

# RELATIVISTIC COUPLED CLUSTER: A POWERFUL COMPUTATIONAL TOOL IN SUPPORT OF EXPERIMENTS

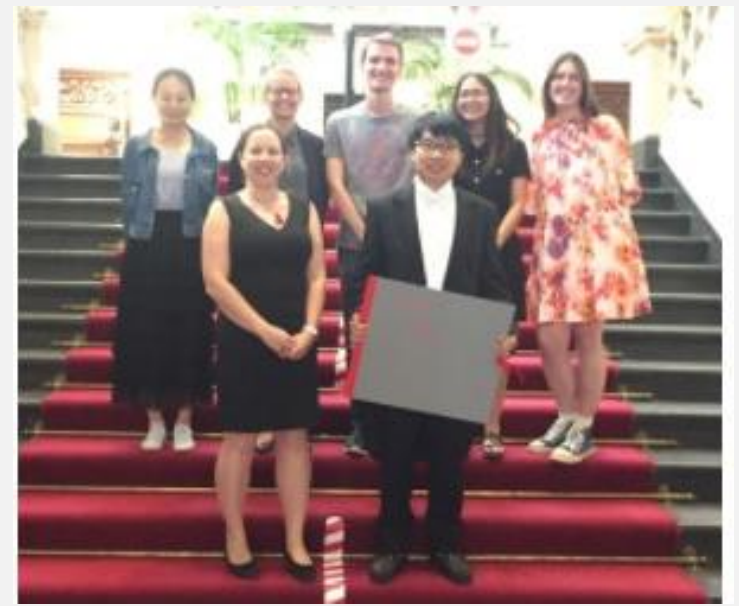
A. Borschevsky



university of  
 groningen

faculty of science  
and engineering

van swinderen institute for  
particle physics and gravity



- University of Groningen

### Research:

Using state of the art methods from computational chemistry to address fundamental problems in physics:

- Search for new physics with low-energy precision measurements
  - Violation of fundamental symmetries in atoms and in molecules
  - Search for variation of fundamental constants
- Highly accurate calculations of spectra and properties of heavy and superheavy atoms and highly charged ions

**Most work done in collaboration with experimental groups**



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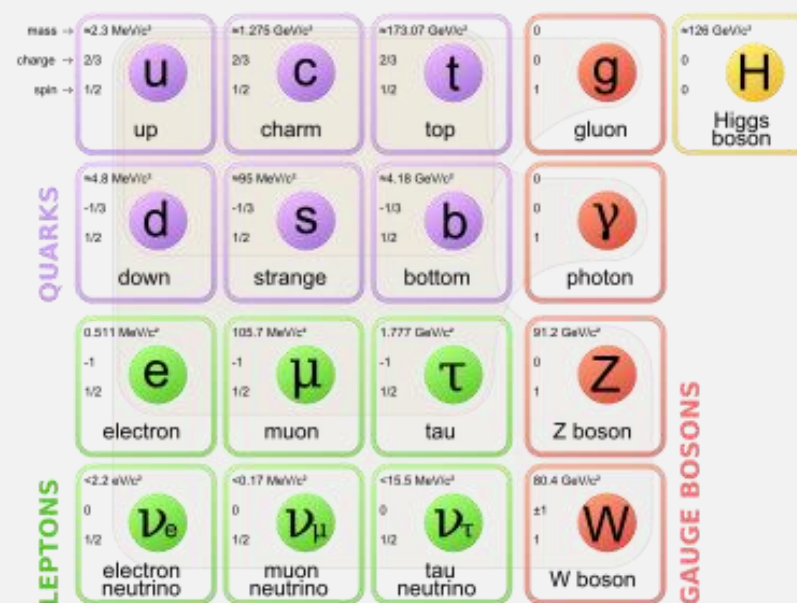
- Search for new physics with low-energy precision measurements
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  - Search for variation of fundamental constants
- Highly accurate calculations of **spectra and properties** of heavy and superheavy atoms and highly charged ions

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MOTIVATION

## Why look for physics beyond the Standard Model (SM)?

- The SM is currently the best fitting physical description of the world around us.
- So far successfully explained the majority of observed natural phenomena and has strong predictive power (Higgs boson, top quark, tau neutrino)
- But... it is incomplete



## Why look for physics beyond the Standard Model (SM)?

- Extensions to the SM attempt to fill these knowledge gaps.
  - Grand Unified Theories, String Theory, SUSY, ...
- These extensions predict new physical phenomena beyond the SM.
  - Variation of fundamental constants (VFC)
  - Violation of fundamental symmetries ( $CP, P, T$ )
- (non) discovery of these phenomena allows to discriminate between extensions or new theories.

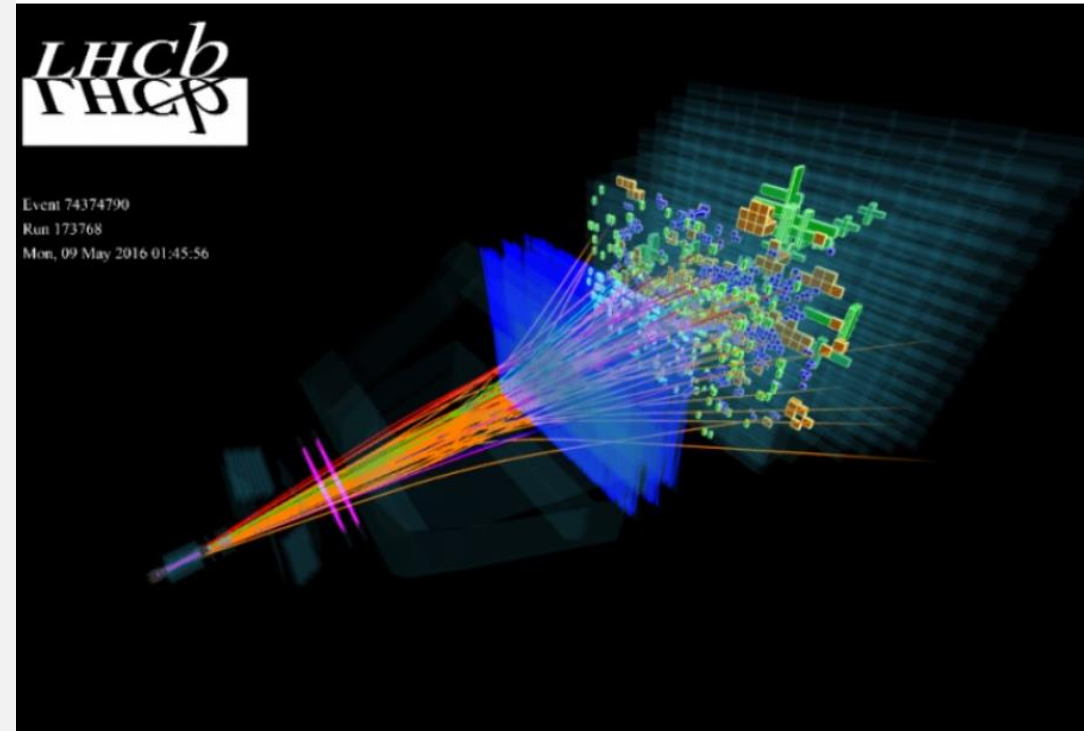


# Why look for physics beyond the SM with atoms and molecules?

- Accelerator research (LHCb, T2K, etc.)
- **Table-top experiments**



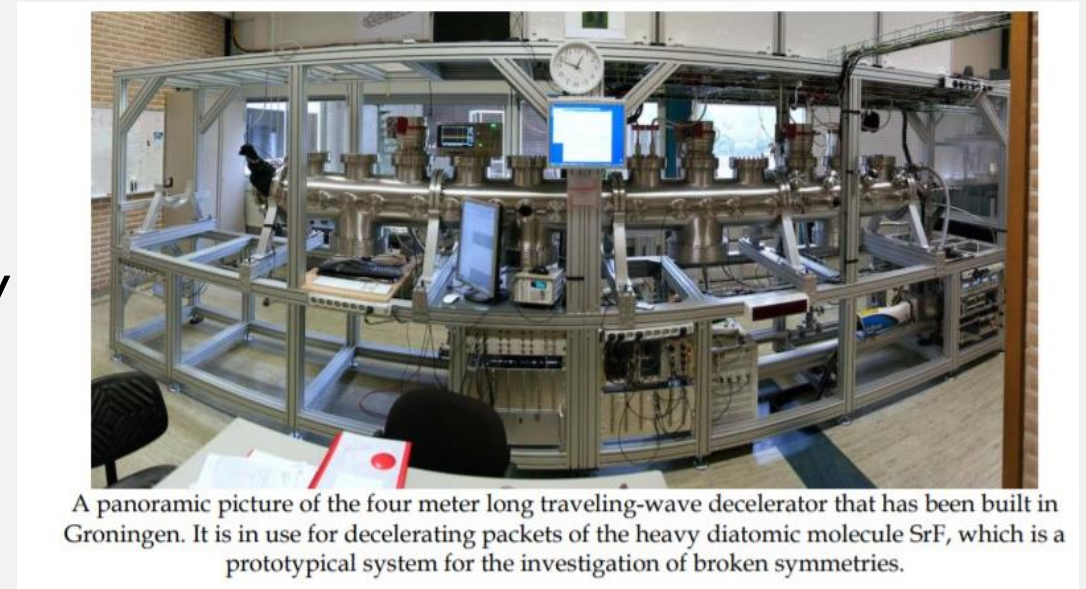
A panoramic picture of the four meter long traveling-wave decelerator that has been built in Groningen. It is in use for decelerating packets of the heavy diatomic molecule SrF, which is a prototypical system for the investigation of broken symmetries.



