Response Functions in Cold atoms and NP
Few to Many-Body and Analogs

J. Carlson (LANL)

Collaborators:
Schiavilla, Gandolfi, Gezerlis, Lovato, Pastore, Reddy, Rocco, Schmidt, Zhang, …

Outline:

- Brief Review of some static properties of the Unitary Fermi Gas
- Review of Dynamics and Response Functions
- Low Momentum transfer
  - rf response and spin response/ν emissivity in NM
- High Momentum Transfer
  - Bragg Scattering compared to e/ν scattering

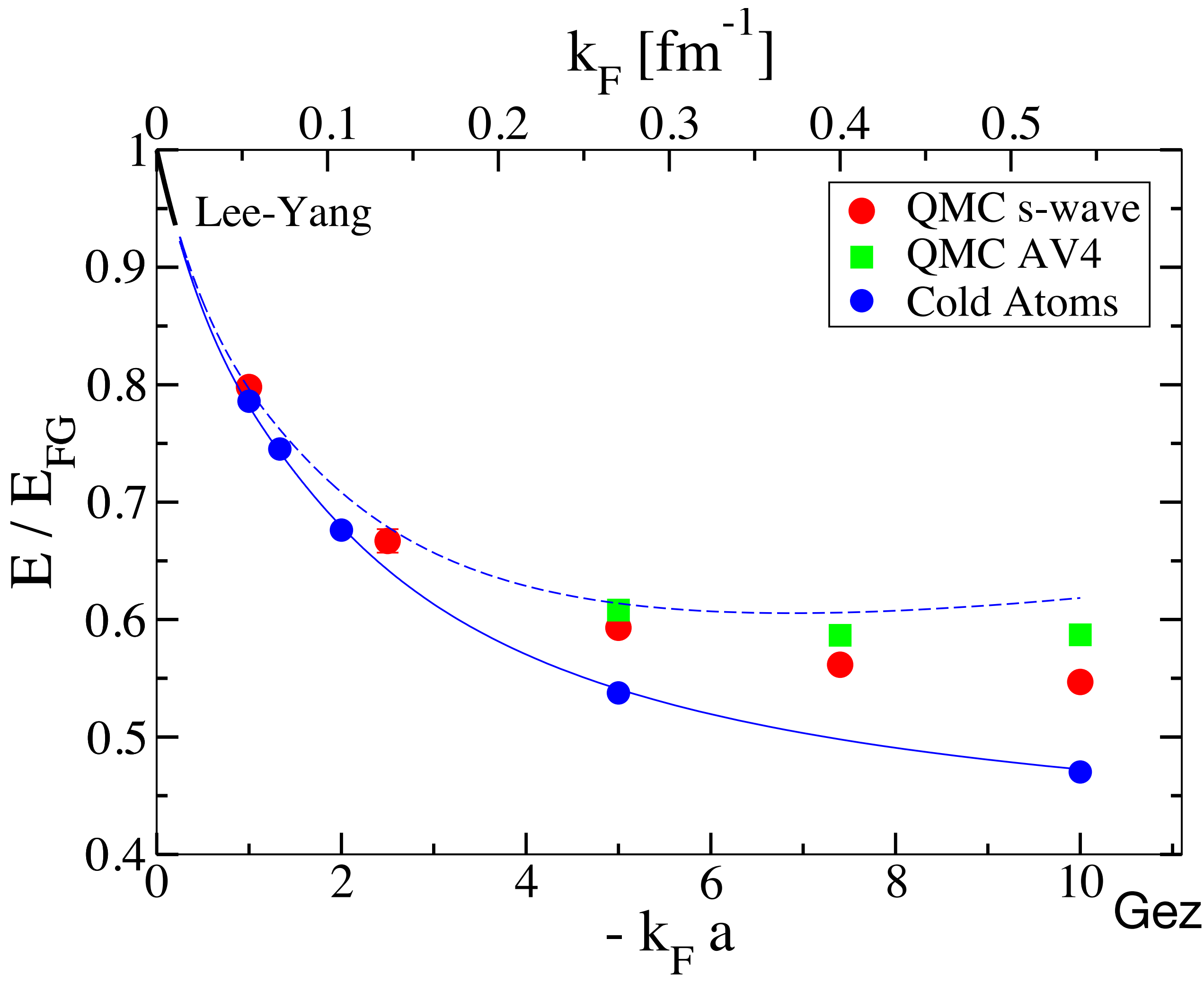
Equation of State
Neutron Matter vs. Cold Atoms

Dilute Neutron Matter vs UFG

Similarity - large scattering length

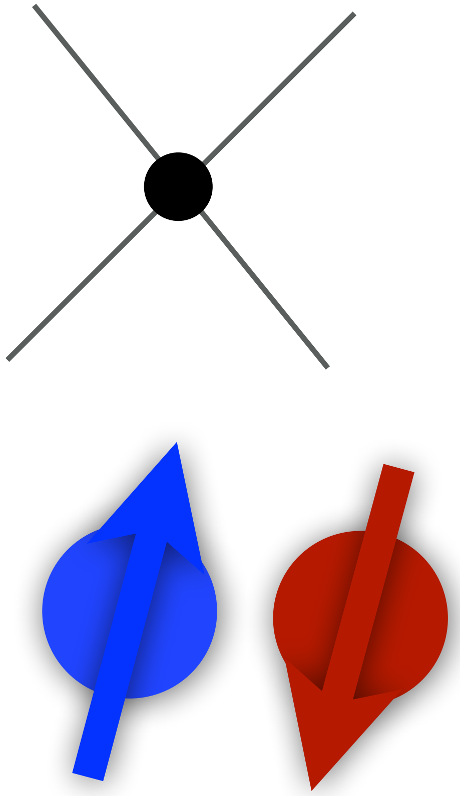
Differences- effective range, p-wave

Neutrons have little interaction in p-waves



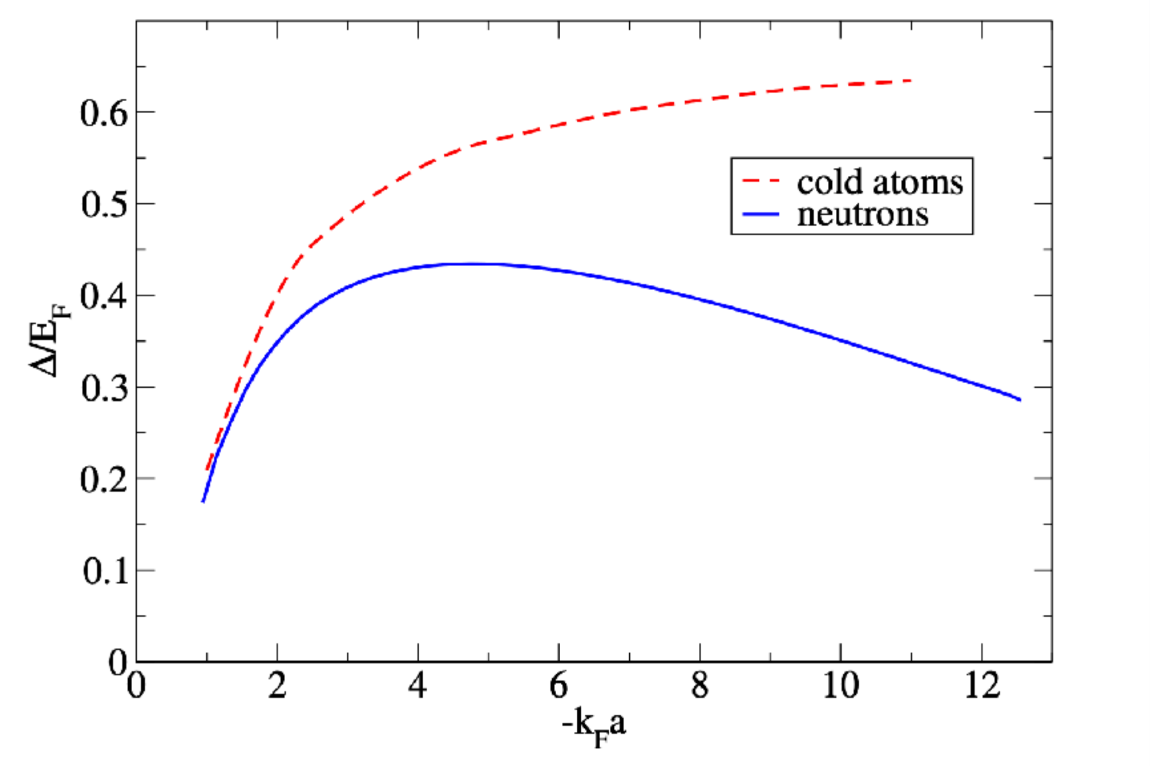
Gezerlis, JC, PRC 2010

Unitary Fermi Gas:
Static Properties



- Density functional from many-body calculations of static response/ neutrons in external field

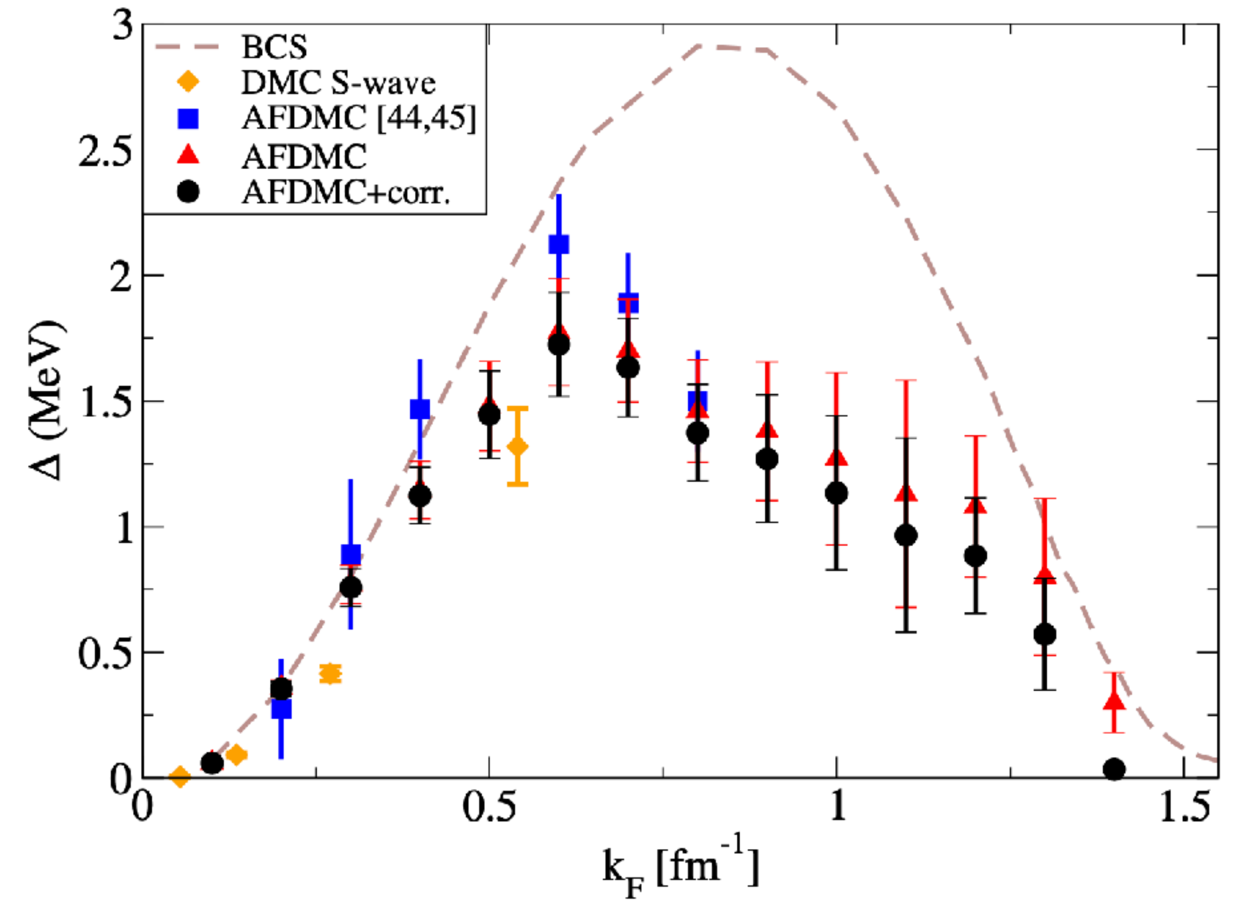
$^1S_0$ gap in neutron matter vs. cold atoms



