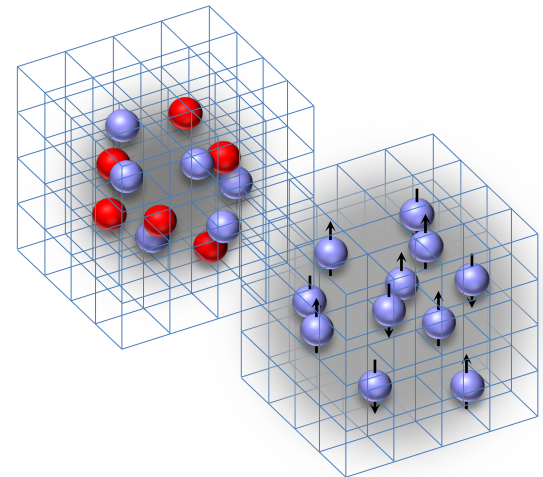


# The unitarity limit, two and four component fermions, and nuclear physics

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Connections between cold atoms and  
nuclear matter: From low to high energies  
ECT\* Workshop  
June 6, 2022



## Outline

Lattice effective field theory

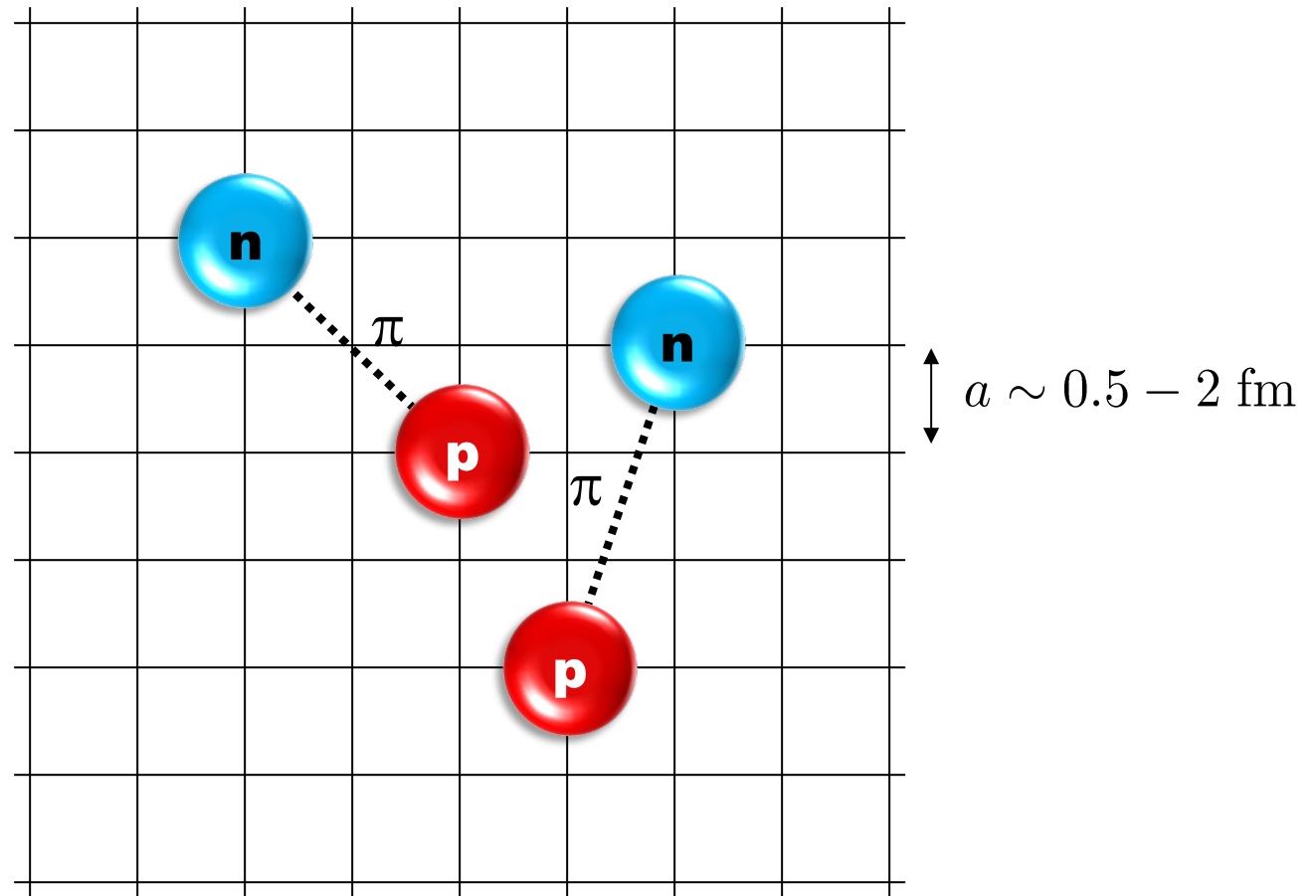
Two-component Fermi gases at unitarity

Is nuclear physics near four-component  
Fermi gas unitarity?

Spectrum and structure of  $^{12}\text{C}$  states

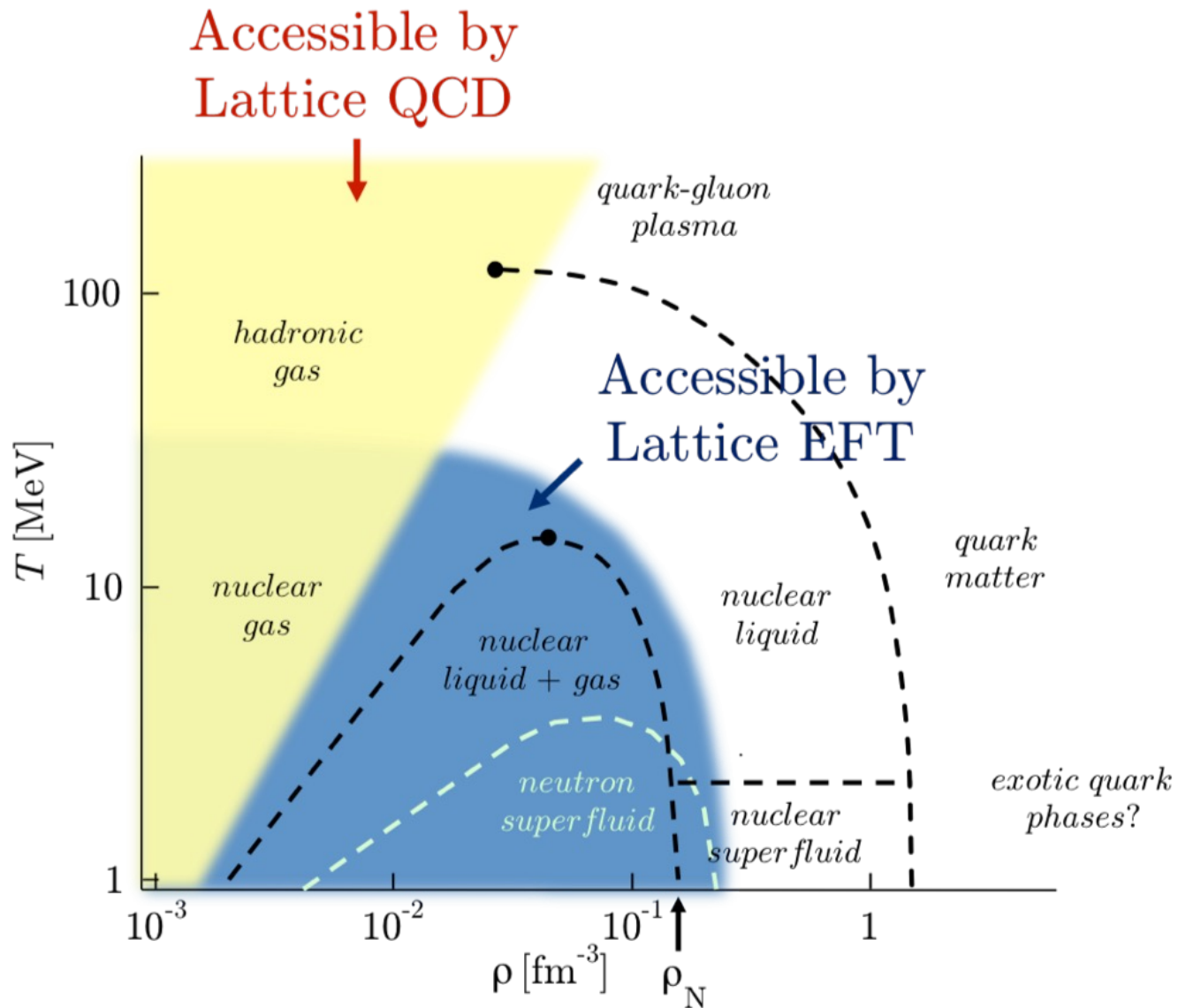
Summary

# Lattice effective field theory



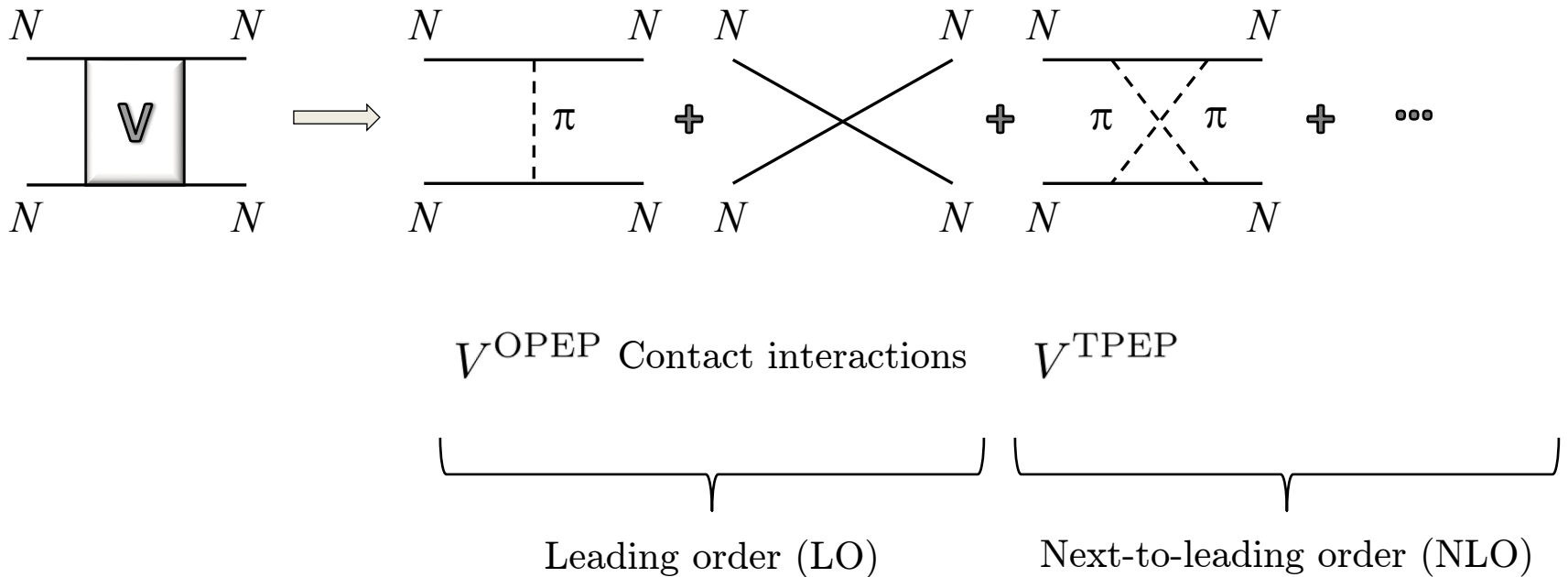
D.L, Prog. Part. Nucl. Phys. 63 117-154 (2009)

