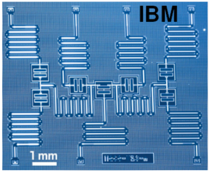
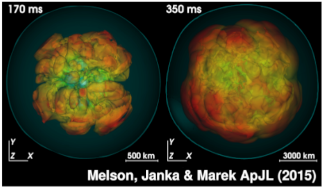
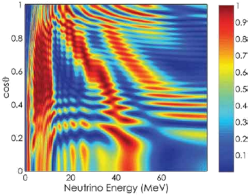
Dynamical phase transitions in collective neutrino oscillations

Alessandro Roggero







Duan et al. (2006)

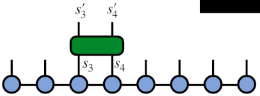
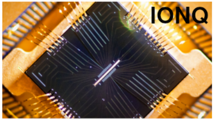
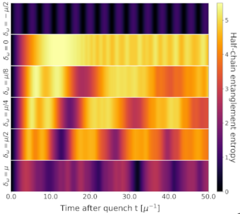


image from itensor.org





Trento Institute for
Fundamental Physics
and Applications

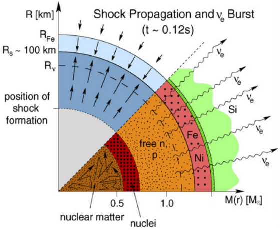
ECT* – 09 June, 2022

UNIVERSITAS ✷ ATHESINA STUDIORUM

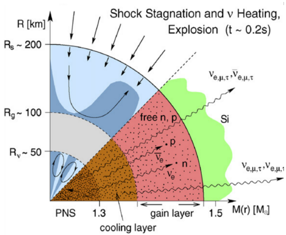
Neutrino's roles in supernovae

figures from Janka et al. (2007)

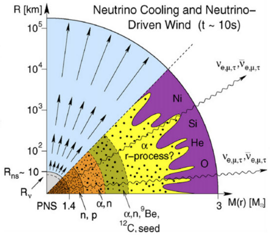
- efficient energy transport away from the shock region (burst)



- energy deposition to revive the stalled shock (explosion)



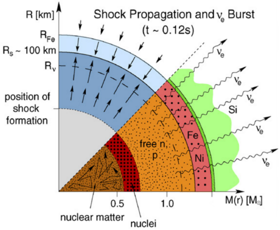
- regulation of electron fraction in ν-driven wind (nucleosynthesis)



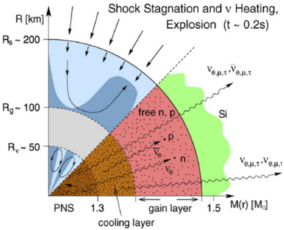
Neutrino's roles in supernovae

figures from Janka et al. (2007)

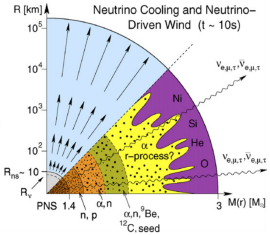
- efficient energy transport away from the shock region (burst)



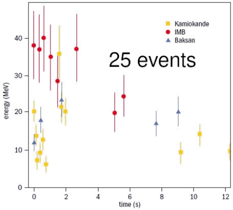
- energy deposition to revive the stalled shock (explosion)

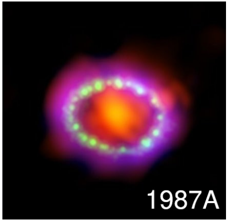


- regulation of electron fraction in ν-driven wind (nucleosynthesis)



$\approx 10^{58}$ neutrinos emitted in few sec.





Neutrino oscillations in astrophysical environments

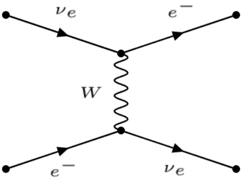
We know that neutrinos can display flavor oscillations in vacuum, does it matter in a core-collapse supernova?

