t_2) \rangle = \delta^{ab} \delta(t_2 - t_1) \delta^{(2)}(\mathbf{q}_1 - \mathbf{q}_2) v(\mathbf{q}_1)$$

In-medium propagator

- Can be formally written in coordinate space in terms of a path integral

$$\mathcal{G}_R(t_2, \mathbf{x}_2; t_1, \mathbf{x}_1; \omega) = \int_{\mathbf{x}_1}^{\mathbf{x}_2} \mathcal{D}\mathbf{r} \exp \left\{ \frac{i\omega}{2} \int_{t_1}^{t_2} d\xi \dot{\mathbf{r}}^2(\xi) \right\} \text{P exp} \left\{ ig \int_{t_1}^{t_2} d\xi A_R^-(\xi, \mathbf{r}(\xi)) \right\}$$

- Satisfies the following Schwinger-Dyson type equation

$$\begin{aligned} \mathcal{G}_R(\mathbf{p}_2, t_2; \mathbf{p}_1, t_1; \omega) &= (2\pi)^2 \delta^{(2)}(\mathbf{p}_2 - \mathbf{p}_1) e^{-i\frac{p_2^2}{2\omega}(t_2 - t_1)} \\ &\quad + ig \int_{t_1}^{t_2} ds e^{-i\frac{p_2^2}{2\omega}(t_2 - s)} \int_{\mathbf{p}'} A_R^-(s, \mathbf{p}_2 - \mathbf{p}') \mathcal{G}_R(\mathbf{p}', s; \mathbf{p}_1, t_1; \omega) \end{aligned}$$

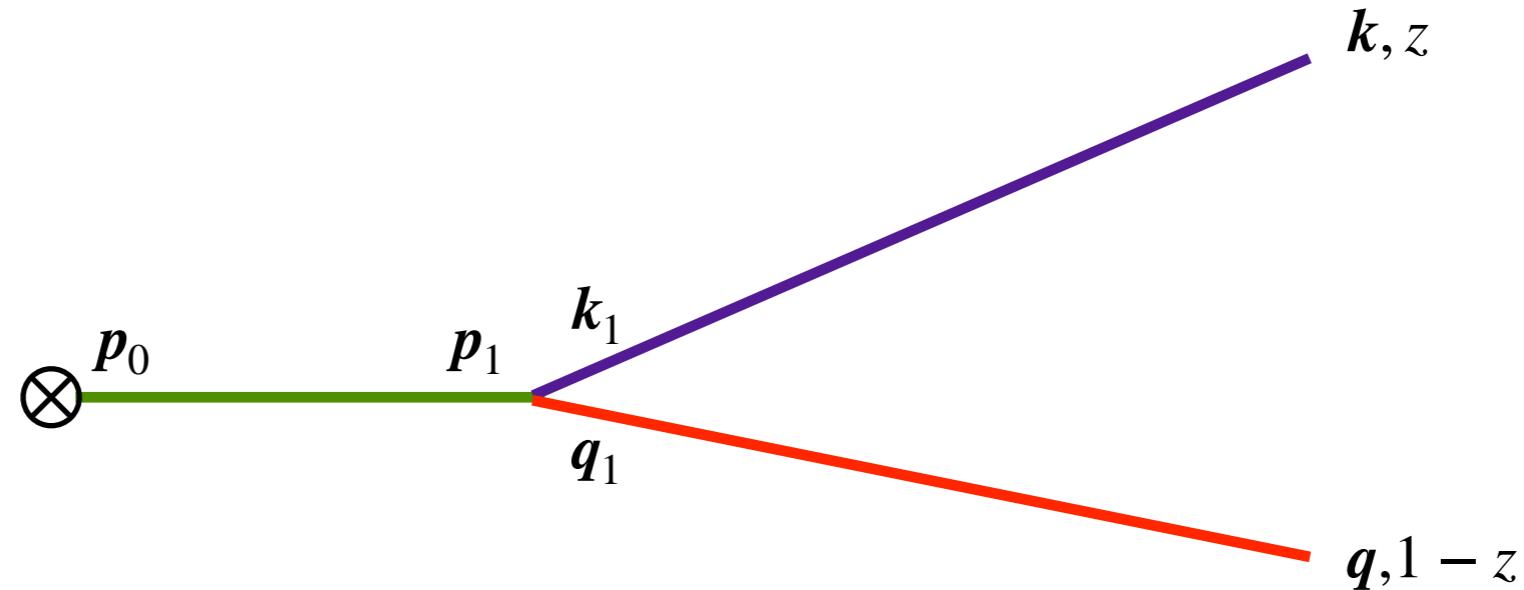


- And convolution relations

$$\int_{\mathbf{p}_2} \mathcal{G}_R(\mathbf{p}_3, t_3; \mathbf{p}_2, t_2; \omega) \mathcal{G}_R(\mathbf{p}_2, t_2; \mathbf{p}_1, t_1; \omega) = \mathcal{G}_R(\mathbf{p}_3, t_3; \mathbf{p}_1, t_1; \omega)$$

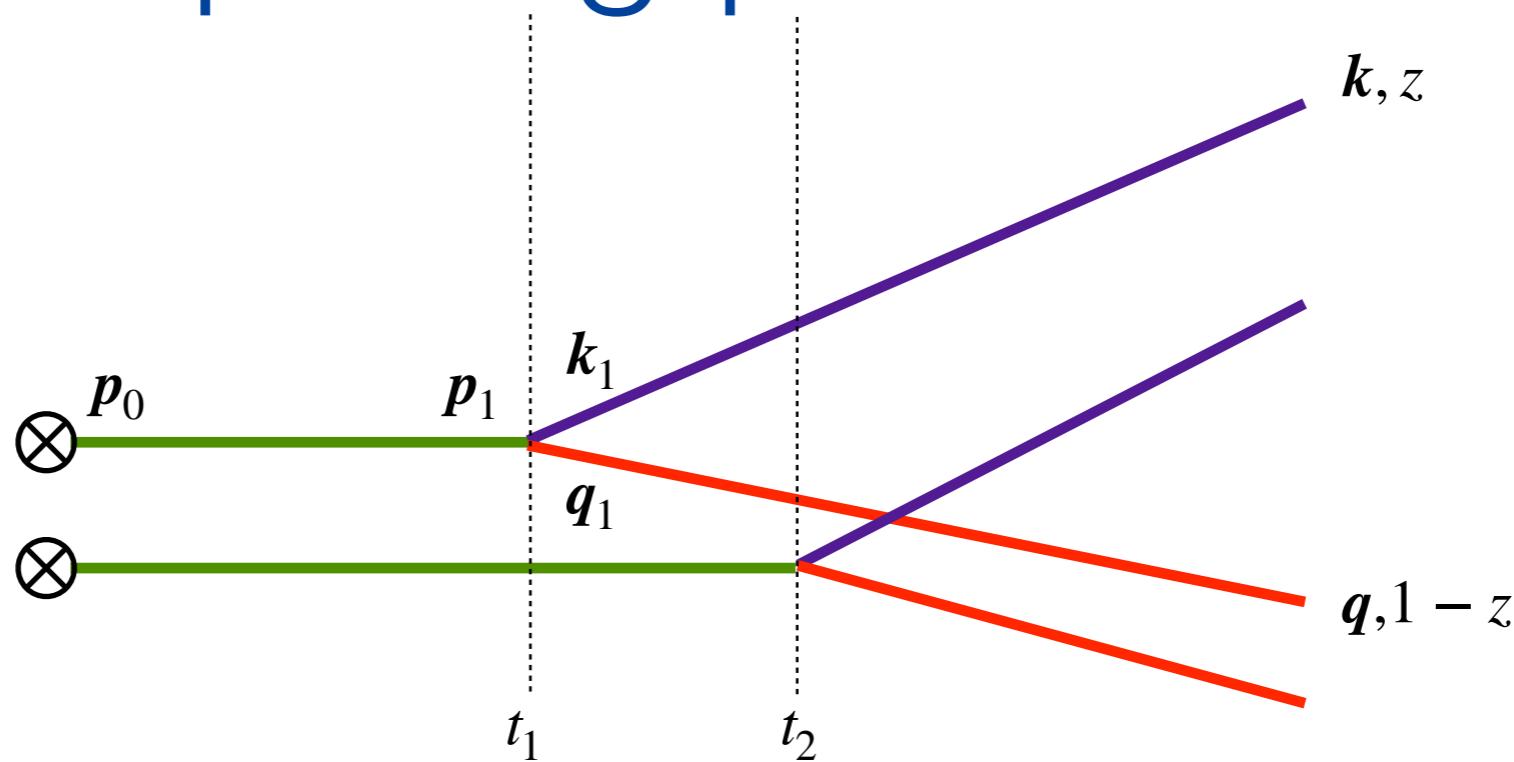
$$\int_{\mathbf{p}_2} \mathcal{G}_R^\dagger(\bar{\mathbf{p}}_1, t_1; \mathbf{p}_2, t_2; \omega) \mathcal{G}_R(\mathbf{p}_2, t_2; \mathbf{p}_1, t_1; \omega) = (2\pi)^2 \delta^{(2)}(\mathbf{p}_1 - \bar{\mathbf{p}}_1)$$

Splitting process



$$\begin{aligned} \mathcal{M}^{\alpha\beta} = & \frac{1}{2E} \int_{\mathbf{p}_0} \int_{\mathbf{p}_1} \int_{\mathbf{k}_1} \int_{\mathbf{q}_1} \int_{t_0}^{\infty} dt_1 (2\pi)^2 \delta^{(2)}(\mathbf{p}_1 - \mathbf{k}_1 - \mathbf{q}_1) \mathcal{G}_{R_b}^{\alpha\alpha_1}(\mathbf{k}, L; \mathbf{k}_1, t_1; zE) \\ & \times \mathcal{G}_{R_c}^{\beta\beta_1}(\mathbf{q}, L; \mathbf{q}_1, t_1; (1-z)E) V(\mathbf{k}_1 - z\mathbf{p}_1, z) T^{\alpha_1\beta_1\gamma_1} \mathcal{G}_{R_a}^{\gamma_1\gamma}(\mathbf{p}_1, t_1; \mathbf{p}_0, t_0; E) \mathcal{M}_0^{\gamma}(E, \mathbf{p}_0) \end{aligned}$$

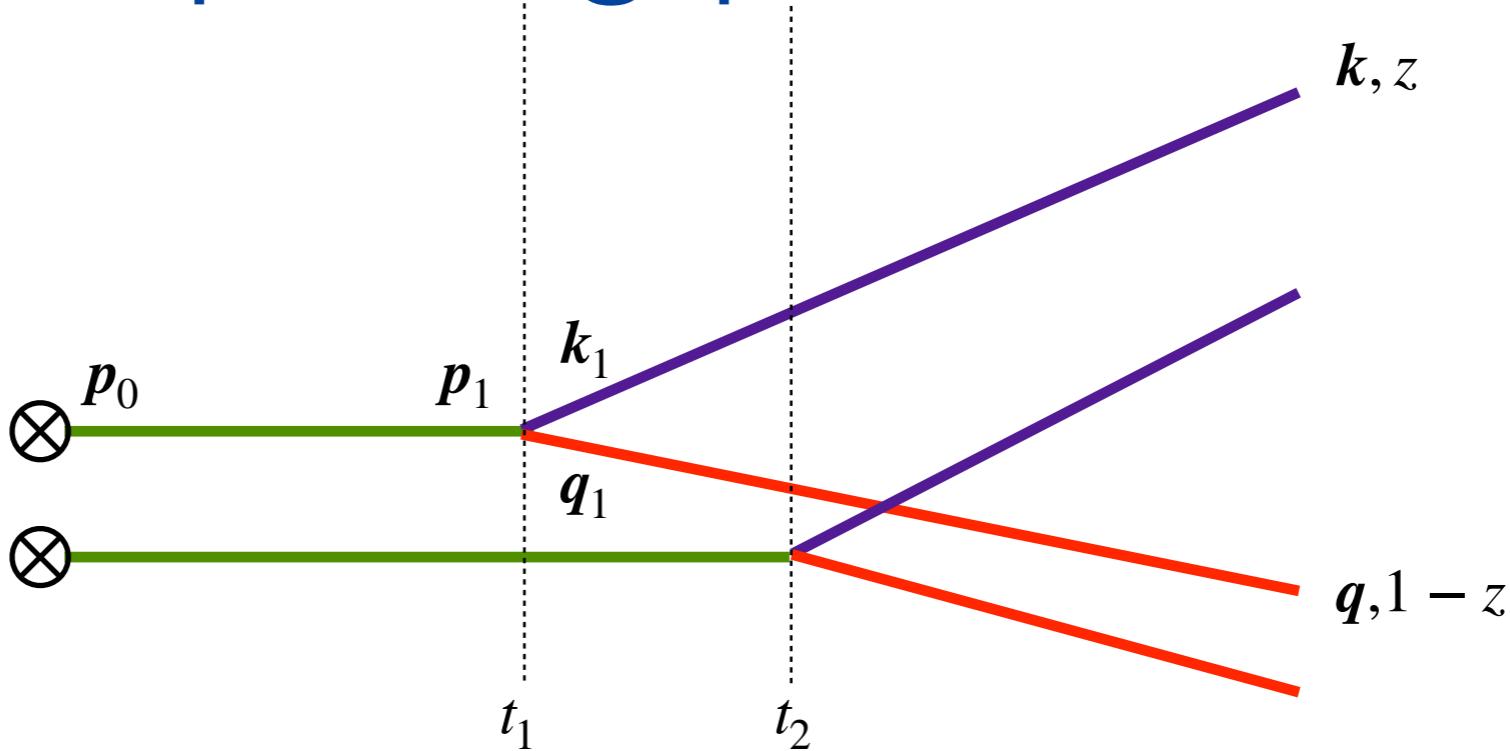
Splitting process



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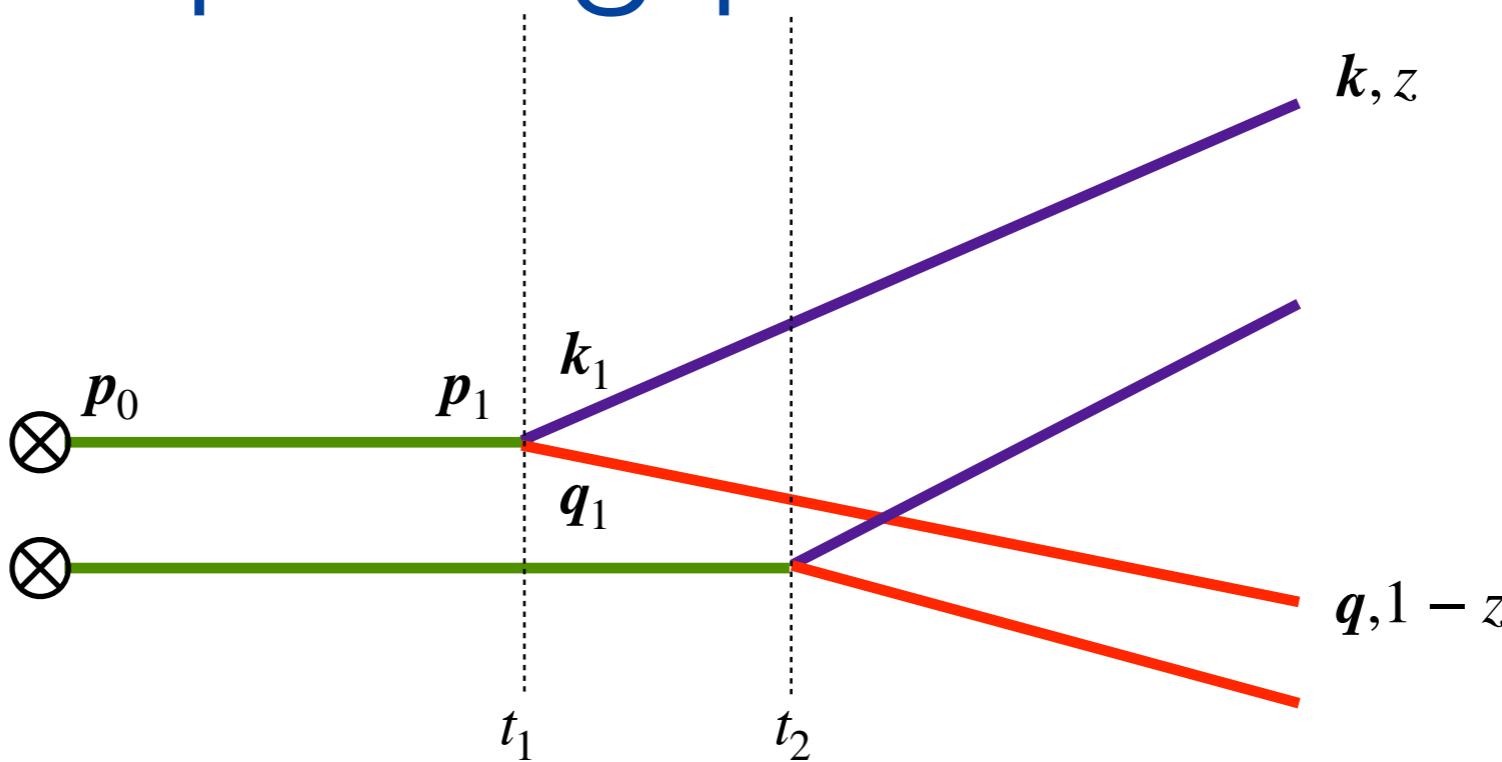
$$\begin{aligned} \langle |\mathcal{M}|^2 \rangle \propto & \left\langle \mathcal{G}_{R_b}^{\alpha\alpha_1}(\mathbf{k}, L; \mathbf{k}_1, t_1; zE) \mathcal{G}_{R_c}^{\beta\beta_1}(\mathbf{q}, L; \mathbf{q}_1, t_1; (1-z)E) \mathcal{G}_{R_b}^{\dagger\bar{\alpha}_2\alpha}(\bar{\mathbf{k}}_2, t_2; \mathbf{k}, L; zE) \right. \\ & \left. \times \mathcal{G}_{R_c}^{\dagger\bar{\beta}_2\beta}(\bar{\mathbf{q}}_2, t_2; \mathbf{q}, L; (1-z)E) \mathcal{G}_{R_a}^{\gamma_1\gamma}(\mathbf{p}_1, t_1; \mathbf{p}_0, t_0; E) \mathcal{G}_{R_a}^{\dagger\bar{\gamma}\bar{\gamma}_2}(\bar{\mathbf{p}}_0, t_0; \bar{\mathbf{p}}_2, t_2; E) \right\rangle \end{aligned}$$

Splitting process



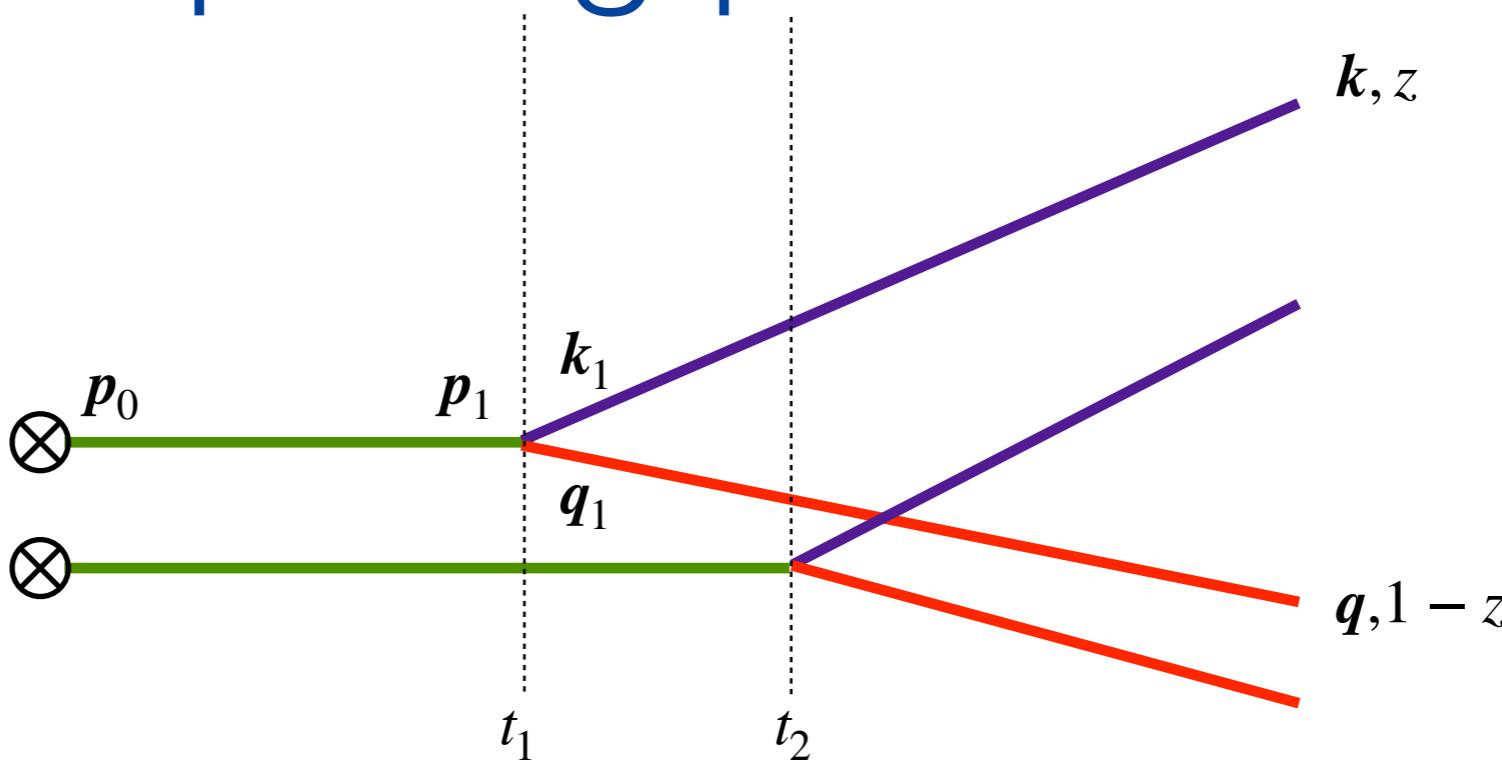
$$\begin{aligned}
 \mathcal{M}^{\alpha\beta} = & \frac{1}{2E} \int_{\mathbf{p}_0 \mathbf{p}_1 \mathbf{k}_1 \mathbf{q}_1} \int_{t_0}^{\infty} dt_1 (2\pi)^2 \delta^{(2)}(\mathbf{p}_1 - \mathbf{k}_1 - \mathbf{q}_1) \mathcal{G}_{R_b}^{\alpha\alpha_1}(\mathbf{k}, L; \mathbf{k}_1, t_1; zE) \\
 & \boxed{\int_{\mathbf{k}_2} \mathcal{G}_{R_b}(\mathbf{k}, L; \mathbf{k}_2, t_2; zE) \mathcal{G}_{R_b}(\mathbf{k}_2, t_2; \mathbf{k}_1, t_1; zE)} T^{\alpha_1 \beta_1 \gamma_1} \mathcal{G}_{R_a}^{\gamma_1 \gamma}(\mathbf{p}_1, t_1; \mathbf{p}_0, t_0; E) \mathcal{M}_0^\gamma(E, \mathbf{p}_0) \\
 \langle |\mathcal{M}|^2 \rangle \propto & \left\langle \mathcal{G}_{R_b}^{\alpha\alpha_1}(\mathbf{k}, L; \mathbf{k}_1, t_1; zE) \mathcal{G}_{R_c}^{\beta\beta_1}(\mathbf{q}, L; \mathbf{q}_1, t_1; (1-z)E) \mathcal{G}_{R_b}^{\dagger \bar{\alpha}_2 \alpha}(\bar{\mathbf{k}}_2, t_2; \mathbf{k}, L; zE) \right. \\
 & \left. \times \mathcal{G}_{R_c}^{\dagger \bar{\beta}_2 \beta}(\bar{\mathbf{q}}_2, t_2; \mathbf{q}, L; (1-z)E) \mathcal{G}_{R_a}^{\gamma_1 \gamma}(\mathbf{p}_1, t_1; \mathbf{p}_0, t_0; E) \mathcal{G}_{R_a}^{\dagger \bar{\gamma}_1 \bar{\gamma}_2}(\bar{\mathbf{p}}_0, t_0; \bar{\mathbf{p}}_2, t_2; E) \right\rangle
 \end{aligned}$$