Lähde, Meißner, Nuclear Lattice Effective Field Theory (2019), Springer

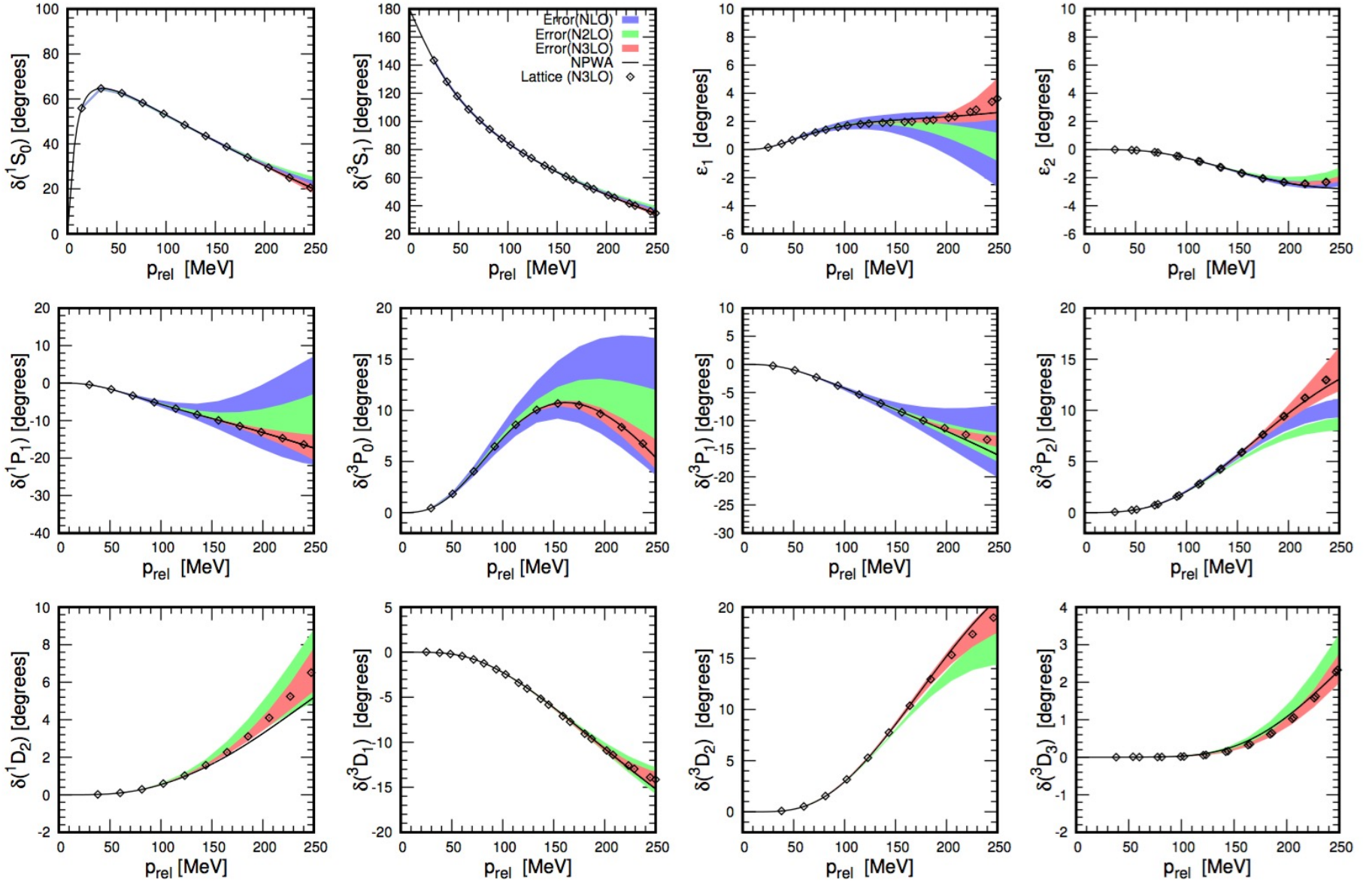


# Chiral effective field theory

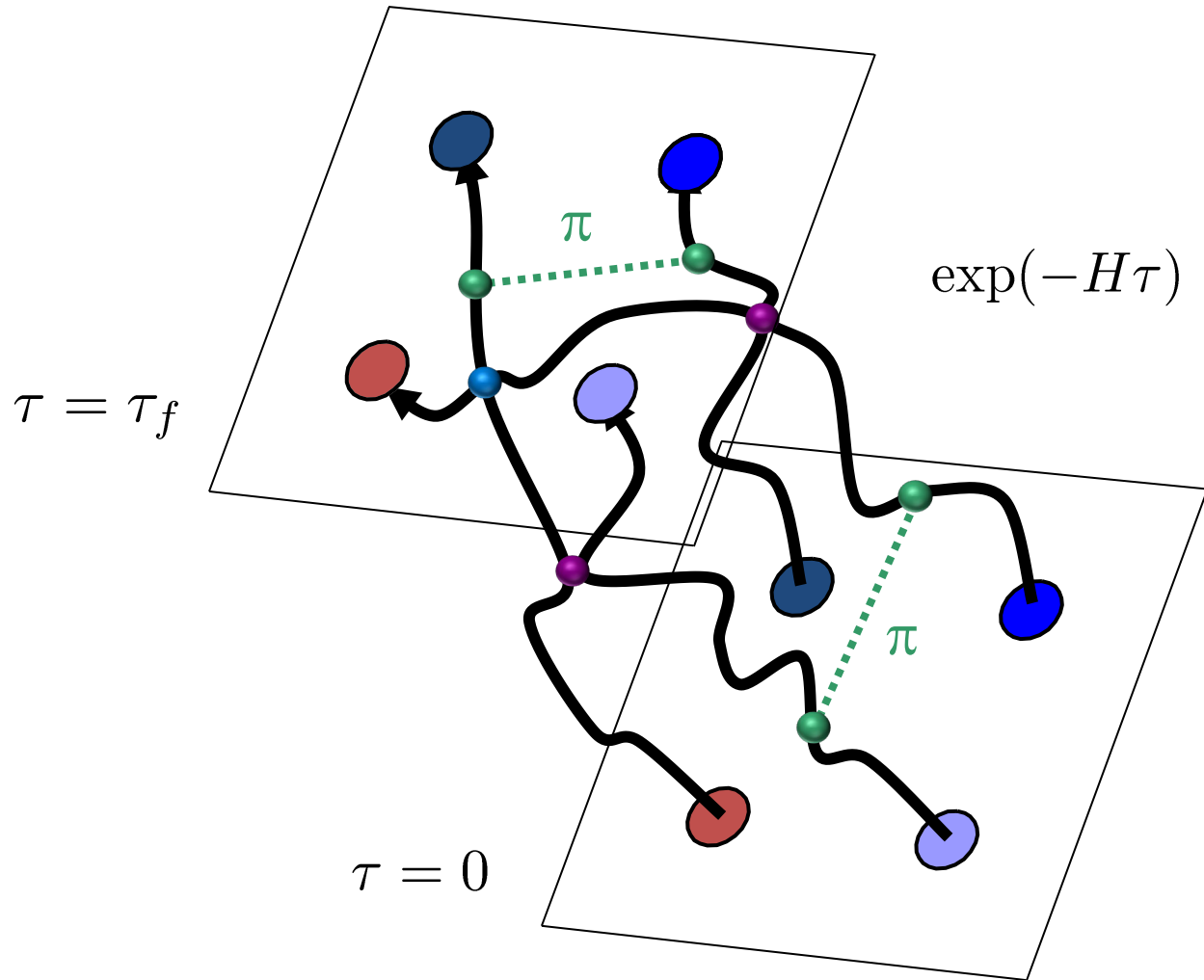
Construct the effective potential order by order



$a = 0.987 \text{ fm}$



# Euclidean time projection



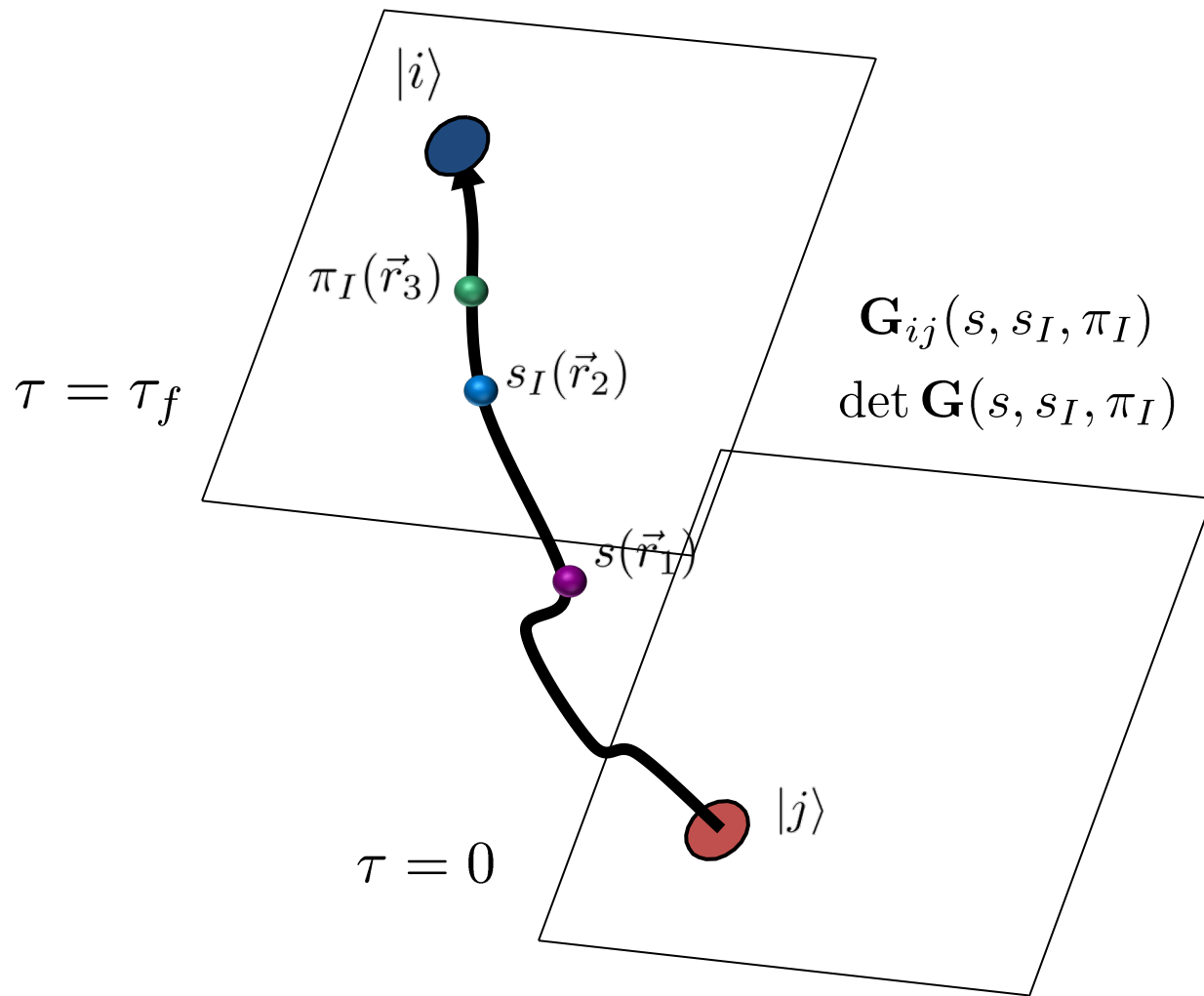
## Auxiliary field method

We can write exponentials of the interaction using a Gaussian integral identity

$$\begin{aligned} & \exp \left[ -\frac{C}{2} (N^\dagger N)^2 \right] \quad \diagdown \quad (N^\dagger N)^2 \\ & = \sqrt{\frac{1}{2\pi}} \int_{-\infty}^{\infty} ds \exp \left[ -\frac{1}{2} s^2 + \sqrt{-C} s (N^\dagger N) \right] \quad \diagup \quad s N^\dagger N \end{aligned}$$

We remove the interaction between nucleons and replace it with the interactions of each nucleon with a background field.





