

$(g - 2)_\mu$  from lattice QCD and experiments:  
4.2 sigma, indeed?

Z. Fodor

Penn State/Wuppertal/FZ Julich/Eotvos Budapest/UC San Diego  
Budapest–Marseille–Wuppertal Collaboration (BMW)

[Nature 593 \(2021\) 7857 51](#)

Trento, May 24, 2022

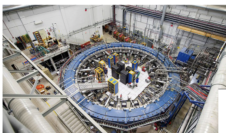


# General interest

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Une anomalie dans le comportement magnétique du muon, une particule élémentaire, est-elle le signe que le cadre conceptuel servant à décrire l'univers petit est en train de craquer ? Des résultats contradictoires relancent le débat.

Par David S. Reardon

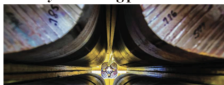
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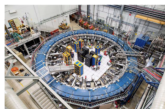
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Proof of new physics from the muon's magnetic moment? Maybe not, according to a new theoretical calculation

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**The Miraculous Measurement of the Muon**

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By Frank Wilczek  
April 13, 2021 12:04 pm ET

WISSENSCHAFT

Die Macht der Myonen

# General interest

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VON MARCO LEBIGER  
Aktualisiert am 12. April 2021, 12:04

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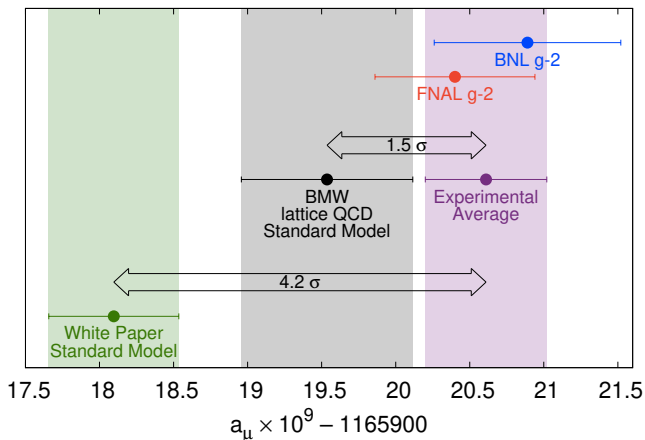
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**Breakthroughs of the year 2021:** “On the same day experimenters released their result, one team of theorists published a calculation that, they argued, increases the standard model prediction and closes the observed gap.”

# Tensions in $(g - 2)_\mu$ : take-home message



[Muon g-2 Theory Initiative, Phys.Rept. 887 (2020) 1-166]

[Budapest–Marseille–Wuppertal-coll., Nature (2021)]

[Muon g-2 coll., Phys. Rev. Lett. 126, 141801 (2021)]

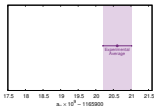
# Outline

## 1. $(g - 2)_\mu$

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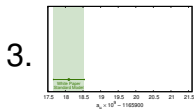
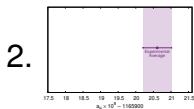
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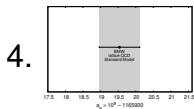
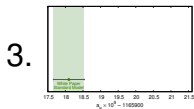
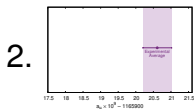
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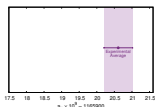