Cold atoms vs. NM

Gap / E_F

Neutron Matter

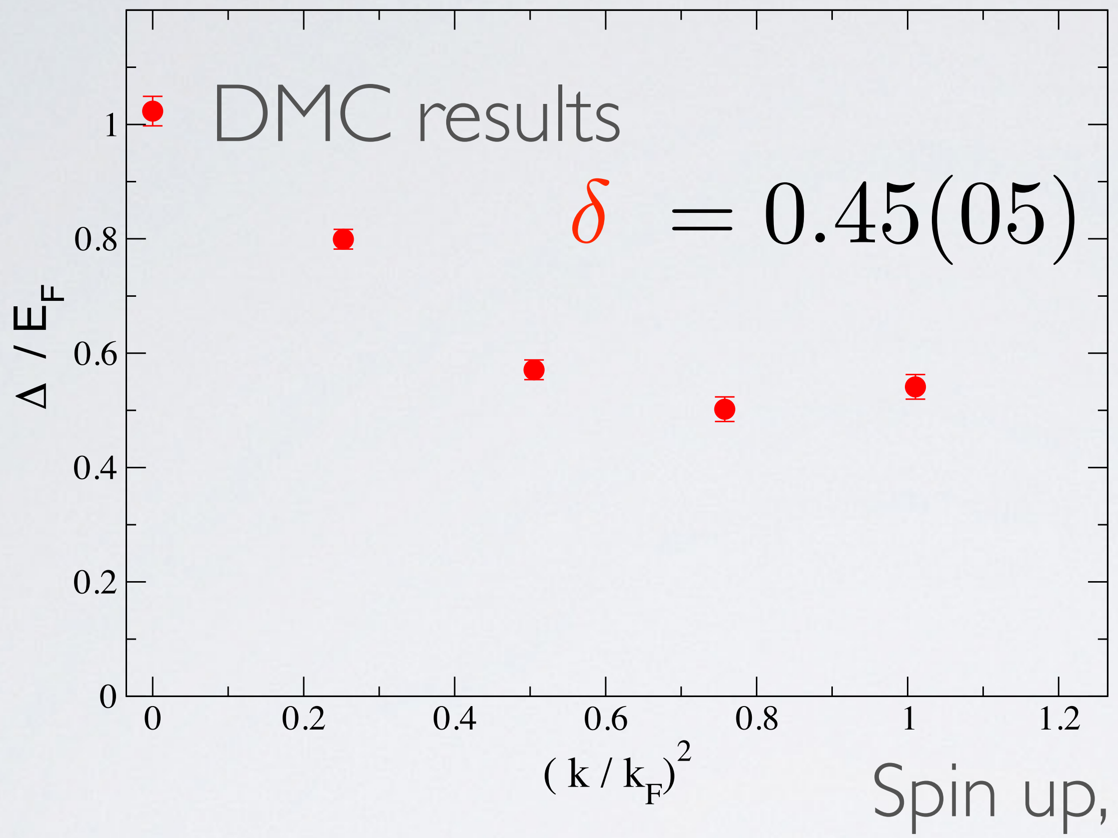


agreement between

different MC methods

Gandolfi, Palkanoglo

Quasiparticle Dispersion and the Gap



$$\delta = 0.45(05)$$

Gap vs. k/k_F

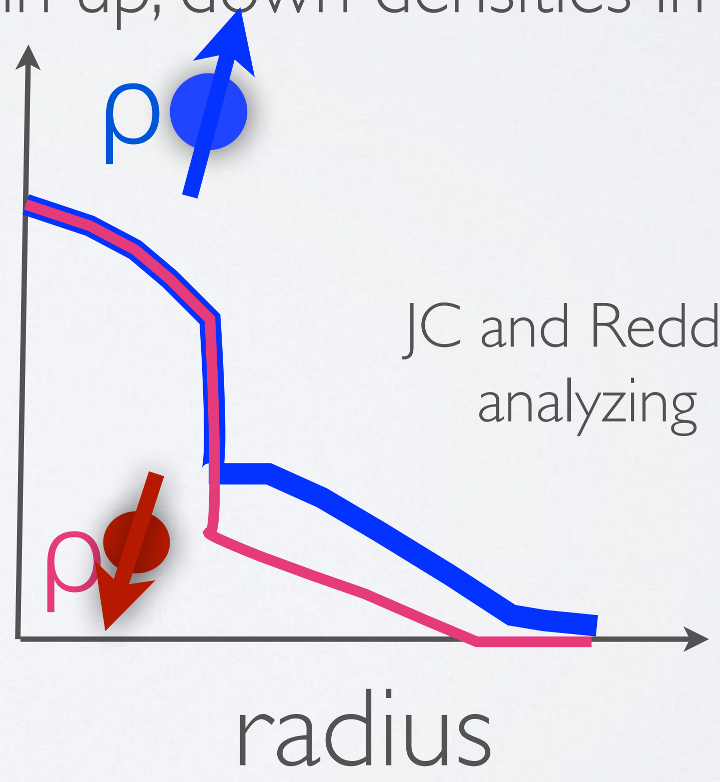
$$\Delta = \delta \frac{\hbar^2 k_F^2}{2m}$$

$$\delta = 0.50\ (03)$$

$$(k_{min}/k_f)^2 = 0.80(10)$$

JC and Reddy, PRL 2005

Spin up, down densities in a trap



JC and Reddy, PRL 2007
analyzing MIT data

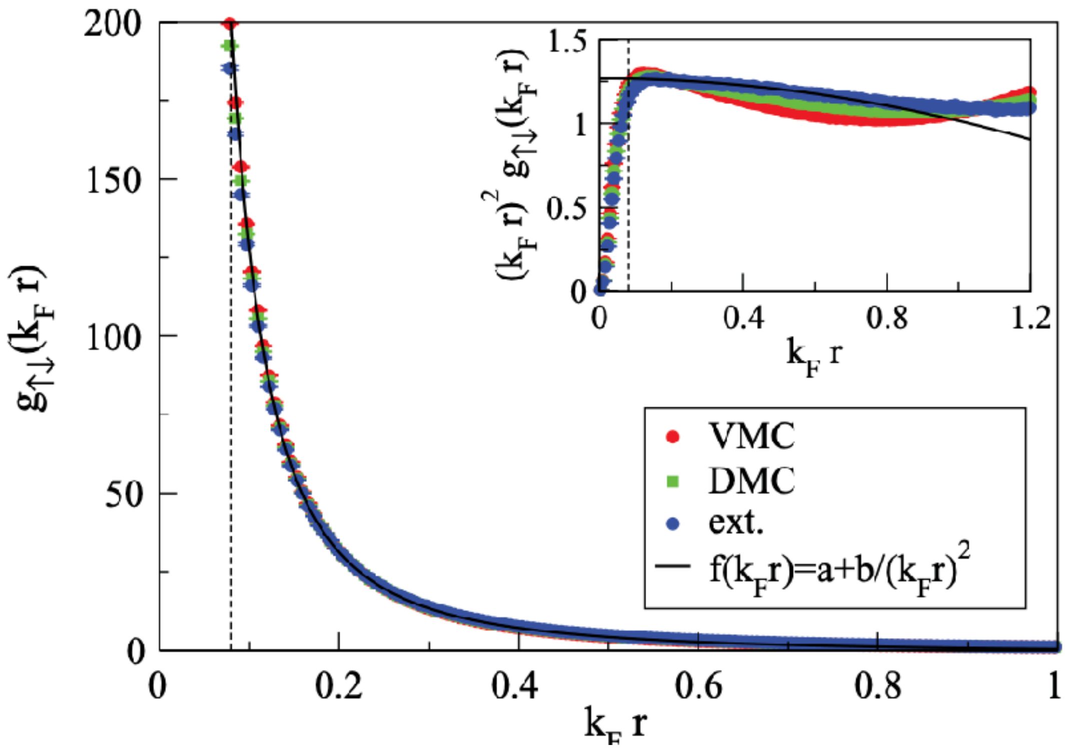
Tan’s contact (from QMC)

Gandolfi, Schmidt, JC; PRA 2011

Tan’s contact in UFG via MC at T=0

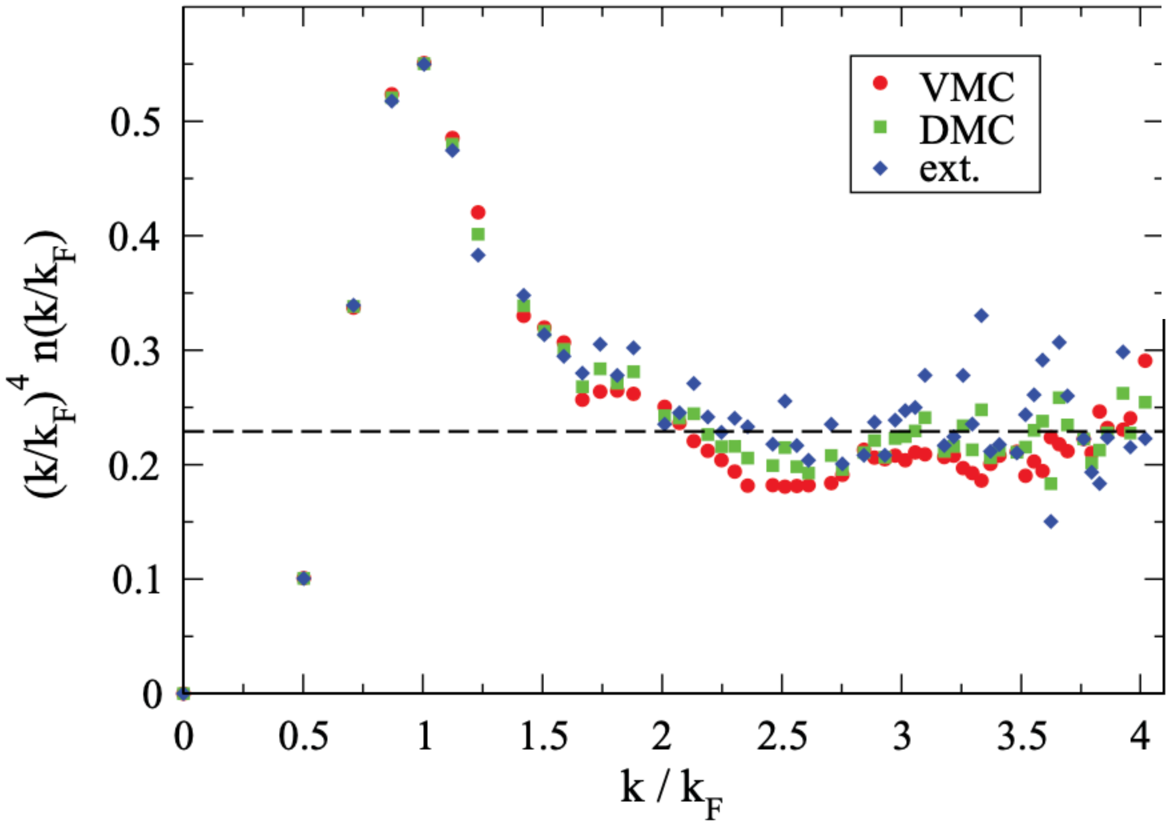
Similar concepts in NP

For e/ν scattering



pair distribution function

$k_F r <\sim 0.8$



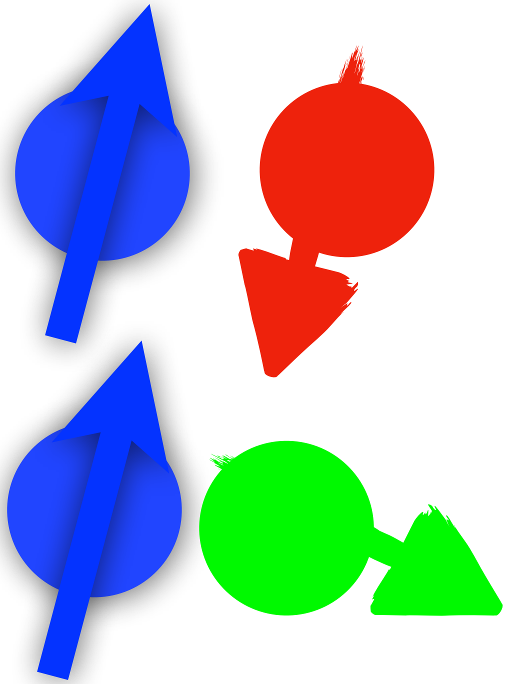
Also related to EOS, …

Momentum distribution function

Asymptotic form at $k >\sim 2 k_F$

Recent Review of RF and Bragg Spectroscopy:

Spectroscopic probes of quantum gases
Chris J. Vale and Martin Zwierlein, Nature Physics17, 1305–1315 (2021)