# Why look for physics beyond the SM with atoms and molecules?

- **Table-top experiments: promising alternative to high energy research**

- Versatile, sensitive to different phenomena
  - Parity violation
  - EDMs (electron, hadronic)
  - Variation of fundamental constants
  - Dark matter
  - ..
- Various enhancement effects → high sensitivity
- Small scale
- (Relatively) inexpensive

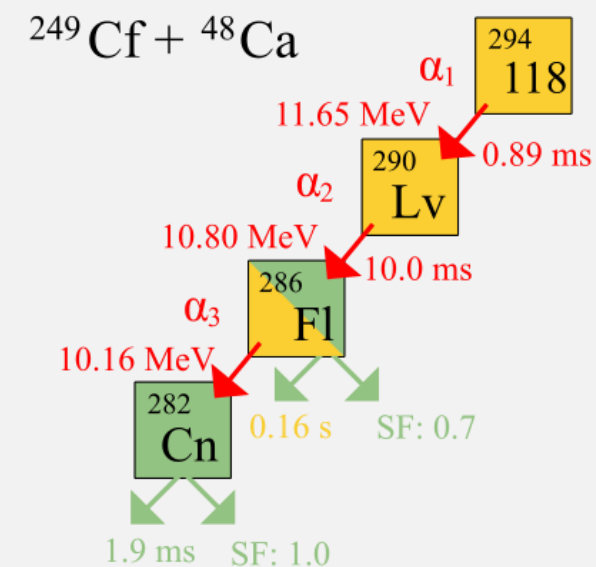


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# Why investigate unstable and artificial elements?

- (Exotic) nuclear structure and properties
- Information about new elements, assignment in Periodic Table
- Behaviour and trends in lower part of the Periodic Table
- Benchmarks for theory (e.g. contribution of QED effects)
- 



## Challenging experiments!

- In one case, unprecedented sensitivity needed to detect the tiny effects of new physics
- In the other, dealing with small amounts of unstable, short lived elements
- Sometimes, combined challenges (e.g. precision measurements on RaF and others)!
- Alongside specially developed experimental techniques, theoretical support becomes crucial

## How can (atomic and molecular) theory be of use?

- Practical parameters for experiments (predictions of transition energies, laser-cooling schemes, etc.)
- Parameters for the interpretation of the results (HFS parameters for extraction of nuclear properties, coupling parameters for new physics phenomena, etc.)
- Identification of promising candidates for precision measurements
  - High sensitivity
  - Experimental considerations (stability, laser-coolability, etc.)

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For use in experiments:

- Reliable predictions based on high accuracy calculations
- Preference for *ab initio* methods (predictive power)
- Possibility of assigning uncertainties

**Choice of computational method becomes important**

# COMPUTATIONAL METHODS

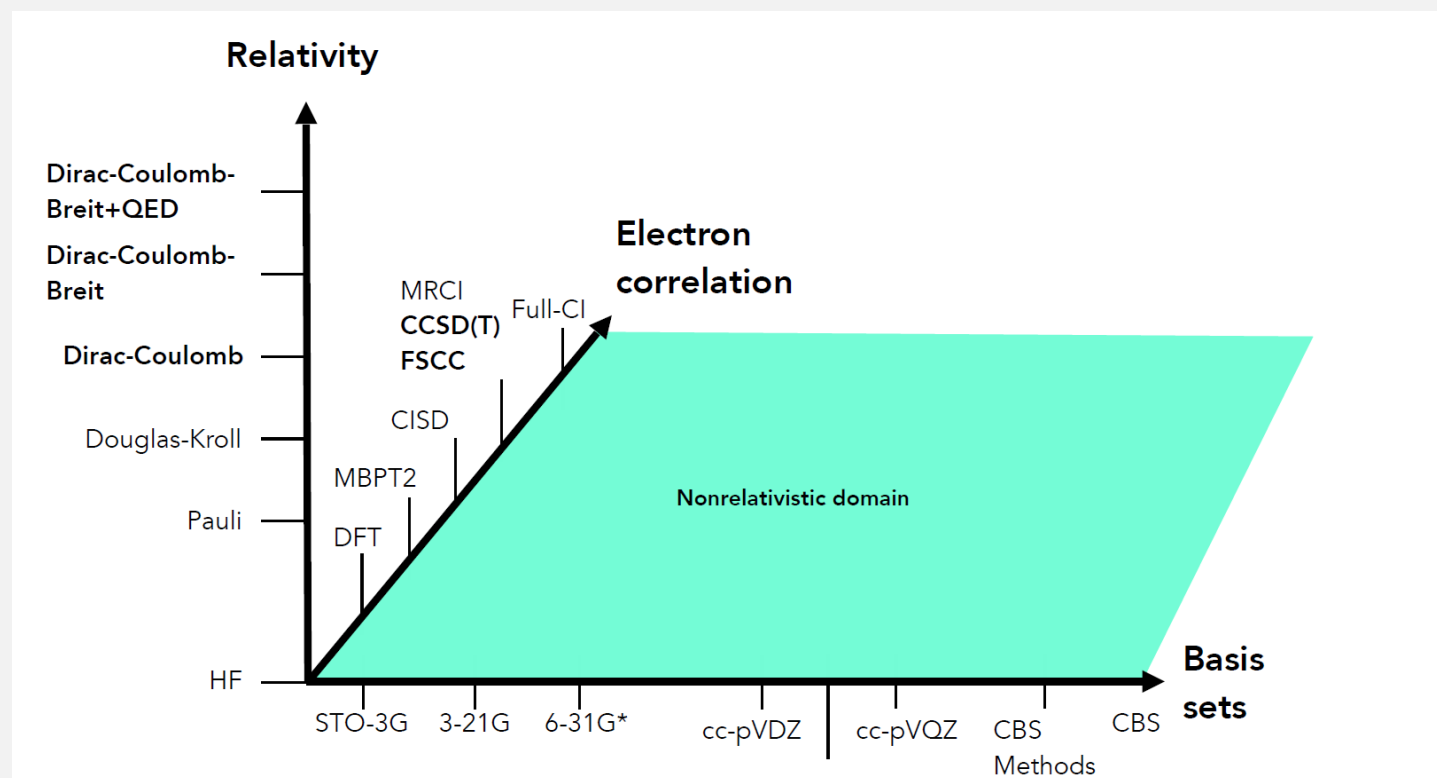
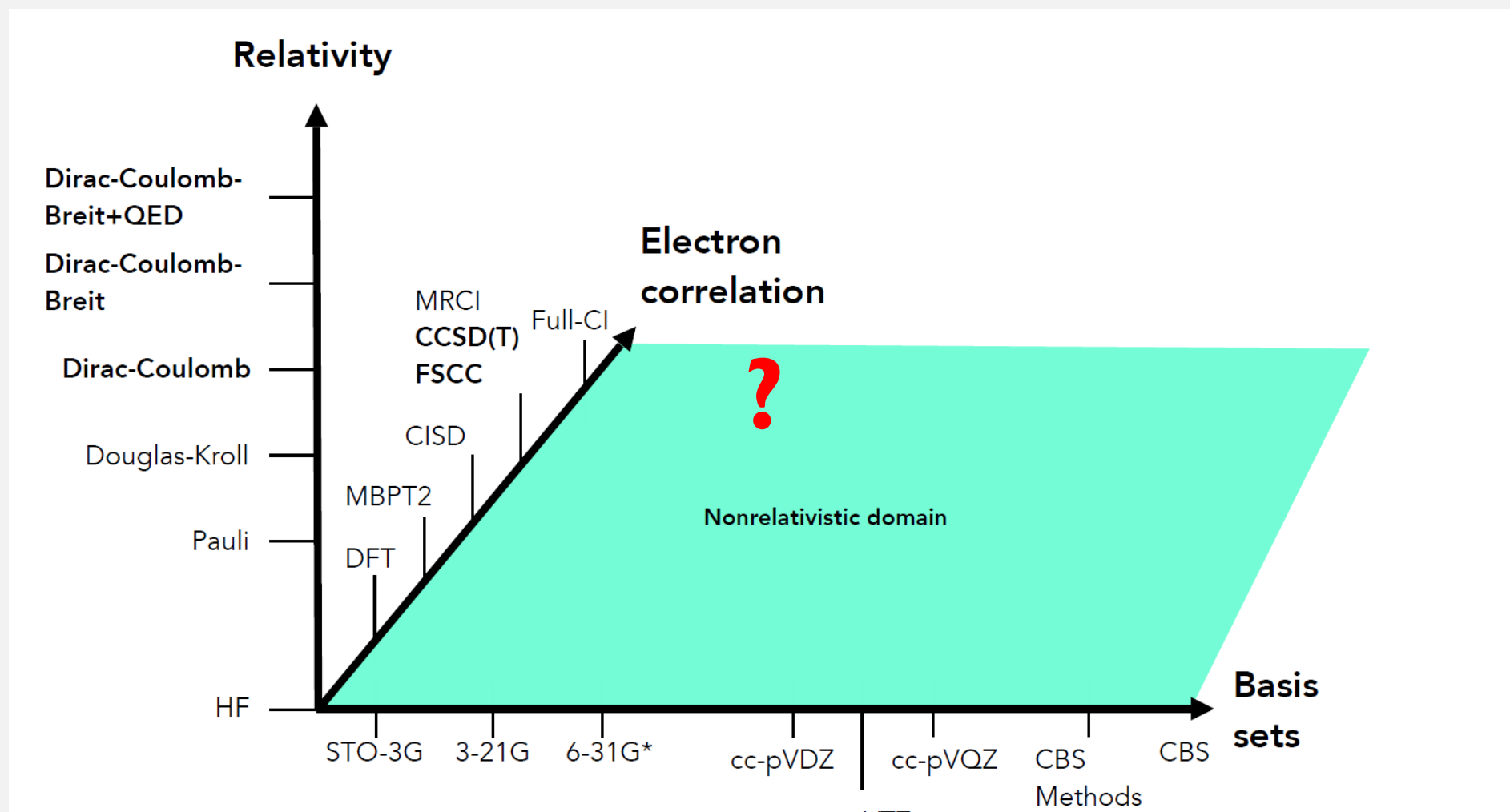