## Unitarity limit

Consider S-wave two-body scattering in three dimensions at low momenta. The scattering amplitude is

$$f_0(p) \propto \frac{1}{p \cot \delta_0(p) - ip}$$

The effective range expansion:  $p \cot \delta_0(p) = -\frac{1}{a_0} + \frac{1}{2}r_0p^2 + \dots$

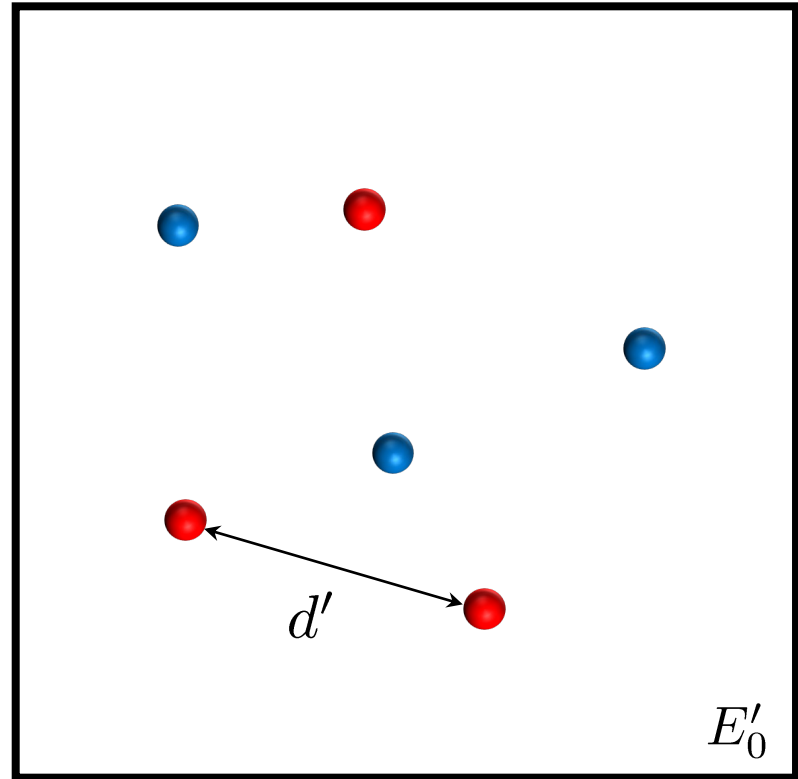
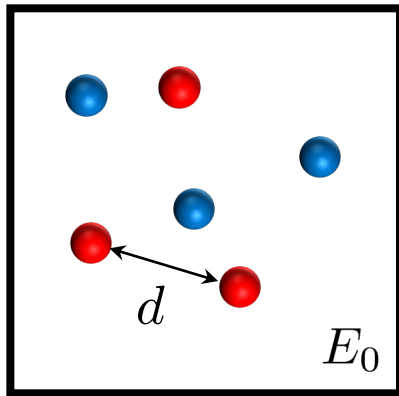
Consider when the scattering length  $a_0$  is infinite. At low momenta, we get

$$p \cot \delta_0(p) \approx 0 \quad f_0(p) \propto \frac{i}{p} \quad \text{scale invariant physics}$$

## Two-component Fermi gases at unitarity

Consider a non-relativistic two-component Fermi gas at unitarity in the limit that the range of the interactions is zero and the scattering length tuned to infinity.

Since the unitarity limit has no intrinsic length scales, all many-body observables must equal some dimensionless number times the appropriate power of the Fermi momentum,  $k_F$ . For energies and temperatures, there is an overall factor of the inverse particle mass.



$$E_0 d^2 = E'_0 d'^2$$

For example, the energy per particle must be proportional the Fermi energy,  $E_F$ . This similar to the case for a free non-interacting Fermi gas. For the free Fermi gas, the ground state energy per particle in the thermodynamic limit is

$$\frac{E_0^{\text{free}}}{A} = \frac{3}{5}E_F$$

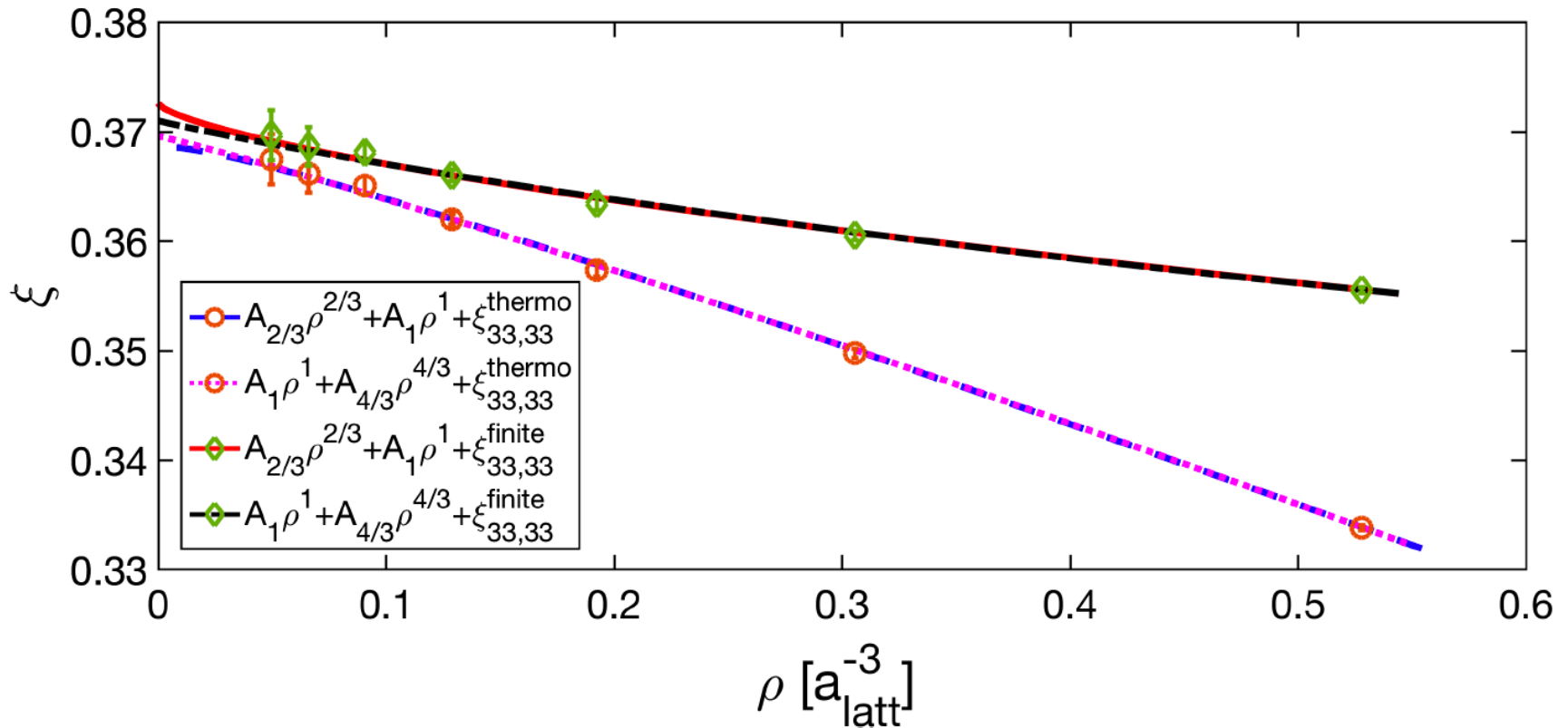
It is conventional to define the Bertsch parameter  $\xi$  for the ratio between the ground state energy per particle in the unitary limit and the ground state energy per particle for the free gas.

$$\frac{E_0}{A} = \xi \frac{E_0^{\text{free}}}{A} = \frac{3}{5}\xi E_F$$

## Lattice simulation results

$$\zeta_{33,33}^{\text{thermo}} = 0.372(2) \quad \zeta_{33,33}^{\text{finite}} = 0.369(2)$$

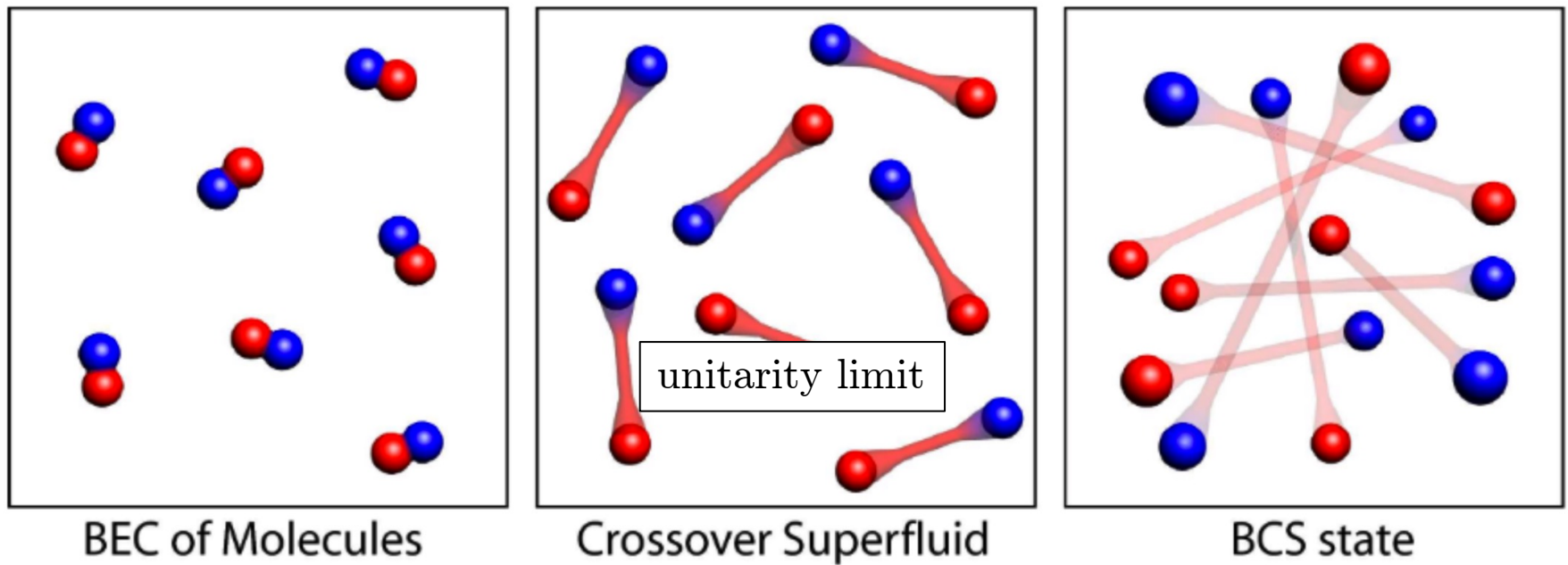
He, Li, Lu, D.L., PRA 101, 063615 (2020)



Other lattice calculations: 0.372(5) Carlson, Gandolfi, Schmidt, Zhang PRA 84 061602(R) (2011)

