ECT* Trento, June 6-10, 2022

Fermi systems with large s-wave scattering length: cold atoms and nuclear matter

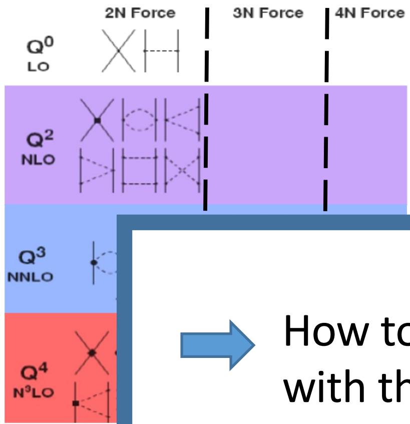
Denis Lacroix (IJCLab, Orsay, France)

Outline:

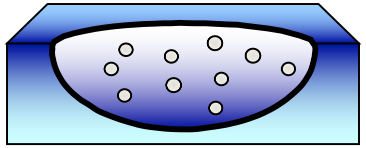
- Brief discussion on Density functional theory for nuclei
- Common DFT for nuclei and cold atoms?
- Applications: energies, static response, …
- Self-energy: quasi-particle properties.

Collaboration: *A. Boulet, J. Bonnard*, M. Grasso, C. Y Yang

Bare int.+Many-Body calculation versus Density Functional theory



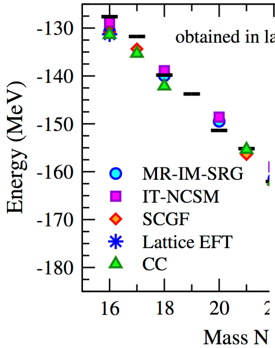
Rather simple smooth properties emerge
in nuclei (Energies, density, shell effects …)
Such simplicity is “easily” grasp by DFT



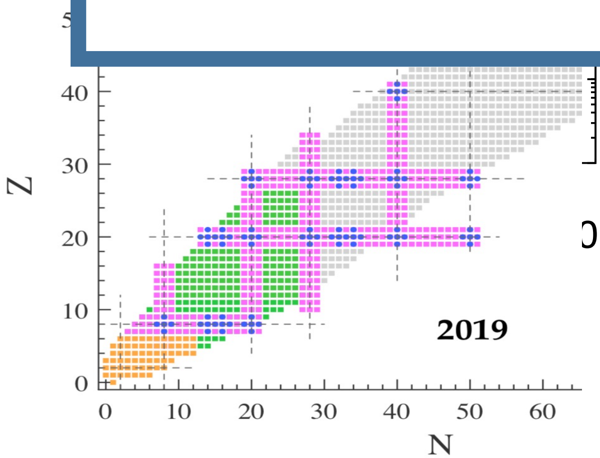
➡ How to reconcile the complexity of the interaction with the apparent simplicity of the density functional Theory to describe the system?

$$\mathcal{E} = \langle \mathrm{Kin} \rangle + \alpha_0 \rho + \alpha_3 \rho^3 + \cdots$$

Favorite
MB



Hebeler et



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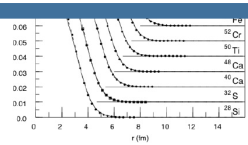
(courtesy V. Soma, T. Duguet)

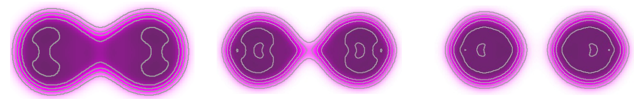
DFT is a simple
nd versatile approach

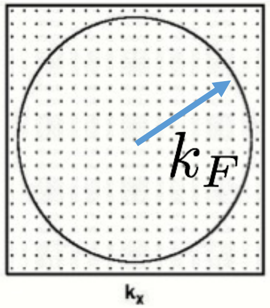
Thermodynamics



Dynamics

Time (fm/c)





General strategy: infinite systems

$$E = E^{\text{HF}} + E^{2^{\text{nd}}} + E^{3^{\text{rd}}} + \ldots$$

$$\Rightarrow \frac{E}{E_{\text{FG}}} = 1 + \frac{10}{9\pi}(\nu - 1)(k_F a_s) + (\nu - 1)\frac{4}{21\pi^2}(11 - 2\ln 2)(k_F a_s)^2 + \cdots$$

$\rho = \frac{\nu}{6\pi^2}k_F^2$ with ν degeneracy

E^{HF} $E^{2^{\text{nd}}}$

Expansion as polynomial of LEC

$(a_s k_F)$ $(r_e k_F)$ $\cdots$

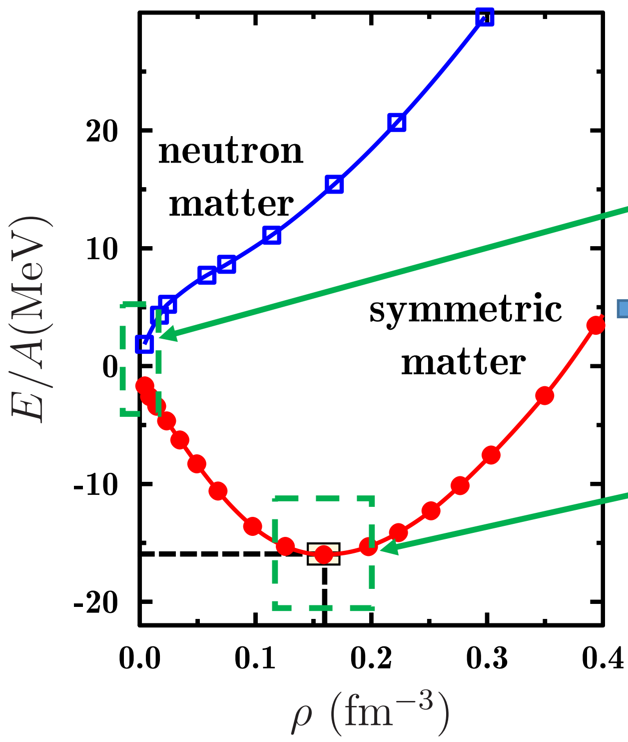
Difficulty

valid for $a_s k_F < 1$

For neutron matter $a_s = -$

$r_e = 2.7$ fm

A difficulty in atomic nuclei:



Here is where $a_s k_F < 1$ $\Rightarrow \rho < 10^{-6}\ \text{fm}^{-3}$

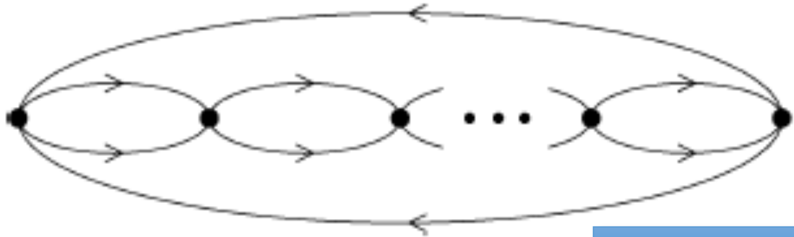
Region of interest

$|a_s k_F| \sim 25$
$r_e k_F \sim 4$
$a_p k_F \leq 1$

The “magic” technique: resummation and phase-space argument

Highlighting work Schaefer, Kao, Cotanch, NPA 762 (2005)

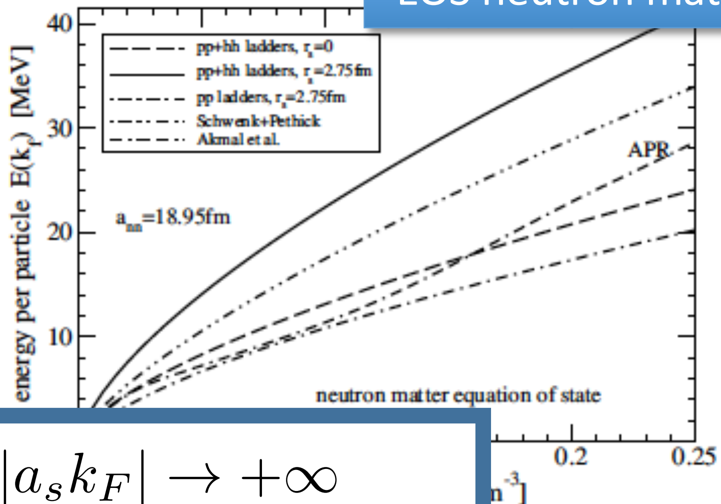
Resummation of particle-particle diagrams



$$\frac{E_{\mathrm{PP}}}{A} = \frac{3(g-1)\pi^2}{k_{\mathrm{F}}^3} \int \frac{d^3P}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} \theta_k^- \frac{4\pi a/M}{1 - \frac{k_{\mathrm{F}} a}{\pi} f_{\mathrm{PP}}(\kappa, s)}.$$

- Contains terms to all order in $(a_s k_F)$
- Results strongly depends on selected diagram

EOS neutron matter



…iser, EPJA 48 (2012)

…oximation

…e

The pragmatic approach

$$E \sim \frac{3}{5}\frac{\hbar^2 k_F^2}{2m} \frac{\frac{10}{9\pi}(a_s k_F)}{1 - \frac{6}{35\pi}(11 - 2\ln 2)(a_s k_F)} \sim \langle f_{\mathrm{PP}} \rangle$$

$$\frac{E}{A} = \left(\frac{3}{5} + \frac{2k_F a_s/3}{\pi - 2k_F a_s}\right)\frac{k_F^2}{2M} \qquad \langle f_{\mathrm{PP}} \rangle \underset{+\infty}{\rightarrow} 2$$

Steele, nucl-th-0010066v2

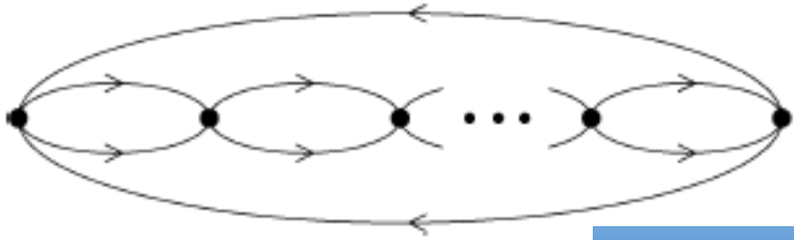
$$|a_s k_F| \rightarrow +\infty$$

- $E/E_{FG} \longrightarrow 0.32$
- $E/E_{FG} \longrightarrow 0.4$

The "magic" technique: resummation and phase-space argument

Highlighting work Schaefer, Kao, Cotanch, NPA 762 (2005)

Resummation of particle-particle diagrams



$$\frac{E_{PP}}{A} = \frac{3(g-1)\pi^2}{k_F^3}\int \frac{d^3P}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}\theta_k^- \frac{4\pi a/M}{1-\ldots}$$

Contains terms to all order in $(a_s k_F \ldots$

Results strongly depends on selected diagram

EOS neutron matter

The pragmatic approach

$$E \sim \frac{3}{5}\frac{\hbar^2 k_F^2}{2m}\frac{\frac{10}{9\pi}(a_s k_F)}{1-\frac{6}{35\pi}(11-2\ln 2)(a_s k_F)} \sim \langle f_{P\ldots}$$

$$\frac{E}{A} = \left(\frac{3}{5} + \frac{2k_F a_s/3}{\pi - 2k_F a_s}\right)\frac{k_F^2}{2M} \qquad \langle f_{PP}\rangle \underset{+\infty}{\rightarrow} 2$$

Steele, nucl-th-00100…

Even more pragmatic approach: take the unitary gas as a reference

$$\frac{E}{E_{FG}} = \left\{1 + \frac{(ak_F)A_0}{1 - A_1(ak_F)}\right\}$$

Two imposed limits:

$$|a_s k_F| \ll 1 \qquad \frac{E}{E_{FG}} = 1 + \frac{10}{9\pi}(\nu - 1)(k_F a_s) + \cdots$$

