6-10 June 2022 ECT* Workshop
Connections Between Cold Atoms and Nuclear Matter

Sound Propagation and Superfluid Density of an Ultra-Cold Quantum Gas

Sandro Stringari

Università di Trento

CNR-INO

Propagation of sound

Propagation of **sound** is an **ubiquitous** feature characterizing many body systems.

In **classical** gases sound propagates as a consequence of **collisions** ensuring the achievement of **hydrodynamic** regime $c = \sqrt{1/(mn\kappa_S)}$

In the **quantum** world sound exhibits novel features. Its propagation is deeply affected by **interactions**, **superfluidity**, **dimensionality**, **breaking** of **Galielean** invariance, **supersolid** effects etc.

Superfluid density

The **superfluid density** is a key parameter characterizing **transport phenomena** in superfluid systems.

Its value cannot be derived from thermodynamic functions at equilibrium.

Superfluid density

The **superfluid density** is a key parameter characterizing **transport phenomena** in superfluid systems.

Its value cannot be derived from thermodynamic functions at equilibrium.

Major question addressed in the talk

- **Can the superfluid density be extracted from the measurement of sound velocity in a quantum gas** ?

Old and Recent Collaborations

Unitary Fermi gas
(IBK-Trento)

BKT in 2D Bose gas
(Tomoki Ozawa)

Collisionless sound
in 2D Bose gas
(Miki Ota)

SOC gases and
Supersolidity
(Giovanni Martone)

Leggett's bound to
superfluid density
(Santo Roccuzzo)

Plan of the talk:

First part (T>0)

- Superfluid equations for first and second sound at finite temperature
- Determination of temperature dependence of the **superfluid density** in 3D and 2D (BKT transition) gases from the measurement of **second sound**

Second part (T=0)

- Behavior of the **superfluid density** in systems **breaking Galilean invariance** (gases in 1D periodic potentials, spin-orbit coupled gases and supersolids)
- Comparison with **Leggett's upper bound** for superfluid density

Dynamic theory of superfluids at finite temperature:

Landau's Two-fluid HD equations for Galilean invariant systems)

(hold in deep collisional regime of thermal non superfluid component $\omega\tau << 1$

Irrotationality of superfluid flow

$$\frac{\partial}{\partial t}\rho + \vec{\nabla}(\vec{j}) = 0$$

$$\frac{\partial}{\partial t}s + \vec{\nabla}(s\vec{v}_N) = 0$$

$$m\frac{\partial}{\partial t}\vec{v}_S + \nabla(\mu(n) + V_{ext}) = 0$$

$$\frac{\partial}{\partial t}\vec{j} + \vec{\nabla}P + n\vec{\nabla}V_{ext} = 0$$

$$\rho = mn = \rho_S + \rho_N$$

$$\vec{j} = \rho_S\vec{v}_S + \rho_N\vec{v}_N$$

s is entropy density
P is local pressure

Ingredients:
- **equation of state**
- **superfluid density**

$$\frac{\partial}{\partial t}\rho + \vec{\nabla}(\vec{j}) = 0$$

$$\frac{\partial}{\partial t}s + \vec{\nabla}(s\vec{v}_N) = 0$$

$$m\frac{\partial}{\partial t}\vec{v}_S + \nabla(\mu(n) + V_{ext}) = 0$$

$$\frac{\partial}{\partial t}\vec{j} + \vec{\nabla}P + n\vec{\nabla}V_{ext} = 0$$

equivalent at T=0

At T=0: $\rho = \rho_S\ ; \vec{j} = \rho\vec{v}_S$
in Galilean invariant systems and eqs. reduce to T=0 irrotational superfluid HD equations

*At T=0 irrotational hydrodynamics follows from superfluidity (role of the phase of the order parameter). Quite successful to describe the macroscopic dynamic behavior of trapped atomic gases (Bose and Fermi) (**expansion**, **collective oscillations**, etc.)*

Landau equations of two fluid hydrodynamics gives rise to two solutions below the critical temperature:

First sound: superfluid and normal fluids move **in phase**

Second sound: superfluid and normal fluids move **out of phase**.

In systems characterized by **small compressibility,**
(like liquid He4 and strongly interacting Fermi gas)
second sound reduces to entropy wave

Second sound velocity
fixed by superfluid density,
hence providing unique
possibility to **measure superfluid density**)

$$c_2^2 = \frac{1}{m}\frac{n_s T s^2}{n_n C_P}$$

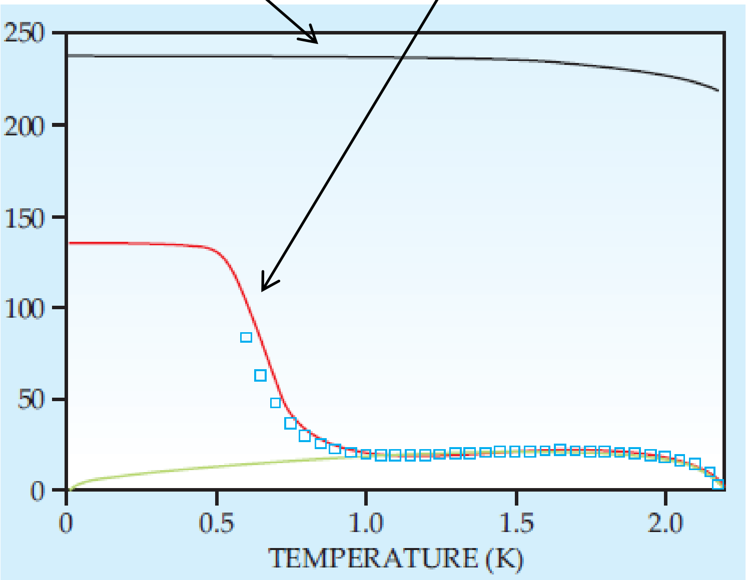
entropy

Specific heat

First and second sound velocities in **superfluid liquid He**

$$c_1^2 = \frac{1}{m}\left(\frac{\partial P}{\partial n}\right)_S$$

$$c_2^2 = \frac{1}{m}\frac{\rho_s T s^2}{\rho_n C_P}$$



Liquid He

(experiment, Peshkov 1946)

Propagation of sound
in the 3D Fermi gas at unitarity

Thermodynamics and Universality of 3D Fermi gas at unitarity

Absence of interaction parameters implies that thermodynamics obeys universal law (Ho, 2004)