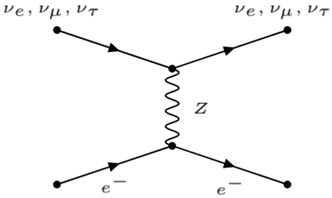
- energy deposition behind shock and in the wind proceeds through charge-current reactions (large differences in $\nu_e - \nu_{\mu/\tau}$)

Neutrino oscillations in astrophysical environments

We know that neutrinos can display flavor oscillations in vacuum, does it matter in a core-collapse supernova?

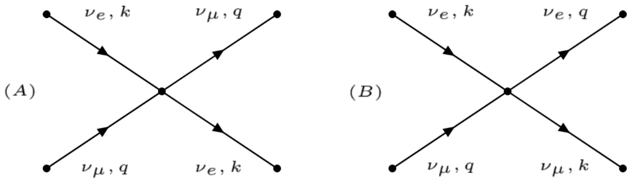
- energy deposition behind shock and in the wind proceeds through charge-current reactions (large differences in $\nu_e - \nu_{\mu/\tau}$)
- neutrino oscillation rates can get enhanced through elastic forward scattering with high density external matter (MSW effect)





Neutrino-neutrino forward scattering

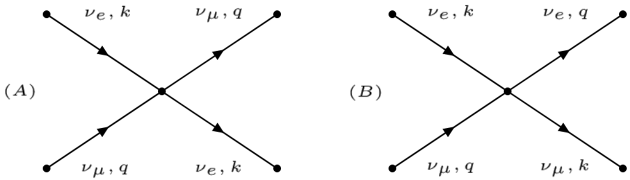
Fuller, Qian, Pantaleone, Sigl, Raffelt, Sawyer, Carlson, Duan, . . .



- diagonal contribution (A) does not impact flavor mixing
- off-diagonal term (B) equivalent to flavor/momentum exchange between two neutrinos
  - total flavor is conserved

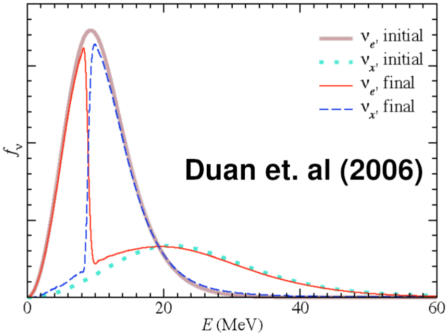
Neutrino-neutrino forward scattering

Fuller, Qian, Pantaleone, Sigl, Raffelt, Sawyer, Carlson, Duan, . . .



- diagonal contribution (A) does not impact flavor mixing
- off-diagonal term (B) equivalent to flavor/momentum exchange between two neutrinos
  - total flavor is conserved

Important effect if initial distributions are strongly flavor dependent



Two-flavor approximation and the iso-spin Hamiltonian

Consider two active flavors (ν_e,ν_x) and encode flavor amplitudes for a neutrino with momentum p_i into an $SU(2)$ iso-spin:

$$|\Phi_i\rangle = \cos(\eta_i)|\nu_e\rangle + \sin(\eta_i)|\nu_x\rangle \equiv \cos(\eta_i)|\uparrow\rangle + \sin(\eta_i)|\downarrow\rangle$$

A system of N interacting neutrinos is then described by the Hamiltonian

$$H = \sum_i \frac{\Delta m^2}{4E_i}\vec{B}\cdot\vec{\sigma}_i + \lambda\sum_i \sigma_i^z + \frac{\mu}{2N}\sum_{i<j}\left(1-\cos(\phi_{ij})\right)\vec{\sigma}_i\cdot\vec{\sigma}_j$$

- vacuum oscillations: $\vec{B} = (\sin(2\theta_{mix}), 0, -\cos(2\theta_{mix}))$
- interaction with matter: $\lambda = \sqrt{2}G_F\rho_e$
- neutrino-neutrino interaction: $\mu = \sqrt{2}G_F\rho_\nu$
  - dependence on momentum direction: $\cos(\phi_{ij}) = \frac{\vec{p}_i}{\|\vec{p}_i\|}\cdot\frac{\vec{p}_j}{\|\vec{p}_j\|}$

for a full derivation, see e.g. Pehlivan et al. PRD(2011)