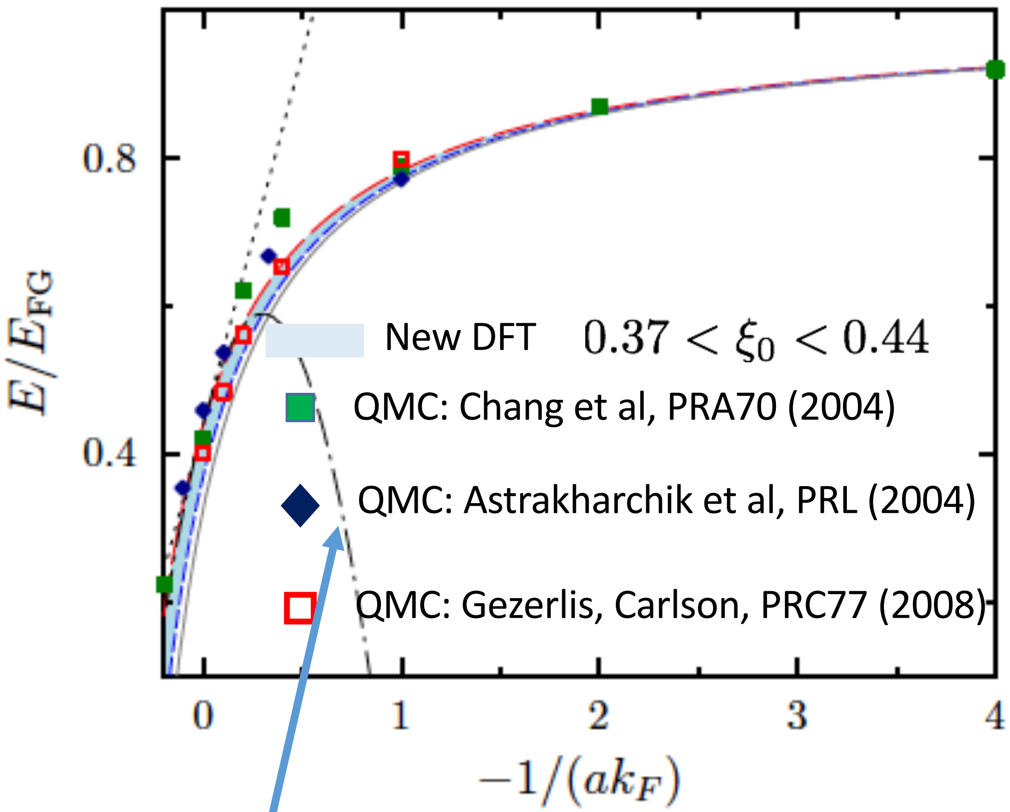
$$|a_s k_F| \gg 1 \qquad \frac{E}{E_{FG}} = \xi_0$$

$$A_0 = \frac{10}{9\pi}(\nu - 1) \qquad 1 - \frac{A_0}{A_1} = \xi_0$$

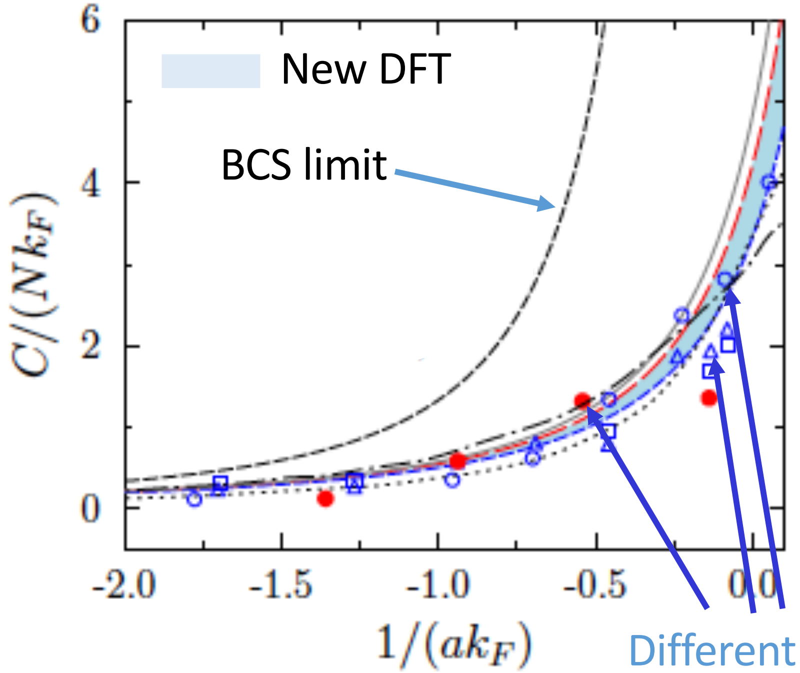
Result of the DFT for at or close to unitarity

Lacroix, PRA 94 (2016)

Energy



Tan contact parameter



Different Measurements in cold atoms

$$\frac{E}{E_{FG}} \simeq \xi_0 - \frac{\zeta}{(ak_F)} - \frac{5}{3}\frac{\nu}{(ak_F)^2} + \cdots \quad \zeta \simeq \nu \simeq 1$$

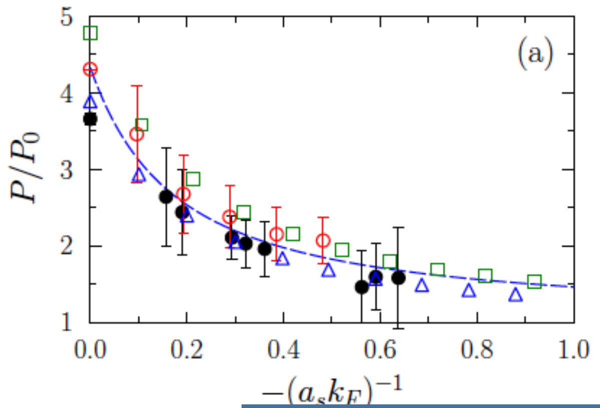
Taylor expansion in $(a_s k_F)^{-1}$: Bulgac and Bertsch, PRL 94 (2005)

Example of applications: thermodynamical quantities around unitarity

A. Boulet, DL, Phys. Rev. C 97 (2018)

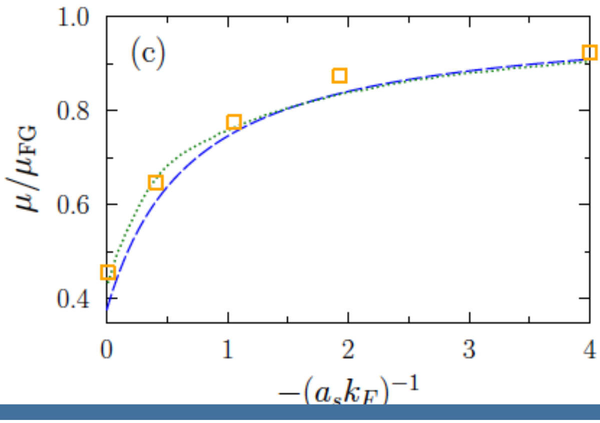
Pressure

$$P = \rho_n^2 \frac{\partial E/N}{\partial \rho_n}\Big|_N$$



Chemical potential

$$\mu = \frac{\partial E}{\partial N}\Big|_V = \frac{\partial \rho_n E/N}{\partial \rho_n}\Big|_V$$



