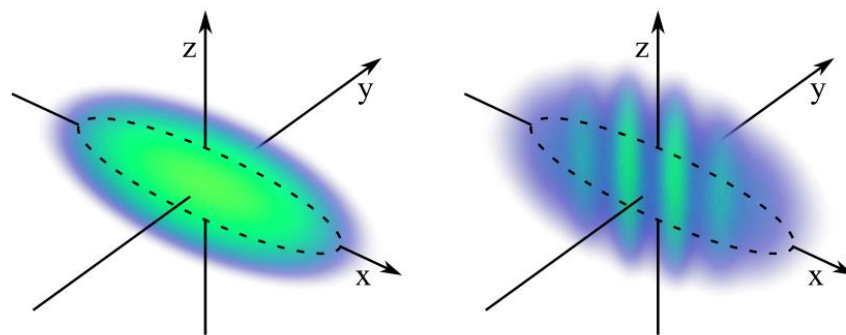


Fundamental properties of supersolids in a dipolar quantum gas



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Department of Physics and Astronomy and LENS, University of Florence,
and CNR-INO, Pisa section

**Connections between cold atoms and nuclear matter:
From low to high energies, Trento, June 2022**

The supersolid team

 CNR-INO (Pisa)



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Antolini

Giulio
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Andrea
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Modugno

Carlo
Gabbanini

Luca
Tanzi



<http://quantumgases.lens.unifi.it/exp/dy>

 @DysprosiumLab

Former members

Julian Maloberti, Francesca Famà,
Eleonora Lucioni, Jacopo Catani

Theory

Michele Modugno, Aitor Alaña (Bilbao)
Beatrice Donelli, Luca Pezzè, Augusto
Smerzi (Firenze)

Santo Roccuzzo, Alessio Recati, Sandro
Stringari (Trento)

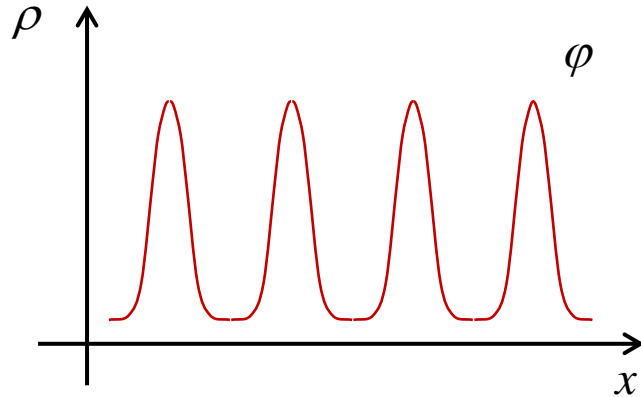
Russell Bisset, Luis Santos (Hannover)

Maria Luisa Chiofalo (Pisa)

Adriano Angelone (Trieste)

Fabio Cinti (Firenze)

The supersolid phase of matter



Spontaneous breaking of gauge and translational invariance.

In the simplest case, bosons with a spatially-modulated macroscopic wavefunction.

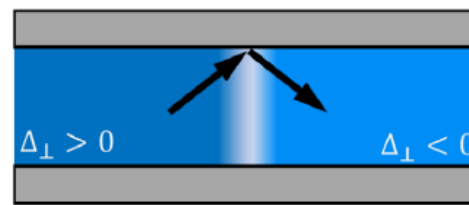
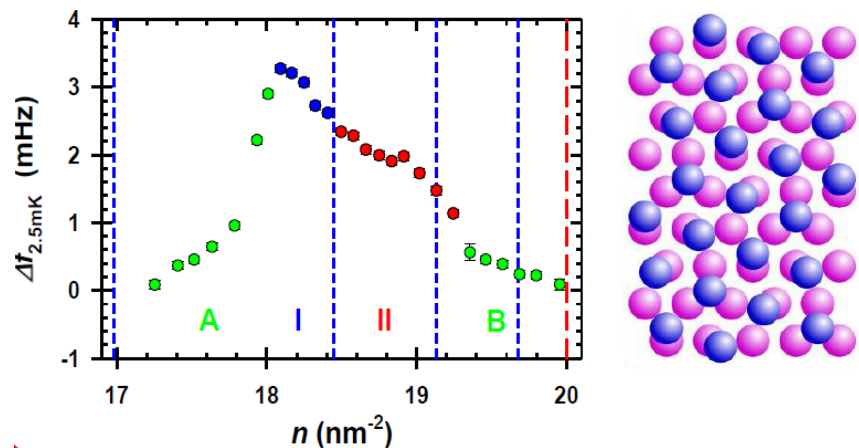
Interaction with finite range is needed.

Unprecedented mixing of superfluid and crystalline properties.

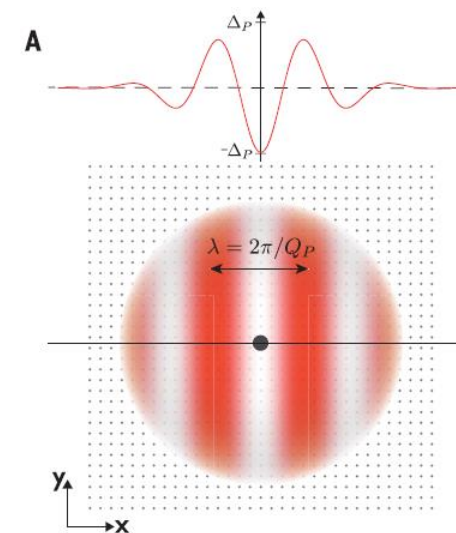
Seminal theory:

- E. P. Gross, Unified theory of interacting bosons, Phys. Rev. 106, 161 (1957).
- A.F. Andreev and L. M. Lifshitz, Quantum theory of defects in crystals, Sov. Phys. JETP 29, 1107–1113 (1969).
- G.V. Chester, Speculations on Bose-Einstein condensation and quantum crystals, Phys. Rev. 2, 161 (1970).
- A.J. Leggett, Can a solid be “superfluid”?, Phys. Rev. Lett. 25, 1543 (1970).

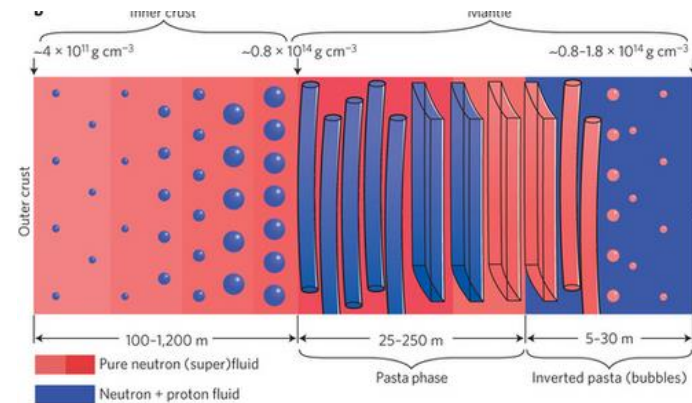
Supersolid-like phases



Helium-3 under confinement: Levitin et al. Phys. Rev. Lett. 122, 085301 (2019); Shook et al. Phys. Rev. Lett. 124, 015301 (2020)



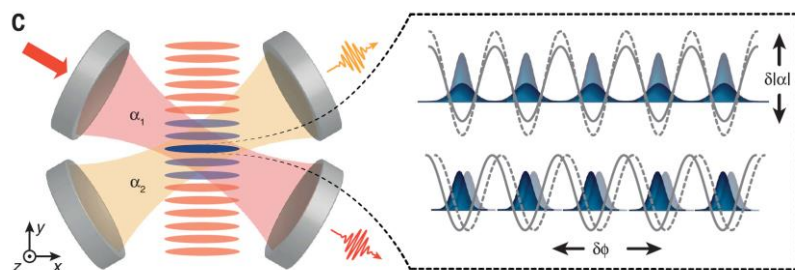
Helium-4 on graphite: Nyeky et al. Nature Phys. 13, 455 (2017)



Pasta phases in neutron stars

Cuprates: Edkins et al., Science 364, 976 (2019).
TMD: Liu et al. Science 372, 1447 (2021).

FFLO: R. Hulet (2021)