Beyond mean field effects

April '03 speedup through entanglement $\tau \sim \mu^{-1}$ Bell et al. PLB (2003)

July '03 in a highly symmetric limit the MF prediction is qualitatively correct $\tau \propto \mu^{-1}\sqrt{N} \to \infty$ Friedland&Lunardini JHEP (2003)

August '04 some models seem to produce $\tau \propto \mu^{-1}\log(N)$ Sawyer (2004)

Summer '19 exact simulations for systems with small N show substantial entanglement buildup Cervia et al. PRD(2019), Rrapaj PRC(2020)

Beyond mean field effects

April '03	speedup through entanglement $\tau \sim \mu^{-1}$	Bell et al. PLB (2003)
July '03	in a highly symmetric limit the MF prediction is qualitatively correct $\tau \propto \mu^{-1}\sqrt{N} \to \infty$	Friedland&Lunardini JHEP (2003)
August '04	some models seem to produce $\tau \propto \mu^{-1}\log(N)$	Sawyer (2004)
Summer '19	exact simulations for systems with small N show substantial entanglement buildup	Cervia et al. PRD(2019), Rrapaj PRC(2020)

- increasing effort in tackling the problem using a variety of methods: diagonalization, tensor networks and semiclassical approaches

Cervia et al. (2021), Patwardhan et al. (2021), AR $(2021)^2$, Xiong (2022), Martin, AR, et al. (2022), AR, Rrapaj, Xiong (2022), Lacroix et al. (2022), . . .

- Great potential for many-body simulations on quantum devices

Hall, AR, et al. (2021), Yeter-Aydeniz et al. (2022), Illa & Savage (2022), Amitrano, AR, et al. (2022)

Dynamical phase transitions

Heyl et al. PRL (2013), Heyl PRL (2015), Heyl RPP (2018)

Quantum quench protocols

1. the system starts as the ground-state of an initial Hamiltonian H_0
2. at time $t = 0$ we switch to a different Hamiltonian H and evolve

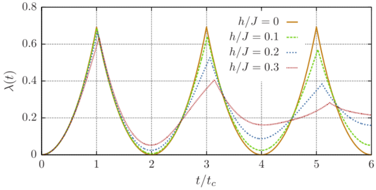
Dynamical critical behavior encoded in Loschmidt echo

$$\mathcal{L}(t) = \left|\langle\Psi_0|e^{-iHt}|\Psi_0\rangle\right|^2 \xrightarrow{N\gg 1} e^{-N\lambda(t)}$$

Loschmidt rate $\lambda(t)$ plays a similar role as the free energy in equilibrium.

$$H(h) = -\sum_{\langle ij\rangle} Z_i Z_j + h\sum_i X_i$$

- start in ground-state for $h \to \infty$
- quench across critical point at $h = 1$



Heyl PRL (2015)

Dynamical phase transitions

Heyl et al. PRL (2013), Heyl PRL (2015), Heyl RPP (2018)

Quantum quench protocols

1. the system starts as the ground-state of an initial Hamiltonian H_0
2. at time $t = 0$ we switch to a different Hamiltonian H and evolve

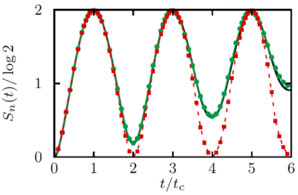
Dynamical critical behavior encoded in Loschmidt echo

$$\mathcal{L}(t) = \left|\langle\Psi_0|e^{-iHt}|\Psi_0\rangle\right|^2 \xrightarrow{N\gg 1} e^{-N\lambda(t)}$$

Loschmidt rate $\lambda(t)$ plays a similar role as the free energy in equilibrium.

$$H(h) = -\sum_{\langle ij\rangle} Z_i Z_j + h \sum_i X_i$$

- start in ground-state for $h \to \infty$
- quench across critical point at $h = 1$



Schmitt & Heyl SciPost Phys (2018)

