A lattice approach to hot neutrons & cold atoms

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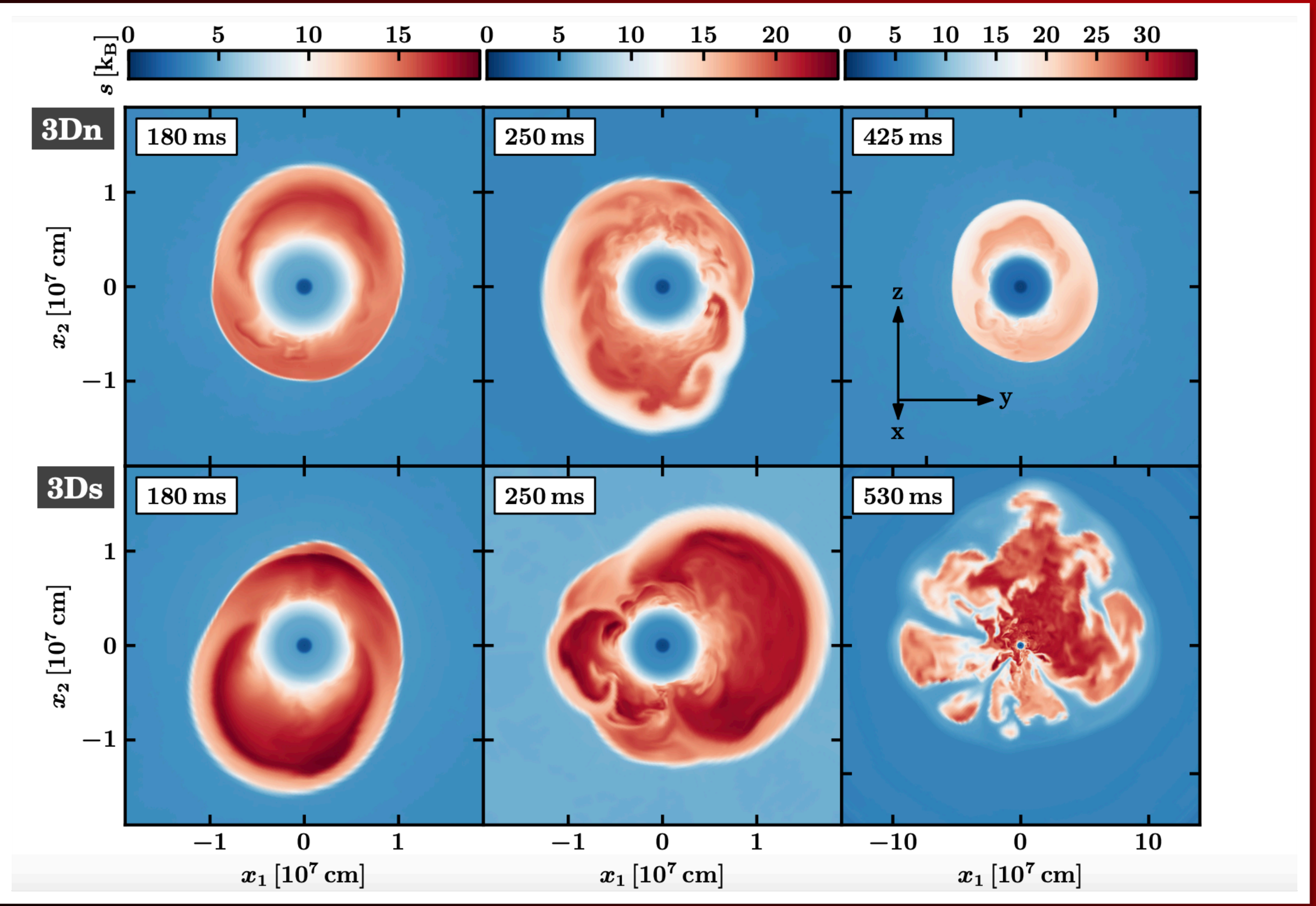
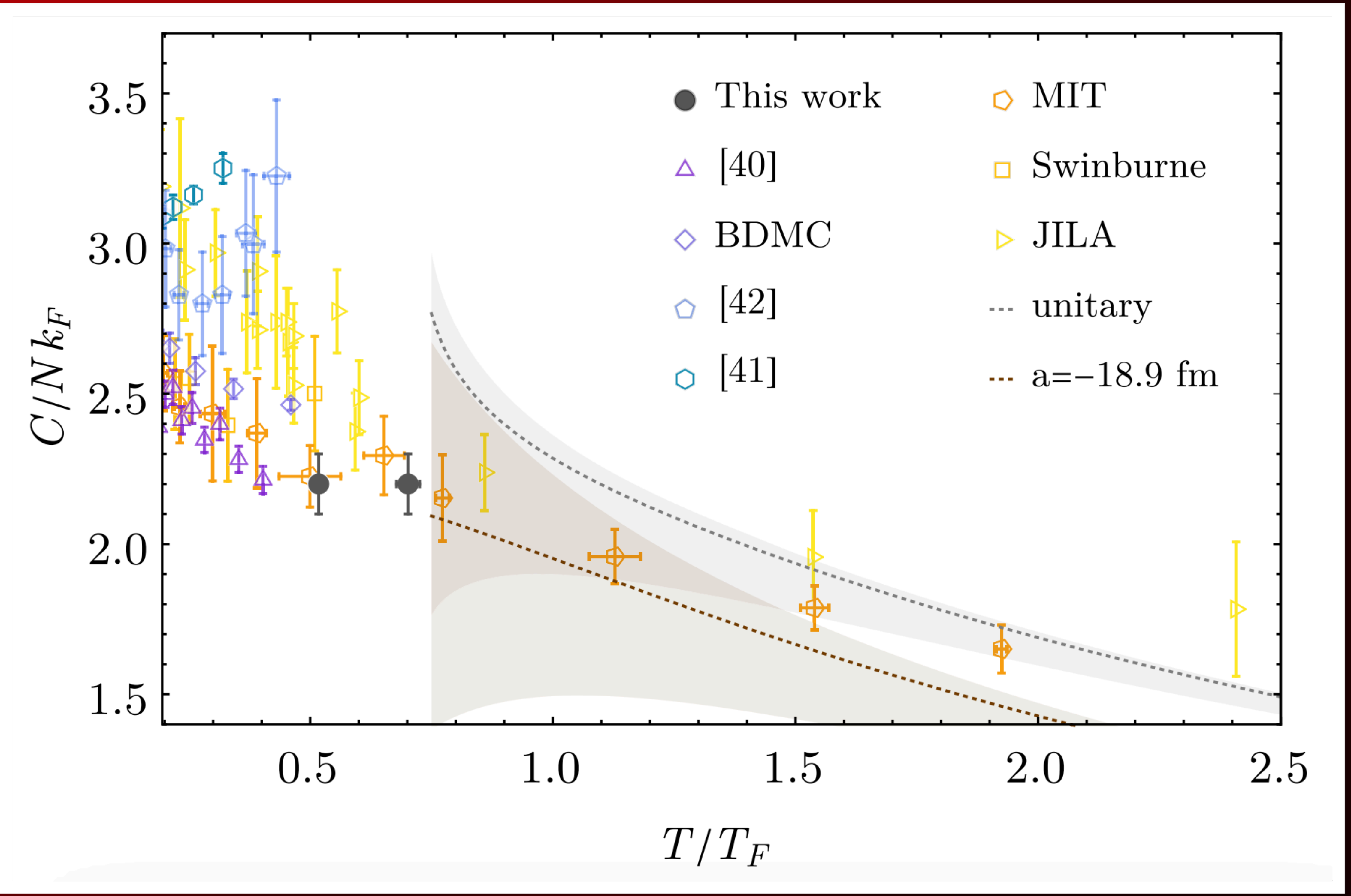
Main Idea:

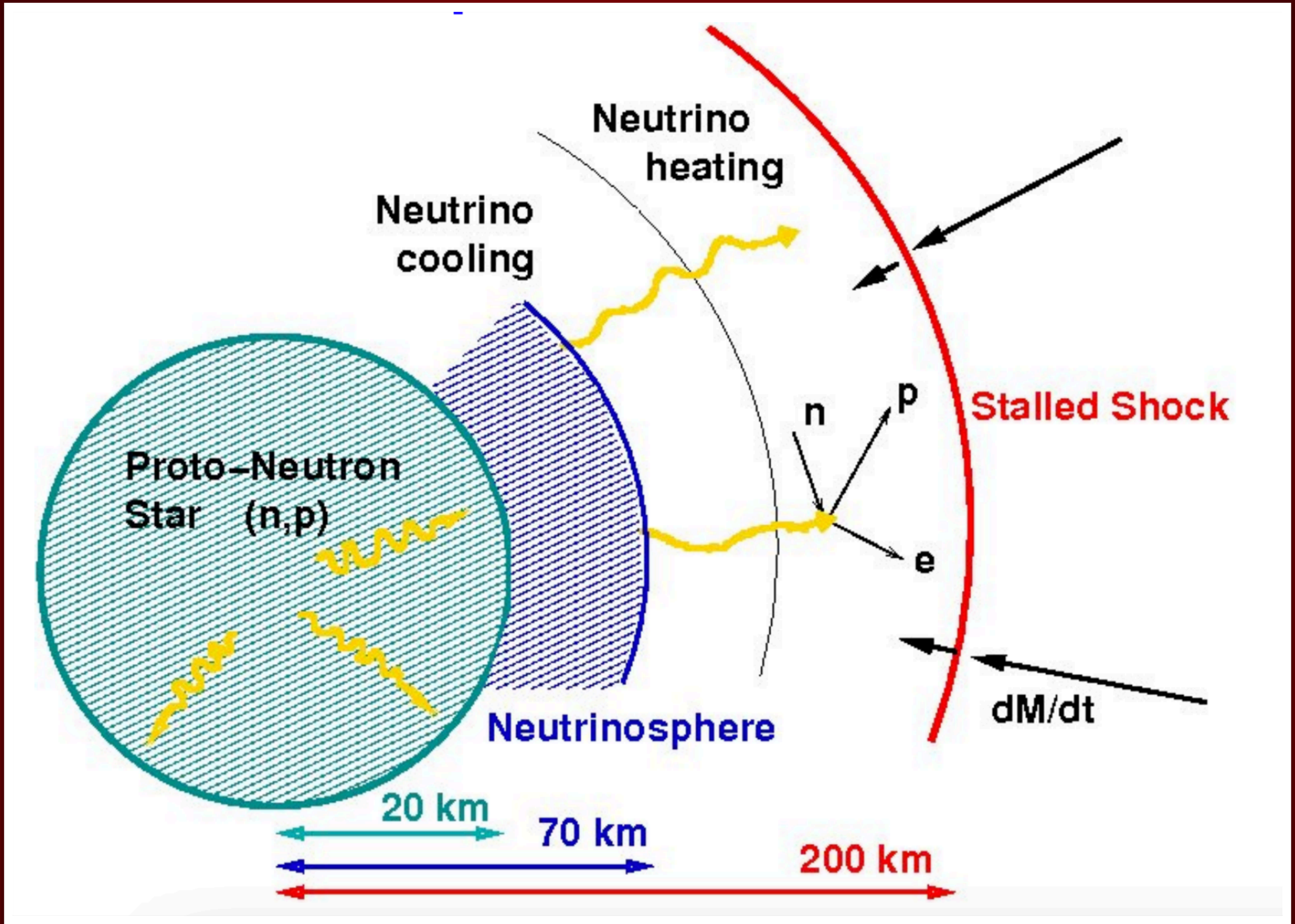
Obtain precision physics where it counts



Melson, Janka et. al., ApJL (2015)