Figure courtesy of P. Schwerdtfeger





## What do we need?

- Coupling parameters describing the effect of  $P(T)$ -violating phenomena (or variation of constants) on electronic structure
  - Relativistic in nature, hence relativistic methods
- Atomic and molecular parameters needed in experiments
  - (usually) heavy (radioactive) systems, hence relativistic methods
- High accuracy
  - State-of-the-art treatment of correlation, large basis sets
- Uncertainty estimates
  - Robust, transparent methods

## Relativistic coupled cluster

- Based on the 4c Dirac Hamiltonian
- Accurate, size-consistent
- **CCSD(T)** - single reference coupled cluster

Closed shell systems/systems with one dominant configuration (good example: BaF,  $X^2\Sigma$ )

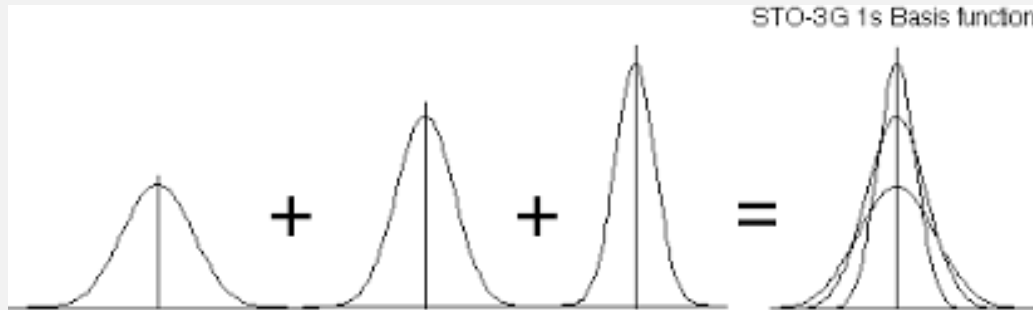
- **FSCC** – multireference Fock space coupled cluster

Open shell systems, excited states, bond dissociation (good example: ThO  $^3\Delta_1$  or any atomic spectrum)

Use the suitable method, or both in complementary manner.

## Basis sets

- Sets of (Gaussian) functions that are used to represent the electronic WF.
- Atom specific, different basis sets for different properties



$$g = x^\ell y^m z^n e^{-\alpha r^2}$$

Gaussian type orbital

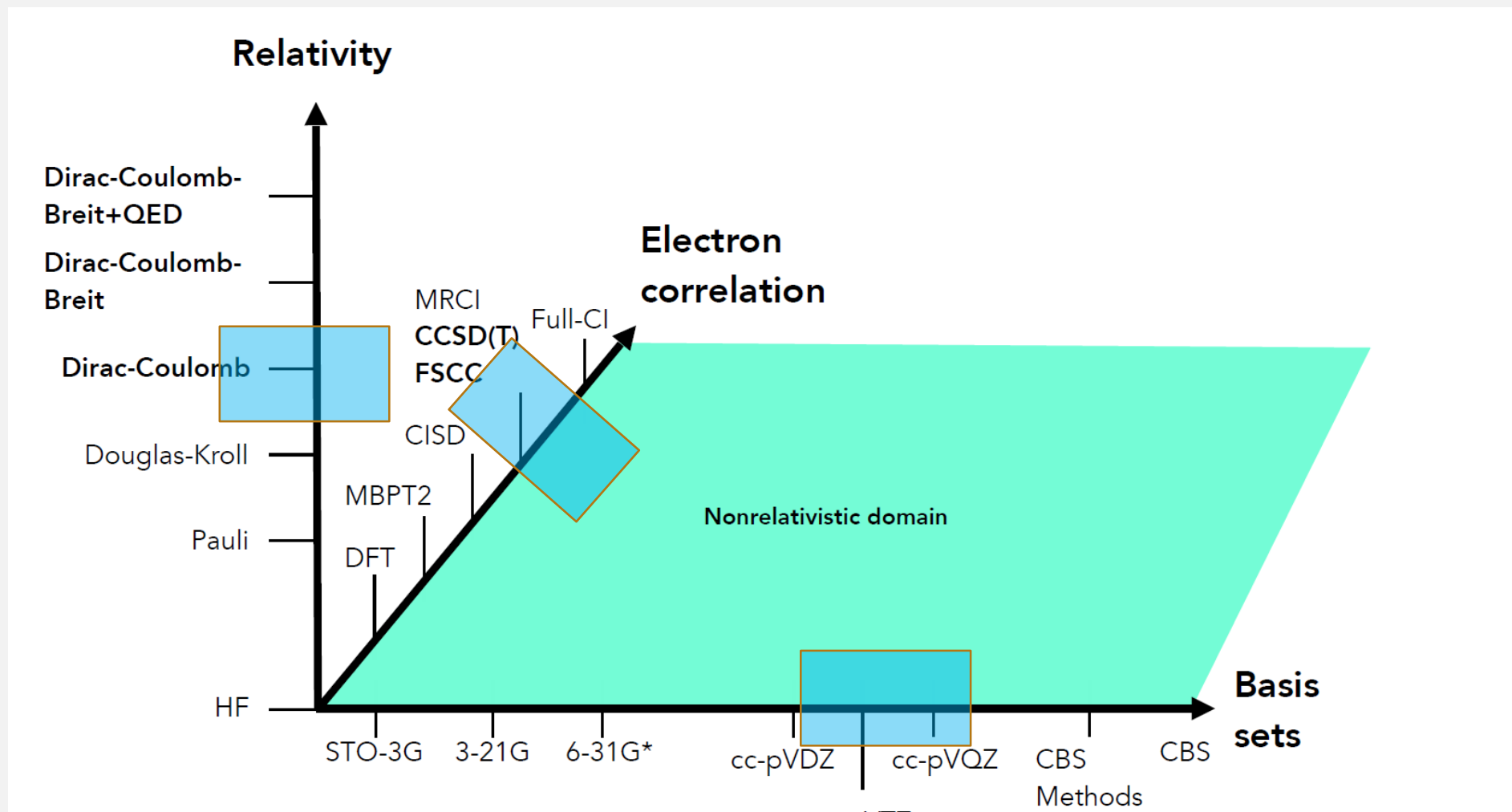
type of function:  
 $\ell = m = n = 0$  for s type  
 $\ell$  or  $m$  or  $n = 1$  for p type  
etc.

'exponent'  $\alpha$

- Dyll's relativistic basis sets; augmented and extended to convergence

(K.G. Dyll, Theor. Chem Acc. 2002, 2004, 2006, 2007, 2009, 2011, 2012, etc.)

