

# Nuclear physics opportunities with electromagnetic moments beyond the quadrupole

Ruben de Groote

ECT\* workshop

April 13, 2022

# Laser spectroscopy and hyperfine structure

- Hyperfine structure links atomic and nuclear properties

$$h\nu \sim \nu_0 + A \frac{C}{2} + B \frac{1}{4} \frac{(3/2)C(C+1) - 2I(I+1)J(J+1)}{I(2I-1)J(2J-1)}$$

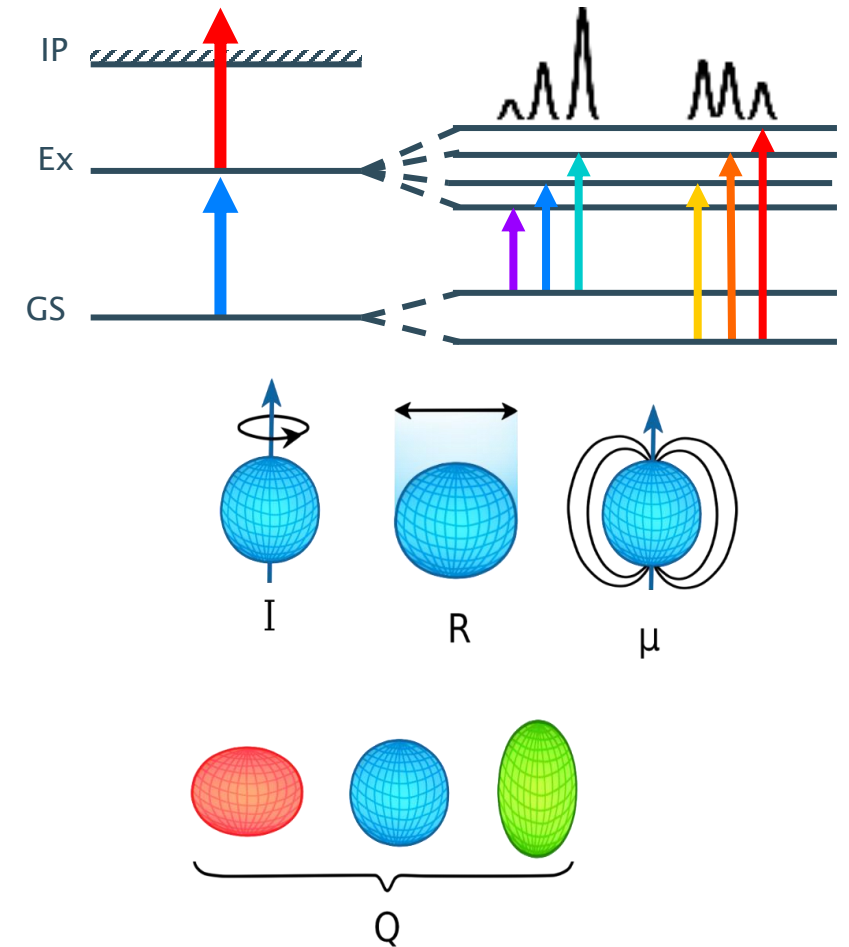
- Nuclear model-independent extraction of:

$$\delta\langle r^2 \rangle$$

$$A = \frac{\mu_I B_J}{IJ}$$

$$B = eQV_{zz}$$

provided the atomic parameters are known (e.g. knowing mass and field shift is required)

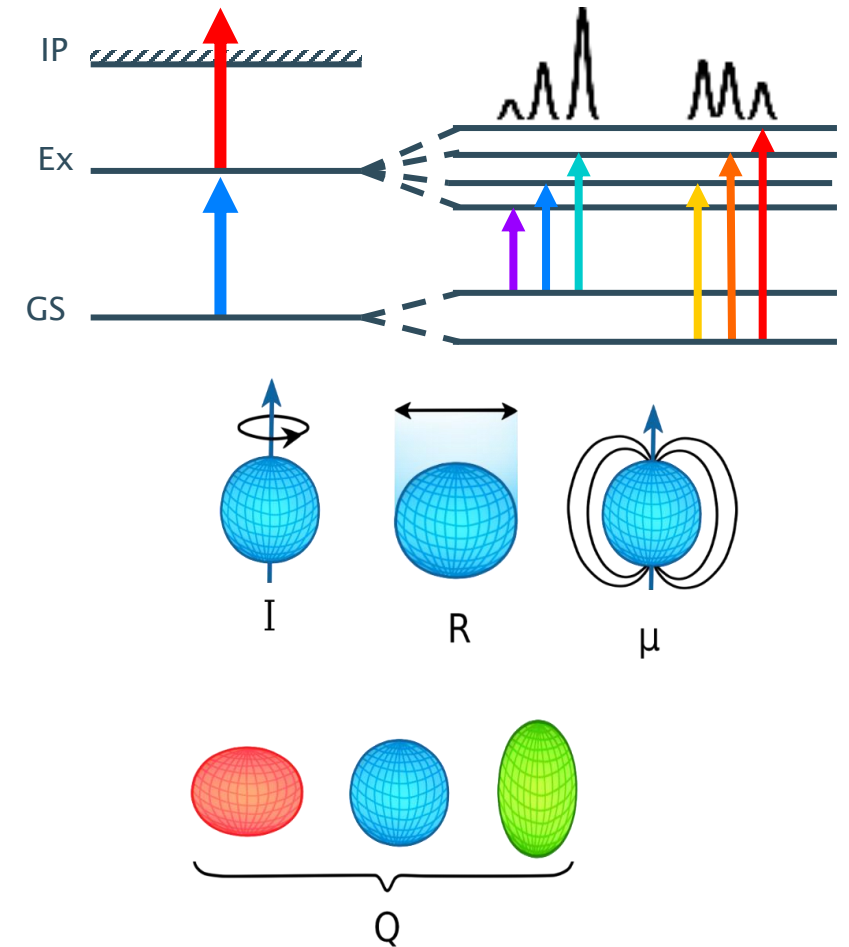


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- Some nuance:
  - / First-order expansion does not stop at 2<sup>nd</sup> multipole order!
  - / Second-order perturbation theory: hyperfine mixing
  - / Hyperfine anomaly



# Laser spectroscopy and hyperfine structure

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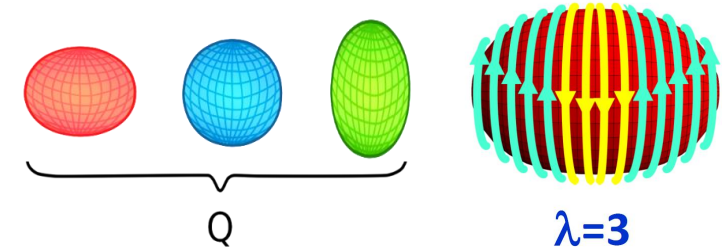
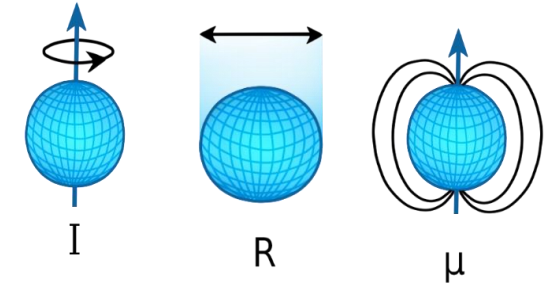
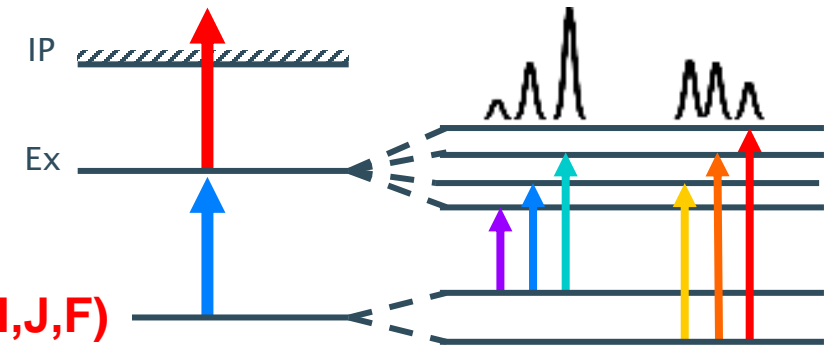
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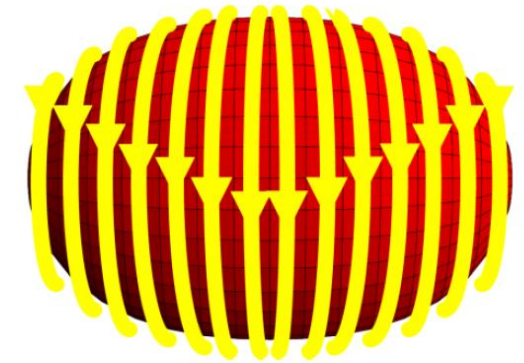
/ Hyperfine anomaly