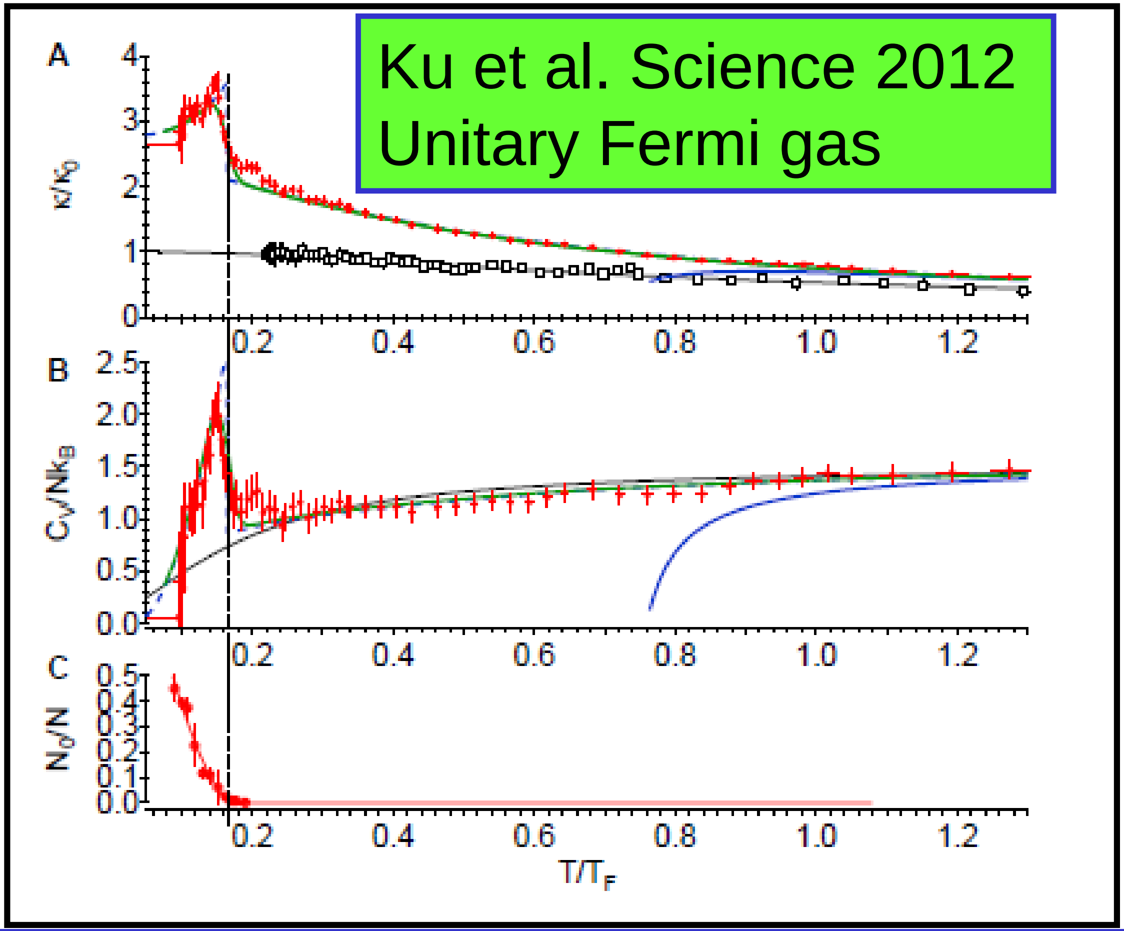
$$\frac{P}{k_B T}\lambda_T^3 = f_p(\mu / k_B T)$$

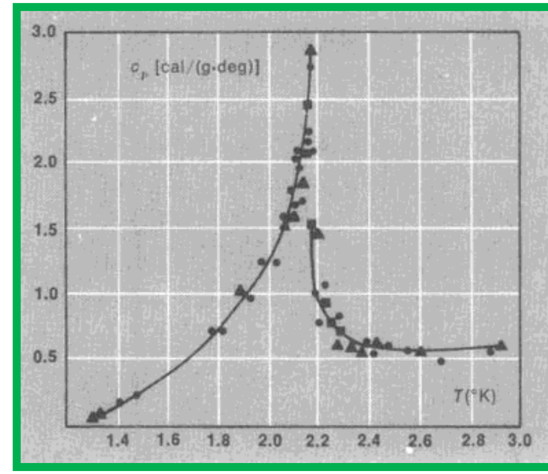
where $\lambda_T = \sqrt{2\pi\hbar^2 / m k_B T}$ is thermal wave-length and $f_p(x)$ is **dimensionless, universal function** (applies to quantum gases and neutron matter).

All thermodynamic functions (entropy, compressibilities, specific heats etc.) can be expressed in terms of the universal function

Calculation of $f_p(x)$ requires however non trivial many-body approaches at finite T.

Universal function $f_p(x)$ and thermodynamic functions now **available experimentally** in a wide range of temperatures





Superfluid He4

Experimental determination of critical temperature

$$T_C / T_F = 0.167(13)$$

(determined by peak in specific heat and onset of BEC) in agreement with many-body predictions (Burowski et al. 2006; Haussmann et al. (2007); Goulko and Wingate 2010)

Universal function $f_p(\mu / k_B T)$ gives access to all thermodynamic quantitities, **except** to **superfluid density**

Question: how to **measure** the **superfluid density** ?

