Nuclear structure effects in light atomic systems

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Nuclear physics from atomic spectroscopy
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Nuclear structure effects in atomic spectra

Simple picture:

- $\rho_E(r)$ and $\rho_M(r)$: the charge and the magnetic moment distribution within the nucleus.
- $G_E(q^2)$, $G_M(q^2)$: corresponding Fourier transform,
- one solves Dirac equation in the modified Coulomb potential $G_E(q^2)/q^2$ and observes energy shift due to finite nuclear size δE
- $\delta E_{\rm fs} = \delta E^{(4)} + \delta E^{(5)} + \delta E^{(6)}$
- $\delta E^{(4)} = \frac{2\,\pi}{3}\,(Z\,\alpha)\,\phi^2(0)\,r_N^2$, where $r_N^2 = \int d^3r\; r^2\,\rho_C(r)$
- $\delta E^{(5)} = -\frac{\pi}{3}\,\phi^2(0)\,(Z\,\alpha)^2\,m\,r_Z^3$, where $r_Z^3 = \int d^3r_1\,d^3r_2\;\rho(r_1)\,\rho(r_2)\,|\vec{r}_1 - \vec{r}_2|^3$
- $\delta E^{(6)} = \ldots$ three-photon exchange

Nuclear structure effects in atomic spectra

More accurate picture:

- $\delta E^{(5)} = \delta E^{(5)}_{\rm pol} + \delta E^{(5)}_{\rm nucleons} \to$ two-photon exchange
- $\delta E^{(5)}_{\rm nucleons} = -\frac{\pi}{3}\,\alpha^2\,\phi^2(0)\,m_e\left[Z\,R^3_{pF} + (A-Z)\,R^3_{nF} \;+ \sum_{i,j=1}^{Z}\langle\phi_N||\vec{R}_i - \vec{R}_j|^3|\phi_N\rangle\right]$
- Friar radii: $R_{pF} = 1.947(75)$ fm, $R_{nF} = 1.43(16)$ fm
- $E^{(5)}_{\rm pol} = -\alpha^2\,\phi^2(0)\,\frac{2}{3}\,m_e\left\langle\phi_N\middle|\vec{d}\,\frac{1}{H_N-E_N}\left[\frac{19}{6} + 5\,\ln\frac{2\,(H_N-E_N)}{m}\right]\vec{d}\middle|\phi_N\right\rangle$ (electronic)
- $E^{(5)}_{\rm pol} = -\frac{4\,\pi\,\alpha^2}{3}\,\phi^2(0)\left\langle\phi_N\middle|\vec{d}\sqrt{\frac{2\,m}{H_N-E_N}}\,\vec{d}\middle|\phi_N\right\rangle + \ldots$ (muonic)
- $\delta E^{(6)}$ not yet calculated, only the elastic part

Nuclear physics from atomic spectroscopy

- Measurements of atomic levels can be very accurate, Garching (2010)

  - $\nu(1S - 2S)_{\rm H} = 2466\,061\,413\,187\,035(10)\ \rm Hz$,

  - $\delta\nu = \frac{7}{6}\ Ry\, c\,(Z\,\alpha)^4\, \frac{r_p^2}{\cancel{\lambda}^2} - 95.5\,\mathrm{Hz}[\sim \alpha] - 929\,\mathrm{Hz}[\sim \alpha^2]$
  - the ultimate theoretical predictions are limited by the proton polarizabilities

- Hydrogen ground state hfs $\delta E_{\rm hfs}(H) = 1\,420\,405.751\,768(1)$ kHz,

  - hadronic contribution 33ppm,

  - agreement with $\delta E_{\rm hfs}(\bar{H})$ up to $3 \cdot 10^{-9}$ ASACUSA (2017)

  - comparison to μH hfs ? (Antognini, PSI+ETH)

- Accurate calculations for determination of the nuclear charge radius is possible only for the hydrogenic system

- For other light systems like: He, Li, Be, B only the charge radii differences between isotopes