RF response: spin flip, essentially zero momentum transfer

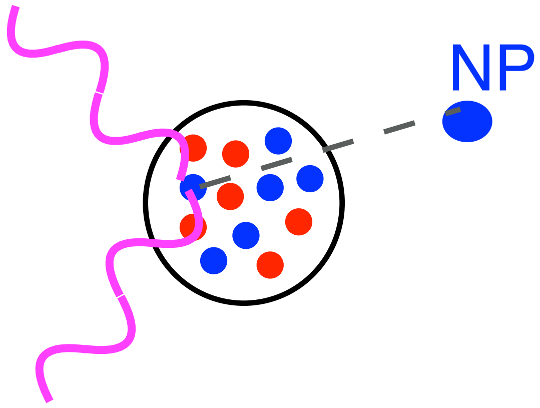
high frequency tail gives contact
beautiful measurements at different T
Can be obtained from spectral function

NP analogs to neutrino emissivity of neutron matter

spin flip response (to leading order)
q small (astrophysical energies) but not zero
But Hamiltonian flips/exchanges spins
in general, low E collective excitations (EW transitions, …)

Bragg spectroscopy: high momentum transfer

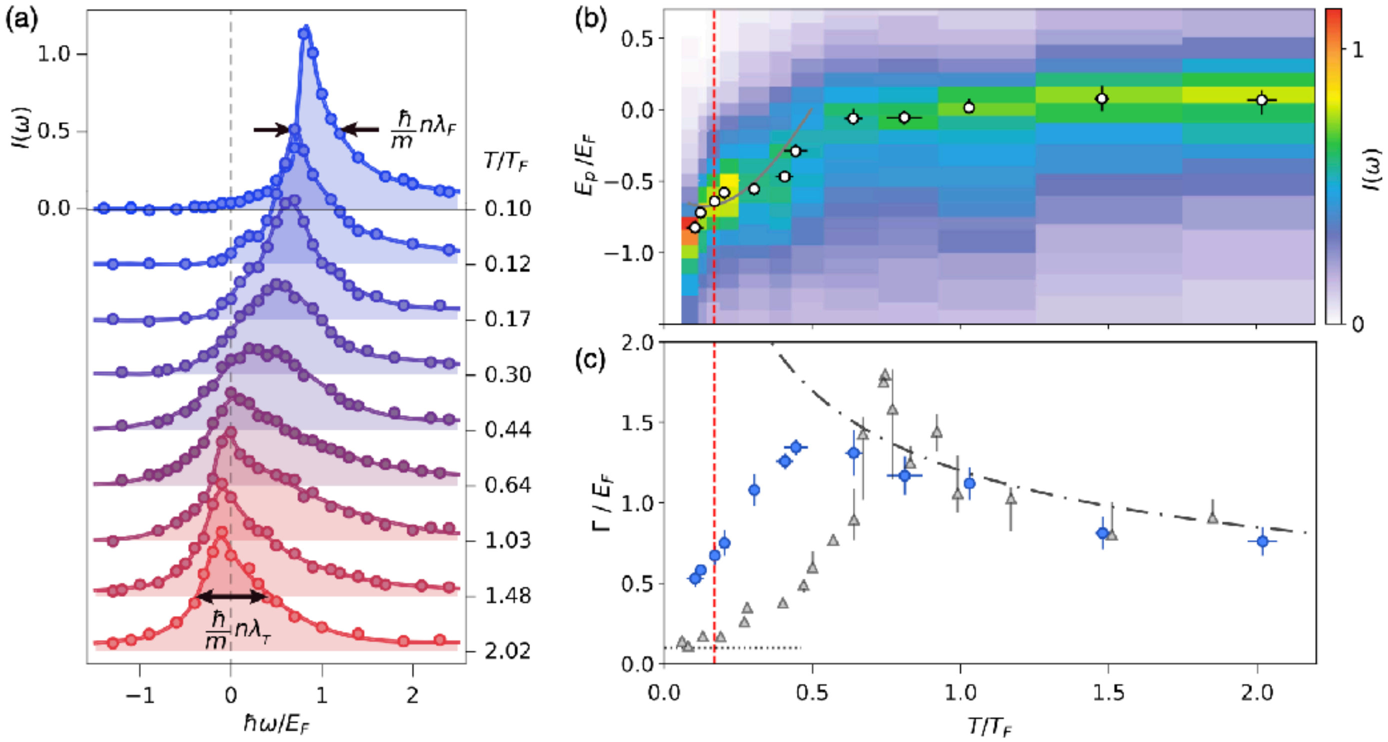
spin (parallel vs. anti-parallel) response can be resolved
Simple single-atom coupling
Can separate into parallel/anti-parallel spin response



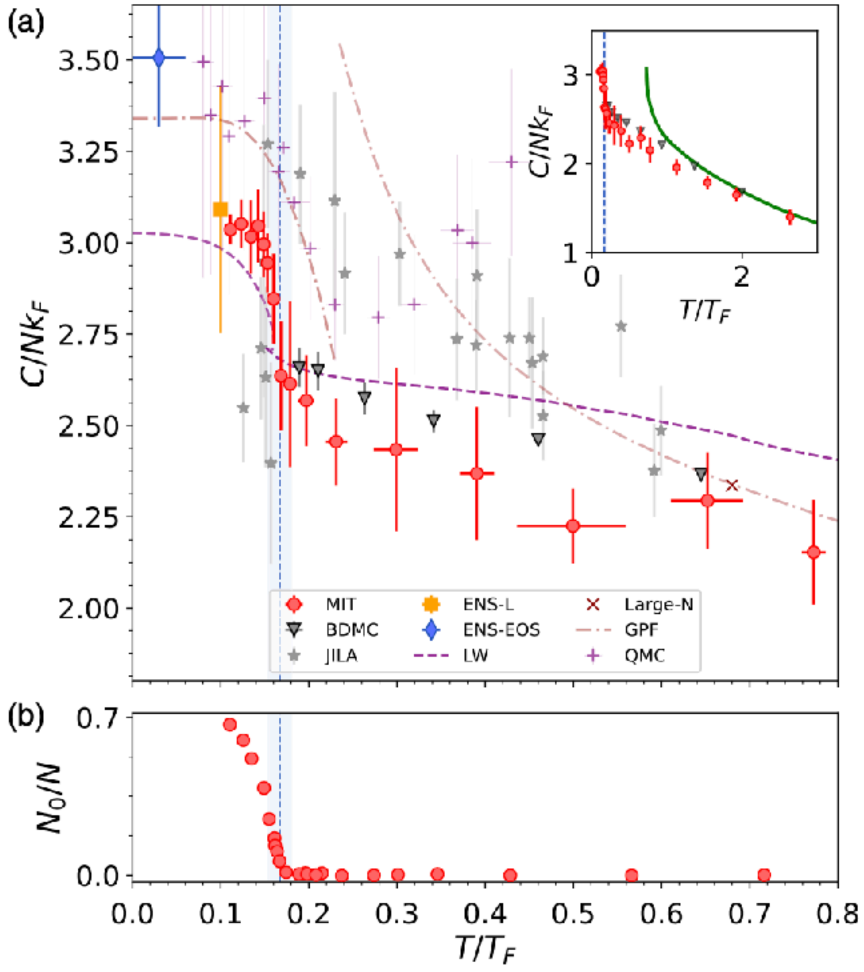
NP analogs to neutrino and electron scattering in QE regime

high momentum transfer
one- and two-nucleon couplings

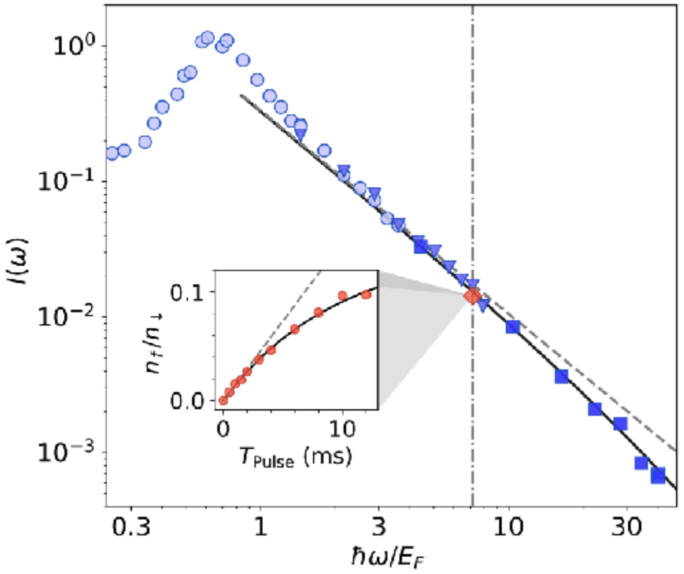
Zwierlein- RF Spectra & contact - PRL 2019
Low (near zero) q



RF response vs. temperature
(Single Peak: Narrow at low T**)**



Contact and condensate fraction vs. T



spectra at low T, peak near $\omega \approx k_F$

RF response: Analogy to neutrino emissivity in neutron matter (cooling of neutron stars)

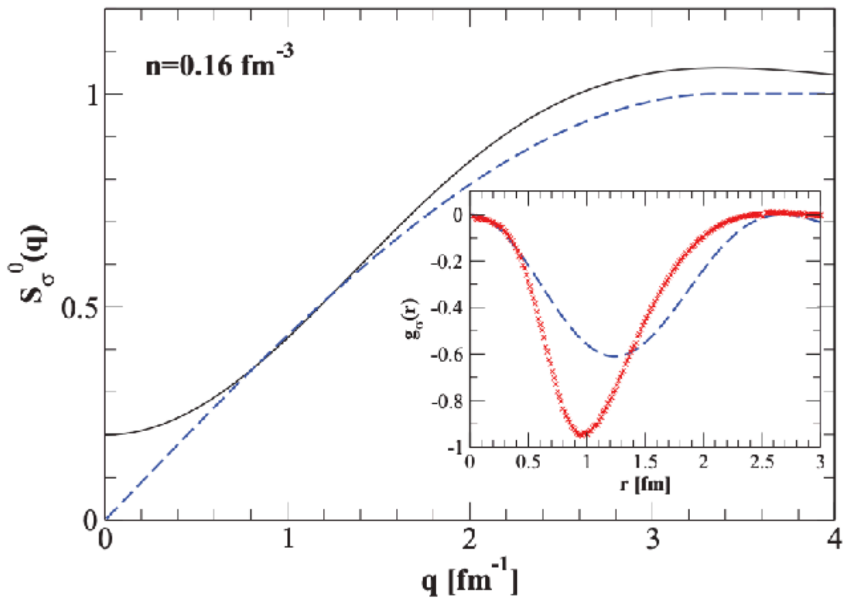
Compute spin susceptibility (1/H sum rule)
Sum Rule and Energy weighted sum rule

$$S_\sigma^{-1} = \frac{\chi_\sigma}{2n},$$

$$S_\sigma^{0} = 1 + \lim_{q\to 0}\frac{4}{3N}\sum_{i\neq j}^{N}\langle 0|e^{-i\mathbf{q}\cdot(\mathbf{r_i}-\mathbf{r_j})}\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j|0\rangle,$$

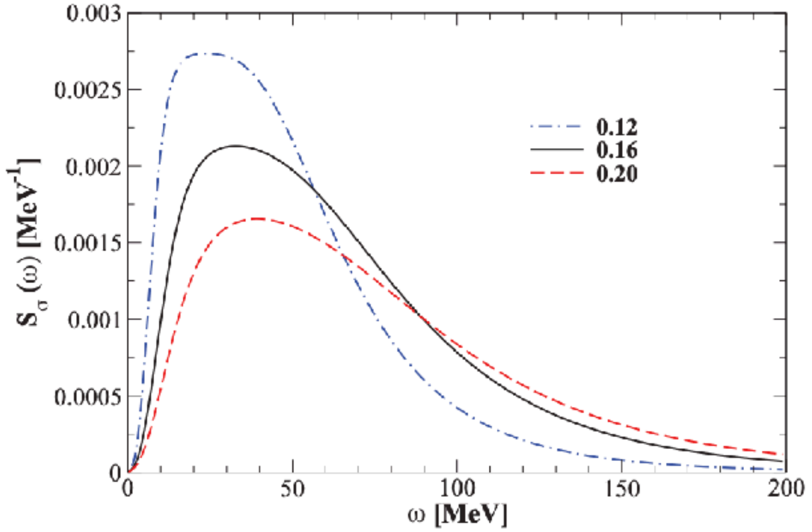
$$S_\sigma^{+1} = -\frac{4}{3N}\lim_{q\to 0}\langle 0|[H_N, \boldsymbol{s}(\mathbf{q})]\cdot\boldsymbol{s}(-\mathbf{q})|0\rangle,$$

Statics: structure factor S(q) in blue
Pair correlation function (spin-spin)
in red (inset)



Ignoring superfluid nature (T> Tc),
two-nucleon currents

Reconstructed response fns
vs. density (note assuming T > Tc



Energy loss rate:

$$Q = \frac{C_A^2 G_F^2 n}{20\pi^3}\int_0^\infty d\omega\,\omega^6 e^{-\omega/T} S_\sigma(\omega),$$

