Nuclear structure corrections in muonic atoms

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Sonia Bacca

Image credit: Oak Ridge National Laboratory, US department of energy;
Conceptual art by LeJean Hardin and Andy Sproles

Outline

- Recap of muonic atoms spectroscopy
- Theory and numerical methods
- Results
- Conclusions

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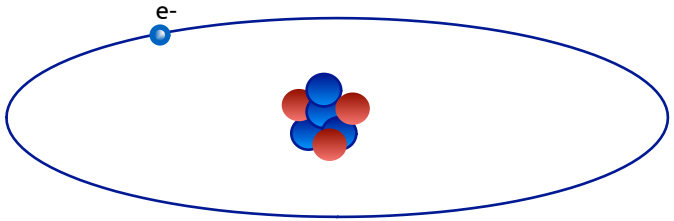
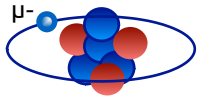
Muonic atoms

Hydrogen-like systems

Ordinary atoms



Muonic atoms



The muon is more sensitive to the nucleus

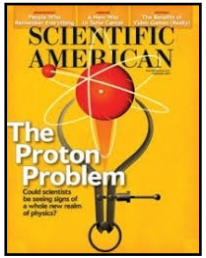
Excellent precision probe for the nucleus

Experimental program at **PSI** of the **CREMA** collaboration

Muonic Hydrogen
- Pohl et al., Nature (2010)
- Antognini et al., Science (2013)

Muonic Deuterium
- Pohl et al., Science (2016)

Muonic Helium-4
- Krauth et al., Nature (2021)



Lamb-shift and charge radius

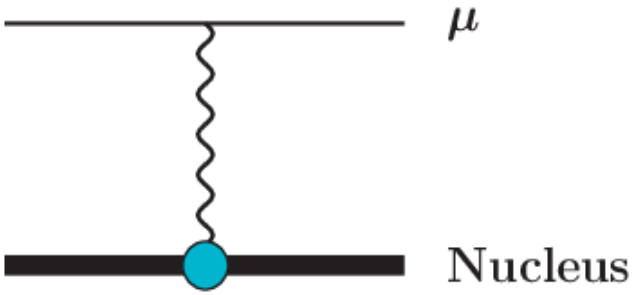
$$\delta_{\mathrm{LS}} = \delta_{\mathrm{QED}} + \mathcal{A}_{\mathrm{OPE}}\, r_c^2 + \delta_{\mathrm{TPE}}$$

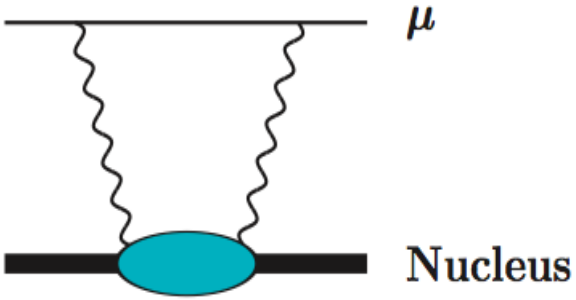
Measured **Extracted**

Lamb-shift and charge radius

$$\delta_{\mathrm{LS}} = \delta_{\mathrm{QED}} + \boxed{\mathcal{A}_{\mathrm{OPE}}\, r_c^2 + \delta_{\mathrm{TPE}}}$$

Nuclear structure corrections

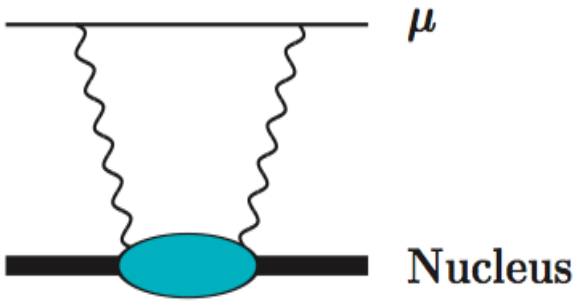




A matter of precision

$$\delta_{\mathrm{LS}} = \delta_{\mathrm{QED}} + \mathcal{A}_{\mathrm{OPE}}\, r_c^2 + \delta_{\mathrm{TPE}}$$