Splitting process



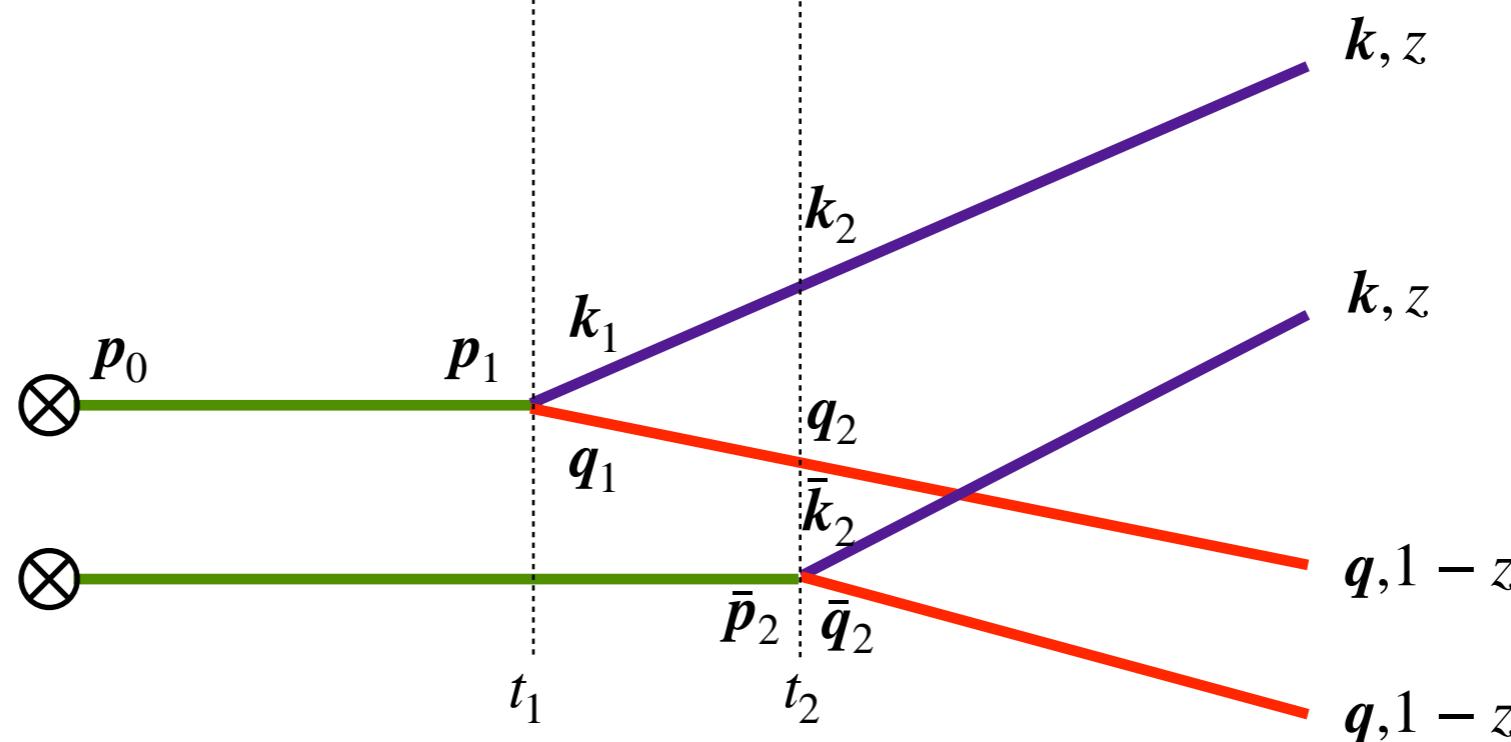
$$\begin{aligned}
 \mathcal{M}^{\alpha\beta} = & \frac{1}{2E} \int_{\mathbf{p}_0 \mathbf{p}_1 \mathbf{k}_1 \mathbf{q}_1} \int_{t_0}^{\infty} dt_1 (2\pi)^2 \delta^{(2)}(\mathbf{p}_1 - \mathbf{k}_1 - \mathbf{q}_1) \mathcal{G}_{R_b}^{\alpha\alpha_1}(\mathbf{k}, L; \mathbf{k}_1, t_1; zE) \\
 & \left[\int_{\mathbf{k}_2} \mathcal{G}_{R_b}(\mathbf{k}, L; \mathbf{k}_2, t_2; zE) \mathcal{G}_{R_b}(\mathbf{k}_2, t_2; \mathbf{k}_1, t_1; zE) \right] T^{\alpha_1 \beta_1 \gamma_1} \mathcal{G}_{R_a}^{\gamma_1 \gamma}(\mathbf{p}_1, t_1; \mathbf{p}_0, t_0; E) \mathcal{M}_0^\gamma(E, \mathbf{p}_0) \\
 \langle |\mathcal{M}|^2 \rangle \propto & \left\langle \mathcal{G}_{R_b}^{\alpha\alpha_1}(\mathbf{k}, L; \mathbf{k}_1, t_1; zE) \mathcal{G}_{R_c}^{\beta\beta_1}(\mathbf{q}, L; \mathbf{q}_1, t_1; (1-z)E) \mathcal{G}_{R_b}^{\dagger \bar{\alpha}_2 \alpha}(\bar{\mathbf{k}}_2, t_2; \mathbf{k}, L; zE) \right. \\
 & \times \left. \mathcal{G}_{R_c}^{\dagger \bar{\beta}_2 \beta}(\bar{\mathbf{q}}_2, t_2; \mathbf{q}, L; (1-z)E) \mathcal{G}_{R_a}^{\gamma_1 \gamma}(\mathbf{p}_1, t_1; \mathbf{p}_0, t_0; E) \mathcal{G}_{R_a}^{\dagger \bar{\gamma}_2 \bar{\gamma}_2}(\bar{\mathbf{p}}_0, t_0; \bar{\mathbf{p}}_2, t_2; E) \right\rangle
 \end{aligned}$$

Splitting process



$$\begin{aligned}
 \mathcal{M}^{\alpha\beta} = & \frac{1}{2E} \int_{\mathbf{p}_0 \mathbf{p}_1 \mathbf{k}_1 \mathbf{q}_1} \int_{t_0}^{\infty} dt_1 (2\pi)^2 \delta^{(2)}(\mathbf{p}_1 - \mathbf{k}_1 - \mathbf{q}_1) \mathcal{G}_{R_b}^{\alpha\alpha_1}(\mathbf{k}, L; \mathbf{k}_1, t_1; zE) \\
 & \left. \int_{\mathbf{k}_2} \mathcal{G}_{R_b}(\mathbf{k}, L; \mathbf{k}_2, t_2; zE) \mathcal{G}_{R_b}(\mathbf{k}_2, t_2; \mathbf{k}_1, t_1; zE) \right) T^{\alpha_1 \beta_1 \gamma_1} \mathcal{G}_{R_a}^{\gamma_1 \gamma}(\mathbf{p}_1, t_1; \mathbf{p}_0, t_0; E) \mathcal{M}_0^\gamma(E, \mathbf{p}_0) \\
 \langle |\mathcal{M}|^2 \rangle \propto & \left\langle \mathcal{G}_{R_b}^{\alpha\alpha_1}(\mathbf{k}, L; \mathbf{k}_1, t_1; zE) \mathcal{G}_{R_c}^{\beta\beta_1}(\mathbf{q}, L; \mathbf{q}_1, t_1; (1-z)E) \mathcal{G}_{R_b}^{\dagger\bar{\alpha}_2\alpha}(\bar{\mathbf{k}}_2, t_2; \mathbf{k}, L; zE) \right. \\
 & \times \left. \mathcal{G}_{R_c}^{\dagger\bar{\beta}_2\beta}(\bar{\mathbf{q}}_2, t_2; \mathbf{q}, L; (1-z)E) \mathcal{G}_{R_a}^{\gamma_1\gamma}(\mathbf{p}_1, t_1; \mathbf{p}_0, t_0; E) \mathcal{G}_{R_a}^{\dagger\bar{\gamma}\bar{\gamma}_2}(\bar{\mathbf{p}}_0, t_0; \bar{\mathbf{p}}_2, t_2; E) \right\rangle
 \end{aligned}$$

Double differential cross section

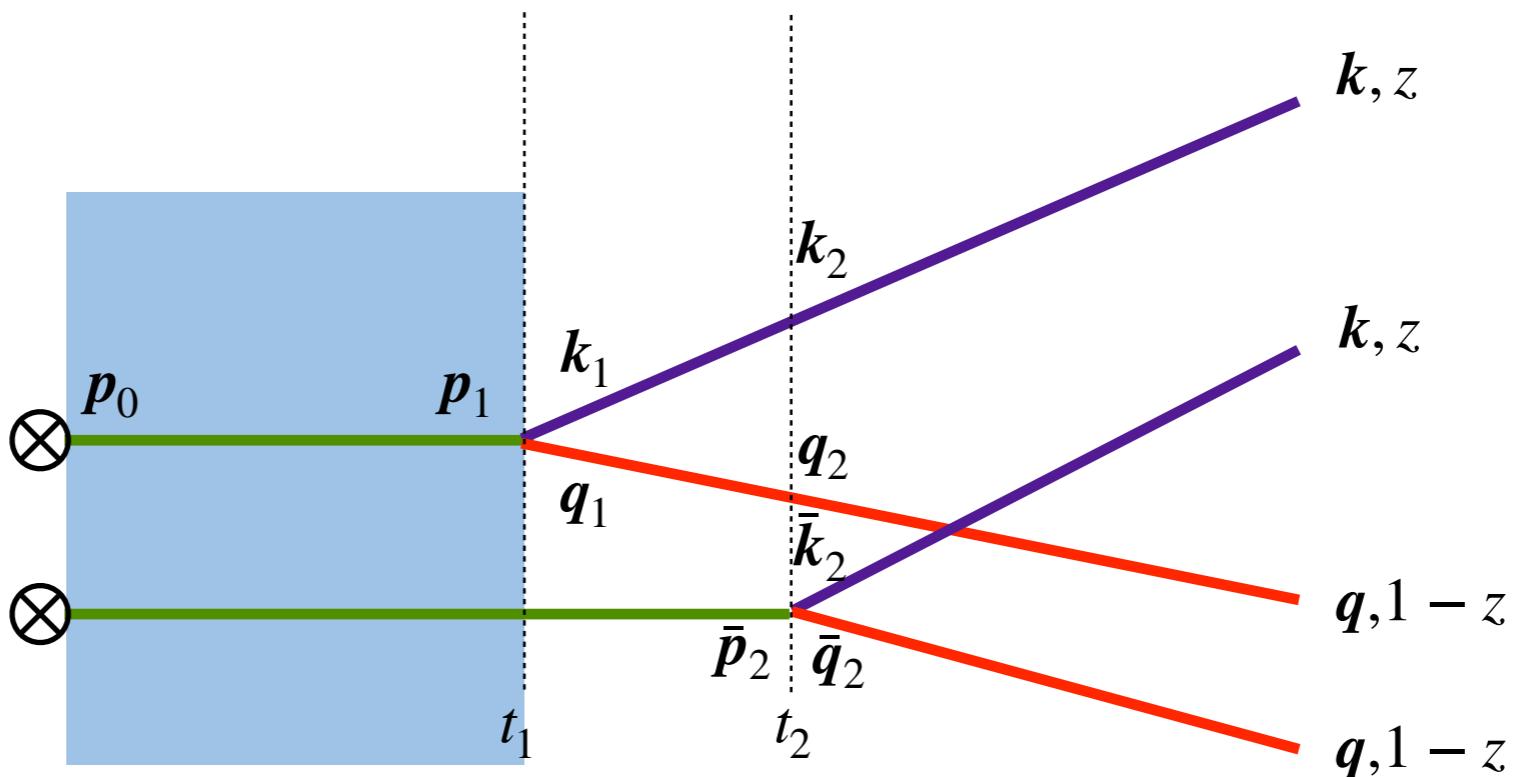


- The locality of the medium averages $\langle A^-(t)A^-(t') \rangle \propto \delta(t - t')$ implies that at any given time:
 - ◆ Averages can be factored into regions with constant number of particles
 - ◆ The sum of all momenta in the amplitude is equal to the sum of all momenta in the conjugate amplitude
 - ◆ When considering the ensemble of all particles in the amplitude and conjugate amplitude, the overall color state is always a singlet

Blaizot, Iancu, FD, Mehtar-Tani [1209.4585](#)

Apolinario, Armesto, Milhano, Salgado [1407.0599](#)

Double differential cross section

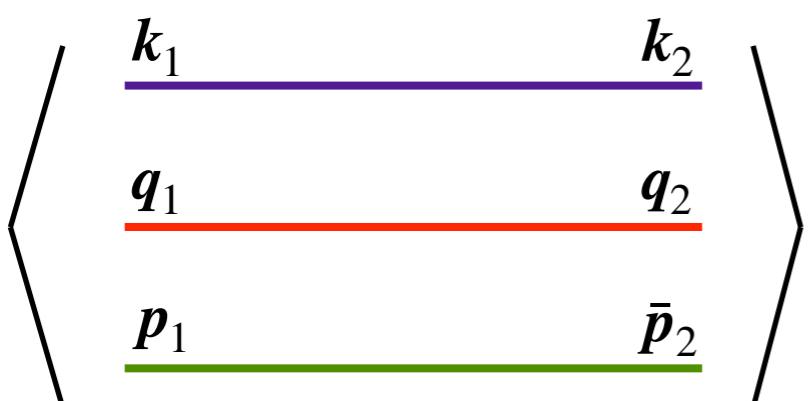
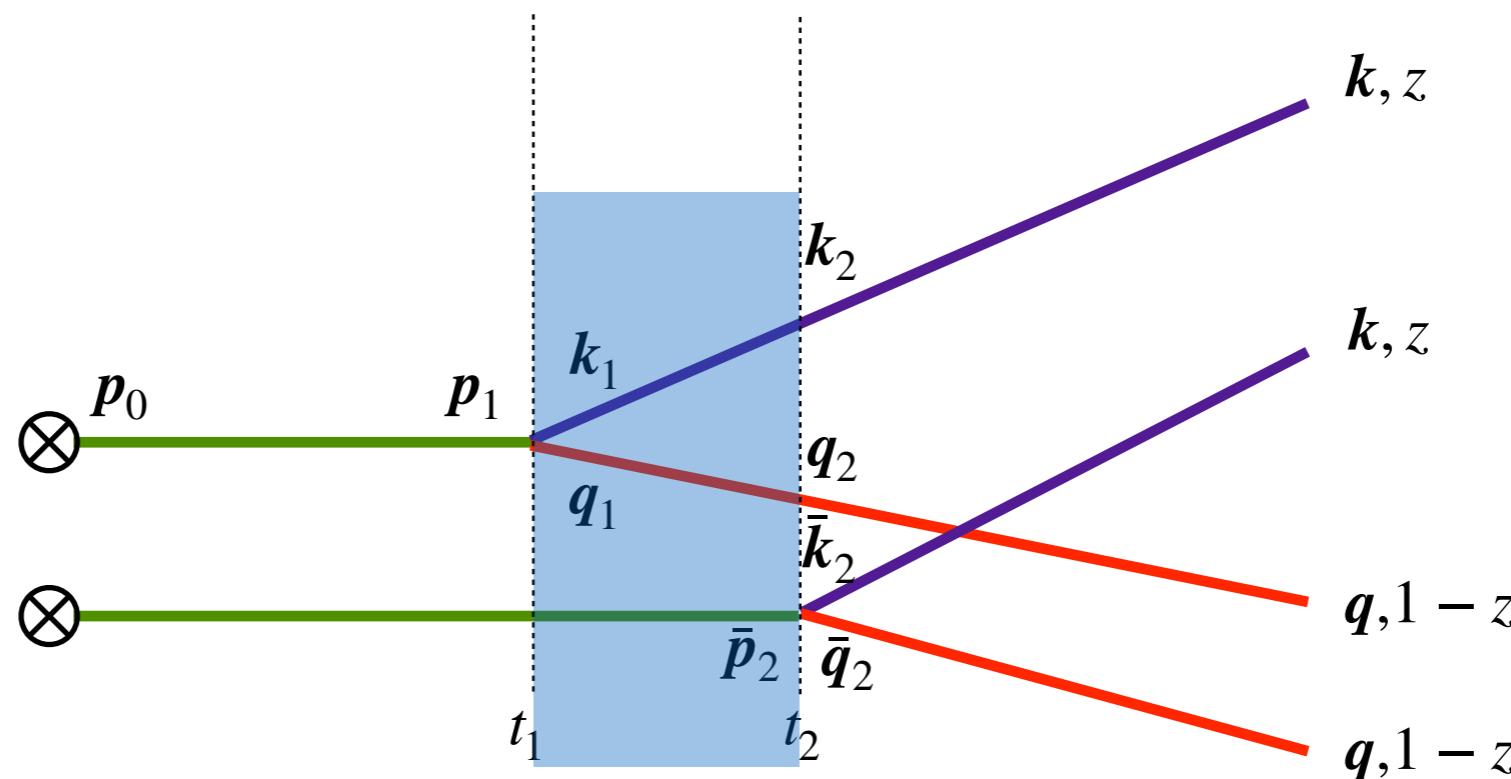


$$\left\langle \frac{\mathbf{p}_0}{\mathbf{p}_0} \frac{\mathbf{p}_1}{\mathbf{p}_1} \right\rangle = \mathcal{P}_{R_a}(\mathbf{p}_1 - \mathbf{p}_0; t_1, t_0)$$

Average depends on total momentum transfer only

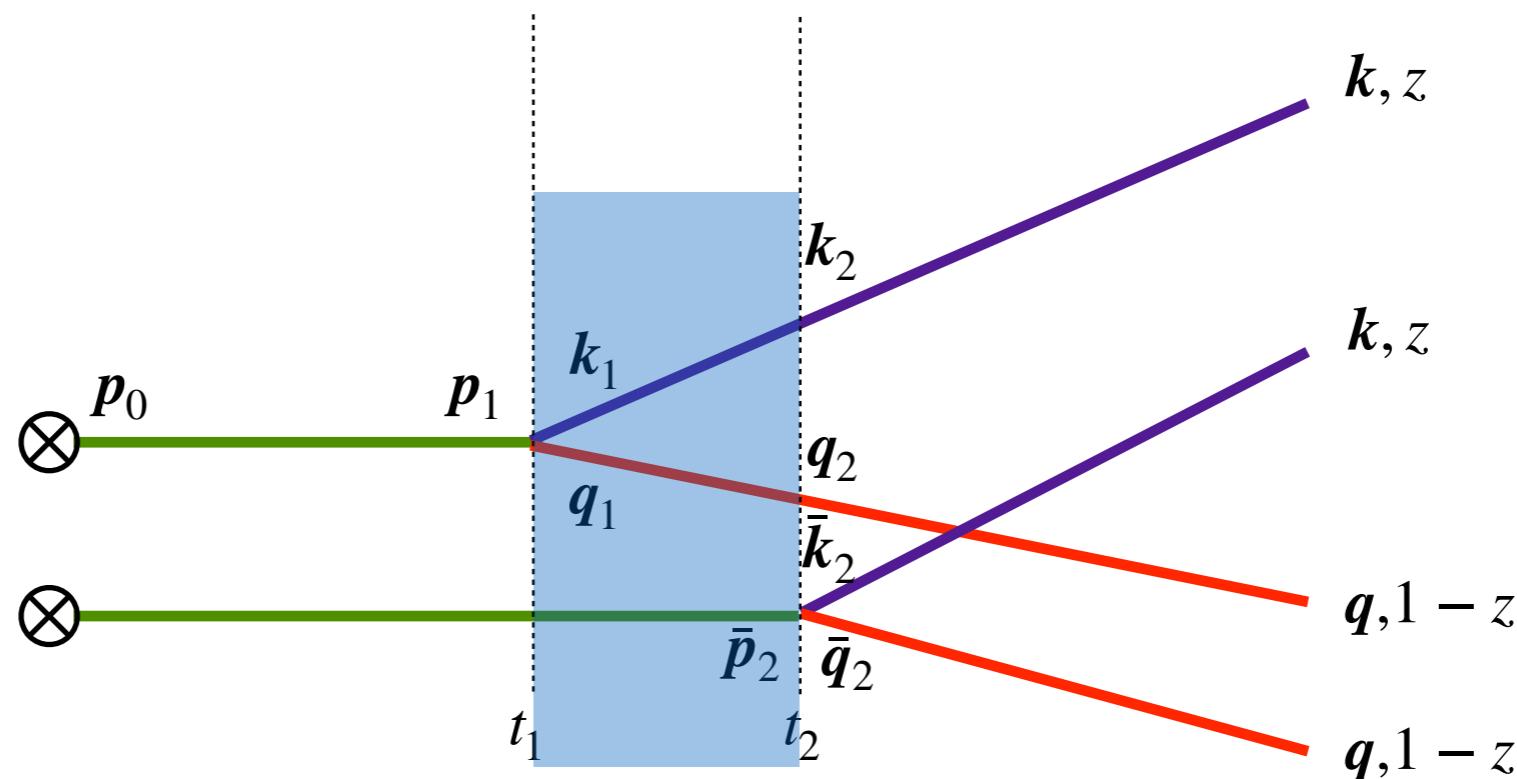
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Double differential cross section

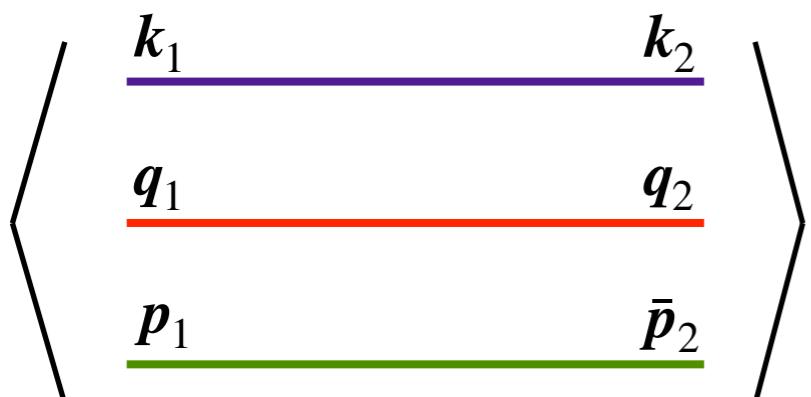


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Double differential cross section