Experiment: 0.376(4) Ku, Sommer, Cheuk, Zwierlein, Science 335 (2012) 563

## Superfluidity pairing and condensate



Ketterle, Zwierlein

## Superfluidity and pairing correlations

The two-body density matrix is defined as

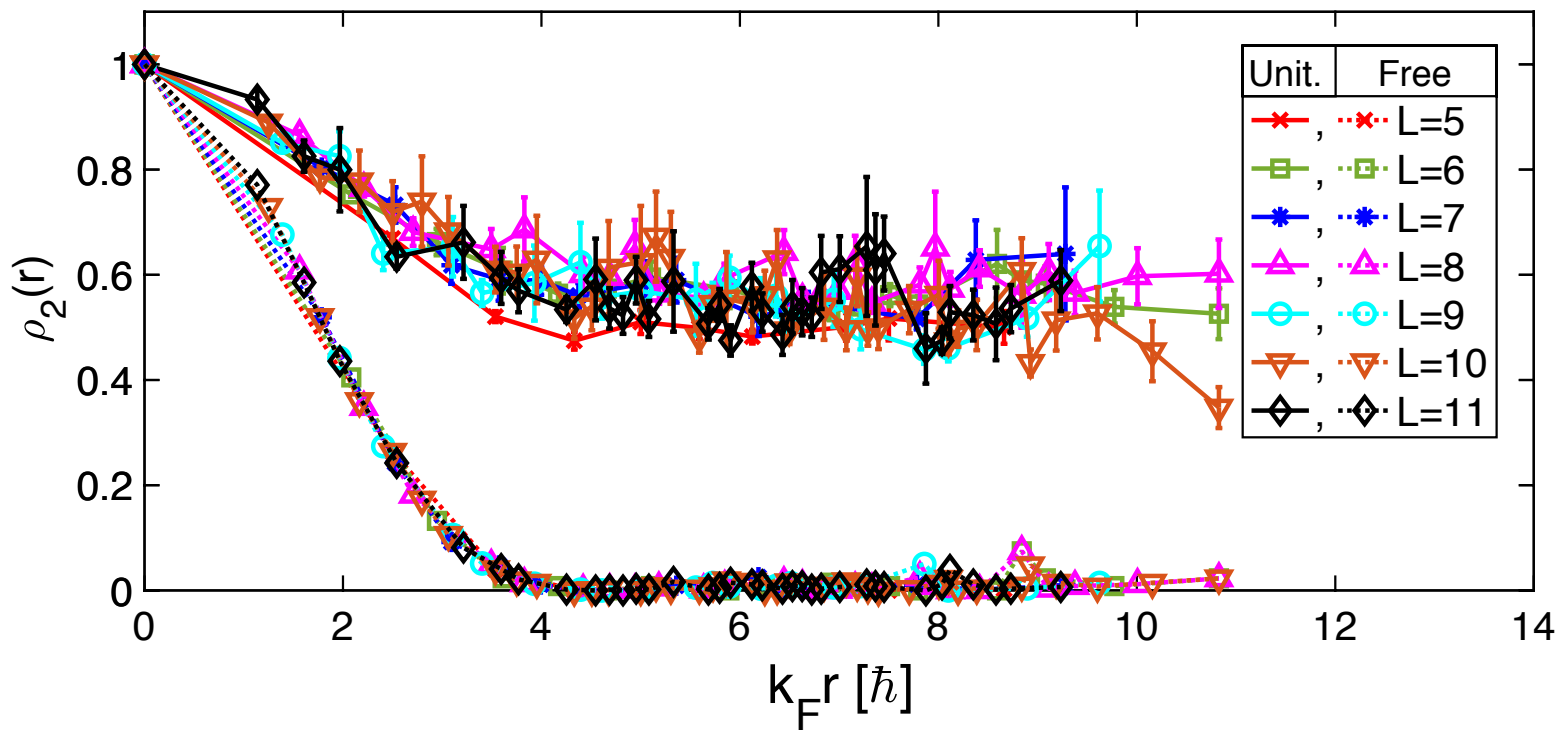
$$\rho_2(\vec{r}'_1, \vec{r}'_2, \vec{r}_1, \vec{r}_2) = \left\langle \psi_{\downarrow}^{\dagger}(\vec{r}'_2) \psi_{\uparrow}^{\dagger}(\vec{r}'_1) \psi_{\uparrow}(\vec{r}_1) \psi_{\downarrow}(\vec{r}_2) \right\rangle$$

Long-range correlations in the two-body density matrix is a signature for pair superfluidity:

$$\begin{aligned} \rho_2(\vec{r}'_1, \vec{r}'_2, \vec{r}_1, \vec{r}_2) &\rightarrow \alpha N/2 \cdot \phi^*(|\vec{r}'_1 - \vec{r}'_2|) \phi(|\vec{r}_1 - \vec{r}_2|) \\ |\vec{r}_1 - \vec{r}'_1|, |\vec{r}_2 - \vec{r}'_2| &\rightarrow \infty \end{aligned}$$

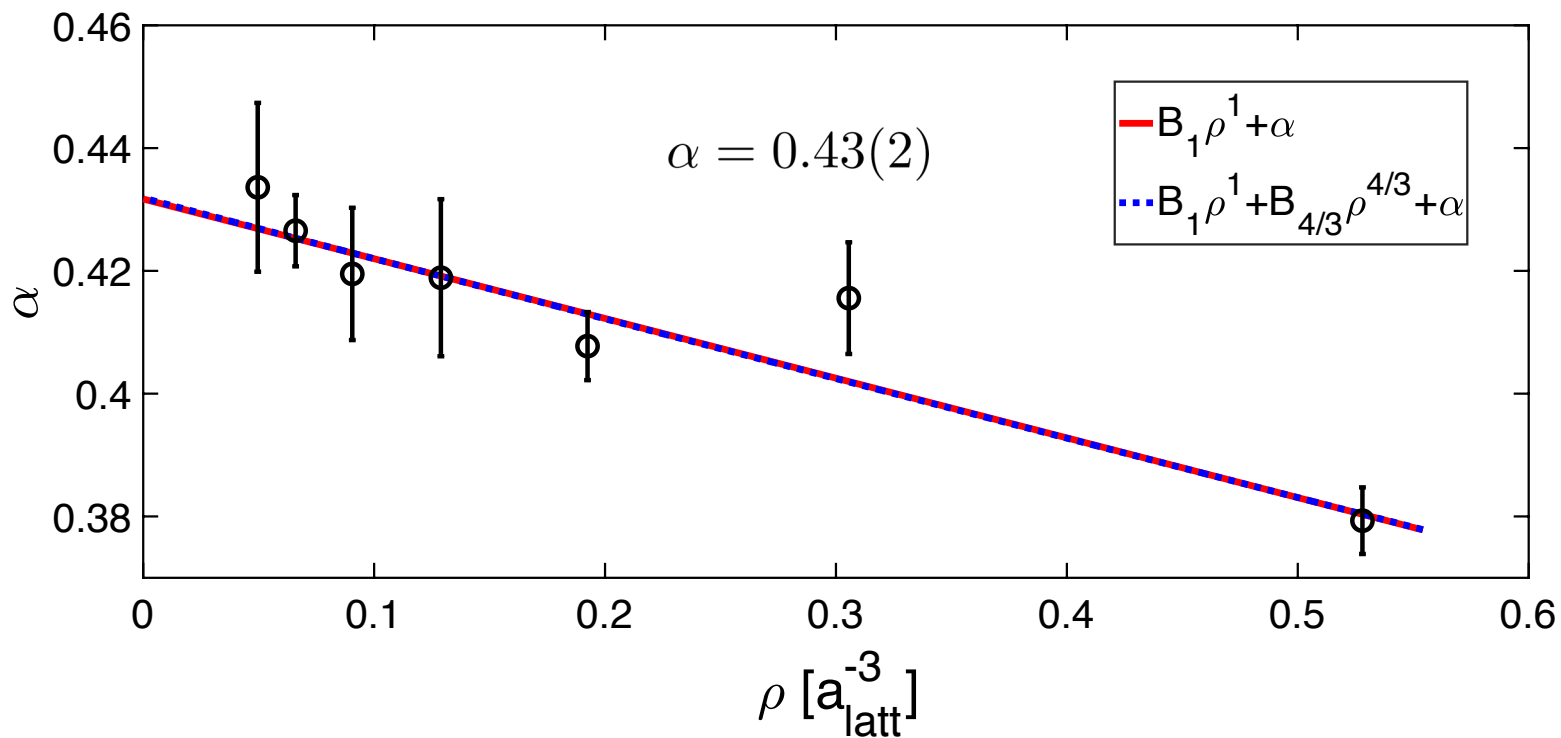
Yang, RMP 34, 694 (1962)





He, Li, Lu, D.L., PRA 101, 063615 (2020)

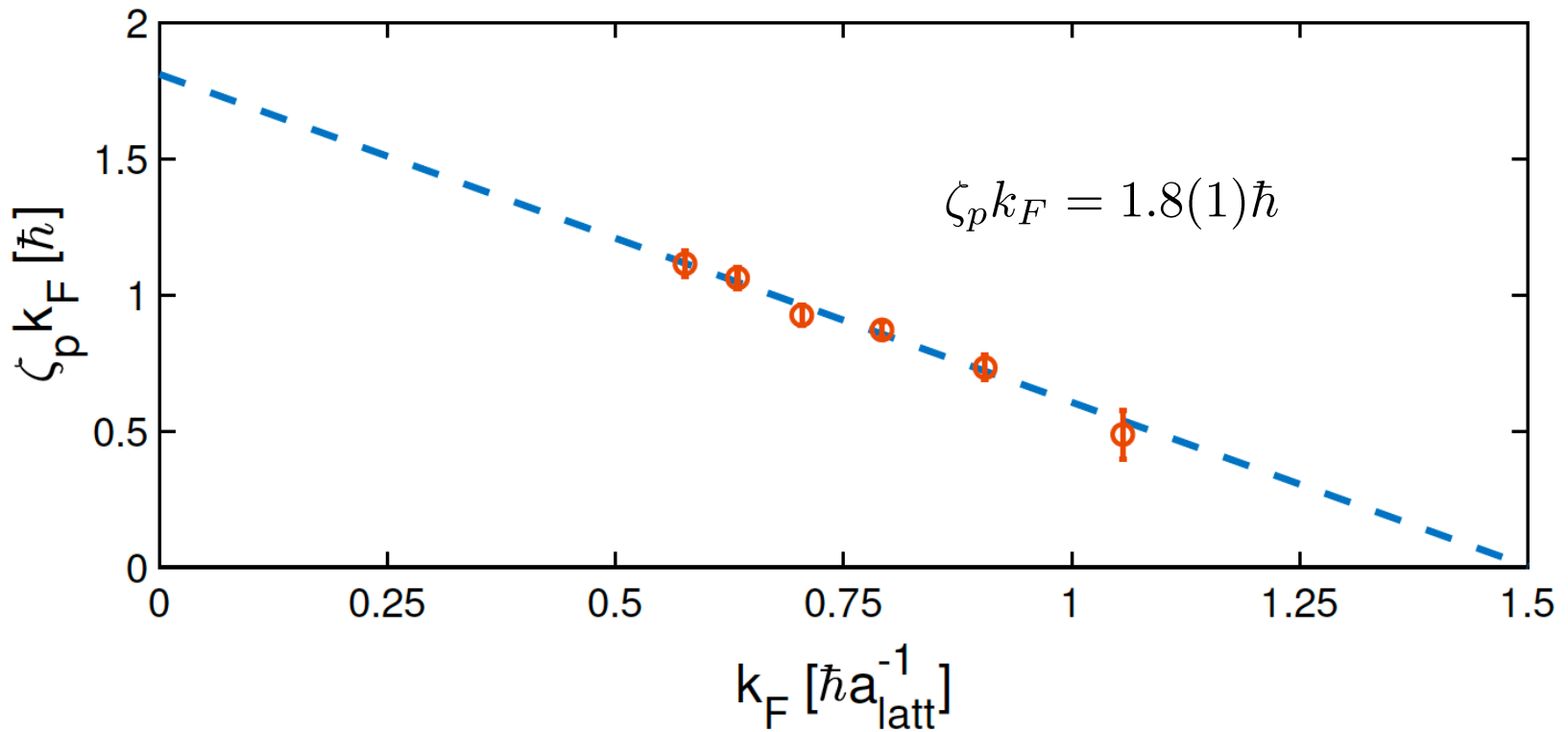
## Superfluid condensate fraction



Experiment: 0.46(7) Zwierlein, Stan, Schunck, Raupach, Kerman, Ketterle, PRL 92 (2004) 120403

## Pair wave function

$$\Phi(r) \propto \frac{e^{-r/\zeta_p}}{r}$$



He, Li, Lu, D.L., PRA 101, 063615 (2020)