Want strength < 50 MeV

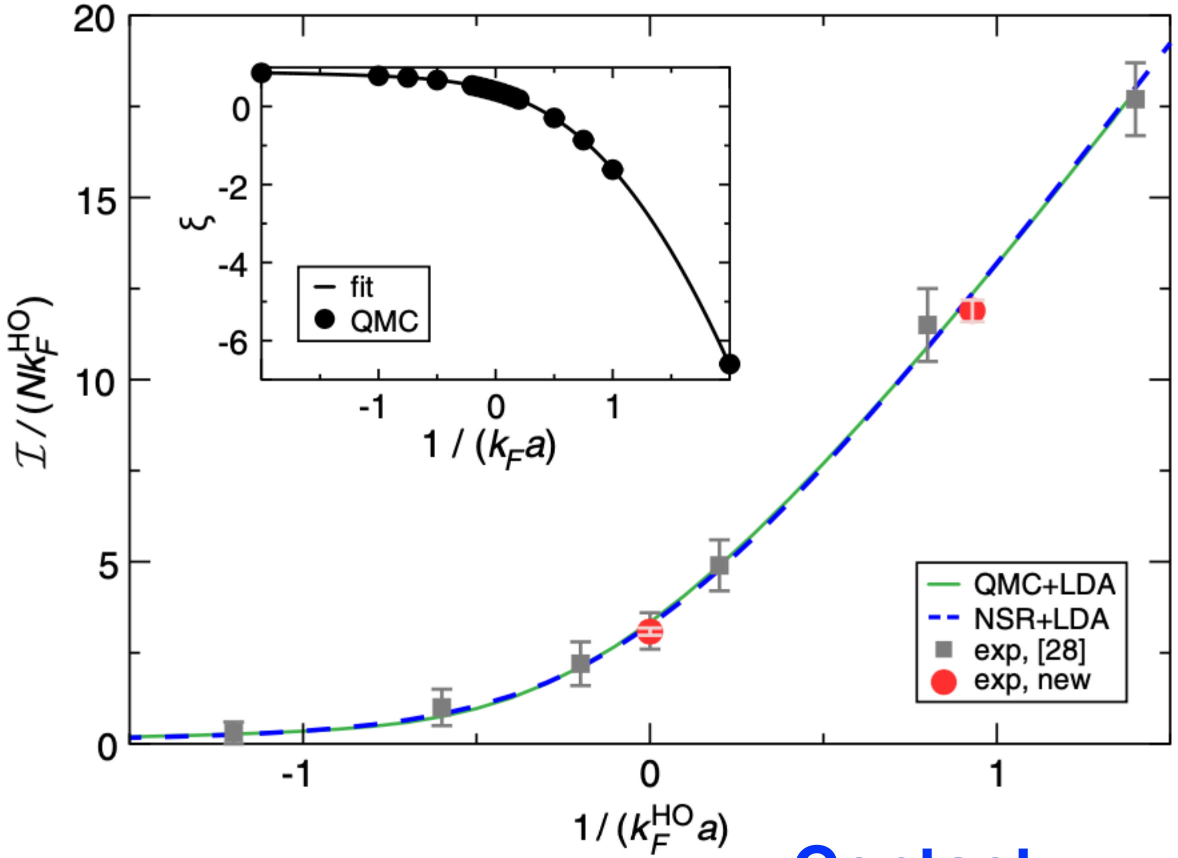
Similarity vs. RF response
Peak near E$_F$, large energy tail
but comes from spin exchange in H

Shen, Gandolfi, Reddy, JC; PRC 2013

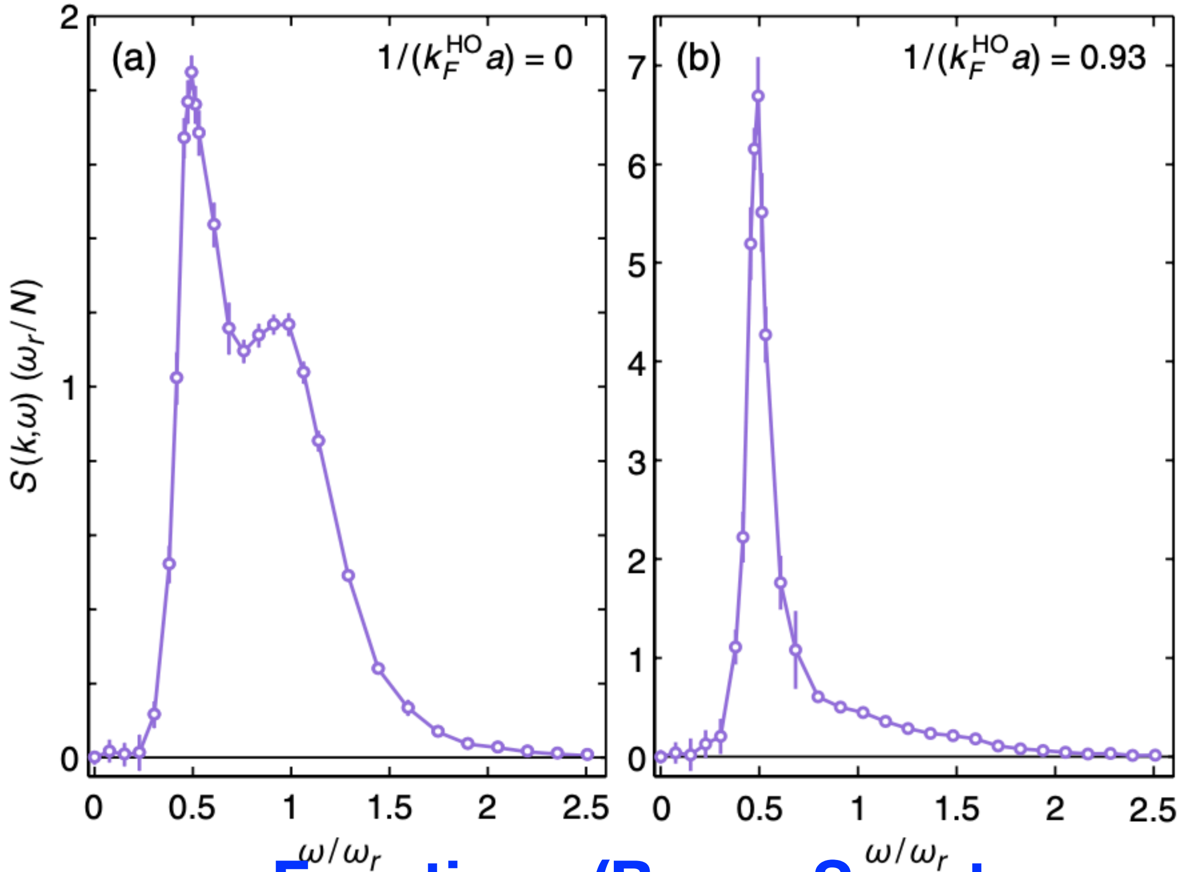
High Momentum Transfer Response (Bragg Spectroscopy)

Experiment: Bragg Spectroscopy and contact ($q \sim 5\ k_F$)

Extracted value at unitarity roughly consistent with experiment
Two distinct peaks near unitary region
Lower energy peak dominates in BEC regime

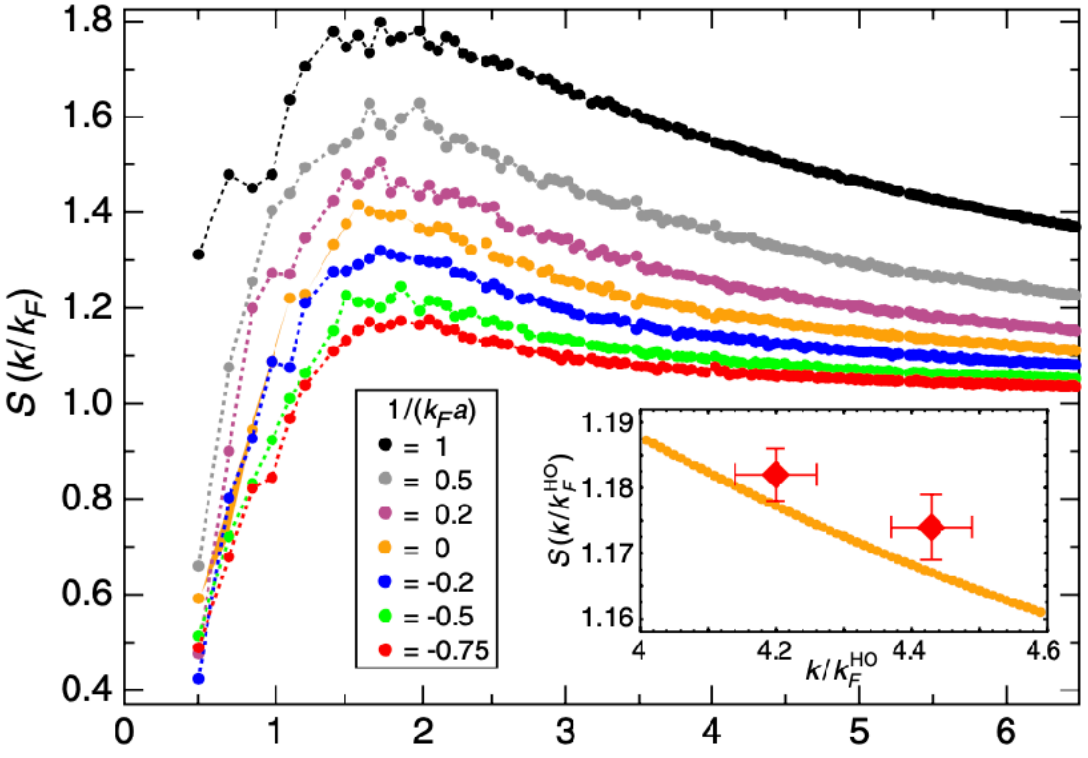


Contact



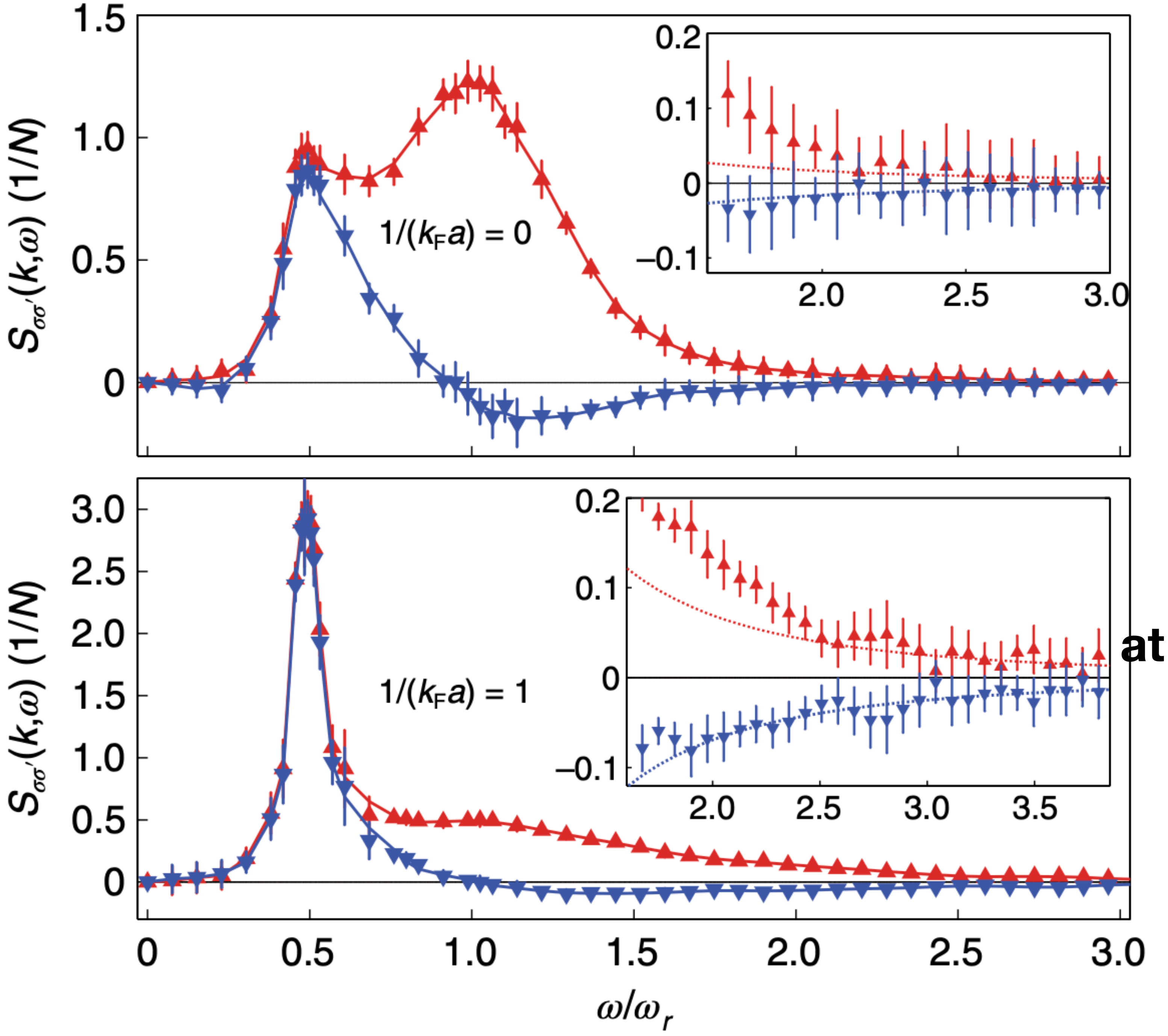
Response Functions (Bragg Spectroscopy
in general two peaks