$$C = -\Omega \langle JJ | T_3^{(e)} | JJ \rangle$$



# Magnetic properties of the nucleus

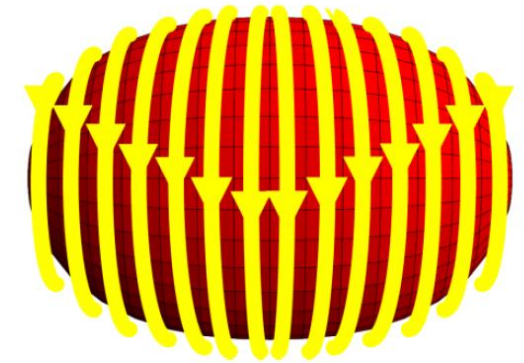
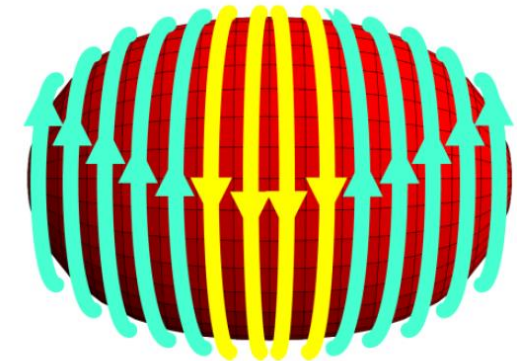
- Magnetic dipole moment:
  - Exceedingly useful probe of nuclear configuration
    - i.e. what orbit is a valence nucleon in?
    - Long chains of radioactive isotopes: study evolution of nuclear properties
  - Comparison with nuclear theory informs on wavefunction composition, purity, ... and thus can test interactions, model spaces, ...
  - **Over 800 measurements** of magnetic dipole moments to date!!



$$\lambda=1$$

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- Magnetic octupole moment:
  - A measure of 'deviation from spherical symmetry' of the distribution of magnetization
  - So far... **only 20 measurements**.

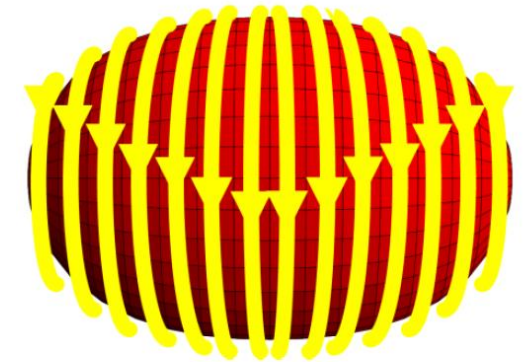
 $\lambda=1$  $\lambda=3$

# Magnetic properties of the nucleus

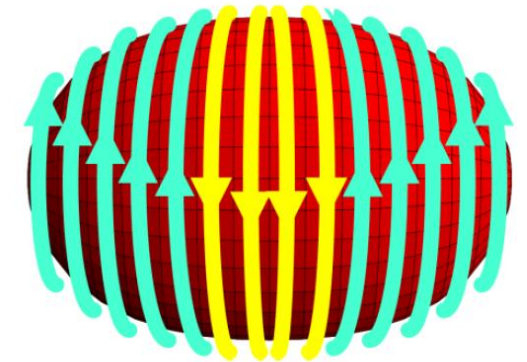
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1A																18 VIIIA															
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$$\lambda=1$$



$$\lambda=3$$



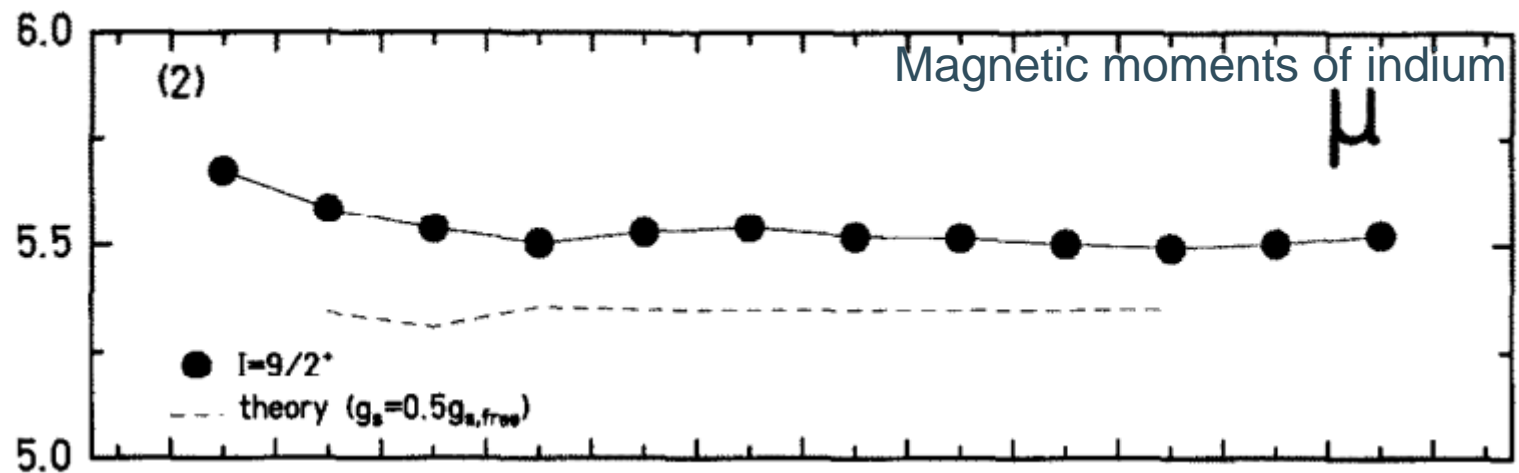
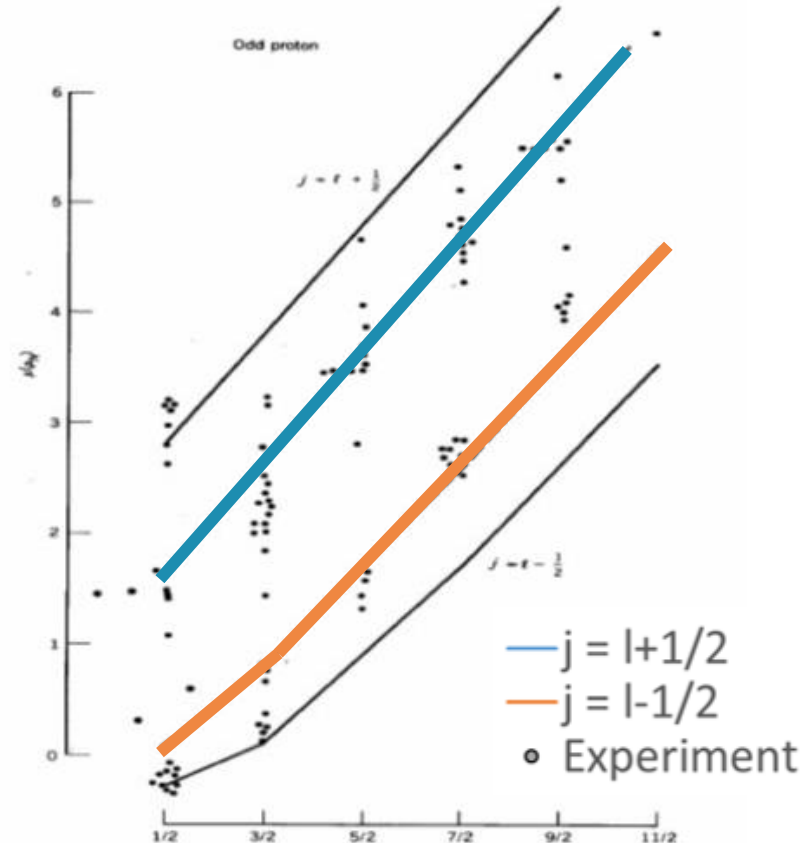
# Dipole vs octupole

Interpretation from theory?

Single-particle estimates

= moment is determined by the valence nucleon

- Global agreement with effective single-particle model
- Example: longer indium chain with near-constant magnetic moments  
= textbook case?