## Going beyond two components

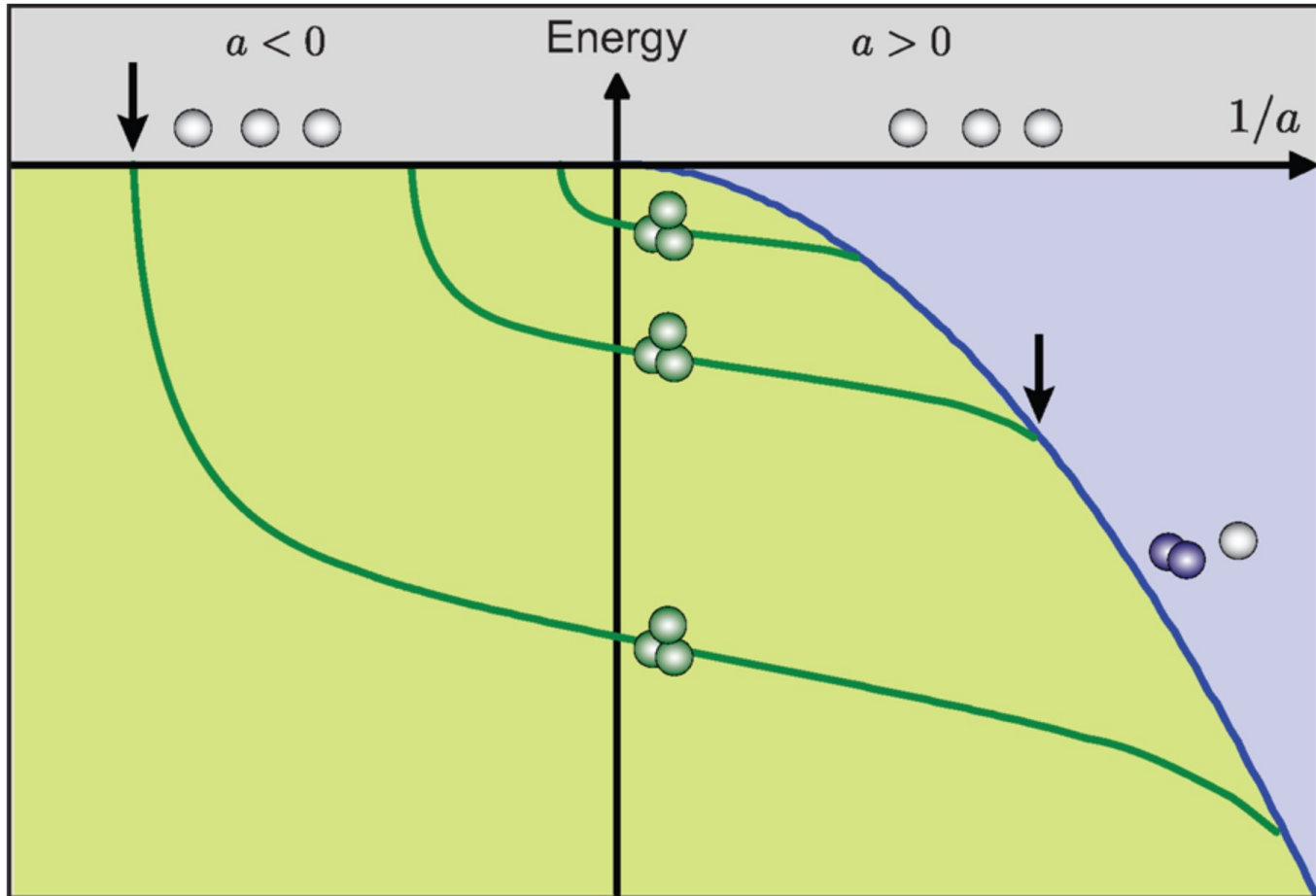
In quantum mechanics and quantum field theory, scale invariance can be spoiled by quantum scale anomalies. This happens when there are bound states, which necessarily correspond to discrete energy levels.

Nevertheless, it may happen that a discrete subgroup of the scale symmetry is preserved for the dynamics of certain sectors of the Hilbert space.

This phenomenon was first noted by Efimov for bound states of three bosons when the two-body interactions are tuned to the unitarity limit.

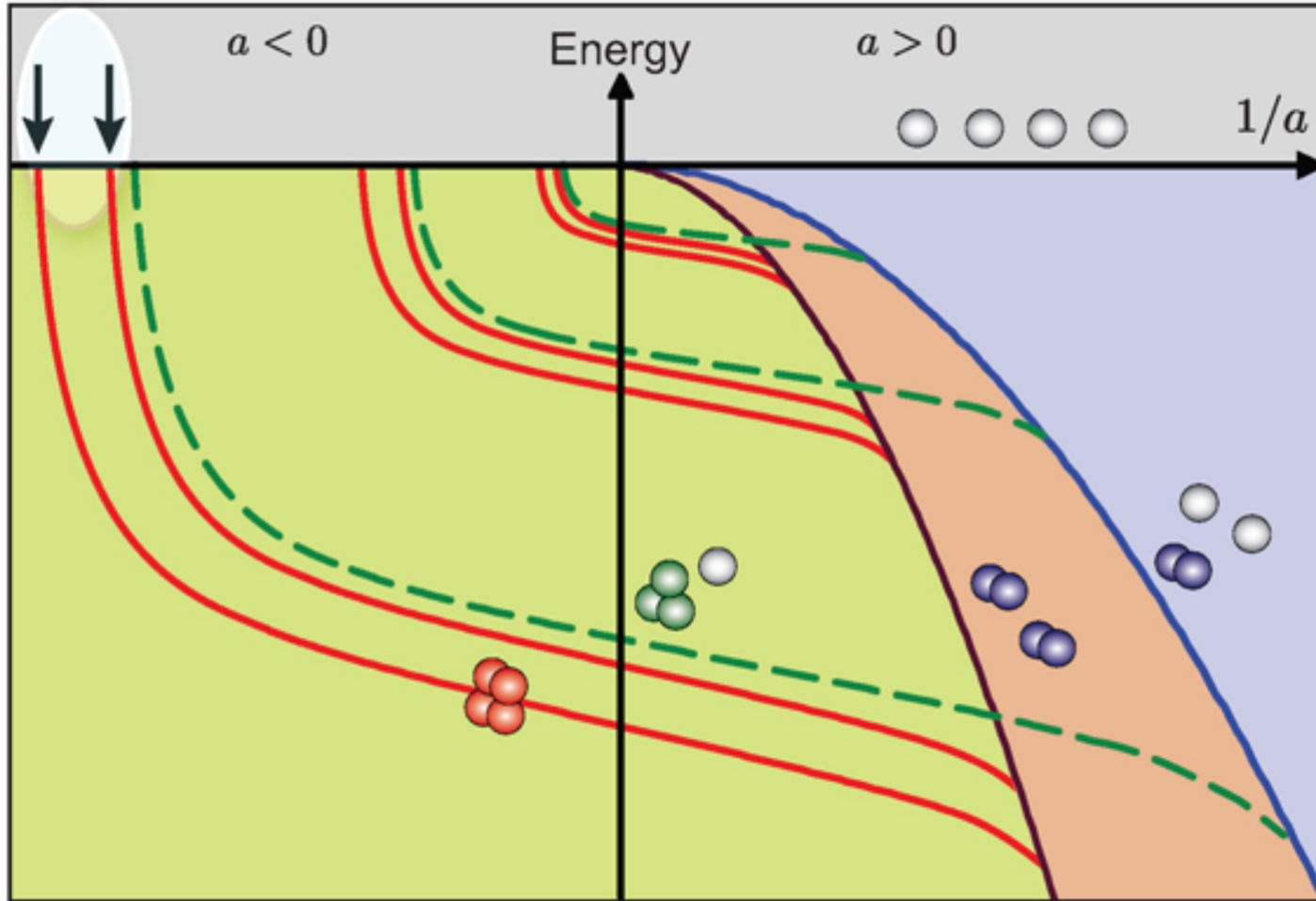
Efimov, Sov. J. Nucl. Phys. 12, 589 (1971); Efimov, Phys. Rev. C47 1876 (1993)  
Bedaque, Hammer, van Kolck, Phys. Rev. Lett. 82 463 (1999)

# Efimov trimers



Ferlaino, Grimm, Physics 3, 9 (2010)

# Efimov tetramers



Platter, Hammer, Meißner, PRA 70, 052101 (2004)

Ferlaino, Koop, Berninger, Harm, D’Incao, Nägerl, Grimm, PRL 102, 140401 (2009)

Ferlaino, Grimm, Physics 3, 9 (2010)

Is nuclear physics near four-component Fermi gas unitarity?

# Nuclear Physics Around the Unitarity Limit

Sebastian König,<sup>1,2,3,\*</sup> Harald W. Griesshammer,<sup>4,†</sup> H.-W. Hammer,<sup>2,3,‡</sup> and U. van Kolck<sup>5,6,§</sup>

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<sup>5</sup>*Institut de Physique Nucléaire, CNRS-IN2P3, Université Paris-Sud, Université Paris-Saclay, 91406 Orsay, France*

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(Received 18 July 2016; revised manuscript received 10 February 2017; published 15 May 2017)

We argue that many features of the structure of nuclei emerge from a strictly perturbative expansion around the unitarity limit, where the two-nucleon  $S$  waves have bound states at zero energy. In this limit, the gross features of states in the nuclear chart are correlated to only one dimensionful parameter, which is related to the breaking of scale invariance to a discrete scaling symmetry and set by the triton binding energy. Observables are moved to their physical values by small *perturbative* corrections, much like in descriptions of the fine structure of atomic spectra. We provide evidence in favor of the conjecture that light, and possibly heavier, nuclei are bound weakly enough to be insensitive to the details of the interactions but strongly enough to be insensitive to the exact size of the two-nucleon system.

DOI: [10.1103/PhysRevLett.118.202501](https://doi.org/10.1103/PhysRevLett.118.202501)

König, Griesshammer, Hammer, van Kolck, PRL 118, 202501 (2017)



# Clustering of Four-Component Unitary Fermions

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<sup>2</sup>*Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA*

<sup>3</sup>*Institut de Physique Nucléaire, CNRS-IN2P3, Université Paris-Sud, Université Paris-Saclay, 91406 Orsay, France*

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(Received 12 August 2019; accepted 13 March 2020; published 9 April 2020)

*Ab initio* nuclear physics tackles the problem of strongly interacting four-component fermions. The same setting could foreseeably be probed experimentally in ultracold atomic systems, where two- and three-component experiments have led to major breakthroughs in recent years. Both due to the problem's inherent interest and as a pathway to nuclear physics, in this Letter we study four-component fermions at unitarity via the use of quantum Monte Carlo methods. We explore novel forms of the trial wave function and find one which leads to a ground state of the eight-particle system whose energy is almost equal to that of two four-particle systems. We investigate the clustering properties involved and also extrapolate to the zero-range limit. In addition to being experimentally testable, our results impact the prospects of developing nuclear physics as a perturbation around the unitary limit.