(measure **second sound** ! $c_2^2 = \frac{1}{m} \frac{\rho_s T s^2}{\rho_n C_P}$)

Second sound and the superfluid fraction in a Fermi gas with resonant interactions

L.A. Sidorenkov, Meng Khoon Tey, R. Grimm, Yan-Hua Hou, L. Pitaevskii & S. Stringari
Nature 498 78 (2013)

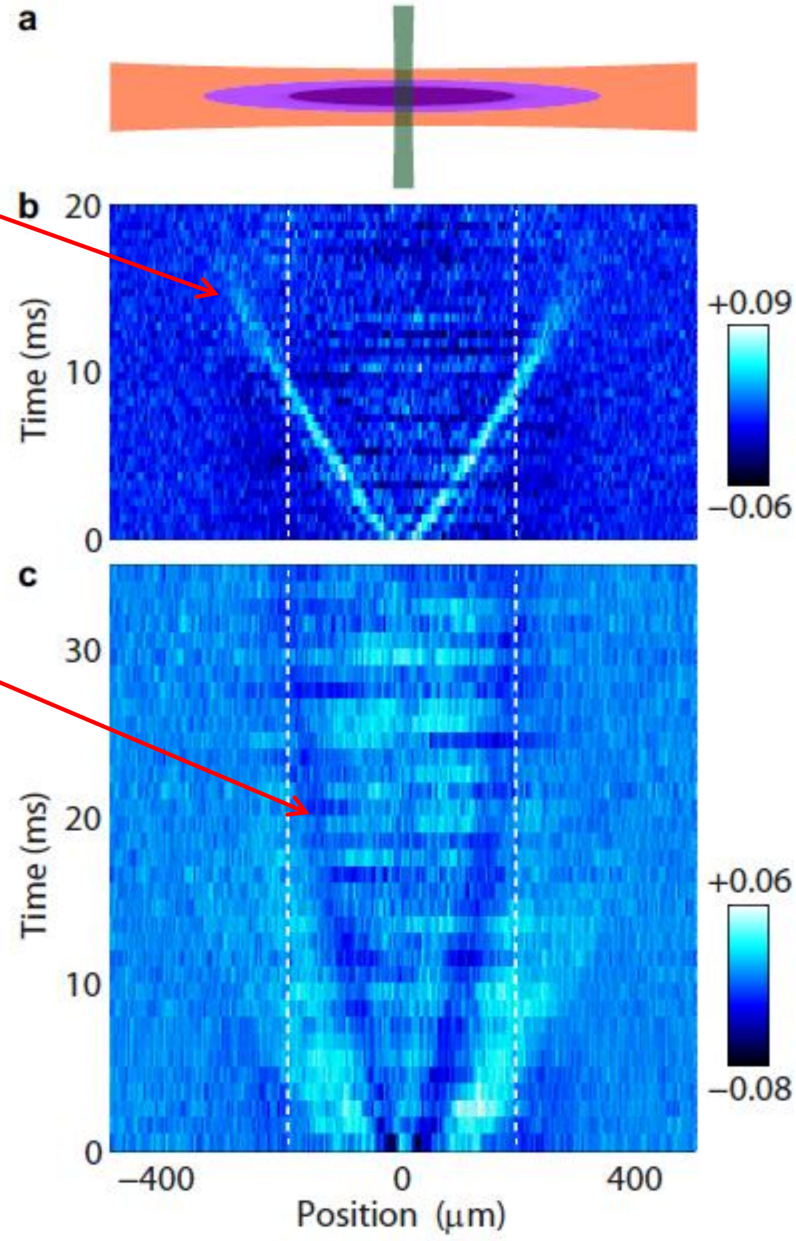
(Innsbruck-Trento collaboration)

In Innsbruck experiment both **first** and **second** sound measured

First sound
propagates also beyond the boundary between the superfluid and the normal parts

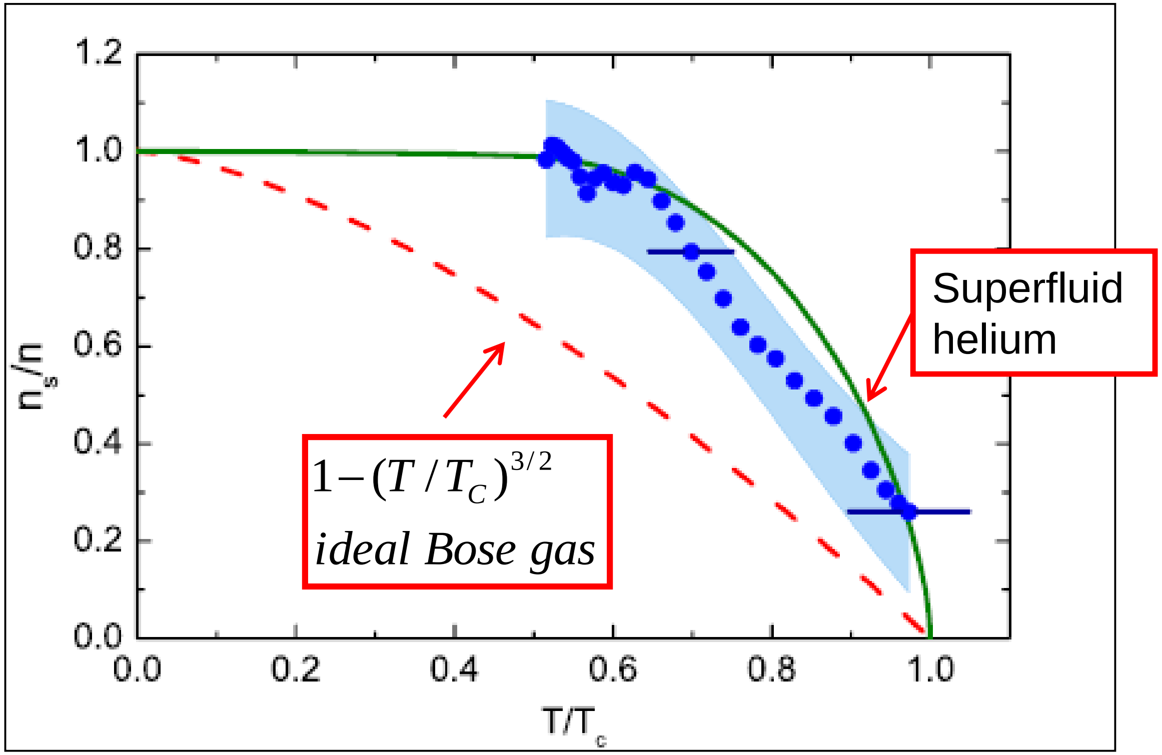
Second sound propagates only within the region of co-existence of the super and normal fluids.

Second sound is basically an isobaric wave, but signal is visbile because of small, but **finite thermal expansion**.