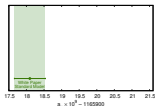
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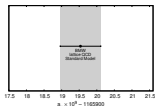
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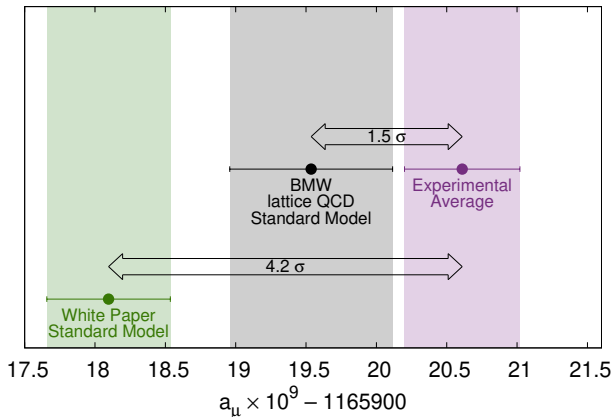
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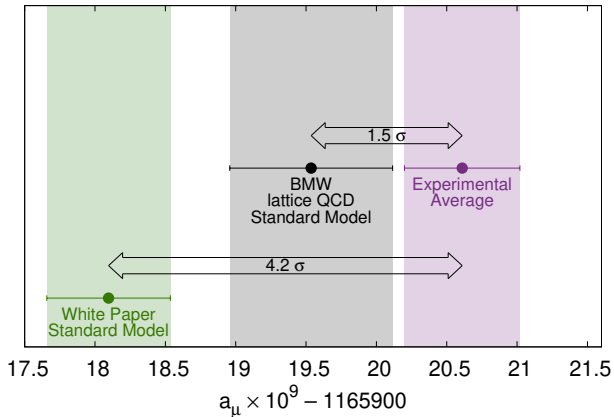
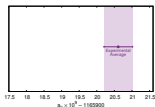


## 5. Summary



## Outline

2.



# Experimental result

- Newly announced result at Fermilab

$$a_{\mu}(\text{FNAL}) = 11\,659\,204.0(5.4) \cdot 10^{-10} \quad (0.46 \text{ ppm})$$

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- Target uncertainty: (1.6)

# Measurement principle

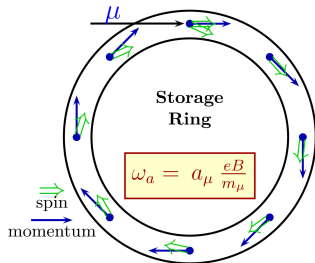
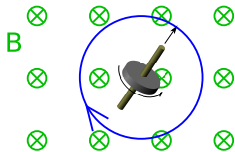
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- Spin precession frequency:  $\omega_s = \frac{eB}{m_\mu \gamma} + a_\mu \frac{eB}{m_\mu}$



- Measure:  $\omega_a = \omega_s - \omega_c = a_\mu \frac{eB}{m_\mu}$
- Gives  $a_\mu = \frac{g_\mu - 2}{2}$  directly
- During each circle spin axis changes 12'

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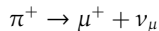
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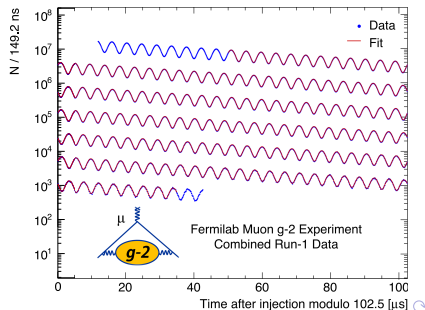
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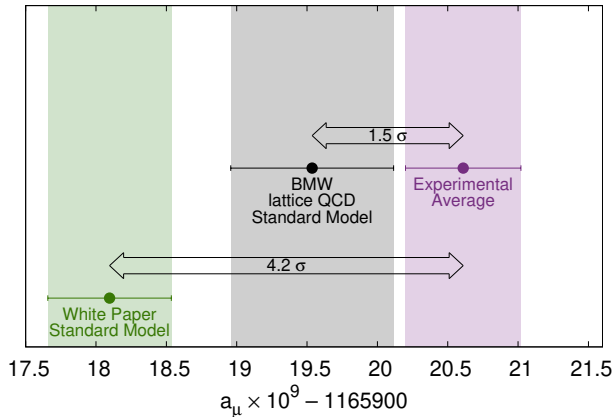
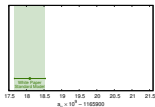


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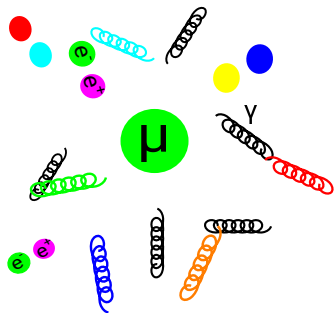
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3.