# Relativistic coupled cluster

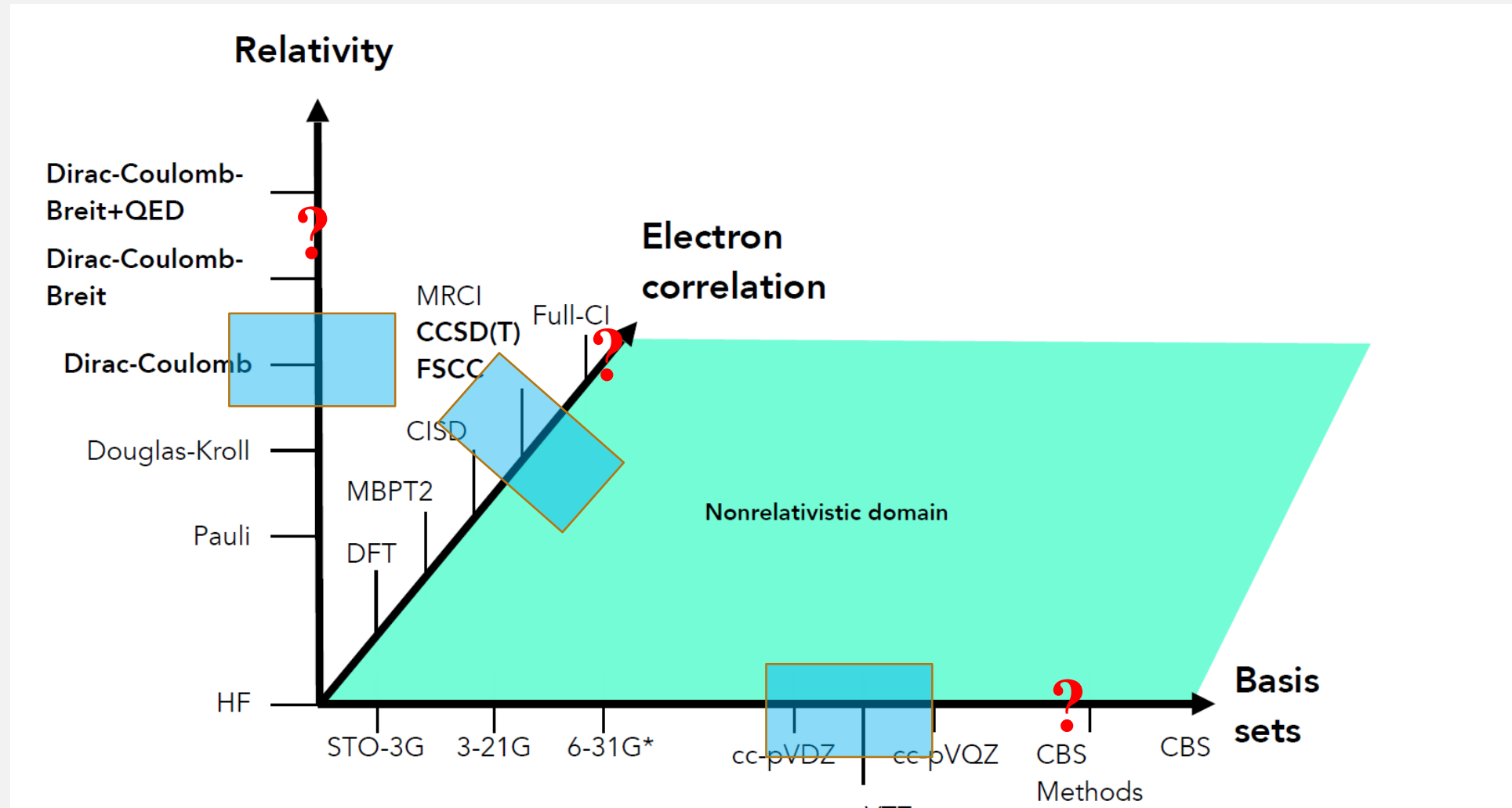




## What can we calculate?

- Atomic properties: energies, IPs, EAs, spectra, **hyperfine structure parameters**, polarisabilities
- Molecular properties: geometries, spectroscopic constants, electronic structure, Franck-Condon Factors (FCFs), transition strengths
- Specific properties:
  - $W_d, W_s$  (eEDM experiments)
  - $W_A$  (NSD-PV, nuclear anapole moments)
  - $W_M$  (nuclear magnetic quadrupole moments)
  - Sensitivity to variation of  $\alpha$
  - ...
- CCSD(T), FSCC (applicable to different systems/states)
- Expected accuracy:  $\sim 10$  meV for energies, single % for properties
- **Systematic investigation of effect of computational parameters and uncertainty evaluation**

# How do we assign uncertainties?



## Software

- Tel Aviv atomic computational package (TRAFS-3C)

Tel-Aviv Relativistic Atomic Fock-Space coupled cluster code, written by E.Eliav and U.Kaldor, with contributions from Y. Ishikawa, A. Landau, A. Borschevsky and H.Yakobi.

- DIRAC18 computational package

DIRAC, a relativistic *ab initio* electronic structure program, release DIRAC18 (2018)

and:

- MRCC code of Kallay et al., [www.mrcc.hu](http://www.mrcc.hu) (higher excitations)
- CFOUR package, <http://www.cfour.de> (geometry optimisation of polyatomic molecules)

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## Any drawbacks?

- ✗ Computationally expensive
- ✗ FSCC is limited to systems with up to two valence electrons/holes
- ✗ Some properties not (yet) available, e.g. TDMs

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Towards High Performance Relativistic Electronic Structure Modelling: The EXP-T Program Package

[Alexander V. Oleynichenko](#) ✉, [Andréi Zaitsevskii](#) & [Ephraim Eliav](#)





# APPLICATIONS

- Hyperfine structure constants:
  - New implementation and test cases
  - Ge
- Nuclear-spin-dependent parity violation measurements:
  - BaF
  - Light triatomic molecules

# HYPERFINE STRUCTURE CONSTANTS

## New implementation: magnetic hyperfine coupling constants

- Expectation values are difficult in CC: use finite field approach

$$\hat{H} = \hat{H}^{(0)} + \lambda \hat{H}_{M,u}^{\text{HFS}}$$

•

$$A_{M,uv} = \frac{g_M}{\Omega} \left. \frac{\partial E_u^{\Omega_v}(\lambda)}{\partial \lambda} \right|_{\lambda=0}$$

$$\begin{aligned} \hat{H}^{M,\text{HFS}} &= g_M \mu_N \vec{I}^M \cdot \sum_i \frac{(\vec{r}_{iM} \times \vec{\alpha}_i)}{r_{iM}^3} \\ &= \sum_u g_M \mu_N I_u^M \sum_i \frac{(\vec{r}_{iM} \times \vec{\alpha}_i)_u}{r_{iM}^3} \\ &= \sum_u I_u^M \hat{H}_u^{M,\text{HFS}}. \end{aligned}$$

- Complementary with already existing implementation for electric quadrupole HFS constants
- Both single reference CCSD(T) and FSCC
- Applicable to atoms and molecules, ground and excited states

# Ground states: Cs and BaF

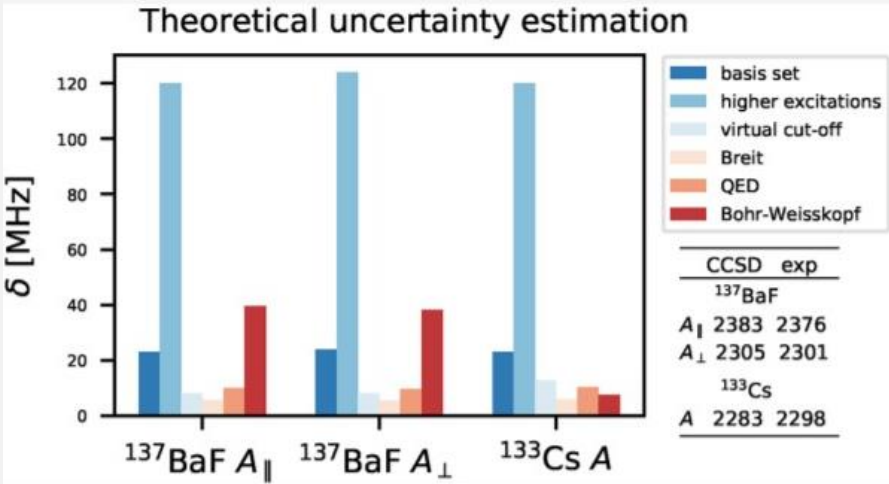
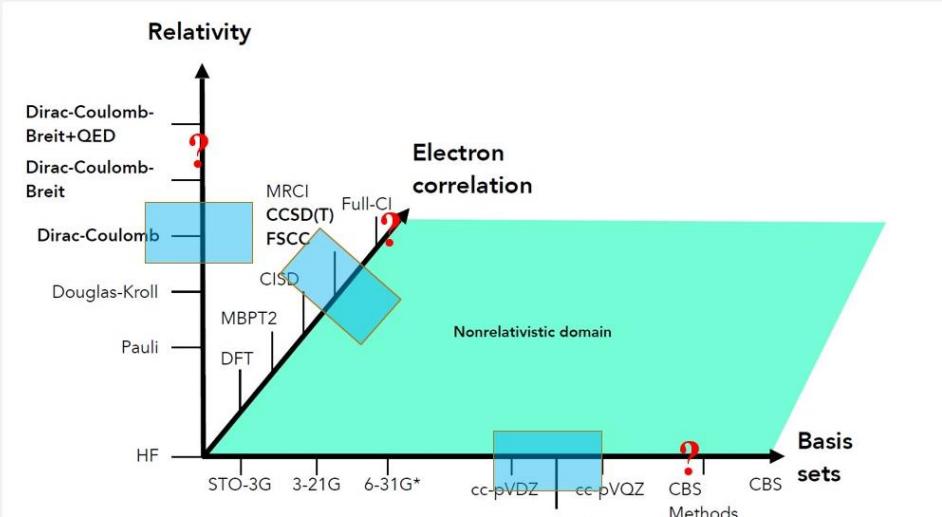
- CCSD/CCSD(T) level of theory
- Uncertainty evaluation:

Table 7. Summary of the Sources of Uncertainty (MHz) of the Calculated  $A_{\parallel}$ ,  $A_{\perp}$ , and  $A$  Constants (MHz) of  $^{137}\text{Ba}$  in BaF and  $^{133}\text{Cs}$

source	$^{137}\text{BaF}$		$^{133}\text{Cs}$
	$\delta A_{\parallel}$	$\delta A_{\perp}$	$\delta A$
basis set			
quality	20.00	20.00	19.0
tight functions	3.00	3.00	4.00
diffuse functions	0.00	1.00	0.00
correlation			
higher order	-120.00	-124.00	-120.00
virtual cutoff	8.18	8.18 <sup>c</sup>	12.78
relativistic effects			
Breit	5.72 <sup>a</sup>	5.53 <sup>a</sup>	6.00 <sup>b</sup>
QED <sup>VP+SE</sup>	-10.01 <sup>a</sup>	-9.68 <sup>a</sup>	-10.30 <sup>b</sup>
Bohr–Weisskopf	-39.56 <sup>a</sup>	-38.26 <sup>a</sup>	-7.60 <sup>b</sup>
quadratic sum	128.74	132.07	123.05
%	5.40	5.73	5.28

<sup>a</sup>Based on  $^{135}\text{Ba}^+$  results from ref 85. <sup>b</sup>Taken directly from ref 85. <sup>c</sup>Used  $A_{\parallel}$  results.

How do we assign uncertainties?



## Ground states: Cs and BaF

**Table 9.  $A$  of Cs in MHz<sup>a</sup>**

method	<sup>133</sup> Cs	%(exp)
MBPT <sup>b</sup> +B <sup>7</sup>	2291.00	−0.31
SDpT+B <sup>8</sup>	2278.5	−0.85
MBPT <sup>b7</sup> +B <sup>10</sup>	2295.87	−0.10
MBPT <sup>b</sup> +OE+G <sup>9</sup>	2302	0.17
CCSDvT <sup>11</sup> +B <sup>10</sup> +QED <sup>VP+SE 87</sup>	2306.6	0.36
CCSD (ECC) <sup>26</sup>	2179.1	−5.18
CCSD (Z-vector) <sup>27</sup>	2218.4	−3.47
MBPT <sup>b</sup> +B+QED <sup>VP+SE 85</sup>	2294.4	−0.16
CCSD (LCCSD) <sup>81</sup>	2345.9	2.08
CCSD (finite field, this work)	2283(123)	−0.66
exp <sup>78</sup>	2298.16	

**Table 8.  $A_{\parallel}$  and  $A_{\perp}$  of <sup>137</sup>Ba in BaF (MHz)**

method	<sup>137</sup> BaF			
	$A_{\parallel}$	%(exp)	$A_{\perp}$	%(exp)
GRECP SCF-EO <sup>90</sup>	2264	−4.71	2186	−5.00
GRECP RASSCF-EO <sup>90</sup>	2272	−4.38	2200	−4.39
DF RASCI <sup>91</sup>	2240	−5.72	2144	−6.82
DF MBPT <sup>91</sup>	2314	−2.61	2254	−2.04
DC CCSD (this work)	2383(129)	0.29	2305(132)	0.17
exp <sup>77</sup>	2376(12)		2301(9)	



## Ground states: Cs and BaF



Pi Haase

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Article

### Hyperfine Structure Constants on the Relativistic Coupled Cluster Level with Associated Uncertainties

Pi A. B. Haase,\* Ephraim Eliav, Miroslav Iliaš, and Anastasia Borschevsky



Cite This: *J. Phys. Chem. A* 2020, 124, 3157–3169



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### in BaF (MHz)

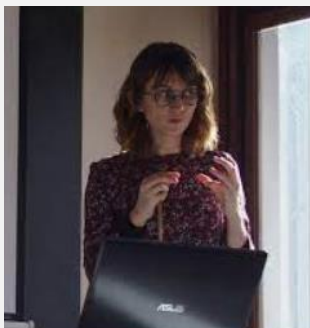
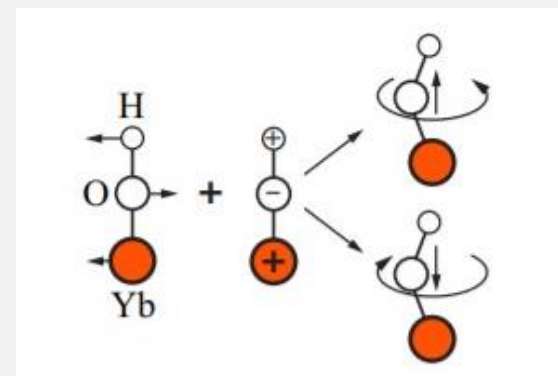
		<sup>137</sup> BaF		
		%(exp)	$A_{\perp}$	%(exp)
DF RASCI <sup>91</sup>		-4.71	2186	-5.00
DF MBPT <sup>91</sup>		-4.38	2200	-4.39
DC CCSD (this work)		-5.72	2144	-6.82
exp <sup>77</sup>		-2.61	2254	-2.04
		0.29	2305(132)	0.17
		2376(12)	2301(9)	

## Predictive power

- YbOH: promising candidate for search for eEDM and other P,T-violating effects.

**TABLE VII.** The  $A_{\parallel}$  constants of the heavy nucleus calculated using the model optimized for  $W_M$  given in (MHz) and comparison with the available experimental data.

System	Calculated $A_{\parallel}$	Expt. $A_{\parallel}$	$\Delta$ (%)
$^{137}\text{BaOH}$	2194.6	2200.2 <sup>42</sup>	0.3
$^{171}\text{YbF}$	7579.0	7429.1 <sup>43</sup>	2
$^{173}\text{YbF}$	-2087.6	-2060.0 <sup>44</sup>	1.3
$^{171}\text{YbOH}$	7174.9		
$^{173}\text{YbOH}$	-1976.3		



Malika Denis

### Enhanced $\mathcal{P}$ , $\mathcal{T}$ -violating nuclear magnetic quadrupole moment effects in laser-coolable molecules

Cite as: J. Chem. Phys. **152**, 084303 (2020); <https://doi.org/10.1063/1.5141065>

Submitted: 03 December 2019 . Accepted: 06 February 2020 . Published Online: 25 February 2020

Malika Denis, Yongliang Hao, Ephraim Eliav, Nicholas R. Hutzler, Malaya K. Nayak, Rob G. E. Timmermans, and Anastasia Borschevsky

# Fine and hyperfine interactions in $^{171}\text{YbOH}$ and $^{173}\text{YbOH}$

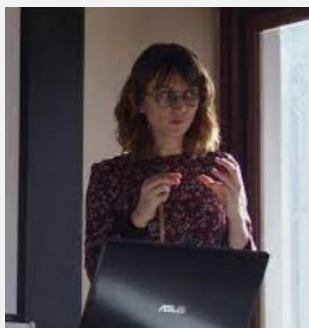
J. Chem. Phys. **154**, 244309 (2021); <https://doi.org/10.1063/5.0055293>

 Nickolas H. Pilgram<sup>1</sup>,  Arian Jadbabaie<sup>1</sup>,  Yi Zeng<sup>1</sup>,  Nicholas R. Hutzler<sup>1</sup>, and  Timothy C. Steimle<sup>2,a)</sup>

**Table II:** Comparison of the measured hyperfine parameters of the  $\tilde{X}^2\Sigma^+(0,0,0)$  state of  $^{171,173}\text{YbOH}$  to calculated values

Isotopologue	Parameter	Measured (MHz)	Theory Ref. <sup>16</sup> (MHz)	Theory Ref. <sup>51</sup> cGHF(MHz)	Theory Ref. <sup>51</sup> cGKS(MHz)	Theory Ref. <sup>52</sup> (MHz)
$^{171}\text{YbOH}$	$A_{  }^a$	6979 (35)	7174.9			
$^{171}\text{YbOH}$	$A_{\perp}^b$	6745(15)				
$^{173}\text{YbOH}$	$A_{  }$	-1929(11)	-1976.3	-1600	-1300	
$^{173}\text{YbOH}$	$A_{\perp}$	-1856 (5)		-1600		
$^{173}\text{YbOH}$	$e^2Qq_0$	-3319 (48)	-3502			-3492

$^{173}\text{YbOH}$  -1976.3



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## Excited states: Ge

- Collinear laser spectroscopy performed at ISOLDE-CERN
- HFS of the  $4s^2 4p^2 \ ^3P_1 \rightarrow 4s^2 4p 5p \ ^3P_1^o$  transition
- $^{69,71,73}\text{Ge}$
- Atomic calculations of  $A_0$  and  $q$  (EFG) parameters used for analysis
- FSCC approach, uncertainty estimation

Configuration	Term	$J$	Level ( $\text{cm}^{-1}$ )	Ref.
$4s^2 4p^2$	$^3P$	0	0.0000	SM93b
		1	557.1341	SM93b
		2	1409.9609	SM93b
$4s^2 4p^2$	$^1D$	2	7125.2989	SM93b
$4s^2 4p^2$	$^1S$	0	16367.3332	SM93b
$4s^2 4p 5s$	$^3P^o$	0	37451.6893	SM93b
		1	37702.3054	SM93b
		2	39117.9021	SM93b



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		1	37702.3054	SM93b
		2	39117.9021	SM93b

Electric field gradient uncertainties

Germanium

	$4p^2 \ ^3P_1$	$4p^2 \ ^3P_2$	$4p 5s \ ^3P_1$
Basis set	0.0043	0.0078	0.0488
Model space	0.0007	0.0016	0.0103
Virtual space	0.0003	0.0004	0.0001
Triples+higher	0.0353	0.0720	0.0266
Gaunt	0.0001	0.0051	0.0016
Total uncertainty	3.0 %	3.0 %	6.4 %
$q^{th}$ (a.u.)	<b>1.178(36)</b>	<b>-2.399(73)</b>	<b>-0.885(56)</b>

Table: Sources of uncertainty and final values of the calculated  $q^{th}$  (a.u.).