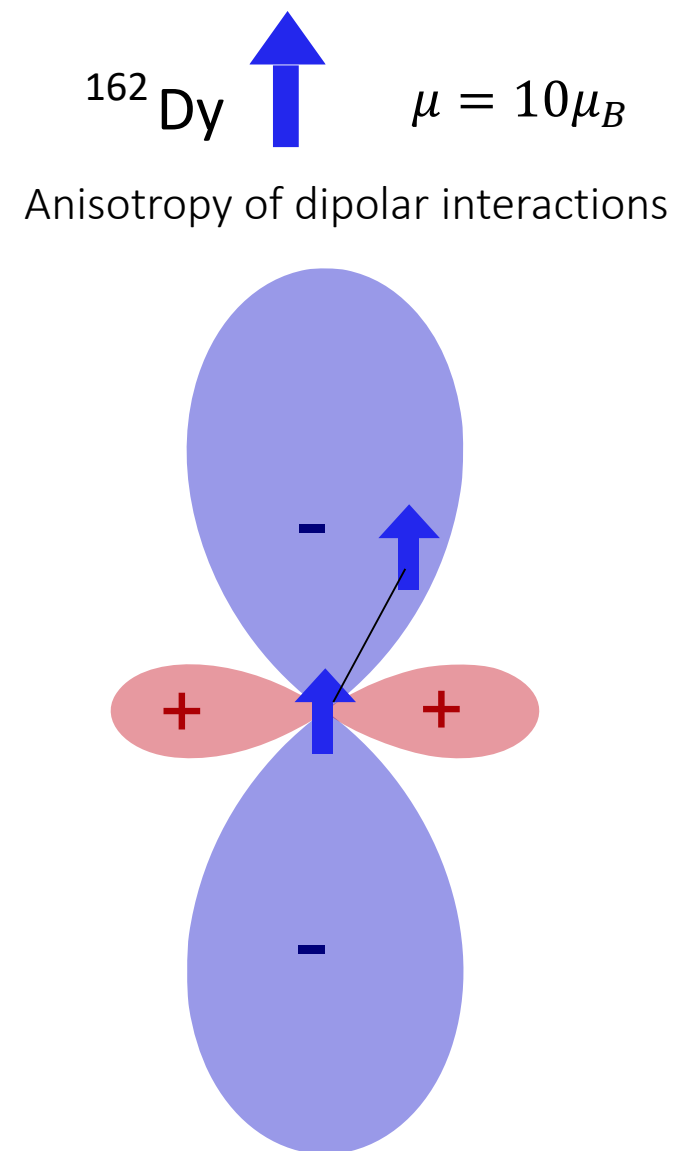
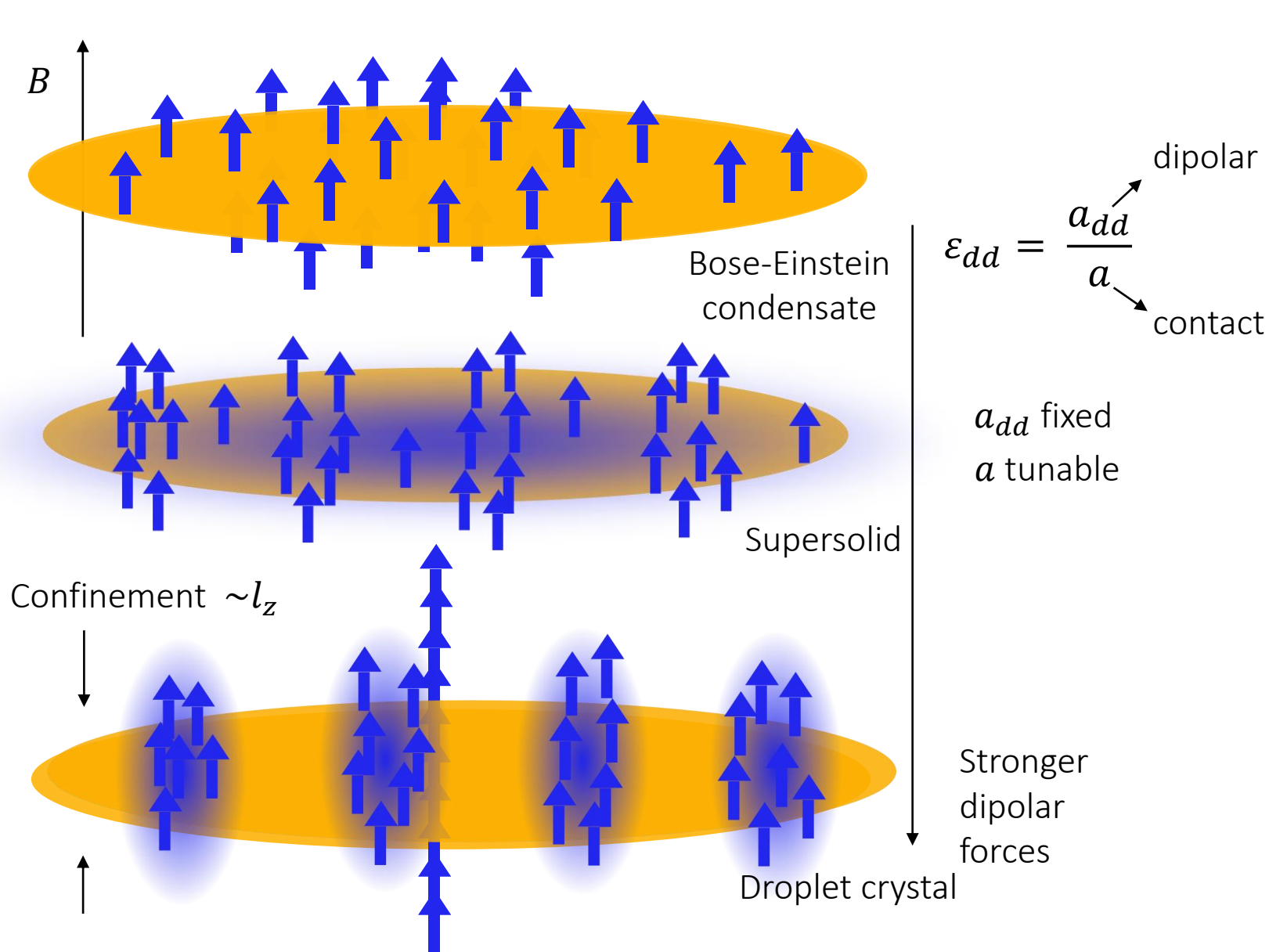


Optical cavities: J. Léonard et al., Science 358, 1415 (2017). **Spin-Orbit coupling:** J. R. Li et al., Nature 543 (2017); theory: Y. Li et al. Phys. Rev. Lett. 110, 235302 (2013).

Many theoretical proposals for quantum gases: e. g. dipoles in lattices or in 2D:

S. Wessel, M. Troyer, Phys. Rev. Lett. 95 127205 (2005); Z.-K. Lu et al. Phys. Rev. Lett. 115, 075303 (2015); ...

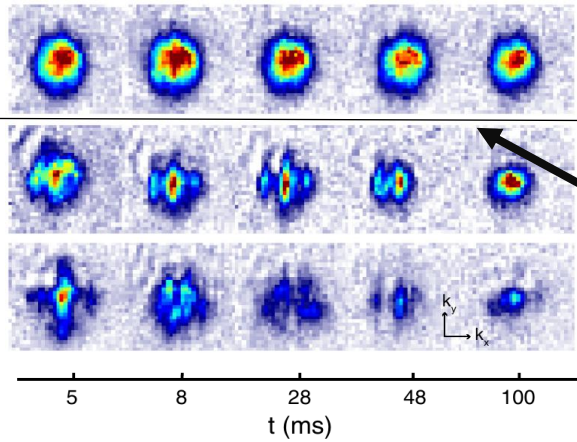
The supersolid in dipolar quantum gases



The supersolid is a new quantum state of matter with a unique mix of properties

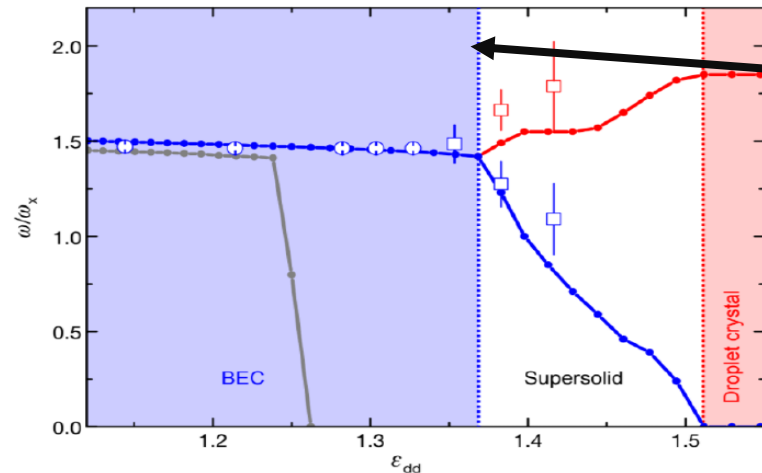
Tanzi L. et al., Observation of a dipolar quantum gas with metastable supersolid properties, *Phys. Rev. Lett.*, **122** 130405 (2019)

Later in this talk

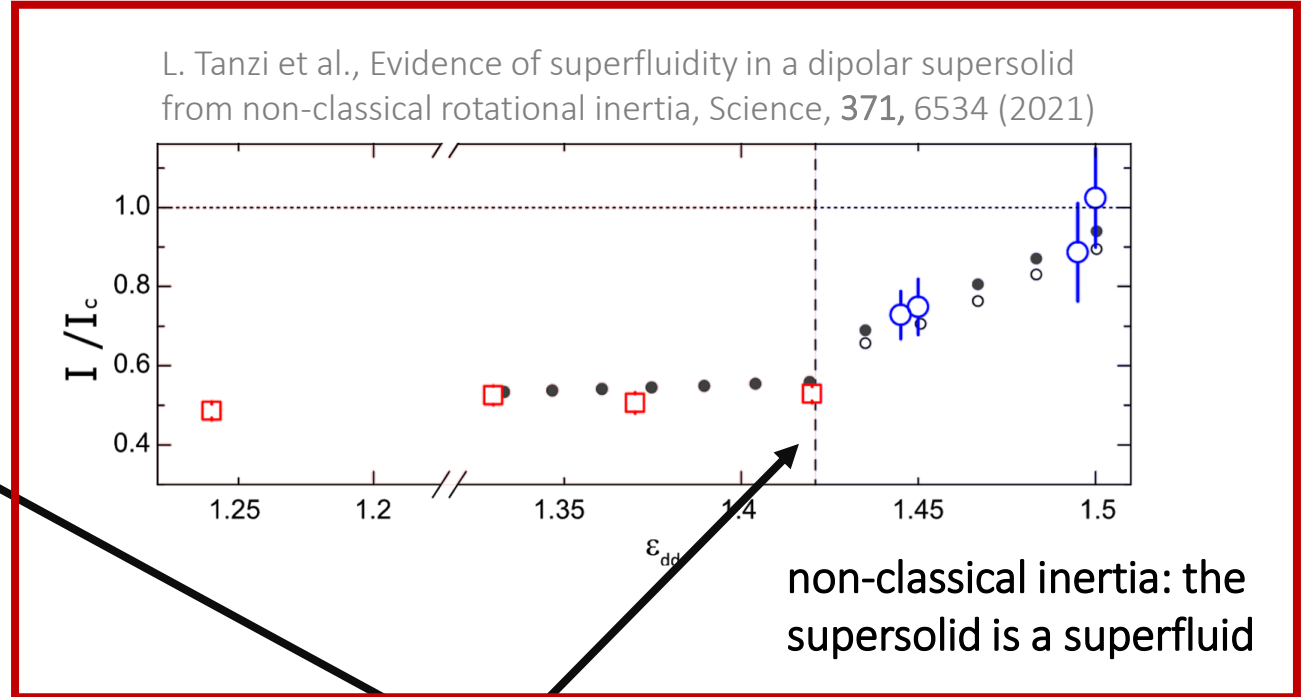


matter wave coherence

Tanzi L. et al., Supersolid symmetry breaking from compressional oscillations in a dipolar quantum gas, *Nature*, **574** 7778 (2019)



spontaneous double symmetry breaking: the supersolid is a solid



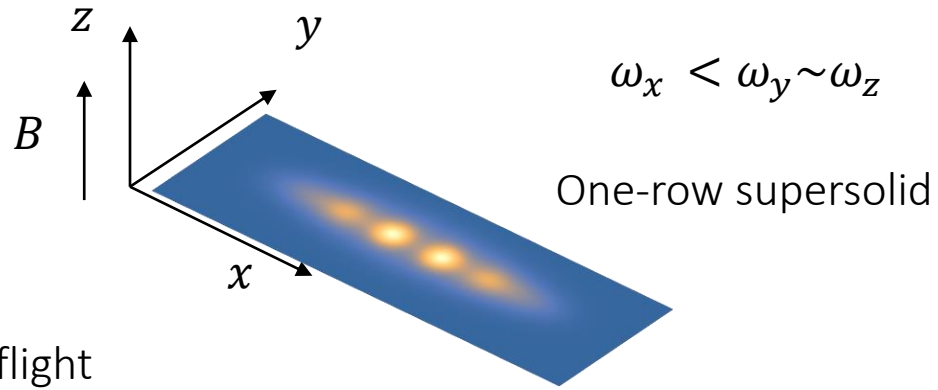
non-classical inertia: the supersolid is a superfluid

.. but how does the quantum phase transition take place?