From measurement of second sound velocity and knowledge of thermodynamic functions one can reconstruct the **3D superfluid fraction**

(Sidorenkov et al. 2013)



First measurement of the T-dependence of the superfluid density **in a Fermi superfluid** (previously measured only in superfluid He4)

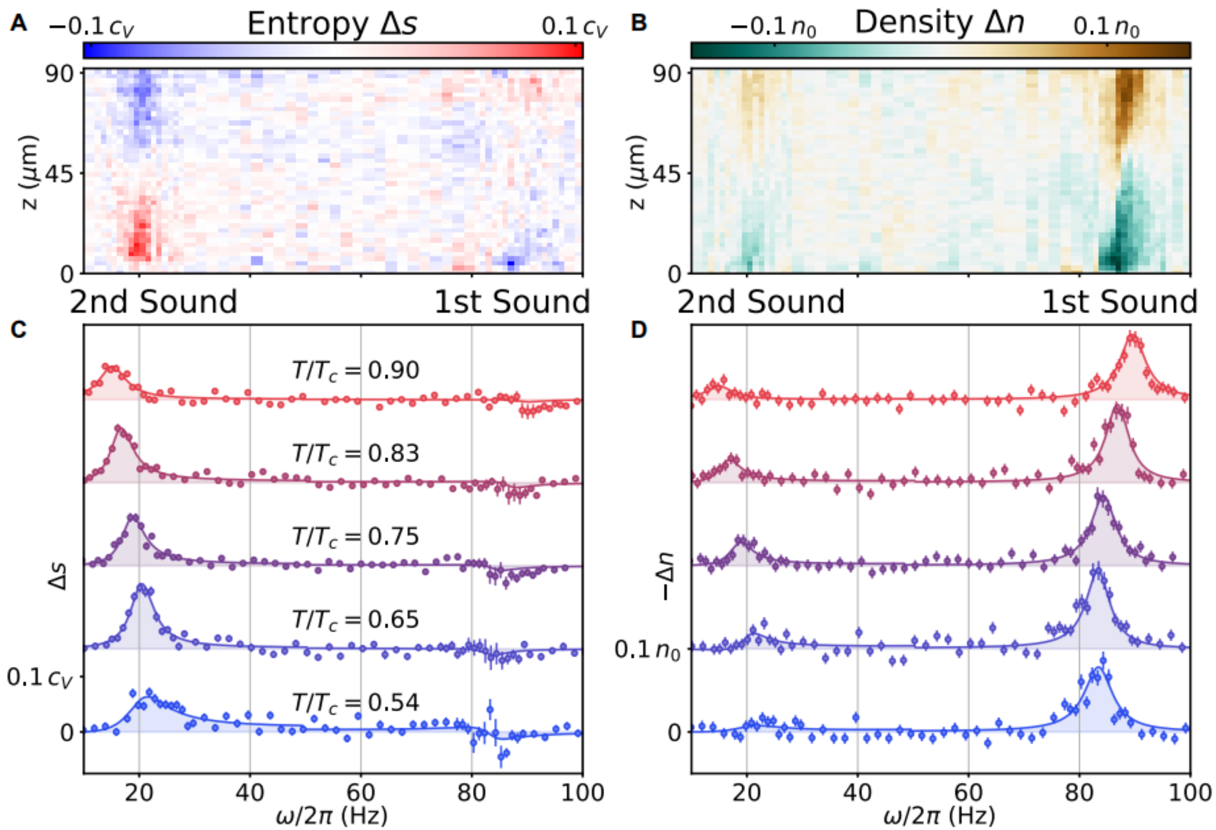
More systematic experimental investigation of the propagation of first and second sound in the unitary Fermi gas recently obtained at MIT using **thermography** techniques

Thermography of two-fluid hydrodynamics in a strongly interacting Fermi gas

Zhenjie Yan, P. B. Patel, B. Mukherjee, Ch. J. Vale, R.J. Fletcher, and M.Zwierlein1

By measuring time dependence of both local temperature and density one obtains direct experimental evidence that **second sound is an entropy wave**, to be compared with **isoentropic nature of first sound**

$$\Delta s = c_V \left(\frac{\Delta T}{T} - \frac{2}{3} \frac{\Delta n}{n_0} \right).$$



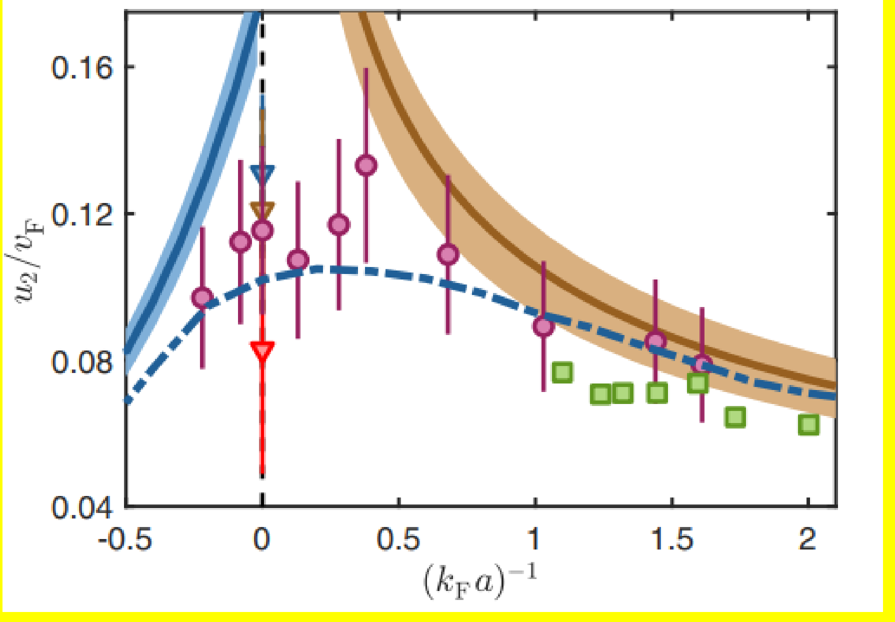
Proof of entropy and density nature of second and first sound, respectively

Zhenjie Yan et al. (MIT, in preparation)

Nature Communications (2021)

Second sound in the crossover from the Bose-Einstein condensate to the Bardeen-Cooper-Schrieffer superfluid

Daniel K. Hoffmann[1], Vijay Pal Singh[2,3], Thomas Paintner[1], Manuel Jäger[1], Wolfgang Limmer[1], Ludwig Mathey[3,4] & Johannes Hecker Denschlag[1]✉