Sascha Hoinka, Marcus Lingham, Kristian Fenech,
Hui Hu, Chris J. Vale, Joaquın E. Drut, and
Stefano Gandolfi, PRL(2013)



Static Structure Factor

Review article by Zwierlein and Vale
Nature Physics 2021



Bragg Spectroscopy
(Vale et al)

Parallel (red)
and anti-parallel (blue)
spin response

at unitarity (top) and in BEC regime
(bottom)

Basic Properties predicted by Tan in OPE

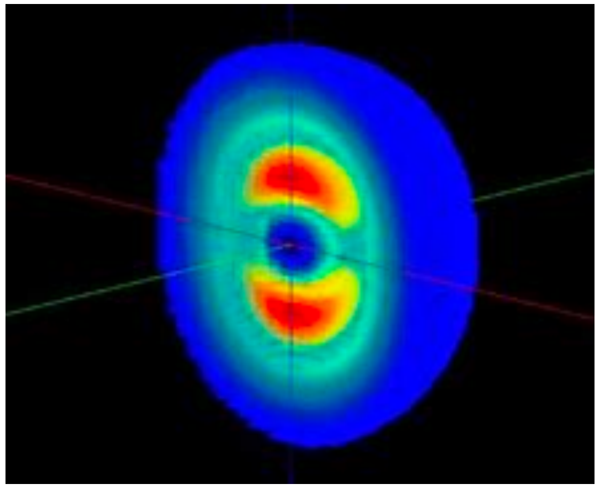
Son and Thompson: Short-distance and short-time structure of the Unitary Fermi Gas
PRA 81, 063634 (2010)
OPE for unitary Fermions; many other papers since

Why study electron scattering?

not to determine properties of electron or photon

Quasi-elastic scattering: higher q, E

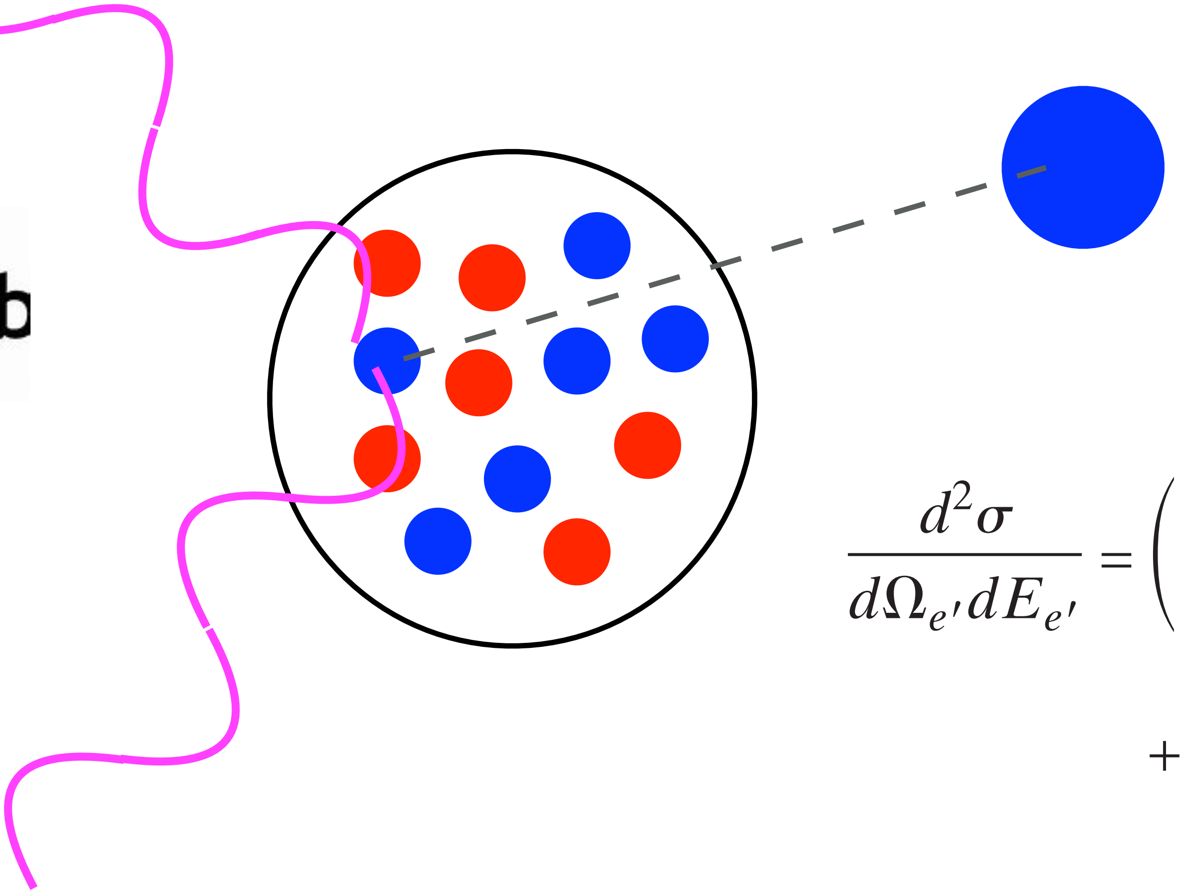
$q \sim k_F \quad E \sim E_F$

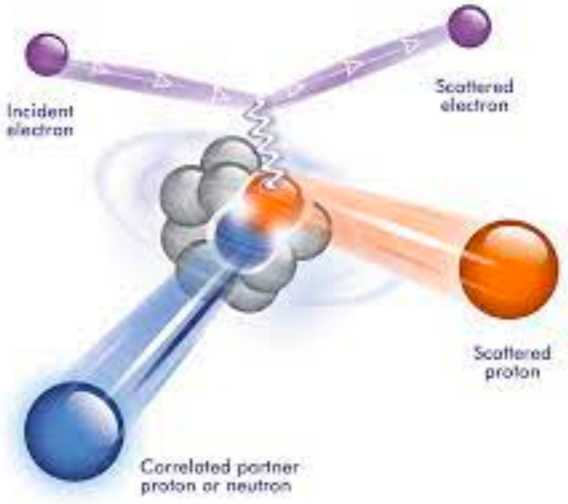


Electron Scattering: 2 response functions

Neutrino/Antineutrinos: 5 response functions

Jefferson Lab

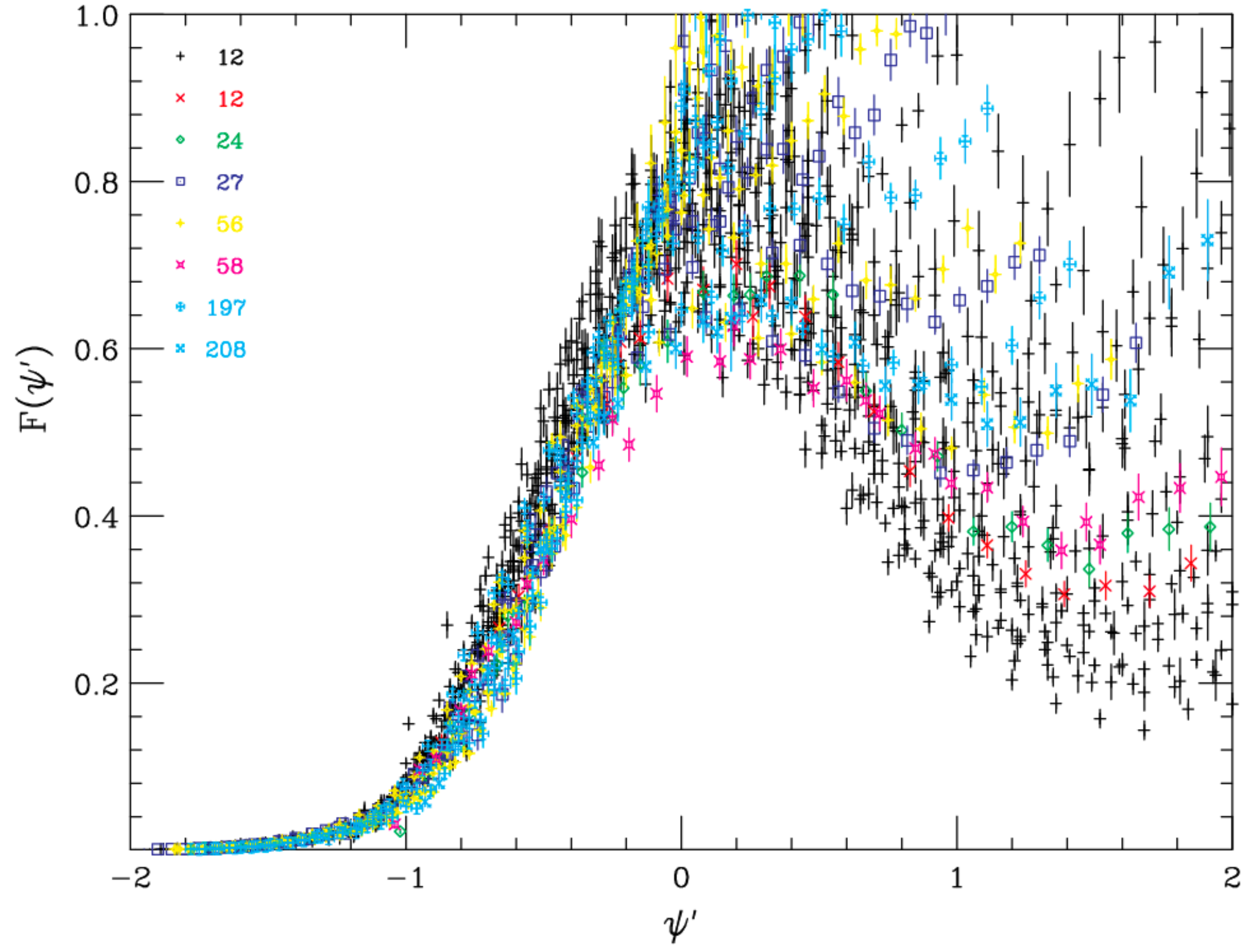




$$\frac{d^2\sigma}{d\Omega_{e'}dE_{e'}} = \left(\frac{d\sigma}{d\Omega_{e'}}\right)_M \left[\frac{Q^4}{|\mathbf{q}|^4}R_L(|\mathbf{q}|,\omega) + \left(\frac{1}{2}\frac{Q^2}{|\mathbf{q}|^2}+\tan^2\frac{\theta}{2}\right)R_T(|\mathbf{q}|,\omega)\right]$$

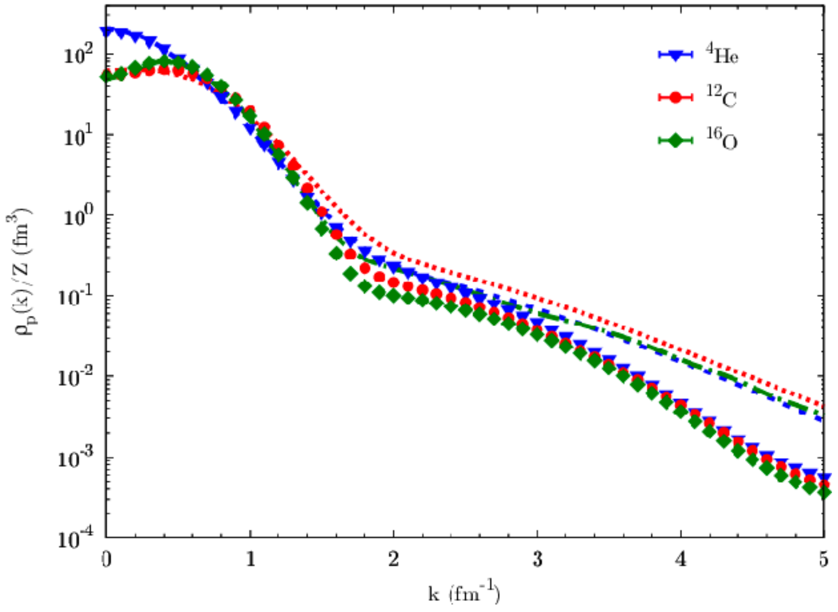
Some basic Observations from Electron Scattering