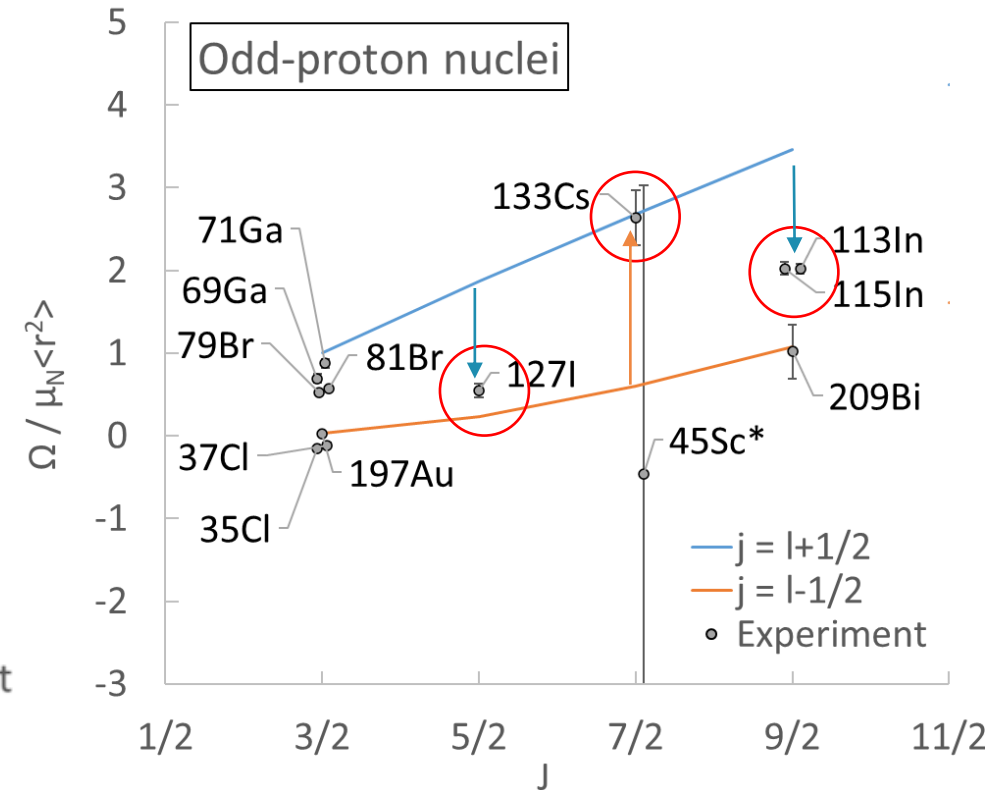
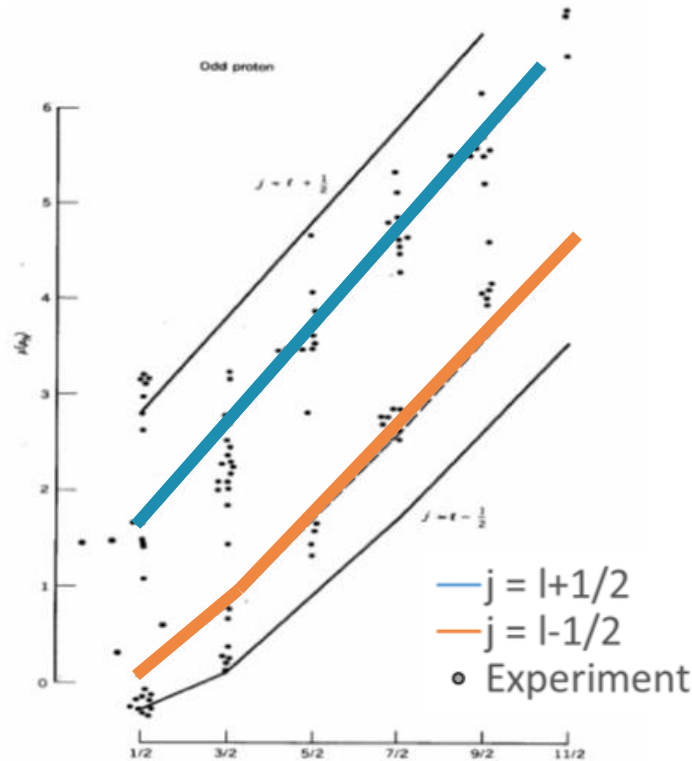


# Dipole vs octupole

Interpretation from theory?

Single-particle estimates

- Single-particle picture works quite well for dipole moments
- Octupole moments: not so much
  - Higher sensitivity to configuration mixing?

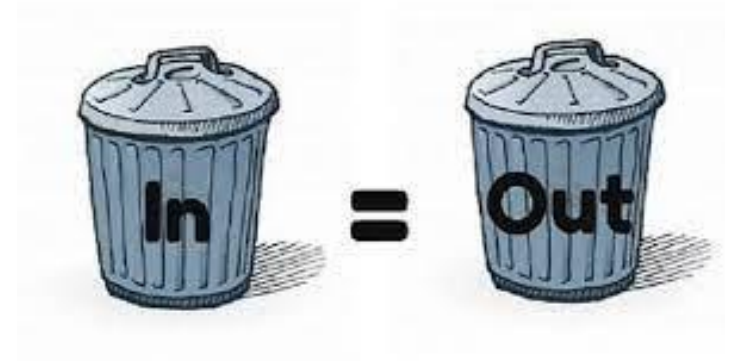


$$\Omega / \mu_N \langle r^2 \rangle = \frac{3}{2} \frac{2I - 1}{(2I + 4)(2I + 2)} \times \begin{cases} (I + 2) \left[ \left( I - \frac{3}{2} \right) g_l + g_s \right], & I = l + \frac{1}{2} \\ (I - 1) \left[ \left( I + \frac{5}{2} \right) g_l - g_s \right], & I = l - \frac{1}{2} \end{cases}$$

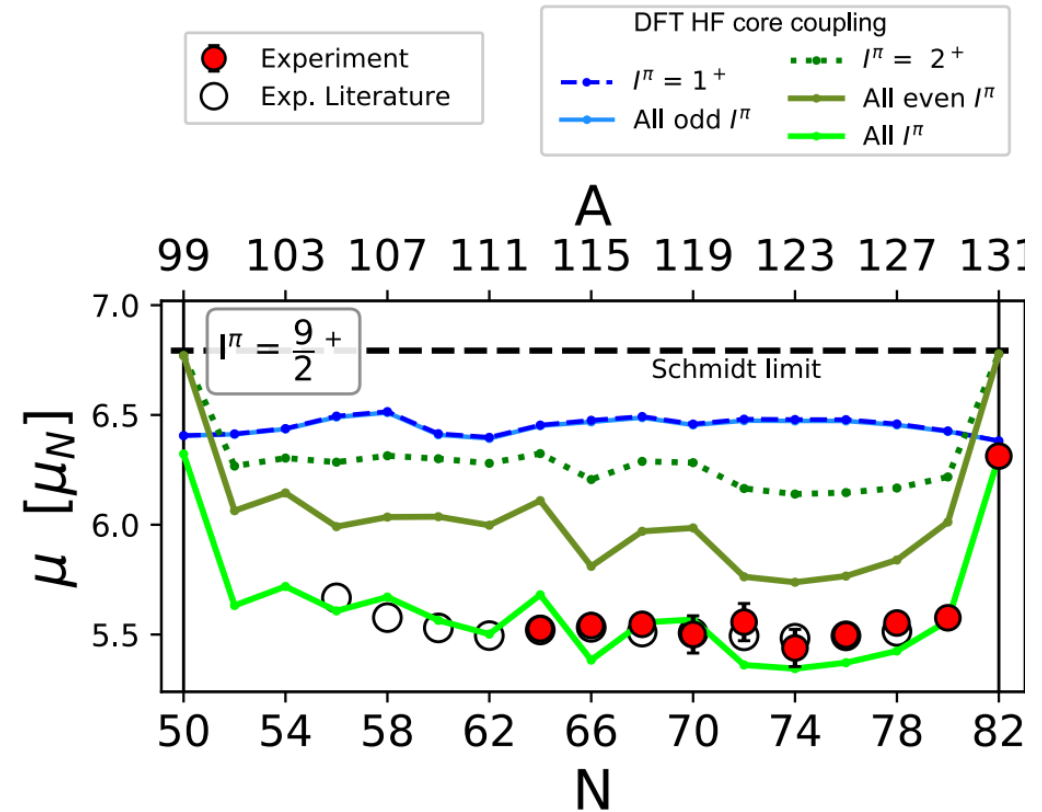
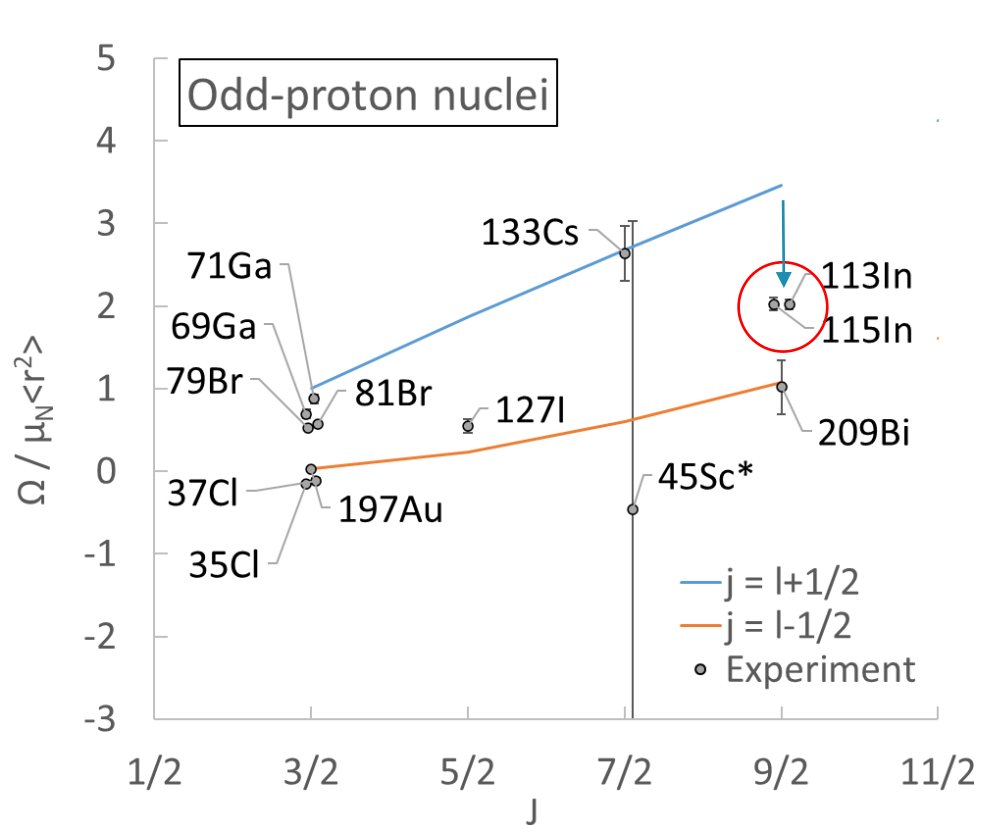
# Some caution...