Innsbruck: L. Chomaz et al., Long-lived and transient supersolid behaviors in dipolar quantum gases, *Phys. Rev. X* **9**, 021012 (2019)

Stuttgart: F. Böttcher et al, Transient supersolid properties in an array of dipolar quantum droplets, *Phys. Rev. X* **9**, 011051 (2019)

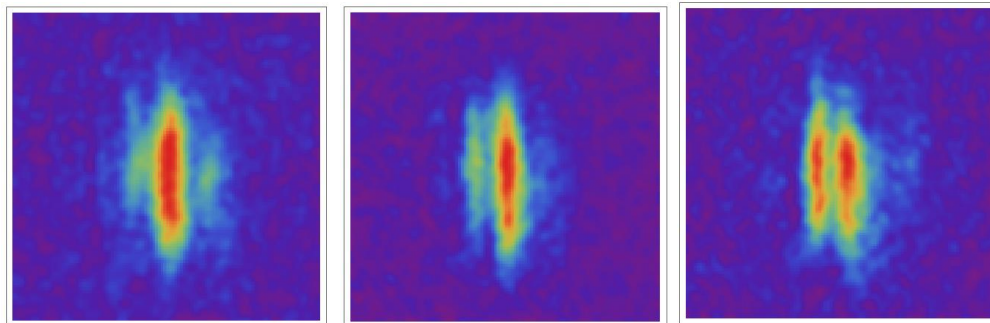
We observe the interference pattern after the “free” expansion of the supersolid



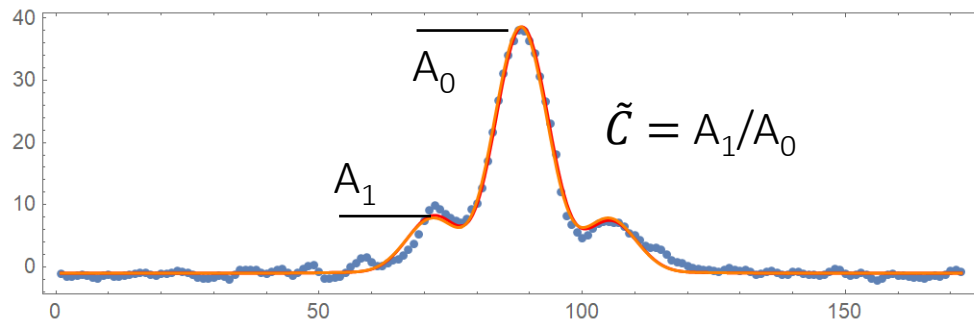
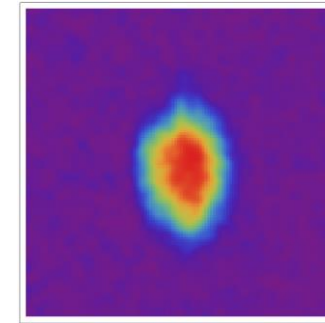
control parameter	scattering length	a_s
order parameter	contrast (in momentum space)	\tilde{C}
other observables	phase, spacing	$\phi, k \dots$
	..	

Time of flight expansion

$\tilde{C} \neq 0$



$\tilde{C} = 0$



weak contact

strong contact

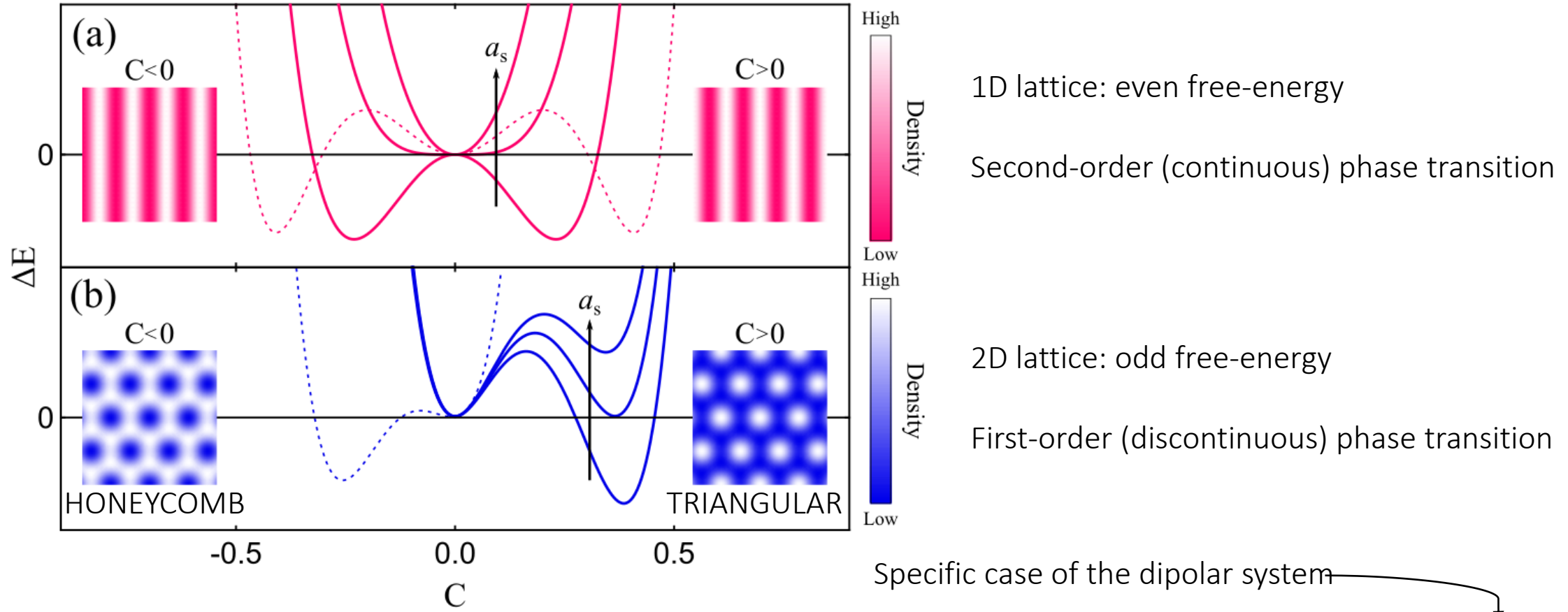
strong dipolar

dipolar/contact

weak dipolar

A simple theoretical framework: the Landau model

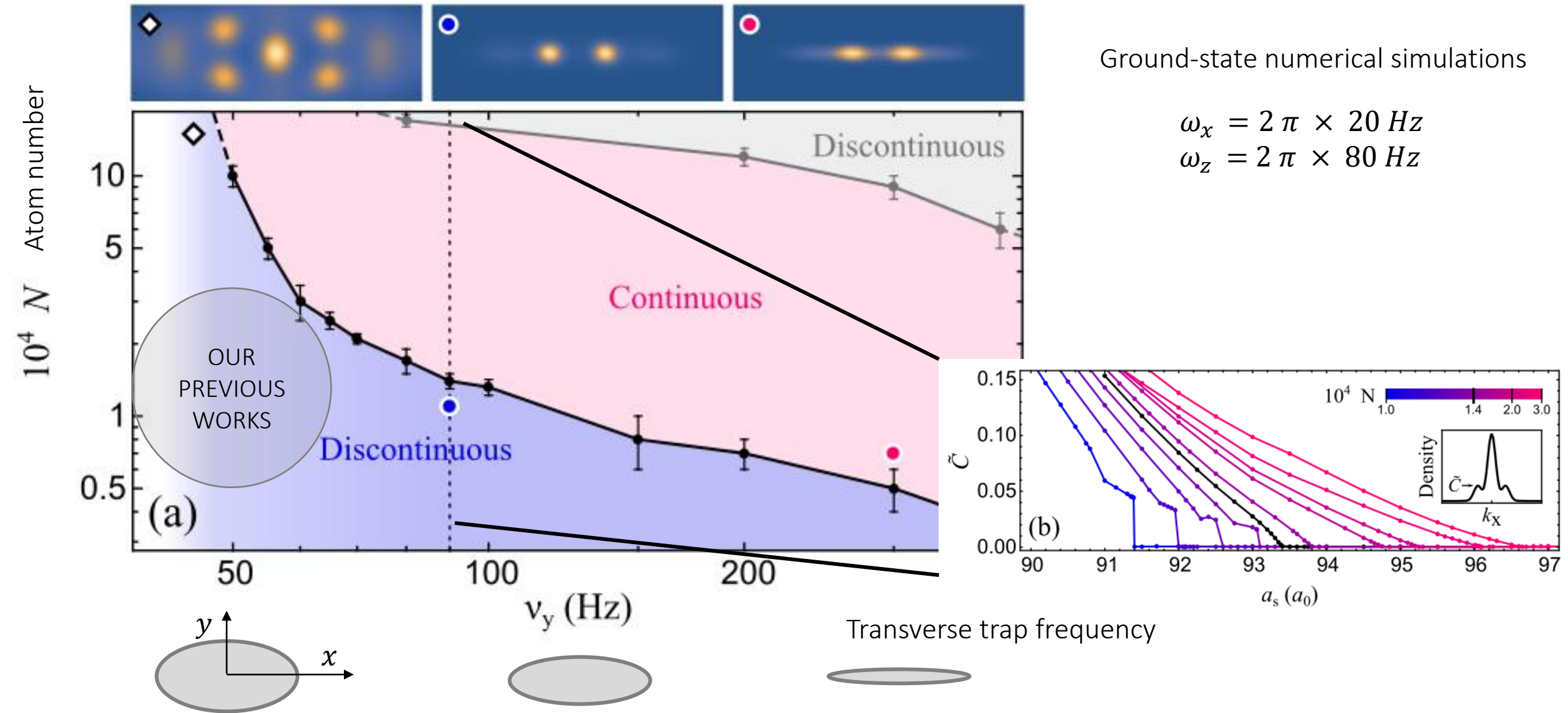
$$\Delta E = a C + b C^2 + c C^3 + d C^4 + \dots$$



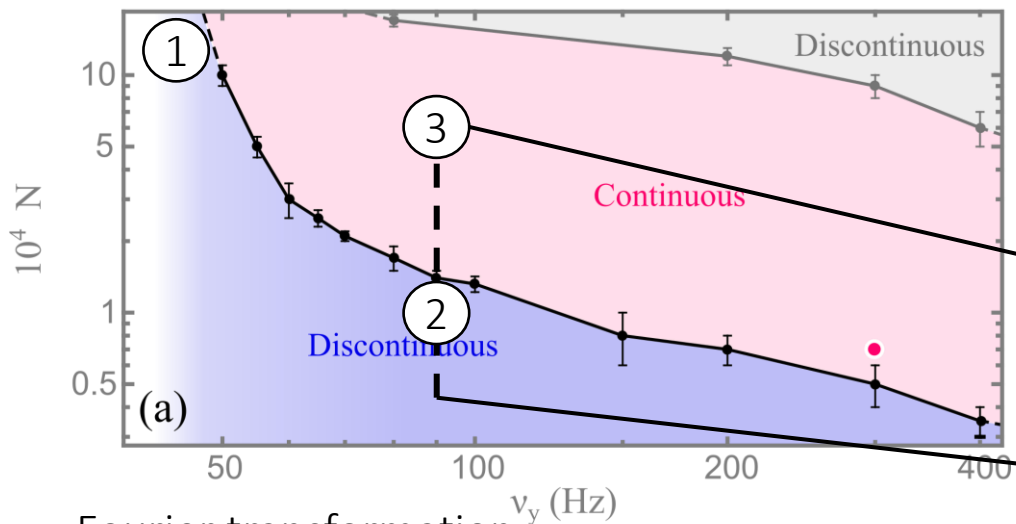
Collaboration with M. Modugno and A. Alana from Bilbao University

