$(g-2)_\mu$ from lattice QCD and experiments: 4.2 sigma, indeed?

Z. Fodor
Penn State/Wuppertal/FZ Julich/Eotvos Budapest/UC San Diego
Budapest–Marseille–Wuppertal Collaboration (BMW)

Nature 593 (2021) 7857 51

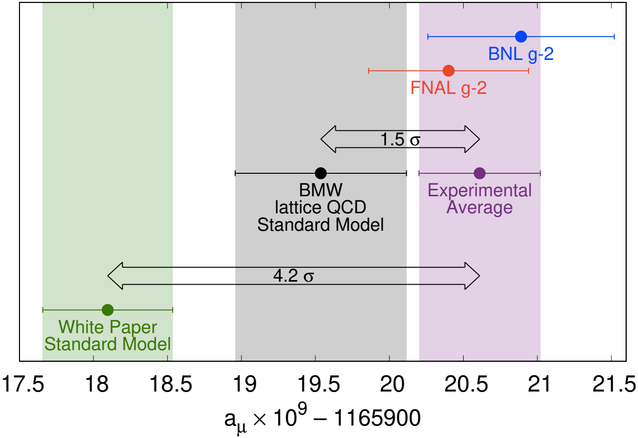
Trento, May 24, 2022

General interest

General interest

Breakthroughs of the year 2021: "On the same day experimenters released their result, one team of theorists published a calculation that, they argued, increases the standard model prediction and closes the observed gap."

Tensions in $(g-2)_\mu$: take-home message



[Muon g-2 Theory Initiative, Phys.Rept. 887 (2020) 1-166]

[Budapest–Marseille–Wuppertal-coll., Nature (2021)]

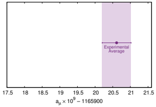
[Muon g-2 coll., Phys. Rev. Lett. 126, 141801 (2021)]

Outline

1. $(g-2)_\mu$

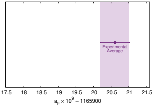
Outline

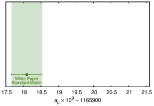
1. $(g-2)_\mu$

2. 

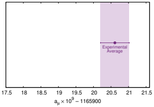
Outline

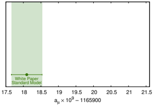
1. $(g-2)_\mu$

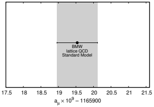
2. 

3. 

Outline

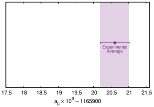
1. $(g-2)_\mu$
2. 

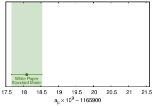
3. 

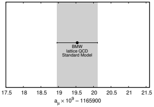
4. 

Outline

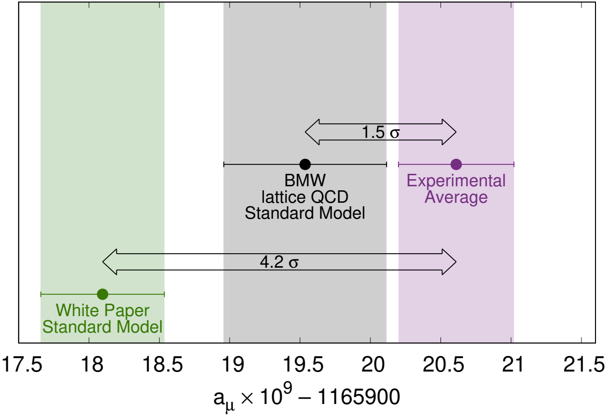
1. $(g-2)_\mu$

2. 

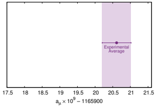
3. 

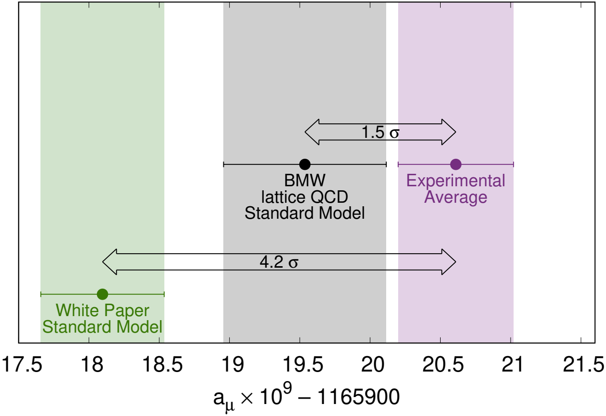
4. 

5. Summary



Outline

2. 
Experimental
Average
17.5
18
18.5
19
19.5
20
20.5
21
21.5
$a_\mu \times 10^9 - 1165900$


1.5 σ
BMW
lattice QCD
Standard Model
Experimental
Average
4.2 σ
White Paper
Standard Model
17.5
18
18.5
19
19.5
20
20.5
21
21.5
$a_\mu \times 10^9 - 1165900$

Experimental result

- Newly announced result at Fermilab

$$a_\mu(\mathrm{FNAL}) = 11\,659\,204.0(5.4)\cdot 10^{-10} \quad (0.46\,\mathrm{ppm})$$

Experimental result

- Newly announced result at Fermilab

$$a_\mu(\text{FNAL}) = 11\,659\,204.0(5.4) \cdot 10^{-10} \quad (0.46\,\text{ppm})$$

- Equivalent to: bathroom scale sensitive to weight of a single eyelash.