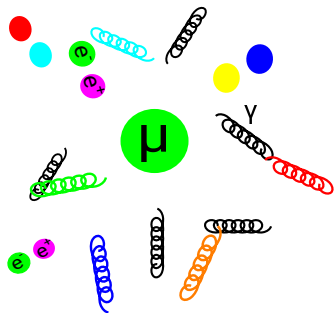


# Theory: Standard Model



Sum over all known physics:

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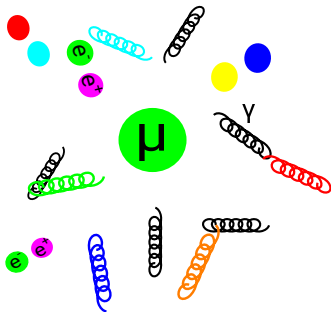


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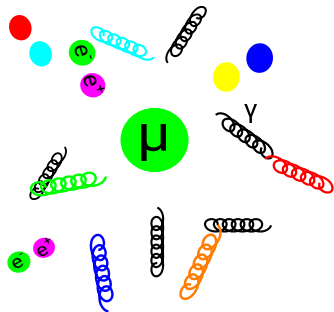


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# Theory: Standard Model



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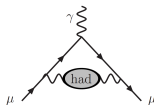
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- [2006.04822] White Paper of Muon  $g-2$  Theory Initiative

	$a_\mu \times 10^{-10}$
QED	11658471.9(0.0)
electroweak	15.4(0.1)
strong	693.7(4.3)
total	11659181.0(4.3)

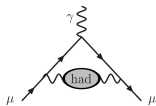
# Hadronic contributions

- LO hadron vacuum polarization (LO-HVP,  $(\frac{\alpha}{\pi})^2$ )

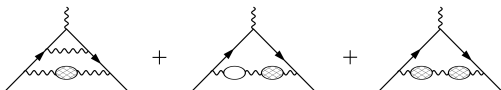


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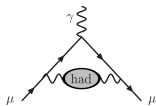
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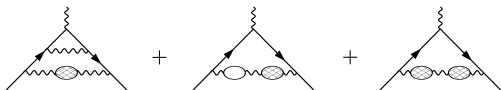


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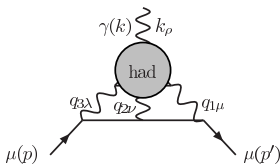
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- Hadronic light-by-light (HLbL,  $(\frac{\alpha}{\pi})^3$ )



- pheno  $a_{\mu}^{\text{HLbL}} = 9.2(1.9)$

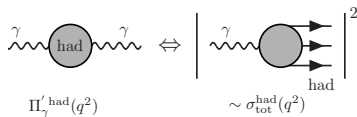
[Colangelo, Hoferichter, Kubis, Stoffer et al '15-'20]

- lattice  $a_{\mu}^{\text{HLbL}} = 7.9(3.1)(1.8)$  or  $10.7(1.5)$

[RBC/UKQCD '19 and Mainz '21]

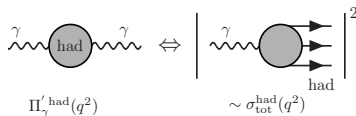
# HVP from R-ratio

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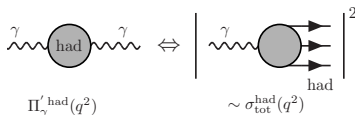
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Use  $e^+e^- \rightarrow \text{had}$  data of CMD, SND, BES, KLOE, BABAR, ...  
systematics limited

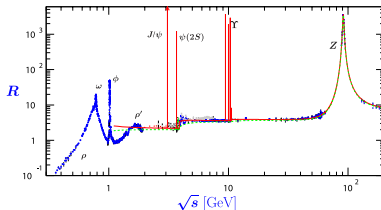
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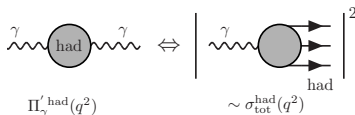
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$$a_\mu^{\text{LO-HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int \frac{ds}{s^2} K_\mu(s) R(s)$$