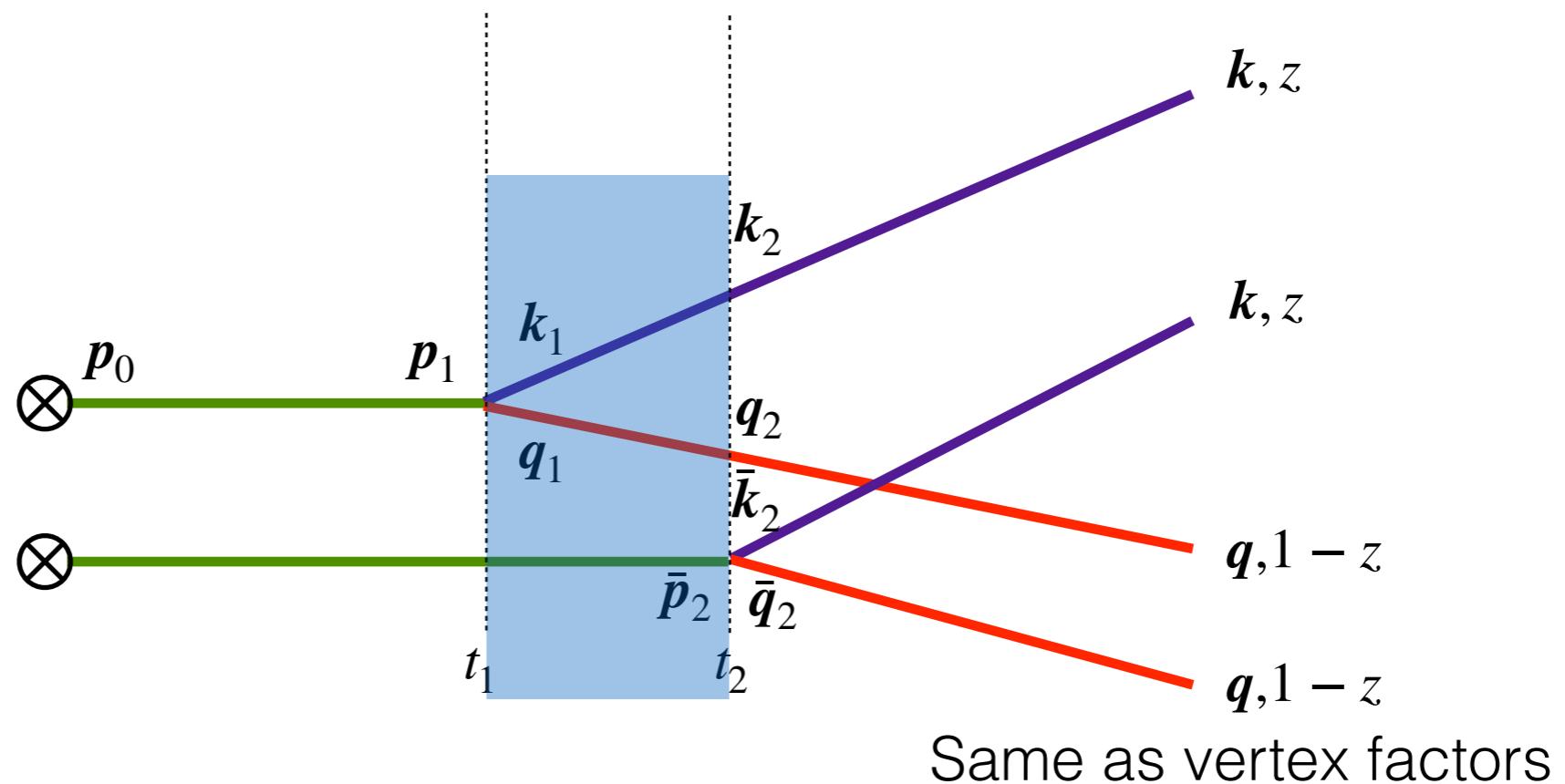


$$k_1 + q_1 = p_1 \quad k_2 + q_2 = \bar{p}_2$$



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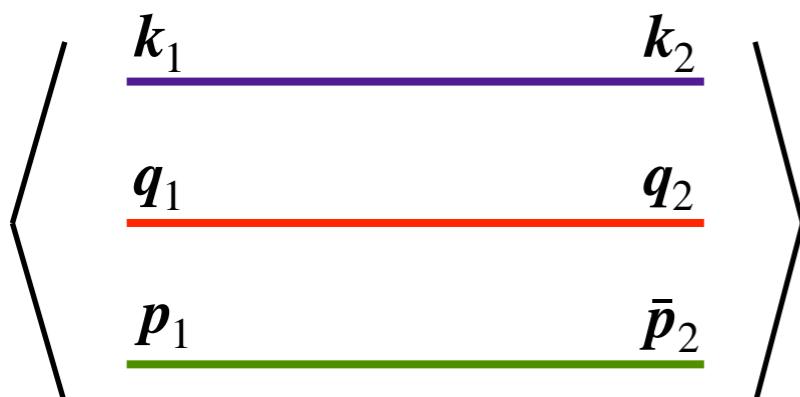
Double differential cross section



$$k_1 + q_1 = p_1$$

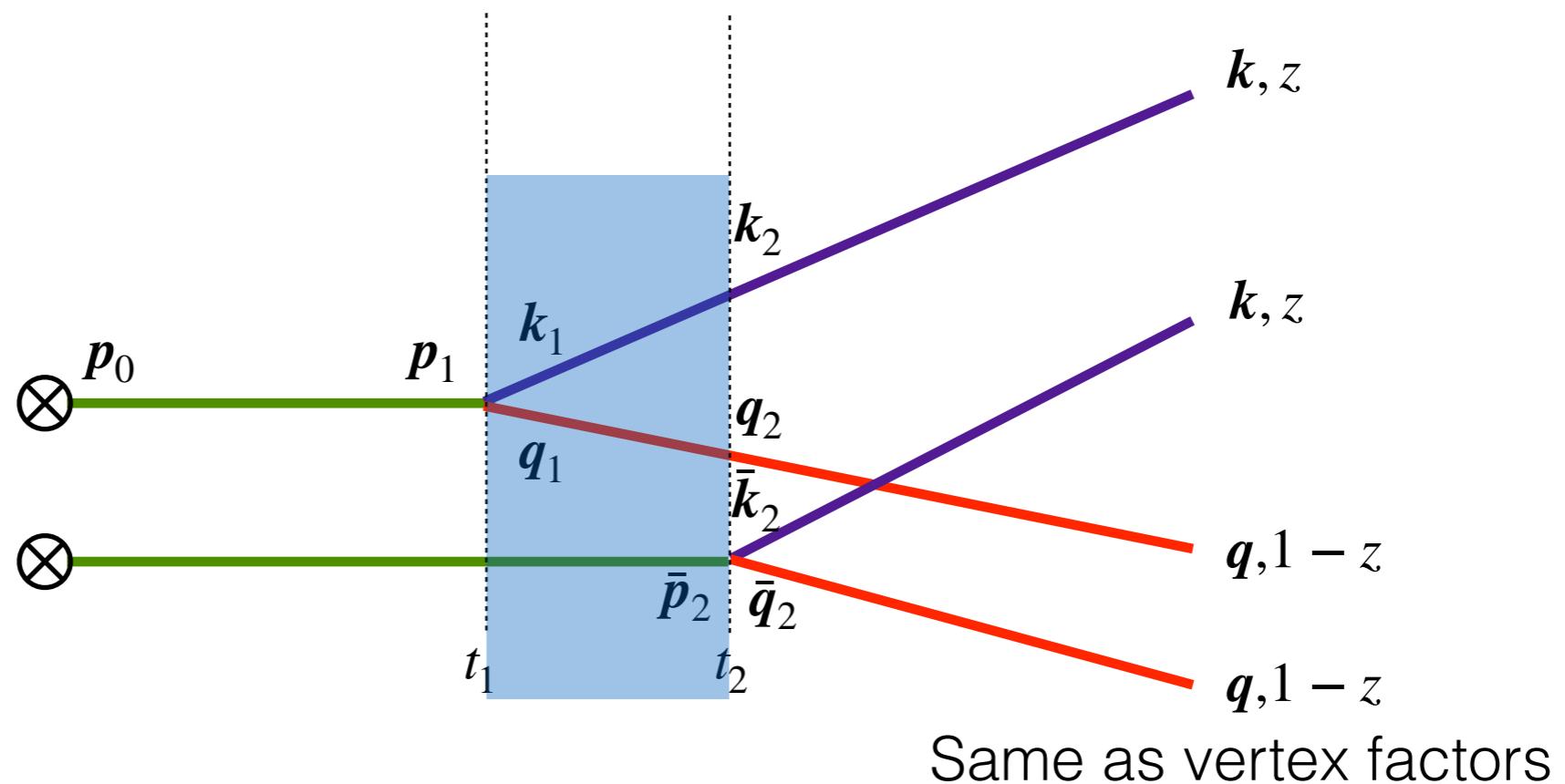
$$k_2 + q_2 = \bar{p}_2$$

$$l_1 = (1 - z)k_1 - zq_1 \quad l_2 = (1 - z)k_2 - zq_2$$



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Double differential cross section



$$k_1 + q_1 = p_1$$

$$k_2 + q_2 = \bar{p}_2$$

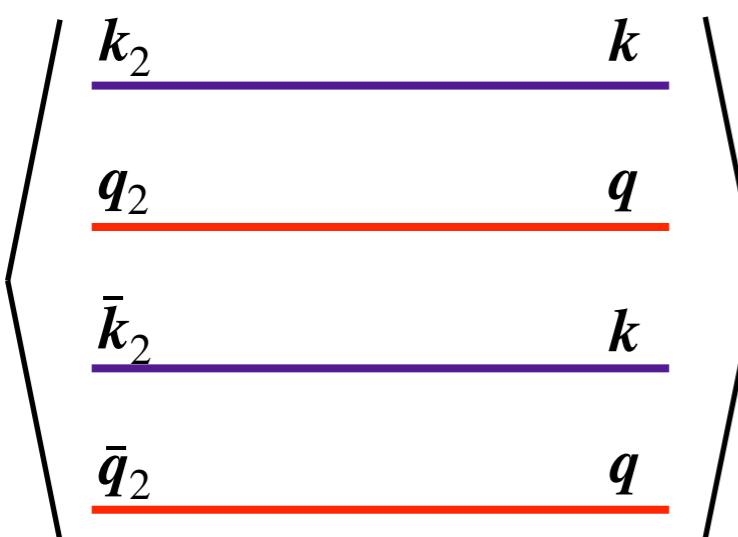
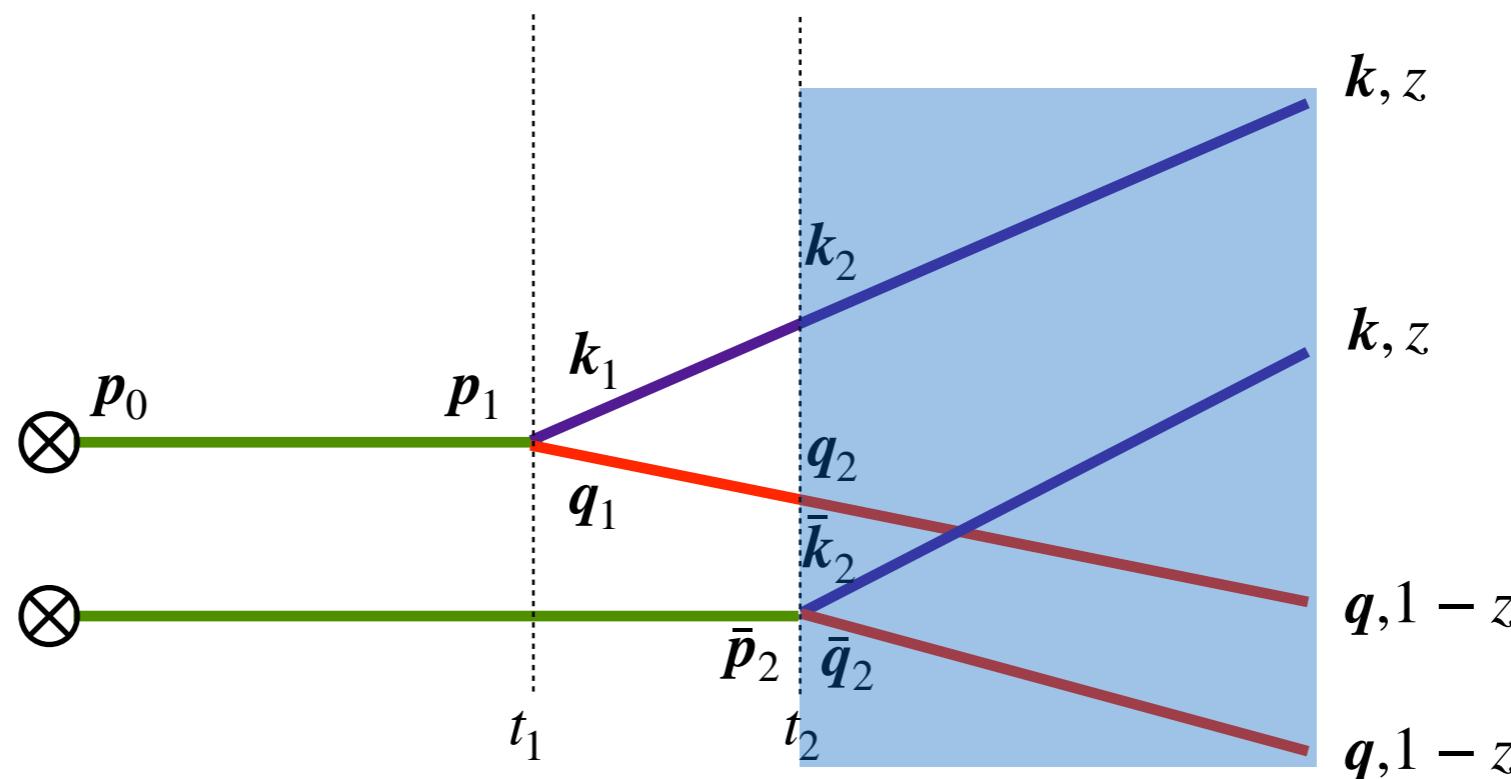
$$l_1 = (1 - z)k_1 - zq_1$$

$$l_2 = (1 - z)k_2 - zq_2$$



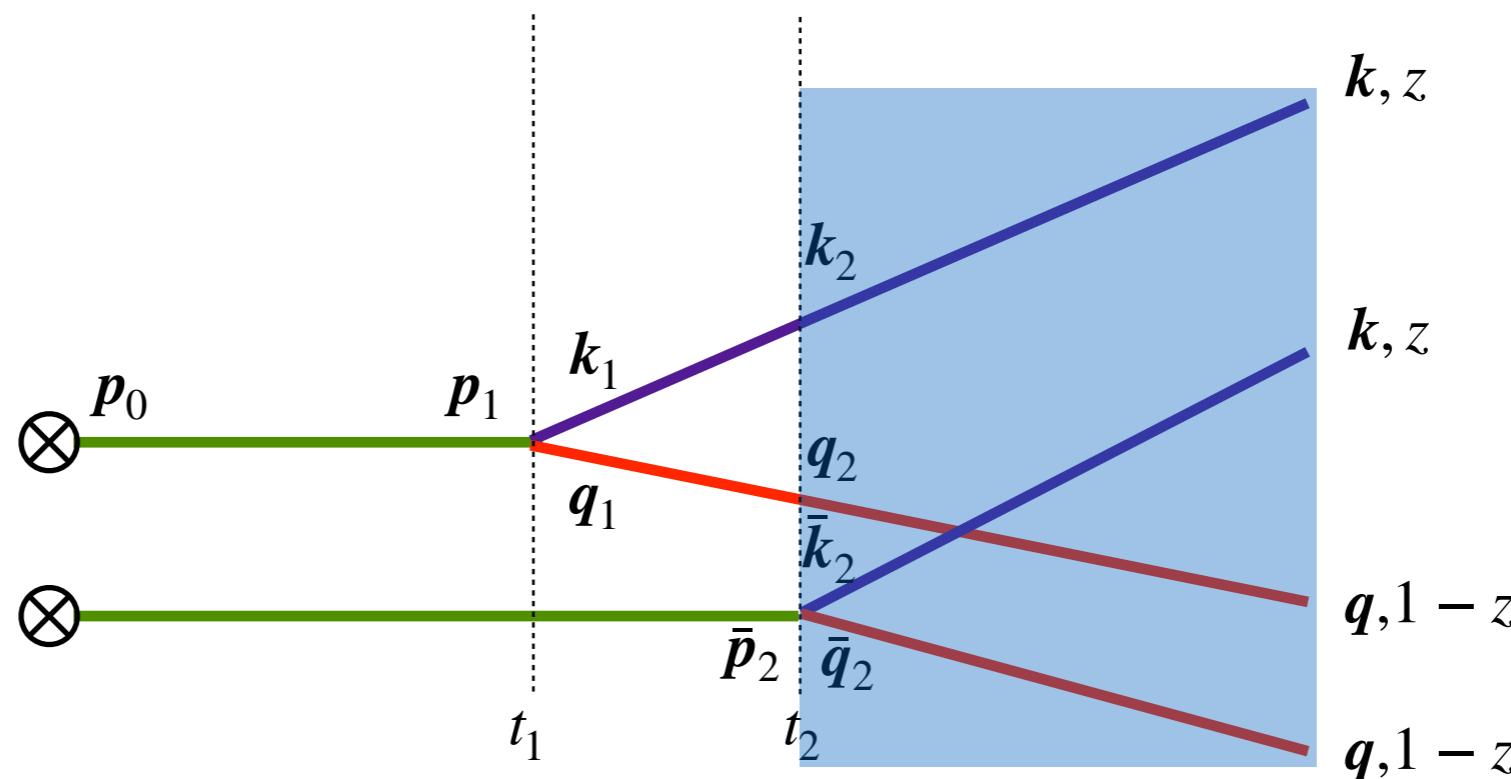
Average depends on $l_1, l_2, \bar{p}_2 - p_1$
 $\mathcal{K}^{(3)}(l_2, t_2; l_1, t_1; \bar{p}_2 - p_1, z)$

Double differential cross section

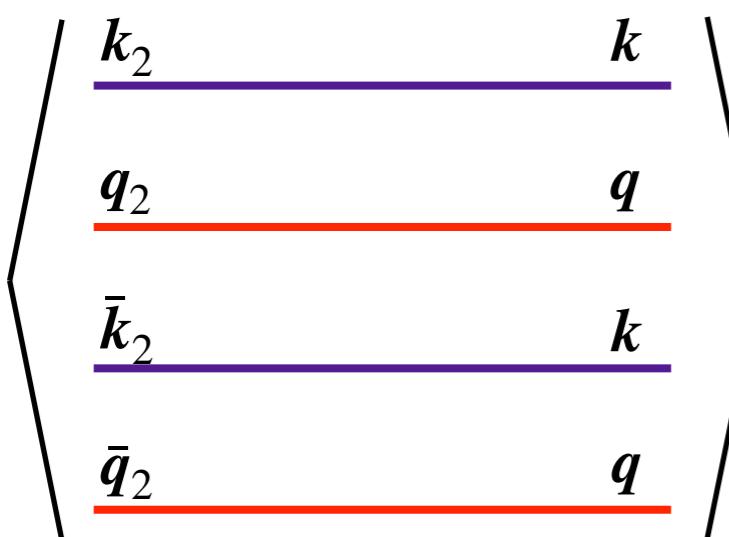


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Double differential cross section

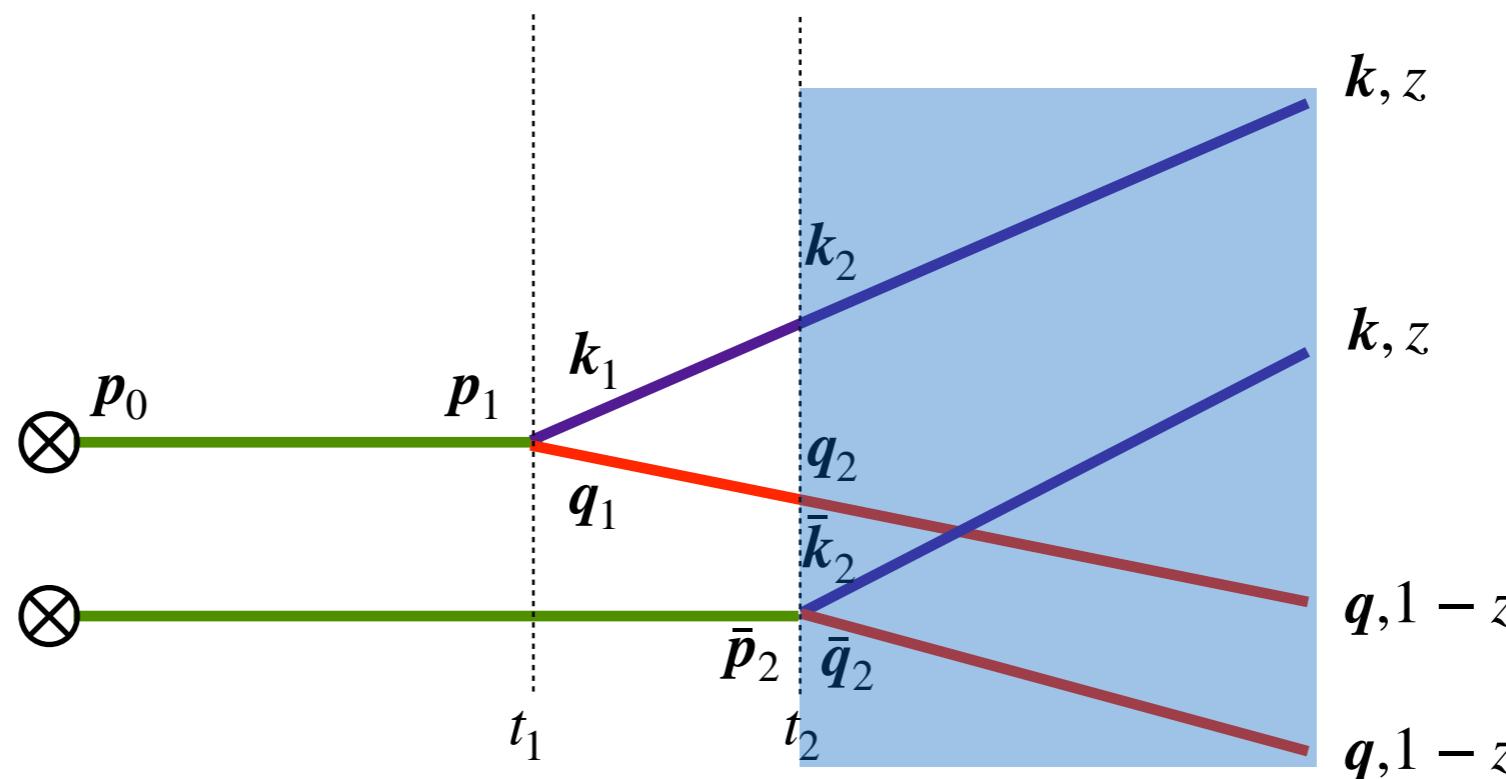


$$k_2 + q_2 = \bar{k}_2 + \bar{q}_2$$

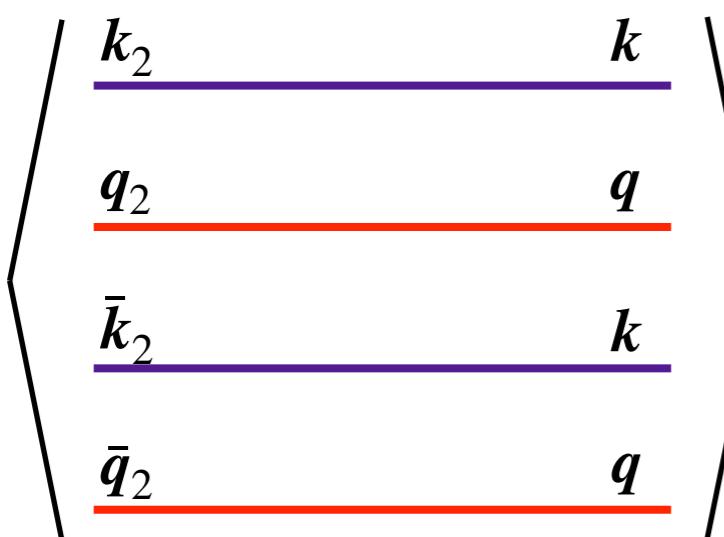


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Double differential cross section