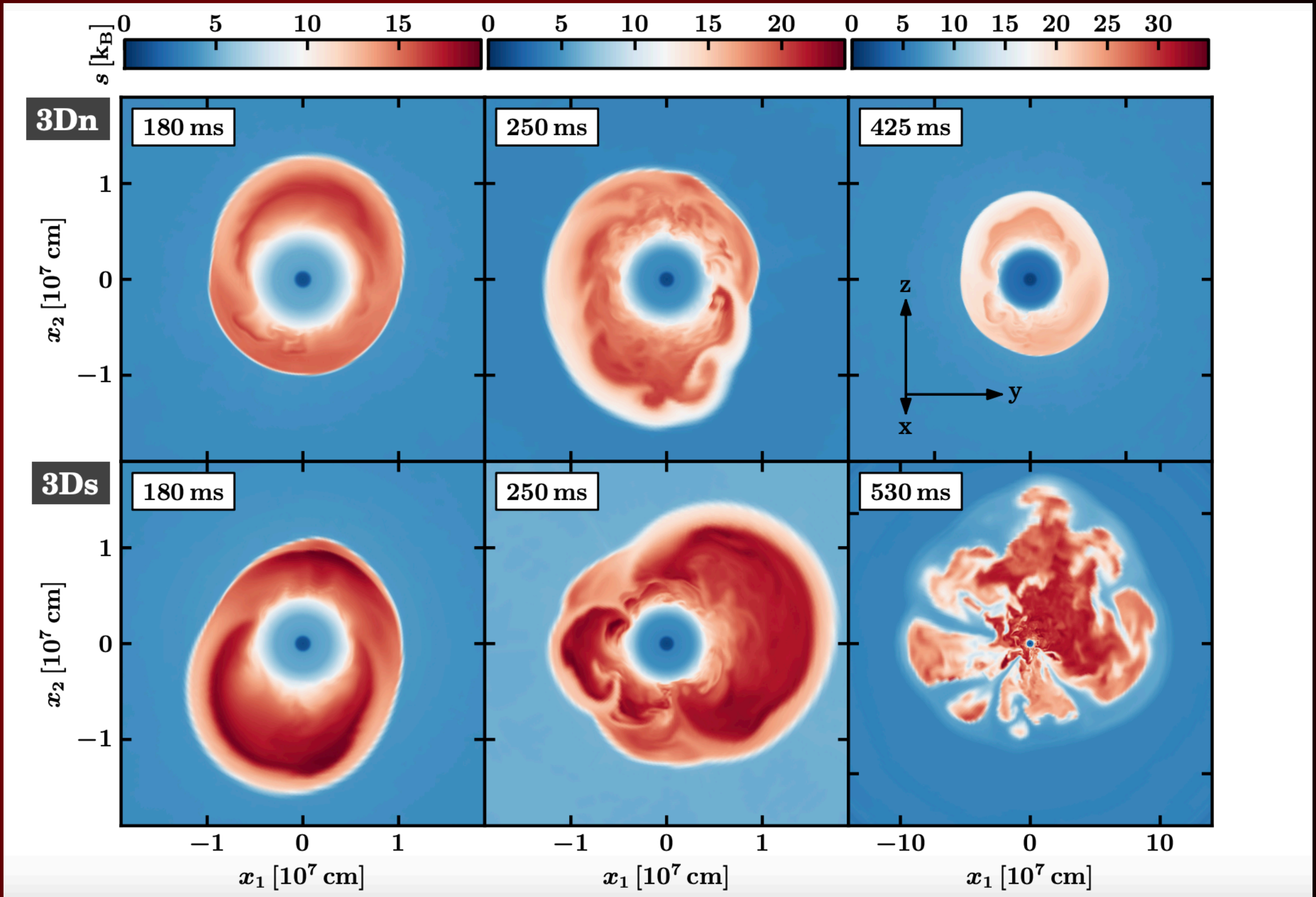
Berkowitz, NCW, et. al. PRL (2022)


Neutrino
heating
Neutrino
cooling
Proto-Neutron
Star (n,p)
n
p
e
Stalled Shock
Neutrinosphere
dM/dt
20 km
70 km
200 km

Dighe (2016)

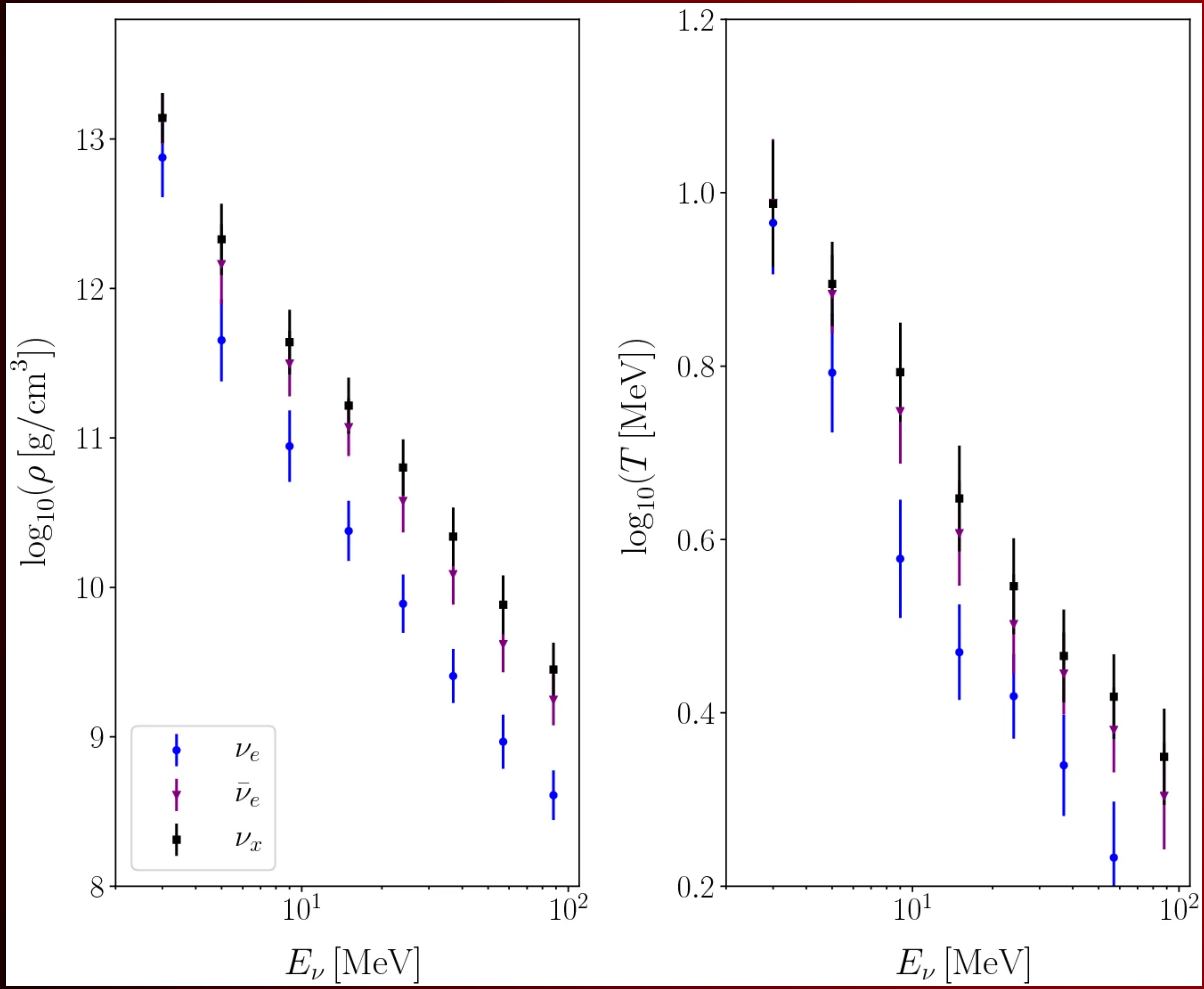
Neglect many-body

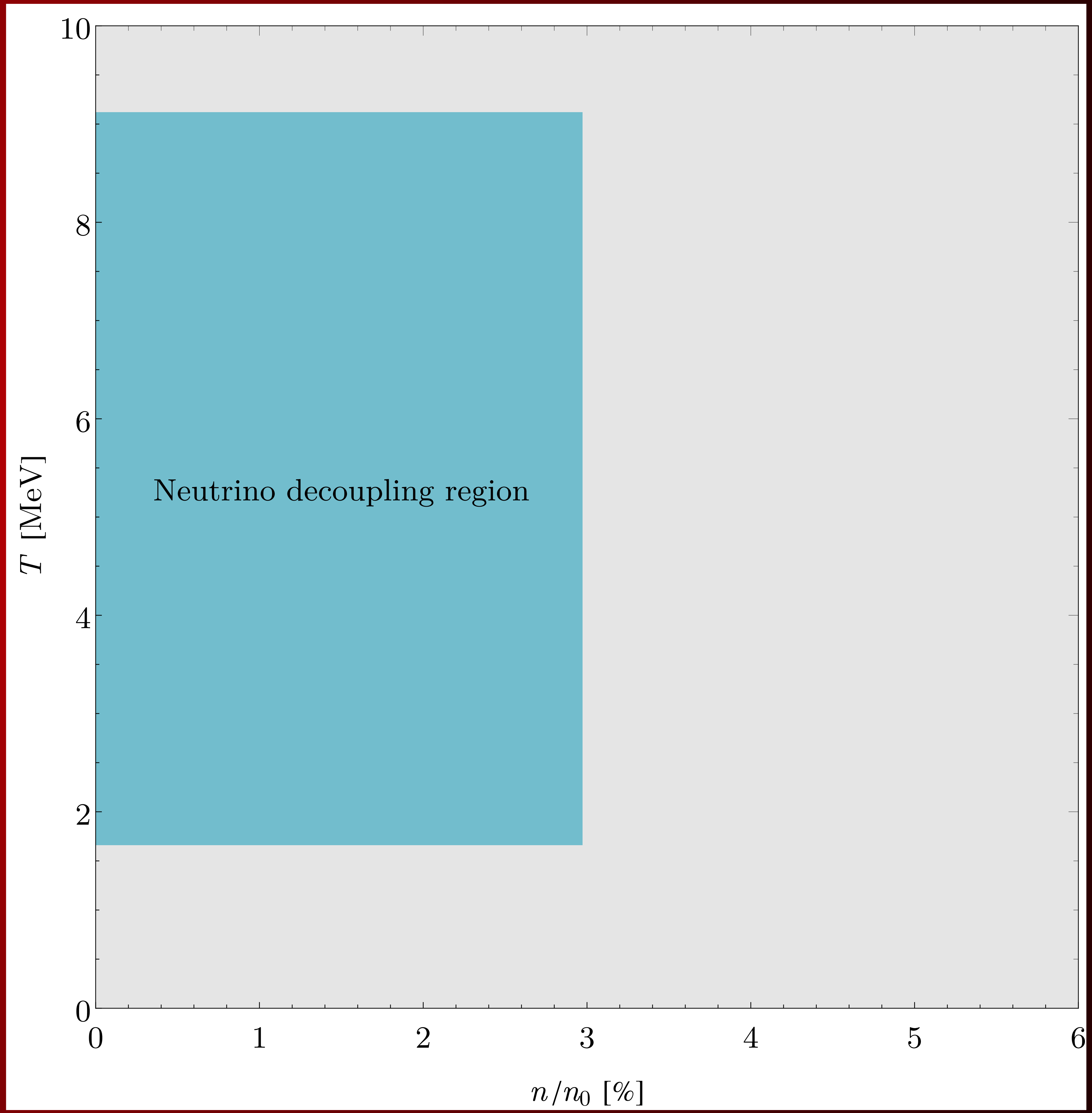
Include many-body



Melson, Janka et. al., ApJL (2015)

The Neutrinosphere





Radice et. al. Eur. Phys. J. (2020)

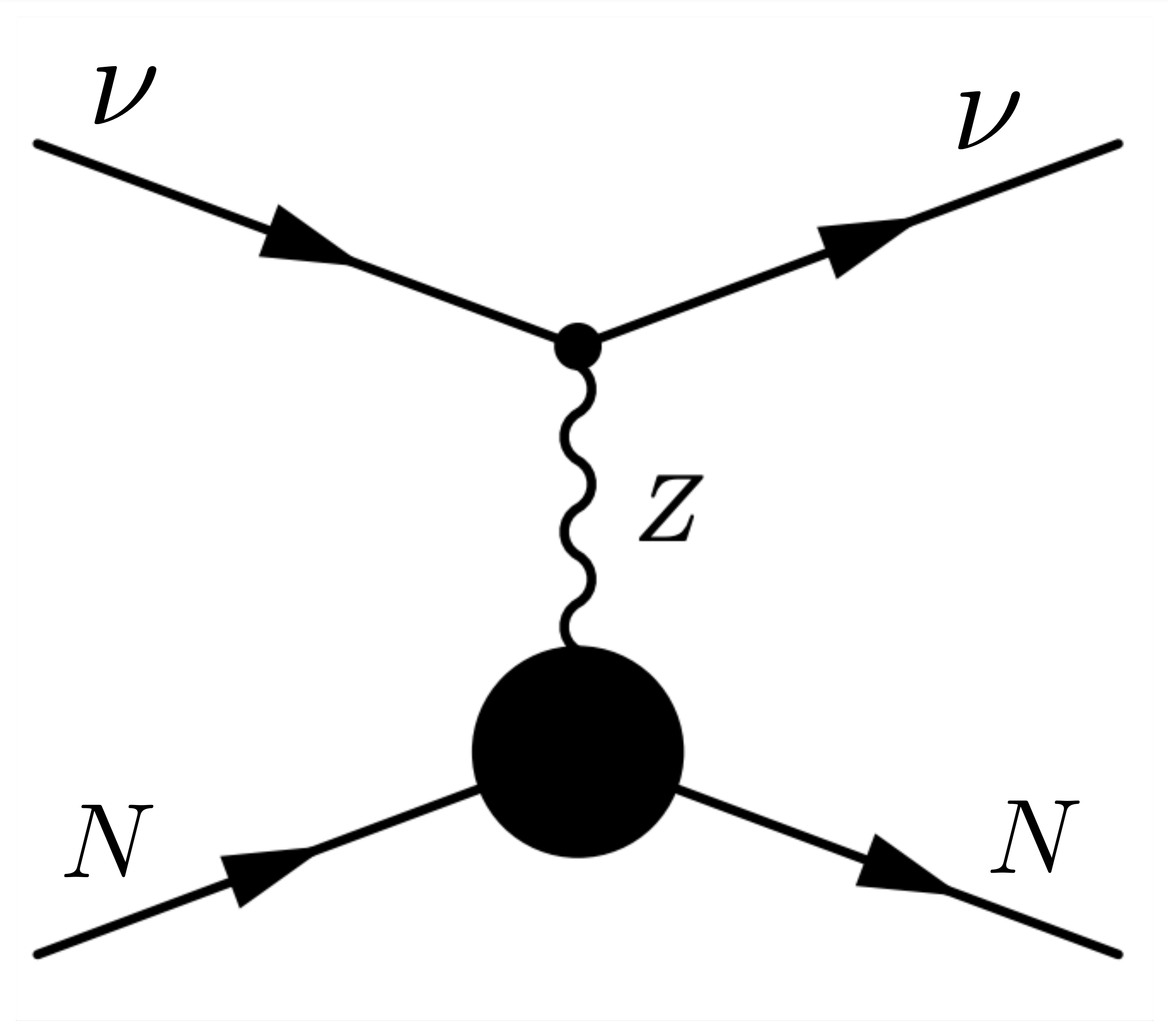
Neutrino Scattering & Structure Factors

Pethick & Iwamoto PRD (1982) , Horowitz & Schwenk PLB (2006)