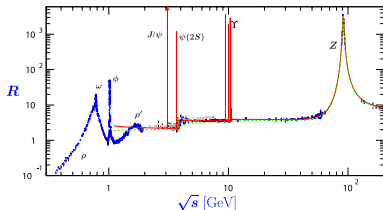
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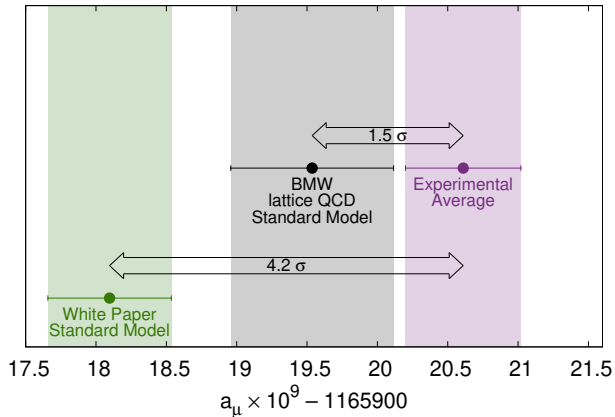
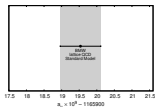
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LO	[Jegerlehner '18]	688.1(4.1)	0.60%
LO	[Davier et al '19]	693.9(4.0)	0.58%
LO	[Keshavarzi et al '19]	692.78(2.42)	0.35%
LO	[Hoferichter et al '19]	692.3(3.3)	0.48%
NLO	[Kurz et al '14]	-9.87(0.09)	
NNLO	[Kurz et al '14]	1.24(0.01)	

## Outline

4.



$a_{\mu}^{\text{LO-HVP}}$  from lattice QCD

Nature 593 (2021) 7857, 51

- Compute electromagnetic current-current correlator

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$$C(t) = \langle J_\mu(t) J_\nu(0) \rangle$$



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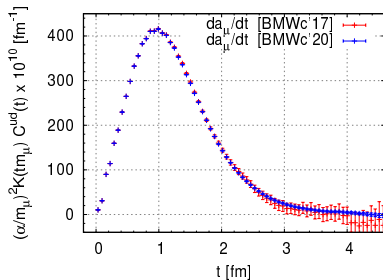
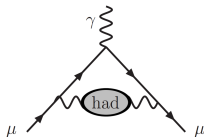
Nature 593 (2021) 7857, 51

- Compute electromagnetic current-current correlator

$$C(t) = \langle J_\mu(t) J_\nu(0) \rangle$$

$$a_\mu^{\text{LO-HVP}} = \alpha^2 \int_0^\infty dt K(t) C(t)$$

$K(t)$  describes the leptonic part of diagram



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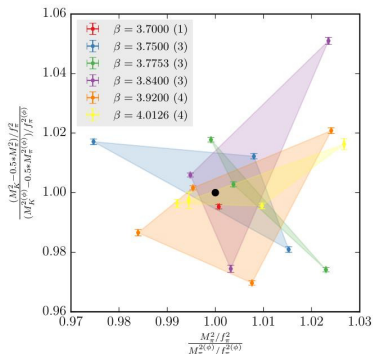
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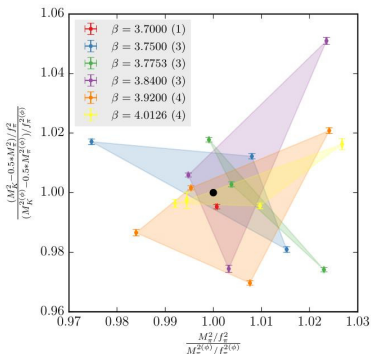
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$\beta$	$a$ [fm]	$L \times T$	#conf
3.7000	0.1315	$48 \times 64$	904
3.7500	0.1191	$56 \times 96$	2072
3.7753	0.1116	$56 \times 84$	1907
3.8400	0.0952	$64 \times 96$	3139
3.9200	0.0787	$80 \times 128$	4296
4.0126	0.0640	$96 \times 144$	6980