DOI: [10.1103/PhysRevLett.124.143402](https://doi.org/10.1103/PhysRevLett.124.143402)

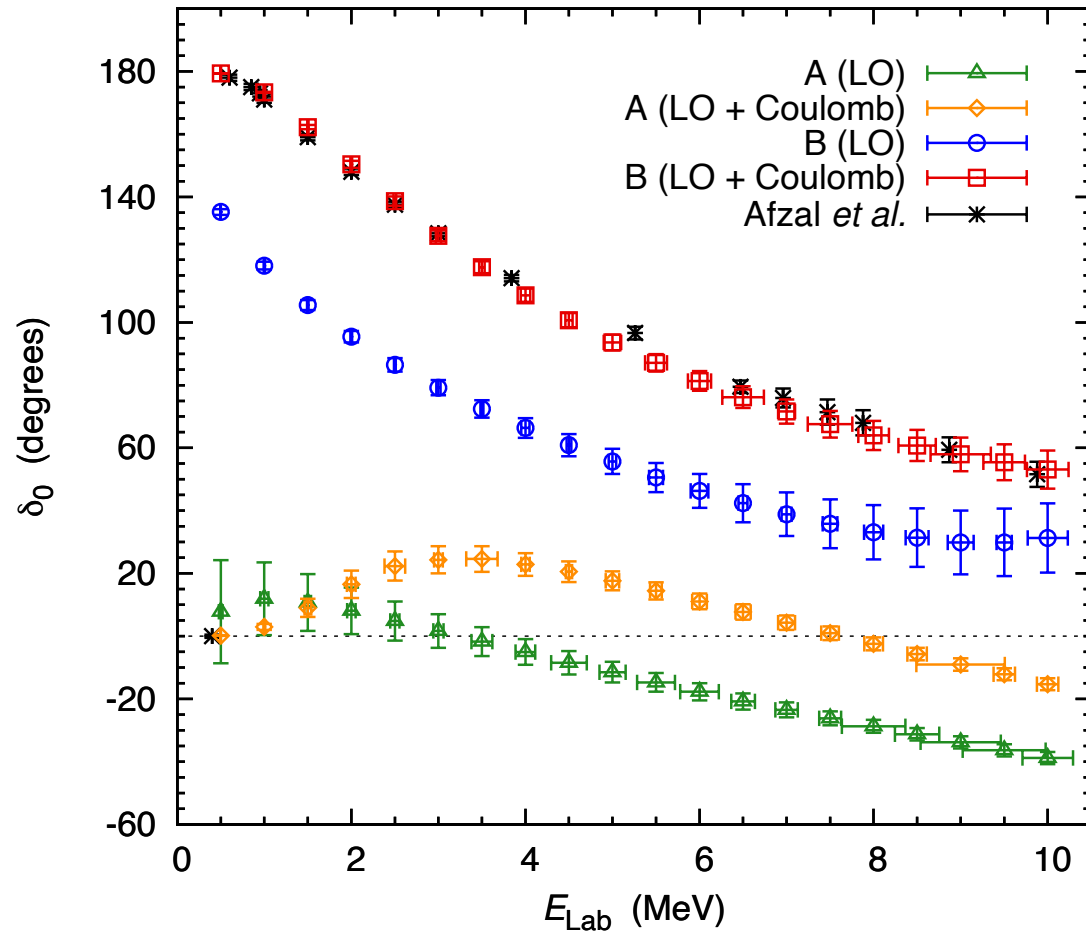
Dawkins, Carlson, van Kolck, Gezerlis, PRL 124, 143402 (2020)

## A tale of two interactions

We consider two different interactions, A and B, at leading order (LO) in chiral effective field theory. They both have the same one-pion exchange potential and Coulomb potential. The difference between A and B resides with their short-range interactions.

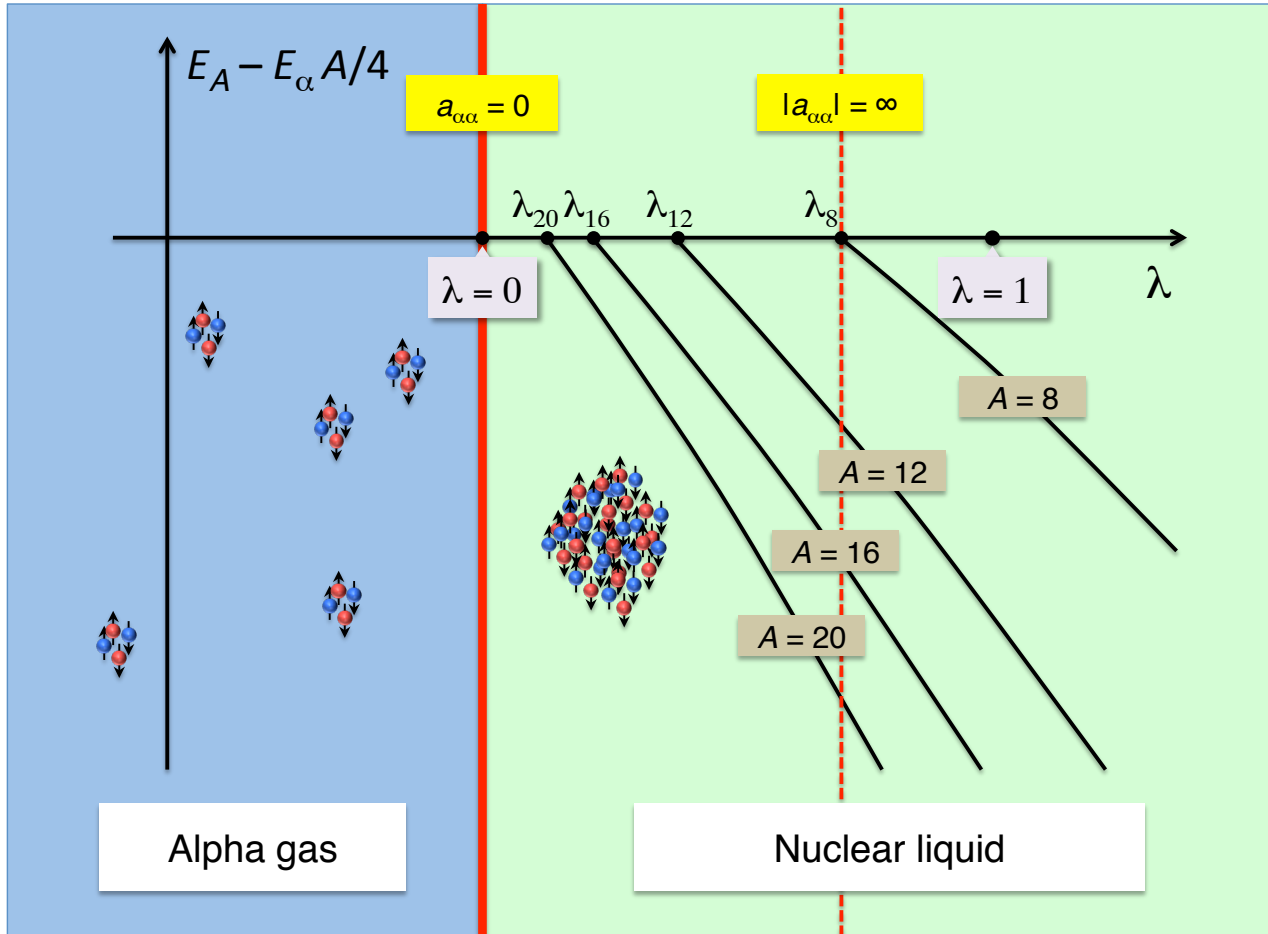
<u>Interaction A</u>	<u>Interaction B</u>
Nonlocal short-range interaction	Nonlocal short-range interaction + Local short-range interaction

## alpha-alpha S-wave scattering



Elhatisari, Li, Rokash, Alarcon, Du, Klein, Lu, Meißner, Epelbaum, Krebs, Lähde, D.L., Rupak, PRL 117, 132501 (2016)

# Nuclear physics near a quantum phase transition



Elhatisari, Li, Rokash, Alarcon, Du, Klein, Lu, Meißner, Epelbaum, Krebs, Lähde, D.L., Rupak, PRL 117, 132501 (2016)

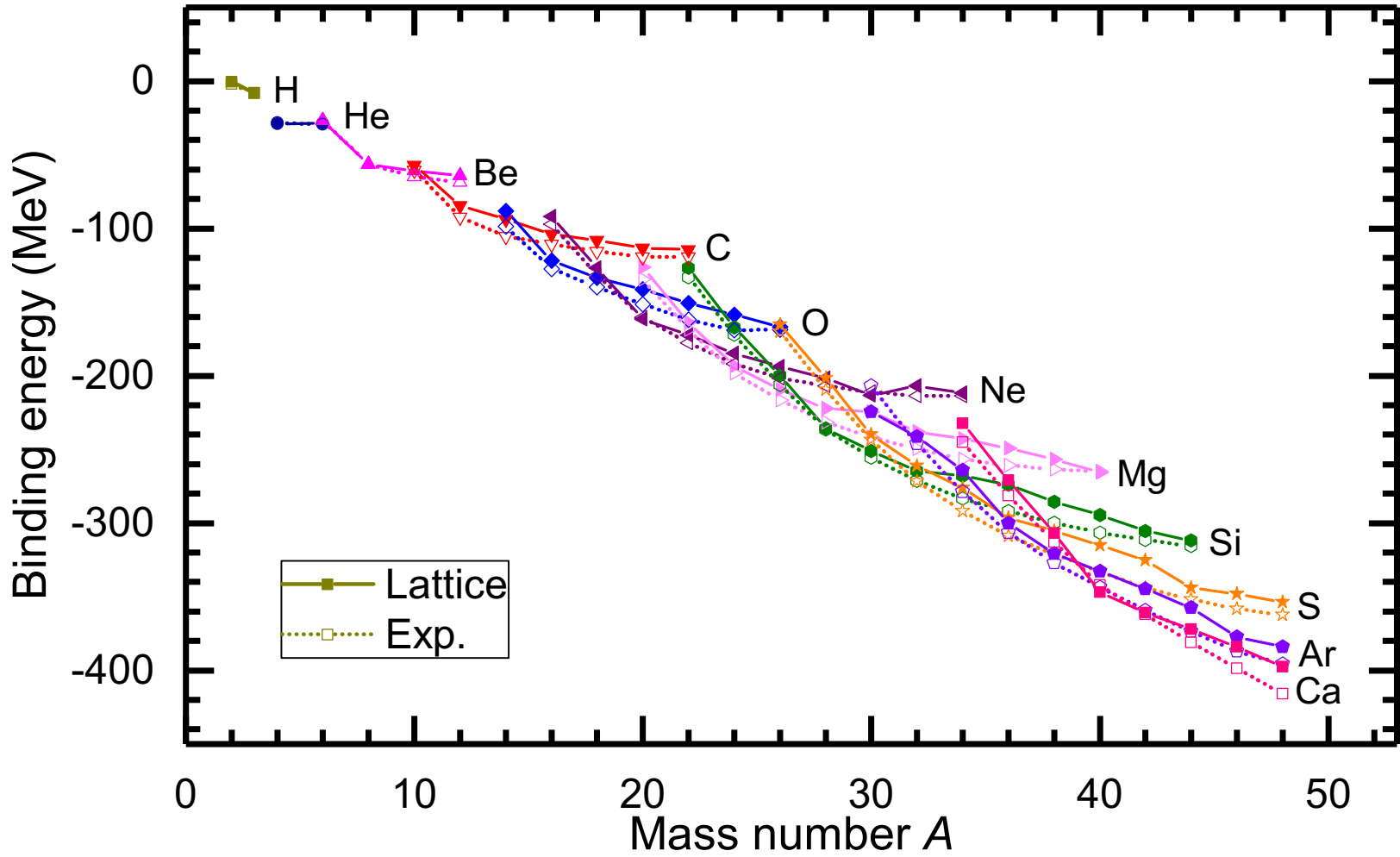
## Essential elements for nuclear binding

What is the minimal nuclear interaction that can reproduce the ground state properties of light nuclei, medium-mass nuclei, and neutron matter simultaneously with no more than a few percent error in the energies and charge radii?