$$k_2 + q_2 = \bar{k}_2 + \bar{q}_2$$



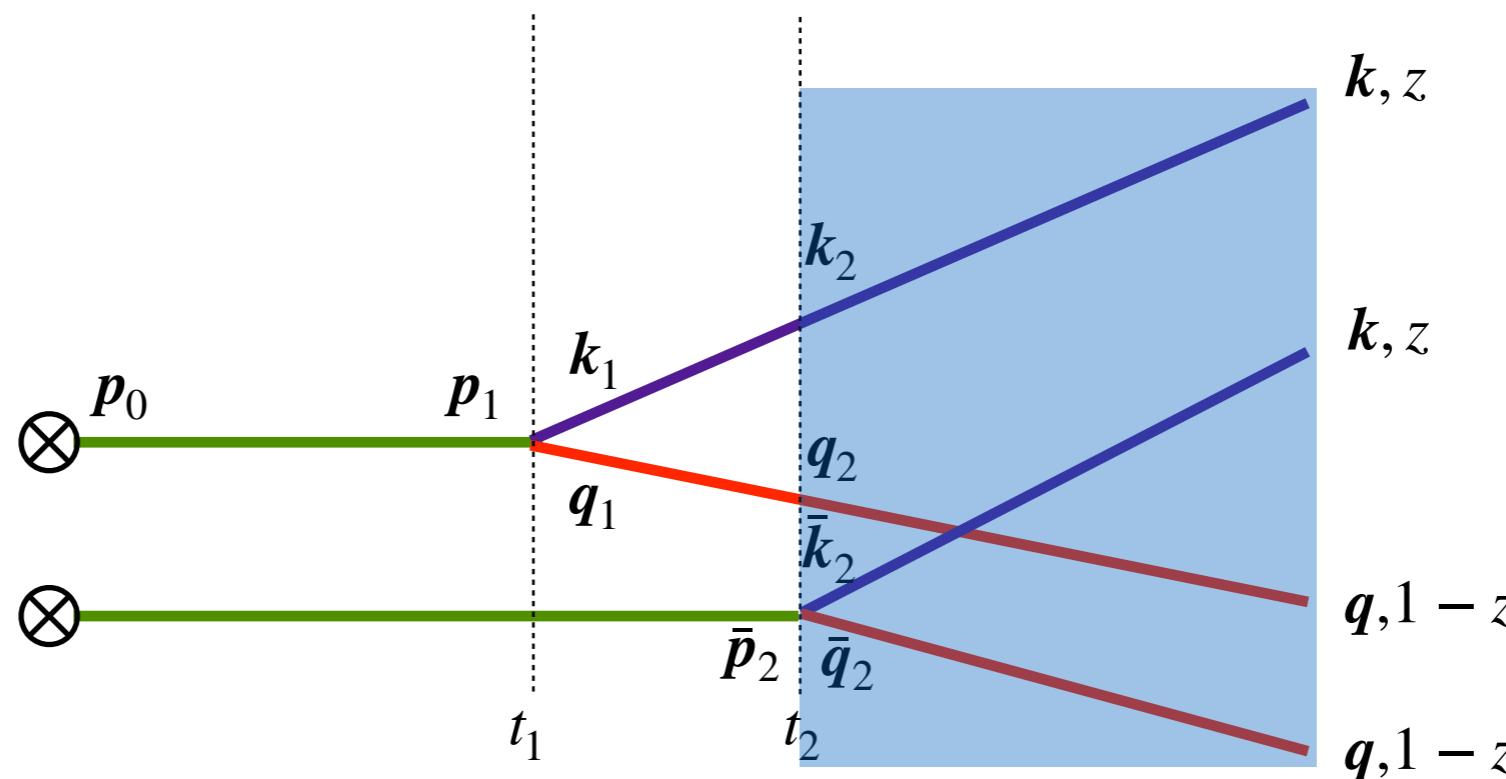
$$\begin{aligned} l_2 &= (1 - z)k_2 - zq_2 \\ \bar{l}_2 &= (1 - z)\bar{k}_2 - z\bar{q}_2 \end{aligned}$$

$$l = (1 - z)k - zq$$

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Apolinario, Armesto, Milhano, Salgado [1407.0599](#)

Double differential cross section



$$k_2 + q_2 = \bar{k}_2 + \bar{q}_2$$

$$\left. \begin{array}{c} k_2 \\ \hline q_2 \\ \hline \bar{k}_2 \\ \hline \bar{q}_2 \end{array} \right\} \quad \left. \begin{array}{c} k \\ \hline q \\ \hline k \\ \hline q \end{array} \right\}$$

$$\begin{aligned} l_2 &= (1 - z)k_2 - zq_2 \\ \bar{l}_2 &= (1 - z)\bar{k}_2 - z\bar{q}_2 \end{aligned}$$

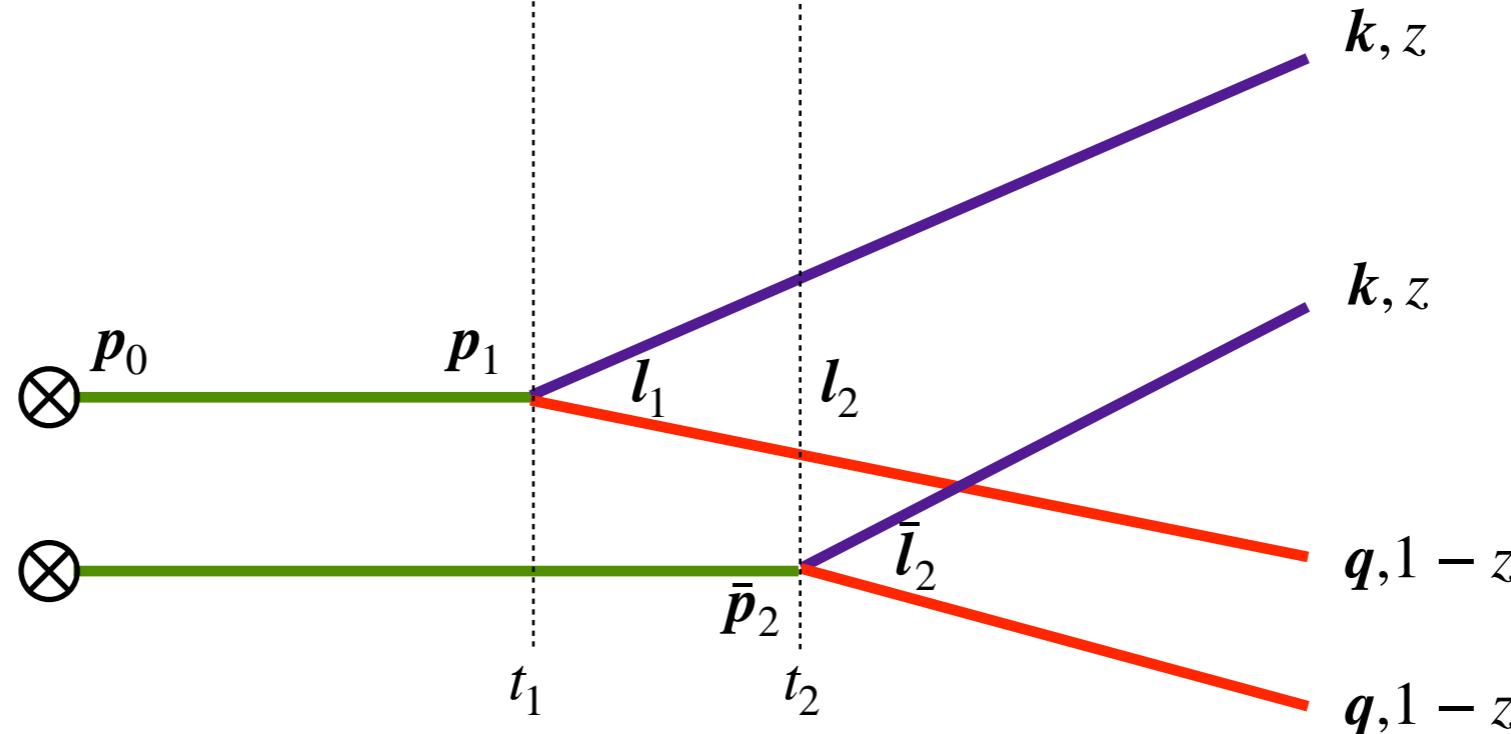
$$l = (1 - z)k - zq$$

Average depends on l, l_2, \bar{l}_2 , and $k + q - k_2 - q_2$

$$\mathcal{S}^{(4)}(l, L; l_2, \bar{l}_2, t_2; k + q - k_2 - q_2, z)$$

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Double differential cross section

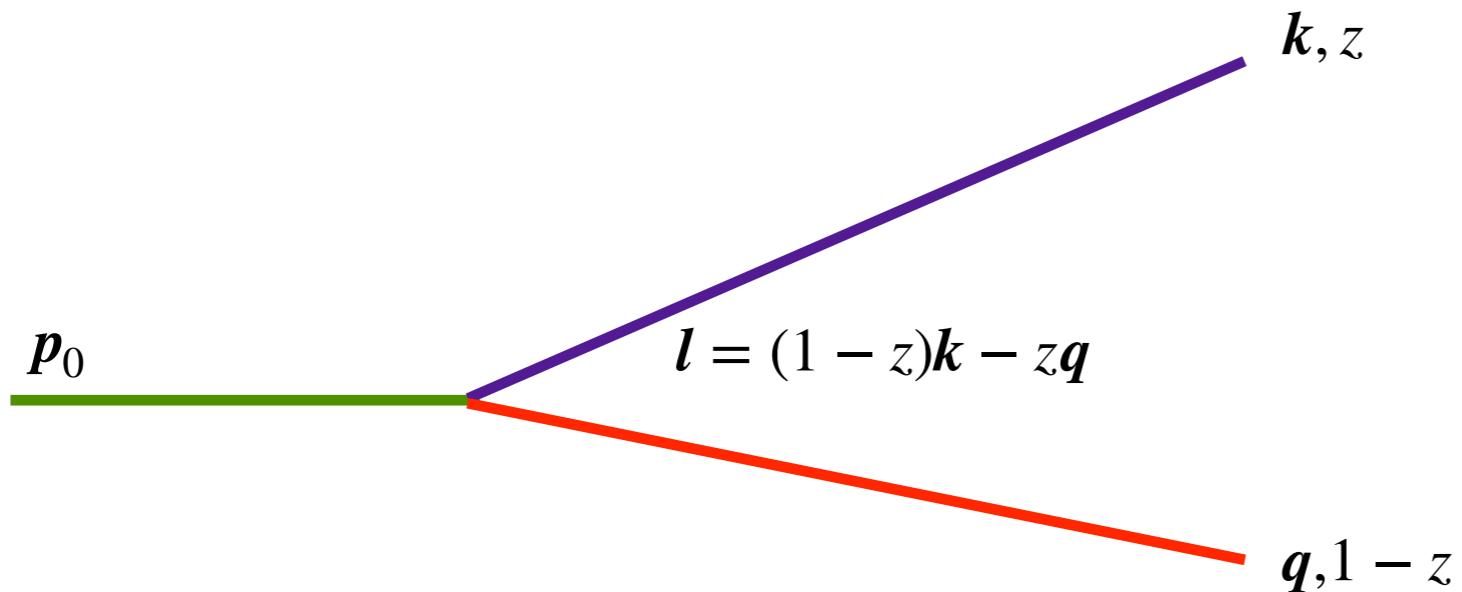


$$\begin{aligned}
 \frac{d\sigma}{d\Omega_k d\Omega_q} = & \frac{g^2}{z(1-z)E^2} P_{a \rightarrow bc}(z) 2\text{Re} \int_{\mathbf{p}_0 \mathbf{p}_1 \bar{\mathbf{p}}_2} \int_{t_0}^{\infty} dt_1 \int_{t_1}^{\infty} dt_2 (\mathbf{l}_1 \cdot \bar{\mathbf{l}}_2) \\
 & \times \mathcal{S}^{(4)}((1-z)\mathbf{k} - z\mathbf{q}, L; \mathbf{l}_2, \bar{\mathbf{l}}_2, t_2; \mathbf{k} + \mathbf{q} - \bar{\mathbf{p}}_2, z) \\
 & \times \mathcal{K}^{(3)}(\mathbf{l}_2, t_2; \mathbf{l}_1, t_1; \bar{\mathbf{p}}_2 - \mathbf{p}_1, z) \mathcal{P}_{R_a}(\mathbf{p}_1 - \mathbf{p}_0; t_1, t_0) \frac{d\sigma_{hard}}{d\Omega_{p_0}}
 \end{aligned}$$

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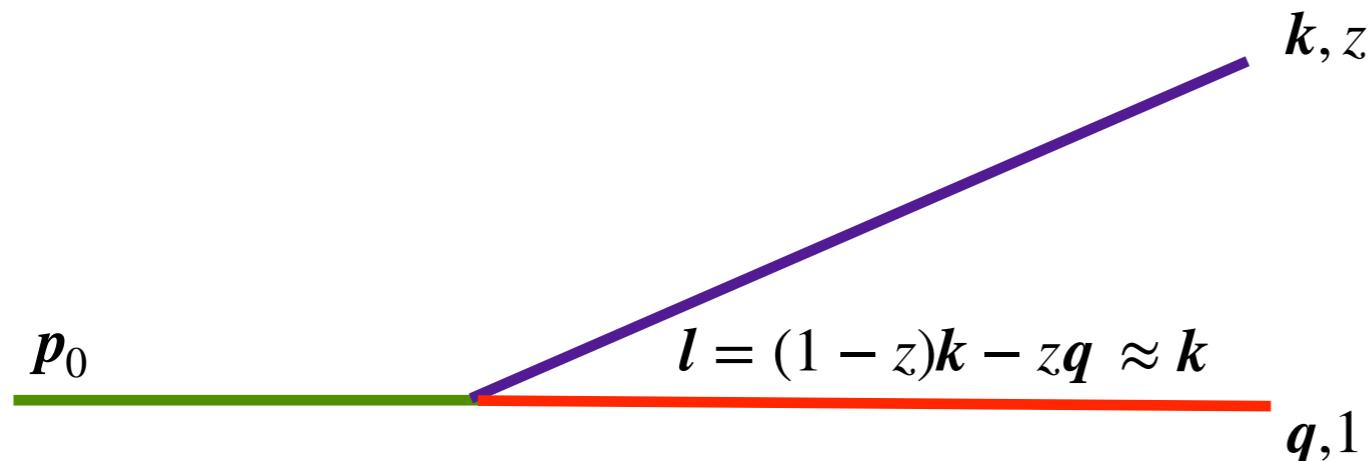
Soft limit

$z \rightarrow 0$ with $\omega = zE$ finite



Soft limit

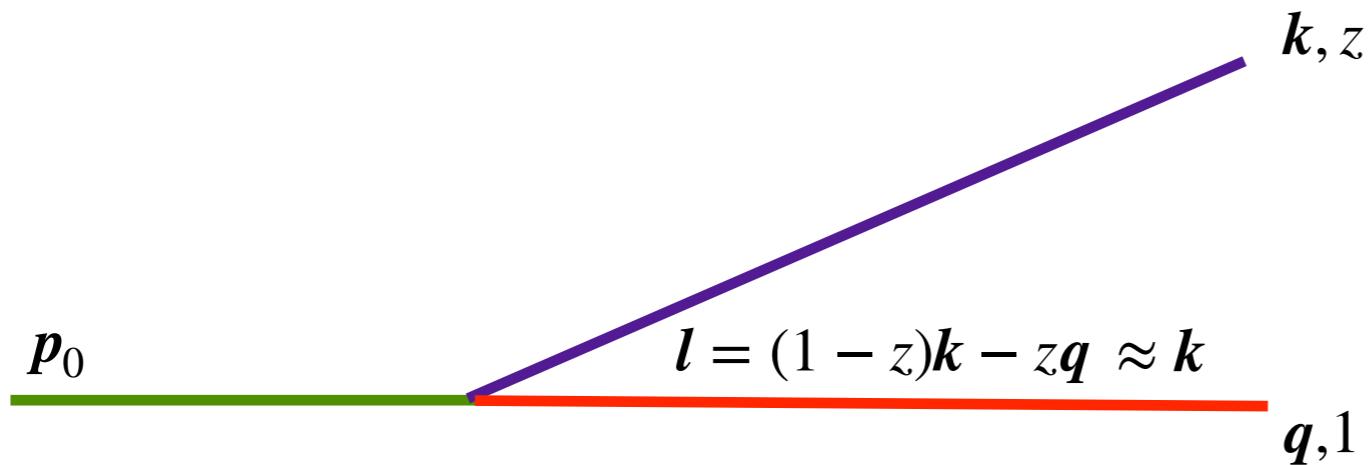
$z \rightarrow 0$ with $\omega = zE$ finite



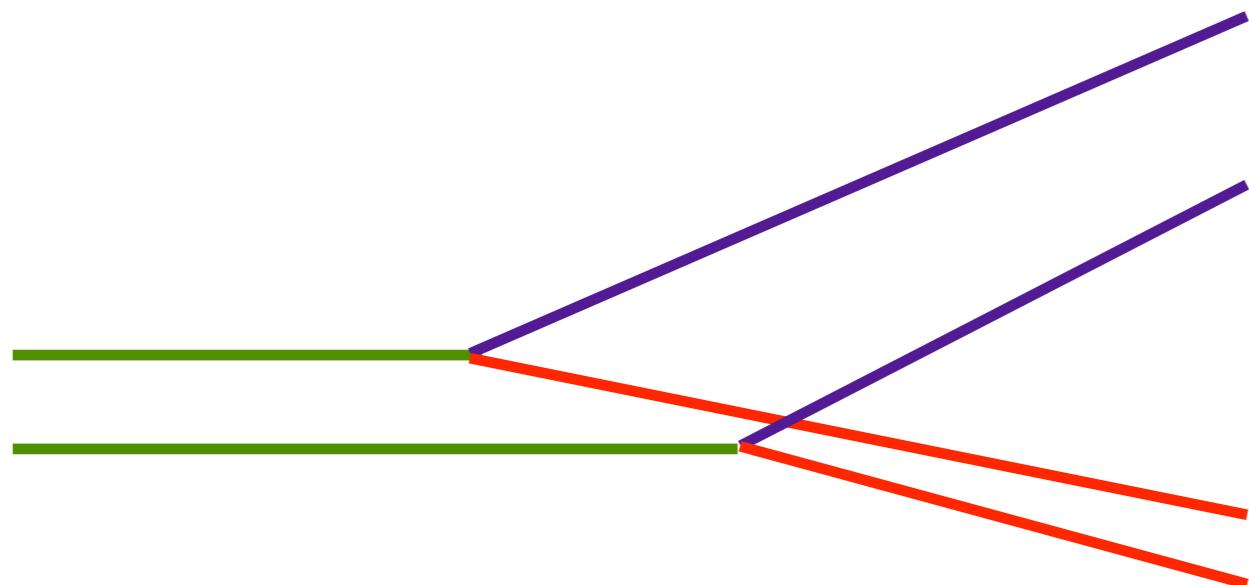
Angle of emission depends
only on transverse momentum
of the soft particle