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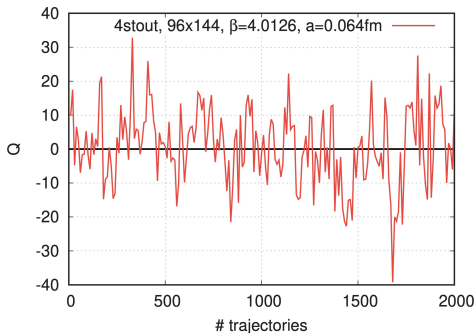
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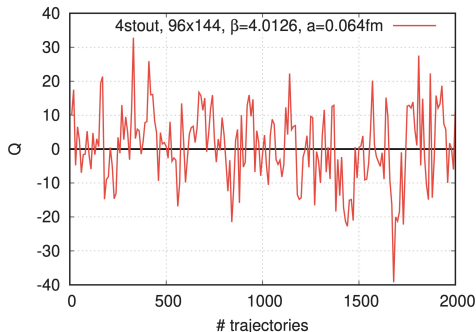
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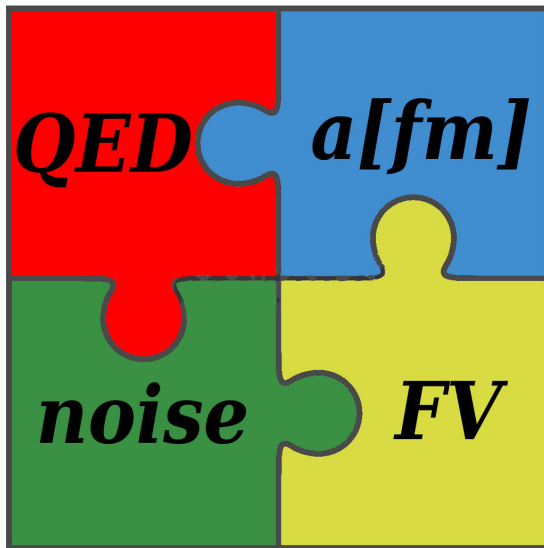
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The integrated autocorrelation time of  $Q$  is 19(2) trajectories.

## New challenges



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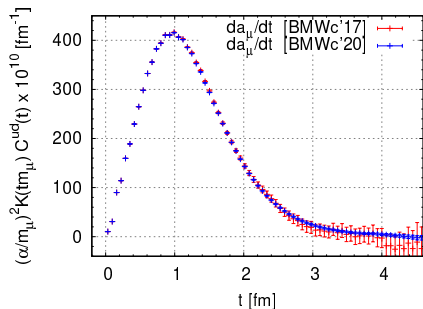
② For separation of isospin breaking effects:  $w_0$  scale setting

- Moderate  $m_q$  dependence
- Can be precisely determined on the lattice
- No experimental value  
→ Determine value of  $w_0$  from  $M_\Omega \cdot w_0$

$$w_0 = 0.17236(29)(63)[70] \text{ fm}$$

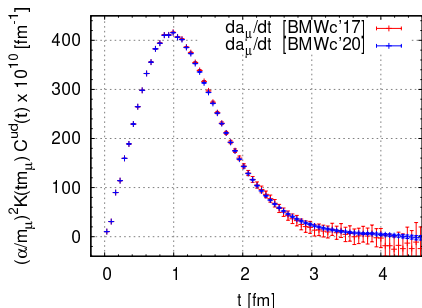
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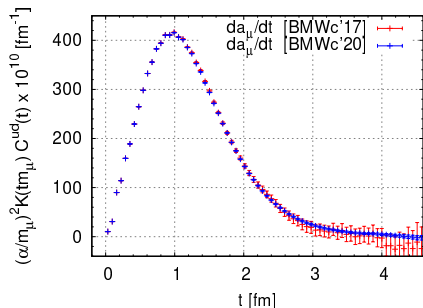


- Low Mode Averaging: use exact (all2all) quark propagator in IR and stochastic in UV
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# Finite-size effects

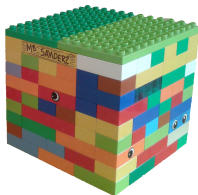
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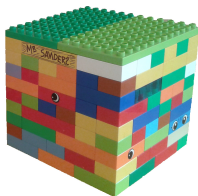