Biagioni G. et al., Dimensional crossover in the superfluid-supersolid quantum phase transition, Phys. Rev. X 12, 021019 (2022)

Numerical phase diagram: how to control the nature of the transition



The crossover is dimensional



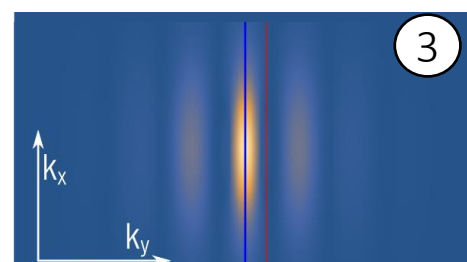
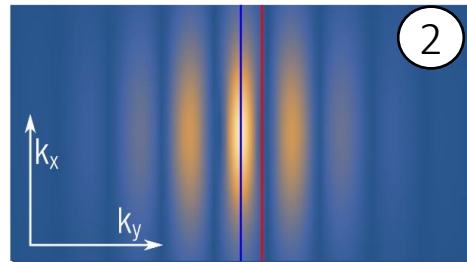
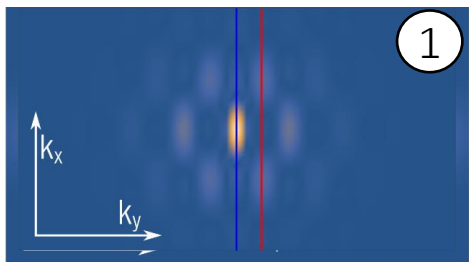
Continuous – 1D structure

The crossover is peculiar to the **cluster solid**

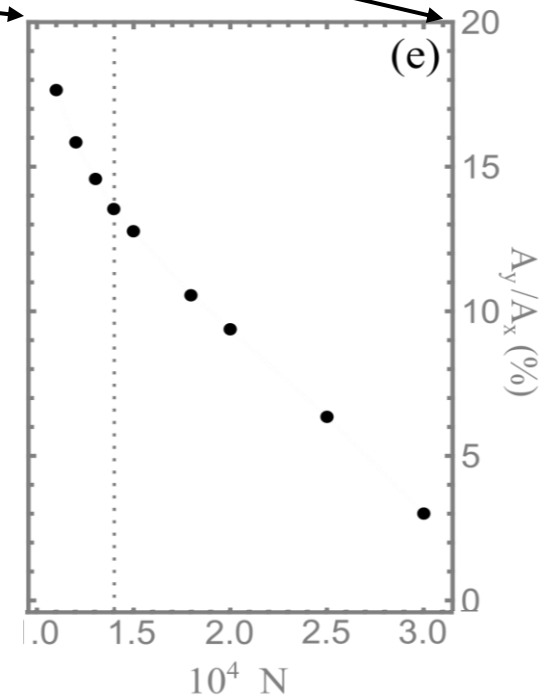
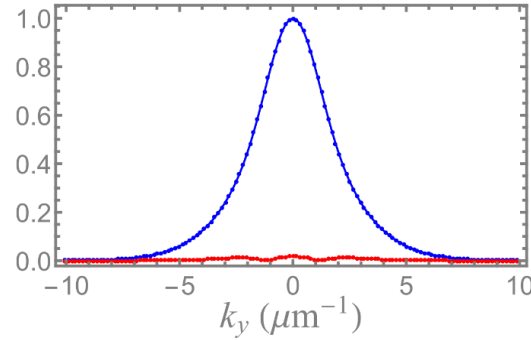
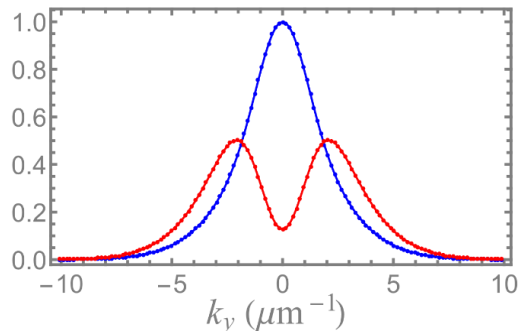
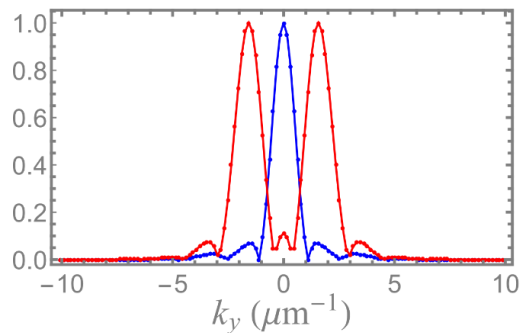
Discontinuous – 2D structure

Few lattice sites, but large N
Fluctuations $\sim 1/\sqrt{N}$

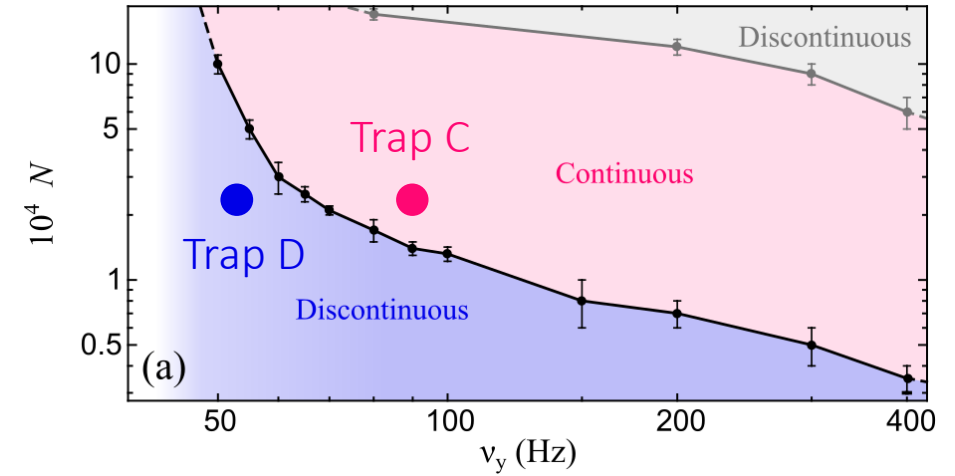
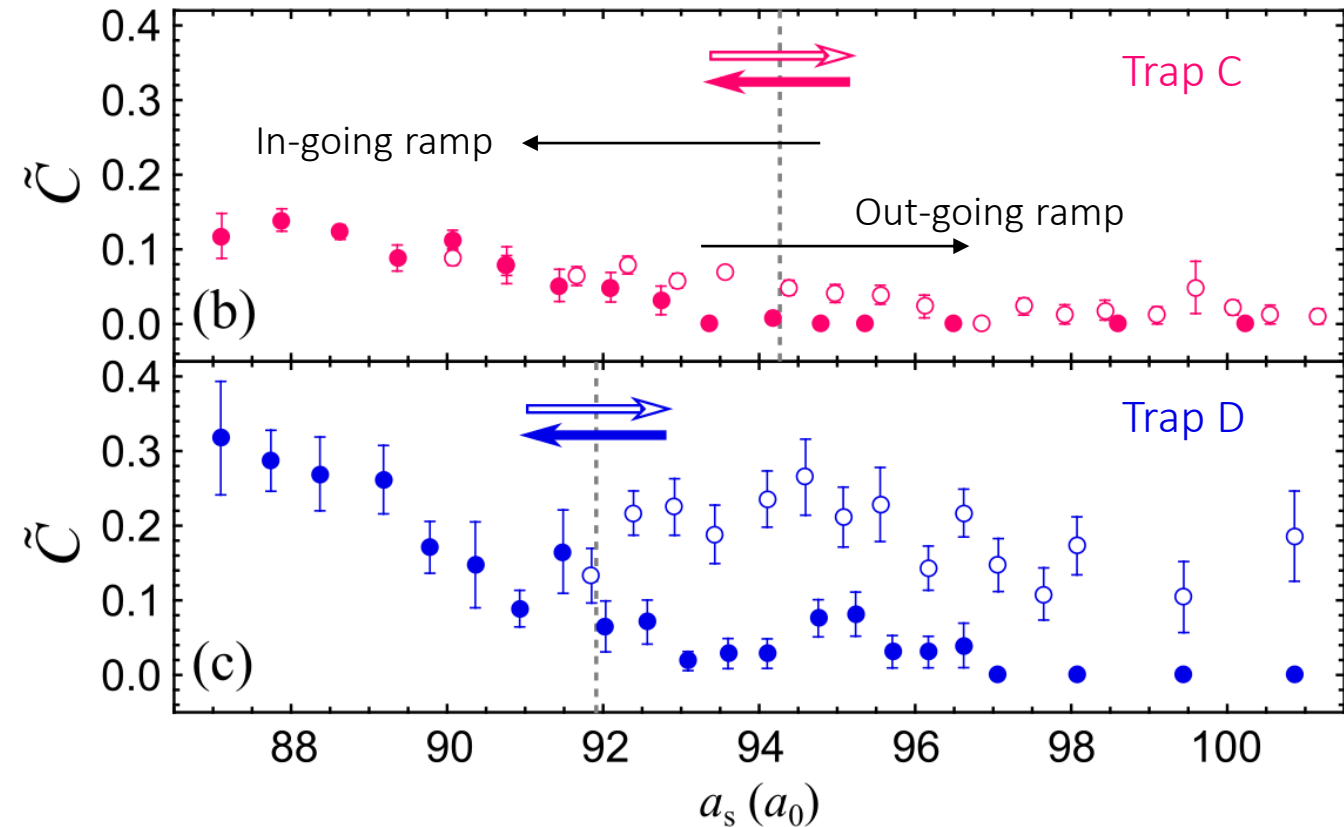
Fourier transformation v_y (Hz)