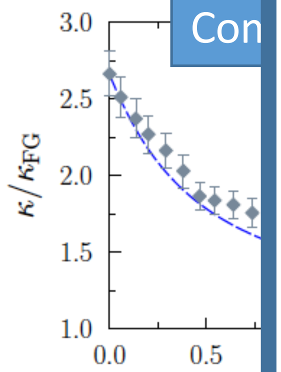
Theories

- ○ [Bulgac *et al.*, PRA **78** (2008)]
- □ [Haussmann *et al.*, PRA **75** (2007)]
- △ [Hu *et al.*, Europhys. Lett. 74 (2006)]
- □ [Pieri *et al.*, PRB **72** (2005)]
- ... [Astrakharchik *et al.*, PRL **93** (2004)]

xperiments

- ● [Navon *et al.*, Science **328** (2010)]
- ◆ [Navon *et al.*, Science **328** (2010)]
  [Ku *et al.*, Science **335** (2012)]
- ■ [Weimer *et al.*, PRL **114** (2015)]
- ▲ [Joseph *et al.*, PRL **98** (2007)]

Con



$\kappa =$

Is it predictive ? (it seems so)

Is it really useful for atomic nuclei ?

➡ Not yet

$$|a_s k_F| \sim 25$$
$$r_e k_F \sim 4$$
$$a_p k_F \leq 1$$

➡ It only give total energy
(no information on quasi-particle properties: effective mass, superfluidity, …)

From cold atom to neutron matter: inclusion of effective range

Lacroix, PRA 94 (2016)

Lacroix, Boulet, Grasso, Yang, PRC 95 (2017)

$$\frac{E}{E_{\rm FG}} = 1 - \frac{U_0}{1-(a_s k_F)^{-1}U_1} + \frac{R_0(r_e k_F)}{[1-R_1(a_s k_F)^{-1}][1-R_1(a_s k_F)^{-1}+R_2(r_e k_F)]}$$

Effective range part (form obtained by resumming effective range effects in HF theory)

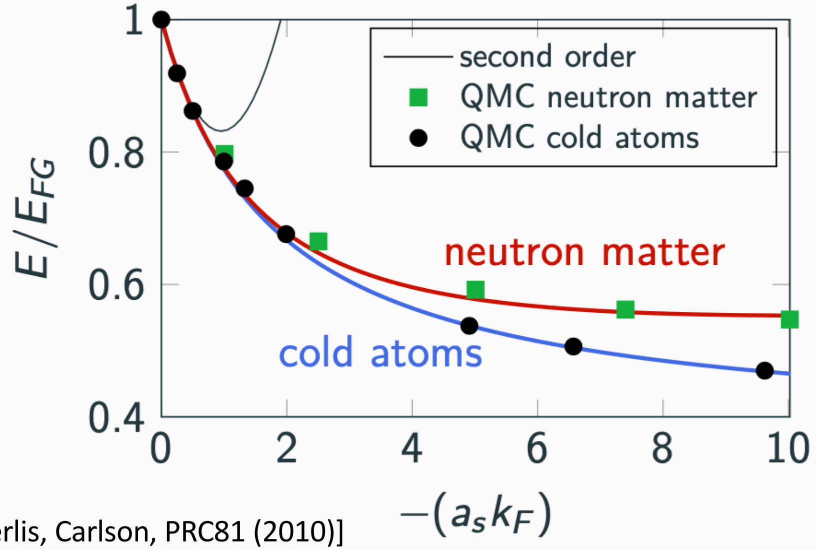
New constraints

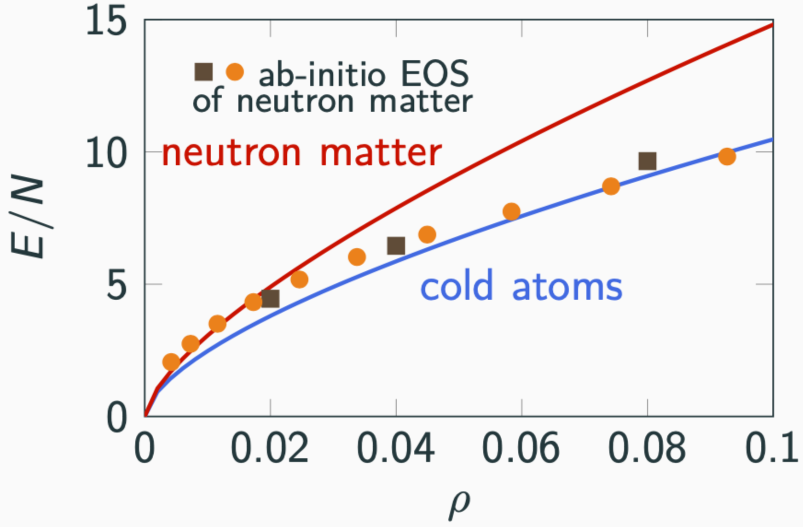
$|a_s k_F| \ll 1$ → $|a_s k_F| \gg 1$

$$\frac{E}{E_{\rm FG}} 1 + \frac{10}{9\pi}(\nu-1)(k_F a_s) + (\nu-1)\frac{1}{6\pi}(k_F r_e)(k_F a_s)^2 + \cdots$$

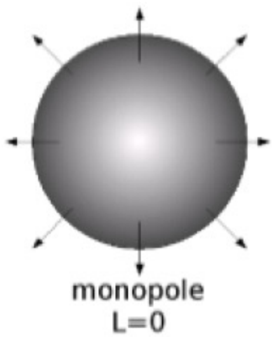
$$\xi(+\infty, r_e k_F) \equiv \xi_0 + (r_e k_F)\eta_e + (r_e k_F)^2 \delta_e$$

Forbes, Gandolfi, Gezerlis, PRA86 (2012)





[QMC: Gezerlis, Carlson, PRC81 (2010)]