Soft limit

$z \rightarrow 0$ with $\omega = zE$ finite

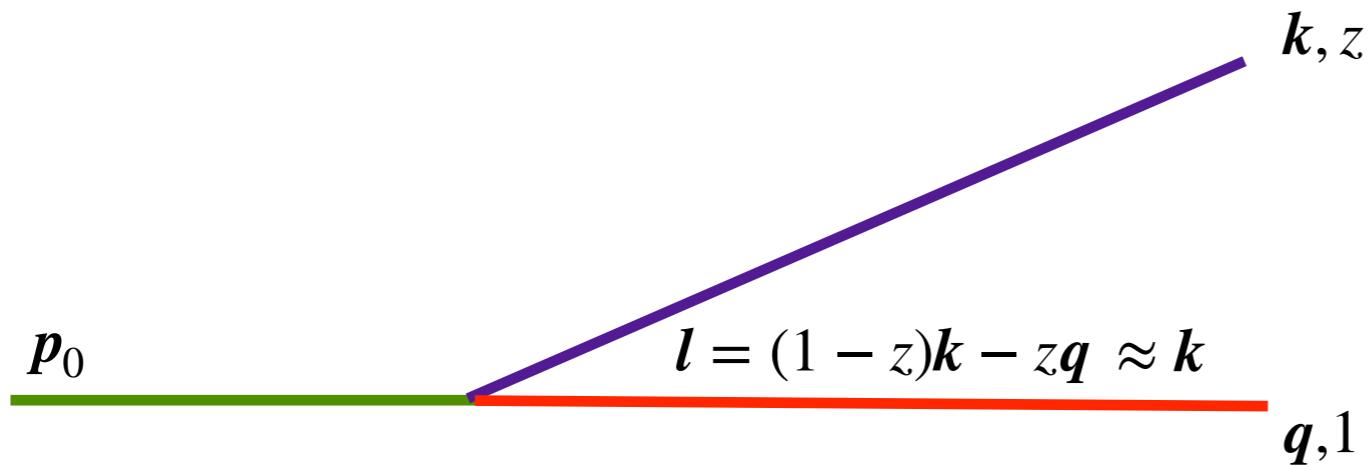


Angle of emission depends
only on transverse momentum
of the soft particle

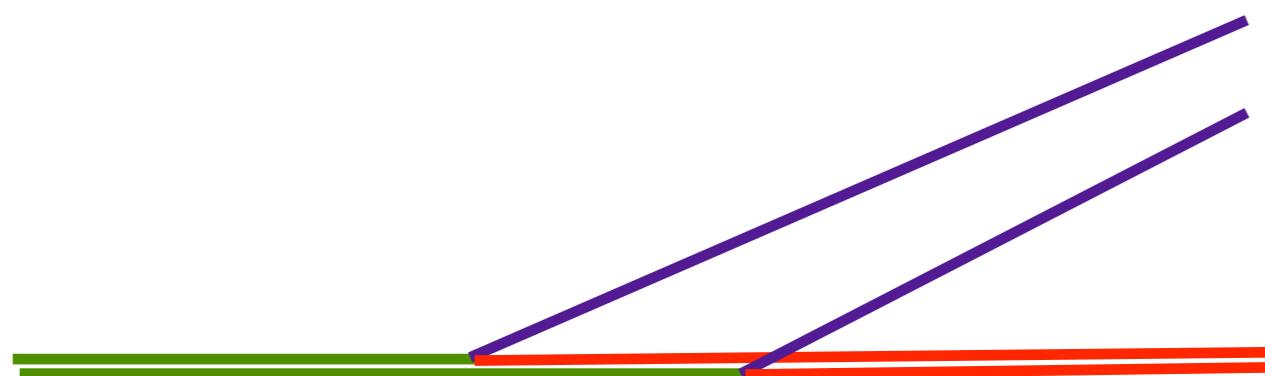


Soft limit

$z \rightarrow 0$ with $\omega = zE$ finite

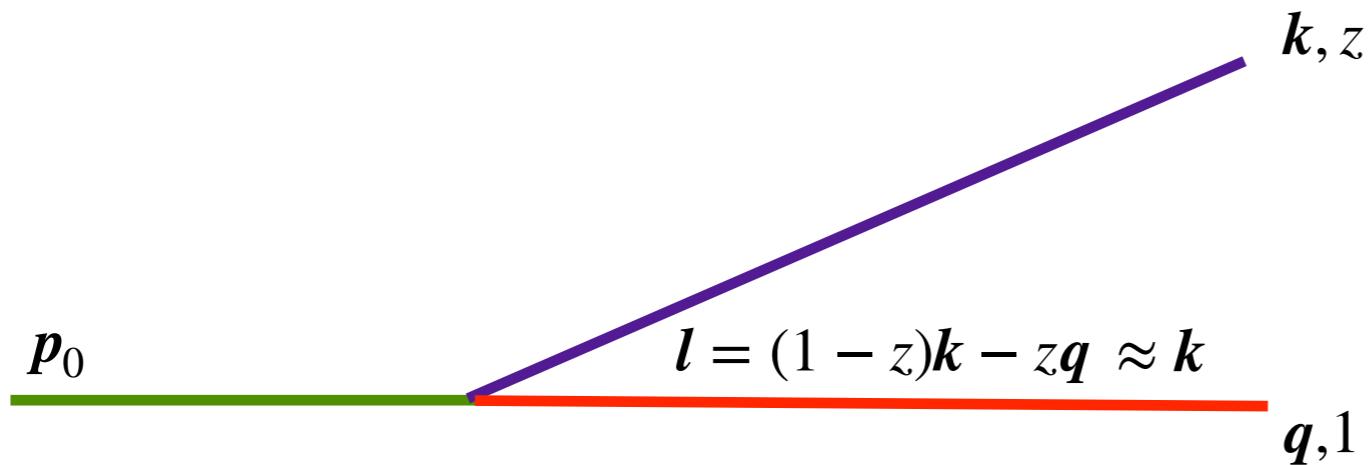


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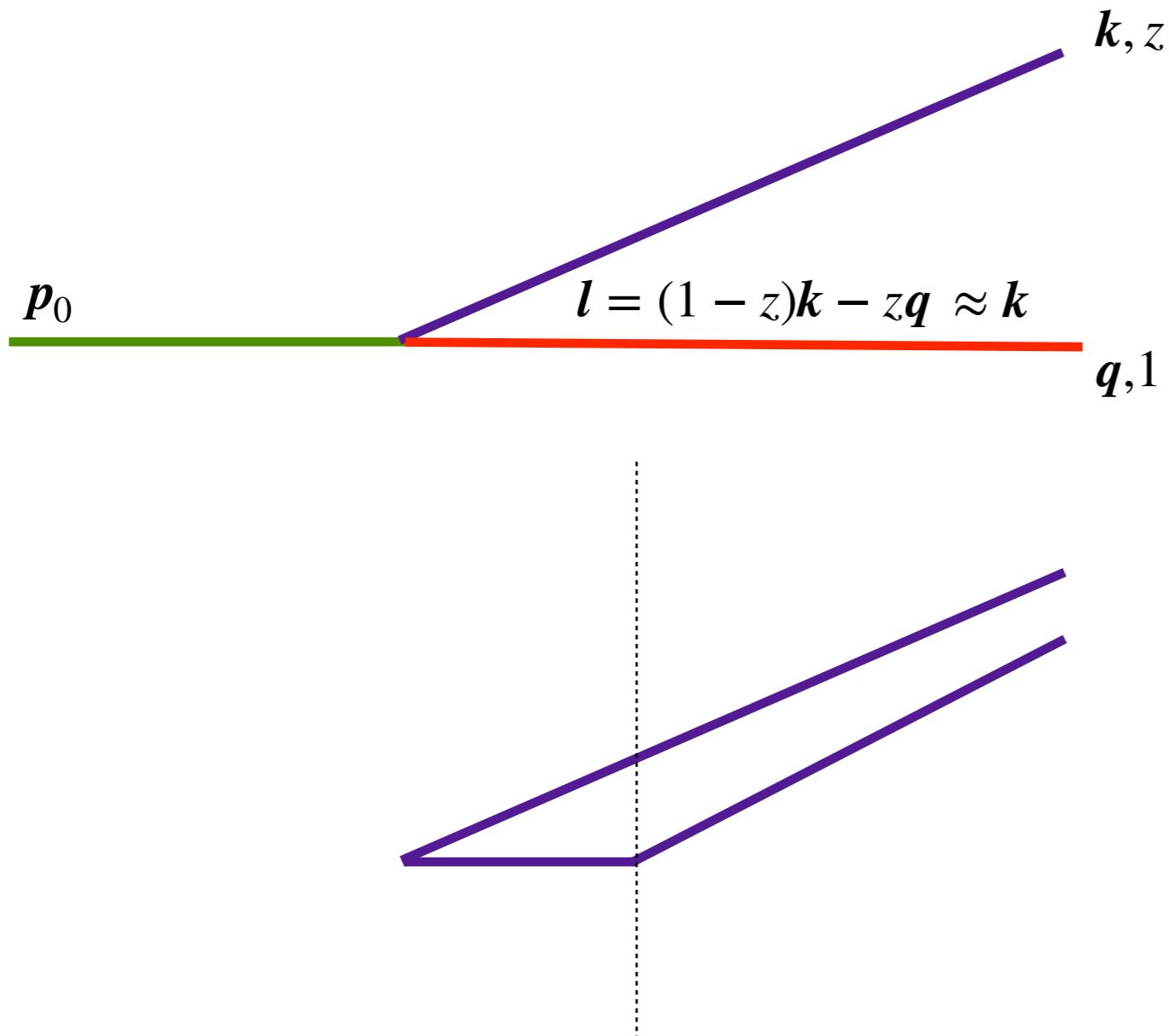
Angle of emission depends
only on transverse momentum
of the soft particle



Initial and final broadening of
the hard particle cancels out

Soft limit

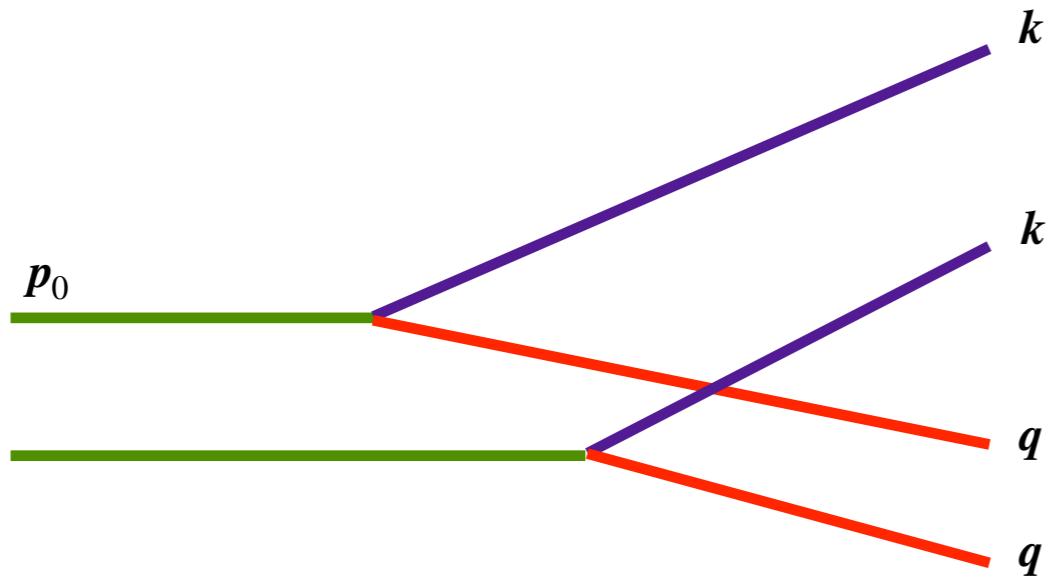
$z \rightarrow 0$ with $\omega = zE$ finite



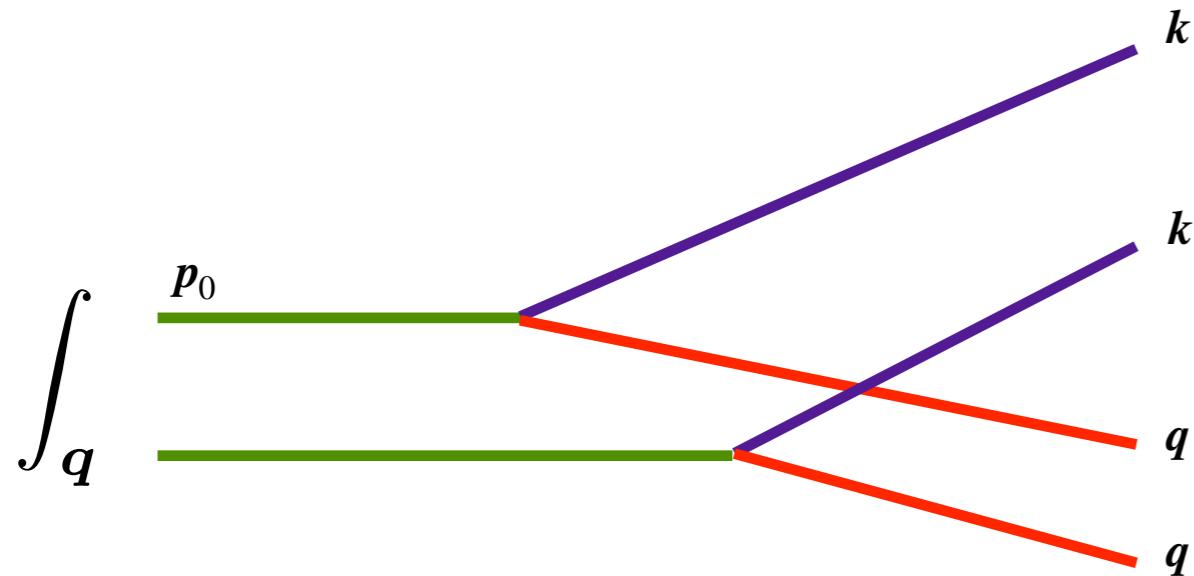
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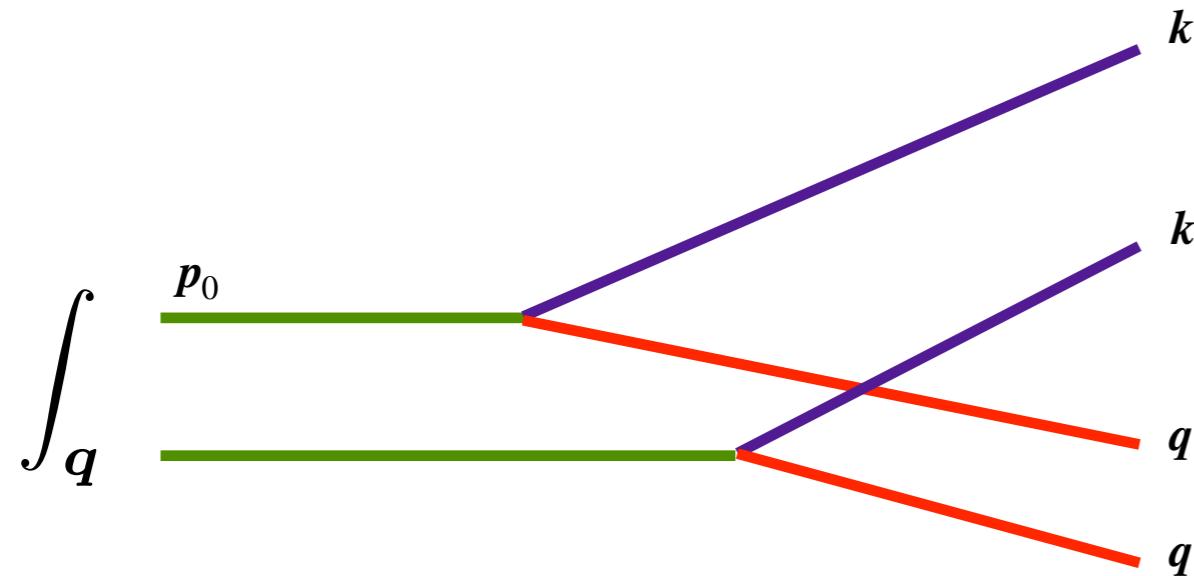
Integrate over final particles



Integrate over final particles

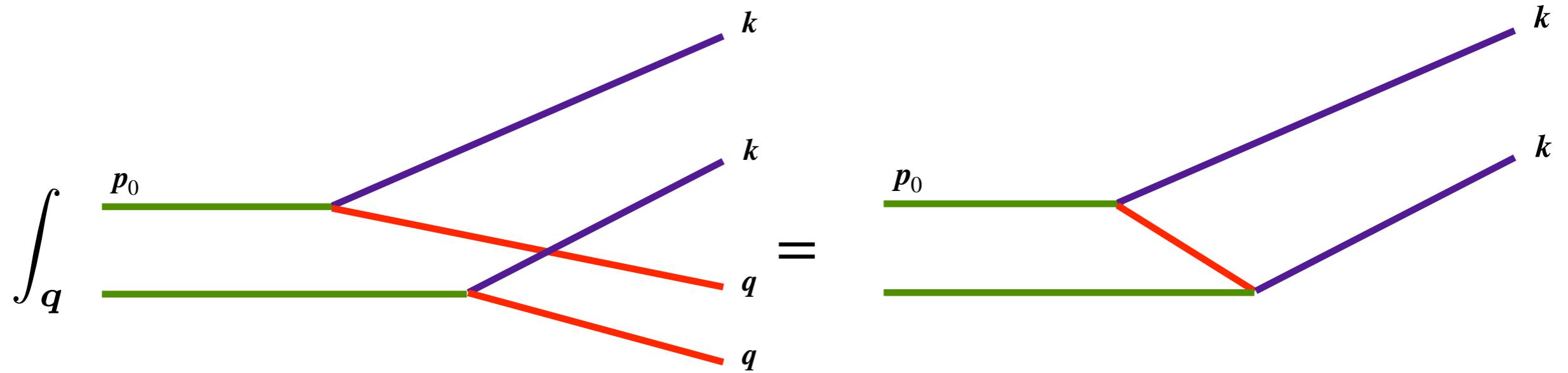


Integrate over final particles



$$\int_{\mathbf{q}} \mathcal{G}_{R_c}^\dagger(\bar{\mathbf{q}}_2, t_2; \mathbf{q}, L; (1-z)E) \mathcal{G}_{R_c}(\mathbf{q}, L; \mathbf{q}_2, t_2; (1-z)E) = (2\pi)^2 \delta^{(2)}(\mathbf{q}_2 - \bar{\mathbf{q}}_2)$$

Integrate over final particles



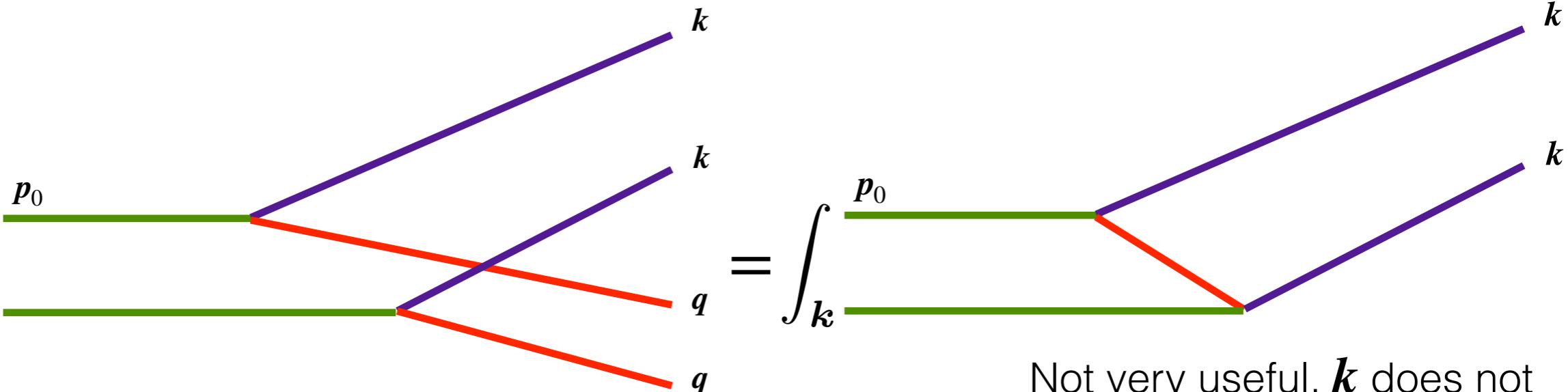
$$\int_{\mathbf{q}} \mathcal{G}_{R_c}^\dagger(\bar{\mathbf{q}}_2, t_2; \mathbf{q}, L; (1-z)E) \mathcal{G}_{R_c}(\mathbf{q}, L; \mathbf{q}_2, t_2; (1-z)E) = (2\pi)^2 \delta^{(2)}(\mathbf{q}_2 - \bar{\mathbf{q}}_2)$$

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Not very useful, \mathbf{k} does not provide information about the splitting

Integrate over final particles

$$\int_{\mathbf{k}} \int_{\mathbf{q}} \frac{p_0}{\textcolor{green}{\mathbf{q}}} = \int_{\mathbf{k}} \frac{p_0}{\textcolor{green}{\mathbf{q}}} = \int_{\mathbf{k}}$$

$$\int_{\mathbf{q}} \mathcal{G}_{R_c}^\dagger(\bar{\mathbf{q}}_2, t_2; \mathbf{q}, L; (1-z)E) \mathcal{G}_{R_c}(\mathbf{q}, L; \mathbf{q}_2, t_2; (1-z)E) = (2\pi)^2 \delta^{(2)}(\mathbf{q}_2 - \bar{\mathbf{q}}_2)$$

Not very useful, \mathbf{k} does not provide information about the splitting

Integrate over final particles

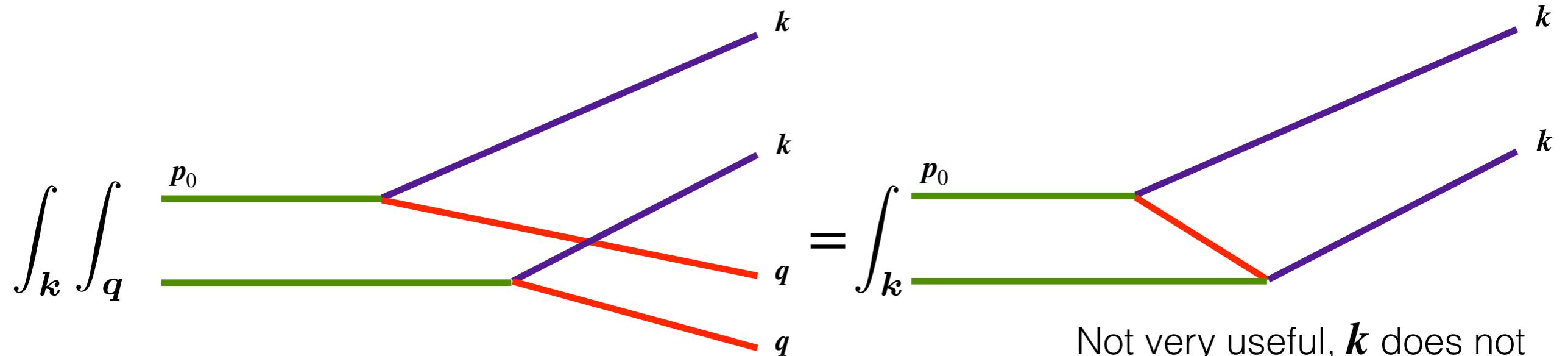
$$\int_{\mathbf{k}} \int_{\mathbf{q}} \frac{p_0}{\textcolor{green}{k}} = \int_{\mathbf{k}} \frac{p_0}{\textcolor{green}{k}} = \int_{\mathbf{k}} \frac{p_0}{\textcolor{purple}{k}}$$

Not very useful, \mathbf{k} does not provide information about the splitting

$$= \textcolor{purple}{\text{circle}}$$

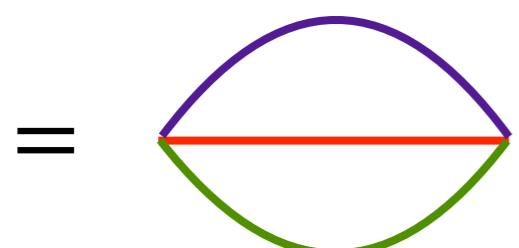
$\int_{\mathbf{q}} \mathcal{G}_{R_c}^\dagger(\bar{\mathbf{q}}_2, t_2; \mathbf{q}, L; (1-z)E) \mathcal{G}_{R_c}(\mathbf{q}, L; \mathbf{q}_2, t_2; (1-z)E) = (2\pi)^2 \delta^{(2)}(\mathbf{q}_2 - \bar{\mathbf{q}}_2)$

Integrate over final particles



$$\int_{\mathbf{q}} \mathcal{G}_{R_c}^\dagger(\bar{\mathbf{q}}_2, t_2; \mathbf{q}, L; (1-z)E) \mathcal{G}_{R_c}(\mathbf{q}, L; \mathbf{q}_2, t_2; (1-z)E) = (2\pi)^2 \delta^{(2)}(\mathbf{q}_2 - \bar{\mathbf{q}}_2)$$

Not very useful, \mathbf{k} does not provide information about the splitting



Gives the energy spectrum $\frac{dI}{dz}$ where all transverse information has been lost

Integrate over final particles

$$\int_{\mathbf{k}} \int_{\mathbf{q}} \frac{p_0}{\text{---}} \text{---} \begin{array}{c} \text{---} \\ \text{---} \end{array} \begin{array}{c} \text{---} \\ \text{---} \end{array} \text{---} \text{---} = \int_{\mathbf{k}} \frac{p_0}{\text{---}} \text{---} \begin{array}{c} \text{---} \\ \text{---} \end{array} \text{---} \text{---}$$

$\int_{\mathbf{q}} \mathcal{G}_{R_c}^\dagger(\bar{\mathbf{q}}_2, t_2; \mathbf{q}, L; (1-z)E) \mathcal{G}_{R_c}(\mathbf{q}, L; \mathbf{q}_2, t_2; (1-z)E) = (2\pi)^2 \delta^{(2)}(\mathbf{q}_2 - \bar{\mathbf{q}}_2)$

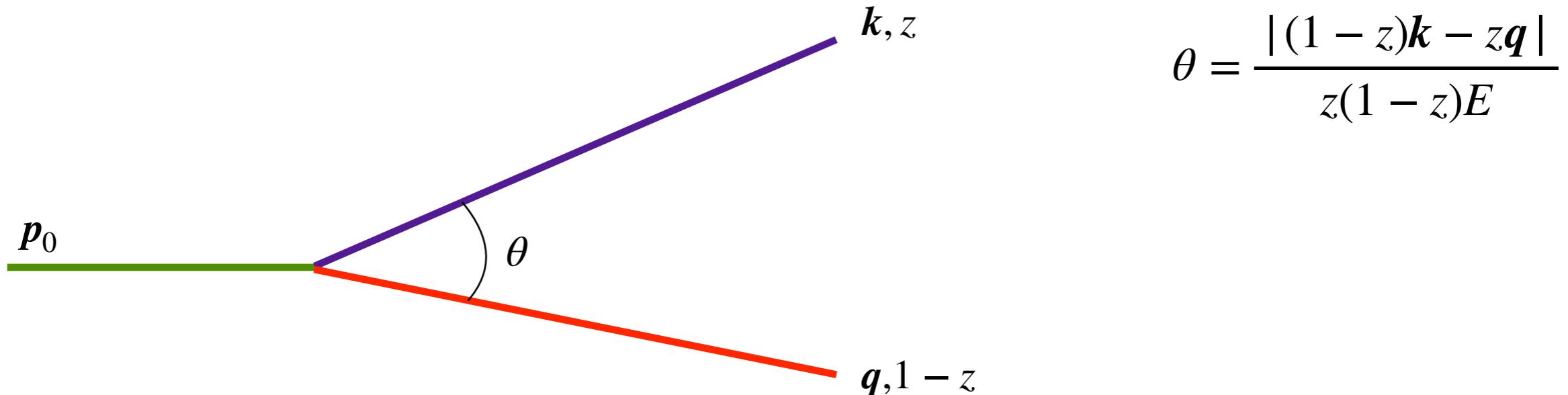
Not very useful, \mathbf{k} does not provide information about the splitting

$$= \text{---} \text{---} \text{---} \text{---} \text{---} \text{---}$$

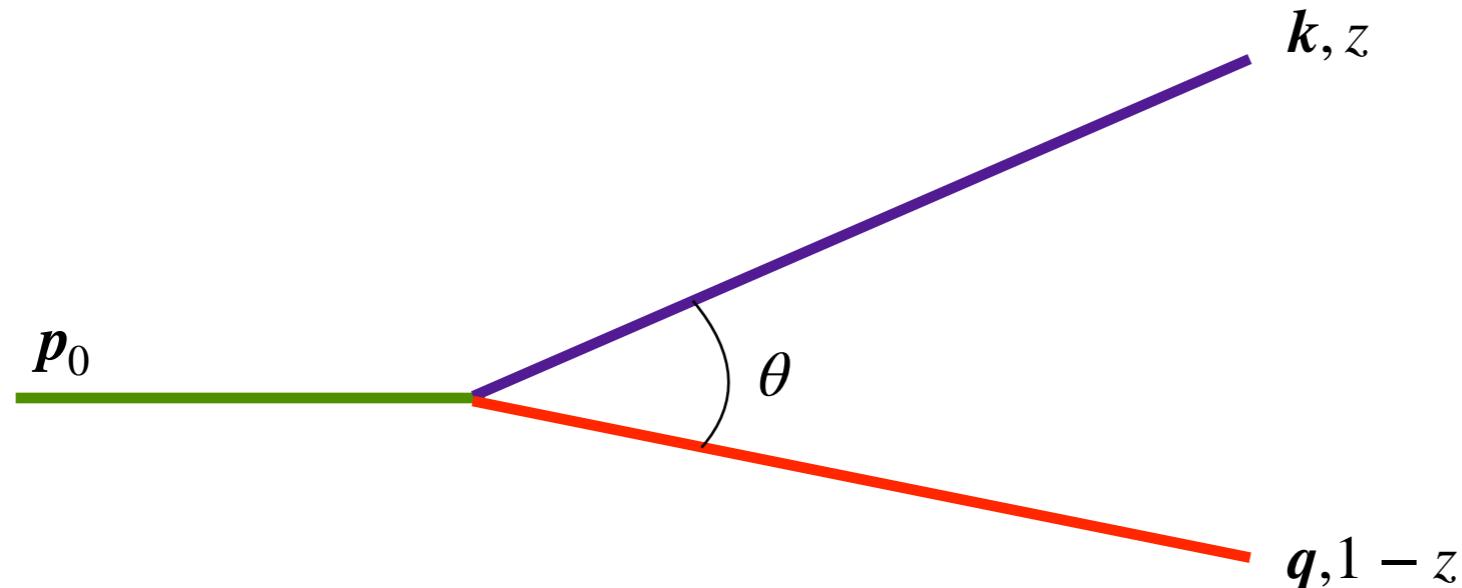
Gives the energy spectrum $\frac{dI}{dz}$ where all transverse information has been lost

None of these simplifications allows us to keep track of the splitting angle while reducing the complexity of the calculation

How can we isolate the angular dependence?