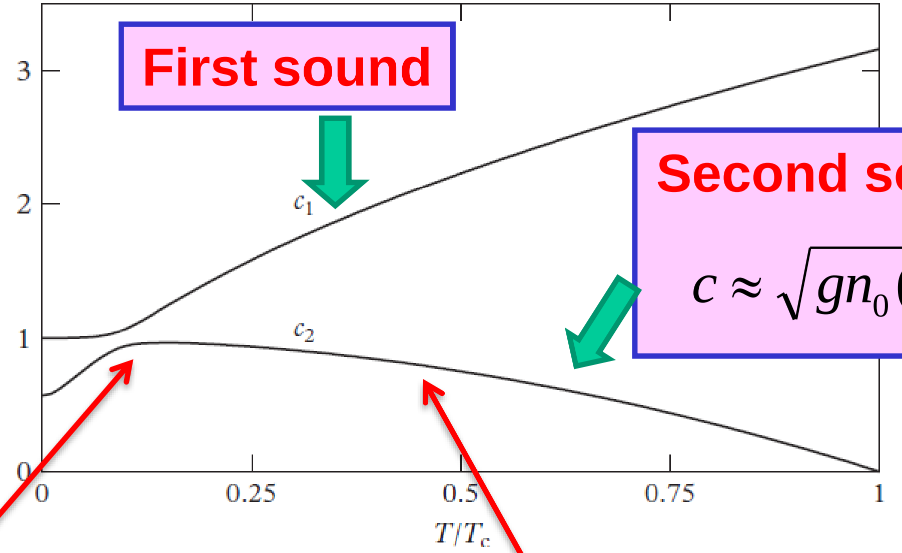


Can second sound propagate
in a weakly interacting Bose gas ?

Weakly interacting **3D Bose gas** is highly **compressible** and behaves **differently** from **Helium** and **Unitary Fermi gas**

- **Superfluid density** coincides with BEC condensate except at very small T and near transition
- **First sound**: oscillation of **thermal** component
- **Second sound**: oscillation of the **condensate**

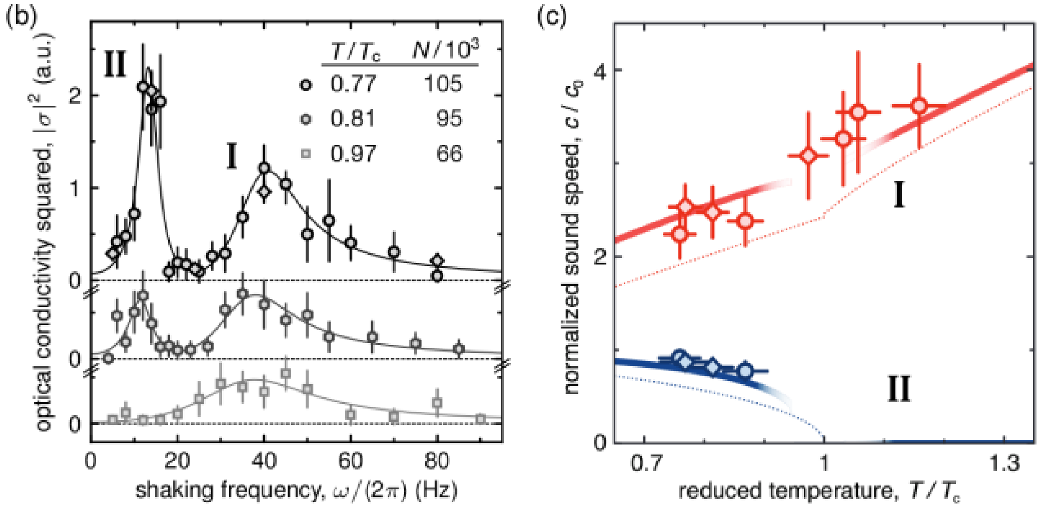
Theory:
Griffin, Nikuni, Zaremba, Pitaevskii, Stringari,….



Hybridization between the two sounds
(Lee and Yang,1959, Vernay et al. 2015)

Continuation of T=0 Bogoliubov sound
First measurement by Meppelink et al. 2009

Theoretical predictions for first and sound velocities in **3D BEC gas** very recently confirmed in Cambridge using a 39K with **large scattering length** to ensure HD collisional regime



Hilker et al., arXiv: 2112.14763

What happens to second sound in a 2D Bose gas ?

2D weakly interacting Bose gas

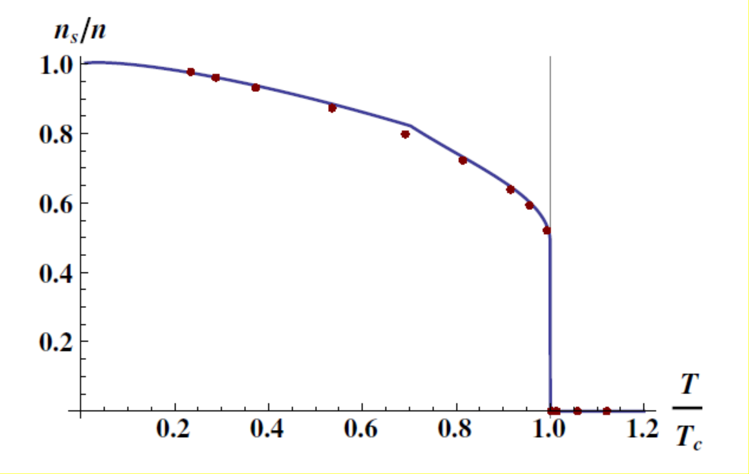
- **Absence of Bose-Einstein Condensation** at finite T (Hohenberg-Mermin-Wagner theorem)

- **Superfluid density** exhibits a **jump** at the Berezinskii - Kosterlitz - Thouless (BKT) transition while all thermodynamic functions are continuous (phase transition of infinite order)