$A_0$  uncertainties

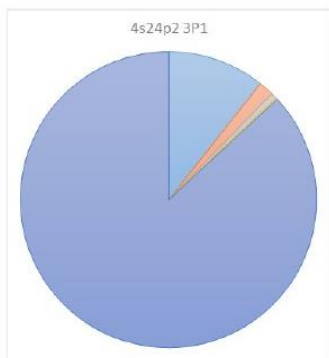
Germanium

	$4p^2 \ ^3P_1$	$4p^2 \ ^3P_2$	$4p 5s \ ^3P_1$
Basis set	2.5	4.3	18.1
Model space	1.7	2.7	9.0
Virtual space	4.2	3.1	4.6
Triple+higher	2.2	9.6	39.4
Gaunt	1.4	0.4	2.7
Total uncertainty	7.8 %	3.5 %	3.4 %
$A_0^{th}$ (MHz)	<b>-74(6)</b>	<b>321(11)</b>	<b>1314(45)</b>

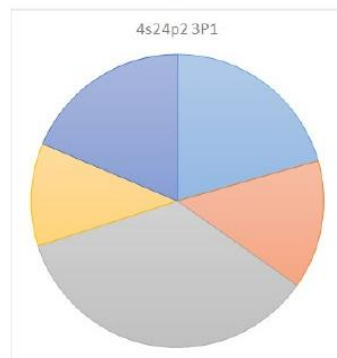
Table: Sources of uncertainty and final values of the calculated  $A_0^{th}$  (MHz).

## Excited states: Ge (uncertainties)

EFG for  $4p^2\ ^3P_1$

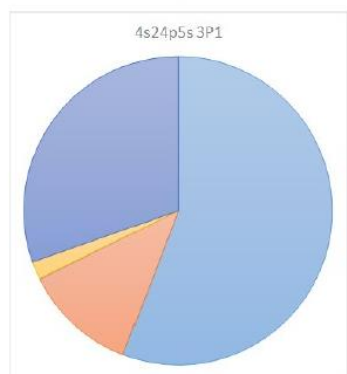


$A_0$  for  $4p^2\ ^3P_1$

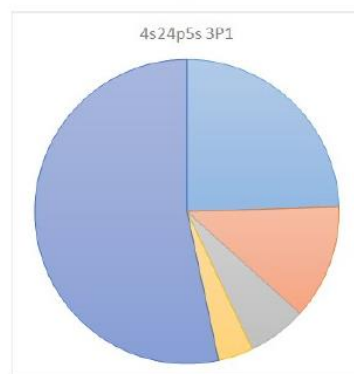


■ Basis ■ Model space ■ Virtual cutoff ■ Gaunt ■ Triples(3%)

EFG for  $4p5s\ ^3P_1$



$A_0$  for  $4p5s\ ^3P_1$



■ Basis ■ Model space ■ Virtual cutoff ■ Gaunt ■ Triples(3%)

- Uncertainties should be evaluated separately for different states and properties.

# Ge moments

	$I^\pi$	$\mu^{lit}(\mu_N)$	$\mu^{exp}(\mu_N)$
$^{69}\text{Ge}$	$5/2^-$	$+0.735(7)$ <sup>1</sup>	$+0.920(5)$
$^{71}\text{Ge}$	$1/2^-$	$+0.54606(7)$ <sup>2</sup>	$+0.547(5)$
$^{73}\text{Ge}$	$9/2^+$	$-0.87824(5)$ <sup>3</sup>	$-0.904(21)$

	$I^\pi$	$Q_s^{lit}(b)$	$Q_s^{exp}(b)$	$Q_s^{JUN45}(b)$
$^{69}\text{Ge}$	$5/2^-$	$+0.027(5)$ <sup>1</sup>	$+0.114(7)$	$+0.150$
$^{73}\text{Ge}$	$9/2^+$	$-0.196(1)$ <sup>4</sup>	$-0.198(4)$	$-0.258$

<sup>1</sup> A. F. Oluwole, S. G. Schmelling, and H. A. Shugart, Phys. Rev. C 2, 228 (1970).

<sup>2</sup> W. J. Childs and L. S. Goodman, Phys. Rev. 141, 15 (1966).

<sup>3</sup> W. Makulski, K. Jackowski, A. Antušek, and M. Jaszunski, J. Phys. Chem. A 110, 11462 (2006).

<sup>4</sup> V. Kellö and A. Sadlej, Mol. Phys. 96, 275 (1999).

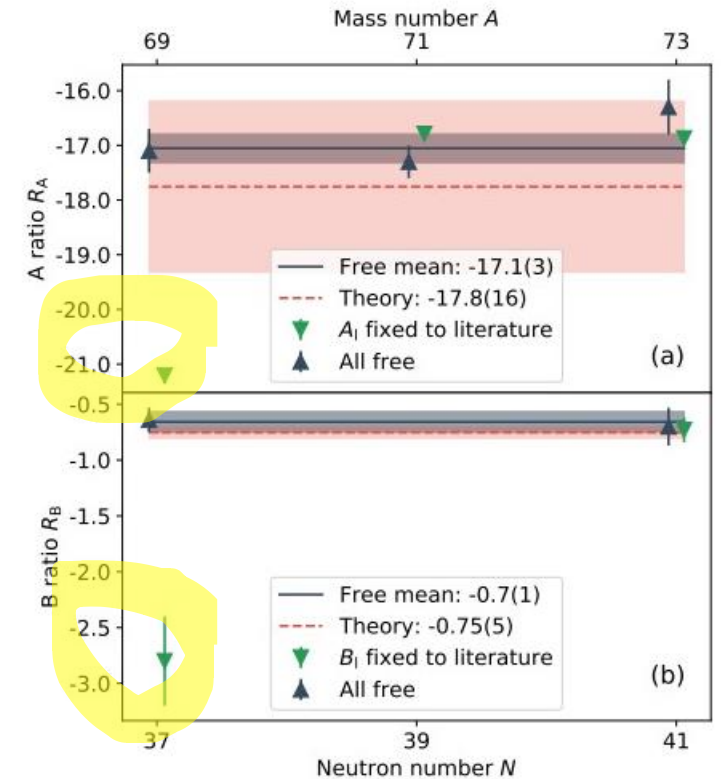



FIG. 4. Ratio of the  $A$  (a) and  $B$  hyperfine constants (b) for the  $4p^{2,3}P_1$  (lower) and  $4p5s^3P_1^o$  (upper) atomic states, deduced with two fitting procedures. One with the  $A_I$  and  $B_I$  values for each isotope fixed (green) to the high-precision value reported in literature [24, 28, 36], and one with all hyperfine constants as free fit parameters (black). The hyperfine constants ratios calculated with atomic relativistic FSCC are shown by red dashed lines.