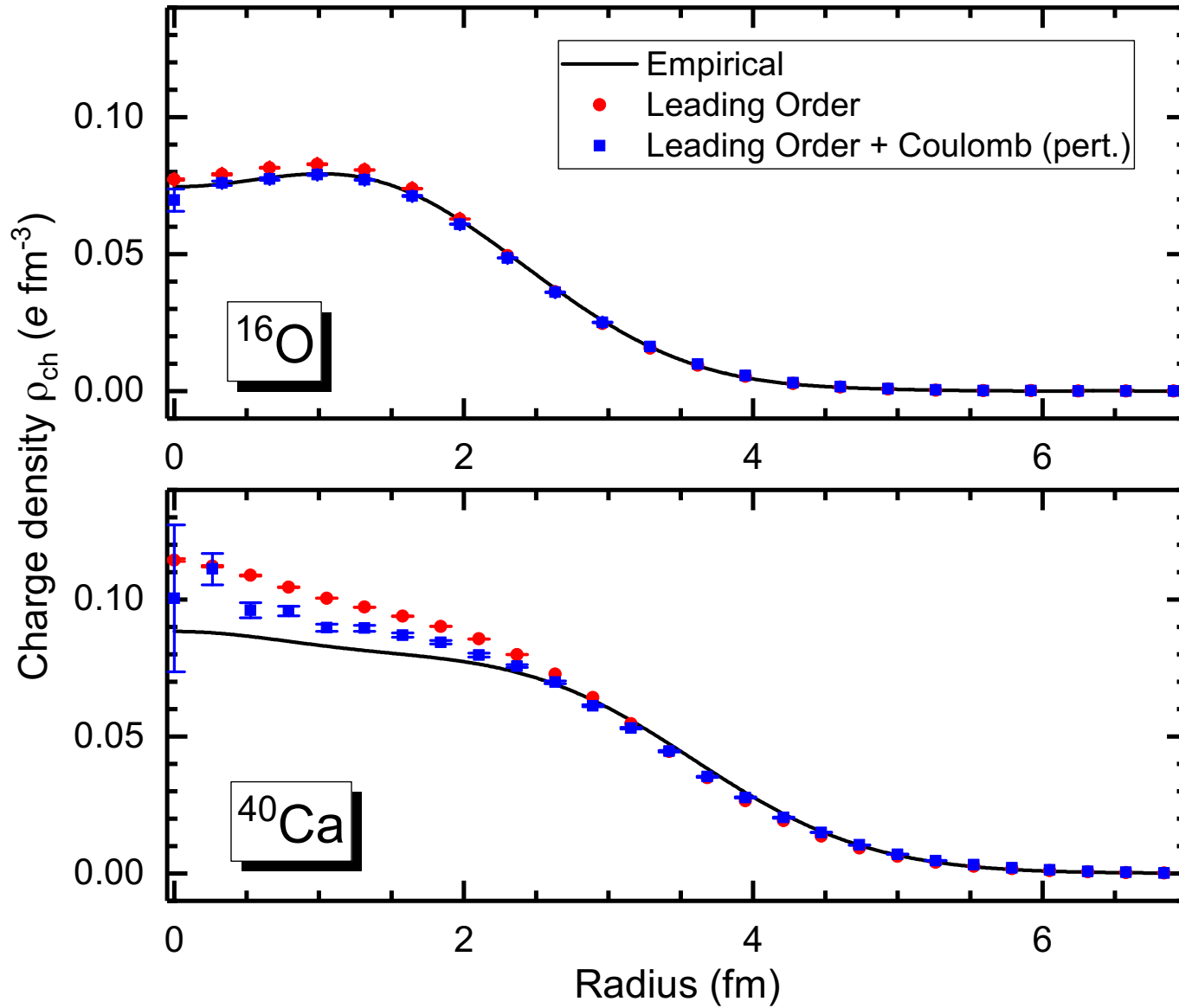
We construct an interaction with only four parameters.

1. Strength of the two-nucleon  $S$ -wave interaction
2. Range of the two-nucleon  $S$ -wave interaction
3. Strength of three-nucleon contact interaction
4. Range of the local part of the two-nucleon interaction

Except for the Coulomb potential, the interaction is invariant under Wigner's  $SU(4)$  symmetry.



	$B$	Exp.	$R_{\text{ch}}$	Exp.
${}^3\text{H}$	8.48(2)(0)	8.48	1.90(1)(1)	1.76
${}^3\text{He}$	7.75(2)(0)	7.72	1.99(1)(1)	1.97
${}^4\text{He}$	28.89(1)(1)	28.3	1.72(1)(3)	1.68
${}^{16}\text{O}$	121.9(1)(3)	127.6	2.74(1)(1)	2.70
${}^{20}\text{Ne}$	161.6(1)(1)	160.6	2.95(1)(1)	3.01
${}^{24}\text{Mg}$	193.5(02)(17)	198.3	3.13(1)(2)	3.06
${}^{28}\text{Si}$	235.8(04)(17)	236.5	3.26(1)(1)	3.12
${}^{40}\text{Ca}$	346.8(6)(5)	342.1	3.42(1)(3)	3.48



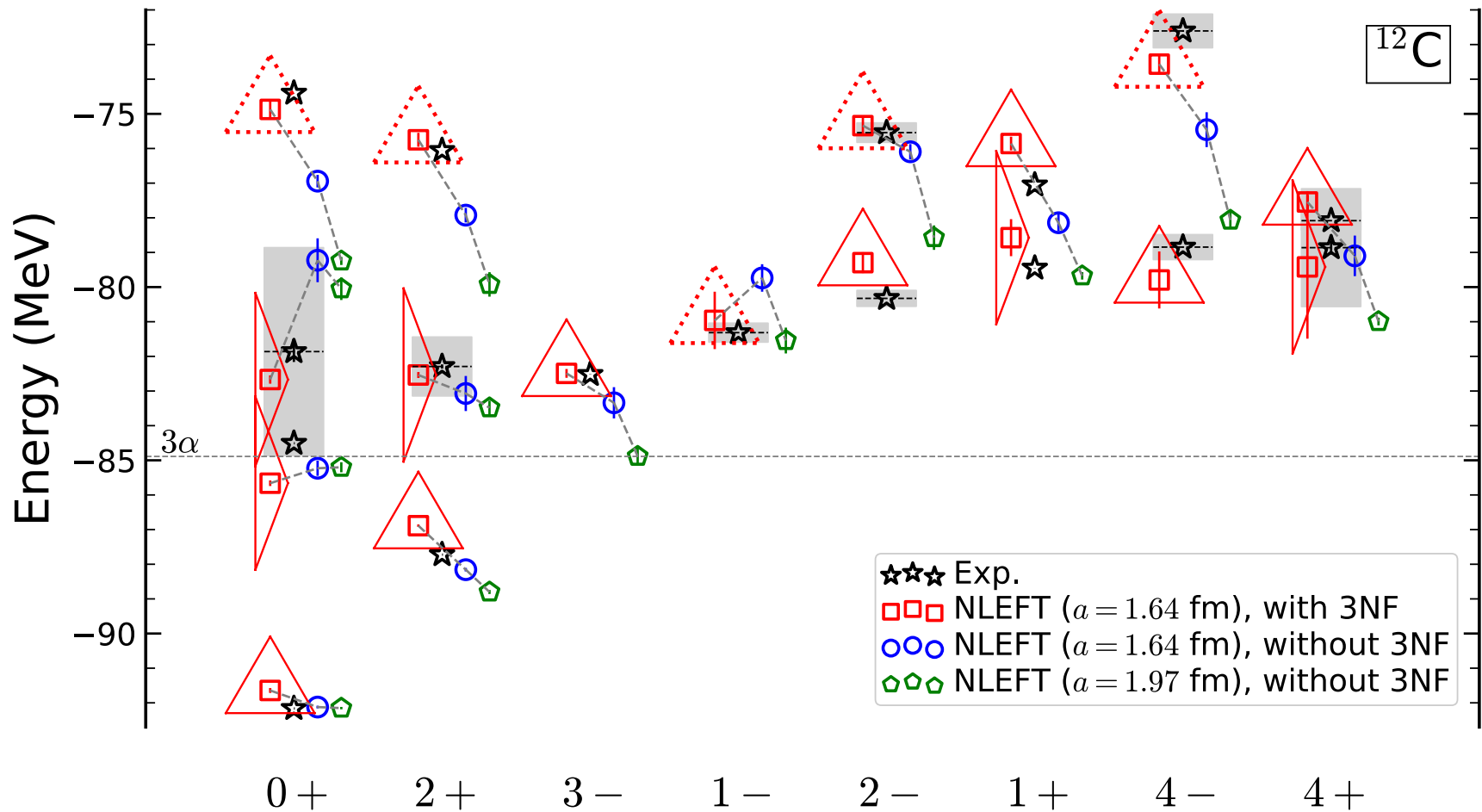


## Spectrum and structure of $^{12}\text{C}$ states

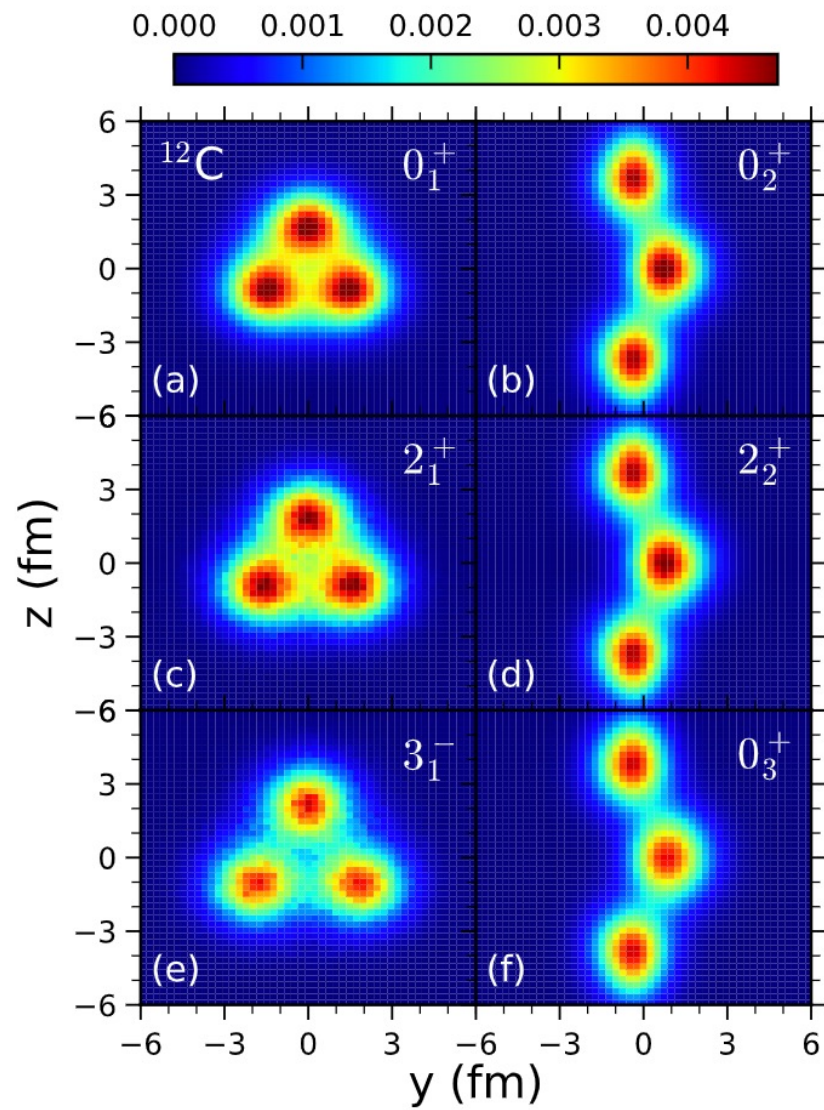
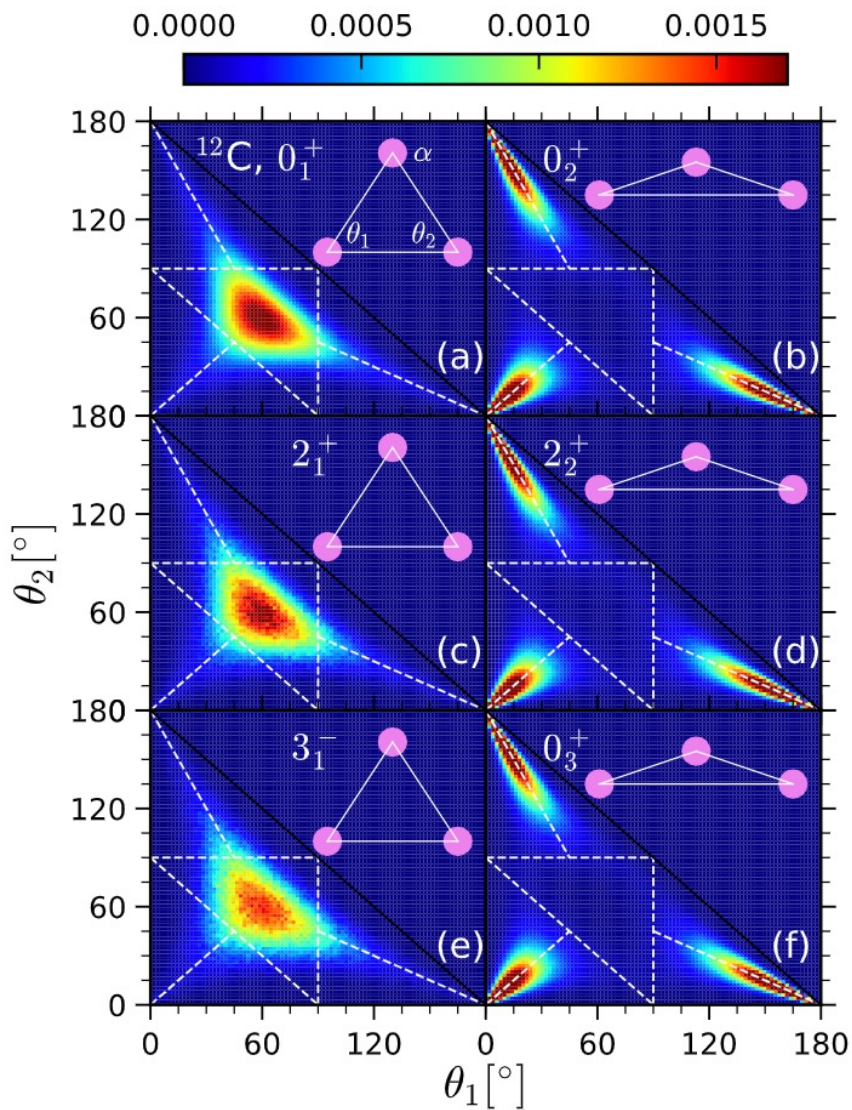
## Structure and spectrum of $^{12}\text{C}$