Cut along k_x



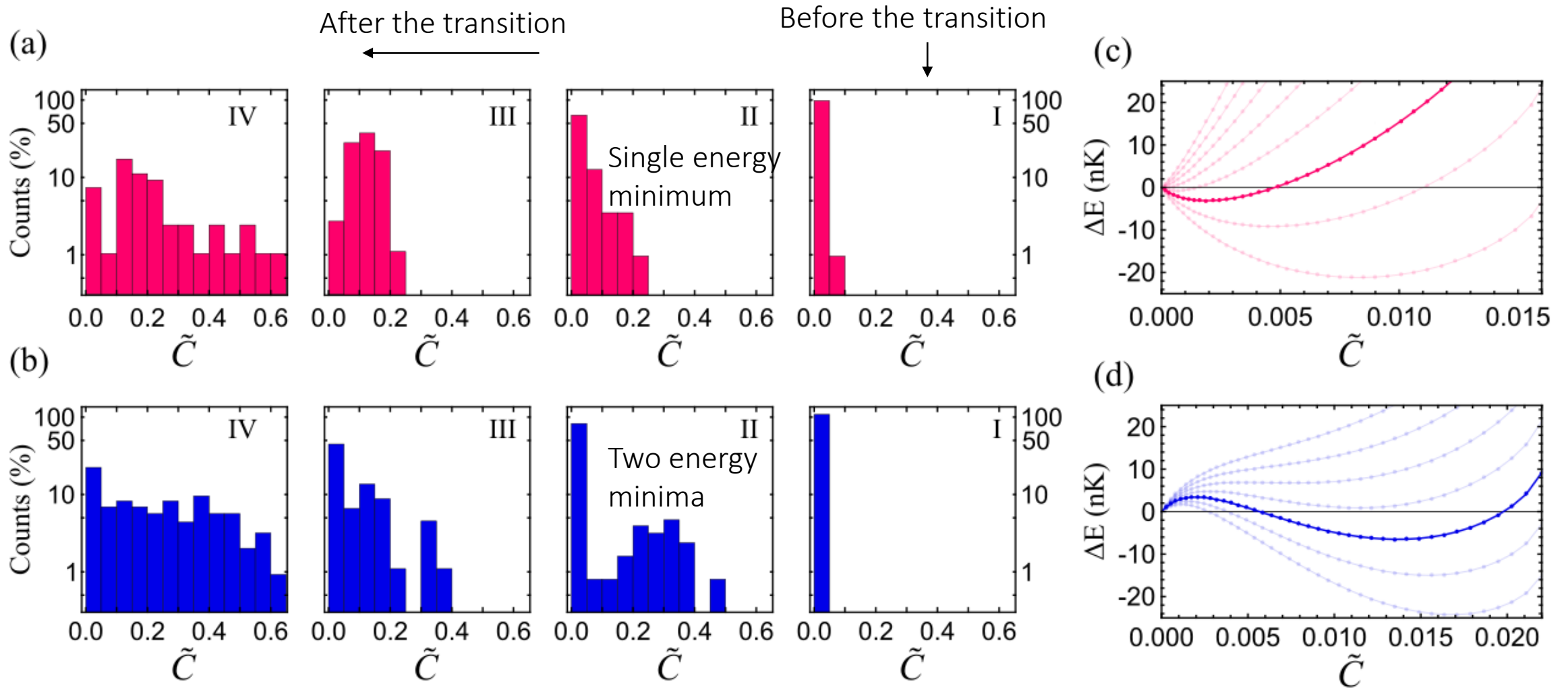
Experimental evidence of continuous and discontinuous phase transitions



Different nature of the transition in the two traps:

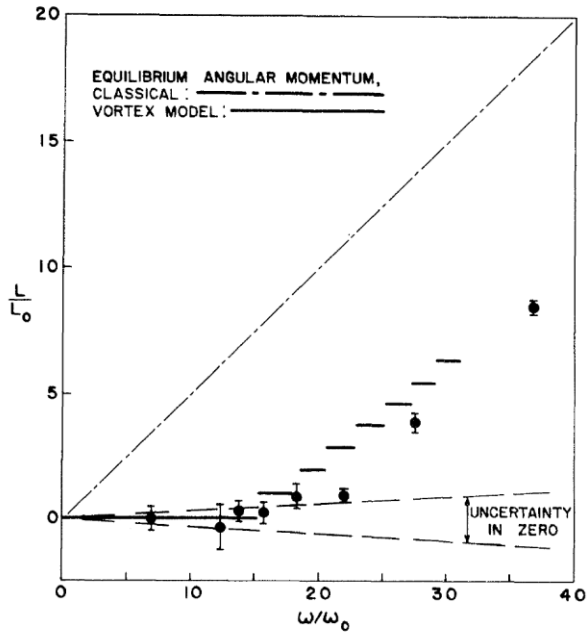
- Strong excitations after the out-going ramp only in trap D
- Strong fluctuations at the transition only in trap D

Fluctuation spectra are affected by the form of the free energy



Free energy calculated from a density ansatz with the experimental parameters

Rotations as a probe of superfluidity

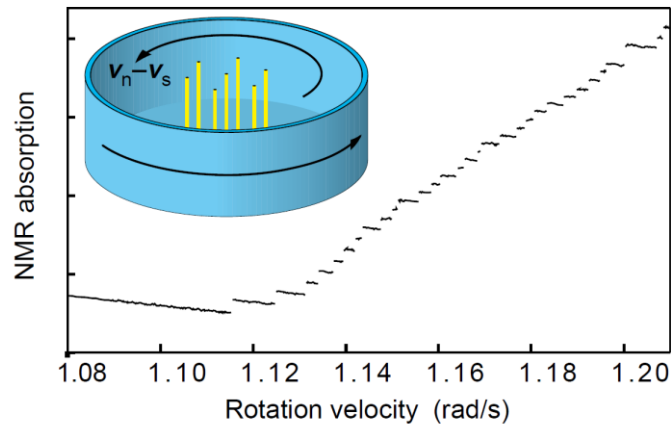


Helium-4 (1967)
Hess-Fairbank

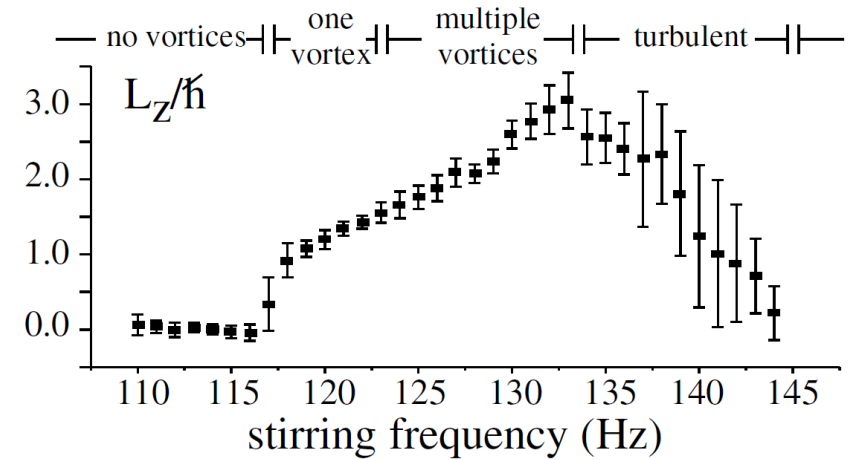
What happens with a supersolid ?

Irrotational velocity field and homogeneous density

$$\oint \vec{v} \cdot d\vec{l} = 0$$

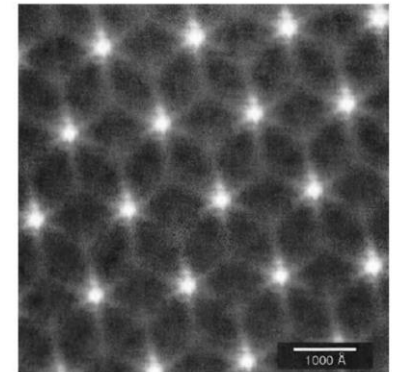


Helium-3 (1982)

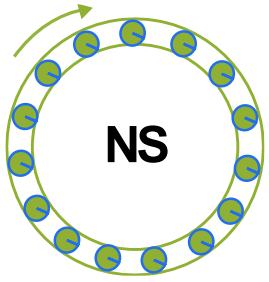


BECs (2000)

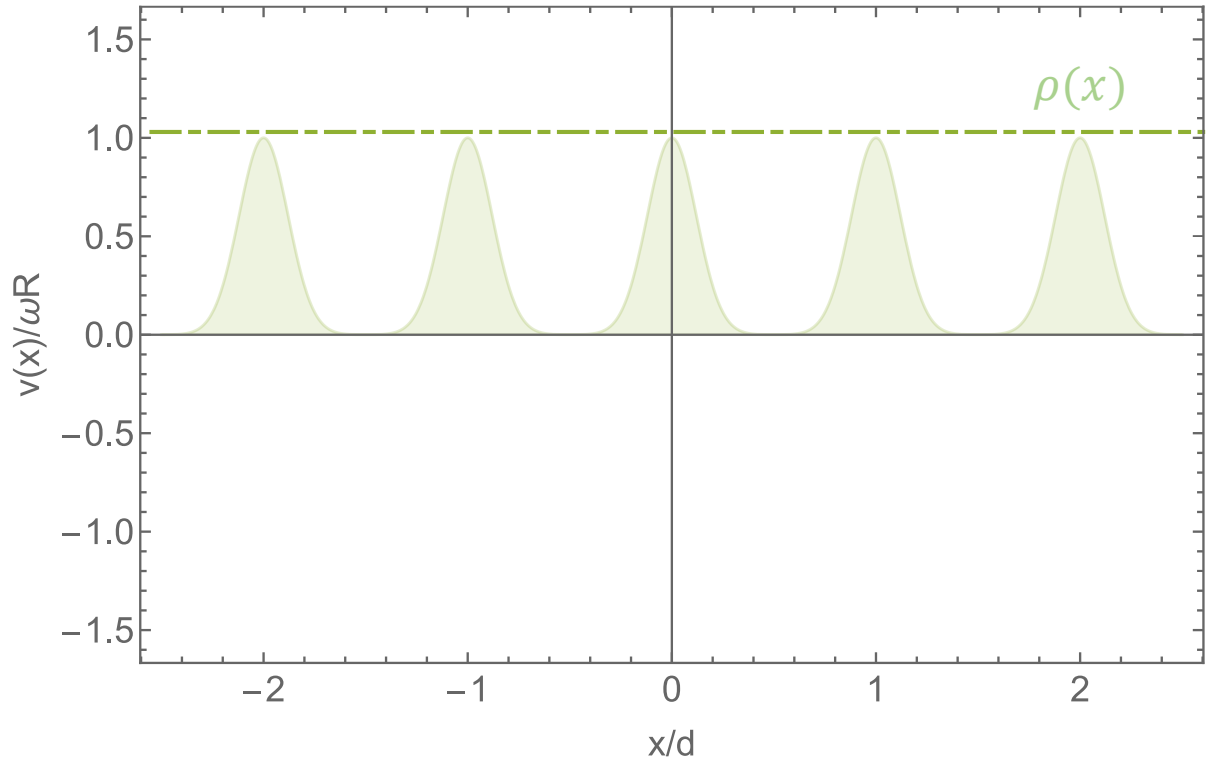
Related Meissner effect in superconductors