DPT for systems with degenerate ground spaces

Heyl PRL (2014)

$$H_{XXZ} = J\sum_i \left[X_i X_{i+1} + Y_i Y_{i+1} + \Delta Z_i Z_{i+1}\right]$$

- disordered gapless phase for $\Delta < 1$
- anti-ferromagnetic phase for $\Delta > 1$
- critical point at $\Delta = 1$

$$|\Psi_0\rangle = |\uparrow\downarrow\uparrow\downarrow\cdots\rangle \qquad |\Psi_0'\rangle = |\downarrow\uparrow\downarrow\uparrow\cdots\rangle$$

Loschmidt Echo for degenerate ground-states

$$\mathcal{L}_0(t) = \left|\langle\Psi_0|e^{-itH}|\Psi_0\rangle\right|^2 \quad \mathcal{L}_1(t) = \left|\langle\Psi_0'|e^{-itH}|\Psi_0\rangle\right|^2 ,$$

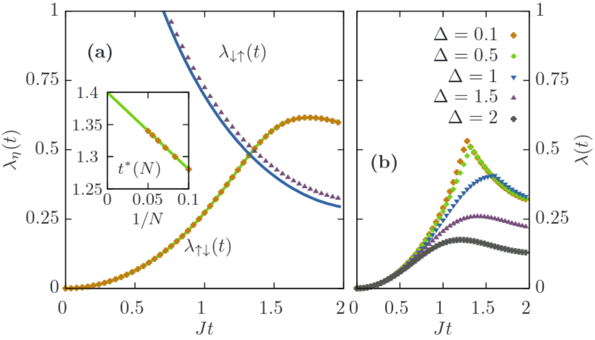
DPT $\Leftrightarrow$ non-analytic behavior of the total echo $\mathcal{L}(t) = \mathcal{L}_0(t) + \mathcal{L}_1(t)$

DPT for systems with degenerate ground spaces II

$$\mathcal{L}_0(t) = \left|\langle\Psi_0|e^{-itH}|\Psi_0\rangle\right|^2 \quad \mathcal{L}_1(t) = \left|\langle\Psi'_0|e^{-itH}|\Psi_0\rangle\right|^2 ,$$

DPT $\Leftrightarrow$ non-analytic behavior of the total echo $\mathcal{L}(t) = \mathcal{L}_0(t) + \mathcal{L}_1(t)$

Both scale exponentially in system size, but with different rates, there is a kink forming if the order between $\mathcal{L}_0(t)$ and $\mathcal{L}_1(t)$ changes at some $t = t^*$

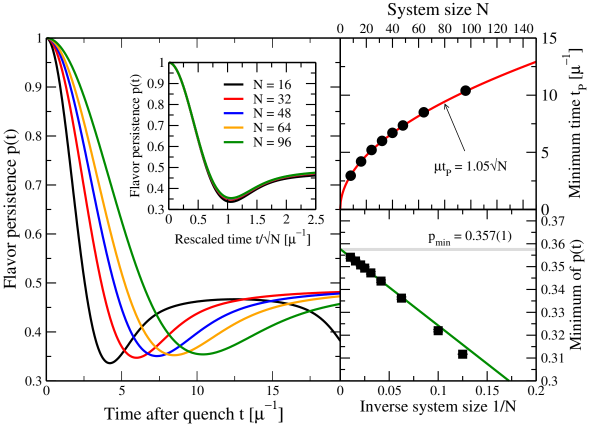


Simple neutrino model

Friedland & Lunardini (2003), AR (2021)

$$H = \frac{1}{2N}\sum_{i<j}\vec{\sigma}_i\cdot\vec{\sigma}_j = \frac{1}{N}S^2 + const. \,.$$