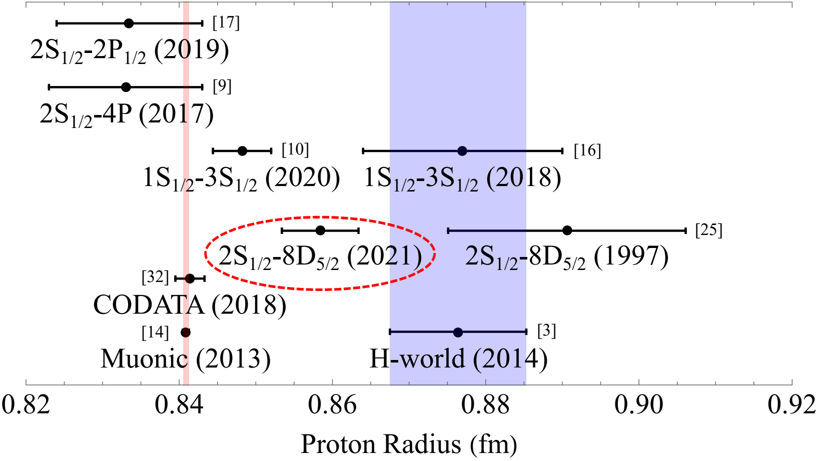
Hydrogen and determination of r_p

- Measurements of transition frequencies can be very accurate, Garching 2010: $\nu(1S-2S)_{\rm H} = 2466\,061\,413\,187\,035(10)\ {\rm Hz}$
- but we need two transitions to determine two unknowns: R_∞ and r_p
- other transitions measured in hydrogen: $2S-2P$, $2S-3S$, $2S-4P$
- hydrogenic systems can be calculated very precisely
  - Dirac equation and finite nuclear mass effects
  - QED radiative corrections
  - nuclear polarizability: limits theory for μH

  up to the finite nuclear size correction: $\delta E = \frac{2\,\pi}{3}\,(Z\,\alpha)\,\phi^2(0)\,\langle r_p^2\rangle$
- for example:

  $r_D^2(\mu D) - r_p^2(\mu H) = 3.817\,47(346)$ fm^2 (Kalinowski 2019)
  $r_D^2(eD) - r_p^2(eH) = 3.820\,70(31)$ fm^2

Proton charge radius r_p : current status



from PRL **128**, 023001 (2022)

μD($2S$) hyperfine splitting

$$E_{\rm hfs}(\rm exp) = 6.2747(70)_{\rm stat}(20)_{\rm syst}\ \rm meV$$
$$E_{\rm hfs}(\rm point) = 6.17815(20)\ \rm meV$$
$$\delta E_{\rm nucl} = E_{\rm hfs}(\rm exp) - E_{\rm hfs}(\rm point) = 0.0966(73)\ \rm meV$$

- The Bohr-Weisskopf effect, charge and magnetic moment distribution within nucleus gives a correction with an opposite sign

$$\delta E_{\rm nucl,BW} = -0.1177(3)\ \rm meV$$

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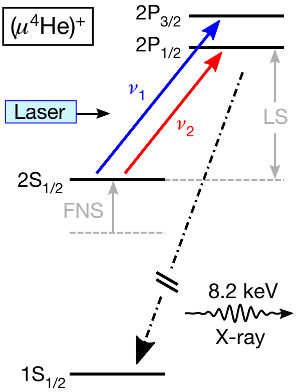
- Nuclear polarizability effects are very important

$$\delta E_{\rm nucl,theo} = 0.0383(86)\ \rm meV$$

in 5 σ disagreement with the experimental value

- lack of good understanding of nuclear structure effects to hfs in muonic atoms

$\mu\,^4$He determination of α-particle charge radius



Nature **589**, 527 (2021): $r_\alpha = 1.67824(13)_{\text{exp}}(82)_{\text{theo}}$ fm
in agreement with elastic electron scattering $r_\alpha = 1.681(4)$ fm

^{4}He atom: theory versus experiments

Very recent measurement of 2^3S_1 ionization energy by F. Merkt *et al.* 2021, and very recent theory V. Patkos *et al.*, Phys. Rev. A 2021.

Table III. Comparison of experimental and theoretical values [12, 47] of the ionization energies of the $2\,^3S_1$, $2\,^3P$ (centroid), $3\,^3D_1$ and $3\,^1D_2$ states in ^{4}He (in MHz) obtained by combining the $2\,^1S_0$ ionization energy with the transition frequencies from Refs. [13, 15, 16, 25, 30, 31].