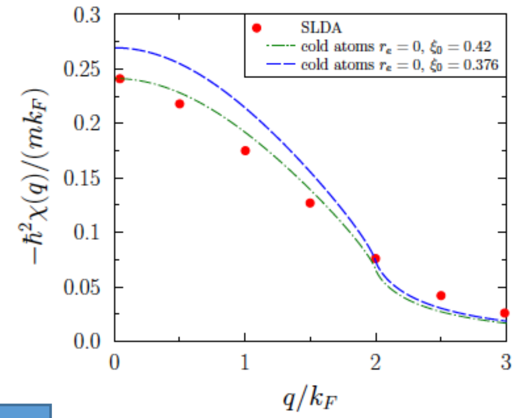
Some successful application

Static response in cold atoms and neutron matter

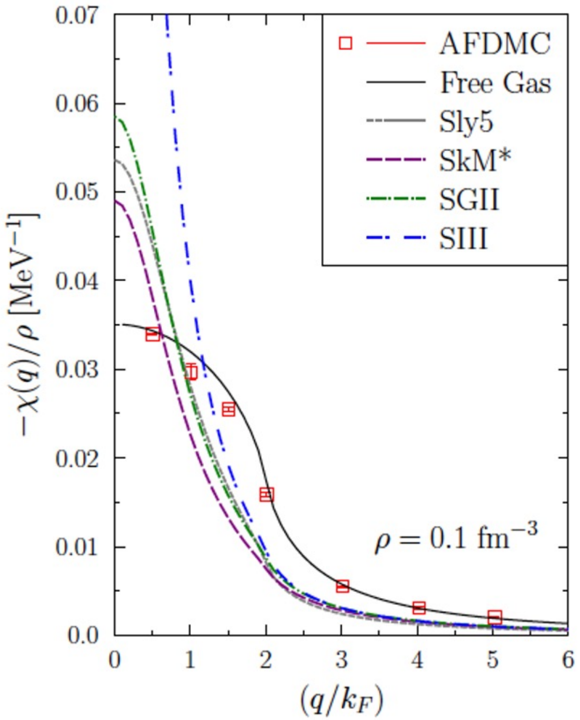
A. Boulet, DL, Phys. Rev. C 97 (2018)

Static response In unitary gas

SLDA: [Forbes and Sharma, PRA **90** (2014)]



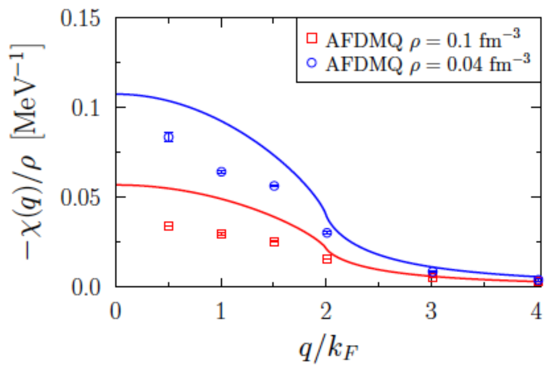
Response in neutron matter



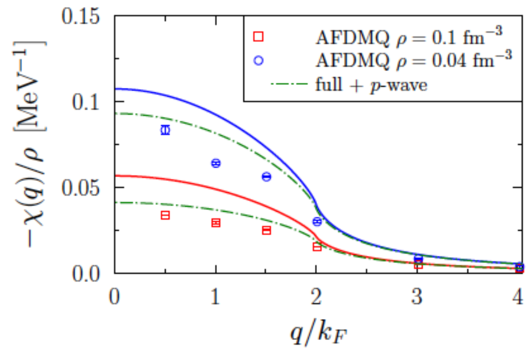
Buraczynski, Gezerlis, PRL 116 (2016), PRC 95 (2017)
Buraczynski, Martinello, Gezerlis, PLB 818 (2021), PRC 105 (2022)

In neutron matter

Non-empirical functional



Non-empirical functional + *p*-wave



Adding *p*-wave
(leading order term only)

$$\frac{E_p}{E_{FG}} = \frac{1}{\pi}(a_p k_F)^3$$

What have we learn beyond having a unitary gas guided DFT?

➡ How to conceal the complexity of the interaction with the apparent simplicity of the density functional Theory to describe the system?

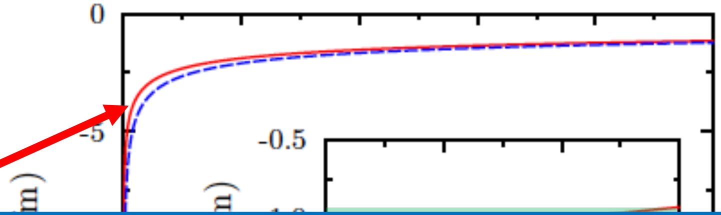
$$\mathcal{E} = \langle \mathrm{Kin} \rangle + \alpha_0 \rho + \alpha_3 \rho^3 + \cdots$$

Restart from the functional

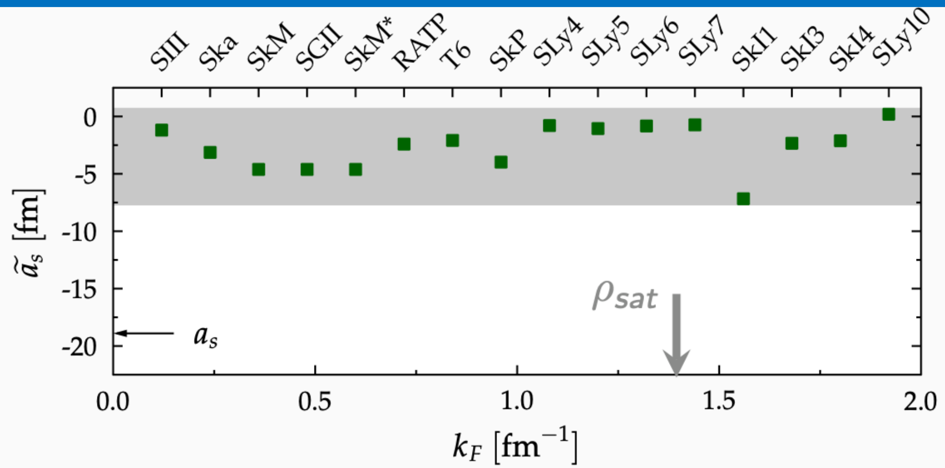
$$\frac{E}{E_{FG}} = 1 - \frac{U_0}{1 - (a_s k_F)^{-1} U_1} + \frac{R_0(r_e k_F)}{[1 - R_1 (a_s k_F)^{-1}][1 - R_1 (a_s k_F)^{-1} + R_2 (r_e k_F)]}$$

Rewrite it as

$$\frac{E}{E_{FG}} = 1 + \frac{k_F^3}{4\pi^2 E_{FG}} \left\{ \frac{\widetilde{C}_0(k_F)}{3} + \frac{k_F^2}{10} [(\nu - 1)\widetilde{C}_2(k_F) + (\nu + 1)\widetilde{C}_2'(k_F)] \right\}$$



Define density dependent scattering length and range



Skyrme functionals:
$\widetilde{a}_s(k_F) = \widetilde{a}_s = m t_0 (1 - x_0)/4\pi$

[Lacroix, AB, et al., PRC 95 (2017)]

What have we learn beyond having a unitary gas guided DFT?