How can we isolate the angular dependence?

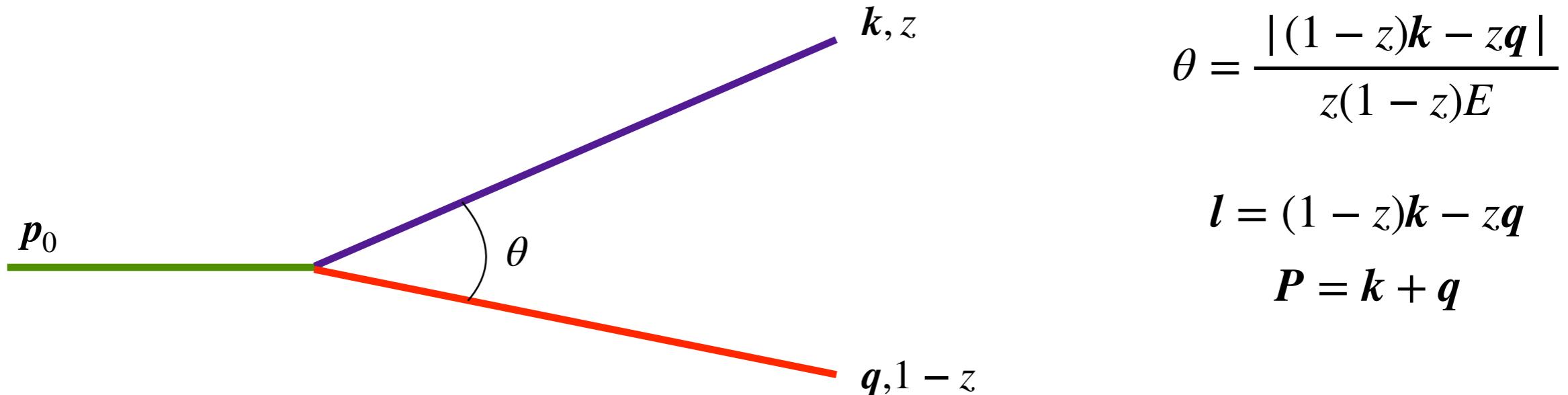


$$\theta = \frac{|(1 - z)\mathbf{k} - z\mathbf{q}|}{z(1 - z)E}$$

$$\mathbf{l} = (1 - z)\mathbf{k} - z\mathbf{q}$$

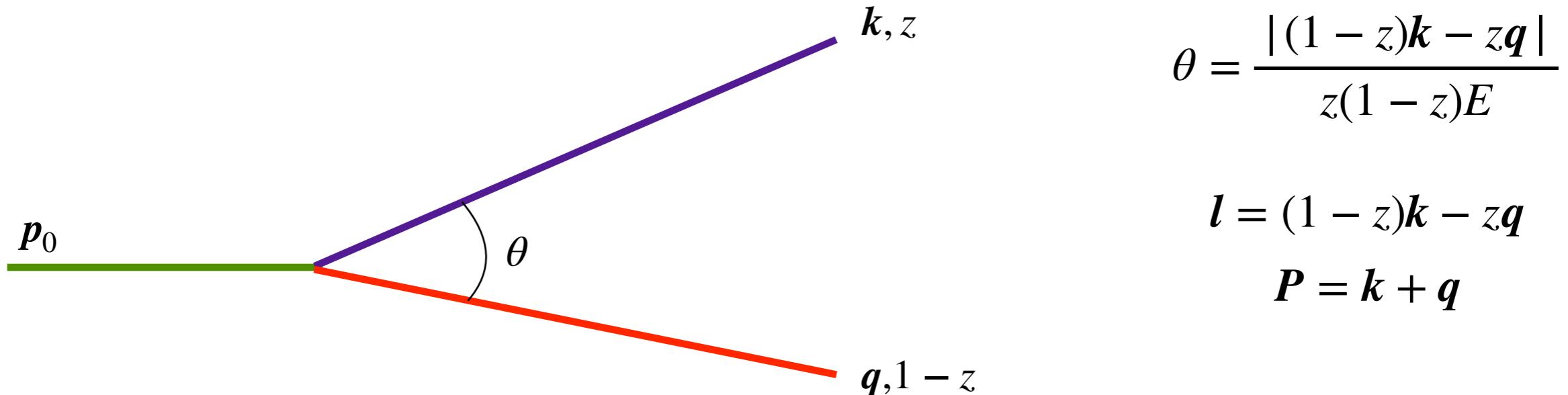
$$\mathbf{P} = \mathbf{k} + \mathbf{q}$$

How can we isolate the angular dependence?



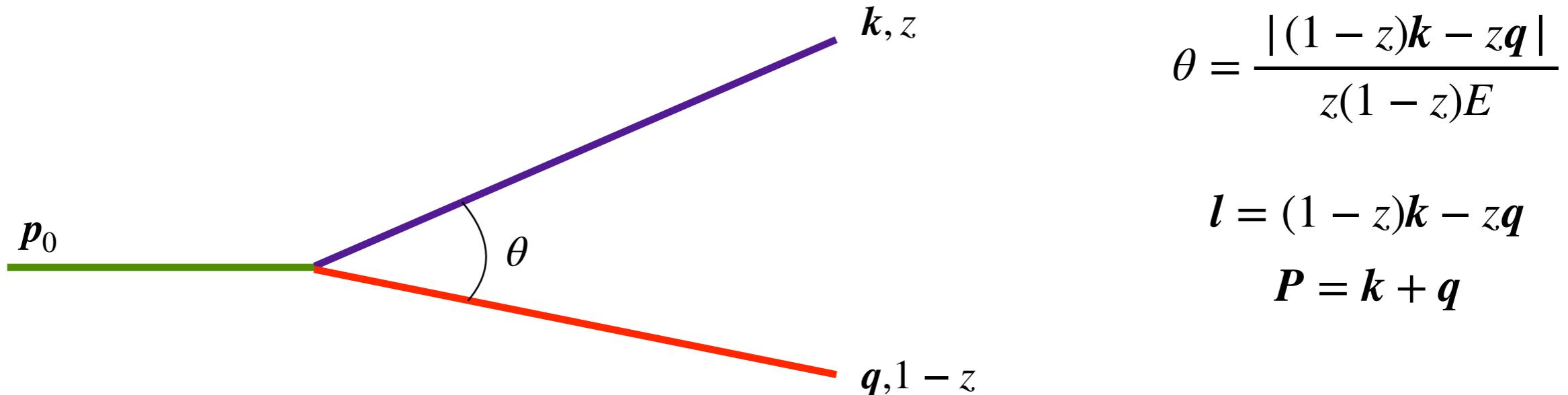
$$\begin{aligned} \frac{d\sigma}{d\Omega_k d\Omega_q} &= \frac{g^2}{z(1-z)E^2} P_{a \rightarrow bc}(z) 2\text{Re} \int_{\mathbf{p}_0 \mathbf{p}_1 \bar{\mathbf{p}}_2} \int_{t_0}^{\infty} dt_1 \int_{t_1}^{\infty} dt_2 (\mathbf{l}_1 \cdot \bar{\mathbf{l}}_2) \\ &\times \mathcal{S}^{(4)}((1-z)\mathbf{k} - z\mathbf{q}, L; \mathbf{l}_2, \bar{\mathbf{l}}_2, t_2; \mathbf{k} + \mathbf{q} - \bar{\mathbf{p}}_2, z) \\ &\times \mathcal{K}^{(3)}(\mathbf{l}_2, t_2; \mathbf{l}_1, t_1; \bar{\mathbf{p}}_2 - \mathbf{p}_1, z) \mathcal{P}_{R_a}(\mathbf{p}_1 - \mathbf{p}_0; t_1, t_0) \frac{d\sigma_{hard}}{d\Omega_{p_0}} \end{aligned}$$

How can we isolate the angular dependence?



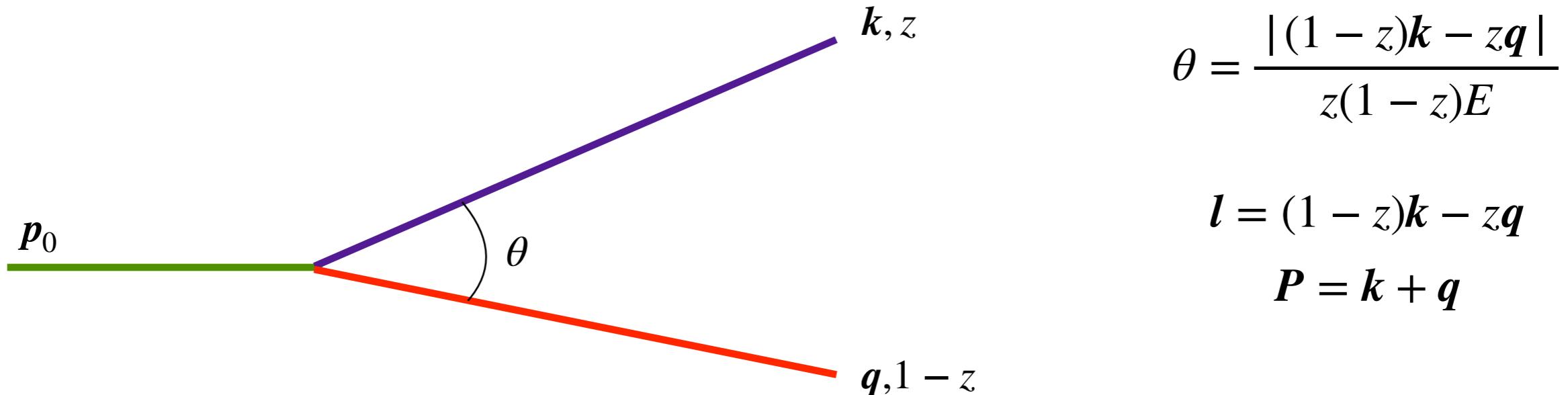
$$\begin{aligned} \frac{d\sigma}{d\Omega_k d\Omega_q} &= \frac{g^2}{z(1 - z)E^2} P_{a \rightarrow bc}(z) 2\text{Re} \int_{\mathbf{p}_0 \mathbf{p}_1 \bar{\mathbf{p}}_2 \mathbf{l}_1 \mathbf{l}_2 \bar{\mathbf{l}}_2} \int_{t_0}^{\infty} dt_1 \int_{t_1}^{\infty} dt_2 (\mathbf{l}_1 \cdot \bar{\mathbf{l}}_2) \\ &\times \mathcal{S}^{(4)}(\mathbf{l}, L; \mathbf{l}_2, \bar{\mathbf{l}}_2, t_2; \mathbf{P} - \bar{\mathbf{p}}_2, z) \\ &\times \mathcal{K}^{(3)}(\mathbf{l}_2, t_2; \mathbf{l}_1, t_1; \bar{\mathbf{p}}_2 - \mathbf{p}_1, z) \mathcal{P}_{R_a}(\mathbf{p}_1 - \mathbf{p}_0; t_1, t_0) \frac{d\sigma_{hard}}{d\Omega_{p_0}} \end{aligned}$$

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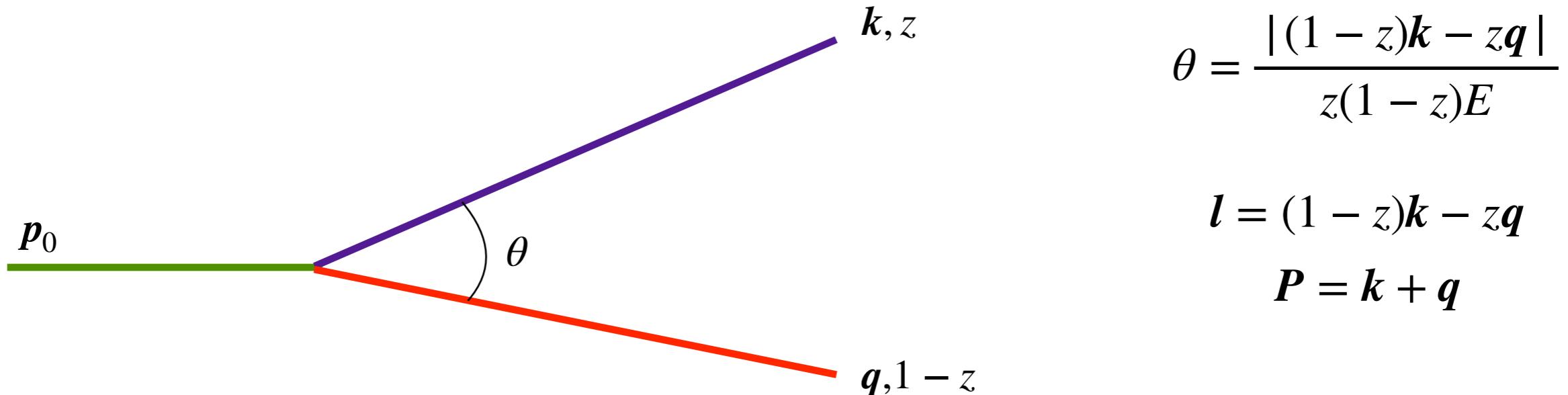
$$\int_{\mathbf{P}} \frac{d\sigma}{d\Omega_k d\Omega_q} = \frac{g^2}{z(1-z)E^2} P_{a \rightarrow bc}(z) 2\text{Re} \int_{\mathbf{p}_0 \mathbf{p}_1 \bar{\mathbf{p}}_2 \mathbf{l}_1 \mathbf{l}_2 \bar{\mathbf{l}}_2} \int_{t_0}^{\infty} dt_1 \int_{t_1}^{\infty} dt_2 (\mathbf{l}_1 \cdot \bar{\mathbf{l}}_2)$$
$$\times \int_{\mathbf{P}} \mathcal{S}^{(4)}(\mathbf{l}, L; \mathbf{l}_2, \bar{\mathbf{l}}_2, t_2; \mathbf{P} - \bar{\mathbf{p}}_2, z)$$
$$\times \mathcal{K}^{(3)}(\mathbf{l}_2, t_2; \mathbf{l}_1, t_1; \bar{\mathbf{p}}_2 - \mathbf{p}_1, z) \mathcal{P}_{R_a}(\mathbf{p}_1 - \mathbf{p}_0; t_1, t_0) \frac{d\sigma_{hard}}{d\Omega_{p_0}}$$

How can we isolate the angular dependence?



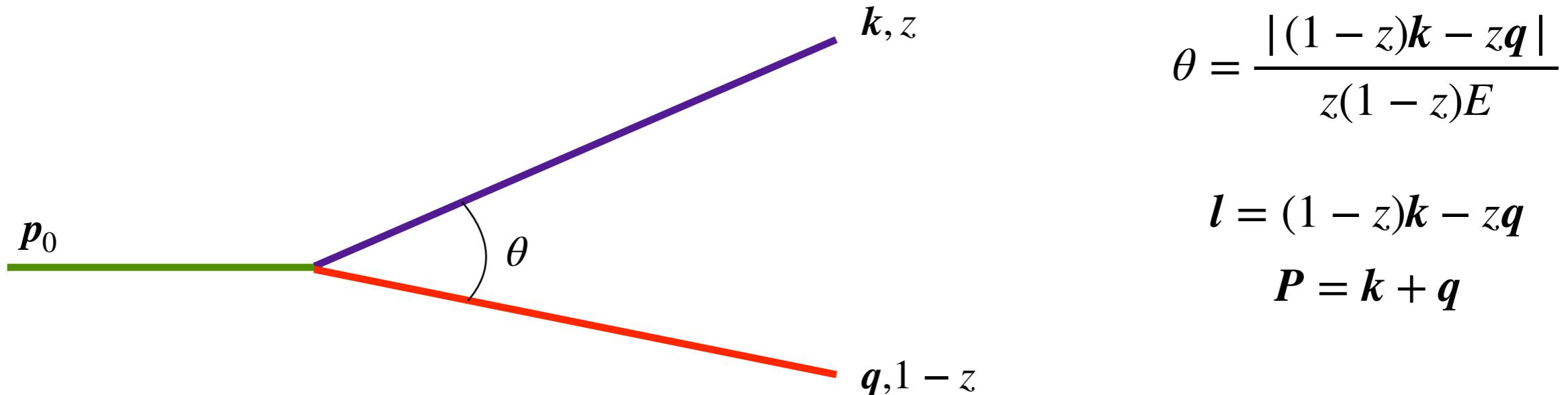
$$\begin{aligned} \int_{\mathbf{P}} \frac{d\sigma}{d\Omega_k d\Omega_q} &= \frac{g^2}{z(1-z)E^2} P_{a \rightarrow bc}(z) 2\text{Re} \int_{\mathbf{p}_0 \bar{\mathbf{p}}_1 \bar{\mathbf{p}}_2 \mathbf{l}_1 \mathbf{l}_2 \bar{\mathbf{l}}_2} \int_{t_0}^{\infty} dt_1 \int_{t_1}^{\infty} dt_2 (\mathbf{l}_1 \cdot \bar{\mathbf{l}}_2) \\ &\times \tilde{\mathcal{S}}^{(4)}(\mathbf{l}, L; \mathbf{l}_2, \bar{\mathbf{l}}_2, t_2; z) \\ &\times \mathcal{K}^{(3)}(\mathbf{l}_2, t_2; \mathbf{l}_1, t_1; \bar{\mathbf{p}}_2 - \mathbf{p}_1, z) \mathcal{P}_{R_a}(\mathbf{p}_1 - \mathbf{p}_0; t_1, t_0) \frac{d\sigma_{hard}}{d\Omega_{p_0}} \end{aligned}$$

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$$\begin{aligned} \int_{\mathbf{P}} \frac{d\sigma}{d\Omega_k d\Omega_q} &= \frac{g^2}{z(1-z)E^2} P_{a \rightarrow bc}(z) 2\text{Re} \int_{\mathbf{p}_0 \mathbf{p}_1 \bar{\mathbf{p}}_2} \int_{t_0}^{\infty} dt_1 \int_{t_1}^{\infty} dt_2 (\mathbf{l}_1 \cdot \bar{\mathbf{l}}_2) \\ &\times \tilde{\mathcal{S}}^{(4)}(\mathbf{l}, L; \mathbf{l}_2, \bar{\mathbf{l}}_2, t_2; z) \\ &\times \mathcal{K}^{(3)}(\mathbf{l}_2, t_2; \mathbf{l}_1, t_1; \bar{\mathbf{p}}_2 - \mathbf{p}_1, z) \mathcal{P}_{R_a}(\mathbf{p}_1 - \mathbf{p}_0; t_1, t_0) \frac{d\sigma_{hard}}{d\Omega_{p_0}} \end{aligned}$$

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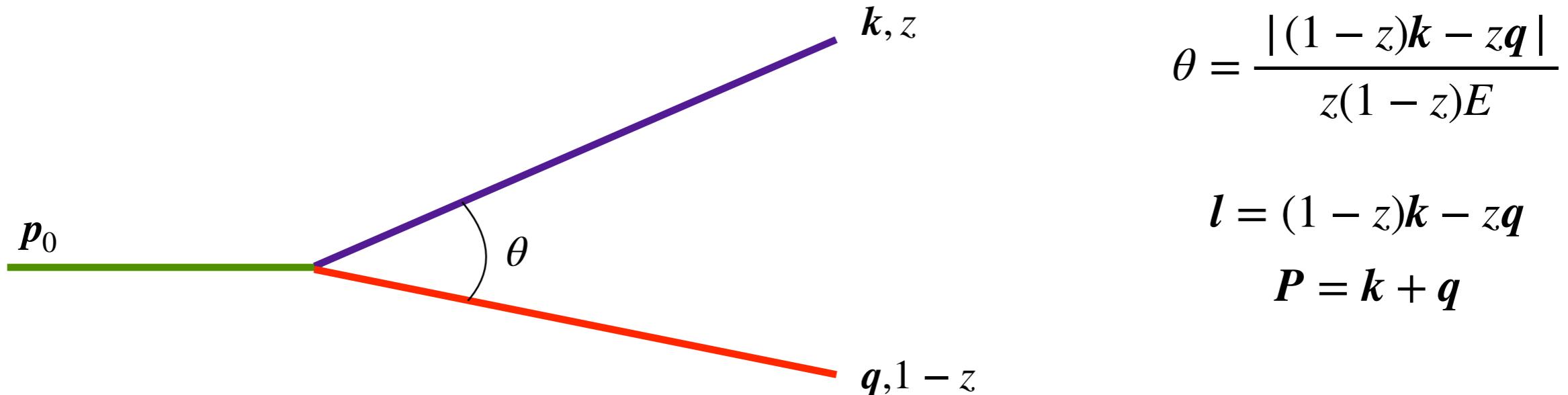


$$\int_{\mathbf{P}} \frac{d\sigma}{d\Omega_k d\Omega_q} = \frac{g^2}{z(1-z)E^2} P_{a \rightarrow bc}(z) 2\text{Re} \int_{\mathbf{p}_0 \mathbf{p}_1 \bar{\mathbf{p}}_2} \int_{t_0}^{\infty} dt_1 \int_{t_1}^{\infty} dt_2 (\mathbf{l}_1 \cdot \bar{\mathbf{l}}_2)$$

$$\times \tilde{\mathcal{S}}^{(4)}(\mathbf{l}, L; \mathbf{l}_2, \bar{\mathbf{l}}_2, t_2; z)$$

$$\times \tilde{\mathcal{K}}^{(3)}(\mathbf{l}_2, t_2; \mathbf{l}_1, t_1; z) \quad \mathcal{P}_{R_a}(\mathbf{p}_1 - \mathbf{p}_0; t_1, t_0) \frac{d\sigma_{hard}}{d\Omega_{p_0}}$$

How can we isolate the angular dependence?