- Systematic effects in the measurements
  - Measurements in lit. are all precise, but are they *accurate*?
- Atomic structure
  - C/Omega
  - Second-order hyperfine structure

Critical survey of literature through the lens of current theory would be of interest.

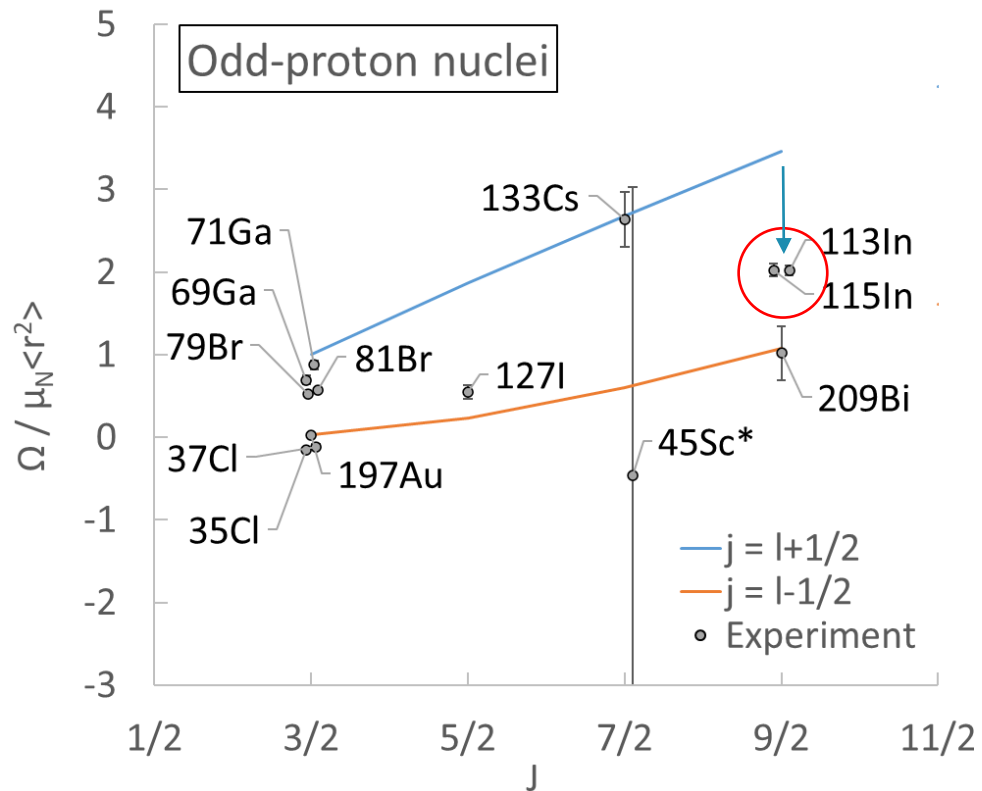


# A textbook case of single-particle structure?

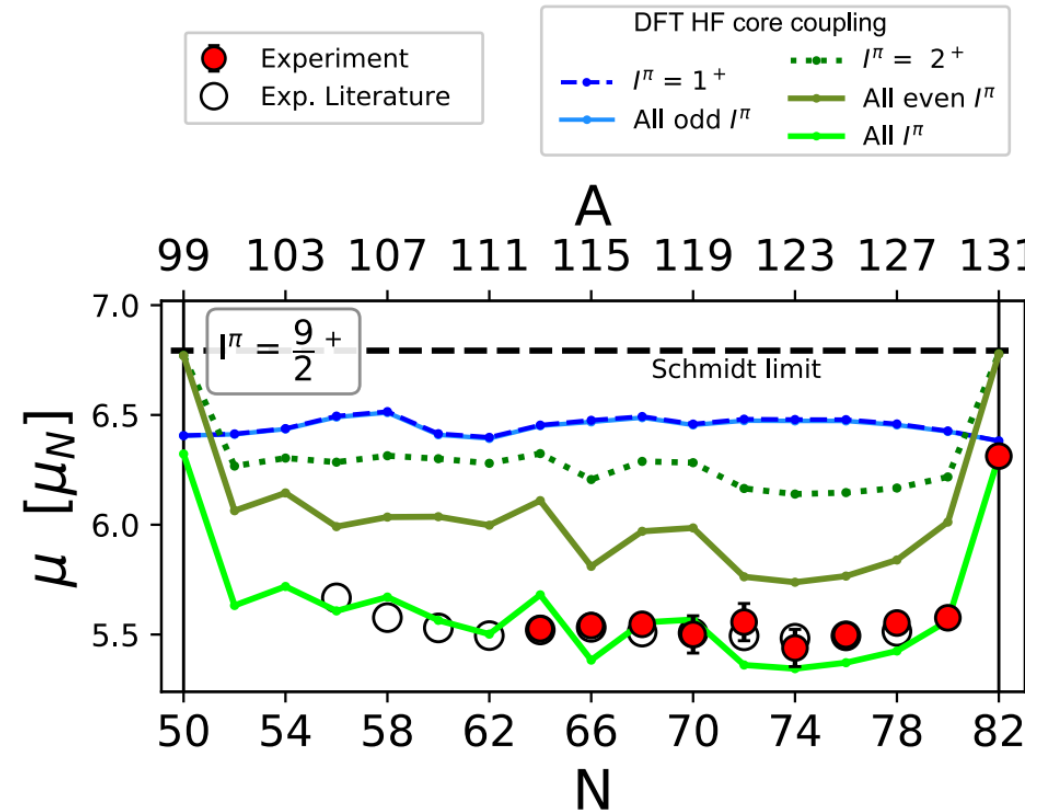


A. Vernon et al, accepted to Nature

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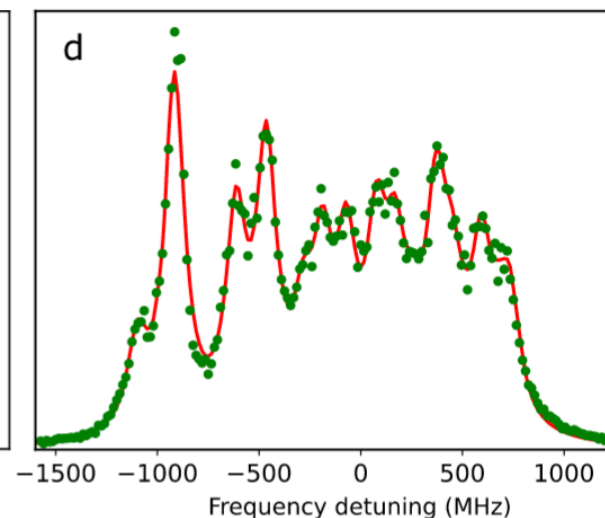
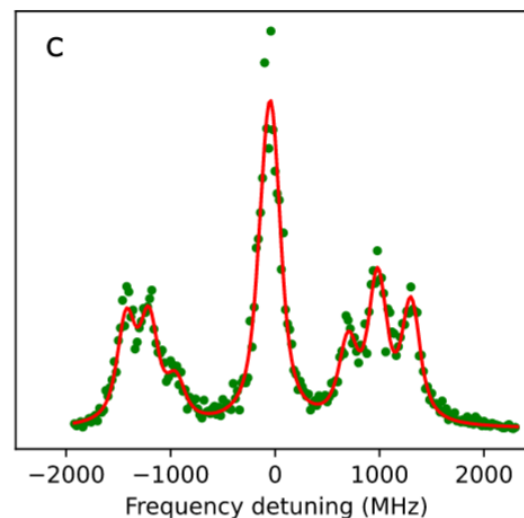
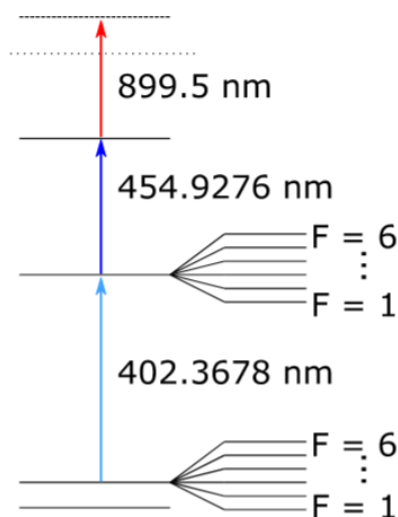
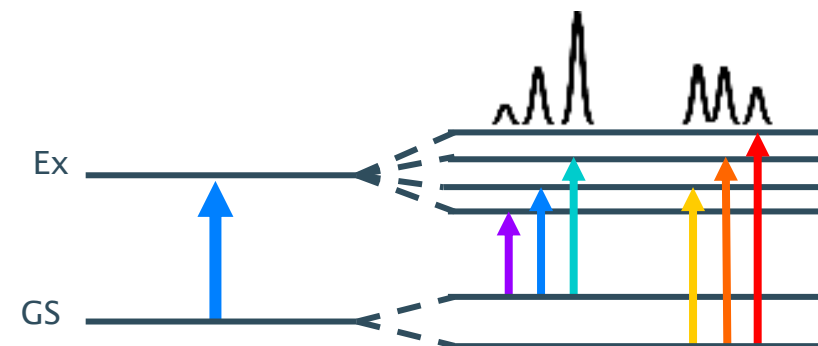
**There is complimentary value in exploring this next-order magnetic observable**