Relative contribution	95 %	4 %	1 %
Uncertainty budget (theo)	5 %	3 %	92 %



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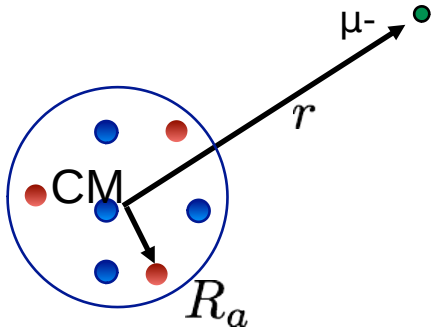
Evaluation of the TPE amplitude

$$\mathrm{H} = \mathrm{H}_N + H_\mu + \Delta V$$

The Nuclear Hamiltonian is included or not depending on whether we look for nuclear sizes or polarizability corrections

$$H_\mu = \frac{\mathbf{p}^2}{2m_r} - \frac{Z\alpha}{r}$$



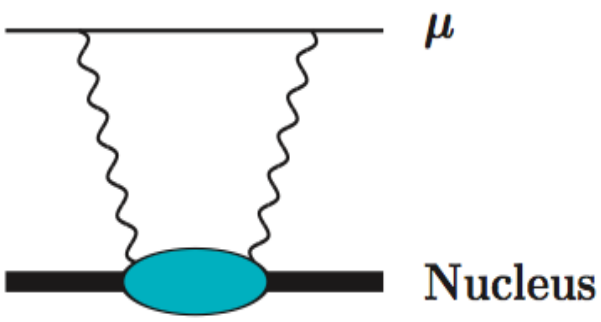
The corrections to the bulk coulomb interactions are included in perturbation theory

$$\Delta V = \sum_a^Z \alpha \left(\frac{1}{r} - \frac{1}{|\mathbf{r} - \mathbf{R}_a|} \right)$$

Evaluation of the TPE amplitude

$$\delta^A_{\rm pol} = \langle N_0 \mu | \, \Delta V \; G \; \Delta V \, | N_0 \mu \rangle$$

$$G = - \sum_{N \neq N_0} \frac{|N\rangle \langle N|}{H_\mu + \omega_N - \epsilon_\mu}$$



Evaluation of the TPE amplitude

- **Non Relativistic corrections**

Non-relativistic states and Green's functions, neglecting Coulomb interactions in intermediate states.

$$\delta^A_{\rm pol} = \sum_{N\neq N_0} \int d^3R\; d^3R'\; \rho^p_N(\mathbf{R})\; W(\mathbf{R},\mathbf{R}',\omega_N)\; \rho^p_N(\mathbf{R}')$$

$W(\mathbf{R},\mathbf{R}',\omega_N)$ **can be expressed as a Taylor expansion**

$$W(\mathbf{R},\mathbf{R}',\omega_N) = \frac{\pi}{6m_r}(Z\alpha)^2\phi^2(0)\left(\frac{2m_r}{\omega_N}\right)^{3/2}\left[\underbrace{\eta^2}_{\text{LO}} - \underbrace{\frac{1}{4}\eta^3}_{\text{NLO}} + \underbrace{\frac{1}{20}\eta^4}_{\text{N2LO}} + \ldots\right]$$

$$\eta = \sqrt{2m_r\omega_N}|\mathbf{R}-\mathbf{R}'| \approx \sqrt{\frac{m_r}{m_N}} \approx 0.33$$

Evaluation of the TPE amplitude

- **Non Relativistic corrections**

LO: **Energy weighted integral**

$$\delta_{\mathrm{D1}}^{(0)} = -\frac{2\pi m_r^3}{9}(Z\alpha)^5 \int_{\omega_{\mathrm{th}}}^{\infty} d\omega \sqrt{\frac{2m_r}{\omega}} S_{D1}(\omega)$$

Of the dipole response function

$$S_{D1}(\omega) = \frac{1}{2J_0+1} \sum_{N \neq N_0} |\langle NJ| |\hat{D}_1| |N_0 J_0\rangle|^2 \delta(\omega - \omega_N)$$

NLO: **Two more corrections related to charge densities**

$$\delta_{\mathrm{Z3}}^{(1)} \quad \delta_{\mathrm{R3}}^{(1)}$$

N2LO: **Three more corrections coming from new response functions**

$$S_{D1D3}(\omega) \qquad S_Q(\omega) \qquad S_{R^2}(\omega)$$

E2 **E0**

Evaluation of the TPE amplitude

- **Non Relativistic corrections**

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E2 E0

- **Coulomb distortion**
- **Relativistic corrections**
- **Nucleon finite-size effects**

Ab-initio Nuclear Theory

Ab-initio methods: Solutions of the time-independent Schrödinger equation for the nuclear states

$$\widehat{H}\Psi = E\Psi$$

With controlled approximations.