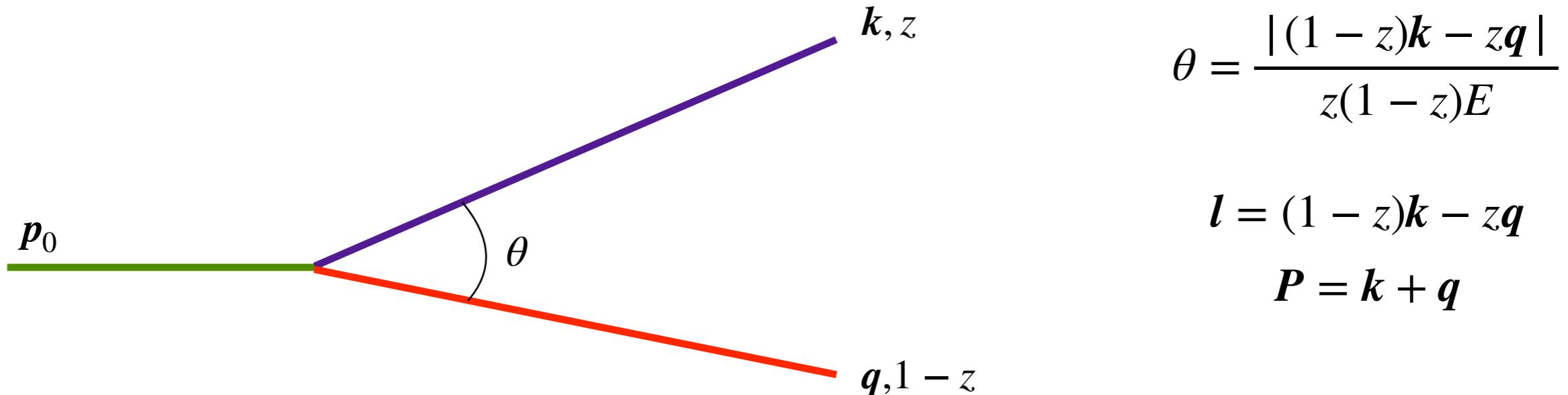


$$\int_{\mathbf{P}} \frac{d\sigma}{d\Omega_k d\Omega_q} = \frac{g^2}{z(1-z)E^2} P_{a \rightarrow bc}(z) 2\text{Re} \int_{\mathbf{p}_0 \mathbf{p}_1} \mathbf{l}_1 \mathbf{l}_2 \bar{\mathbf{l}}_2 \int_{t_0}^{\infty} dt_1 \int_{t_1}^{\infty} dt_2 (\mathbf{l}_1 \cdot \bar{\mathbf{l}}_2)$$

$$\times \tilde{\mathcal{S}}^{(4)}(\mathbf{l}, L; \mathbf{l}_2, \bar{\mathbf{l}}_2, t_2; z)$$

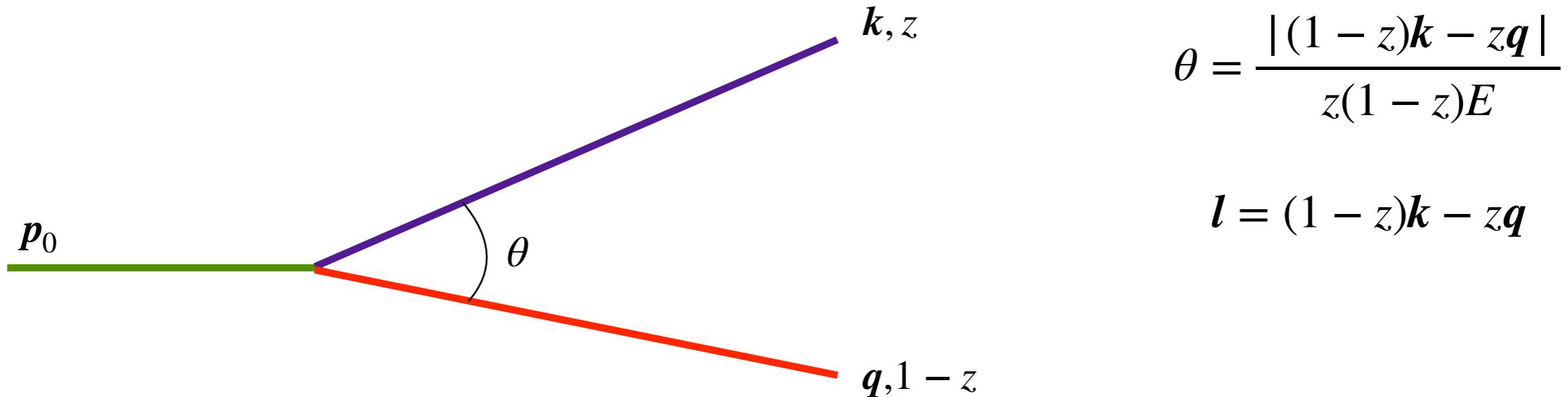
$$\times \tilde{\mathcal{K}}^{(3)}(\mathbf{l}_2, t_2; \mathbf{l}_1, t_1; z) \quad \mathcal{P}_{R_a}(\mathbf{p}_1 - \mathbf{p}_0; t_1, t_0) \frac{d\sigma_{hard}}{d\Omega_{p_0}}$$

How can we isolate the angular dependence?



$$\int_P \frac{d\sigma}{d\Omega_k d\Omega_q} = \frac{g^2}{z(1-z)E^2} P_{a \rightarrow bc}(z) 2\text{Re} \int_{p_0 p_1} \int_{t_0}^{\infty} dt_1 \int_{t_1}^{\infty} dt_2 (\mathbf{l}_1 \cdot \bar{\mathbf{l}}_2)$$
$$\times \tilde{\mathcal{S}}^{(4)}(\mathbf{l}, L; \mathbf{l}_2, \bar{\mathbf{l}}_2, t_2; z)$$
$$\times \tilde{\mathcal{K}}^{(3)}(\mathbf{l}_2, t_2; \mathbf{l}_1, t_1; z) \frac{d\sigma_{hard}}{d\Omega_{p_0}}$$

How can we isolate the angular dependence?



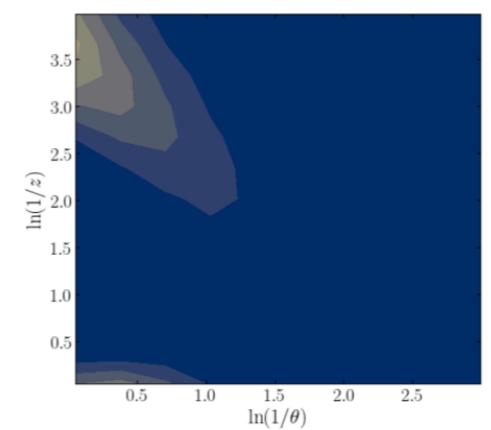
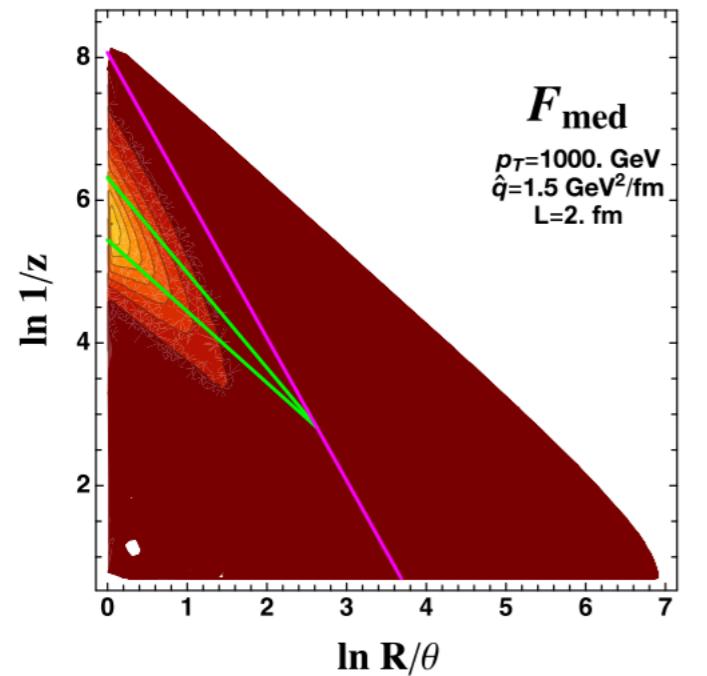
$$E \frac{d\sigma}{dz dE d^2\mathbf{l}} = \frac{g^2}{2(2\pi)^3(z(1-z)E)^2} P_{a \rightarrow bc}(z) 2\text{Re} \int_{\mathbf{l}_1 \mathbf{l}_2 \bar{\mathbf{l}}_2} \int_{t_0}^{\infty} dt_1 \int_{t_1}^{\infty} dt_2 (\mathbf{l}_1 \cdot \bar{\mathbf{l}}_2)$$
$$\times \tilde{\mathcal{S}}^{(4)}(\mathbf{l}, L; \mathbf{l}_2, \bar{\mathbf{l}}_2, t_2; z) \tilde{\mathcal{K}}^{(3)}(\mathbf{l}_2, t_2; \mathbf{l}_1, t_1; z) E \frac{d\sigma_{hard}}{dE}$$

One can go one step further and integrate the angle of \mathbf{l} , since we only care about its magnitude

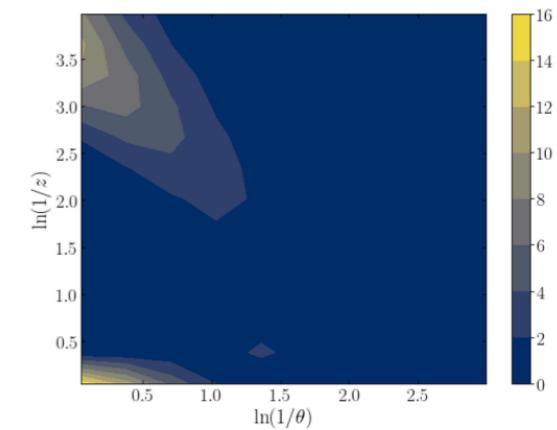
Lund plane with tilted Wilson lines

- In the hard splitting case, one can simplify the structure of the in-medium propagators using the so-called “tilted Wilson lines” where the particles are assumed to go on straight lines
- Results in the harmonic approximation

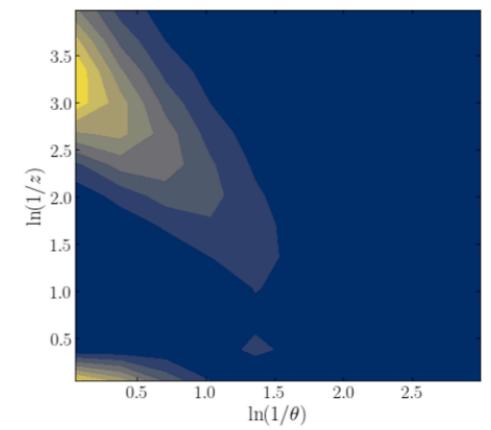
$$\gamma \rightarrow q\bar{q}$$



(a) Photon splitting.



(b) Quark-gluon splitting.



(c) Gluon-gluon splitting.

Calculating medium averages

- Use integral equation for the in-medium propagator

$$\begin{aligned}\mathcal{G}_R(\mathbf{p}_2, t_2; \mathbf{p}_1, t_1; \omega) = & (2\pi)^2 \delta^{(2)}(\mathbf{p}_2 - \mathbf{p}_1) e^{-i\frac{\mathbf{p}_2^2}{2\omega}(t_2 - t_1)} \\ & + ig \int_{t_1}^{t_2} ds e^{-i\frac{\mathbf{p}_2^2}{2\omega}(t_2 - s)} \int_{\mathbf{p}'} A_R^-(s, \mathbf{p}_2 - \mathbf{p}') \mathcal{G}_R(\mathbf{p}', s; \mathbf{p}_1, t_1; \omega)\end{aligned}$$



- Take averages by pairing explicit insertions of the background field

$$\langle A^{a-}(\mathbf{q}_1, t_1) A^{b-\dagger}(\mathbf{q}_2, t_2) \rangle = \delta^{ab} \delta(t_2 - t_1) \delta^{(2)}(\mathbf{q}_1 - \mathbf{q}_2) v(\mathbf{q}_1)$$

- Take advantage of momentum conservation and color conservation (always an overall color singlet state)

Average of two propagators

$$\begin{aligned}\mathcal{G}_R(\mathbf{p}_2, t_2; \mathbf{p}_1, t_1; \omega) = & (2\pi)^2 \delta^{(2)}(\mathbf{p}_2 - \mathbf{p}_1) e^{-i\frac{\mathbf{p}_2^2}{2\omega}(t_2 - t_1)} \\ & + ig \int_{t_1}^{t_2} ds e^{-i\frac{\mathbf{p}_2^2}{2\omega}(t_2 - s)} \int_{\mathbf{p}'} A_R^-(s, \mathbf{p}_2 - \mathbf{p}') \mathcal{G}_R(\mathbf{p}', s; \mathbf{p}_1, t_1; \omega)\end{aligned}$$



$$\mathcal{P}_{R_a}(\mathbf{p}_1 - \mathbf{p}_0; t_1, t_0) = (2\pi)^2 \delta^{(2)}(\mathbf{p}_1 - \mathbf{p}_0) - \frac{1}{2} \int_{t_0}^{t_1} ds n(s) \int_{\mathbf{p}'} \sigma(\mathbf{p}_1 - \mathbf{p}') \mathcal{P}_{R_a}(\mathbf{p}' - \mathbf{p}_0; t_1, t_0)$$

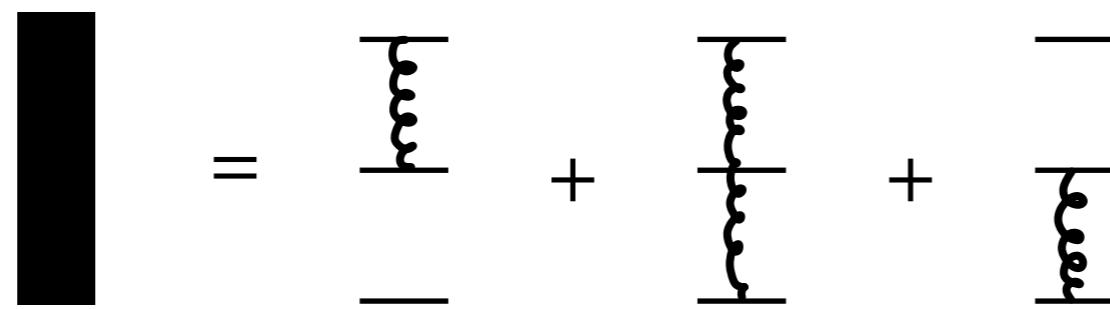
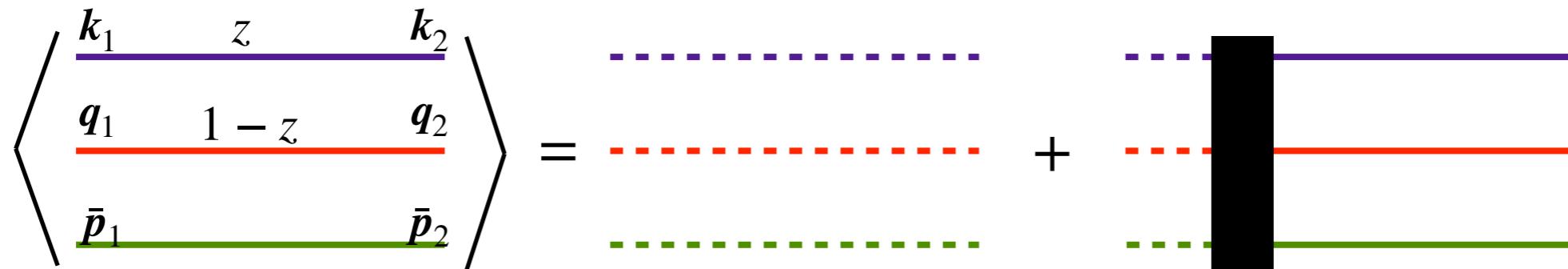
$$\partial_{t_1} \mathcal{P}_{R_a}(\mathbf{p}_1 - \mathbf{p}_0; t_1, t_0) = -\frac{1}{2} n(t_1) \int_{\mathbf{p}'} \sigma(\mathbf{p}_1 - \mathbf{p}') \mathcal{P}_{R_a}(\mathbf{p}' - \mathbf{p}_0; t_1, t_0)$$

Average of three propagators

$$\left\langle \begin{array}{c} k_1 \\ q_1 \\ \bar{p}_1 \end{array} \middle| \begin{array}{c} z \\ 1-z \\ \bar{p}_2 \end{array} \begin{array}{c} k_2 \\ q_2 \\ \bar{p}_2 \end{array} \right\rangle = \text{---} + \text{---} + \text{---}$$

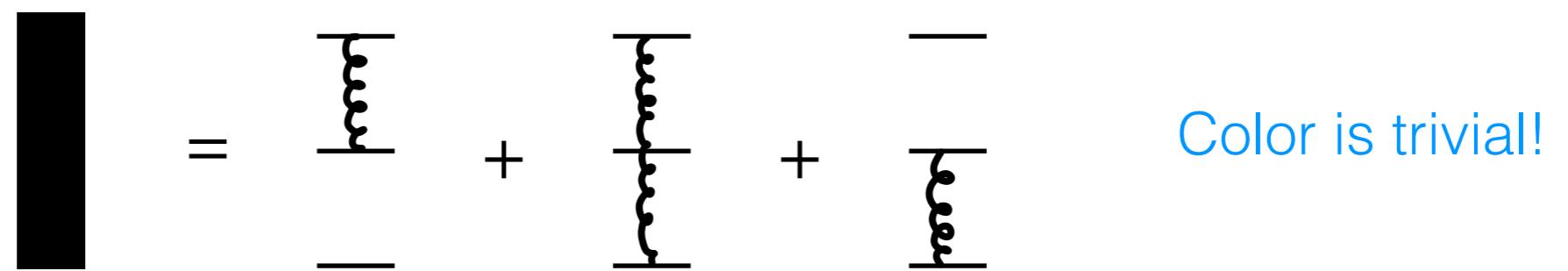
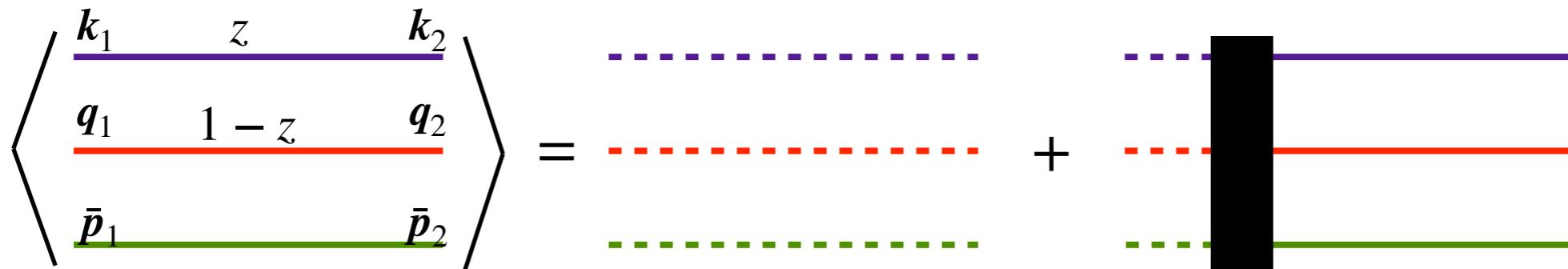
$$\text{---} = \text{---} + \text{---} + \text{---} \quad \text{Color is trivial!}$$

Average of three propagators



Phases: $-i \frac{k_2^2}{2zE} - i \frac{q_2^2}{2(1-z)E} + i \frac{\bar{p}_2^2}{2E} = -i \frac{((1-z)k_2 - zq_2)^2}{2z(1-z)E} = -i \frac{l_2^2}{2z(1-z)E}$

Average of three propagators



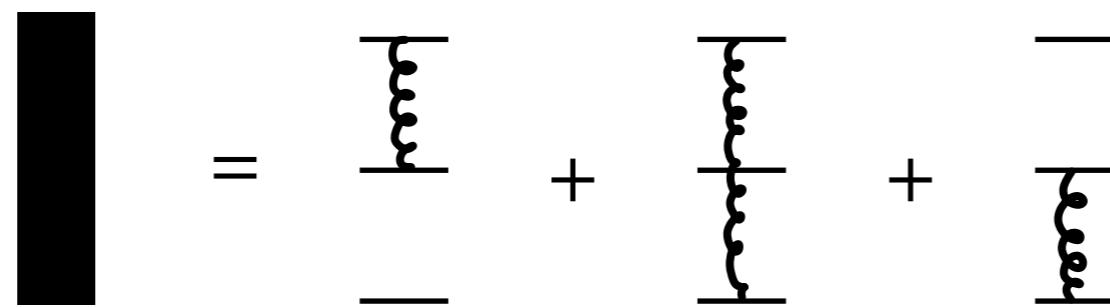
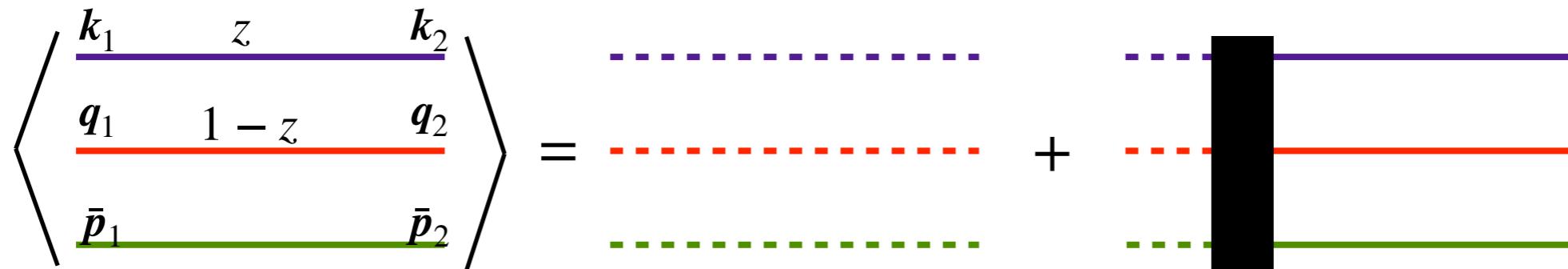
Phases:

$$-i \frac{\mathbf{k}_2^2}{2zE} - i \frac{\mathbf{q}_2^2}{2(1-z)E} + i \frac{\bar{\mathbf{p}}_2^2}{2E} = -i \frac{((1-z)\mathbf{k}_2 - z\mathbf{q}_2)^2}{2z(1-z)E} = -i \frac{l_2^2}{2z(1-z)E}$$

Interaction:

$$\begin{aligned} & \sigma(\mathbf{q}') \left[-\delta^{(2)}(\mathbf{k}_2 - \mathbf{k}_1 - \mathbf{q}') \delta^{(2)}(\mathbf{q}_2 - \mathbf{q}_1 + \mathbf{q}') \delta^{(2)}(\bar{\mathbf{p}}_2 - \bar{\mathbf{p}}_1) \right. \\ & + \delta^{(2)}(\mathbf{k}_2 - \mathbf{k}_1 - \mathbf{q}') \delta^{(2)}(\mathbf{q}_2 - \mathbf{q}_1) \delta^{(2)}(\bar{\mathbf{p}}_2 - \bar{\mathbf{p}}_1 - \mathbf{q}') \\ & \left. + \delta^{(2)}(\mathbf{k}_2 - \mathbf{k}_1) \delta^{(2)}(\mathbf{q}_2 - \mathbf{q}_1 - \mathbf{q}') \delta^{(2)}(\bar{\mathbf{p}}_2 - \bar{\mathbf{p}}_1 - \mathbf{q}') \right] \end{aligned}$$

Average of three propagators

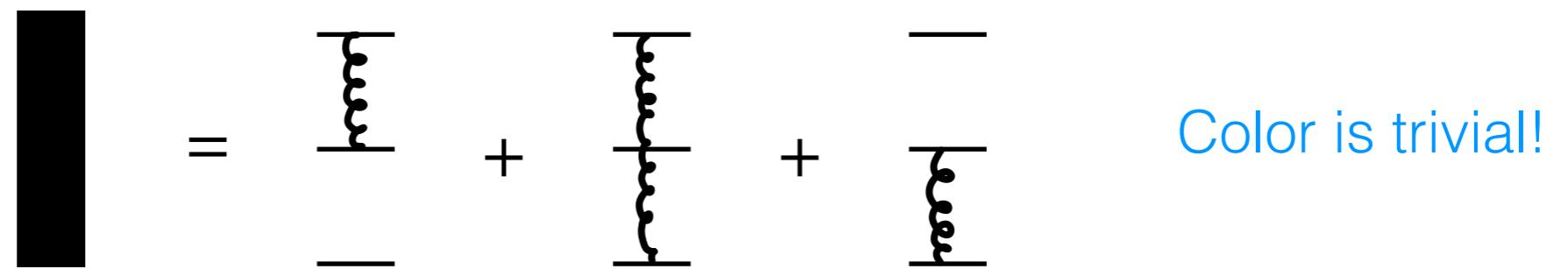
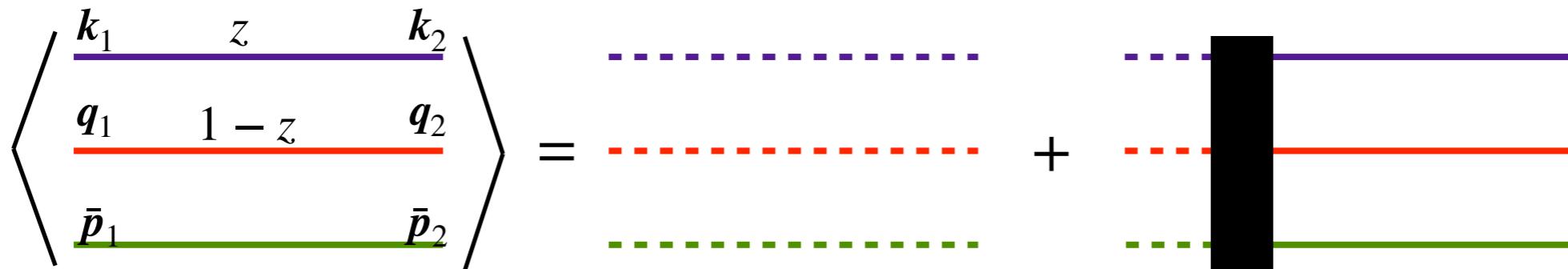


Color is trivial!