A. Vernon et al, accepted to Nature

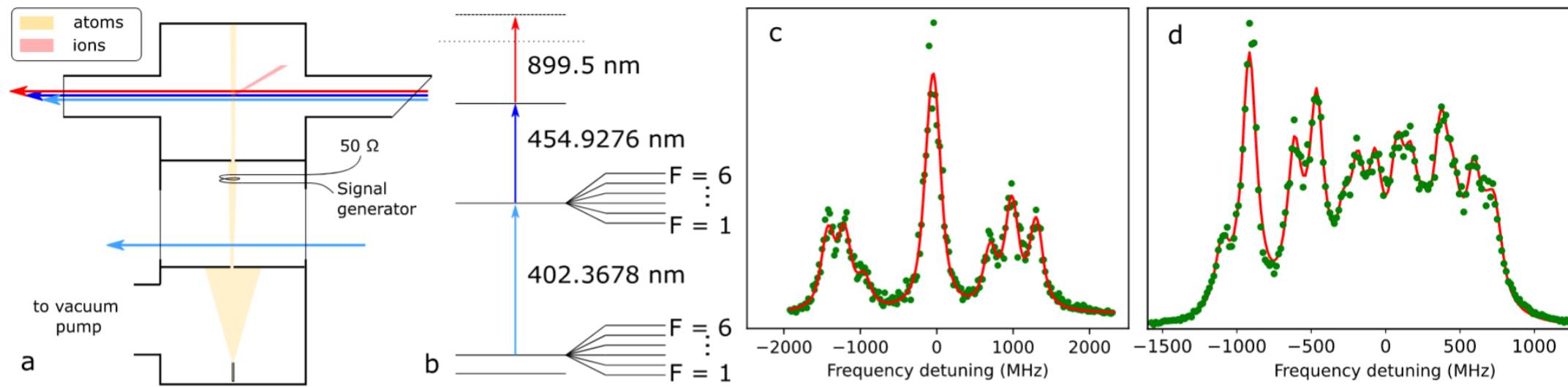
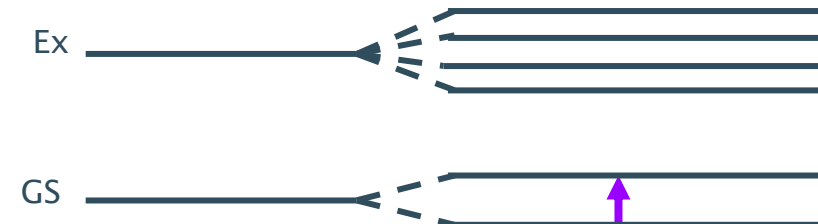
# Moving forwards: highly precise and accurate hyperfine structure measurements

- ‘Standard’ laser spectroscopy is not sufficiently precise
- Example  $^{45}\text{Sc}$ : Ca core + 1 proton
- Linewidth: MHz level (limited by lifetime of excited state)

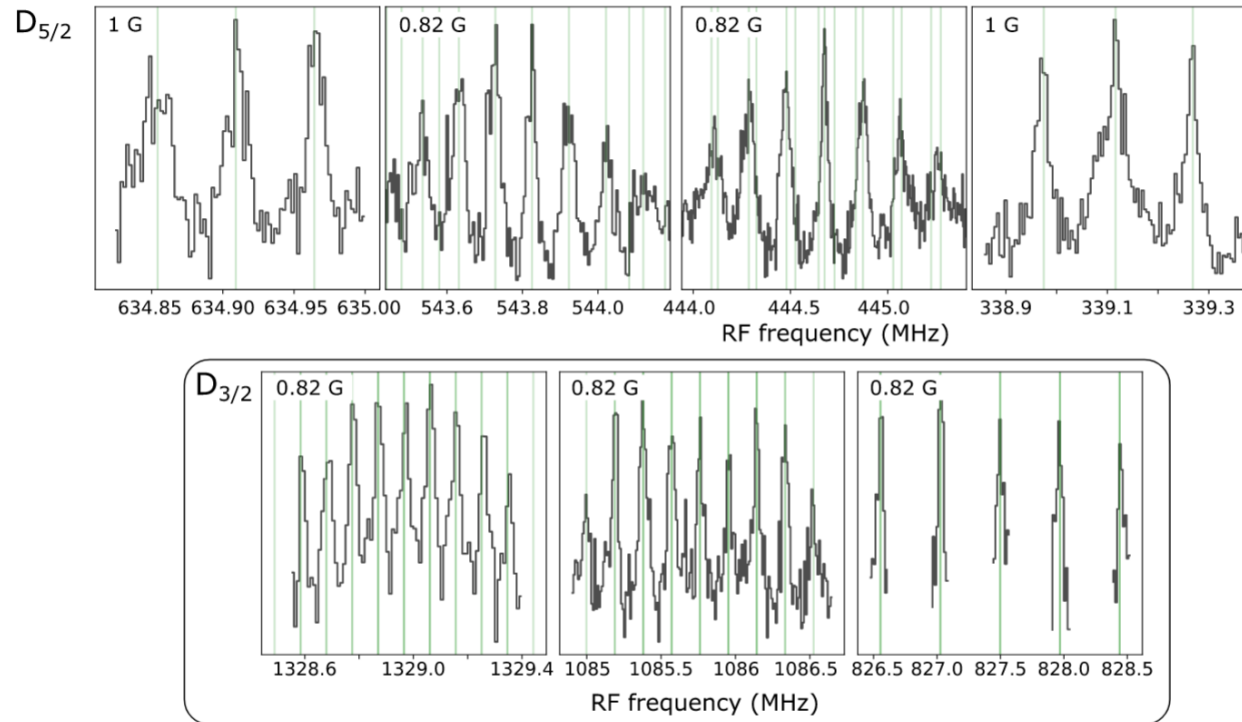


# Moving forwards: highly precise and accurate hyperfine structure measurements

- 'Standard' laser spectroscopy is not sufficiently precise
- Example  $^{45}\text{Sc}$ : Ca core + 1 proton
- Linewidth: MHz level (limited by lifetime of excited state)
- Direct excitations within hyperfine manifold with rf/microwave required
  - Linewidth  $\sim 1 / \text{interaction time}$