$$\frac{d\Gamma(E_\nu)}{d\cos\theta} = \frac{G_F^2 E_\nu^2}{4\pi^2}\left[c_V^2(1+\cos\theta)S_V(\mathbf{q}) + c_A^2(3-\cos\theta)S_A(\mathbf{q})\right]$$

$$S_V(\mathbf{q}) = \int d^3x\, \mathrm{e}^{-i\mathbf{q}\cdot\mathbf{x}} \langle \delta n(\mathbf{x},0)\delta n(\mathbf{0},0)\rangle$$

$$S_A(\mathbf{q}) = \int d^3x\, \mathrm{e}^{-i\mathbf{q}\cdot\mathbf{x}} \langle \delta s(\mathbf{x},0)\delta s(\mathbf{0},0)\rangle$$



Pionless EFT:

Bedaque & van Kolck Annu. Rev. Nucl. Part. Sci. (2002)

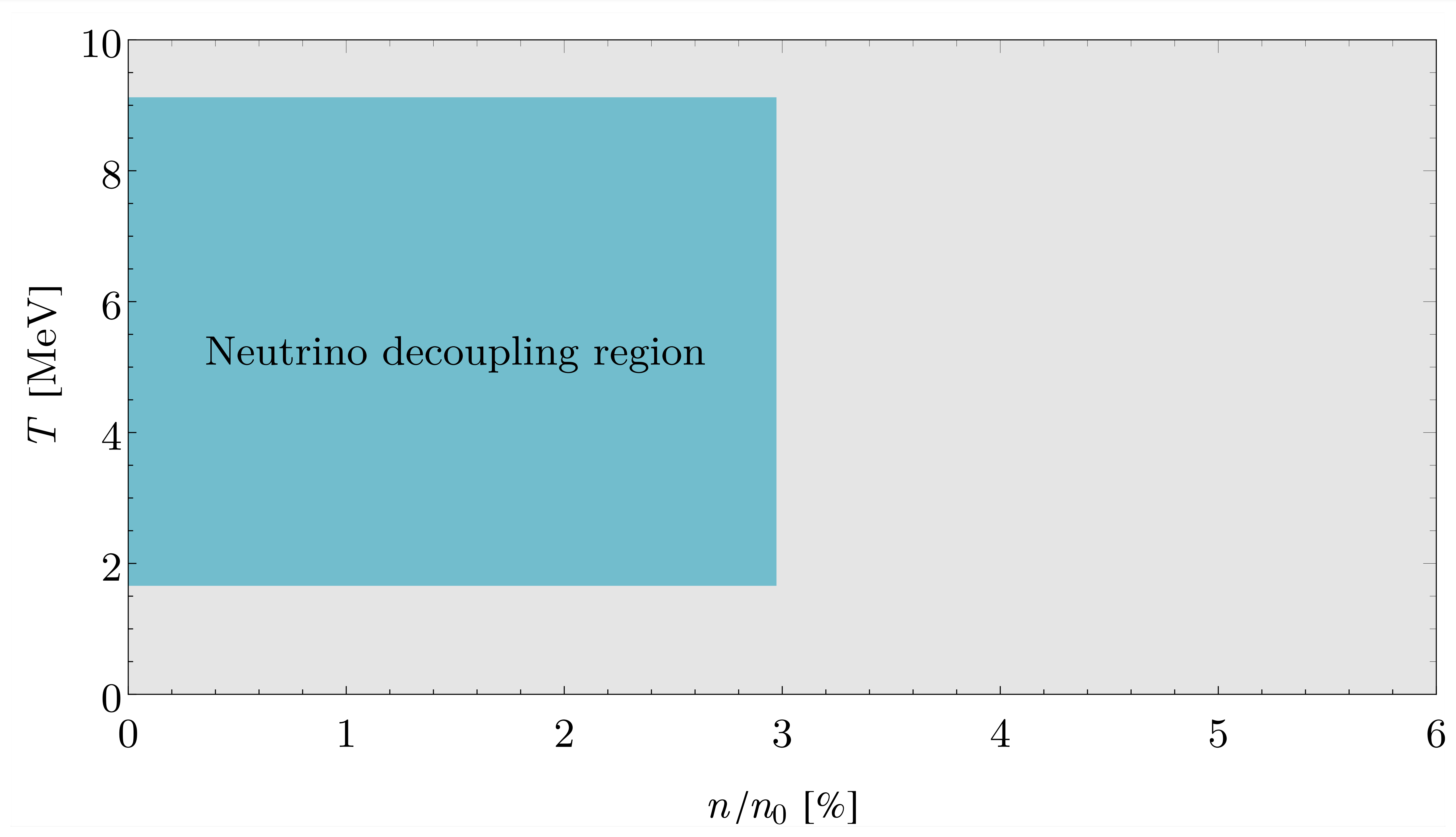
$$H = \int d^3x \; \frac{|\nabla\psi|^2}{2M} + C_0\left(\psi^\dagger\psi\right)^2 + C_2\left(\psi^\dagger\psi\right)\nabla^2\left(\psi^\dagger\psi\right) + \ldots$$

Converges for $k < m_\pi$

$$\Updownarrow$$

$$T \lesssim 10 \text{ MeV}$$

$$n \lesssim 6\% n_0$$



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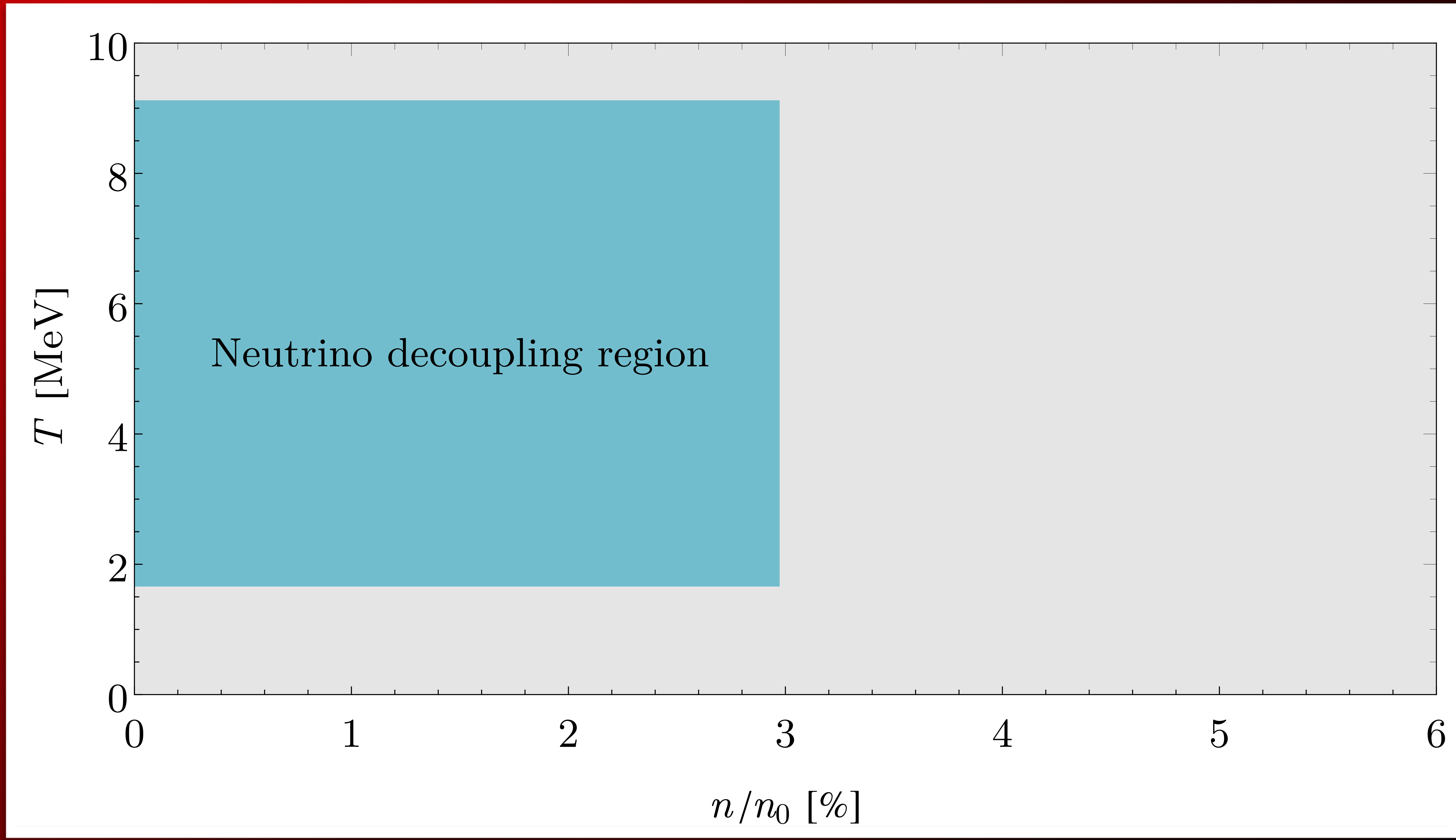


Lattice Methods

Goal: $\mathrm{tr}\left(\mathcal{O}\ \rho\right)$, where $\rho = \dfrac{\mathrm{e}^{-\beta(H-\mu N)}}{\mathrm{tr}\left(\mathrm{e}^{-\beta(H-\mu N)}\right)}$

Step 1: Discretize to finite cubic lattice

$$H = \sum_x \Delta x^3 \left\{ \frac{|\nabla\psi(x)|^2}{2M} + C_0\left(\psi^\dagger\psi\right)^2 \right\}$$

Lattice Methods

Goal: $\mathrm{tr}\left(\mathcal{O}\ \rho\right)$, where $\rho = \dfrac{\mathrm{e}^{-\beta(H-\mu N)}}{\mathrm{tr}\left(\mathrm{e}^{-\beta(H-\mu N)}\right)}$

Step 2: Discretize in “time”

$$\mathrm{tr}\left(\mathrm{e}^{-\beta(H-\mu N)}\right) = \mathrm{tr}\left(\mathrm{e}^{-\Delta t(K-\mu N)}\mathrm{e}^{-\Delta t V}\ldots\mathrm{e}^{-\Delta t(K-\mu N)}\mathrm{e}^{-\Delta t V}\right) + \mathcal{O}(\Delta t^2)$$

where $\beta = \Delta t N_t$

Lattice Methods

Goal: $\mathrm{tr}\big(\mathcal{O}\ \rho\big)$, where $\rho = \dfrac{\mathrm{e}^{-\beta(H-\mu N)}}{\mathrm{tr}\left(\mathrm{e}^{-\beta(H-\mu N)}\right)}$

Step 3: Hubbard-Stratanovich

$$\mathrm{tr}\left(\mathrm{e}^{-\Delta t(K-\mu N)}\mathrm{e}^{-\Delta t V}\ldots\mathrm{e}^{-\Delta t(K-\mu N)}\mathrm{e}^{-\Delta t V}\right) = \int \prod_{xt} dA_{xt}\mathrm{e}^{-\sum_{xt} A_{xt} M_{xy} A_{yt}}\ \mathrm{det} D(A)$$

Lattice Methods

Goal: $\mathrm{tr}\big(\mathcal{O}\ \rho\big)$, where $\rho = \dfrac{\mathrm{e}^{-\beta(H-\mu N)}}{\mathrm{tr}\big(\mathrm{e}^{-\beta(H-\mu N)}\big)}$