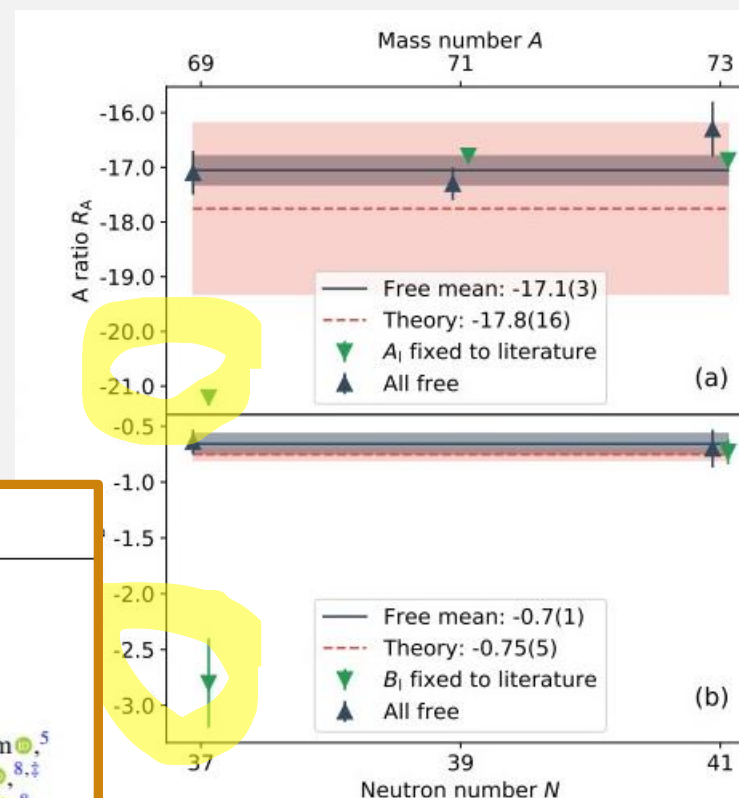


# Ge moments



Martijn Reitsma

	$I^\pi$	$\mu^{lit}(\mu_N)$	$\mu^{exp}(\mu_N)$	
$^{69}\text{Ge}$	$5/2^-$	$+0.735(7)^1$	$+0.920(5)$	 <b>Martijn Reitsma</b>
$^{71}\text{Ge}$	$1/2^-$	$+0.54606(7)^2$	$+0.547(5)$	
$^{73}\text{Ge}$				
PHYSICAL REVIEW C <b>102</b> , 054331 (2020)				
<b>Nuclear moments of germanium isotopes near <math>N = 40</math></b>				
A. Kanellakopoulos <sup>1</sup> , X. F. Yang <sup>2,1,*</sup> , M. L. Bissell <sup>3</sup> , M. L. Reitsma <sup>4</sup> , S. W. Bai <sup>2</sup> , J. Billowes, A. Borschevsky <sup>4</sup> , B. Cheal <sup>6</sup> , C. S. Devlin <sup>6</sup> , R. F. Garcia Ruiz <sup>7,†</sup> , H. Heylen <sup>7</sup> , S. Kaufmann, <sup>8,9</sup> Á. Koszorús, <sup>1,§</sup> S. Lechner <sup>7,10</sup> , S. Malbrunot-Ettenauer <sup>7</sup> , R. Neugart, <sup>5,9</sup> G. Neyens <sup>1,7</sup> , W. Nört T. Ratajczyk, <sup>8</sup> L. V. Rodríguez <sup>5,11,  </sup> , S. Sels, <sup>7,¶</sup> S. J. Wang, <sup>2</sup> L. Xie, <sup>3</sup> Z. Y. Xu, <sup>1,‡</sup> and D. T. Yo				
$^{69}\text{Ge}$	$5/2^-$	$+0.027(5)^1$	$+0.114(7)$	$+0.150$
$^{73}\text{Ge}$	$9/2^+$	$-0.196(1)^4$	$-0.198(4)$	$-0.258$



4. Ratio of the  $A$  (a) and  $B$  hyperfine constants (b) for the  $4p^{23}P_1$  (lower) and  $4p5s^3P_1^o$  (upper) atomic states, deduced with two fitting procedures. One with the  $A_1$  and  $B_1$  values for each isotope fixed (green) to the high-precision value reported in literature [24, 28, 36], and one with all hyperfine constants as free fit parameters (black). The hyperfine constants ratios calculated with atomic relativistic FSCC are shown by red dashed lines.

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<sup>4</sup> V. Kellö and A. Sadlej, Mol. Phys. 96, 275 (1999).



# NUCLEAR SPIN DEPENDENT PV EFFECTS

# Anapole moment measurement in BaF

- NSD PV effects in atoms/molecules:

$$H_{\text{NSD}} = \frac{G_F}{\sqrt{2}I} \sum_i (\kappa_A + \kappa_{\text{ax}} + \kappa_{\text{hfs}}) (\boldsymbol{\alpha}_i \cdot \mathbf{I}) \rho(\mathbf{r}_i)$$

- Nuclear anapole moment is the dominant NSD-PV effect in heavy systems
- Measurement can provide stringent test of SM and nuclear theory
- Planned measurement on BaF\*
- $W_A$  coefficient can be used to extract anapole from measurements

$$W_A = \frac{G_F}{\sqrt{2}} \left\langle +\frac{1}{2} \left| \sum_i \rho(\mathbf{r}_i) \alpha_+ \right| -\frac{1}{2} \right\rangle.$$

$$\alpha_+ = \alpha_x + i\alpha_y = \begin{pmatrix} 0 & \sigma_x \\ \sigma_x & 0 \end{pmatrix} + i \begin{pmatrix} 0 & \sigma_y \\ \sigma_y & 0 \end{pmatrix}$$

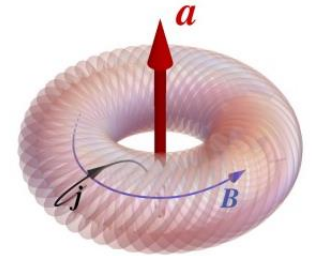


FIG. 7 (Color online) The toroidal component of current density  $\mathbf{j}$  produces anapole moment  $\mathbf{a}$ , with magnetic field  $\mathbf{B}$  that is entirely confined inside the “doughnut”. The azimuthal component of current density generates magnetic dipole moment aligned with  $\mathbf{a}$ , with its associated conventional dipolar magnetic field not shown.

## Anapole moment measurement in BaF

- | Method               | $W_A$ (Hz)     | Ref.  |
|----------------------|----------------|---|
| <b>CCSD(T)+Gaunt</b> | <b>147.7±2</b> | <b>Present</b>                                    |
| RECP+SCF+EO          | 181            | Kozlov et al., PRA <b>56</b> R3326 (1997)         |
| Semiempirical        | 164            | DeMille et al., PRL <b>100</b> , 023003 (2008)    |
| 4c-RASCI             | 160            | Nayak and Das, PRA <b>79</b> , 060502 (2009)      |
| Scaled ZORA-HF       | 190            | Isaev & Berger, PRA <b>86</b> , 062515 (2012)     |
| DHF/DFT+CP           | 146            | Borschevsky et al., PRA <b>88</b> , 022125 (2013) |

# Anapole moment measurement in BaF

Method	$W_A$ (Hz)	Ref.
<b>CCSD(T)+Gaunt</b>	<b>147.7±2</b>	<b>Present</b>
RECP+SCF+EO	181	Kozlov et al., PRA <b>56</b> R3326 (1997)
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DHF/DFT+CP	146	Borschevsky et al., PRA <b>88</b> , 022125 (2013)

- Quick sanity check:

Table 8.  $A_{\parallel}$  and  $A_{\perp}$  of  $^{137}\text{Ba}$  in BaF (MHz)

method	$^{137}\text{BaF}$			
	$A_{\parallel}$	% (exp)	$A_{\perp}$	% (exp)
GRECP SCF-EO <sup>90</sup>	2264	-4.71	2186	-5.00
GRECP RASSCF-EO <sup>90</sup>	2272	-4.38	2200	-4.39
DF RASCI <sup>91</sup>	2240	-5.72	2144	-6.82
DF MBPT <sup>91</sup>	2314	-2.61	2254	-2.04
DC CCSD (this work)	2383(129)	0.29	2305(132)	0.17
exp <sup>77</sup>	2376(12)		2301(9)	

# Anapole moment measurement in BaF



Yongliang Hao

Method	$W_A$ (Hz)	Ref.
<b>CCSD(T)+Gaunt</b>	<b>147.7±2</b>	<b>Present</b>
RECP+SCF+EO	181	Kozlov et al., PRA <b>56</b> R3326 (1997)
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4c-RASCI	160	Nayak and Das, PRA <b>79</b> , 060502 (2009)

Scal

DH

PHYSICAL REVIEW A **98**, 032510 (2018)

**Nuclear anapole moment interaction in BaF from relativistic coupled-cluster theory**

Yongliang Hao,<sup>1</sup> Miroslav Iliaš,<sup>2</sup> Ephraim Eliav,<sup>3</sup> Peter Schwerdtfeger,<sup>4,5</sup> Victor V. Flambaum,<sup>6,7</sup> and Anastasia Borschevsky<sup>1,\*</sup>

• Qu

GRECP SCF-EO <sup>90</sup>	2264	−4.71	2186	−5.00
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exp <sup>77</sup>	2376(12)		2301(9)	

# NSD-PV effects in light polyatomic molecules

- NSD PV effects in atoms/molecules:

$$H_{\text{NSD}} = \frac{G_F}{\sqrt{2}I} \sum_i (\kappa_A + \kappa_{\text{ax}} + \kappa_{\text{hfs}}) (\boldsymbol{\alpha}_i \cdot \boldsymbol{I}) \rho(\boldsymbol{r}_i)$$

- In light systems these contributions are similar in size
- Measurement can provide stringent tests of SM, nuclear theory, probe the Z-boson exchange between the electrons and the nuclei, and offer the possibility to search for new particles, such as the Z' boson
- Measurements planned on BeNC, BeCN, MgNC, and MgCN
- Presence of different atoms: possibility to disentangle the contributions