## RF scans: (F, m) -> (F', m) resonances



20 kHz width

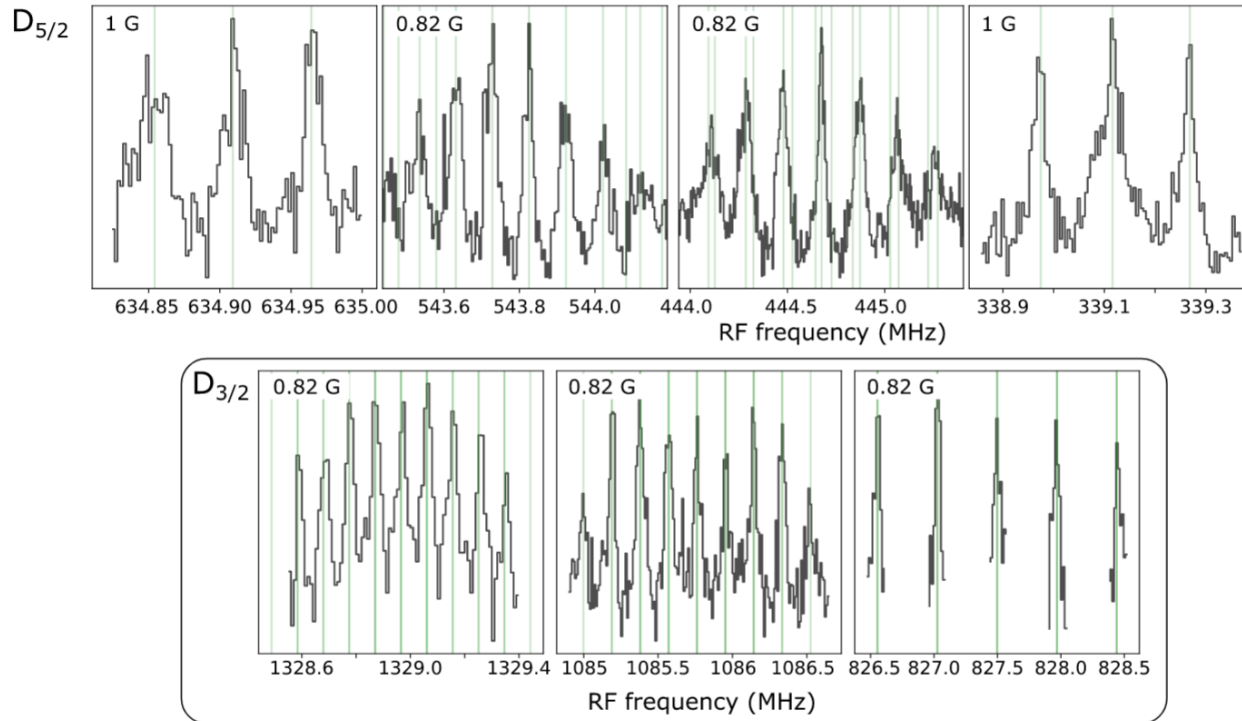
Many resonances due to Zeeman splitting

Hyperfine constants ~ 100x more precise

		Expt. this work	
		Uncorrected	Corrected
$D_{3/2}$	A [MHz]	269.55817(5)	269.55844(7)[3]
	B [MHz]	-26.3531(9)	-26.3596(5)[5]
	C [kHz]	-0.010(22)	0.039(28)[2]
	$\Omega$ [ $\mu_N$ b]	0.17(38)	-0.68(49)[6]
$D_{5/2}$	A [MHz]	109.03275(7)	109.03297(5)[3]
	B [MHz]	-37.3954(12)	-37.3745(8)[15]
	C [kHz]	0.31(8)	-0.062(59)[17]
	$\Omega$ [ $\mu_N$ b]	-1.92(51)	0.39(37)[11]

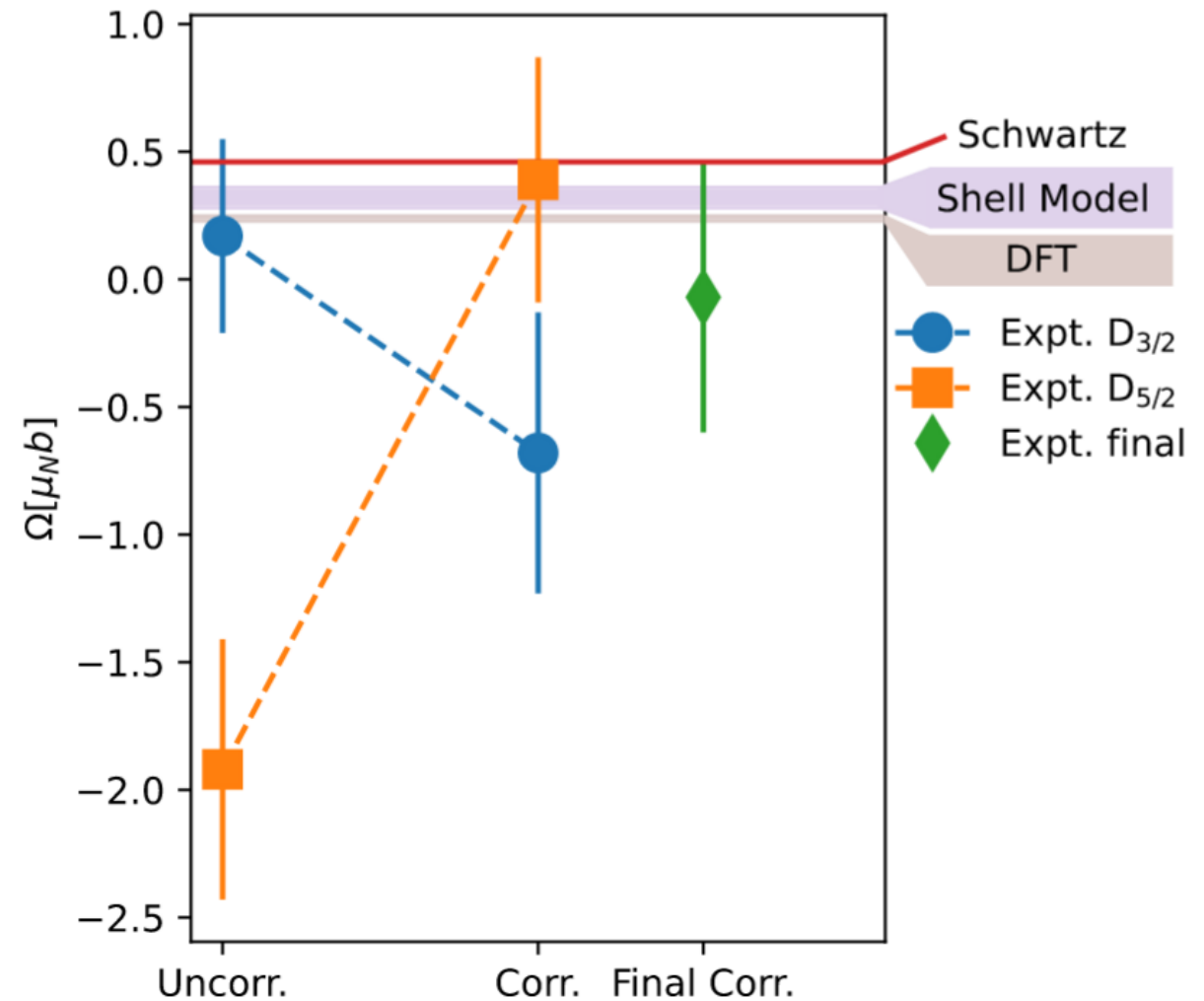


## RF scans: $(F, m) \rightarrow (F', m)$ resonances



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DFT calculations by Jacek (talk Friday)

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  - Room to explore stable elements!

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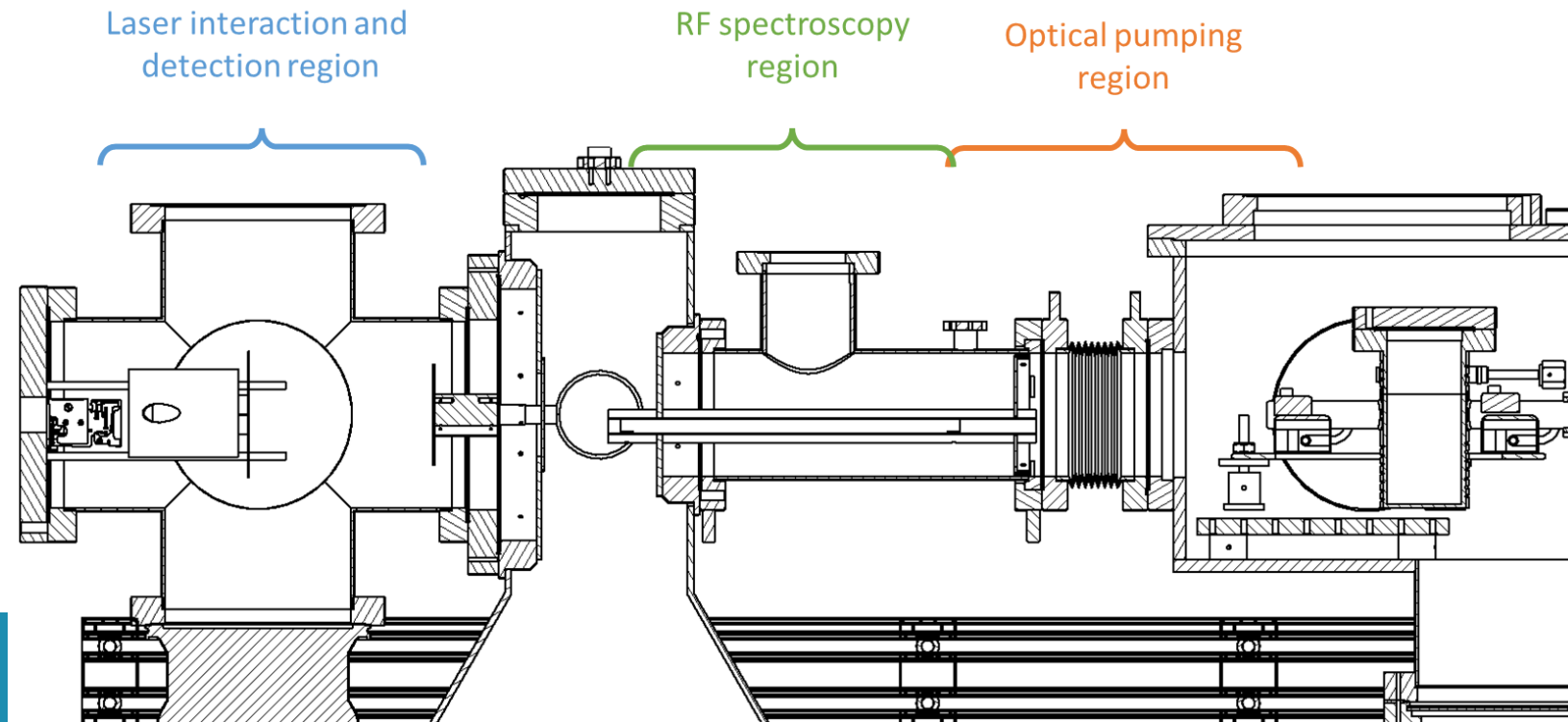
# Future perspectives

- ‘Conventional’ laser double-resonance experiments
  - Room to explore stable elements!
  - Implementations optimized for radioactive species (e.g. in collinear geometry)