➡ How to conceal the complexity of the interaction with the apparent simplicity of the density functional Theory to describe the system?

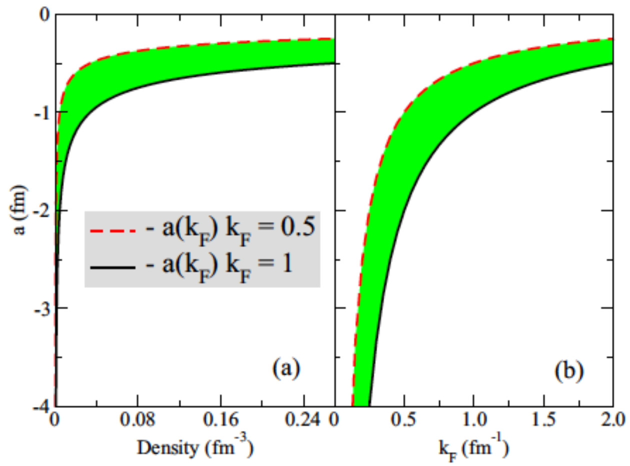
$$\mathcal{E} = \langle \mathrm{Kin} \rangle + \alpha_0 \rho + \alpha_3 \rho^3 + \cdots$$

The simple DFT has guided us to design new functional theory valid at low density up to densities of interest.

The ELYO functional

*ELYO : Extended Lee-Yang Orsay

$$\frac{E}{N} = \frac{\hbar^2 k_F^2}{2m}\left[\frac{3}{5} + \frac{2}{3\pi}(k_F a) + \frac{4}{35\pi^2}(11 - 2\ln 2)(k_F a)^2 + \frac{1}{10\pi}(k_F r_s)(k_F a)^2 + 0.019(k_F a)^3\right],$$

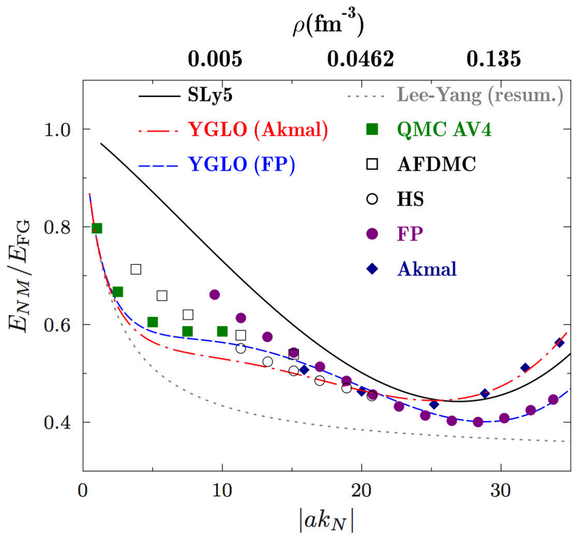


Grasso, Lacroix, Yang, PRC 95 (2017)

The YGLO functional

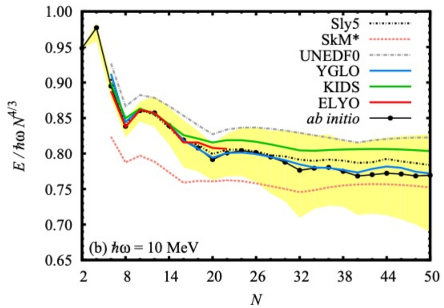
*YGLO : Yang Grasso Lacroix Orsay

$$\frac{E}{A} = K_\beta + \frac{B_\beta \rho}{1 - R_\beta \rho^{1/3} + C_\beta \rho^{2/3}} + D_\beta \rho^{5/3} + F_\beta \rho^{\alpha+1}$$



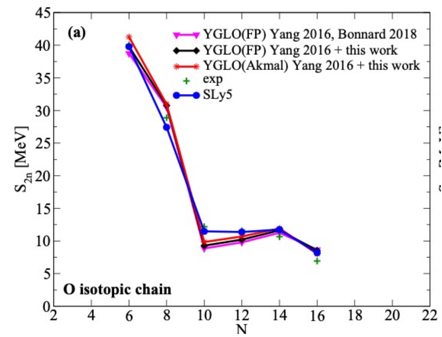
Yang, Grasso, Lacroix PRC94 (2016)

Neutron drops



Bonnard, Grasso, lacroix, PRC98 (2018), PRC101 (2020)

Atomic Nuclei



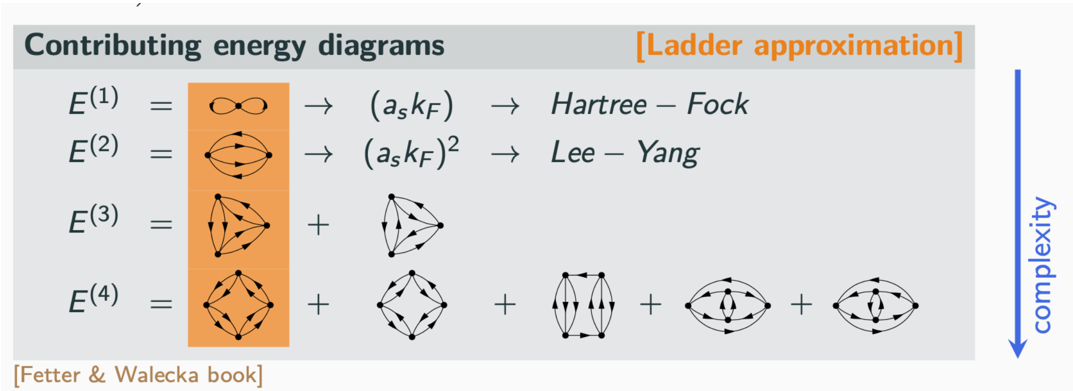
Burrello, Bonnard, Grasso, PRC103 (2021)

Can we get quasi-particle properties using the same strategy?