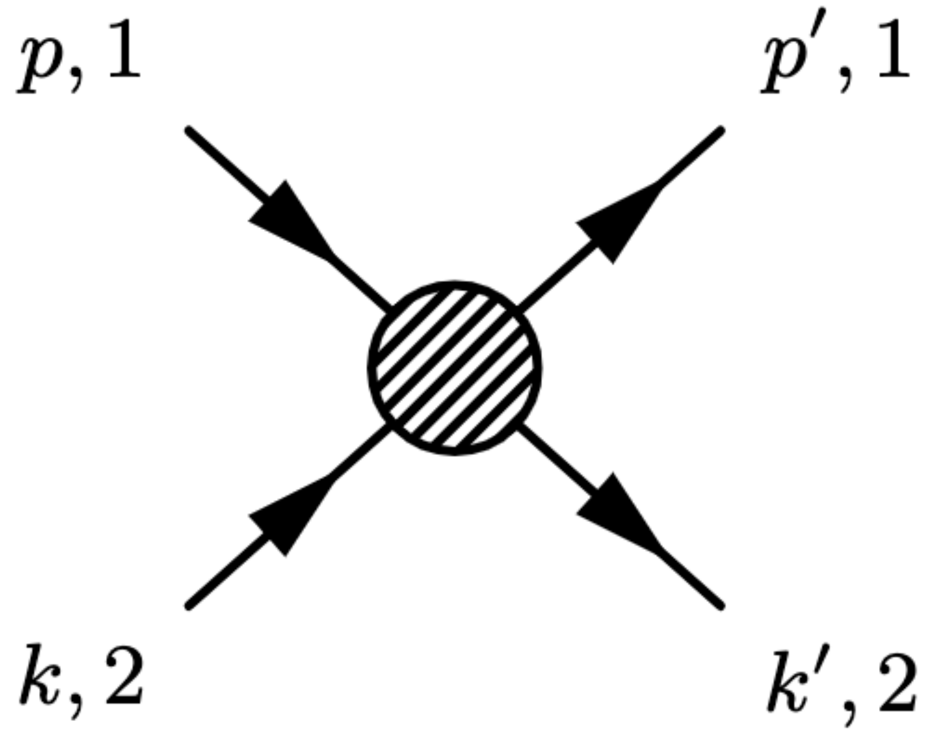
Step 4: Monte Carlo

$$\frac{\mathrm{tr}(\mathcal{O}\prod_t \mathrm{e}^{-\Delta t V}\mathrm{e}^{-\Delta t(K-\mu N)})}{\mathrm{tr}(\prod_t \mathrm{e}^{-\Delta t V}\mathrm{e}^{-\Delta t(K-\mu N)})} = \int DA\; p(A)\mathcal{O}(A)$$

where $p(A) = \dfrac{\mathrm{e}^{-\sum_{xt} A_{xt}M_{xy}A_{yt}}\ \det D(A)}{\int DA\ \mathrm{e}^{-\sum_{xt} A_{xt}M_{xy}A_{yt}}\ \det D(A)}$

Renormalization

Kaplan, Savage & Wise (1998), Bedaque & van Kolck (2002)



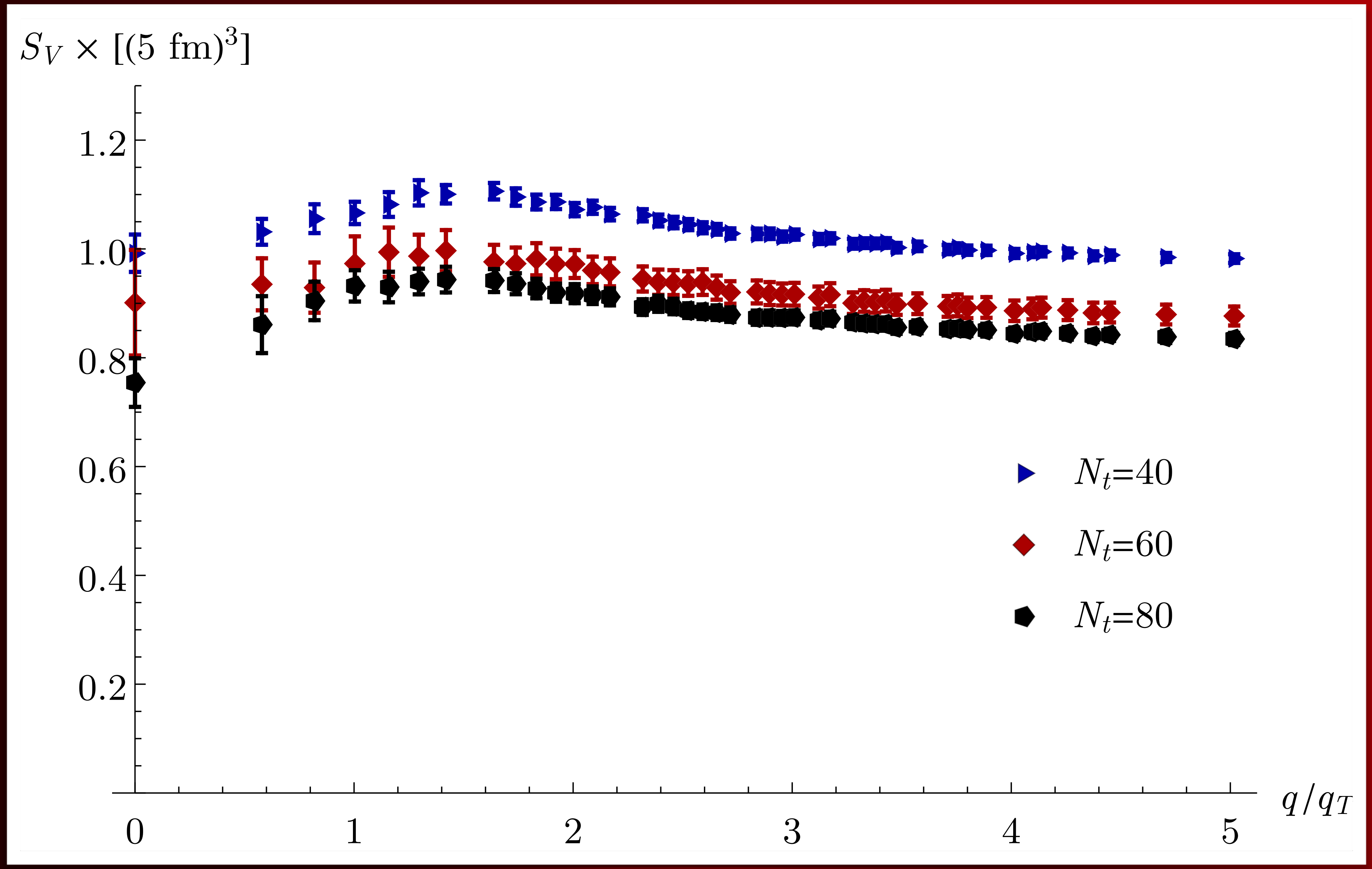
$$= \left(\frac{\Delta x}{M C_0} + \mathcal{P} \int \frac{d^3 q}{(2\pi)^3} \frac{1}{q^2 - E M \Delta x^2} + i \frac{\sqrt{E M \Delta x^2}}{4\pi} \right)^{-1}$$

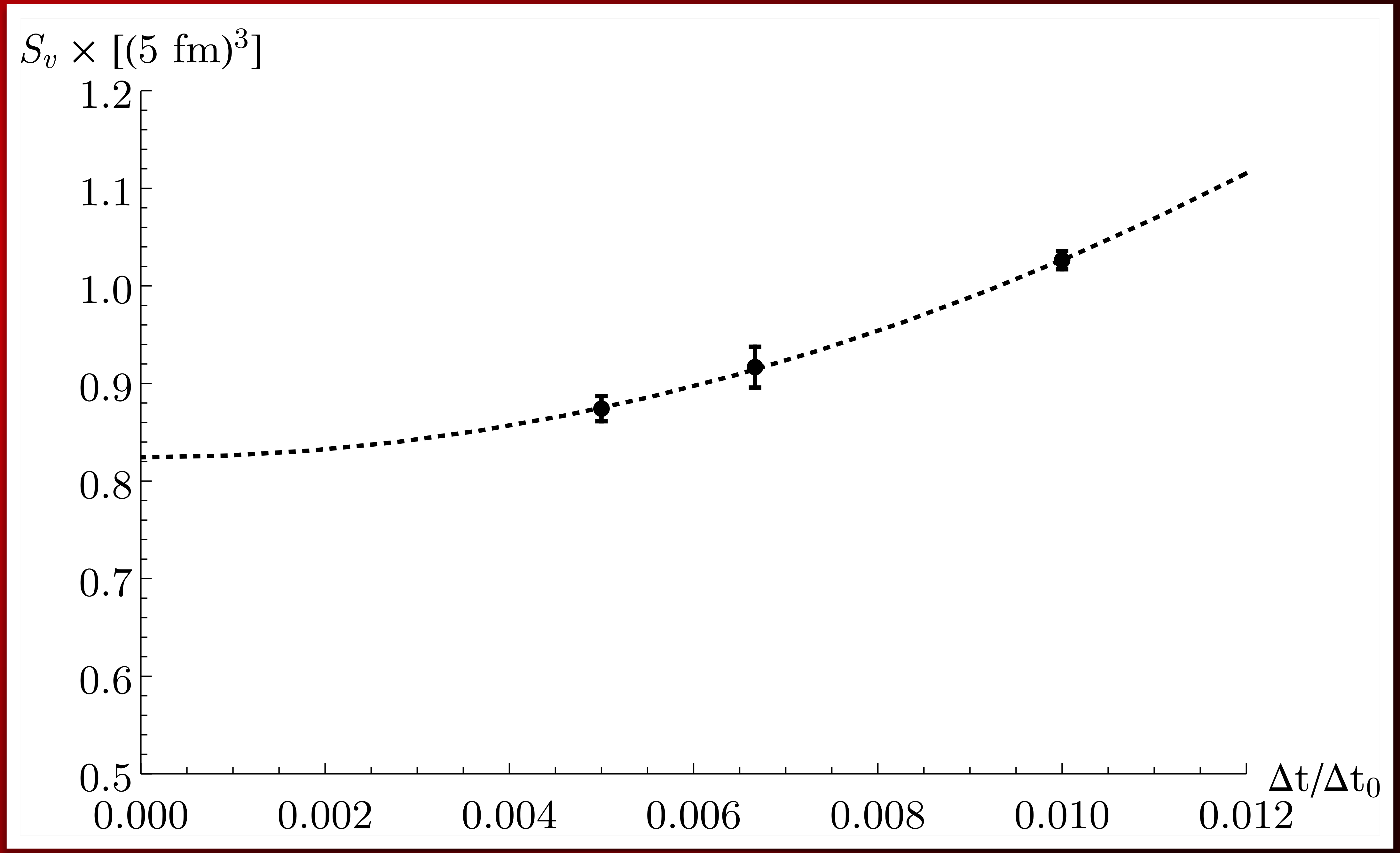
$$iA = \frac{4\pi/M}{k \cot\delta(k) - ik} = \frac{4\pi/M}{(-1/a + rk^2/2 + \ldots) - ik}$$

$$\Longrightarrow \quad C_0(\Delta x) = -\frac{\Delta x}{M}\left(\alpha - \frac{\Delta x}{4\pi a}\right)^{-1}$$