Nuclear physics approximations come mainly from two sides:

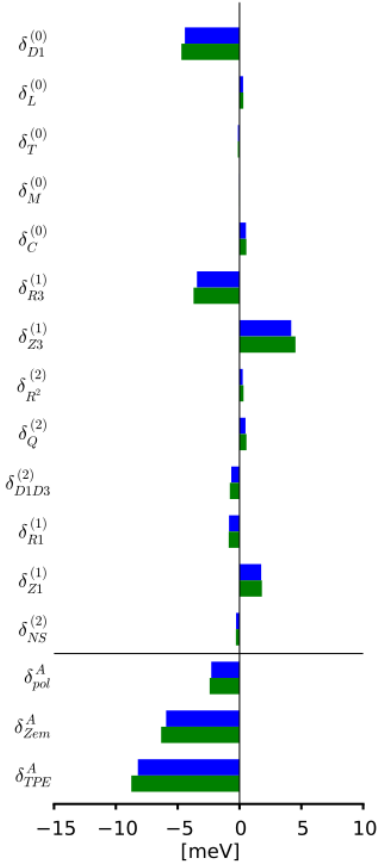
1) Hamiltonian $\widehat{H}$

2) Nuclear wavefunction Ψ **(with the Hyperspherical Harmonics method)**

Previous work on Helium-4

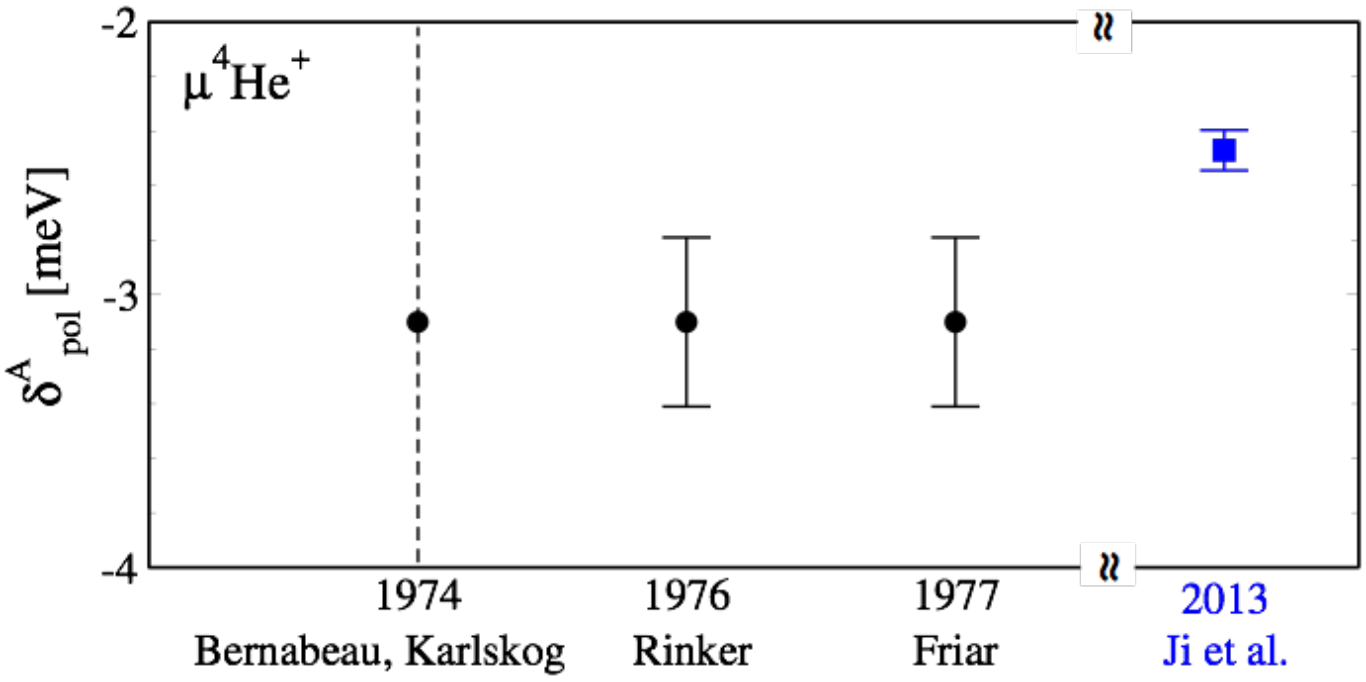
C. Ji, et al. PRL 143402-1 (2013)

C. Ji, et al. J. Phys. G: Nucl. Part. Phys. 45 (2018)



	AV18/UIX	χEFT
$\delta^{(0)}_{D1}$	-4.418	-4.701
$\delta^{(0)}_{L}$	0.289	0.308
$\delta^{(0)}_{T}$	-0.126	-0.134
$\delta^{(0)}_{M}$	0.008	0.006
$\delta^{(0)}_{C}$	0.519	0.553
$\delta^{(1)}_{R3}$	-3.442	-3.717
$\delta^{(1)}_{Z3}$	4.183	4.526
$\delta^{(2)}_{R^2}$	0.259	0.324
$\delta^{(2)}_{Q}$	0.484	0.561
$\delta^{(2)}_{D1D3}$	-0.666	-0.784
$\delta^{(1)}_{R1}$	-0.848	-0.874
$\delta^{(1)}_{Z1}$	1.753	1.811
$\delta^{(2)}_{NS}$	-0.279	-0.293
δ^{A}_{pol}	-2.284	-2.414
δ^{A}_{Zem}	-5.936	-6.337
δ^{A}_{TPE}	-8.220	-8.751