Temperature dependence of **superfluid density**

$$g = \sqrt{8\pi} a / a_z^{ho} = 0.1$$

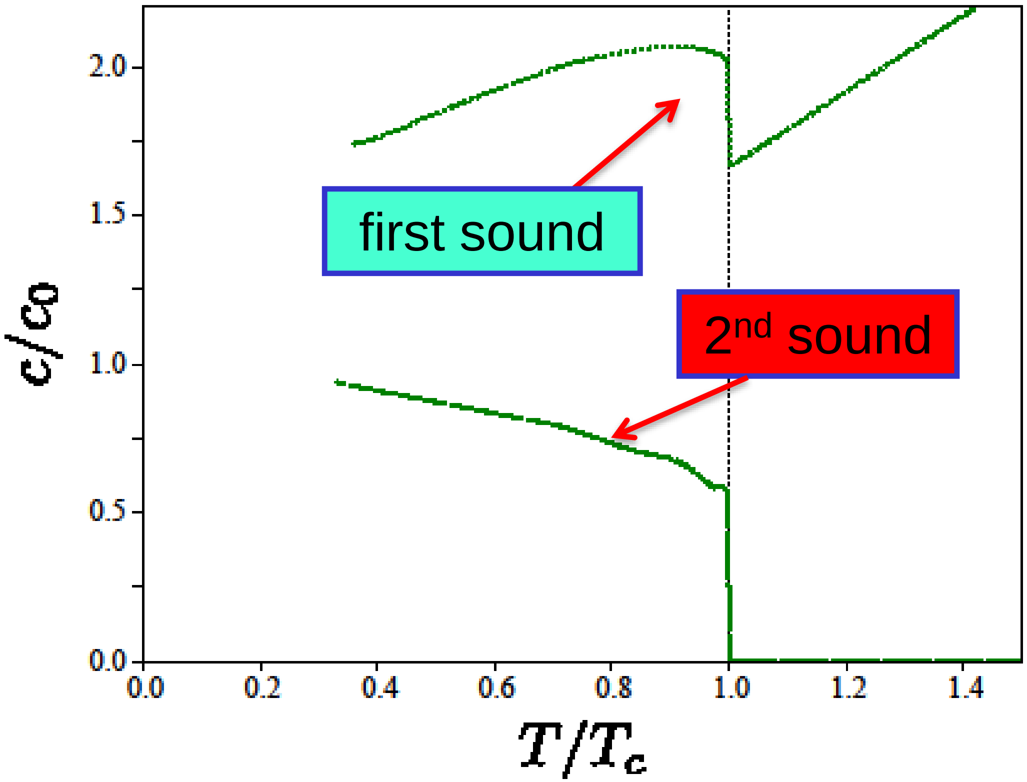
Prokofeev and Svistunov 2001)



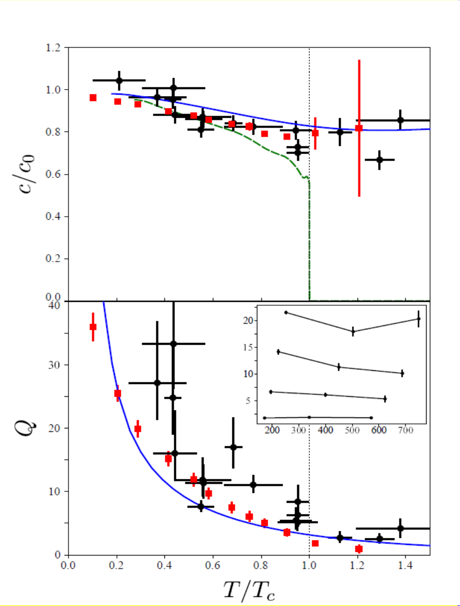
- Nelson-Kosterlitz relationship (1977) $k_B T_C = \pi\hbar^2 n_S / 2m$ between critical temperature and superfluid density at the transition

Prediction for second sound in a 2D Bose gas

As a consequence of discontinuity of superfluid density **both first** and **second** sound in a 2D Bose gas are **discontinuous** at the BKT transition (T. Ozawa and S.S, PRL 112, 025302 2014)



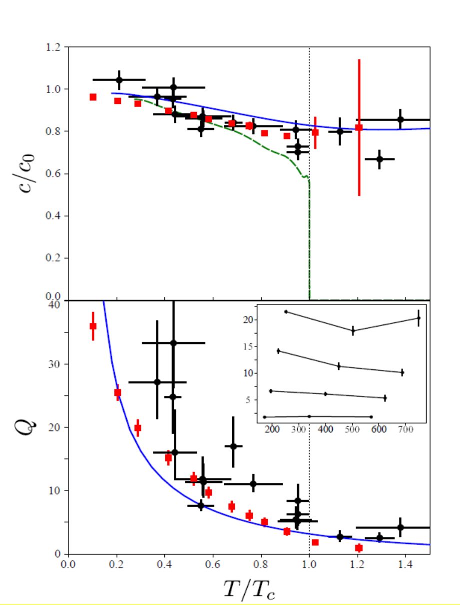
First experiments on sound propagation in 2D dilute Bose gases failed to observe second sound because the collisional HD regime is difficult to achieve without the use of Feshbach resonances.



Collisionless sound was actually **observed** at finite T also **beyond Tc** (large **Landau** damping)

- exp: J. Ville et al. PRL 121, 145301 (2018)
- theory: M. Ota et al. PRL 121 145302 (2018)

First experiments on sound propagation in 2D dilute Bose gases failed to observe second sound because the collisional HD regime is difficult to achieve without the use of Feshbach resonances.



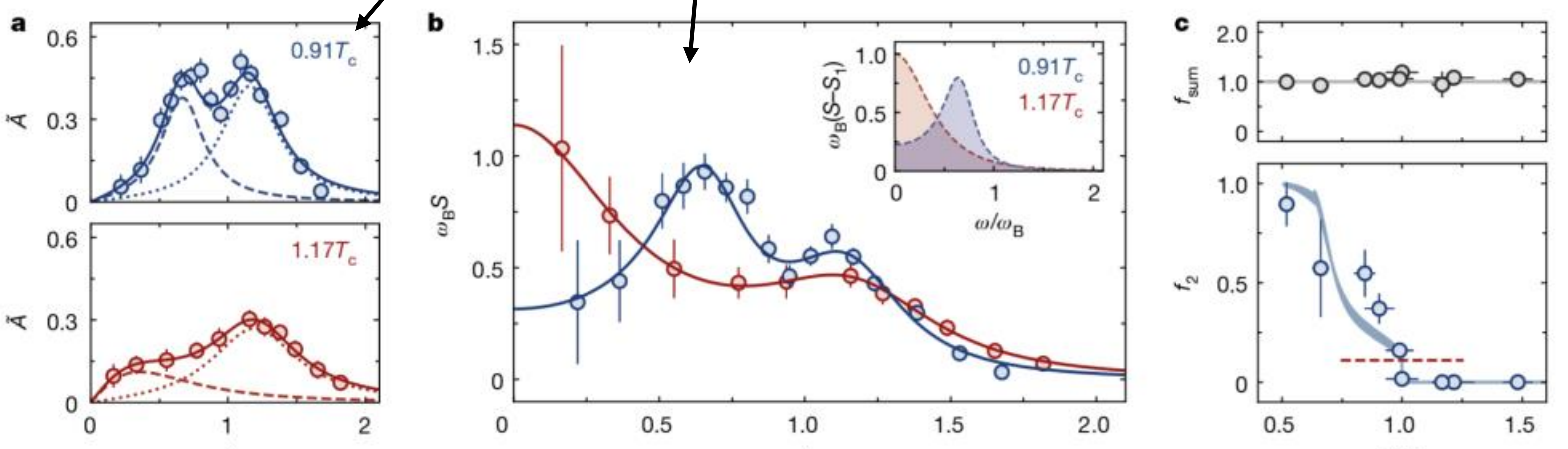
Collisionless sound was actually **observed** at finite T also **beyond Tc** (large **Landau** damping)
- exp: J. Ville et al. PRL 121, 145301 (2018)
- theory: M. Ota et al. PRL 121 145302 (2018)