Rotation of a supersolid: Leggett's model



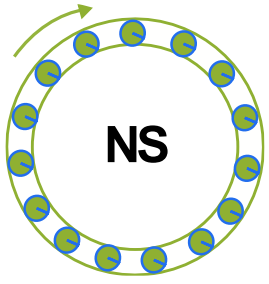
$$I = I_c$$



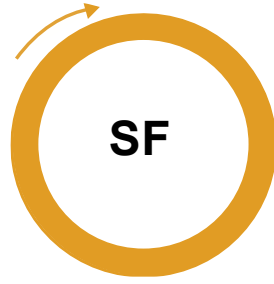
A.J. Leggett, Can a solid be superfluid?, Phys. Rev. Lett. 25, 1543 (1970)

Biagioni G., Rotation of a dipolar supersolid, Il nuovo Cimento **44 C** (2021) 107

Rotation of a supersolid: Leggett's model



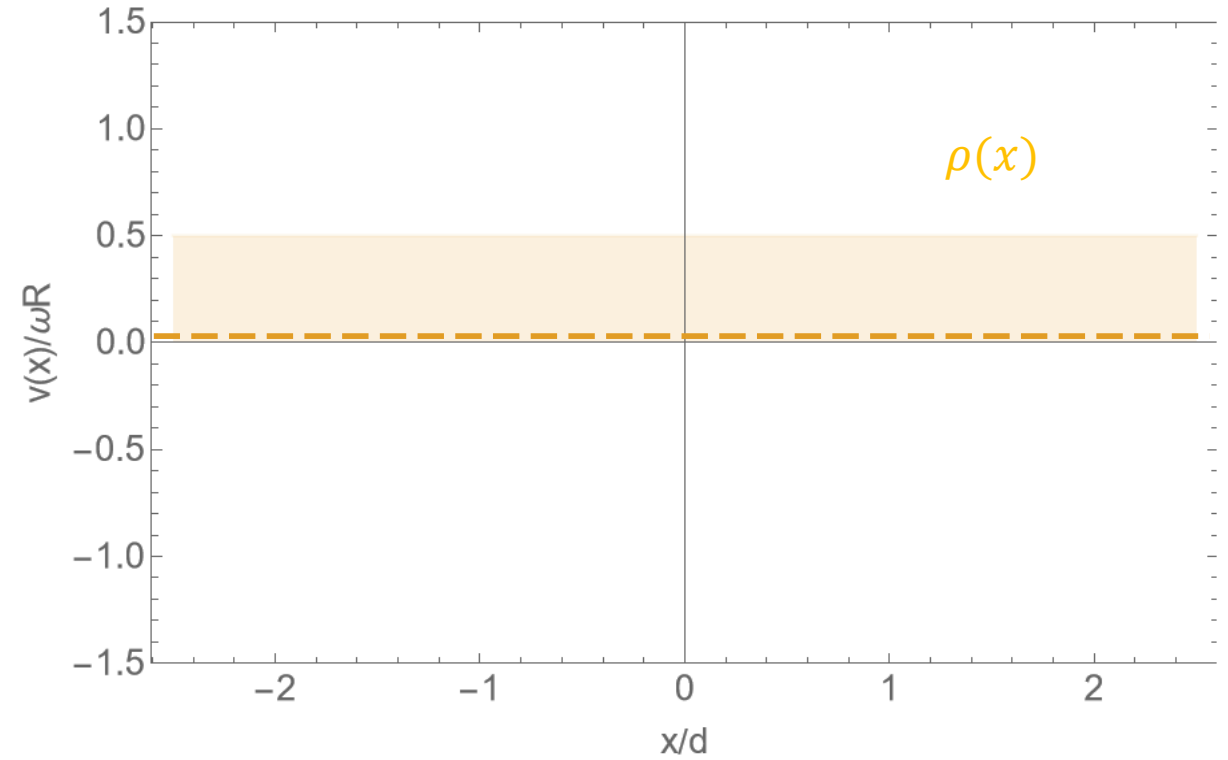
$$I = I_c$$



$$I = 0$$

- Irrotational condition

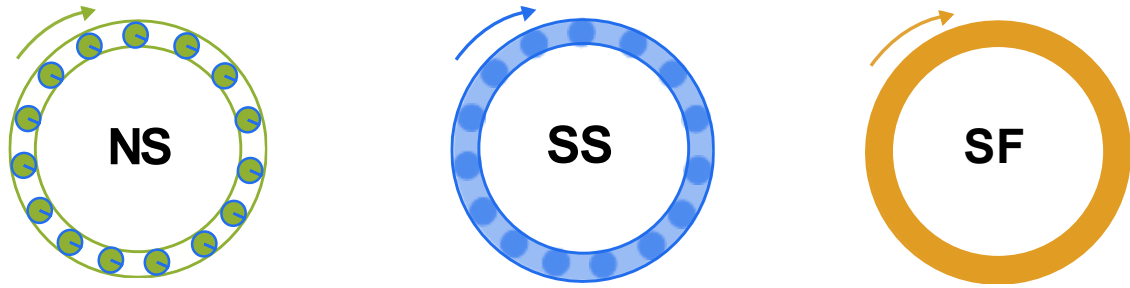
$$\oint \vec{v} \cdot d\vec{l} = 0$$



A.J. Leggett, Can a solid be superfluid?, Phys. Rev. Lett. 25, 1543 (1970)

Biagioni G., Rotation of a dipolar supersolid, Il nuovo Cimento 44 C (2021) 107

Rotation of a supersolid: Leggett's model



$$I = I_c \quad I = (1 - f_s)I_c \quad I = 0$$

Superfluid fraction: $f_s = \left(\int dx / \bar{\rho}(x)\right)^{-1}$

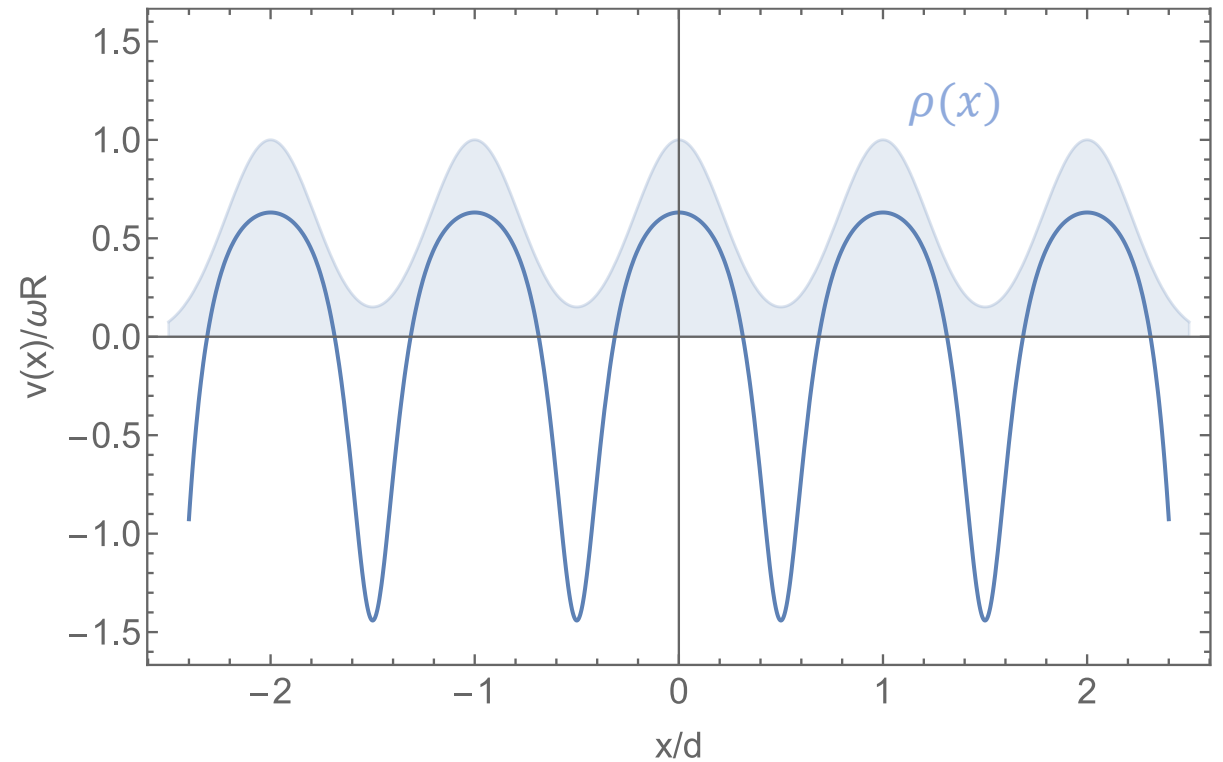
- Irrotational condition

$$\oint \vec{v} \cdot d\vec{l} = 0$$

- Continuity equation (steady state)

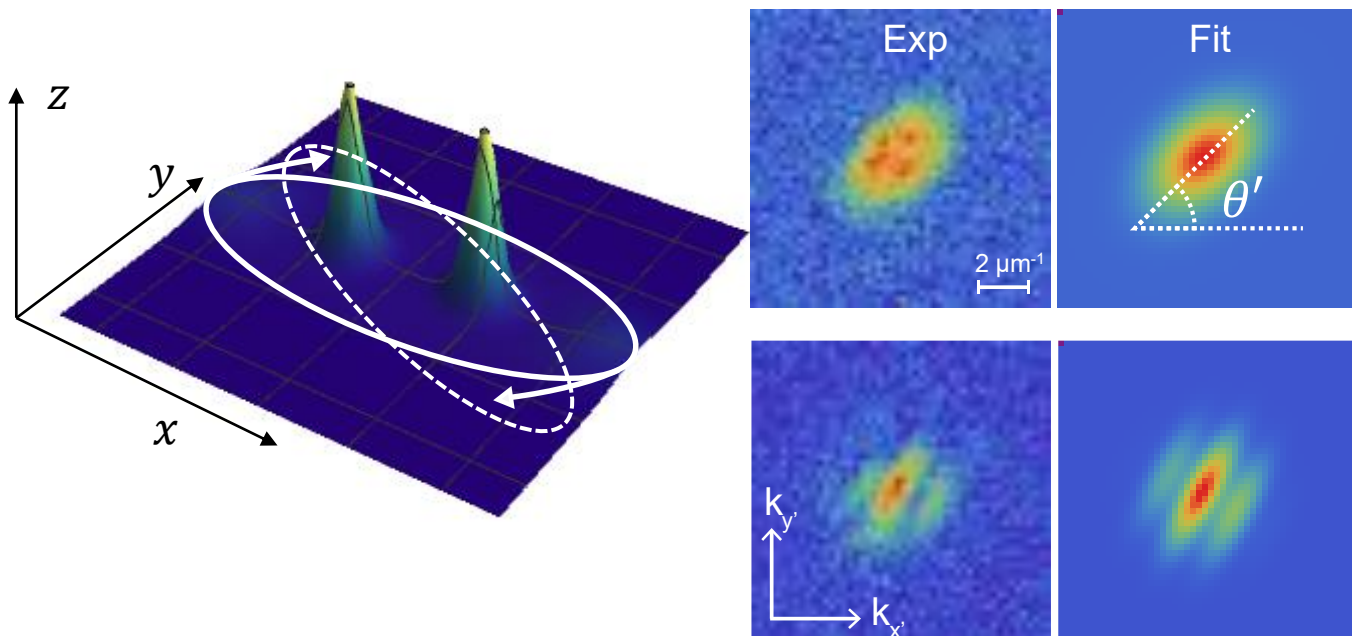
$$\nabla \cdot (\rho v) = 0 \quad v \propto 1/\rho$$