Impact of ab-initio theory



Uncertainty

Uncertainty sources

- **Numerical**
- **Nuclear model**
- **Nucleon model**
- **Truncation of EM multipoles**
- **η-expansion**
- **Expansion in $(Z\alpha)$**

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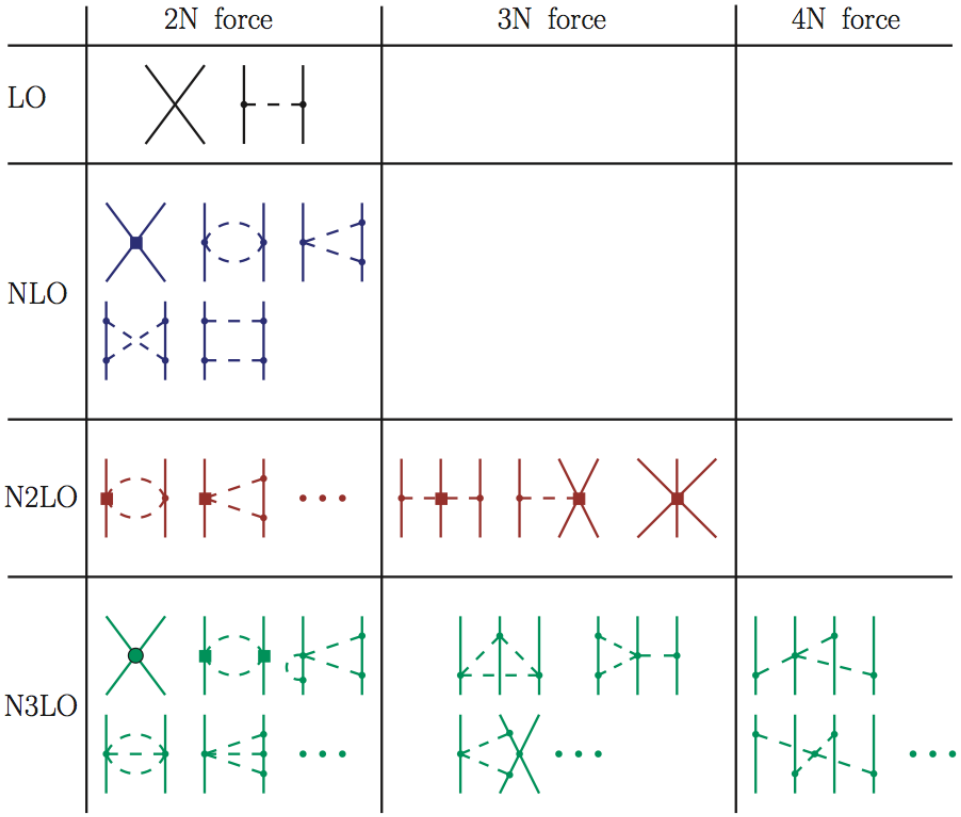
The Hamiltonians



Interactions derived from the Chiral effective field theory and written in coordinate space.

A. Gezerlis et. al. PRC 90, 054323 (2014)
J. Lynn et. al. PRL 116, 062501 (2016)

Hierarchy among different operators decided after specifying a power counting scheme