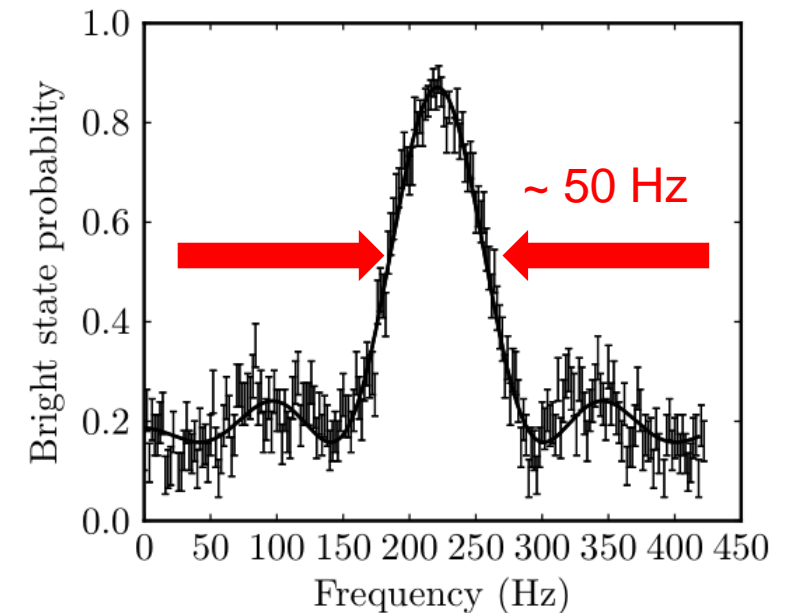
**MARIE CURIE ACTIONS**

Proof-of-concept experiments  
coming soon (?) @ JYU ACCLAB



# Future perspectives

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  - Room to explore stable elements!
  - Implementations optimized for radioactive species (e.g. in collinear geometry)
- Experiments in atom and ion traps
  - Precision  $\sim 1/\text{interaction time}$ ...
  - Demonstration with stable barium

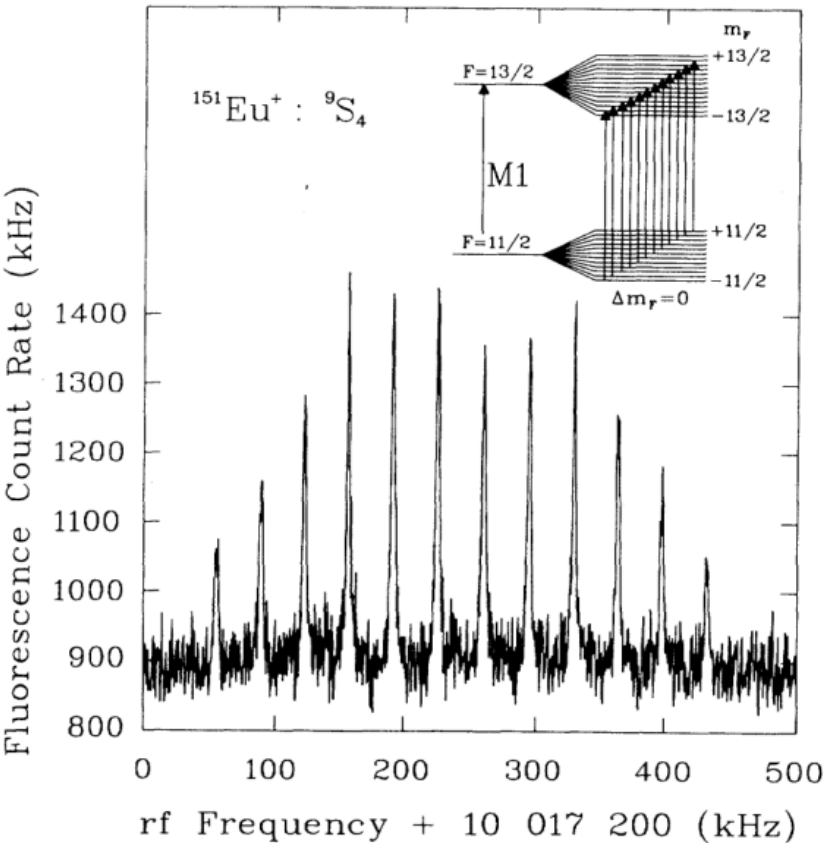


$$\Omega \left( {}^{137}\text{Ba}_{\text{D}_{3/2}}^+ \right) = 0.05057(54) \text{ } (\mu_{\text{N}} \times \text{b}),$$
$$\Omega \left( {}^{137}\text{Ba}_{\text{D}_{5/2}}^+ \right) = 0.0496(37) \text{ } (\mu_{\text{N}} \times \text{b}),$$

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- Experiments in atom and ion traps
  - Precision ~ 1/interaction time...
  - Demonstration with stable barium, europium

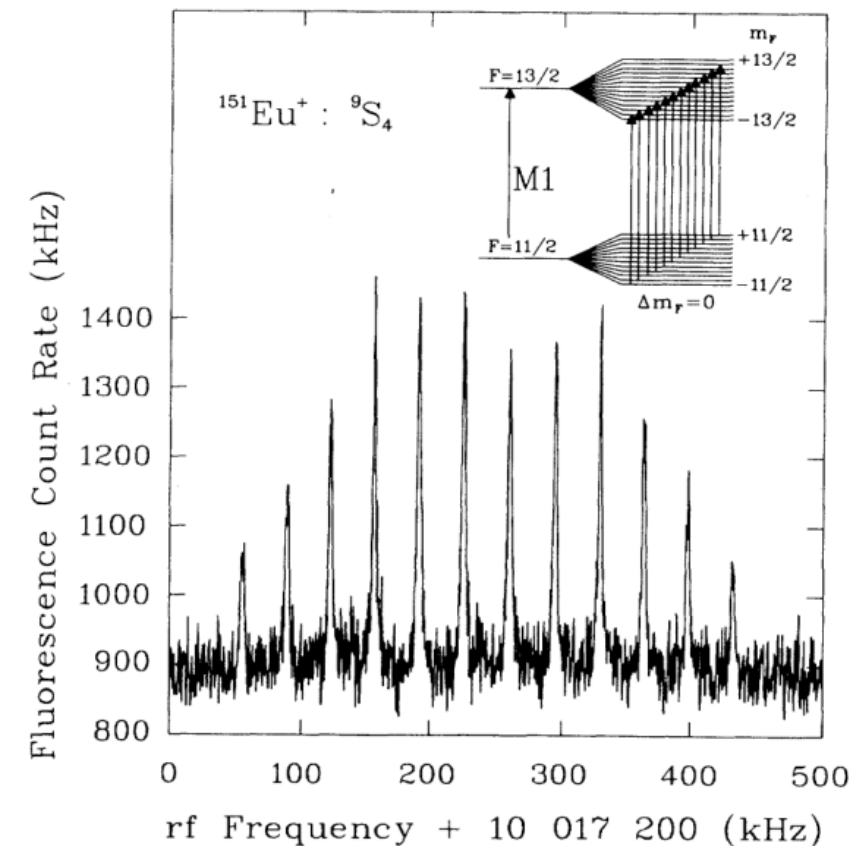
Fit No.	hfs constant	$^{151}\text{Eu}^+$ (Hz)	$^{153}\text{Eu}^+$ (Hz)	$^{151}\text{Eu}^+; ^{153}\text{Eu}^+$
VI	<i>A</i>	1 540 297 394(13)	684 565 993(9)	2.250 034 927(35)
	<i>B</i>	−660 862(231)	−1 752 868(84)	0.377 02(13)
	<i>C</i>	26(23)	3(7)	9(22)
	<i>D</i>	−6(5)	−5(2)	1.2(1.1)