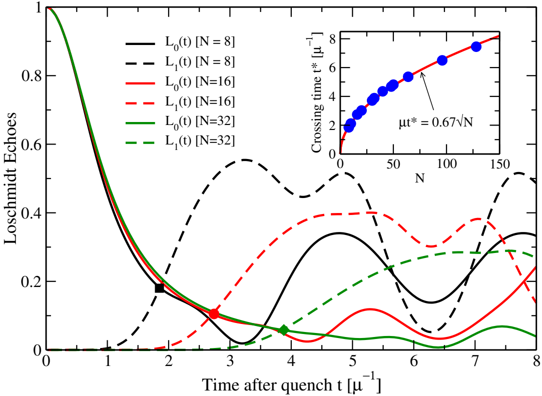
Initialize system in $|\Psi(0)\rangle = |\downarrow\rangle^{\otimes N/2}\otimes|\uparrow\rangle^{\otimes N/2}$ and compute the flavor persistence $p(t) = (1 - \langle\Psi(t)|\sigma_1|\Psi(t)\rangle)/2$ for increasing system size



Simple neutrino model II

$$H(x) = \frac{x}{2N} S^2 + (1-x) \sum_{a\in\mathcal{A}} \sum_{b\in\mathcal{B}} Z_a Z_b \ ,$$

start at $x = 0$ and evolve with $x = 1$. State is $|\Psi_0\rangle = |\downarrow\rangle^{\otimes N/2} \otimes |\uparrow\rangle^{\otimes N/2}$.



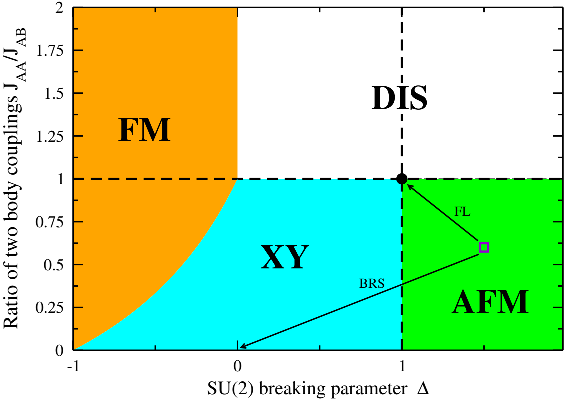
Crossing time t^* diverges as $\sqrt{N}$ $\Rightarrow$ no evolution for a large system!

Many-body speedup in unphysical model

Bell,Rawlinson,Sawyer PLB(2003), AR (2021)

$$H_{BRS} = \frac{1}{2N} \sum_{i<j} \mathcal{J}_{ij} \left(X_i X_j + Y_i Y_j + \Delta Z_i Z_j \right)$$

with $\mathcal{J}_{ij} = J_{AA}$ for (i,j) in $\mathcal{A}$ or $\mathcal{B}$ and $\mathcal{J}_{ij} = J_{AB}$ otherwise. Our initial state is (degenerate) gs of H_{BRS} in the limit $\Delta \gg 1$ and $J_{AA} < J_{AB}$

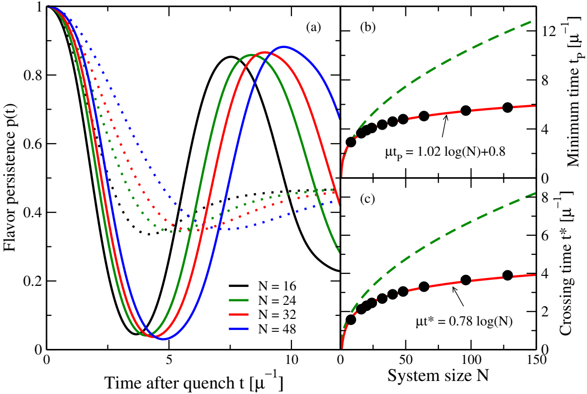


Many-body speedup in unphysical model II

Bell,Rawlinson,Sawyer PLB(2003), AR (2021)

$$H_{BRS} = \frac{1}{2N}\sum_{i<j}\mathcal{J}_{ij}\left(X_iX_j + Y_iY_j + \Delta Z_iZ_j\right)$$

with $\mathcal{J}_{ij} = J_{AA}$ for (i,j) in $\mathcal{A}$ or $\mathcal{B}$ and $\mathcal{J}_{ij} = J_{AB}$ otherwise. Our initial state is (degenerate) gs of H_{BRS} in the limit $\Delta \gg 1$ and $J_{AA} < J_{AB}$