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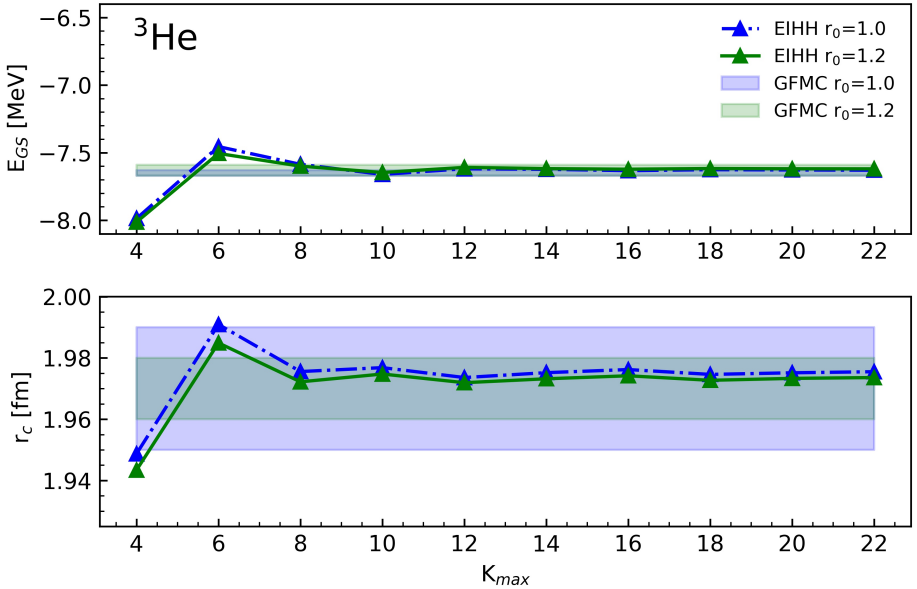
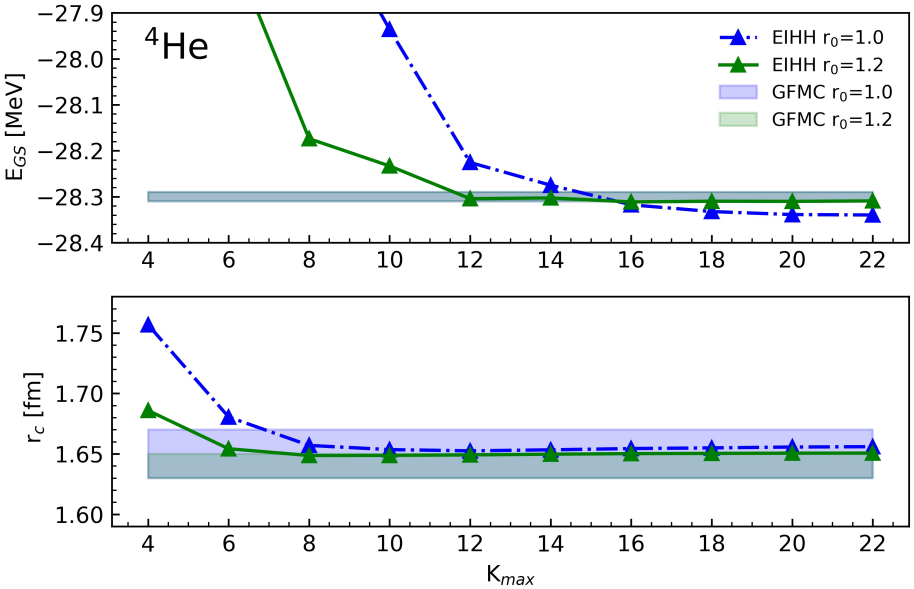
Benchmark tests

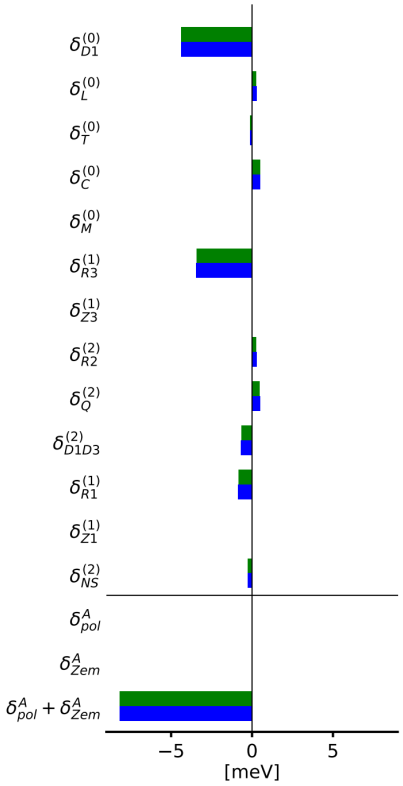
S.S. LM, S. Bacca , N. Barnea, Front. Phys. 9, 671869 (2021)