# Future perspectives

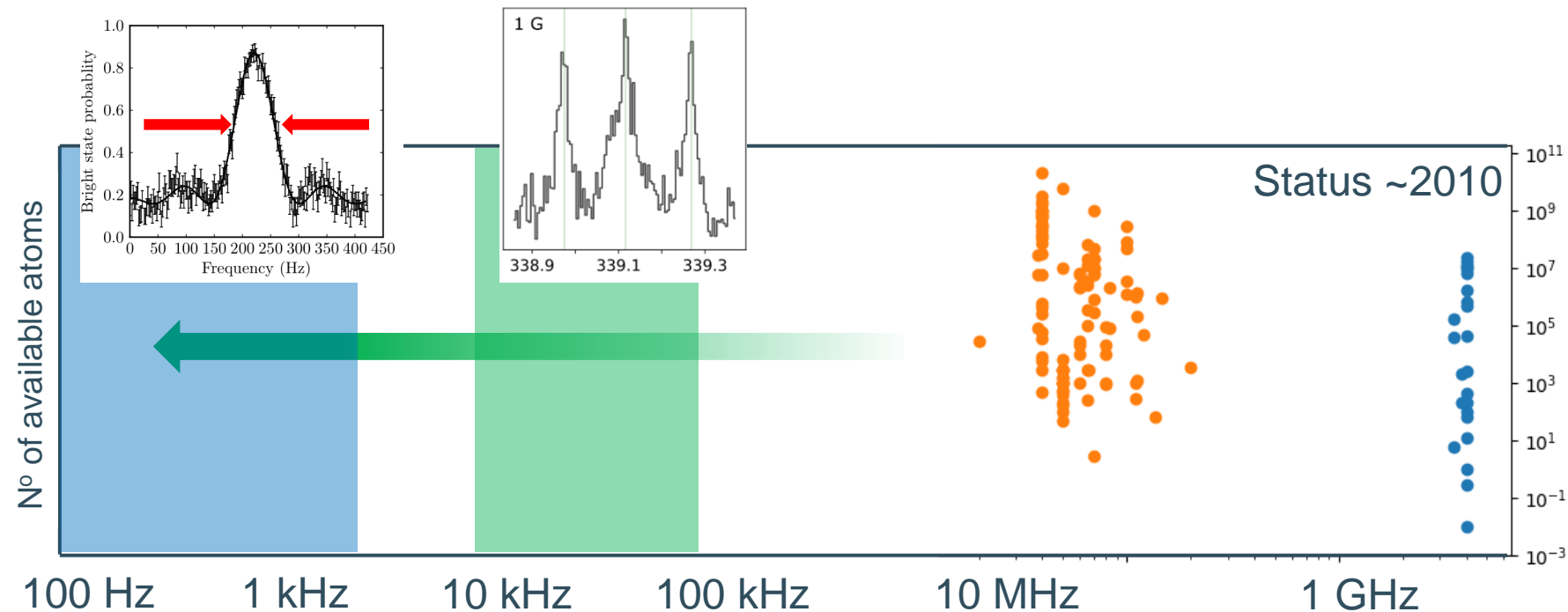
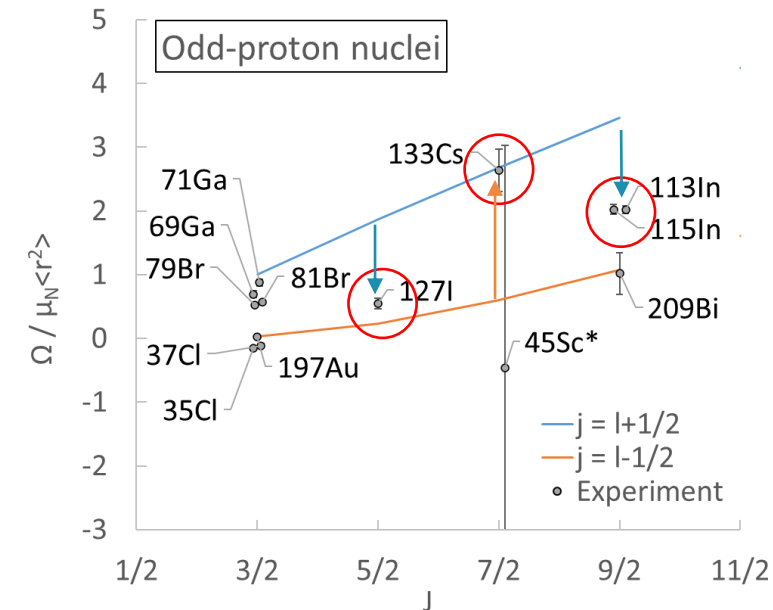
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Fit No.	hfs constant	<sup>151</sup> Eu <sup>+</sup> (Hz)	<sup>153</sup> Eu <sup>+</sup> (Hz)	<sup>151</sup> Eu <sup>+</sup> ; <sup>153</sup> Eu <sup>+</sup>
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	B	−660 862(231)	−1 752 868(84)	0.377 02(13)
	C	26(23)	3(7)	9(22)
	D	−6(5)	−5(2)	1.2(1.1)



# In conclusion...

- Higher-order moments have only scarcely been explored
- Significant potential for new information on the nucleus
- Lots of experimental opportunity
- Nuclear theory ready for the challenge(?)
- Atomic theory required for accurate results and to find most sensitive states to probe





# Atomic theory is warming up...

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## Appraising nuclear-octupole-moment contributions to the hyperfine structures in $^{211}\text{Fr}$

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*(Received 10 July 2015; published 3 November 2015)*

## Hyperfine structure of $^{173}\text{Yb}^+$ : Toward resolving the $^{173}\text{Yb}$ nuclear-octupole-moment puzzle

Di Xiao<sup>1</sup>, Jiguang Li (李冀光)<sup>2</sup>, Wesley C. Campbell,<sup>3</sup> Thomas Dellaert<sup>3</sup>, Patrick McMillin<sup>3</sup>, Anthony Ransford,<sup>3</sup> Conrad Roman,<sup>3</sup> and Andrei Derevianko<sup>1,\*</sup>

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<sup>2</sup>*Institute of Applied Physics and Computational Mathematics, Beijing 100088, China*

## Hyperfine structure of the metastable $^3P_2$ state of alkaline-earth-metal atoms as an accurate probe of nuclear magnetic octupole moments

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## Relativistic coupled-cluster-theory analysis of the hyperfine interaction of $\text{Ra}^+$ isotopes

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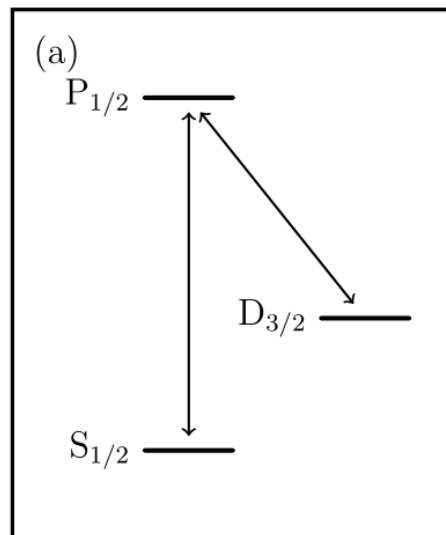
*(Dated: March 16, 2021)*

Hyperfine-structure constants of odd  $\text{Ra}^+$  due to the interactions of nuclear magnetic dipole, electric quadrupole, and magnetic octupole moments with the electrons are investigated in the frame-

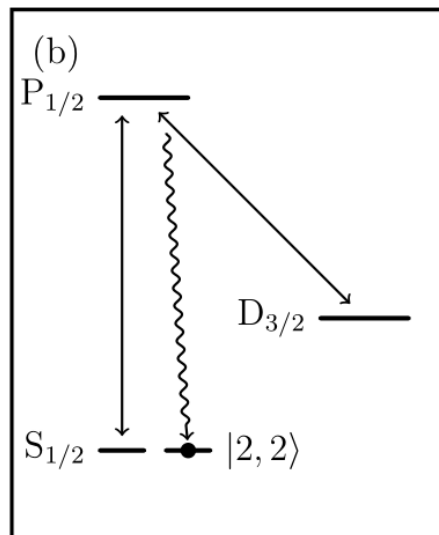
# Thanks for your attention!



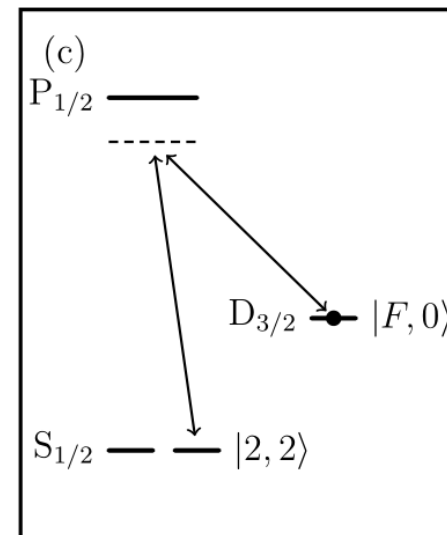
Doppler cooling



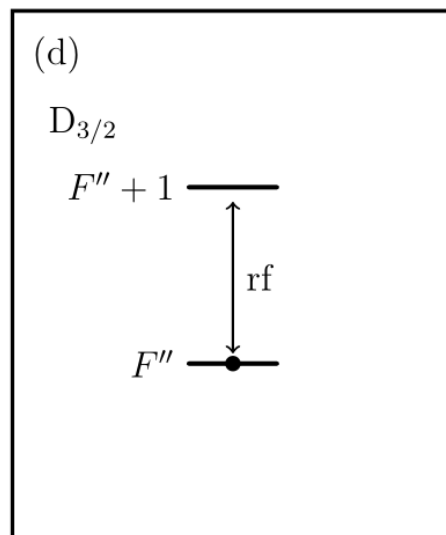
Optical pumping



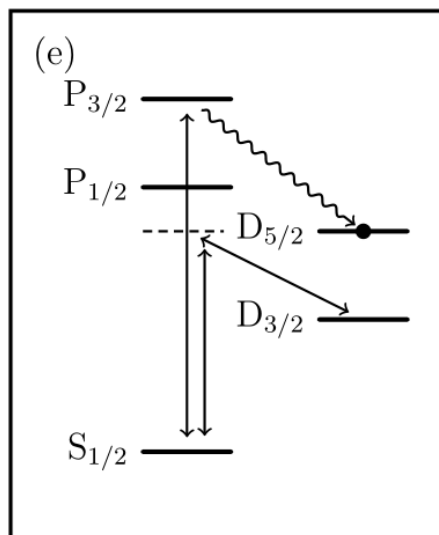
Raman transition



Rf transition



Detection shelving



Fluorescence detection