The **collisional HD** regime in a 2D Bose gas was recently **achieved** by the **Cambridge team** (Christodoulou et al. Nature 594, 191 (2021)) using a 39K gas with large 2D coupling constant $g = \sqrt{8\pi}(a/a_z) = 0.64$ as compared to value $g = 0.17$ of previous Paris Rb experiment

Second sound measured in 2D

An applied spatially **uniform driving force** of the form $F \propto \sin(\omega t)$ **excites** the lowest **phonon mode(s)** with wavevector equal to $q = \pi / L$

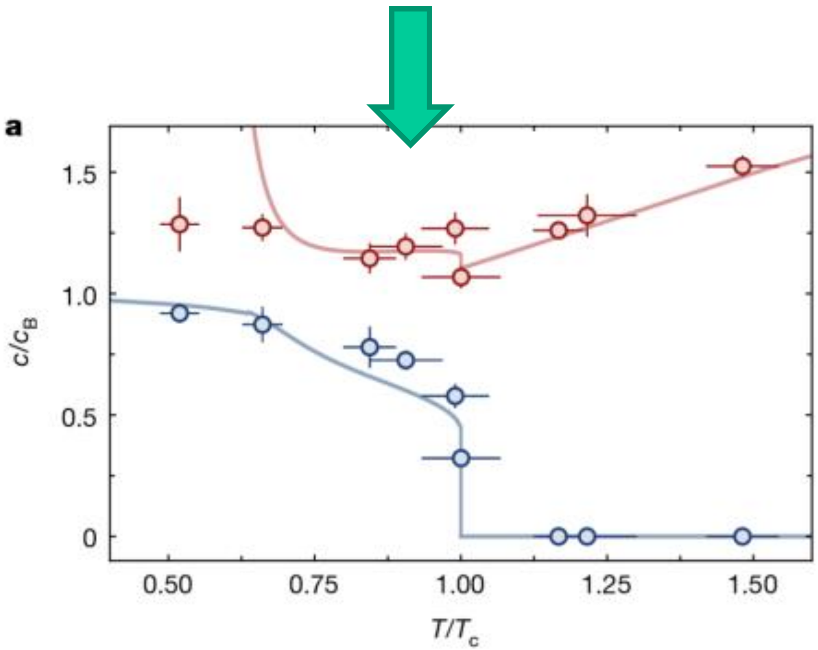
From the measurement of the resulting perturbation one extracts the imaginary part A of the response and the dynamic structure factor $S = k_B T A / \omega$, below (**blue**) and above (**red**) critical temperature

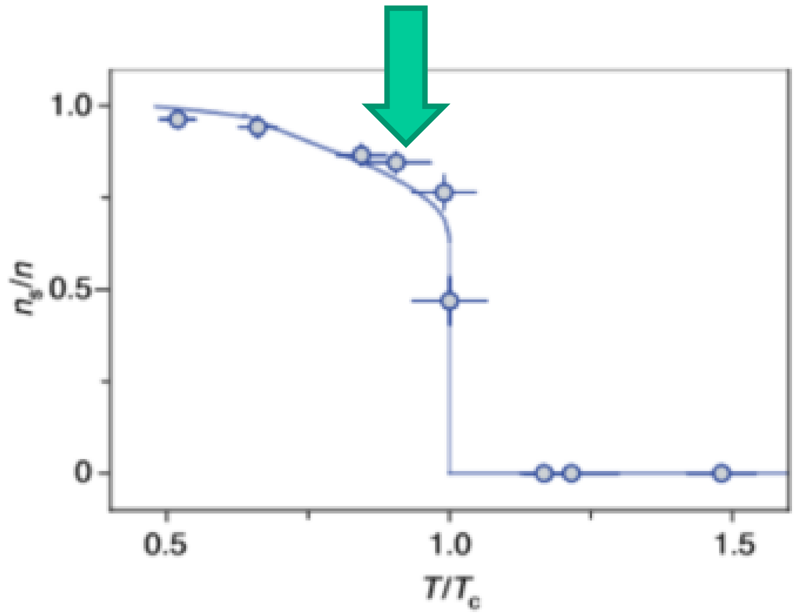


Below Tc one finds **two resonances** (first and second sound)
Above Tc the **low frequency signal** has clear **diffusive** nature.

Measured values of **first** and **second** sound velocities

T-dependence of superfluid density extracted from measured sound velocities





First experimental confirmation (Christodoulou et al. Nature, 2021) of the predicted (Ozawa and S.S.,PRL 2014) **jump of second sound** velocity **at the BKT transition**

Second part: T=0

Superfluid density at zero temperature in systems breaking Galilean invariance

In a **Galilean** invariant superfluid the **superfluid density coincides with total density at zero temperature**.

We consider two examples of systems breaking Galilean invariance along x-direction.