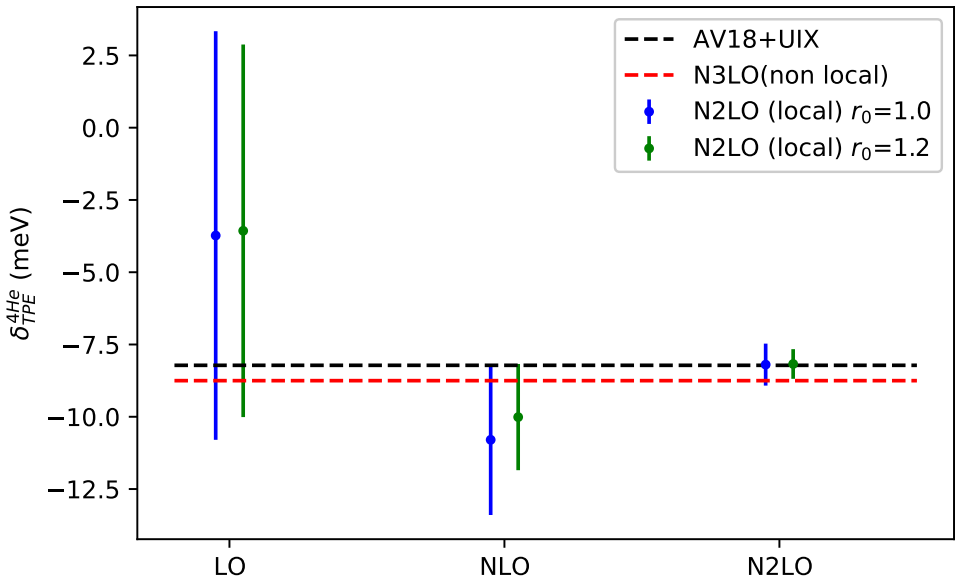


TPE in He-4 (EKM)

S.S. LM, et al. In preparation

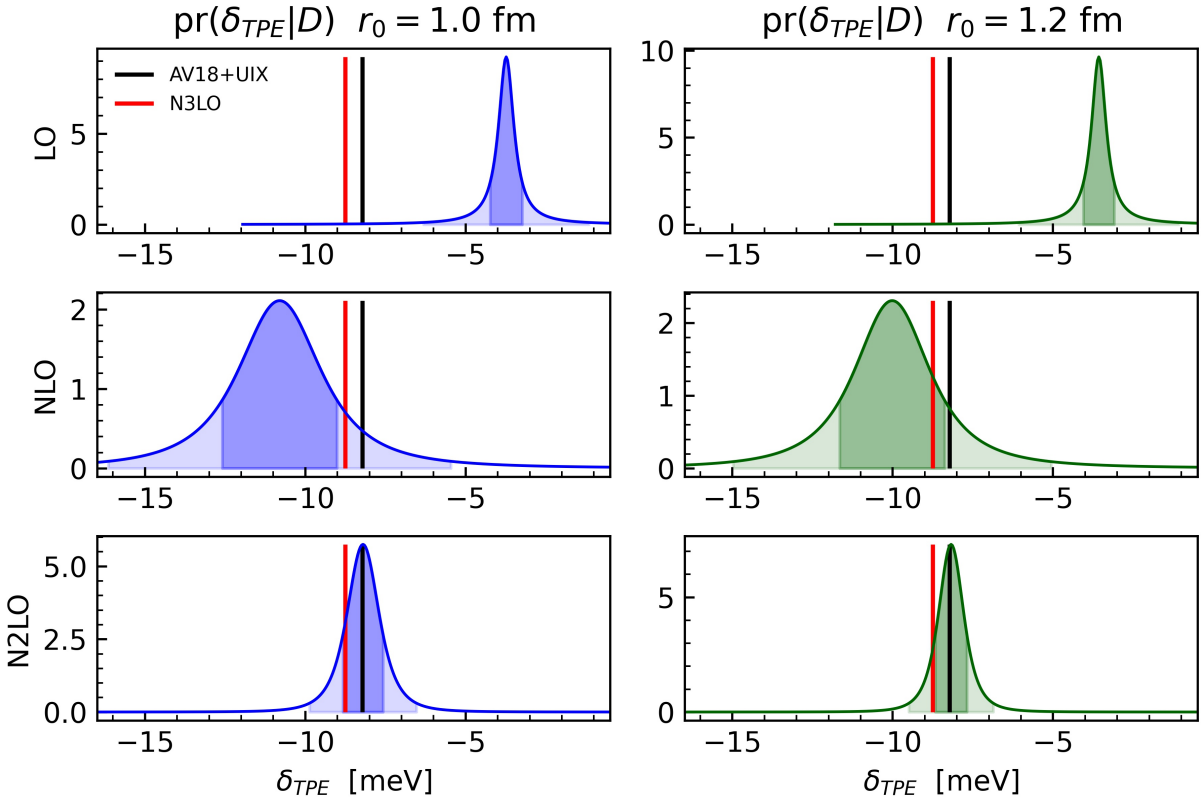


	$r_0 = 1.2$	$r_0 = 1.0$
$\delta^{(0)}_{D1}$	-4.386	-4.373
$\delta^{(0)}_{L}$	0.272	0.287
$\delta^{(0)}_{T}$	-0.124	-0.125
$\delta^{(0)}_{C}$	0.517	0.514
$\delta^{(0)}_{M}$	0.011	0.011
$\delta^{(1)}_{R3}$	-3.422	-3.477
$\delta^{(1)}_{Z3}$	-	-
$\delta^{(2)}_{R2}$	0.267	0.285
$\delta^{(2)}_{Q}$	0.484	0.505
$\delta^{(2)}_{D1D3}$	-0.668	-0.69
$\delta^{(1)}_{R1}$	-0.846	-0.856
$\delta^{(1)}_{Z1}$	-	-
$\delta^{(2)}_{NS}$	-0.272	-0.277
δ^{A}_{pol}	-	-
δ^{A}_{Zem}	-	-
$\delta^{A}_{pol} + \delta^{A}_{Zem}$	-8.174	-8.196