Phases: $-i \frac{k_2^2}{2zE} - i \frac{q_2^2}{2(1-z)E} + i \frac{\bar{p}_2^2}{2E} = -i \frac{((1-z)\mathbf{k}_2 - z\mathbf{q}_2)^2}{2z(1-z)E} = -i \frac{\mathbf{l}_2^2}{2z(1-z)E}$

Interaction: $\sigma(\mathbf{q}') \left[-\delta^{(2)}(\mathbf{l}_2 - \mathbf{l}_1 - \mathbf{q}') + \delta^{(2)}(\mathbf{l}_2 - \mathbf{l}_1 - (1-z)\mathbf{q}') + \delta^{(2)}(\mathbf{l}_2 - \mathbf{l}_1 + z\mathbf{q}') \right]$

Average of three propagators



Phases:

$$-i \frac{\mathbf{k}_2^2}{2zE} - i \frac{\mathbf{q}_2^2}{2(1-z)E} + i \frac{\bar{\mathbf{p}}_2^2}{2E} = -i \frac{((1-z)\mathbf{k}_2 - z\mathbf{q}_2)^2}{2z(1-z)E} = -i \frac{\mathbf{l}_2^2}{2z(1-z)E}$$

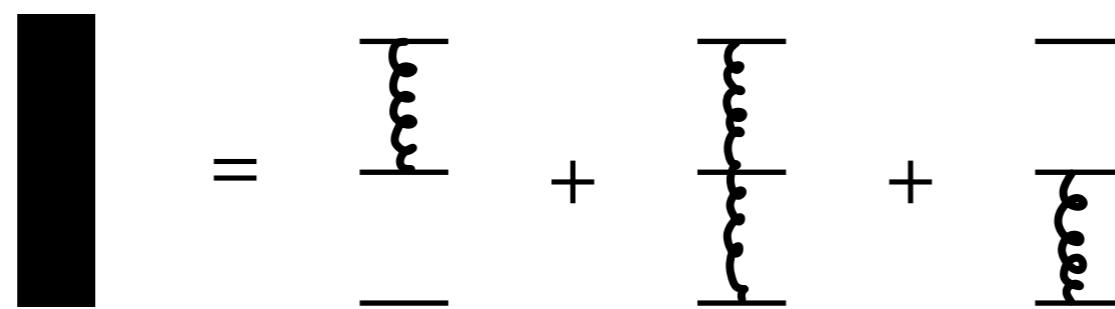
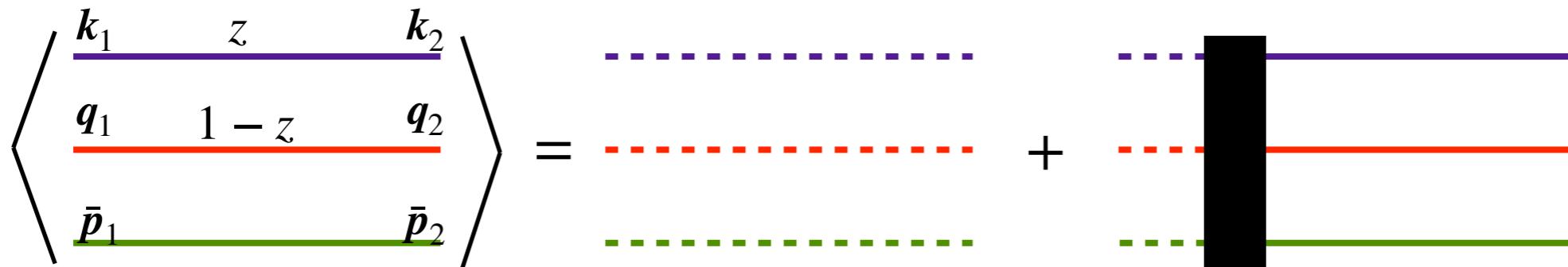
Interaction:

$$\sigma(\mathbf{q}') \left[-\delta^{(2)}(\mathbf{l}_2 - \mathbf{l}_1 - \mathbf{q}') + \delta^{(2)}(\mathbf{l}_2 - \mathbf{l}_1 - (1-z)\mathbf{q}') + \delta^{(2)}(\mathbf{l}_2 - \mathbf{l}_1 + z\mathbf{q}') \right]$$

$$\begin{aligned} \tilde{\mathcal{K}}^{(3)}(\mathbf{l}_2, t_2; \mathbf{l}_1, t_1; z) &= (2\pi)^2 \delta^{(2)}(\mathbf{l}_2 - \mathbf{l}_1) e^{-i \frac{\mathbf{l}_2^2}{2z(1-z)E} (t_2 - t_1)} \\ &\quad - \frac{1}{2} \int_{t_1}^{t_2} ds n(s) e^{-i \frac{\mathbf{l}_2^2}{2z(1-z)E} (t_2 - s)} \int_{\mathbf{l}'} \sigma^{(3)}(\mathbf{l}_2 - \mathbf{l}'; z) \tilde{\mathcal{K}}^{(3)}(\mathbf{l}', s; \mathbf{l}_1, t_1; z) \end{aligned}$$

$$\sigma^{(3)}(\mathbf{q}; z) = \frac{1}{2N_c} \left[(C_b + C_c - C_a) \sigma(\mathbf{q}) + (C_a + C_b - C_c) \frac{1}{(1-z)^2} \sigma \left(\frac{\mathbf{q}}{(1-z)} \right) - (C_a + C_c - C_b) \frac{1}{z^2} \sigma \left(\frac{\mathbf{q}}{z} \right) \right]$$

Average of three propagators



Color is trivial!

$$\text{Phases: } -i\frac{\mathbf{k}_2^2}{2zE} - i\frac{\mathbf{q}_2^2}{2(1-z)E} + i\frac{\bar{\mathbf{p}}_2^2}{2E} = -i\frac{((1-z)\mathbf{k}_2 - z\mathbf{q}_2)^2}{2z(1-z)E} = -i\frac{\mathbf{l}_2^2}{2z(1-z)E}$$

$$\text{Interaction: } \sigma(\mathbf{q}') \left[-\delta^{(2)}(\mathbf{l}_2 - \mathbf{l}_1 - \mathbf{q}') + \delta^{(2)}(\mathbf{l}_2 - \mathbf{l}_1 - (1-z)\mathbf{q}') + \delta^{(2)}(\mathbf{l}_2 - \mathbf{l}_1 + z\mathbf{q}') \right]$$

$$\begin{aligned} \partial_{t_2} \tilde{\mathcal{K}}^{(3)}(\mathbf{l}_2, t_2; \mathbf{l}_1, t_1; z) &= -i \frac{\mathbf{l}_2^2}{2z(1-z)E} \tilde{\mathcal{K}}^{(3)}(\mathbf{l}_2, t_2; \mathbf{l}_1, t_1; z) \\ &\quad - n(t_2) \int_{\mathbf{l}'} \sigma^{(3)}(\mathbf{l}_2 - \mathbf{l}'; z) \tilde{\mathcal{K}}^{(3)}(\mathbf{l}', t_2; \mathbf{l}_1, t_1; z) \end{aligned}$$

$$\sigma^{(3)}(\mathbf{q}; z) = \frac{1}{2N_c} \left[(C_b + C_c - C_a)\sigma(\mathbf{q}) + (C_a + C_b - C_c) \frac{1}{(1-z)^2} \sigma\left(\frac{\mathbf{q}}{(1-z)}\right) - (C_a + C_c - C_b) \frac{1}{z^2} \sigma\left(\frac{\mathbf{q}}{z}\right) \right]$$

Average of four propagators

$$\left\{ \begin{array}{c} k_1 \quad z \quad k_2 \\ \hline q_1 \quad 1-z \quad q_2 \\ \hline \bar{k}_1 \quad z \quad \bar{k}_2 \\ \hline \bar{q}_1 \quad 1-z \quad \bar{q}_2 \end{array} \right\} = \dots + \dots + \dots + \dots$$

The diagram shows a bracketed set of four propagators on the left, each with two horizontal lines and a central vertical line labeled with variables. To the right of an equals sign, there are four dashed lines (purple and red) representing the individual propagators. A plus sign follows, and then a large black rectangle is placed over the four dashed lines, representing their average.

$$\text{[Large Black Box]} = \text{[Diagram 1]} + \text{[Diagram 2]} + \text{[Diagram 3]} + \text{[Diagram 4]} + \text{[Diagram 5]} + \text{[Diagram 6]}$$

The large black box is decomposed into six smaller diagrams, each consisting of a horizontal line with a curly brace above it, representing a specific color component of the average propagator.

Color is NOT trivial!

- There are several overall singlets for all cases
 - ◆ For $q\bar{q}$ there are 2 singlets. $3 \otimes \bar{3} \otimes 3 \otimes \bar{3} = 1 \oplus 1 \oplus 8 \oplus \dots$
 - ◆ For qg there are 3 singlets. $3 \otimes 8 \otimes \bar{3} \otimes 8 = 1 \oplus 1 \oplus 1 \oplus 8 \oplus \dots$
 - ◆ For gg there are 8 singlets. $8 \otimes 8 \otimes 8 \otimes 8 = \dots$
- One must define an orthonormal basis for the singlet space, project all averages into that basis, and then calculate all possible transitions

Average of four propagators

$$\left\langle \begin{array}{ccc} \frac{k_1}{q_1} & z & \frac{k_2}{q_2} \\ \hline \frac{\bar{k}_1}{\bar{q}_1} & 1-z & \frac{\bar{k}_2}{\bar{q}_2} \end{array} \right\rangle$$

Average of four propagators

$$\left\langle \begin{array}{ccc} \underline{k_1} & z & \underline{k_2} \\ \underline{q_1} & 1-z & \underline{q_2} \\ \bar{k}_1 & z & \bar{k}_2 \\ \bar{q}_1 & 1-z & \bar{q}_2 \end{array} \right\rangle$$

Remember change of variables

$$l_i = (1 - z)k_i - zq_i \quad \bar{l}_i = (1 - z)\bar{k}_i - z\bar{q}_i$$
$$\Delta p_i = k_i + q_i - \bar{k}_i - \bar{q}_i \quad P_i = \frac{1}{2}(k_i + q_i + \bar{k}_i + \bar{q}_i)$$

Average of four propagators

$$\left\{ \begin{array}{ccc} \underline{k_1} & z & \underline{k_2} \\ \underline{q_1} & 1-z & \underline{q_2} \\ \underline{\bar{k}_1} & z & \underline{\bar{k}_2} \\ \underline{\bar{q}_1} & 1-z & \underline{\bar{q}_2} \end{array} \right\}$$

Remember change of variables

$$l_i = (1 - z)k_i - zq_i \quad \bar{l}_i = (1 - z)\bar{k}_i - z\bar{q}_i$$
$$\Delta p_i = k_i + q_i - \bar{k}_i - \bar{q}_i \quad P_i = \frac{1}{2}(k_i + q_i + \bar{k}_i + \bar{q}_i)$$

We always take $\Delta p_i = 0$ and integrate over P_i

Average of four propagators

$$\left\langle \begin{array}{ccc} \underline{k_1} & z & \underline{k_2} \\ \underline{q_1} & 1-z & \underline{q_2} \\ \bar{k}_1 & z & \bar{k}_2 \\ \bar{q}_1 & 1-z & \bar{q}_2 \end{array} \right\rangle$$

Remember change of variables

$$\begin{aligned} l_i &= (1-z)\mathbf{k}_i - z\mathbf{q}_i & \bar{l}_i &= (1-z)\bar{\mathbf{k}}_i - z\bar{\mathbf{q}}_i \\ \Delta\mathbf{p}_i &= \mathbf{k}_i + \mathbf{q}_i - \bar{\mathbf{k}}_i - \bar{\mathbf{q}}_i & P_i &= \frac{1}{2}(\mathbf{k}_i + \mathbf{q}_i + \bar{\mathbf{k}}_i + \bar{\mathbf{q}}_i) \end{aligned}$$

We always take $\Delta\mathbf{p}_i = 0$ and integrate over \mathbf{P}_i

$$\left\langle \mathcal{G}_{R_b}(\mathbf{k}_2; \mathbf{k}_1; zE) \otimes \mathcal{G}_{R_c}(\mathbf{q}_2; \mathbf{q}_1; (1-z)E) \otimes \mathcal{G}_{R_b}^\dagger(\bar{\mathbf{k}}_1; \bar{\mathbf{k}}_2; zE) \otimes \mathcal{G}_{R_c}^\dagger(\bar{\mathbf{q}}_1; \bar{\mathbf{q}}_1; (1-z)E) \right\rangle \Big|_{(t_2, t_1)}$$

$$\downarrow \\ S(l_2, \bar{l}_2, t_2; l_1, \bar{l}_1, t_1; z)$$

Average of four propagators

$$\left\langle \begin{array}{ccc} \underline{k_1} & z & \underline{k_2} \\ \underline{q_1} & 1-z & \underline{q_2} \\ \bar{k}_1 & z & \bar{k}_2 \\ \bar{q}_1 & 1-z & \bar{q}_2 \end{array} \right\rangle$$

Remember change of variables

$$\begin{aligned} \boldsymbol{l}_i &= (1-z)\boldsymbol{k}_i - z\boldsymbol{q}_i & \bar{\boldsymbol{l}}_i &= (1-z)\bar{\boldsymbol{k}}_i - z\bar{\boldsymbol{q}}_i \\ \Delta\boldsymbol{p}_i &= \boldsymbol{k}_i + \boldsymbol{q}_i - \bar{\boldsymbol{k}}_i - \bar{\boldsymbol{q}}_i & \boldsymbol{P}_i &= \frac{1}{2}(\boldsymbol{k}_i + \boldsymbol{q}_i + \bar{\boldsymbol{k}}_i + \bar{\boldsymbol{q}}_i) \end{aligned}$$

We always take $\Delta\boldsymbol{p}_i = 0$ and integrate over \boldsymbol{P}_i

$$\left\langle \mathcal{G}_{R_b}(\boldsymbol{k}_2; \boldsymbol{k}_1; zE) \otimes \mathcal{G}_{R_c}(\boldsymbol{q}_2; \boldsymbol{q}_1; (1-z)E) \otimes \mathcal{G}_{R_b}^\dagger(\bar{\boldsymbol{k}}_1; \bar{\boldsymbol{k}}_2; zE) \otimes \mathcal{G}_{R_c}^\dagger(\bar{\boldsymbol{q}}_1; \bar{\boldsymbol{q}}_1; (1-z)E) \right\rangle \Big|_{(t_2, t_1)}$$



$$S(\boldsymbol{l}_2, \bar{\boldsymbol{l}}_2, t_2; \boldsymbol{l}_1, \bar{\boldsymbol{l}}_1, t_1; z)$$

Defining $|s_i\rangle$ as an orthonormal basis for the singlet space, take $S_{ij} = \langle s_i | S | s_j \rangle$

Average of four propagators

$$\left\langle \begin{array}{ccc} \underline{k_1} & z & \underline{k_2} \\ \underline{q_1} & 1-z & \underline{q_2} \\ \bar{k}_1 & z & \bar{k}_2 \\ \bar{q}_1 & 1-z & \bar{q}_2 \end{array} \right\rangle$$

Remember change of variables

$$l_i = (1 - z)k_i - zq_i \quad \bar{l}_i = (1 - z)\bar{k}_i - z\bar{q}_i$$

$$\Delta p_i = k_i + q_i - \bar{k}_i - \bar{q}_i \quad P_i = \frac{1}{2}(k_i + q_i + \bar{k}_i + \bar{q}_i)$$

We always take $\Delta p_i = 0$ and integrate over P_i

$$\left\langle \mathcal{G}_{R_b}(\mathbf{k}_2; \mathbf{k}_1; zE) \otimes \mathcal{G}_{R_c}(\mathbf{q}_2; \mathbf{q}_1; (1-z)E) \otimes \mathcal{G}_{R_b}^\dagger(\bar{\mathbf{k}}_1; \bar{\mathbf{k}}_2; zE) \otimes \mathcal{G}_{R_c}^\dagger(\bar{\mathbf{q}}_1; \bar{\mathbf{q}}_1; (1-z)E) \right\rangle \Big|_{(t_2, t_1)}$$

$$\downarrow$$

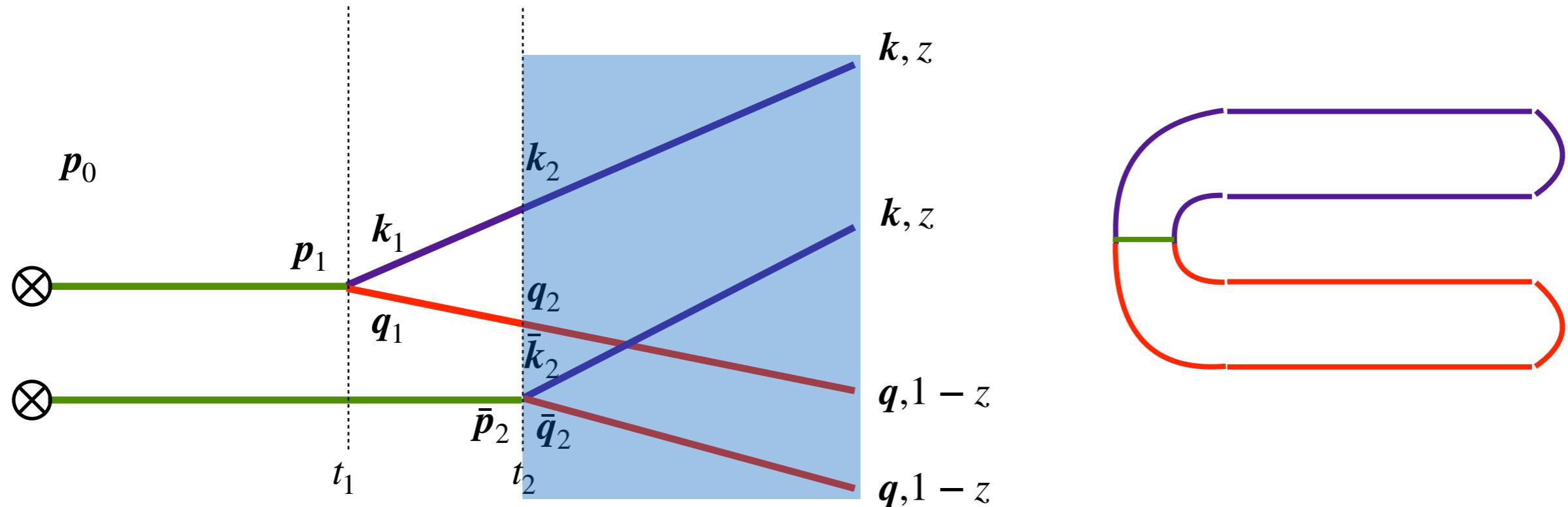
$$S(l_2, \bar{l}_2, t_2; l_1, \bar{l}_1, t_1; z)$$

Defining $|s_i\rangle$ as an orthonormal basis for the singlet space, take $S_{ij} = \langle s_i | S | s_j \rangle$

$$\partial_{t_1} S_{ij}(l_2, \bar{l}_2, t_2; l_1, \bar{l}_1, t_1; z) = \frac{i \left(l_1^2 - \bar{l}_1^2 \right)}{2z(1-z)E} S_{ij}(l_2, \bar{l}_2, t_2; l_1, \bar{l}_1, t_1; z)$$

$$- \int_{l'_1, \bar{l}'_1} S_{ik}(l_2, \bar{l}_2, t_2; l'_1, \bar{l}'_1, t_1; z) \mathcal{T}_{kj}(l'_1, \bar{l}'_1; l_1, \bar{l}_1; t_1)$$

Average of four propagators



$$\tilde{\mathcal{S}}^{(4)}(\mathbf{l}, L; \mathbf{l}_2, \bar{\mathbf{l}}_2, t_2; z) = c_j \mathcal{S}_{1j}(\mathbf{l}, \mathbf{l}, L; \mathbf{l}_2, \bar{\mathbf{l}}_2, t_2; z)$$

Summary

- Double differential cross sections can be simplified to calculable objects which still capture the dynamics of the splitting
- Calculating the Lund plane for in medium splitting reduced to solving a set of coupled first order differential equations
- Very demanding in terms of computational resources. We are currently working on making the code more efficient
- Full calculation will provide the building block for jet substructure studies

Thank you!