Many-body speedup in a physical model

To engineer a “DPT” we can ensure the system crosses a critical point

$$H = -\frac{\delta_\omega}{2}\left(\sum_{i\in\mathcal{A}}\sigma_i^z - \sum_{i\in\mathcal{B}}\sigma_i^z\right) + \frac{\mu}{2N}\sum_{i<j}\vec{\sigma}_i\cdot\vec{\sigma}_j\ ,$$

AFM ($\mu > 0$) transition at $\delta_\omega = 0$ between gapped phases

FM ($\mu < 0$) transitions at $\delta_\omega = \pm\mu$ between gapped and gapless phases

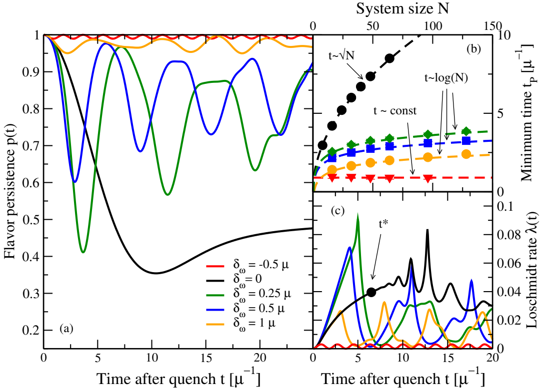
Many-body speedup in a physical model

To engineer a "DPT" we can ensure the system crosses a critical point

$$H = -\frac{\delta_\omega}{2}\left(\sum_{i\in\mathcal{A}}\sigma_i^z - \sum_{i\in\mathcal{B}}\sigma_i^z\right) + \frac{\mu}{2N}\sum_{i<j}\vec{\sigma}_i\cdot\vec{\sigma}_j \ ,$$

AFM $(\mu > 0)$ transition at $\delta_\omega = 0$ between gapped phases

FM $(\mu < 0)$ transitions at $\delta_\omega = \pm\mu$ between gapped and gapless phases



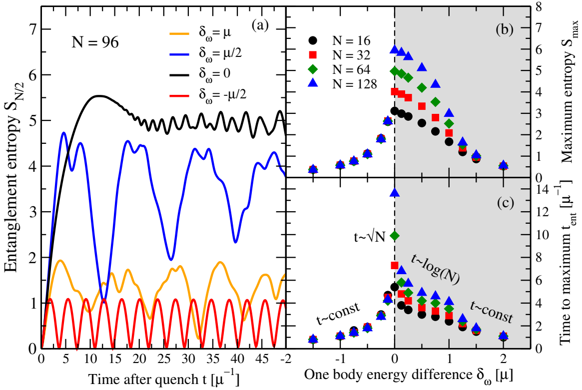
Summary and perspectives

- collective neutrino oscillations are an interesting **strongly coupled** many-body system driven by the **weak interaction**
- there seems to be a strong connection between oscillation time scales and dynamical phase transitions
  - explains seemingly conflicting results from the past
  - alternative way of understanding the appearance of bipolar modes
- can be generalized to more complicated geometries
  - "fast" modes with three beams can also be understood in terms of a DPT almost identical to the one present in the two beam case
- useful to exploit semi-classical methods to have a better understanding of the DPT in these systems
- great system to explore with fast advancing quantum technologies, Hamiltonian is two-local but all-to-all $\rightarrow$ best suited for trapped-ions

Many-body speedup in a physical model II

To engineer a "DPT" we can ensure the system crosses a critical point

$$H = -\frac{\delta_\omega}{2}\left(\sum_{i\in\mathcal{A}}\sigma_i^z - \sum_{i\in\mathcal{B}}\sigma_i^z\right) + \frac{\mu}{2N}\sum_{i<j}\vec{\sigma}_i\cdot\vec{\sigma}_j\ ,$$



Phase diagram for the BRS model

$$M^Z_{AB} = \frac{1}{N}\langle Z_A Z_B\rangle \qquad M^{XY}_{AB} = \frac{1}{N}\left(\langle X_A X_B\rangle + \langle Y_A Y_B\rangle\right)$$