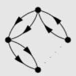
Strategy used for the energy:

Select a class of diagrams → Perform resummation → Use a "Phase-space" Argument to simplify the expression → Check the result and eventually Use it as a guidance for predictive functional

Can we do the same for the self-energy ?
(here restricted to non-superfluid systems, only s-wave)



Example of expressions (for the energies)

$E_{int} = \sum_{n=1}^{\infty}$  $= \frac{80 E_{FG}}{\pi k_F^5} \int_0^{k_F} s^2 ds \int_0^{\sqrt{k_F^2 - s^2}} t dt \ \mathrm{atan} \frac{(a_s k_F) \pi I_*(s,t)}{\pi - (a_s k_F) R(s,t)}$

[Kaiser, NPA860 (2011)] (no pairing, no self-consistency)

$$F(s,t) = 1 + \frac{s}{k_F} - \frac{t}{k_F} \ln\left|\frac{k_F + s + t}{k_F + s - t}\right| + \frac{k_F^2 - s^2 - t^2}{2 s k_F} \ln\left|\frac{(k_F + s)^2 - t^2}{k_F - s^2 - t^2}\right|$$

$$R(s,t) = F(s,t) + F(-s,t)$$

$$I_*(s,t) = \begin{cases} t/k_F & \text{for } 0 \leq t < k_F - s \\ (k_F^2 - s^2 - t^2)/2 s k_F & \text{for } k_F - s \leq t < \sqrt{k_F^2 - s^2} \end{cases}$$

Phase-space ansatz

$$\frac{E}{E_{\rm FG}} = \left\{ 1 + \frac{(a k_F) A_0}{1 - A_1 (a k_F)} \right\}$$

A. Boulet, PhD Université Paris-Saclay (2019)

Can we get quasi-particle properties using the same strategy?

Strategy used for the energy:

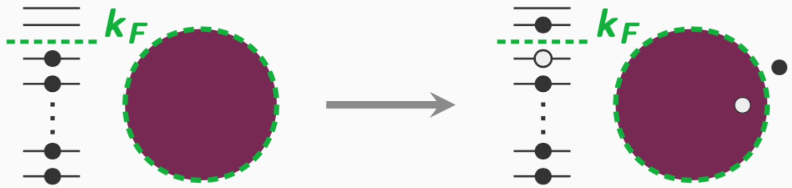
Select a class of diagrams → Perform resummation → Use a "Phase-space" Argument to simplify the expression → Check the result and eventually Use it as a guidance for predictive functional

Can we do the same for the self-energy ?
(here restricted to non-superfluid systems, only s-wave)

$$E_{int} = \sum_{kk'} V_{eff}(k,k') n_k n_{k'}$$

Low-lying excited states: $n_k \to n_k + \delta n_k$

$$\delta E = \sum_k \Sigma^\star(k) \delta n_k \longmapsto$$
$$\Sigma^\star(k) = U(k) + iW(k) = \frac{\delta E}{\delta n_k}$$



$\epsilon_k = \frac{k^2}{2m} + U(k)$ (single-particle energy)

$\frac{1}{2\gamma_k} = -W(k)$ (life-time)

Close to Fermi surface: $v_{k_F} \equiv \partial_k \epsilon_k \big|_{k=k_F} \equiv k_F/m^\star$

$$\epsilon_k = \epsilon_{k_F} + (k - k_F)\frac{k_F}{m^\star} + \cdots$$

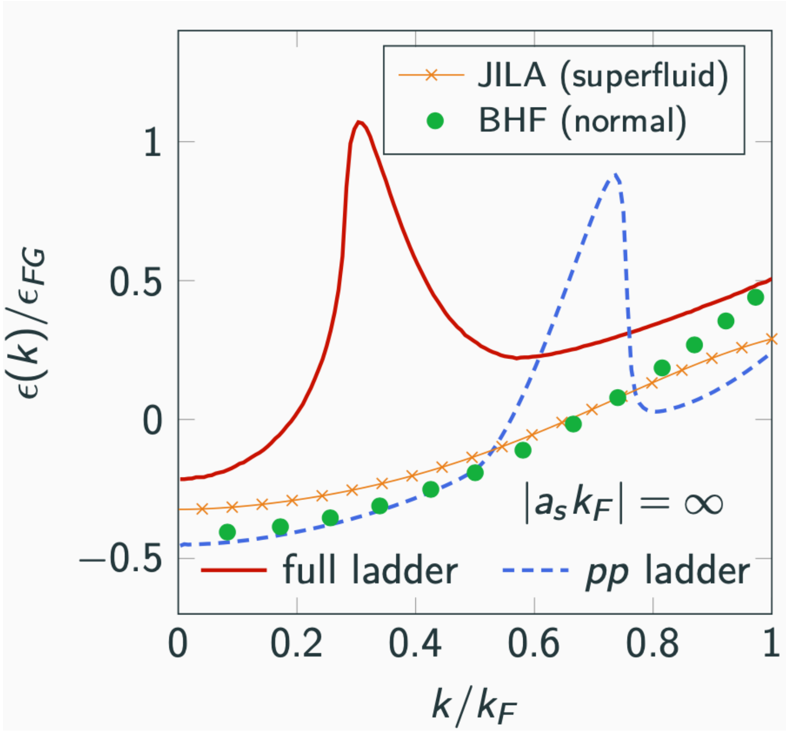
A. Boulet, PhD Université Paris-Saclay (2019)

Can we get quasi-particle properties using the same strategy?

Exact formula of the self-energy after resummation:

$$\Sigma^\star(k) = U(k) + iW(k) \qquad U(k<k_F) = \frac{8}{m\pi^2}\int_0^{k_F} s^2 ds \int_0^{\sqrt{k_F^2-s^2}} tdt\, \mathcal{U}(s,t,k<k_F)$$

[Kaiser, EPJA49 (2013)]



- ✓ valid at low density (Galitskii formula)
- ✓ finite limit at unitarity $|a_s k_F| \to \infty$
- ✗ bad predictivity power for $|a_s k_F| \gg 1$
- ✗ strong dependence of retained diagrams (cf. energy)

BHF: [Doggen & Kinnumen (2015)]
JILA exp.: [Stewart et al., Nature **454** (2008)]

➡ Performing a "phase-space" like average does not seem so evident to us

Can we get quasi-particle properties using the same strategy?