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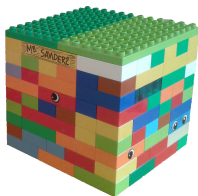
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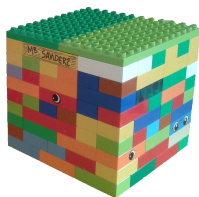
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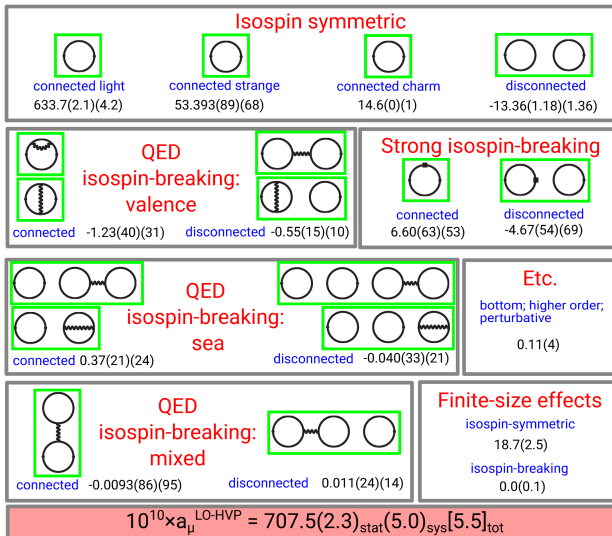
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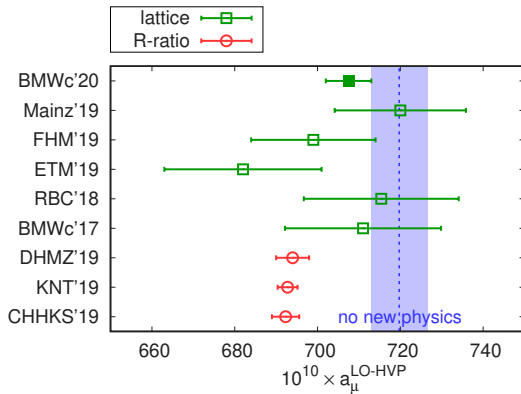
- use models for remnant finite-size effect of “big”  $\sim 0.1\%$

# Isospin breaking effects

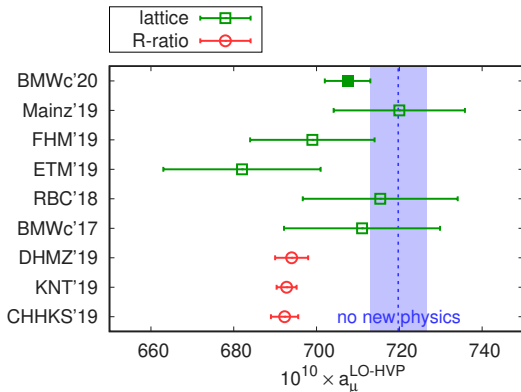
- Include leading order IB effects:  $O(e^2)$ ,  $O(\delta m)$



## Final result

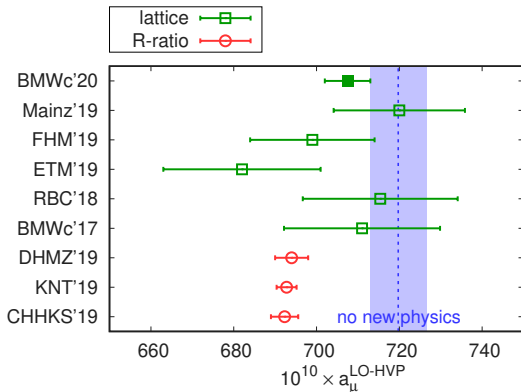


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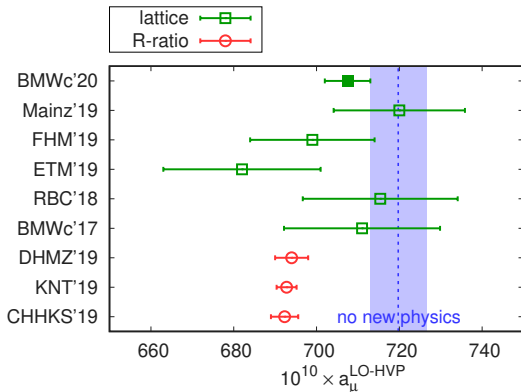
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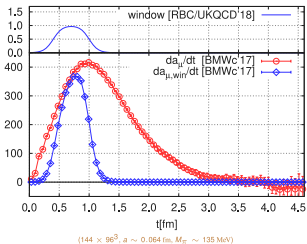


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- $2.0\sigma$  larger than [DHMZ'19],  $2.5\sigma$  than [KNT'19]