Superscaling: for the same kinematics, response looks similar for different nuclei ($q > k_F$)

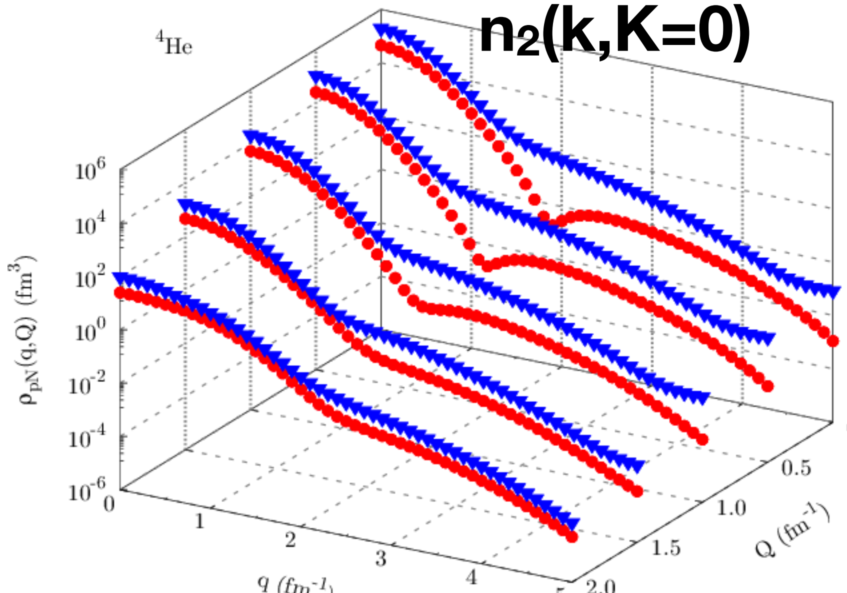


Superscaling in inclusive e-nucleus scattering
Different nuclei at the same kinematics
Same kinematics: same ratio of L/T response
Donnelly, Sick PRL (1999)

n(k)

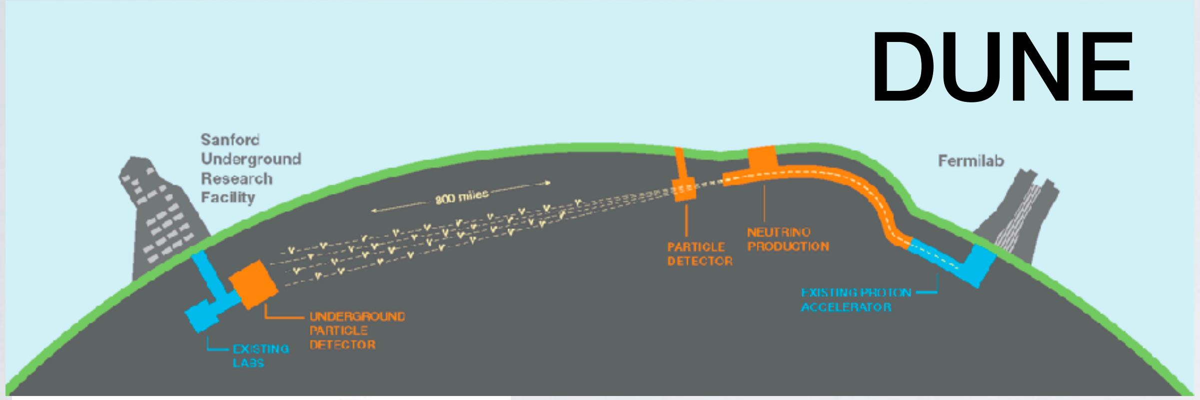


Gives y-scaling

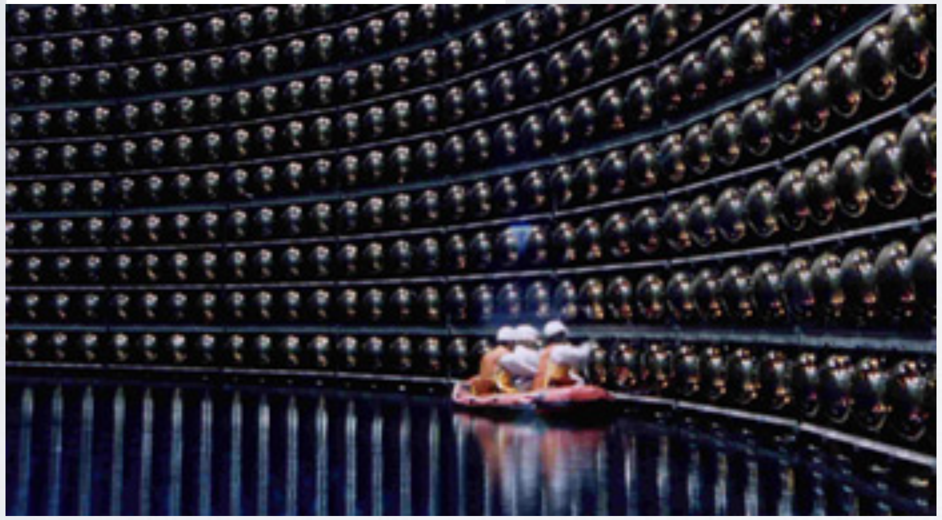

$n_2(k,K=0)$

Lonardoni, Gandolfi, Wang, JC (2018)
Related to back-to-back pairs

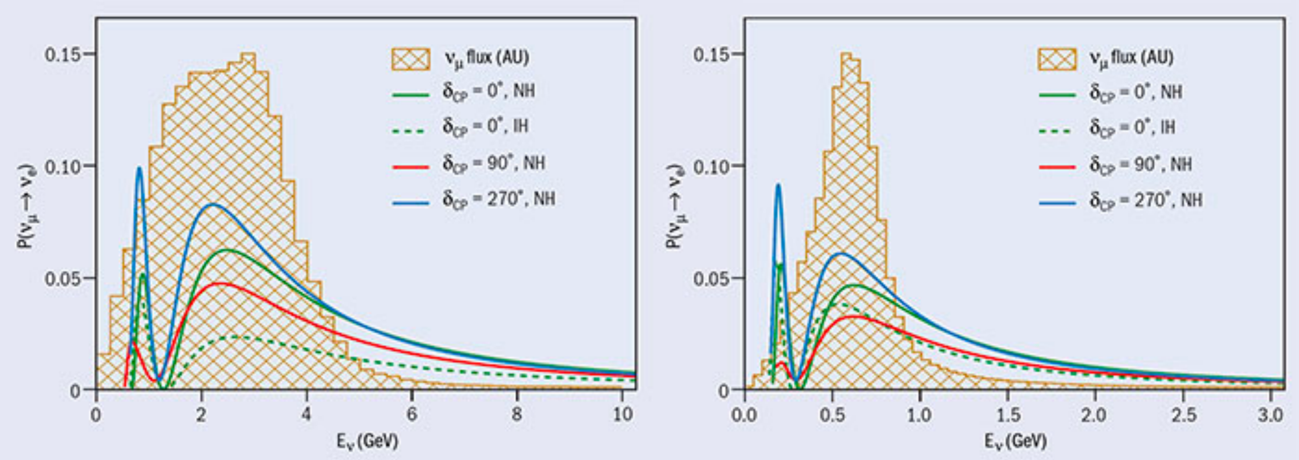
Neutrino Scattering



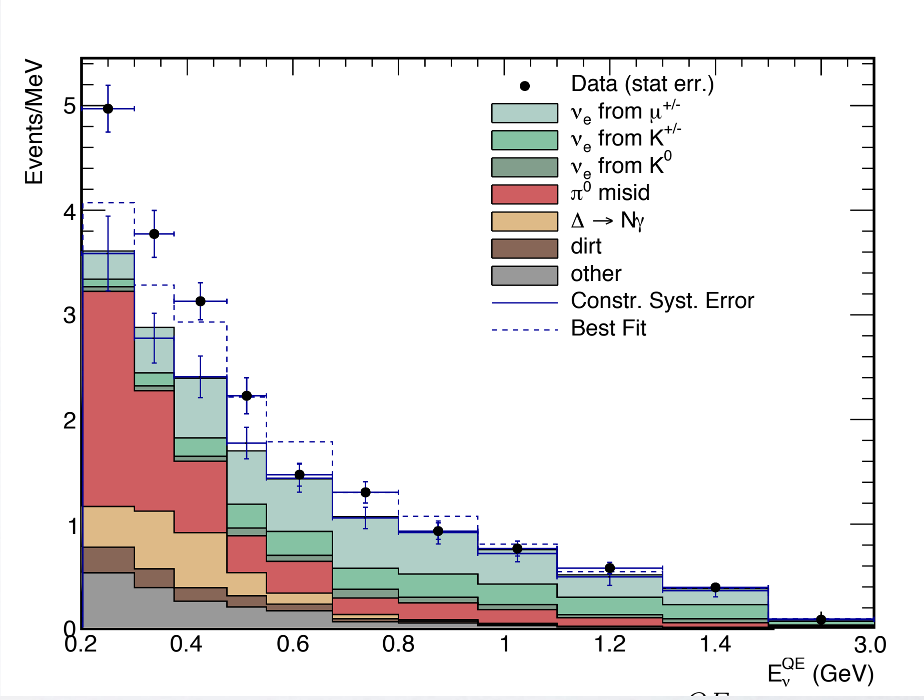
T2K

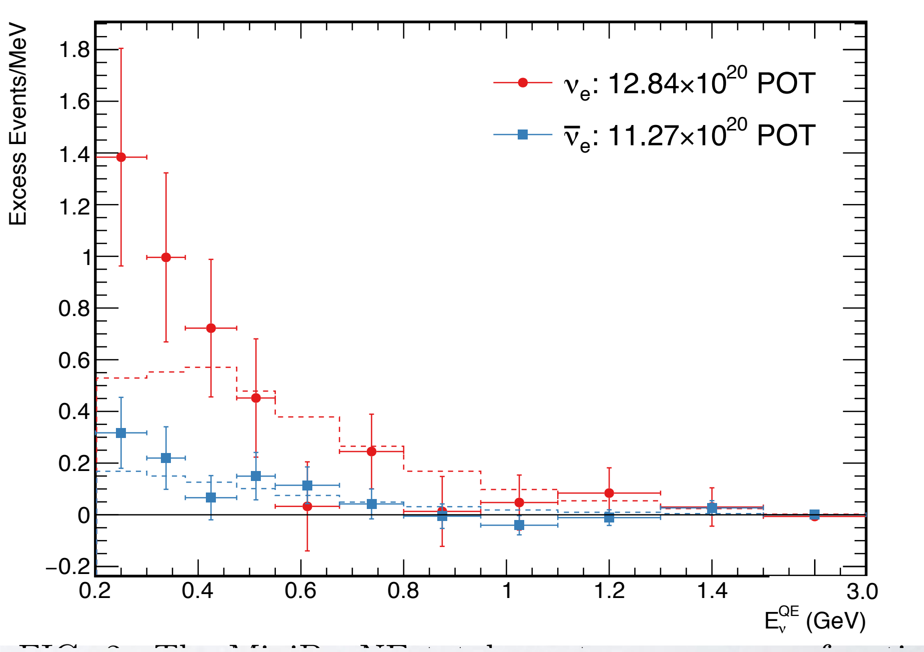


Need energy (L/E) for oscillation analysis



MiniBoone (2018)