Strategy

1. $\Delta t \to 0$ first

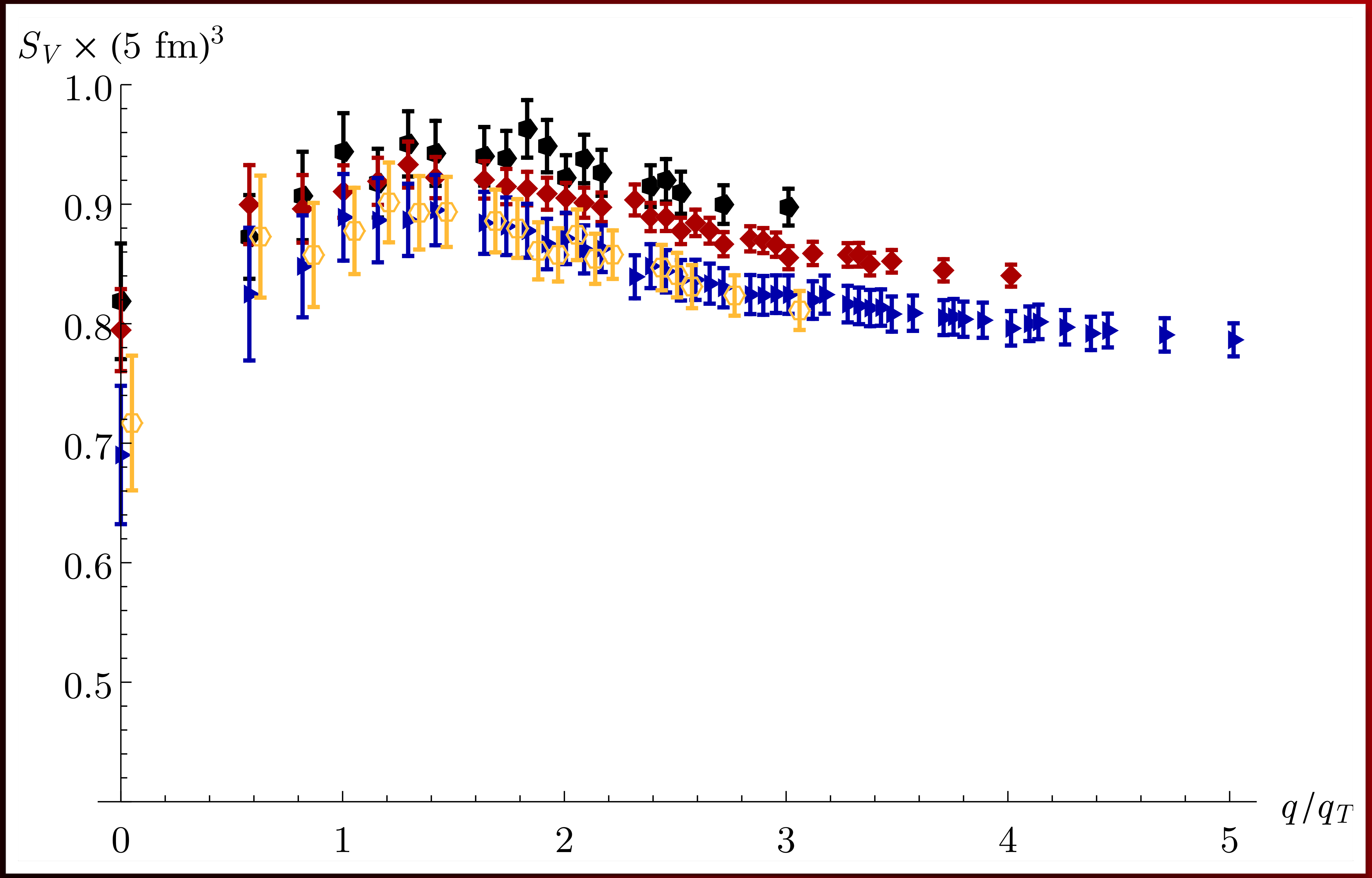


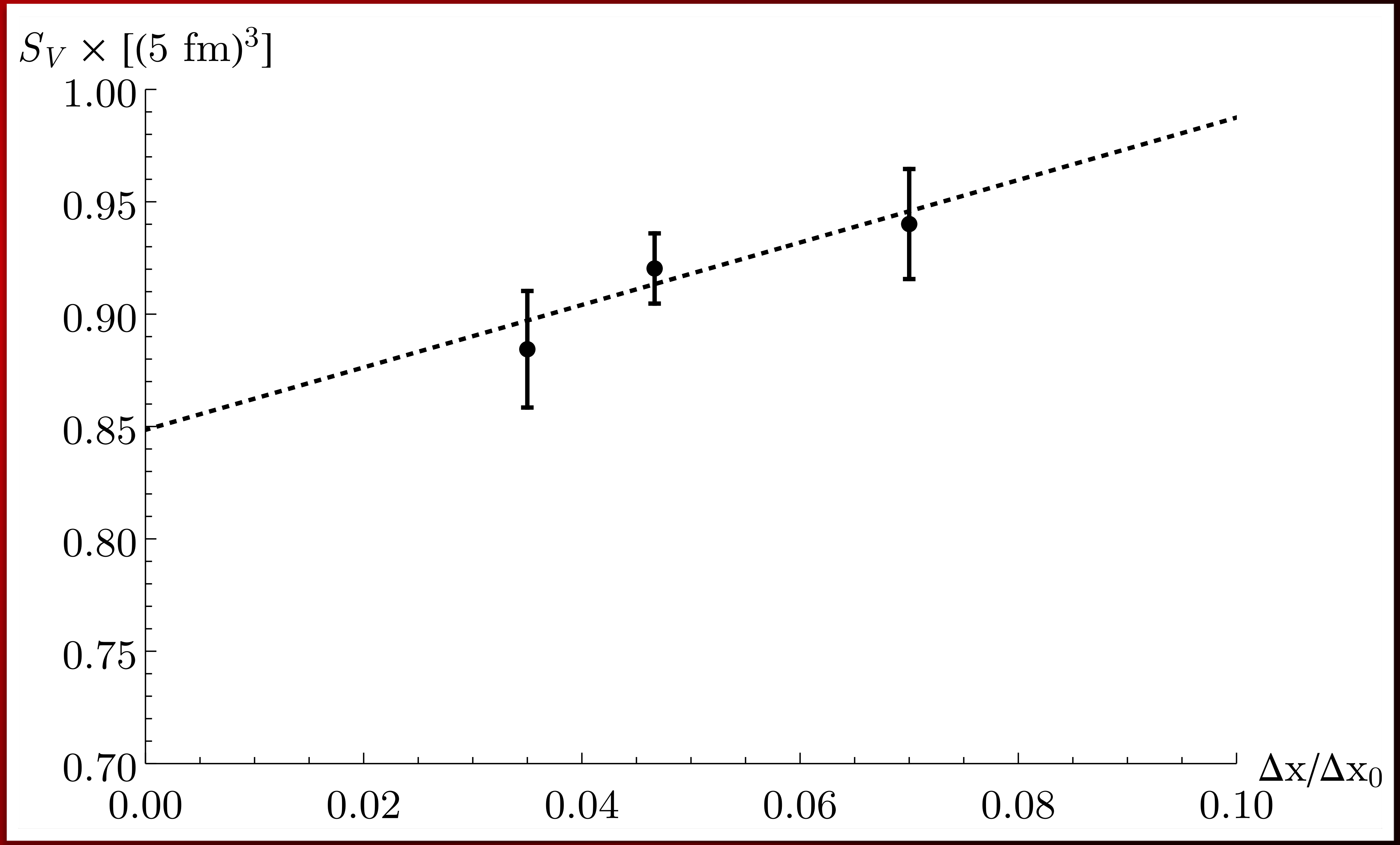


Identical scaling demonstrated by Alhassid et. al., PRL (2020)

Strategy

2. $\Delta x \to 0$ second

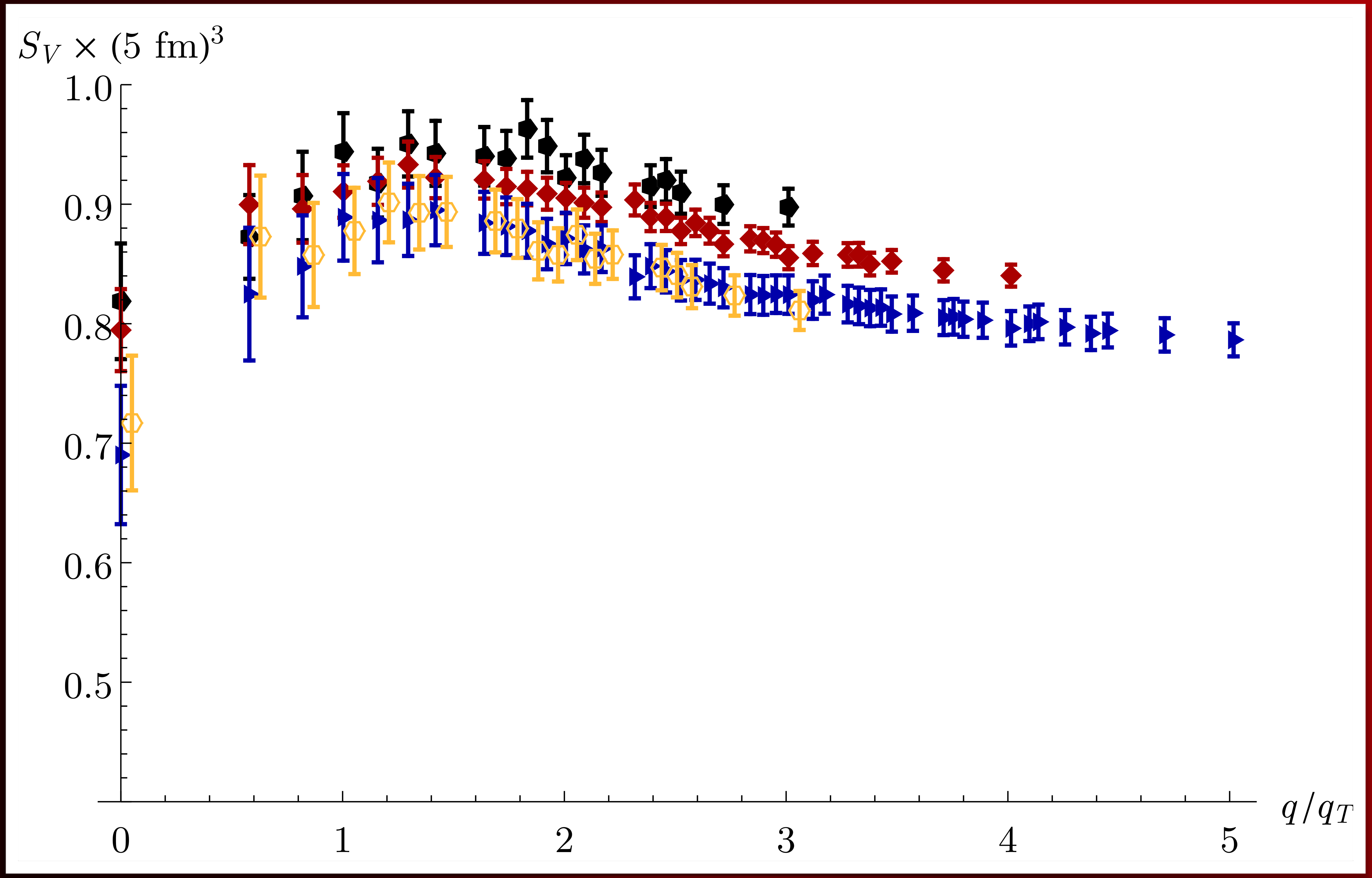


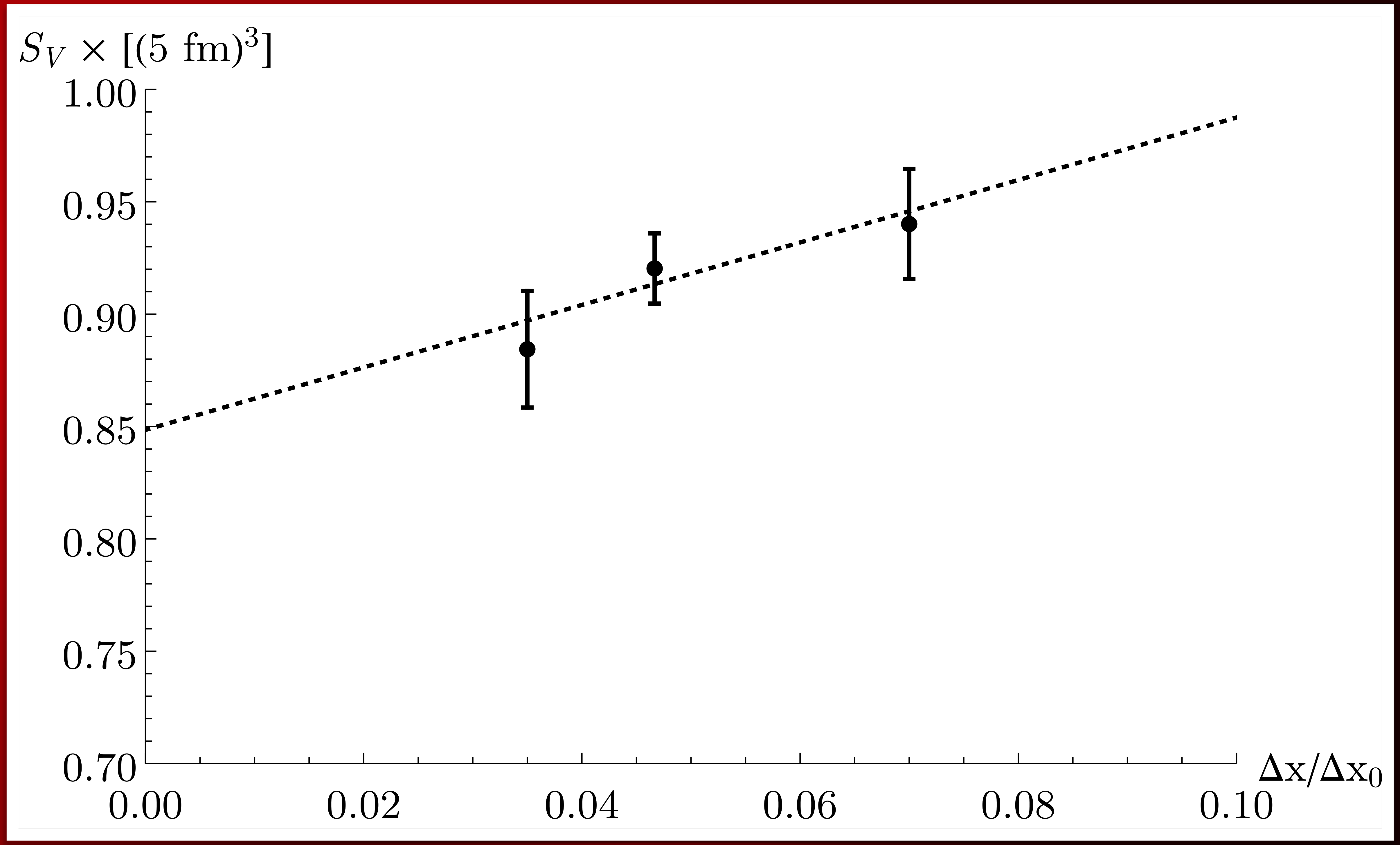


Identical scaling demonstrated by Alhassid et. al., PRL (2020)