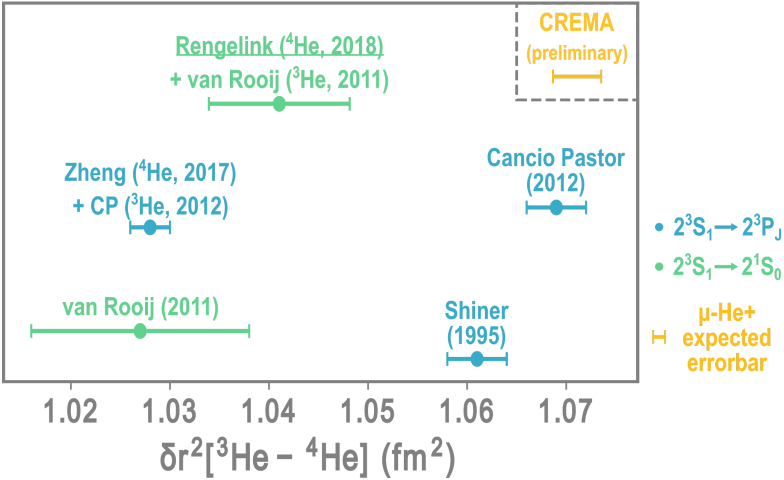
	Experiment	Reference	Theory	Reference	$\Delta E_{I,\text{exp.}-\text{calc.}}$
$2\,^3S_1$	1 152 842 742.637(32)	[13]	1 152 842 742.231(52)	[12]	0.406(61)
$2\,^3P$	876 106 247.017(32)	[13, 15, 30, 31]	876 106 246.611(16)	[12]	0.406(36)
$3\,^3D_1$	366 018 892.635(65)	[13, 25]	366 018 892.691(23)	[47]	-0.056(69)
$3\,^1D_2$	365 917 748.688(34)	[16]	365 917 748.661(19)	[47]	0.027(38)

but a very good agreement with $2^3S_1 - 2^3P$ transition frequency with the charge radius from μHe Lamb shift

$$E(2^3S - 2^3P)_{\text{theo}} = 276\,736\,495.620\,(54)\ \text{MHz}$$

$$E(2^3S - 2^3P)_{\text{exp}} = 276\,736\,495.600\,0\,(14)\ \text{MHz, Zheng } et\ al\ 2017.$$

^{4}He - ^{3}He isotope shift of nuclear charge radii difference



picture by Youri van der Werf

^{6}Li-^{7}Li isotope shift and the charge radii diff.

$$\delta_{\rm fs} E = \frac{2\,\pi\,Z\,\alpha}{3}\,\left\langle\sum_a \delta^3(r_a)\right\rangle\,\langle r^2\rangle$$

$$\delta r^2 = r^2(^6\mathrm{Li}) - r^2(^7\mathrm{Li}) = \begin{cases} 0.705(3)\ \mathrm{fm}^2 \\ \qquad 2P_{1/2} - 2S_{1/2},\ \text{NIST (2013)} \\ \\ 0.700(9)\ \mathrm{fm}^2 \\ \qquad 2P_{3/2} - 2S_{1/2},\ \text{NIST (2013)} \\ \\ 0.731(22)\ \mathrm{fm}^2 \\ \qquad 3S_{1/2} - 2S_{1/2},\ \text{Nörtershäuser } \textit{et al} \text{ (2011)} \end{cases}$$

Li: ground state hyperfine structure

Fermi contact interaction

$$H_{\rm hfs} = \frac{2\, g_N\, Z\, \alpha}{3\, m\, M} \sum_a \vec{I} \cdot \vec{\sigma}_a\, \pi\, \delta^3(r_a)\,.$$

Finite nuclear size effect:

$$H_{\rm size} = -H_{\rm hfs}\, 2\, Z\, \alpha\, m\, r_Z$$

where

$$r_Z = \int d^3r\, d^3r'\, \rho_E(r)\, \rho_M(r')\, |\vec{r} - \vec{r}'|$$

Li: hyperfine structure

$$A = A^{(4)} + \alpha\, A^{(5)} + \alpha^2\, A^{(6)} + \alpha^3\, A^{(7)} + \dots$$

	^{7}Li[MHz]	^{6}Li[MHz]
$A^{(4)}$	401.654 08(21)	152.083 69(11)
$A^{(5)}_{\rm rec}$	−0.004 14	−0.001 80
$A^{(6)}$	0.260 08(2)	0.098 48(1)
$A^{(7)}$	−0.010 2(13)	−0.003 9(5)
$A_{\rm the}$ (point nucleus)	401.899 8(13)	152.176 5(5)
$A_{\rm exp}$	401.752 043 3(5)	152.136 839(2)
$(A_{\rm exp} - A_{\rm the})/A_{\rm exp}$	−368(3) ppm	−261(3) ppm
r_Z	3.25(3) fm	2.30(3) fm
r_E	2.390(30) fm	2.540(28) fm

significant dependence of r_Z on the isotope, confirmed by measurements in Li^+

Expected and planned measurements

- $\mu\,^3$He Lamb shift, PSI
- $He^+(1S - 2S)$ Garching and Amsterdam
- μH ground state hyperfine splitting, ETH
- $\mu^+ e^-$ ETH
- $\mu - p$ scattering with high sensitivity to r_p, AMBER collaboration at CERN, Na66
- $e - p$ versus $\mu - p$ scattering, MUSE collaboration at PSI