TPE in He-4 (Bayesian)

S.S. LM, et al. In preparation



C. Ji, et al. J. Phys. G: Nucl. Part. Phys. 45 (2018)

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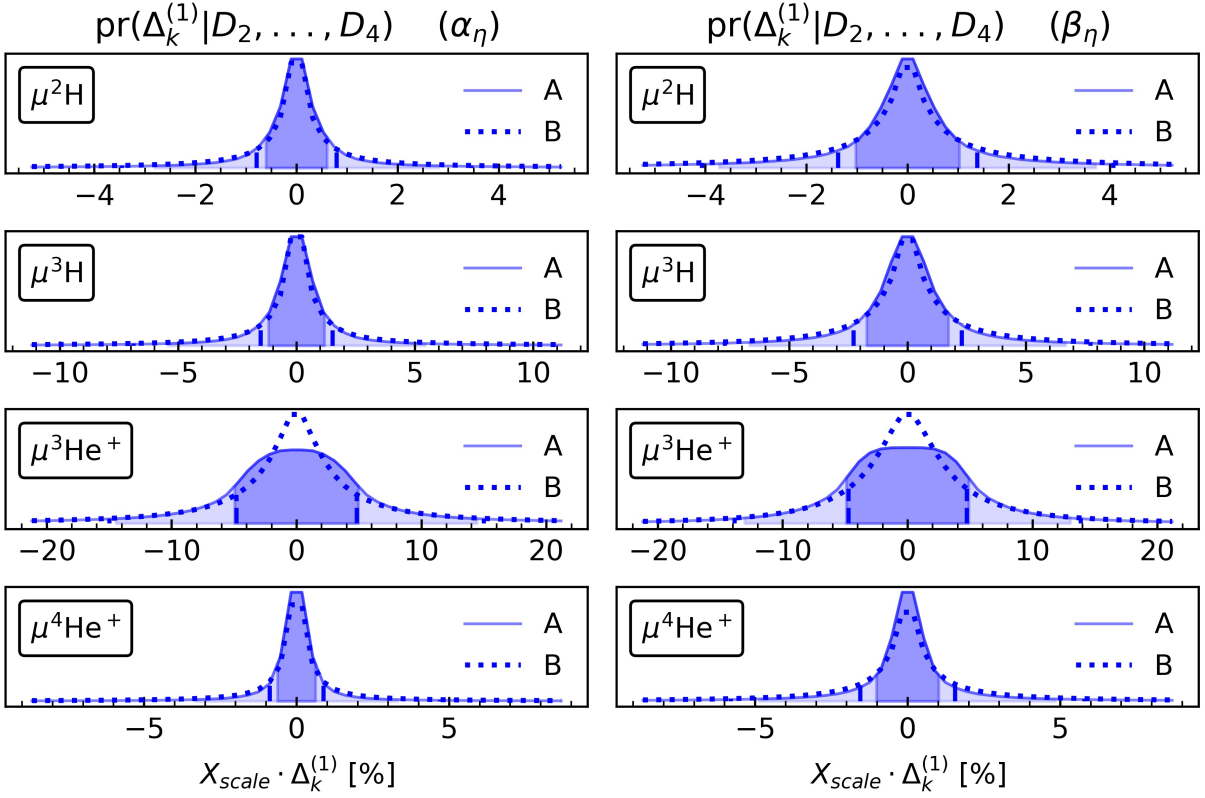
Eta-expansion uncert. (Bayesian)



S.S.LM, et al. ArXiv:2203.10792

	μ^2H	μ^3H	μ^3He	μ^4He
Liter.	0.4%	1.3%	1.1%	0.8%
This work	0.8%	1.5%	4.8%	0.9%

Conclusions

- **Ab-initio theories represent an excellent framework to calculate nuclear structure effects in muonic atoms.**
- **We re-evaluated the uncertainties coming from the nuclear model and from the η-expansion using Bayesian techniques.**
- **This work paves the way for future improvements, higher-order computations are needed to improve the precision.**

Backup

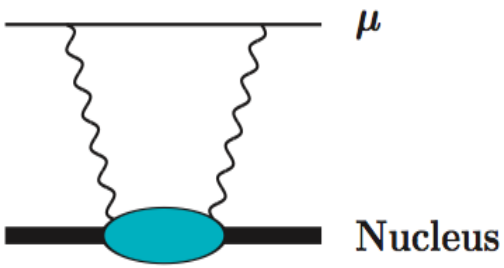
Evaluation of the TPE amplitude

- **Coulomb Distortion**

Contribution coming from intermediate interactions between lepton and nucleus during the TPE process.