Shen, Lähde, D.L. Meißner, EPJA 57, 276 (2021)

State	$a = 1.97 \text{ fm}$	$a = 1.64 \text{ fm}$	Experiment
$0_1^+$	−92.15(3)	−92.12(4)	−92.162
$2_1^+$	−88.87(4)	−88.19(17)	−87.722
$0_2^+$	−85.20(15)	−85.23(22)	−84.508
$3_1^-$	−84.9(2)	−83.3(5)	−82.521(5)
$2_2^+$	−83.5(2)	−83.1(5)	−82.29(6)
$0_3^+$	−80.0(3)	−79.2(6)	−81.9(3)
$1_1^-$	−81.5(4)	−79.7(4)	−81.315(4)
$2_1^-$	−78.6(2)	−76.1(2)	−80.326(4)
$1_1^+$	−79.67(11)	−78.14(24)	−79.452(6)
$4_1^-$	−78.1(2)	−75.5(5)	−78.846(20)
$4_1^+$	−80.99(11)	−79.1(6)	−78.083(5)
$2_3^+$	−79.9(4)	−77.9(2)	−76.056
$0_4^+$	−79.25(11)	−76.94(18)	−74.402



Shen, Lähde, D.L. Meißner, arXiv:2202.13596



Shen, Lähde, D.L. Meißner, arXiv:2202.13596

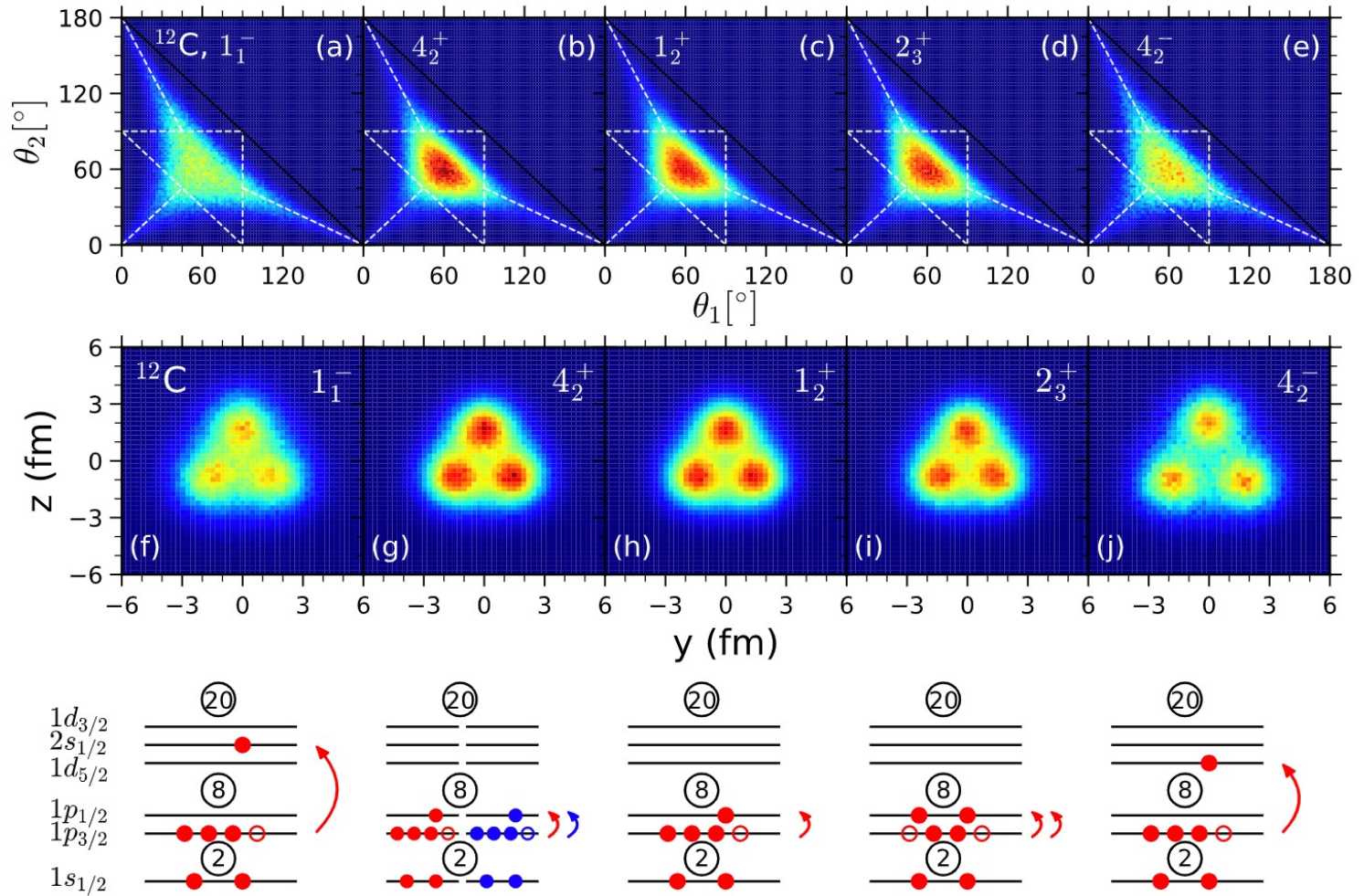


Figure S3: **Top Panel:** Density distribution for the two inner angles of the triangle formed by the three alpha clusters. **Middle Panel:** Tomographic projection of the nuclear density. **Lower Panel:** Sketch of the orbitals for the shell model initial states used in each of these calculations.

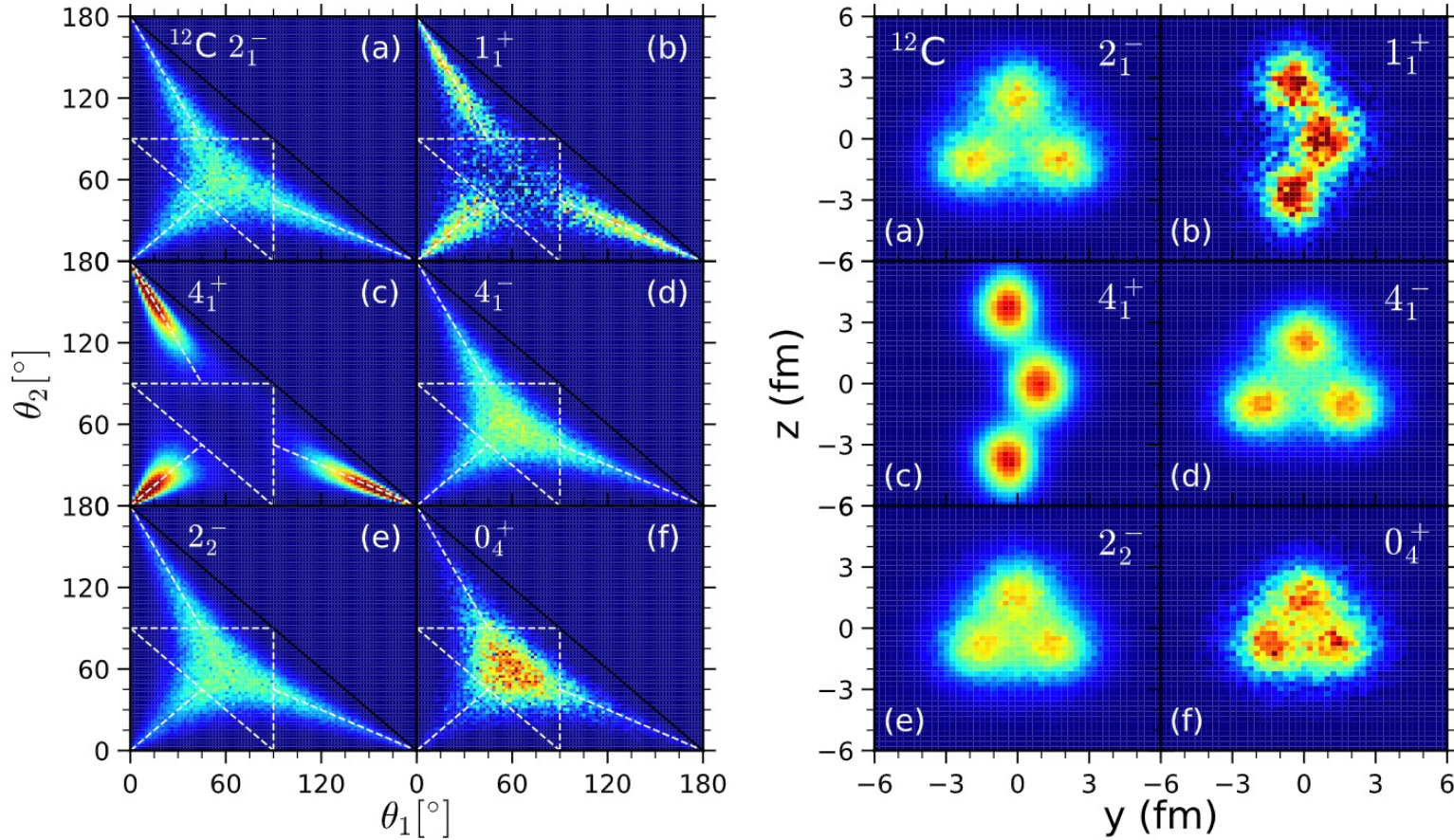


Figure S4: **Left Panel:** Density distribution for the two inner angles of the triangle formed by the three alpha clusters. **Right Panel:** Tomographic projection of the nuclear density. From (a) to (f), the selected states are ordered by their energies from low to high.

## Summary

We started with an introduction to lattice effective field theory. We then discussed the unitarity limit and properties of two-component Fermi gases at unitarity.

We then moved on to fermionic systems with more than two components and asked the question whether nuclear physics is near the unitarity limit. We found that symmetric nuclear matter is near a quantum phase transition, suggesting that the unitarity limit for nucleons likely produces a Bose gas of alpha particles rather than a nuclear liquid.

However, we found numerical evidence that nuclear physics is close to Wigner's SU(4) symmetric limit with alpha-alpha scattering near the unitarity limit. We then discussed the intrinsic structure of the low-lying  $^{12}\text{C}$  states.