## COMMUNICATIONS PHYSICS

ARTICLE

<https://doi.org/10.1038/s42005-019-0181-1>

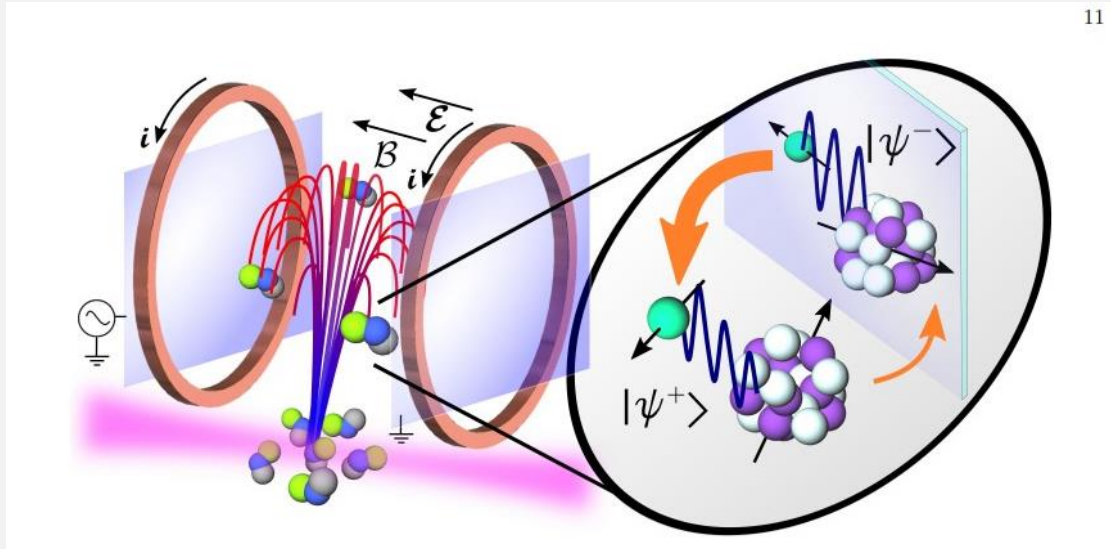
OPEN

Nuclear-spin dependent parity violation in optically trapped polyatomic molecules

E.B. Norrgard<sup>1</sup>, D.S. Barker<sup>1</sup>, S. Eckel<sup>1</sup>, J.A. Fedchak<sup>1</sup>, N.N. Klimov<sup>1</sup> & J. Scherschligt<sup>1</sup>

# NSD-PV effects in light polyatomic molecules

- In light system accurate nuclear theory predictions tangible:
- Together with predicted molecular enhancement factors can be used to estimate the expected measurable effect.



11

**TABLE I:** Magnetic moments (in units of nuclear magneton) [28, 61–65], anapole-moment coupling constants, spin operator matrix elements, and  $\kappa_{\text{ax}}$  coupling constants for  $^9\text{Be}$ ,  $^{13}\text{C}$ ,  $^{14,15}\text{N}$  and  $^{25}\text{Mg}$  obtained within NCSM. The results obtained using the single-particle model are also shown, along with the valence particle (V.p.) and the valence orbital (V.o) for each nucleus.

	$^9\text{Be}$	$^{13}\text{C}$	$^{14}\text{N}$	$^{15}\text{N}$	$^{25}\text{Mg}$
$I^\pi$	$3/2^-$	$1/2^-$	$1^+$	$1/2^-$	$5/2^+$
$\mu^{\text{exp.}}$	-1.177 <sup>a</sup>	0.702 <sup>b</sup>	0.404 <sup>c</sup>	-0.283 <sup>d</sup>	-0.855 <sup>e</sup>
NCSM calculations					
$\mu$	-1.05	0.44	0.37	-0.25	-0.50
$\kappa_A$	0.016	-0.028	0.036	0.088	0.035
$\langle s_{p,z} \rangle$	0.009	-0.049	-0.183	-0.148	0.06
$\langle s_{n,z} \rangle$	0.360	-0.141	-0.1815	0.004	0.30
$\kappa_{\text{ax}}$	0.035	-0.019	0.0002	0.015	0.024
$\kappa$	0.050	-0.046	0.037	0.103	0.057
Single-particle model calculations					
V. p.	$n$	$n$	$n, p$	$p$	$n$
V. o.	$p_{3/2}$	$p_{1/2}$	$p_{1/2}$	$p_{1/2}$	$d_{5/2}$
$K$	-2	1	1	1	-3
$\kappa_A$	0.007	-0.007	0.035	0.044	0.014
$\kappa_{\text{ax}}$	0.050	-0.017	0.0	0.017	0.050
$\kappa_{\text{hfs}}$	-0.001	0.001	0.0006	-0.0004	-0.002
$\kappa$	0.056	-0.023	0.036	0.060	0.062

**TABLE V:** Recommended values of the  $W_{\text{PV}}$  parameters (Hz) with corresponding uncertainties.

Atom	Atom 1	Atom 2	Atom 3
Mol.	Be	Mg	
BeNC	0.50	—	0.34
BeCN	0.54	—	0.28
MgNC	—	5.3	0.45
MgNC <sup>†</sup>	—	5.3	0.47
MgCN	—	5.4	0.37
Uncertainty (%)	6.3	4.9	7.6



# NSD-PV effects in light polyatomic molecules

- In light system accurate nuclear theory predictions tangible:

**TABLE I:** Magnetic moments (in units of nuclear magneton) [28, 61–65], anapole-moment coupling constants, spin operator matrix elements, and  $\kappa_{\text{ax}}$  coupling constants for  $^9\text{Be}$ ,  $^{13}\text{C}$ ,  $^{14,15}\text{N}$  and  $^{25}\text{Mg}$  obtained within NCSM. The results obtained using the single-particle model are also shown, along with the valence particle (V.p.) and the valence orbital (V.o) for each nucleus.

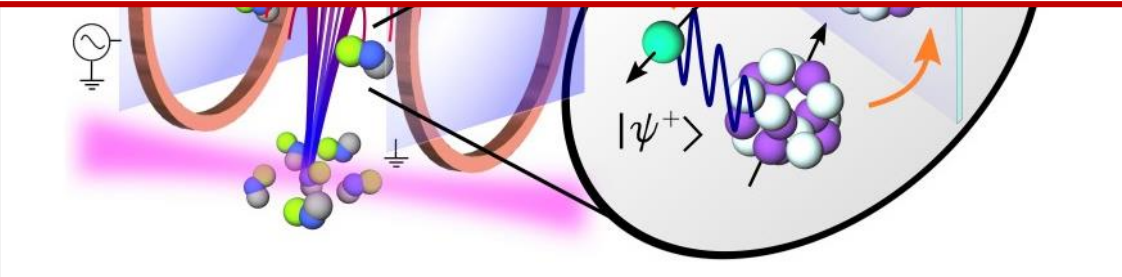
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$\langle s_{p,z} \rangle$	0.009	-0.049	-0.183	-0.148	0.06
Model calculations					
				$p$	$n$
				$p_{1/2}$	$d_{5/2}$
				1	-3
				0.044	0.014
				0.017	0.050
				-0.0004	-0.002
				0.060	0.062

Editors' Suggestion

## Nuclear spin-dependent parity-violating effects in light polyatomic molecules

Yongliang Hao, Petr Navrátil, Eric B. Norrgard, Miroslav Iliaš, Ephraim Eliav, Rob G. E. Timmermans, Victor V. Flambaum, and Anastasia Borschevsky

Phys. Rev. A **102**, 052828 – Published 25 November 2020



values of the  $W_{\text{PV}}$   
bonding uncertainties.

Atom	Atom 1	Atom 2	Atom 3
Mol.	Be	Mg	
BeNC	0.50	—	0.34
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MgCN	—	5.4	0.37
Uncertainty (%)	6.3	4.9	7.6



# CONCLUSIONS

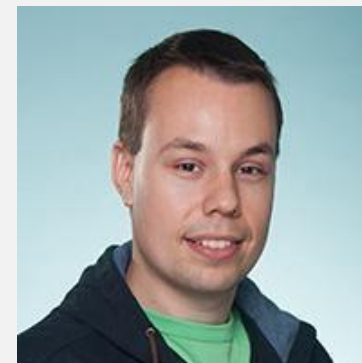
- State of the art high accuracy computational approach
- Versatile method: many possible applications
- Reliable predictions, uncertainty estimates possible
- Close collaborations with experimental groups
- These are very exciting times!



Ephraim Eliav



Miroslav Ilias



Lukas Pasteka



Victor Flambaum



Peter Schwerdtfeger



# RELATIVISTIC COUPLED CLUSTER

- Based on the 4c Dirac Hamiltonian
- Exponential wave operator:

$$\Psi = \exp(S)\Psi_0 = \left(1 + S + \frac{S^2}{2!} + \cdots\right)\Psi_0$$

- $S$  is the excitation operator:

$$S = S_1 + S_2 + \cdots + S_N; \quad S_1 = \sum_{ia} s_i^a a_a^\dagger a_i; \quad S_2 = \sum_{ijab} s_{ij}^{ab} a_a^\dagger a_b^\dagger a_j a_i$$

- CC energy equations:

$$\langle \Phi_0 | (H - E_{\text{CCSD}}) \exp(S_1 + S_2) | \Phi_0 \rangle = 0$$

- Accurate, all-order in PT, size-extensive, and size-consistent

# Reaching meV accuracy

Complete basis set limit extrapolation