The droplets move in the direction opposite to the background



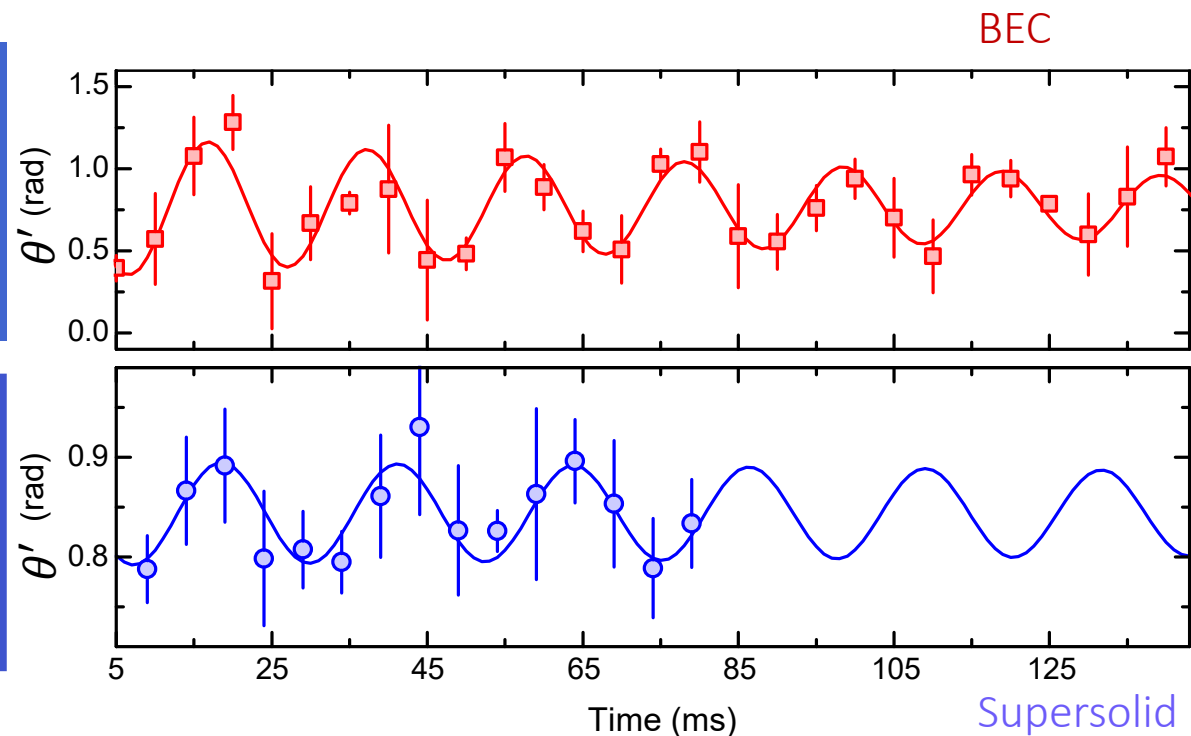
Similar concepts in the physics of neutron-stars crust (entrainment)

In the harmonic trap: the scissors mode



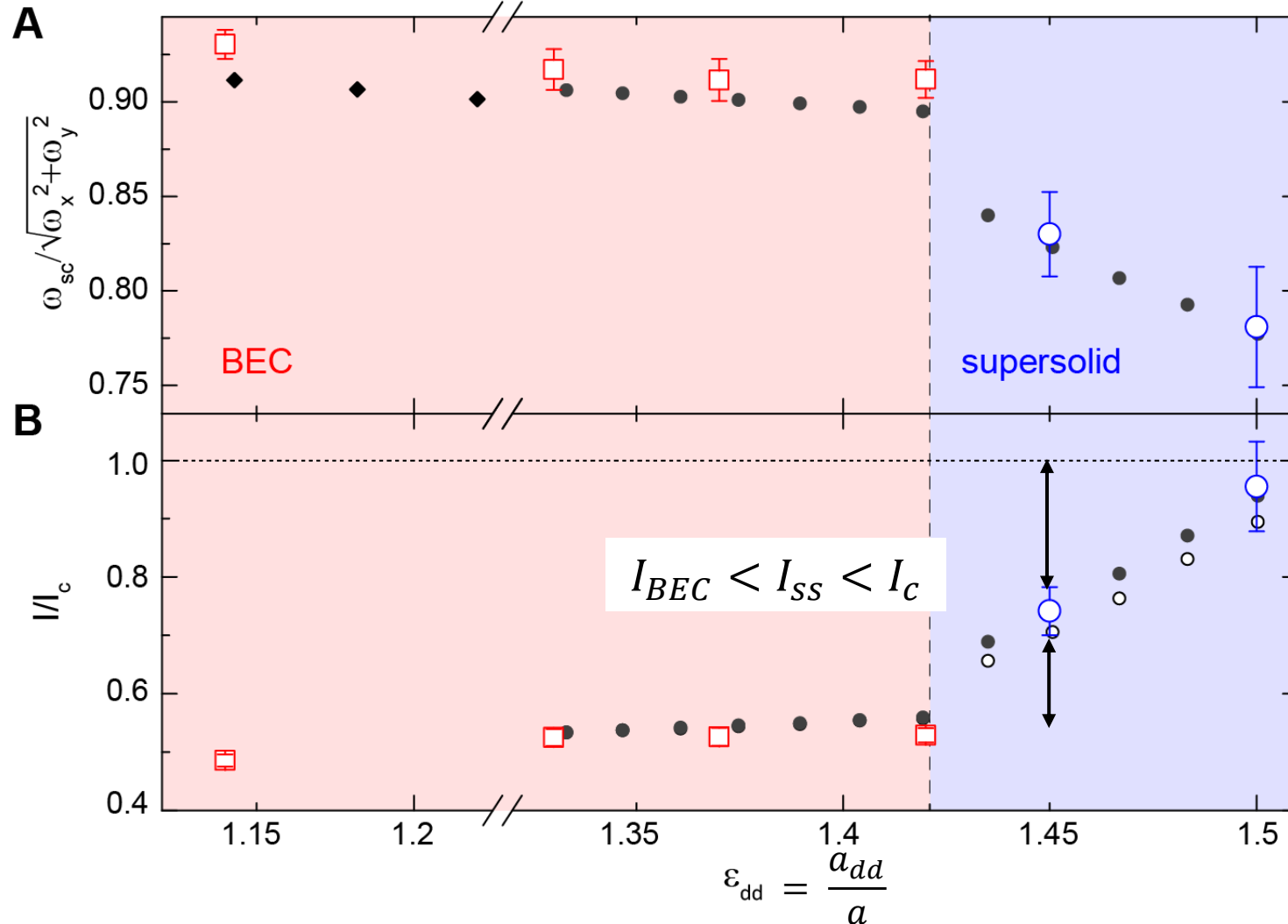
We excite the scissors mode changing temporarily the trapping frequencies ..

.. then we extract the angle at different times with a 2D fit



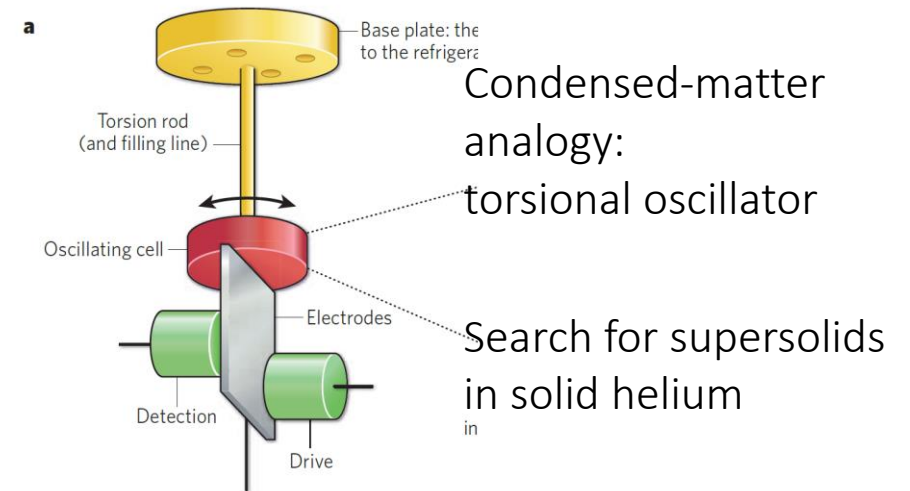
Single-frequency oscillation in both the BEC and the supersolid regime