# Window observable

- Restrict correlator to window between  $t_1 = 0.4$  fm and  $t_2 = 1.0$  fm

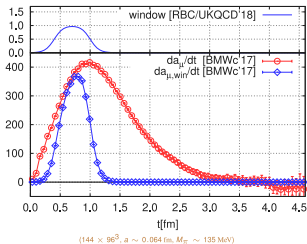
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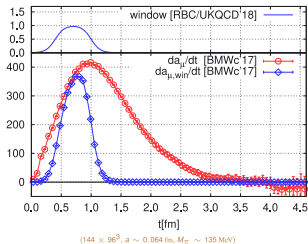


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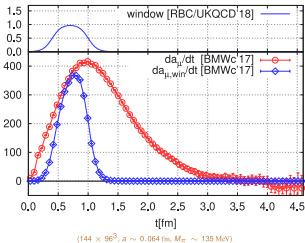


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  - signal/noise
  - finite size effects
  - lattice artefacts (short & long)

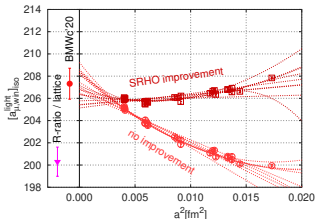
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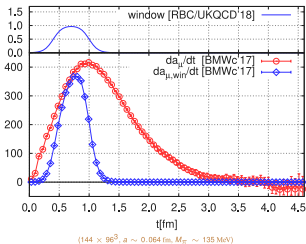
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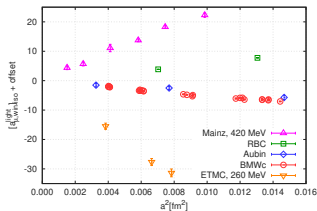
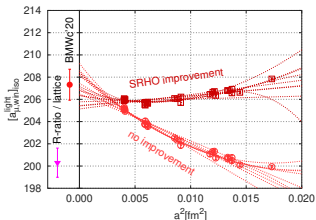
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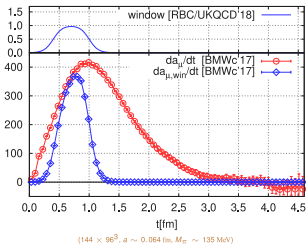
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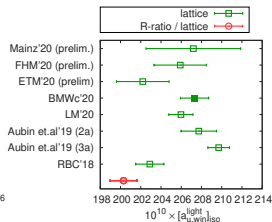
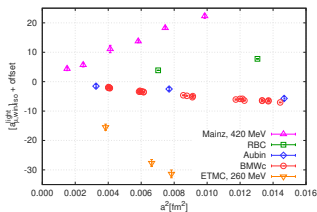
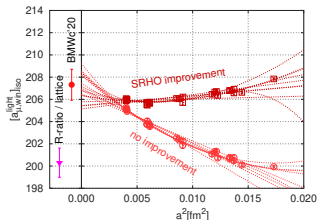
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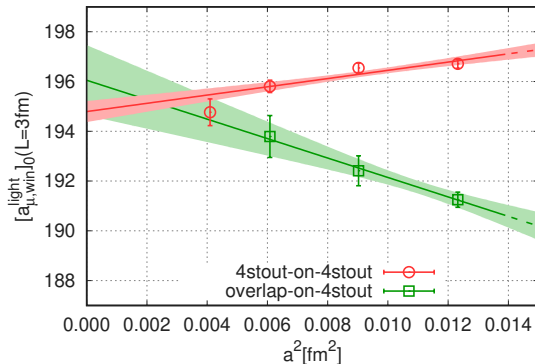


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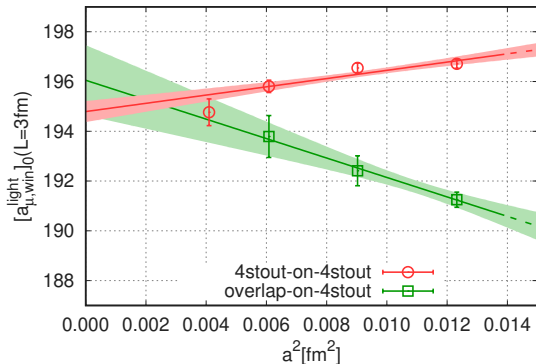