Strategy

3. $V \to \infty$ third





Structure Factors

Berkowitz, NCW, et. al. PRL (2022)
NCW, et. al. PRC (2020)

OPE Prediction:

Braaten & Platter PRL (2008)

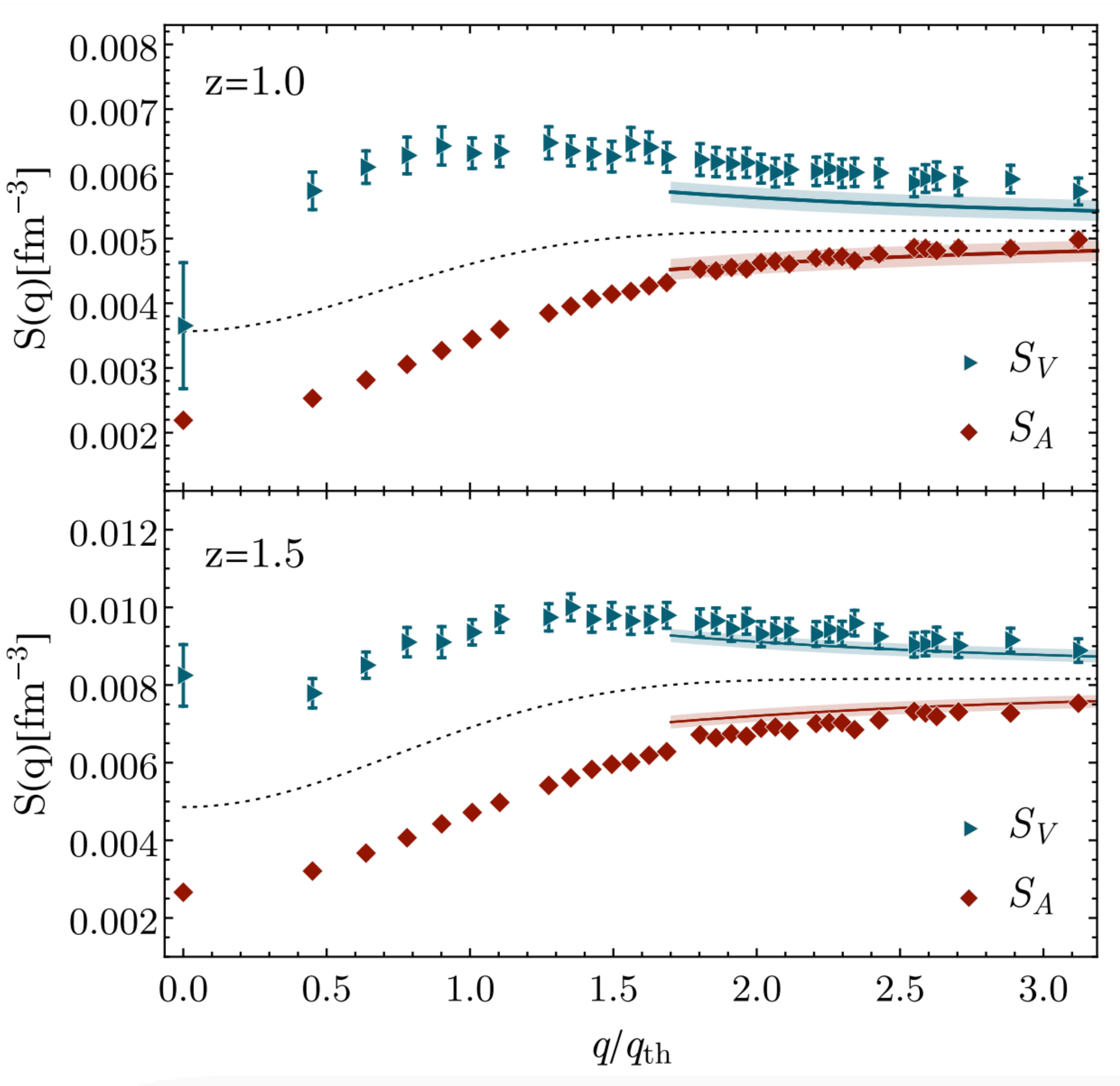
$$S_V(q) = \langle n \rangle + \frac{C}{8Vq} + \mathcal{O}(q^{-2})$$

$$S_A(q) = \langle n \rangle - \frac{C}{8Vq} + \mathcal{O}(q^{-2})$$

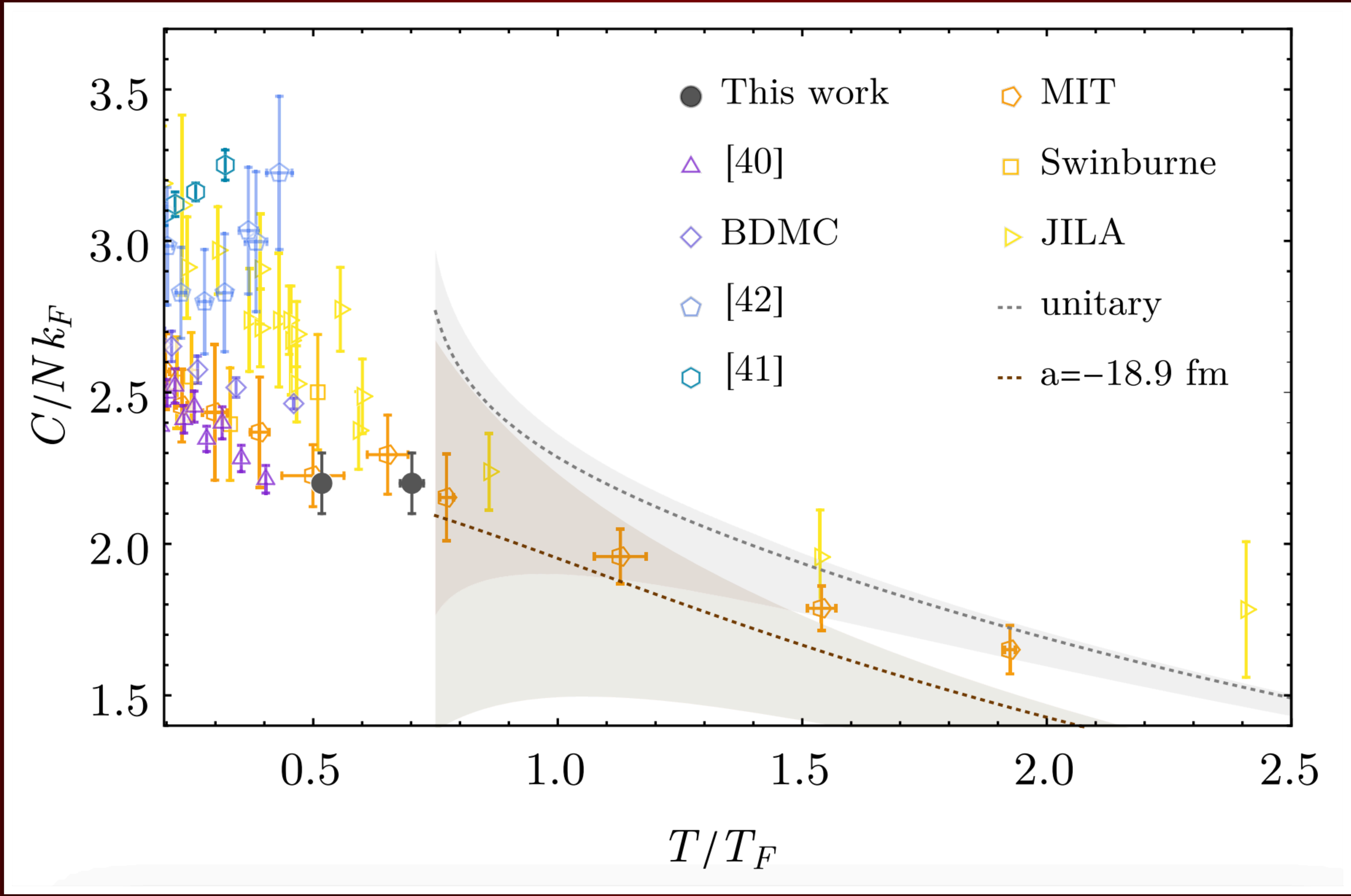
Fluctuations:

$$S_V(0) = T\partial n/\partial \mu$$

$$S_A(0) = T\partial s/\partial h$$



Symanzik Improvement for the unitary gas



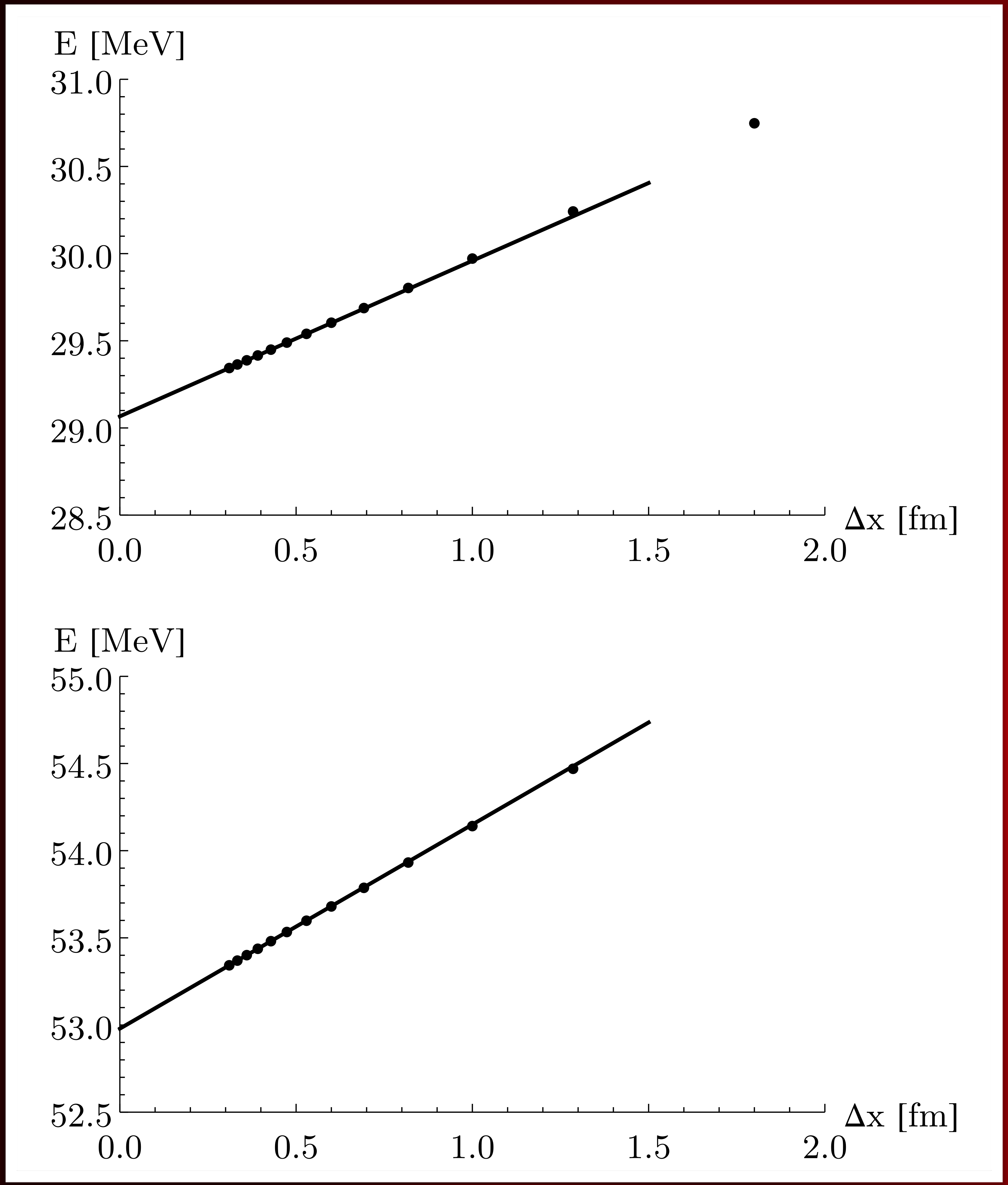
Berkowitz, NCW, et. al. PRL (2022)