Reduced moment of inertia: the supersolid is superfluid



Theory, including β , by the Trento group: S. M. Roccuzzo et al., Rotating a supersolid dipolar gas, Phys. Rev. Lett., **124**, 045702 (2020)

$$I = I_c \alpha \beta \frac{(\omega_x^2 + \omega_y^2)}{\omega_{sc}^2}$$



$$I = \frac{K}{\omega_{osc}^2}$$

E. Kim and M. H. W. Chan, Nature 427, 225 (2004)
 D. Y. Kim and M. H. W. Chan, Phys. Rev. Lett. 109, 155301 (2012)

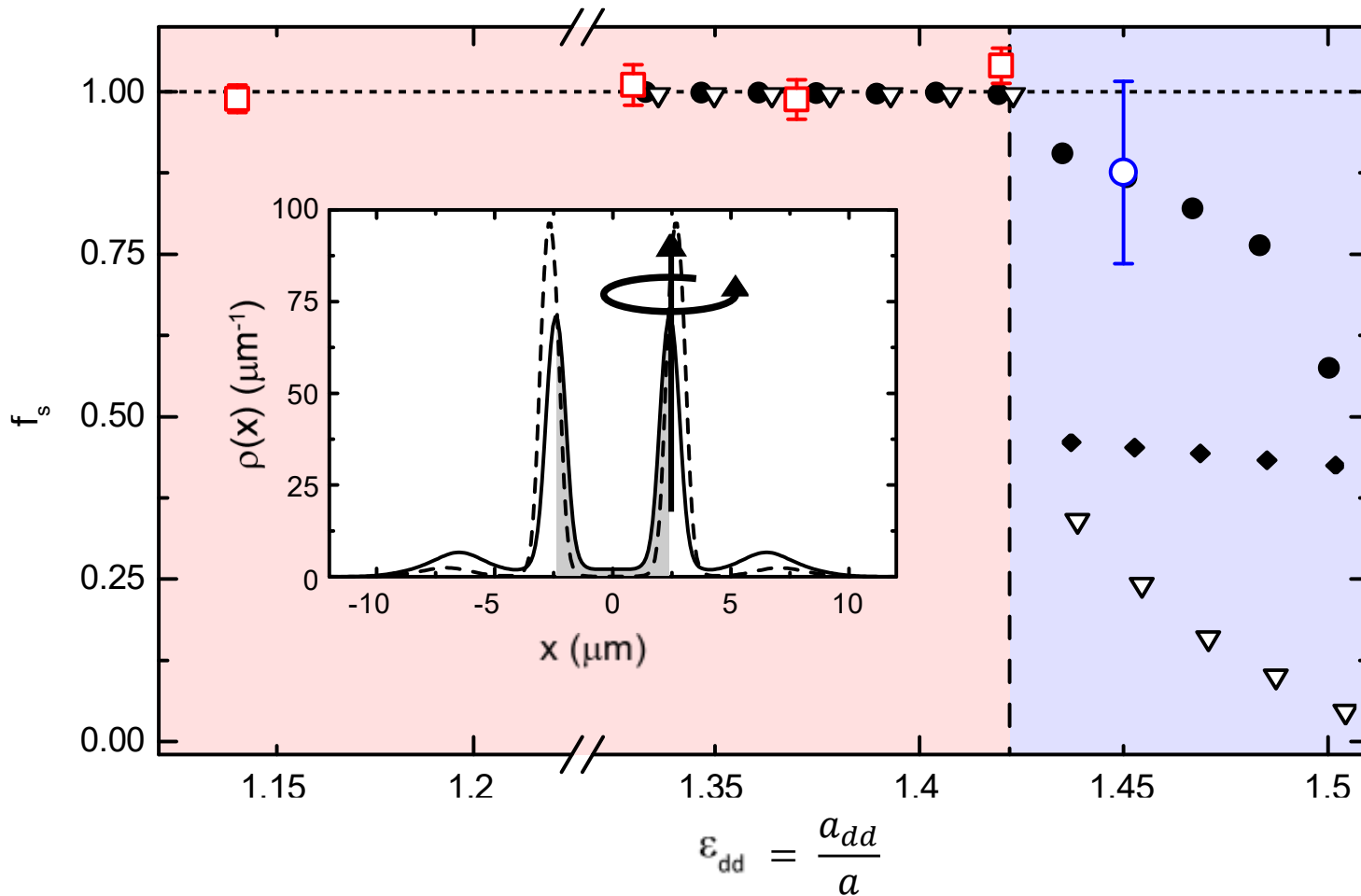
L. Tanzi et al., Evidence of superfluidity in a dipolar supersolid from non-classical rotational inertia Science, **371**, 6534 (2021)

Superfluid fraction

Our definition of superfluid fraction for an anisotropic system:

$$I = (1 - f_s)I_c + f_s\beta^2 I_c$$

$f_s \sim 1$ (cluster supersolid)



Each droplet is cylindrical and superfluid

$$f_s^{drop} = \frac{1}{1 + \beta} \quad \blacklozenge$$

Leggett mechanism

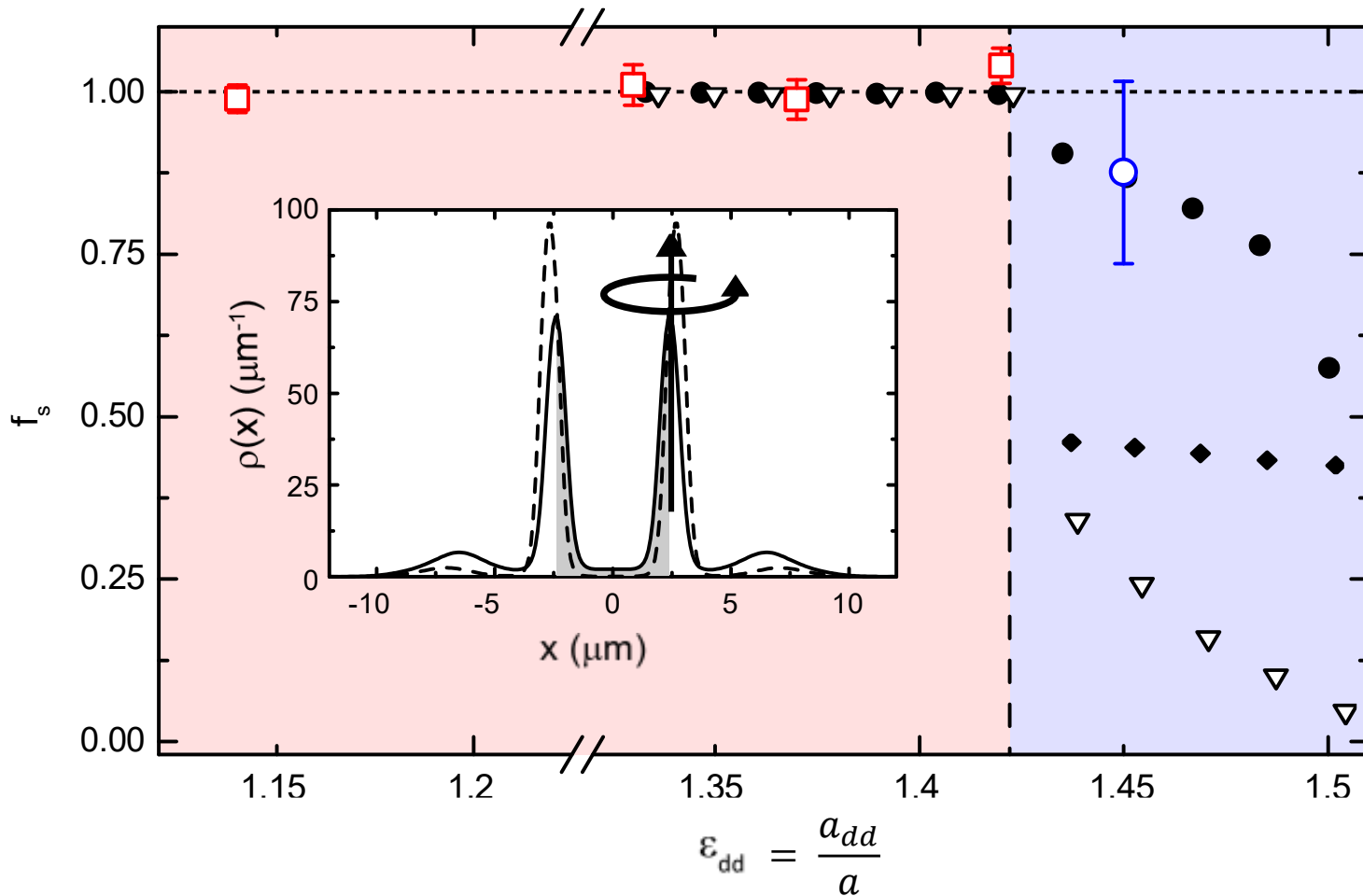
$$f_s = \left(\int dx / \bar{\rho}(x) \right)^{-1} \quad \blacktriangledown$$

Superfluid fraction

Our definition of superfluid fraction for an anisotropic system:

$$I = (1 - f_s)I_c + f_s\beta^2 I_c$$

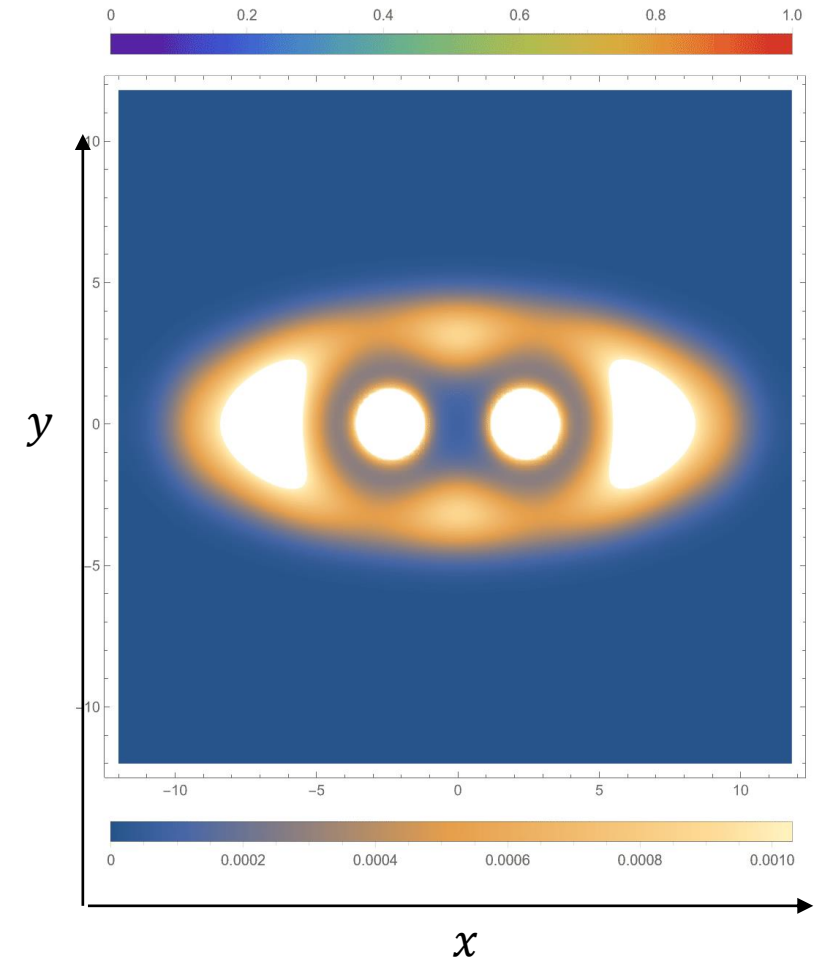
$f_s \sim 1$ (cluster supersolid)



M. Norcia et al., [arXiv:2111.07768](https://arxiv.org/abs/2111.07768)

Is it possible to speak about superfluid fraction in the trapped system?

Our simulations:



Outlook

The dipolar supersolid is an exciting phase of matter which rises many questions of fundamental physics in a novel type of environment.

Open questions:

As a superfluid:

- Self-induced Josephson effect (also superfluid fraction)
- Entanglement properties (crossing a second order PT)
- Persistent currents
- Vortices
- ...

As a solid:

- Dissipationless deformation
- ...