## Crosscheck – overlap



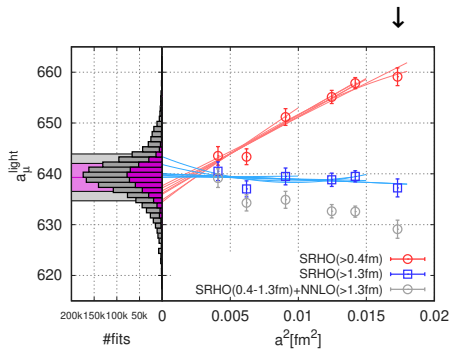


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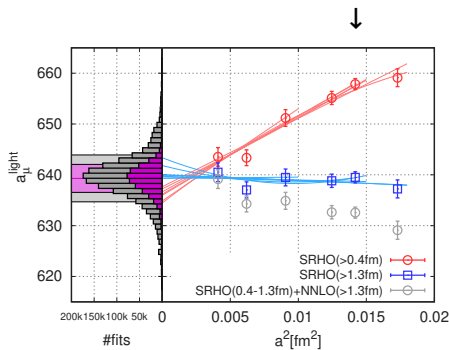


- compute  $a_{\mu,win}$  with overlap valence
- local current instead of conserved  $\rightarrow$  had to compute  $Z_V$
- cont. limit in  $L = 3$  fm box consistent with staggered valence

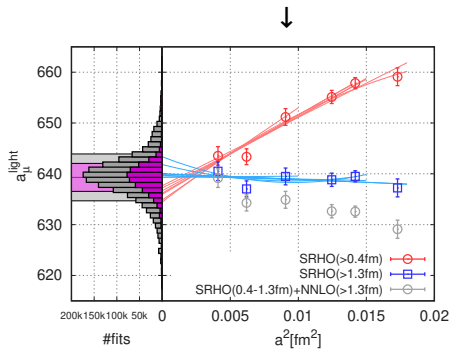
## Continuum limit



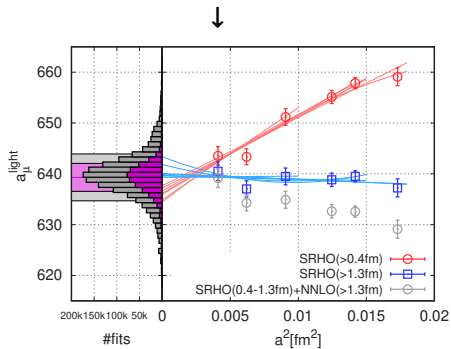
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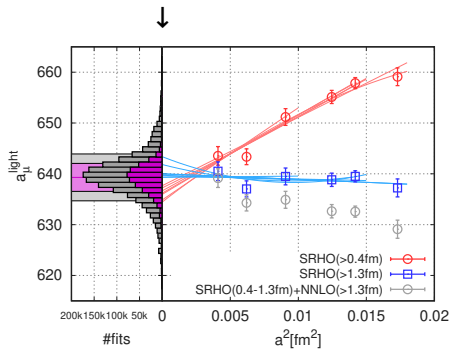
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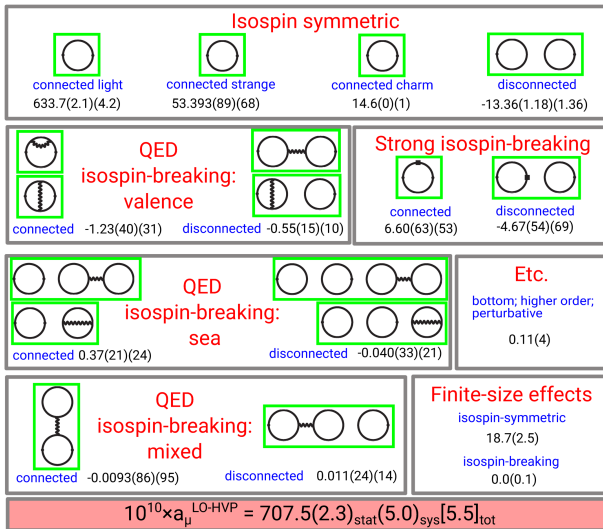
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# Outline

## 5. Summary

## Final result





## Tensions: take-home message