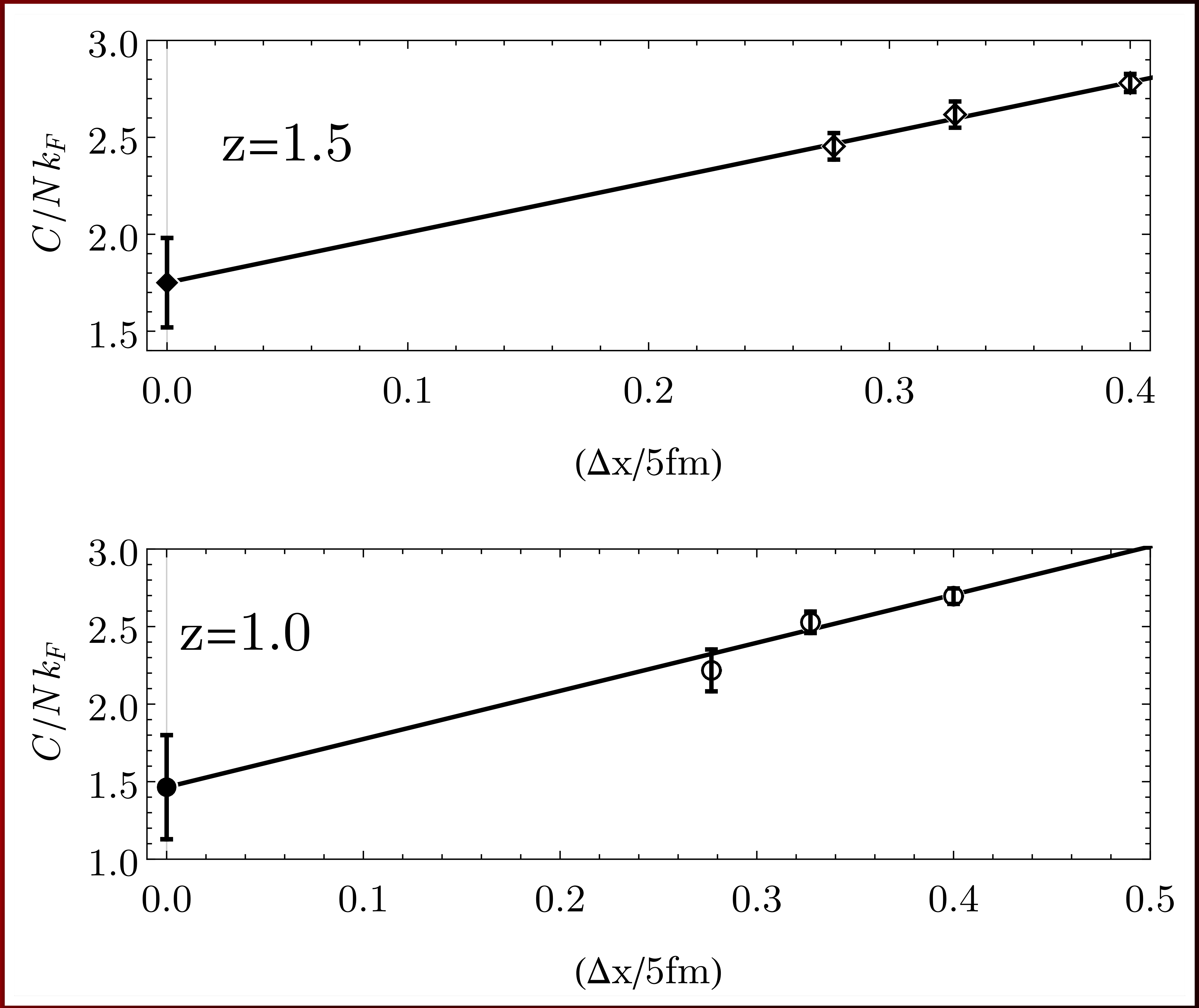
Symanzik Improvement speeds-up convergence

K. Symanzik, NPB 226, 187 (1983)

$$H \to H + \sum_i \underbrace{C_i}\mathcal{O}_i$$

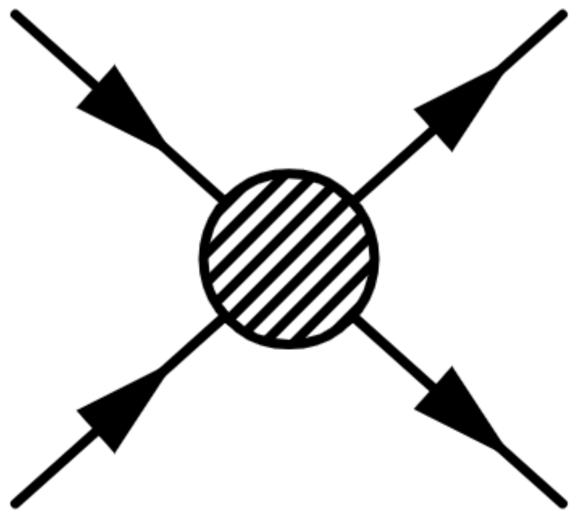
Tuned to cancel leading order Δx dependence





Two-body

Many-body



$$= \left(\frac{\Delta x}{M C_0} + \mathcal{P} \int \frac{d^3 q}{(2\pi)^3} \frac{1}{q^2 - EM\Delta x^2} + i \frac{\sqrt{EM\Delta x^2}}{4\pi} \right)^{-1}$$

$$= \left(\frac{\Delta x}{M C_0} + \alpha + \beta(EM\Delta x^2) + i \frac{\sqrt{EM\Delta x^2}}{4\pi} + \mathcal{O}\big((EM\Delta x^2)^2\big) \right)^{-1}$$

$$iA = \frac{4\pi/M}{k\cot\delta(k) - ik}$$

$$= \frac{4\pi/M}{(-1/a + rk^2/2 + \ldots) - ik}$$

$$\Longrightarrow$$

$$C_0(\Delta x) = -\frac{\Delta x}{M}\left(\alpha - \frac{\Delta x}{4\pi a}\right)^{-1}$$

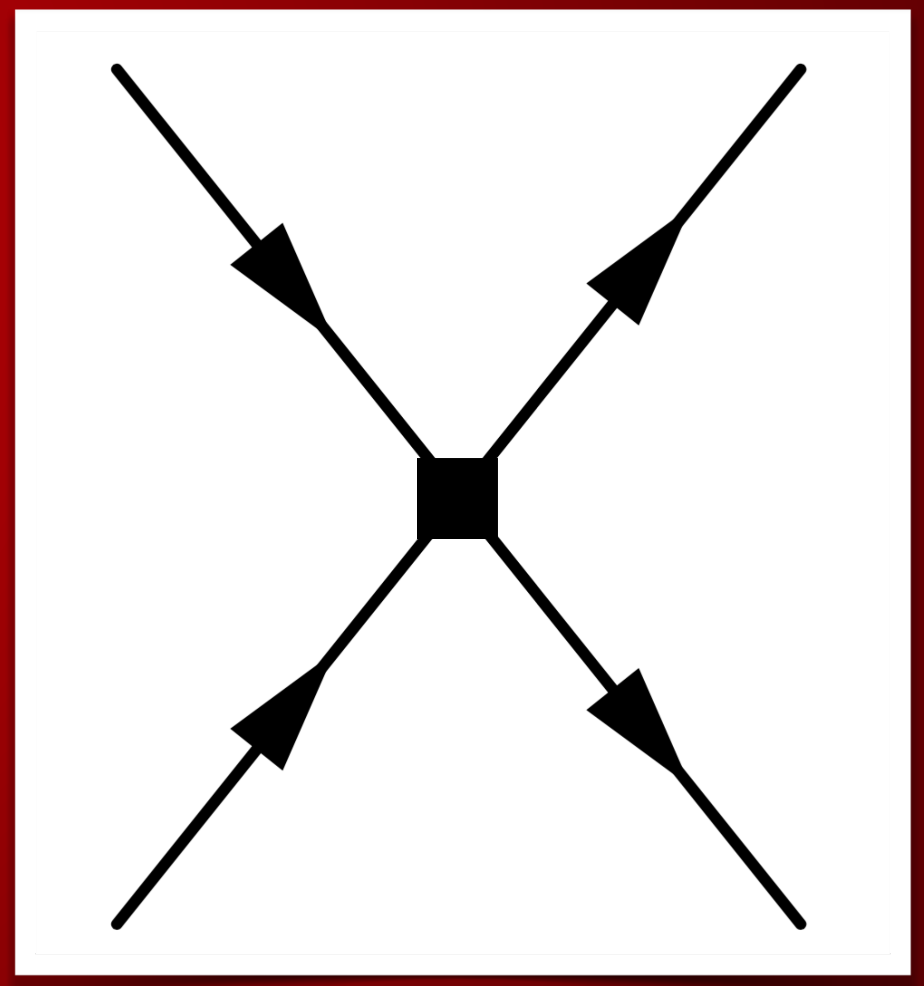
$$r(\Delta x) = 0.337\Delta x$$

A solution: $H \to H + C_2 \sum_{xi} n(x)n(x+\delta x_i)$

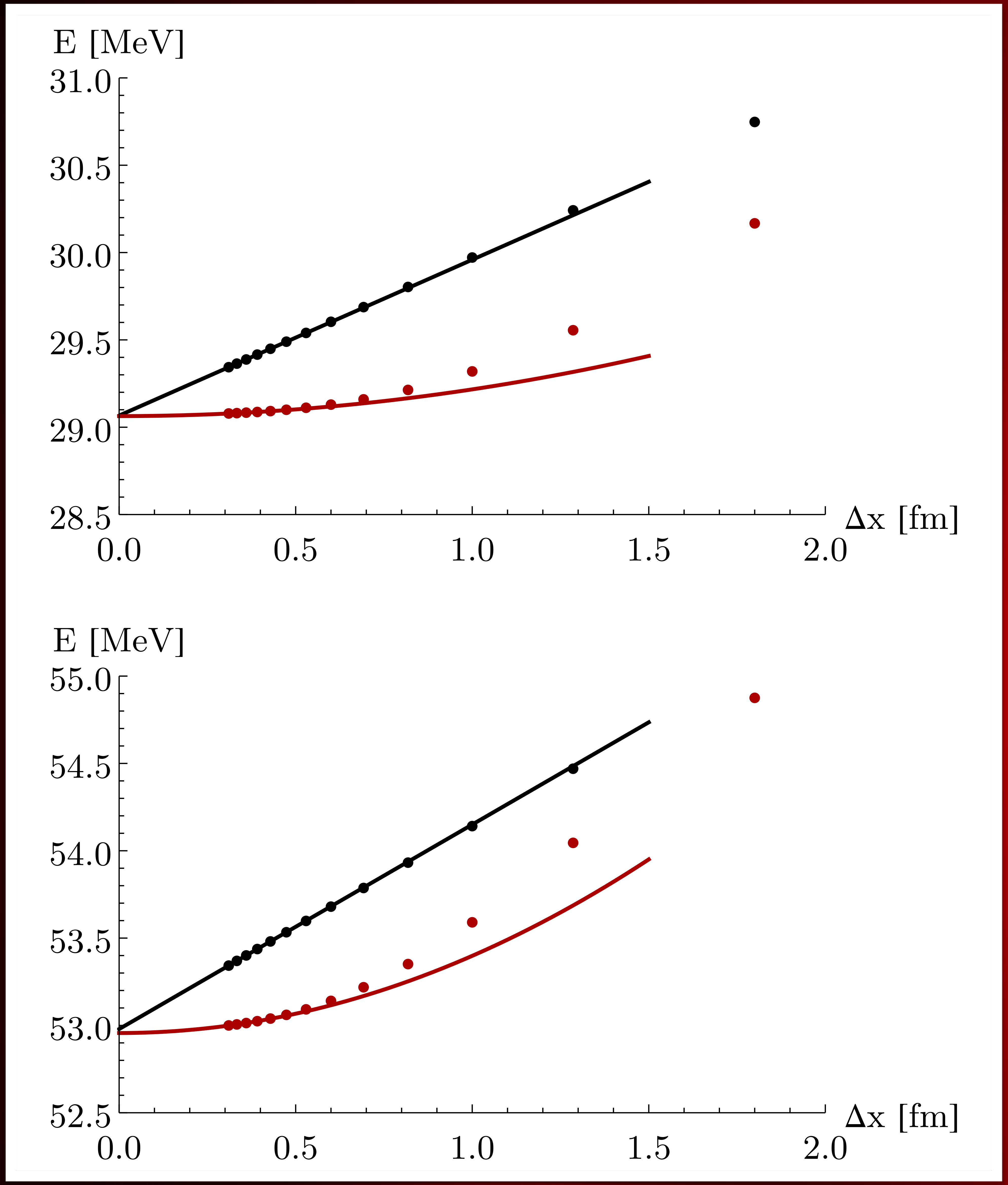
Reason:

$$C_2 \sum_{xi} n(x)n(x+\delta x_i)$$

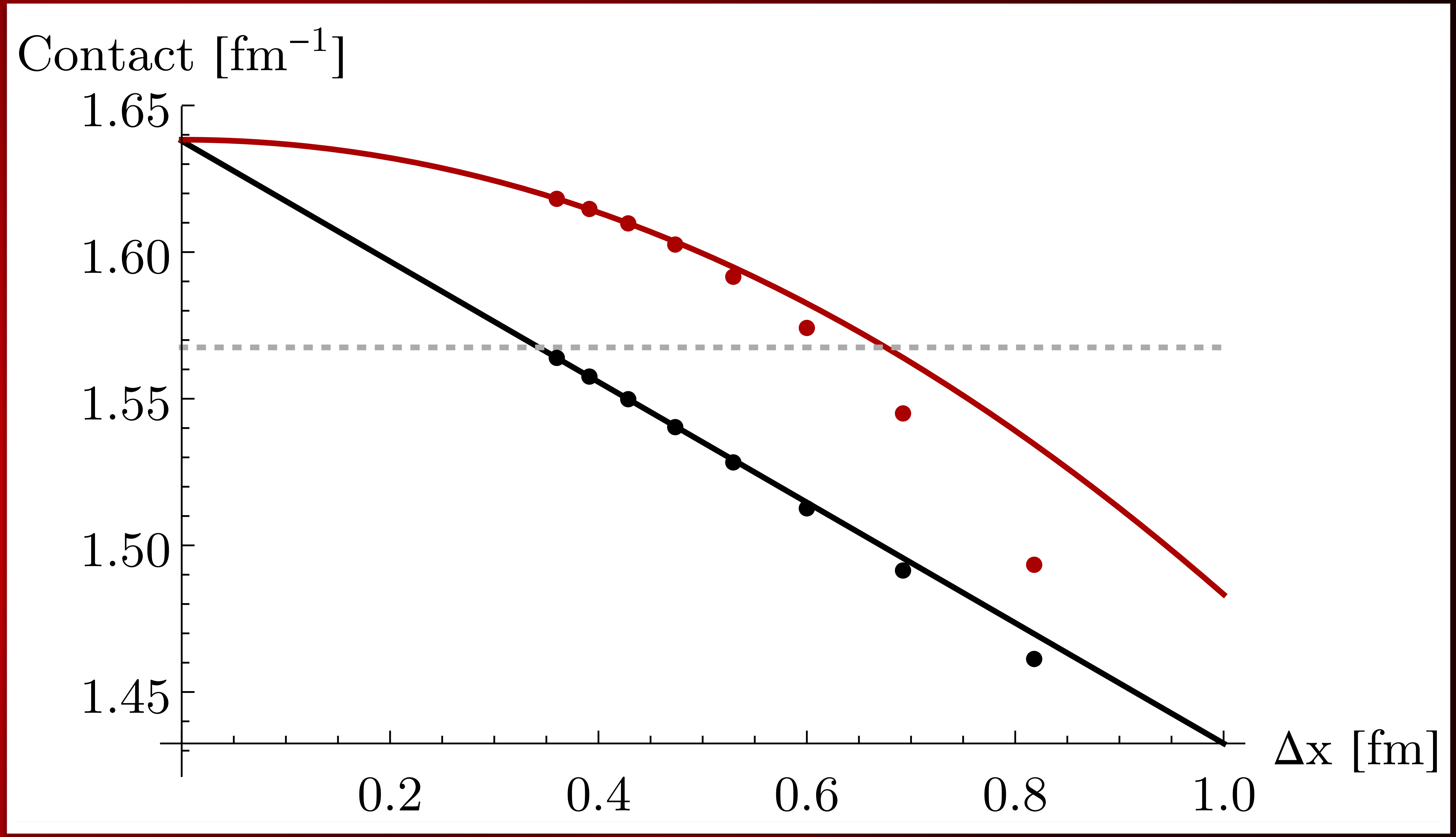
$$= C_2 \sum_{xi} n(x)\left[n(x) + \Delta x^2 \nabla^2 n(x) + \ldots\right]$$



$$= C_2 \Delta x^2 k^2$$



E [MeV]
31.0
30.5
30.0
29.5
29.0
28.5
0.0
0.5
1.0
1.5
2.0
Δx [fm]
E [MeV]
55.0
54.5
54.0
53.5
53.0
52.5
0.0
0.5
1.0
1.5
2.0
Δx [fm]



Contact [fm^{-1}]
1.65
1.60
1.55
1.50
1.45
0.2
0.4
0.6
0.8
1.0
Δx [fm]

Modification of Tan Contact

Tan Energy Relation: $$-\frac{C}{4\pi M} = \frac{dE}{da^{-1}}$$

Tan, Annals of Physics (2008)

Braaten showed: $$-\frac{C}{4\pi M} = \langle \frac{\partial H}{\partial a^{-1}} \rangle = \langle \frac{\partial H}{\partial C_0} \frac{\partial C_0}{\partial a^{-1}} \rangle$$

Braaten & Platter PRL (2008)

$$= C_0^2 M^2 \langle \int d^3x \; n_1(x) n_2(x) \rangle$$

Modification of Tan Contact

Contact in improved theory:

$$-\frac{C}{4\pi M} = \langle \frac{\partial H}{\partial a^{-1}} \rangle = \langle \frac{\partial H}{\partial C_0} \frac{\partial C_0}{\partial a^{-1}} + \frac{\partial H}{\partial C_2} \frac{\partial C_2}{\partial a^{-1}} \rangle$$

Braaten operator

new operator

The Team

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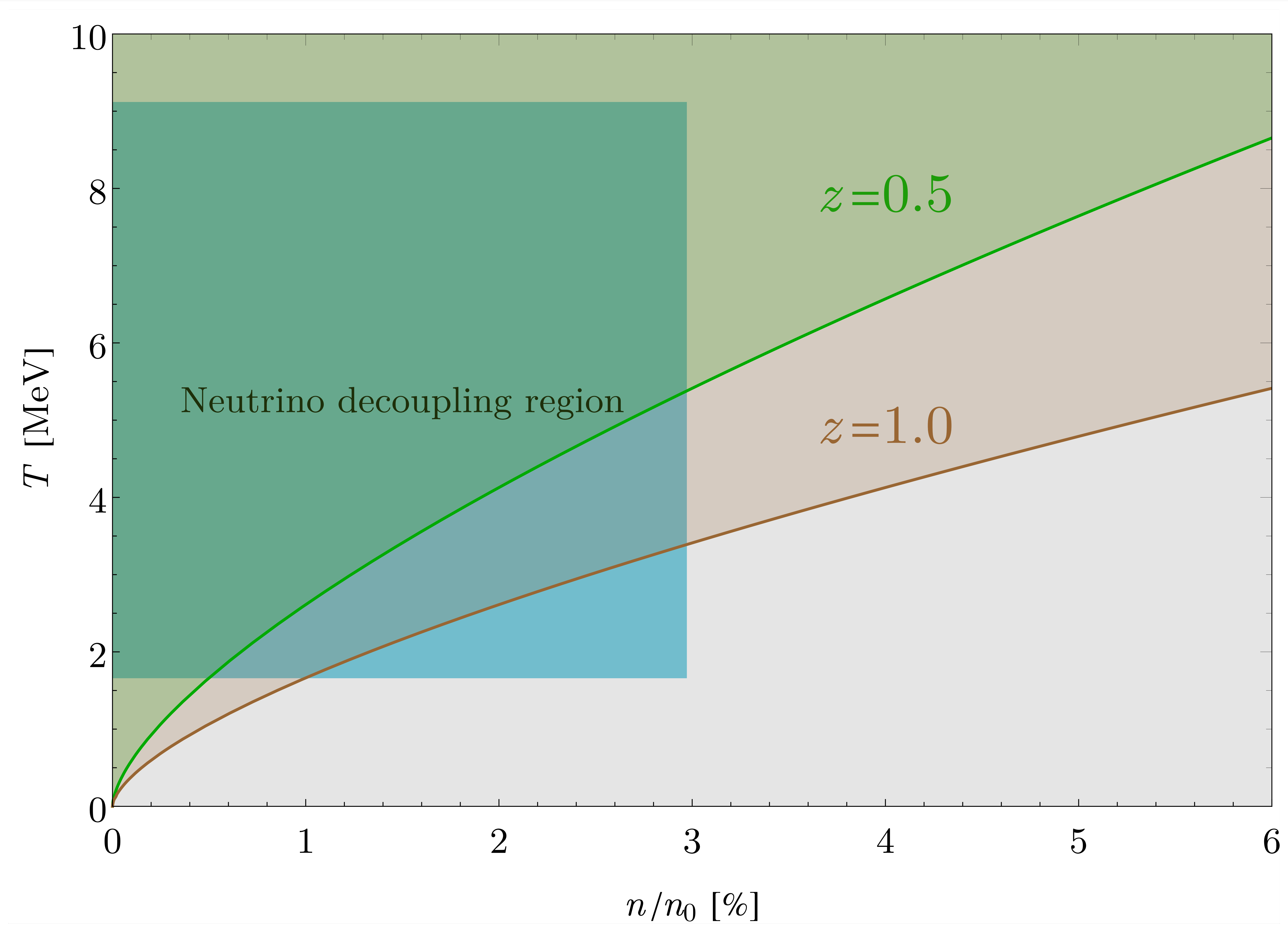
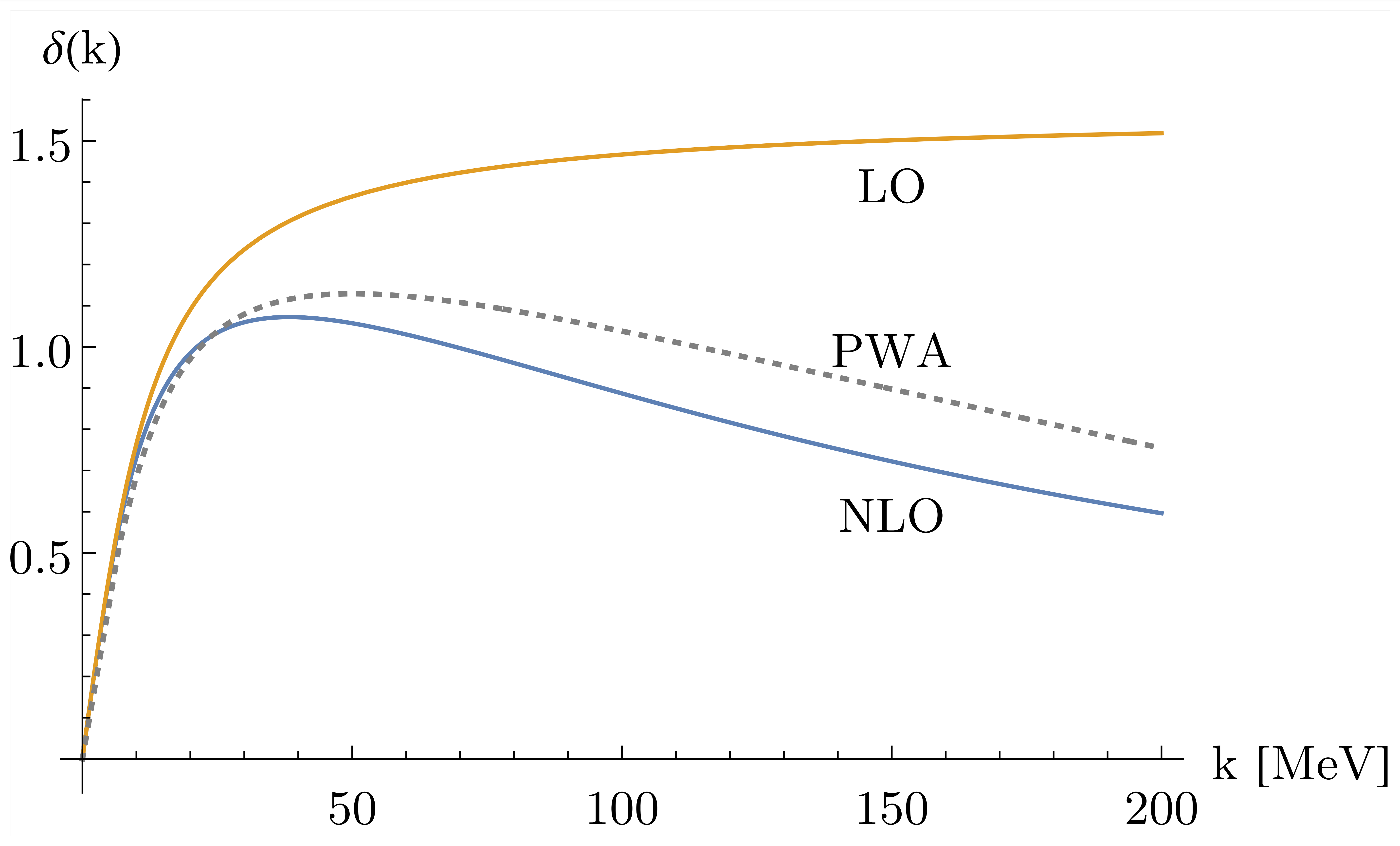
Forschungszentrum Jülich

Andrei Alexandru
GW University



Virial comparison



1S0 nn phase shifts