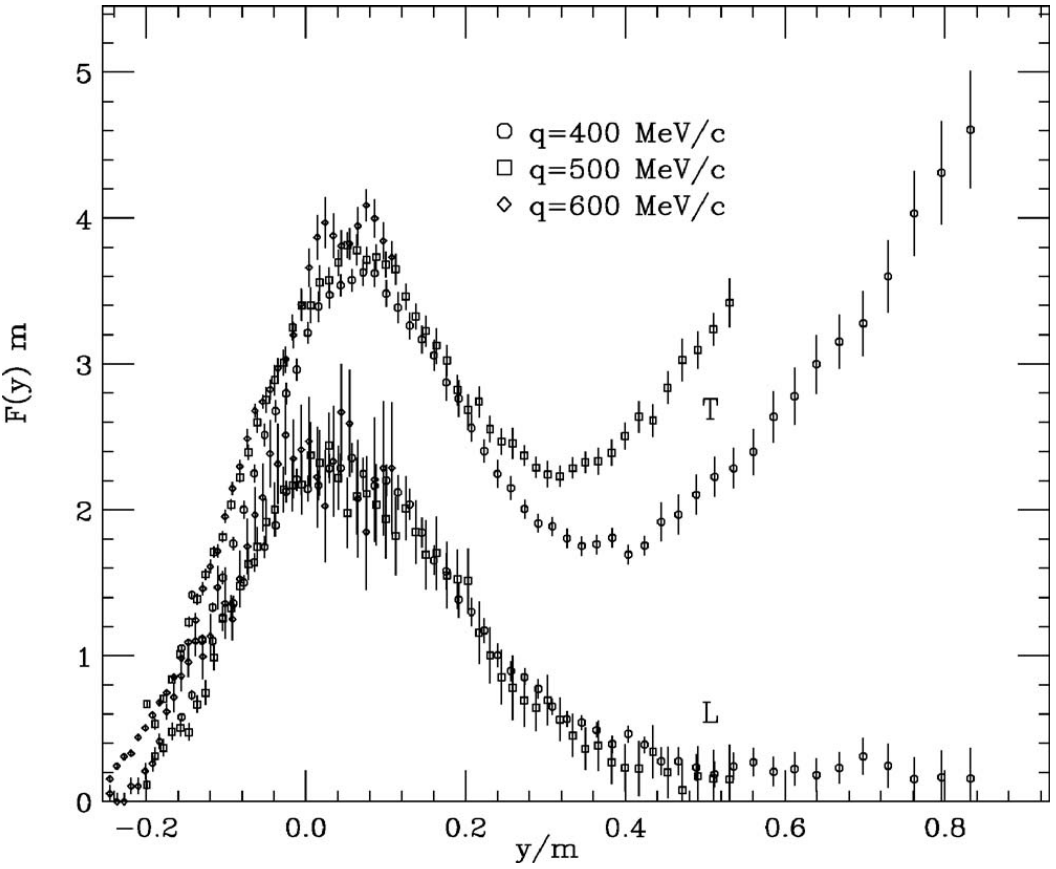




But, incoherent scaling from single nucleons is not the full picture

Longitudinal (charge) vs Transverse (current) scattering

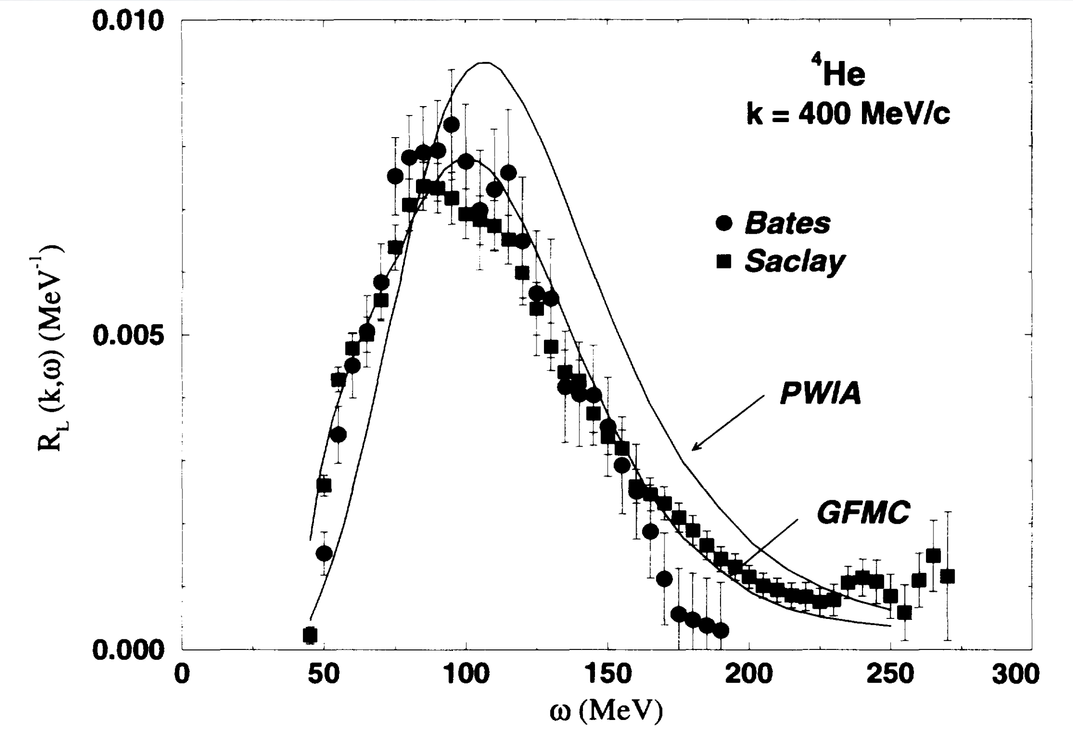
from Benhar, Day, Sick,
RMP 2008
data Finn, et al 1984



Scaled longitudinal vs.
transverse scattering from ^{12}C

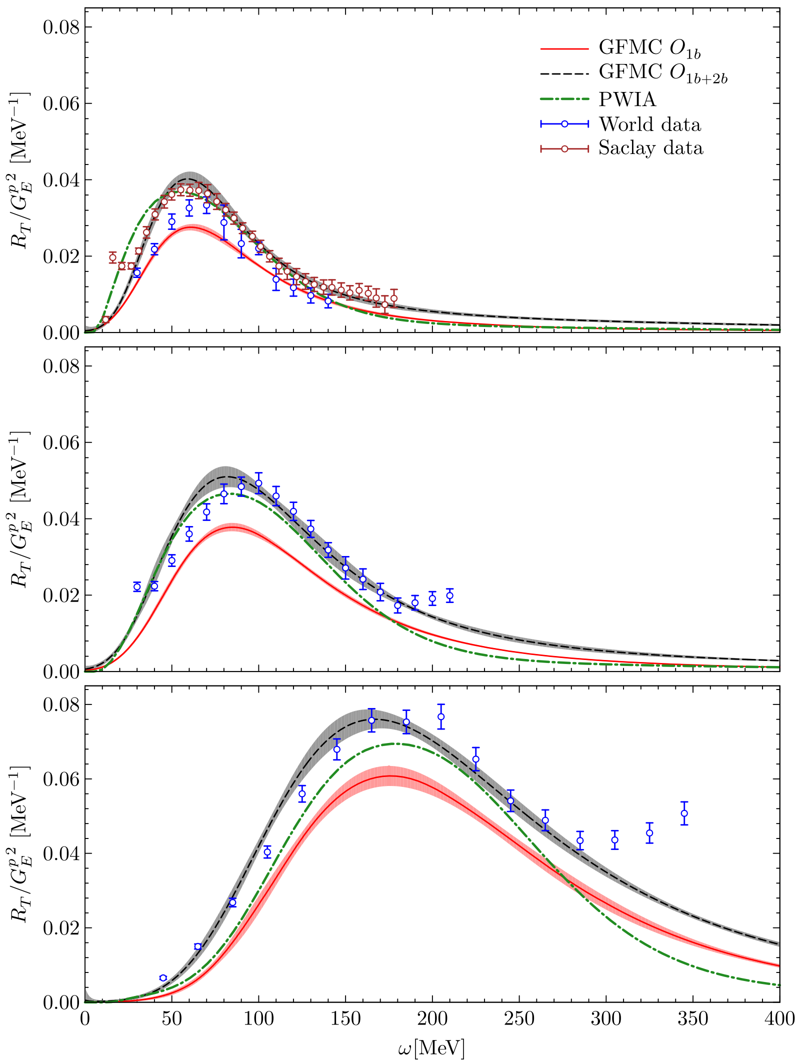
Electron Scattering ($q > k_F$)

Longitudinal (charge) scattering in ^{4}He



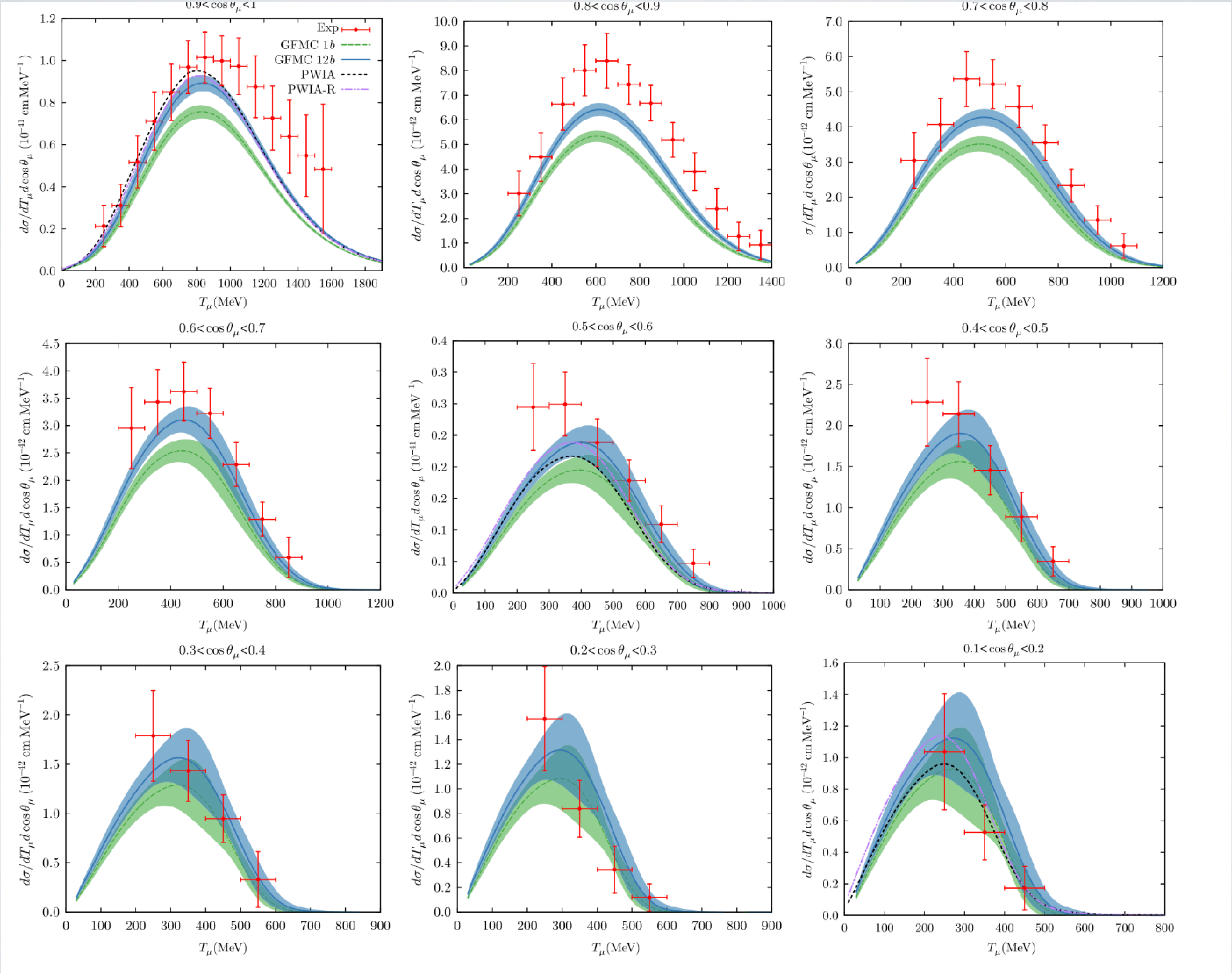
JC & Schiavilla, PRL (1992)

Note: Interaction moves charge
NN currents are important



Transverse (current) scattering in ^{12}C
Lovato, et al, PRL 2016

Neutrino & Anti-Neutrino scattering



Short-Time approximation (STA):

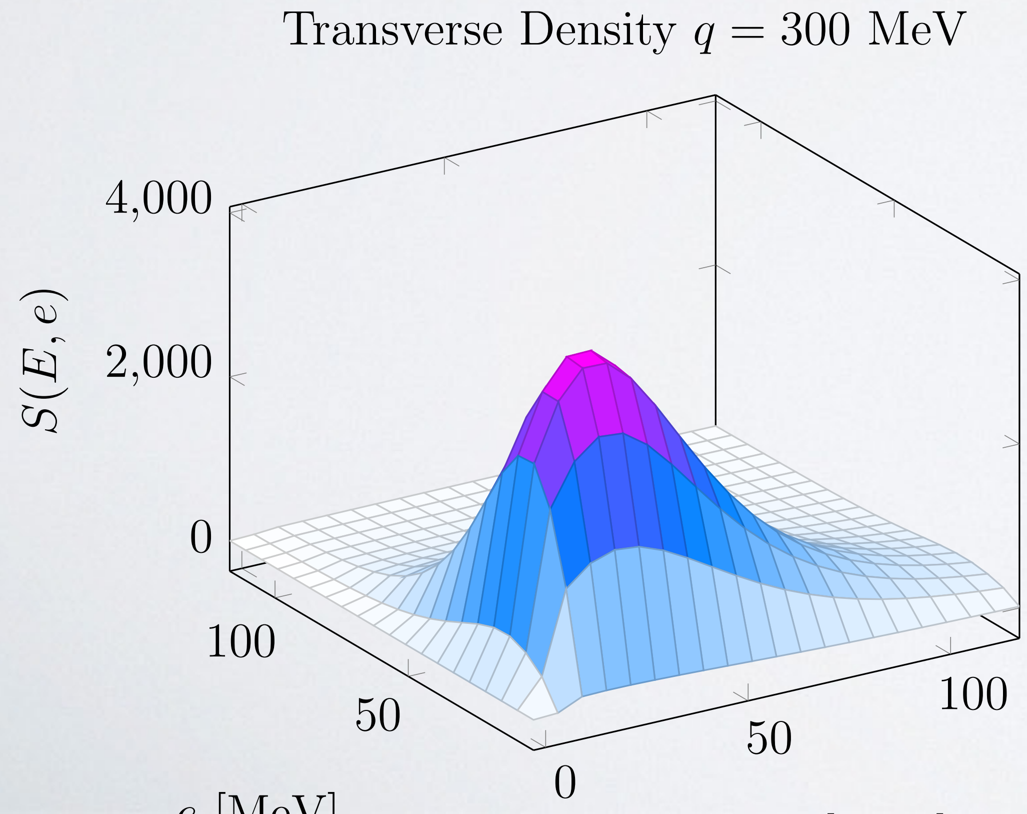
Factorization: 1- or 2-nucleon at vertex
Two nucleons included in propagation:

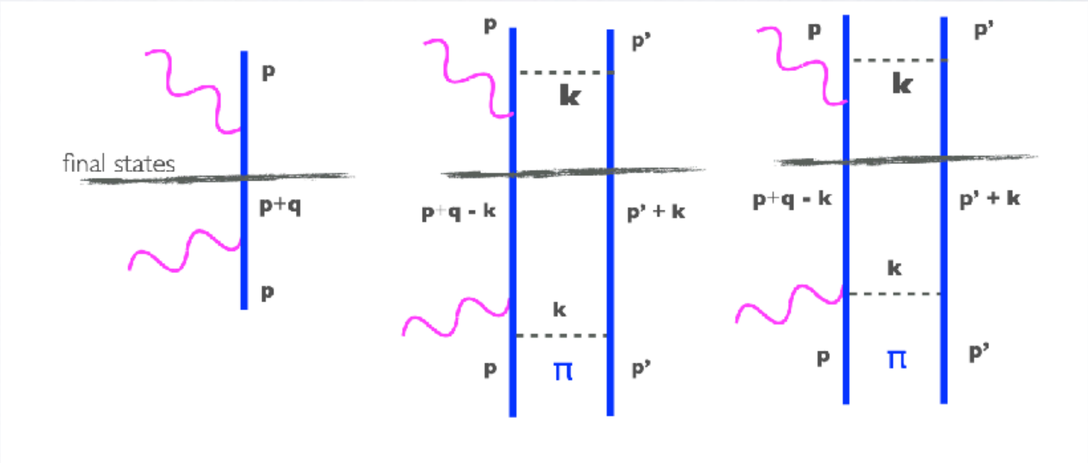
$$R^O(q,\omega) \;=\; \frac{\int d\Omega_q}{4\pi}\sum_f \langle\Psi_0|\mathcal{O}^\dagger(\mathbf{q})|\Psi_f\rangle\langle\Psi_f|\mathcal{O}(\mathbf{q})|\Psi_0\rangle\delta(E_f-E_0-\omega),$$

$$R^O(q,\omega) \;=\; \frac{\int d\Omega_q}{4\pi}\int\frac{dt}{2\pi}\exp[i\omega t]\langle\Psi_0|\mathcal{O}^\dagger(\mathbf{q},t')\exp[-iHt]\mathcal{O}(\mathbf{q},t=0)\Psi_0\rangle,$$

At short time evolution can be described as a product of NN propagators

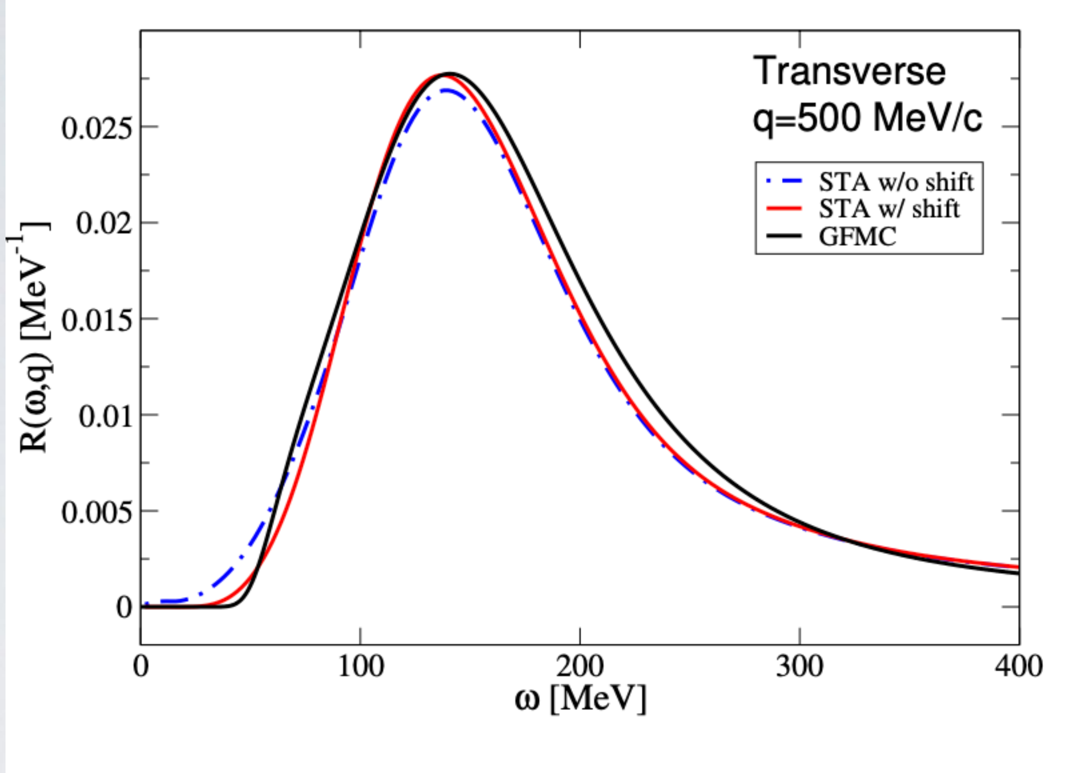
$$\langle\mathbf{R}',\sigma',\tau'|\exp[-iHt]|\mathbf{R},\sigma,\tau\rangle \approx \langle\mathbf{R}',\sigma',\tau'|\prod_i \exp[-iH_i^0 t]\frac{\mathcal{S}\prod_{i<j}\exp[-iH_{ij}t]}{\prod_{i<j}\exp[-iH_{ij}^0 t]}|\mathbf{R},\sigma,\tau\rangle$$

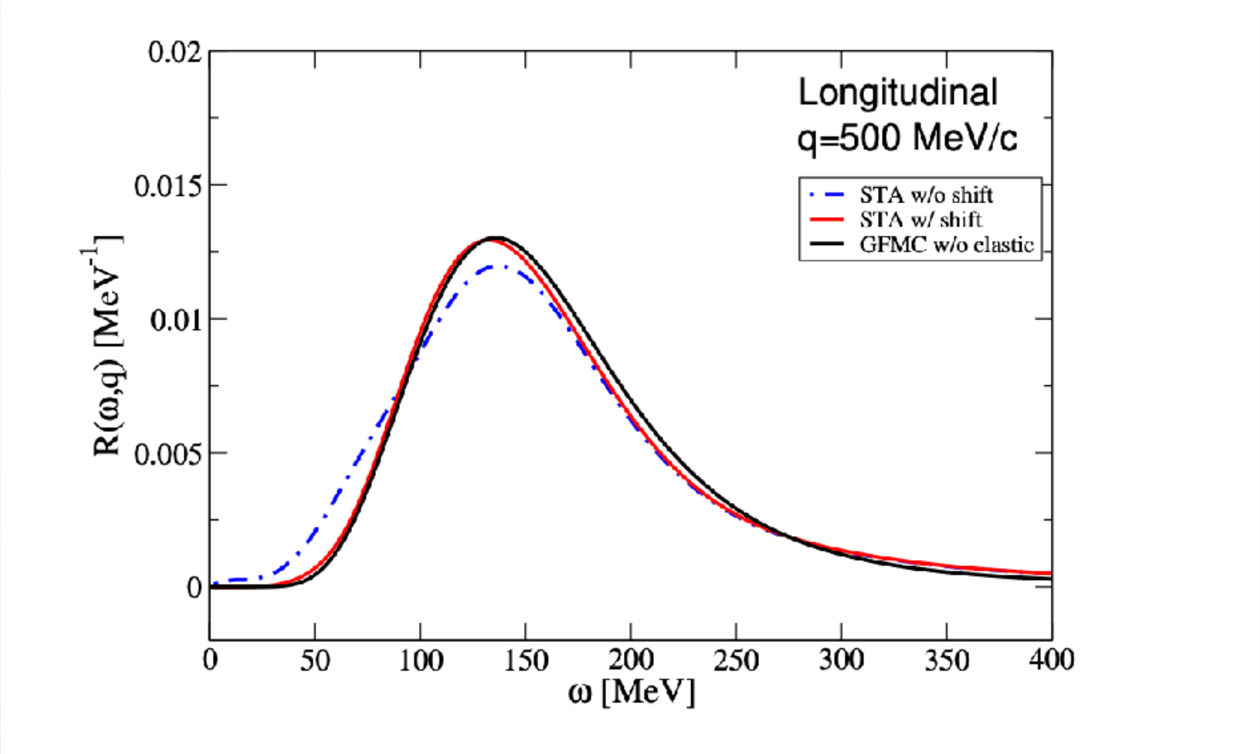




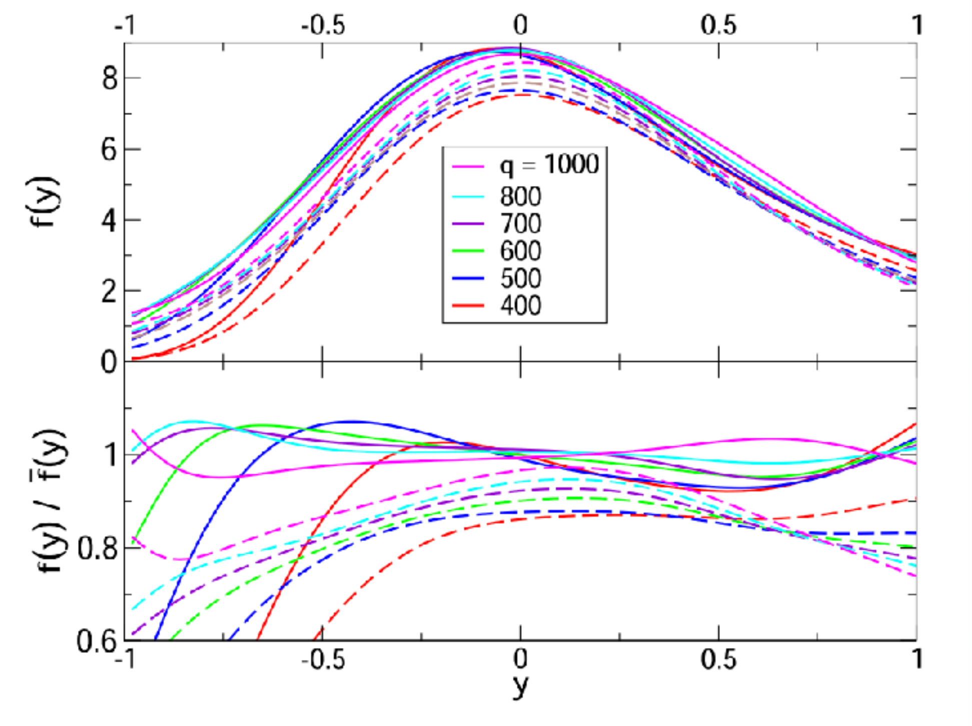
Saori Pastore, et al, 2019

Comparison of STA and GFMC (4He electron scattering)





Scaling in Transverse response improved with two-nucleon currents



dashed lines- 1 body current
full lines - 1+ 2-body currents

Summary & Outlook

- *Quantum dynamics in strongly-correlated systems is an important (and difficult) problem,*
  Linear response somewhat easier than general problem with many important applications in HENP and cold atoms, …
- *Many analogies between NP applications and cold atom (both static and dynamic properties)*
- *Few-body dynamics can be visible in response of many-body systems*
- *Improved theories of response has very many important applications, in nuclear (and nuclear astrophysics), neutrino physics, and cold atom physics.*
  Often advances can impact multiple fields both