We include only the correction of order $(Z\alpha)^6 log(Z\alpha)$

Since we work in the leading dipole approximation, the correction is related to the electric dipole response



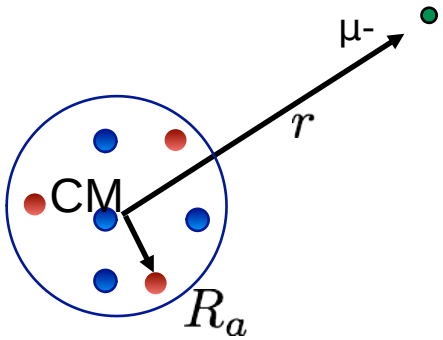
- **Relativistic effects**

They are smaller by factors $\frac{\omega_{th}}{m_r}$

Includes also first effects from electromagnetic currents. Are evaluated in the leading dipole approximation

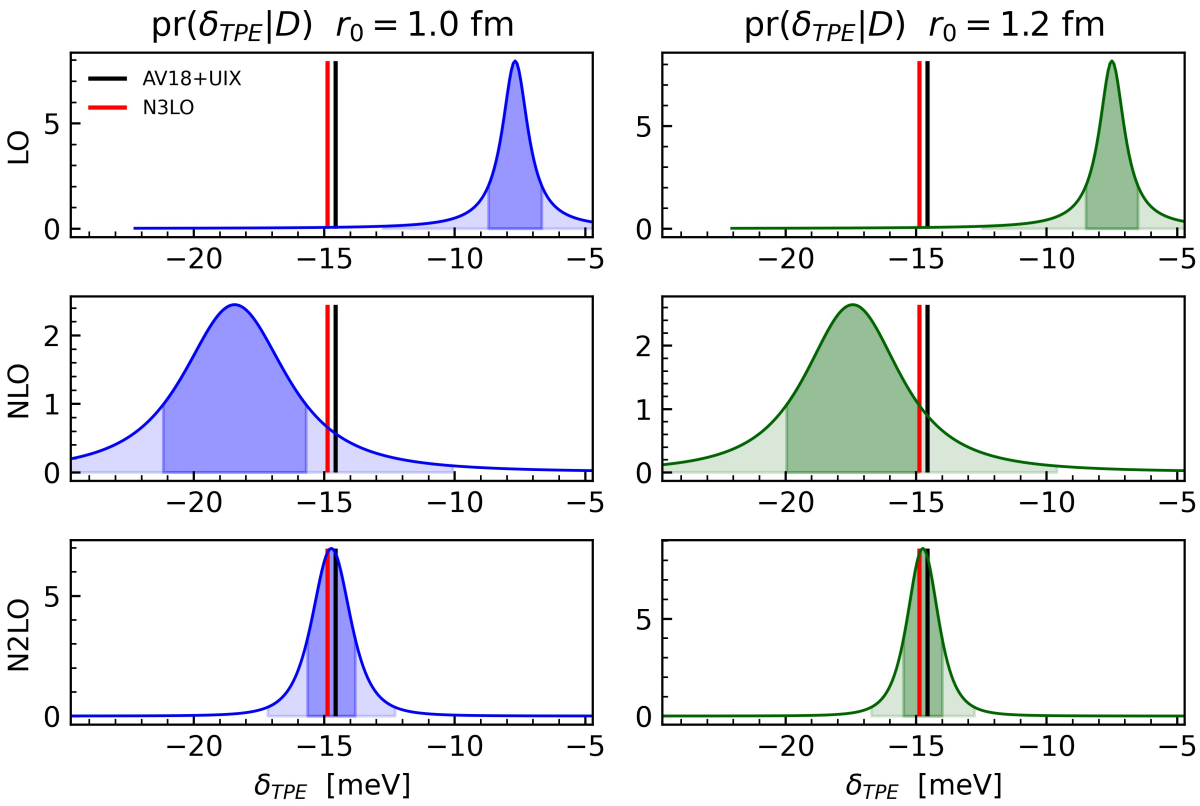
$$\delta^{(0)}_{L(T)} = \frac{2m_r^3}{9}(Z\alpha)^5 \int_{\omega_{th}}^{\infty} K_{L(T)}\left(\frac{\omega}{m_r}\right) S_{D1}(\omega)$$

- **Nucleon size effects**



TPE in He-3

S.S. LM, et al. In preparation



C. Ji, et al. J. Phys. G: Nucl. Part. Phys. 45 (2018)