1) 3D Superfluid in the presence of an external 1D periodic potential of the form $V(x) = V_0 \cos(2\pi x / L)$

2) 3D Bose-Einstein condensed gas with 1D spin orbit coupling along x

$$h_{SO} = \frac{(p_x - \hbar k_R \sigma_z)^2}{2m} + \frac{p_\perp^2}{2m} + \frac{\hbar \Omega_R}{2} \sigma_x .$$

In both cases **superfluid density** along x-direction **is quenched,** reflecting the **dynamic anisotropy** of the problem

The **T=0 superfluid fraction** $f_S = \rho_S / \rho$ along the x-direction can be determined (**theoretically and experimentally**) by measuring the **sound velocities** along **x** and **y** directions (superfluid flow unaffected by external potential along y)

$$mc_x^2 = f_S \bar{n} \partial_{\bar{n}} \mu$$
$$mc_y^2 = \bar{n} \partial_{\bar{n}} \mu$$

$$f_S = c_x^2 / c_y^2$$

G. Martone and S.S. SciPost Phys. 11, **092** (2021)

We have further compared the ratio $f_S = c_x^2 / c_y^2$ with the **Leggett's upper bound**

$$f_s^L = \left(\frac{1}{L} \int_0^L dx \frac{\bar{n}}{n(x)} \right)^{-1}$$

Journal of Statistical Physics, Vol. 93, Nos. 3/4, 1998

On the Superfluid Fraction of an Arbitrary Many-Body System at $T=0$

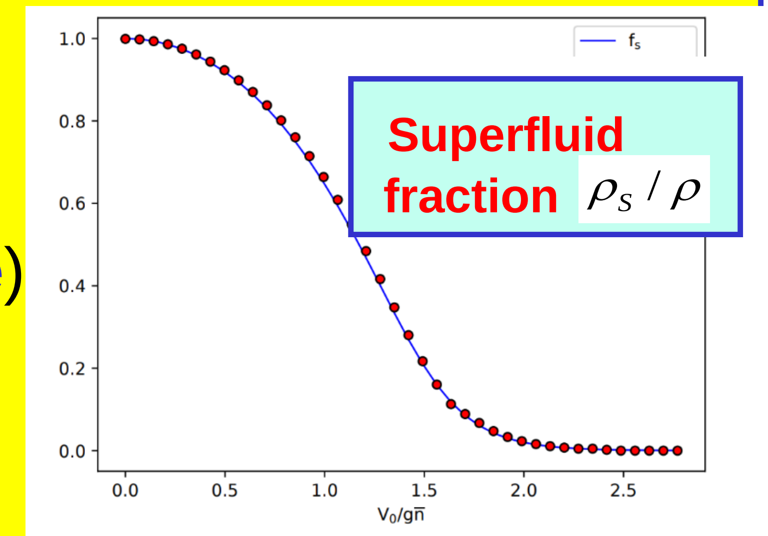
A. J. Leggett[1]

Leggett's upper bound is

- $f_s^L = 1$ for uniform configurations
- $f_s^L = 0$ if density contrast is total

BEC gas in presence of periodic potential $V(x) = V_0 \cos(2\pi x / L)$ at T=0

Superfluid fraction (along x) calculated through the square c_x^2 / c_y^2 of the ratio of sound velocities (**red**) and Leggett bound f_s^L (**blue**)



(Santo Roccuzzo et al. in preparation

Agreement is excellent for all values of $V_0 / g\bar{n}$, showing that Leggett's bound coincides with superfluid fraction in systems with a single phase in the many-body wave function.

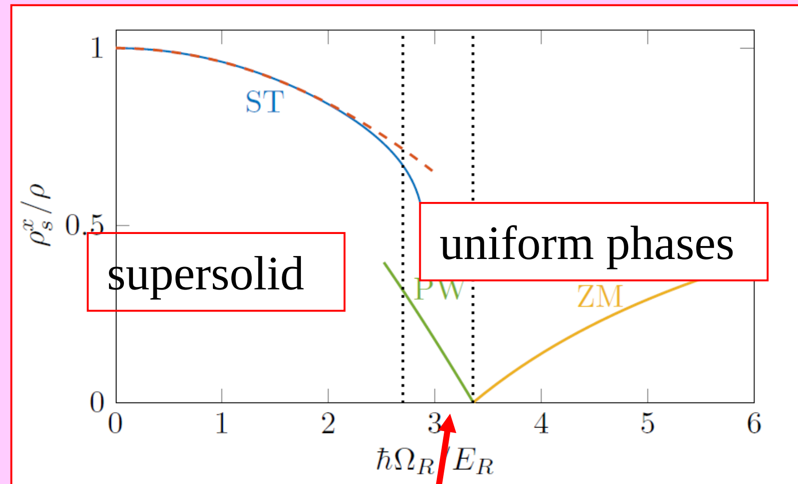
Present collaboration with Dalibard's team in Paris aimed to measure sound velocities c_x, c_y and Leggett's integral and confirm theory predictions for superfluid fraction.

BEC gas in presence spin-orbit coupling (T=0)

Superfluid fraction calculated through the square of the ratio c_x^2 / c_y^2 as a function of Raman coupling across the three quantum phases (supersolid stripe (**ST**), Plane Wave (**PW**) and Single Minimum (**SM**)) predicted
By SOC Hamoltonian

G. Martone and S.S.
(SciPost Phys. 11, **092** (2021)

[See also
Y.C. Zhang et al. (PRA 2016)
X.L. Chen et al. (PRA 2018)
J. Sanchez-Baena et al. (PRA 2020)]



Results are particularly striking near the transition between **PW** and **SM** phases, where the system is **uniform** but the **superfluid density is vanishingly small** (Y.C. Zhang et al. (PRA 2016))
(uselessness of Leggett's upper bound $f_s^L = 1$)

Main conclusions

- Recent experiments on sound propagation in quantum gases confined in box potentials have opened new perspectives for the measurement of superfluid density in systems of different dimensionality (including BKT transition)

- At T>0 relevant information arises from second sound
- At T=0 relevant effects concerns the reduction of superfluid fraction in systems breaking Galilean invariance

Main conclusions

- Recent experiments on sound propagation in quantum gases confined in box potentials have opened new perspectives for the measurement of superfluid density in systems of different dimensionality (including BKT transition)
- At T>0 relevant information arises from second sound
- At T=0 relevant effects concerns the reduction of superfluid fraction in systems breaking Galilean invariance

Some perspectives

- Behavior of second sound in 2D Fermi superfluids
- Sound and superfluid density in quantum mixtures (applicability of 3 fluid hydrodynamics)
- Role of disorder and propagation of sound in 1D systems
- Applicability of Leggett's upper bound in Fermi superfluids