We used a different strategy

$$\frac{E}{E_{FG}} = 1 + \frac{10}{9\pi} \frac{(a_s k_F)}{1 - (a_s k_F/\pi)\frac{9\pi^2}{14}\phi_2(k_F)}$$

$$\mu = \left.\frac{\partial E}{\partial N}\right|_V$$

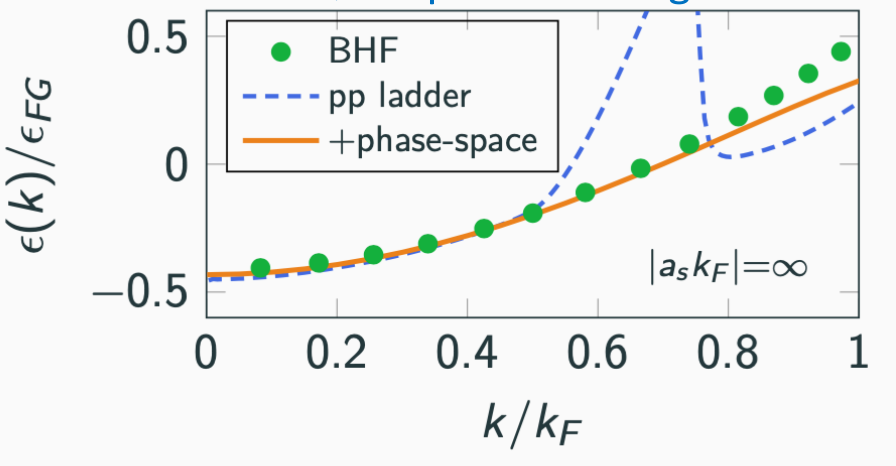
$$\frac{\mu}{\mu_{FG}} = 1 + \frac{4}{3}\frac{(a_s k_F)}{\pi - (a_s k_F)\frac{9\pi^2}{14}\phi_2(k_F)} + \frac{2}{9}\frac{(a_s k_F)^2 \frac{9\pi^2}{14}\phi_2(k_F)}{\left[\pi - (a_s k_F)\frac{9\pi^2}{14}\phi_2(k_F)\right]^2}$$

$$\phi_2(k_F) \to \phi_2(k)$$

$$\frac{\epsilon(k)}{\epsilon_{FG}} = \frac{k^2}{k_F^2} + \frac{4}{3}\frac{(a_s k_F)}{\pi - (a_s k_F)\frac{9\pi^2}{14}\phi_2(k)} + \frac{2}{9}\frac{(a_s k_F)^2 \frac{9\pi^2}{14}\phi_2(k)}{\left[\pi - (a_s k_F)\frac{9\pi^2}{14}\phi_2(k)\right]^2}$$

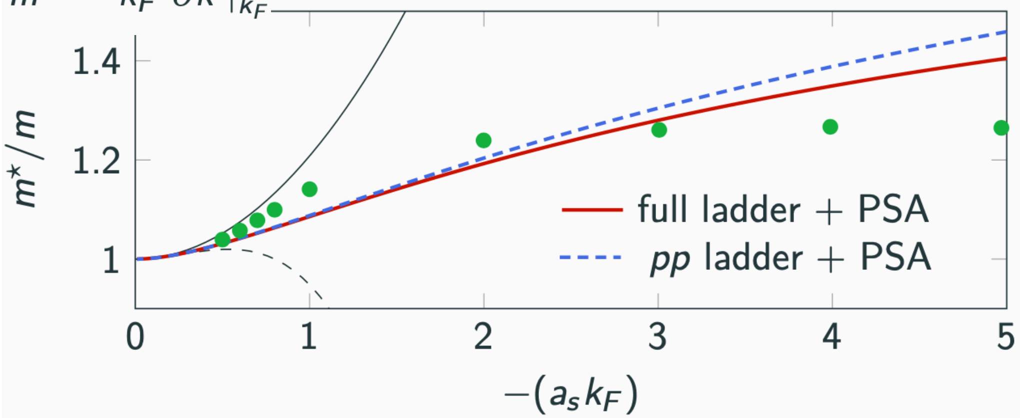
Gives the proper Galiitski formula at low density

Quasi-particle energies

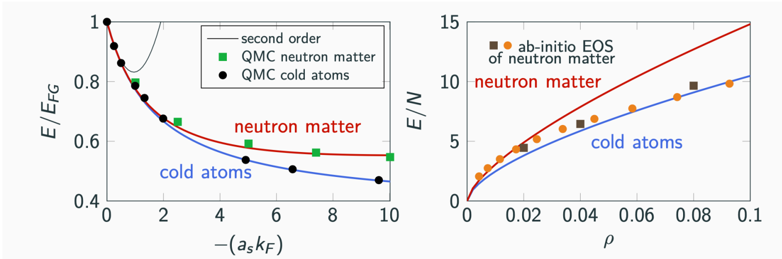


$$\frac{m}{m^\star} = \frac{m}{k_F}\left.\frac{\partial \epsilon_k}{\partial k}\right|_{k_F}$$

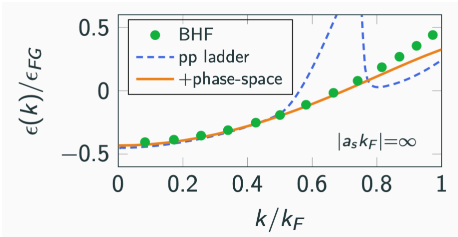
Effective mass



- We have developed rather simple Density functional theory that applies to both cold atoms and nuclei



- These functionals seems useful to understand the apparent simplicity of other historical successful functionals.
- This has also initiated new types of functionals that respects low density limit
- The problem is much harder on self-energy (without superfluidity and selected diagrams)



Local energy density functional for superfluid Fermi gases from effective field theory

Antoine Boulet[1,*] Gabriel Wlazłowski[1,2,†] and Piotr Magierski[1,2,‡]

[1] *Faculty of Physics, Warsaw University of Technology, Ulica Koszykowa 75, 00-662 Warsaw, Poland*
[2] *Department of Physics, University of Washington, Seattle, Washington 98195–1560, USA*

(Dated: January 20, 2022)

arXiv:2201.07626