Experimental result

- Newly announced result at Fermilab

$$a_\mu(\text{FNAL}) = 11\,659\,204.0(5.4) \cdot 10^{-10} \quad (0.46\,\text{ppm})$$

- Equivalent to: bathroom scale sensitive to weight of a single eyelash.

- Fully agrees with the BNL E821 measurement

$$\begin{aligned} a_\mu(\text{BNL}) &= 11\,659\,209.1(6.3) \cdot 10^{-10} \quad (0.54\,\text{ppm}) \\ a_\mu(\text{combined}) &= 11\,659\,206.1(4.1) \cdot 10^{-10} \quad (0.35\,\text{ppm}) \end{aligned}$$

Experimental result

- Newly announced result at Fermilab

$$a_\mu(\text{FNAL}) = 11\,659\,204.0(5.4) \cdot 10^{-10} \quad (0.46\,\text{ppm})$$

- Equivalent to: bathroom scale sensitive to weight of a single eyelash.

- Fully agrees with the BNL E821 measurement

$$\begin{aligned} a_\mu(\text{BNL}) &= 11\,659\,209.1(6.3) \cdot 10^{-10} \quad (0.54\,\text{ppm}) \\ a_\mu(\text{combined}) &= 11\,659\,206.1(4.1) \cdot 10^{-10} \quad (0.35\,\text{ppm}) \end{aligned}$$

- Target uncertainty: (1.6)

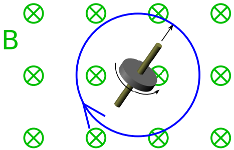
Measurement principle

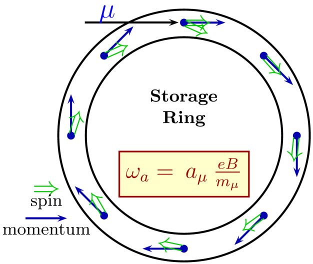
Measurement principle

- Cyclotron motion frequency: $\omega_c = \dfrac{eB}{m_\mu \gamma}$ with $\gamma = \dfrac{1}{\sqrt{1-v^2}}$

Measurement principle

- Cyclotron motion frequency: $\omega_c = \frac{eB}{m_\mu \gamma}$ with $\gamma = \frac{1}{\sqrt{1-v^2}}$
- Spin precession frequency: $\omega_s = \frac{eB}{m_\mu \gamma} + a_\mu \frac{eB}{m_\mu}$





- Measure: $\omega_a = \omega_s - \omega_c = a_\mu \frac{eB}{m_\mu}$
- Gives $a_\mu = \frac{g_\mu - 2}{2}$ directly
- During each circle spin axis changes 12′

Measurement principle

- Pions produced at fixed target $p + p \rightarrow p + n + \pi^+$

Measurement principle

- Pions produced at fixed target $p + p \to p + n + \pi^+$
- Pion decays through weak interaction $\longrightarrow$ highly polarized muons

$$\pi^+ \to \mu^+ + \nu_\mu$$

Measurement principle

- Pions produced at fixed target $p + p \to p + n + \pi^+$
- Pion decays through weak interaction $\longrightarrow$ highly polarized muons

$$\pi^+ \to \mu^+ + \nu_\mu$$

- Muons decay after several circles: $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$

Measurement principle

- Pions produced at fixed target $p + p \rightarrow p + n + \pi^+$
- Pion decays through weak interaction $\longrightarrow$ highly polarized muons

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

- Muons decay after several circles: $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$
- Strong correlation between μ^+ spin and e^+ momentum

Measurement principle

- Pions produced at fixed target $p + p \rightarrow p + n + \pi^+$
- Pion decays through weak interaction $\longrightarrow$ highly polarized muons

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

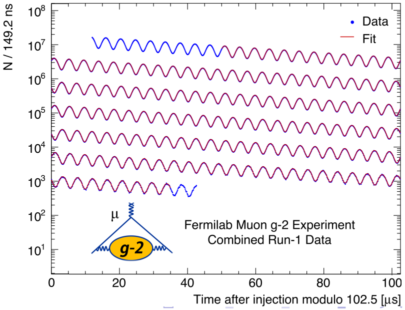
- Muons decay after several circles: $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$
- Strong correlation between μ^+ spin and e^+ momentum
- Detect emitted e^+

Measurement principle

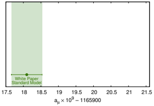
- Pions produced at fixed target $p + p \to p + n + \pi^+$
- Pion decays through weak interaction $\longrightarrow$ highly polarized muons

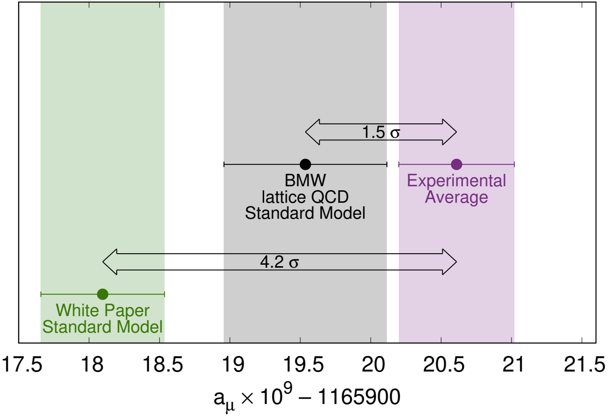
$$\pi^+ \to \mu^+ + \nu_\mu$$

- Muons decay after several circles: $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$
- Strong correlation between μ^+ spin and e^+ momentum
- Detect emitted e^+

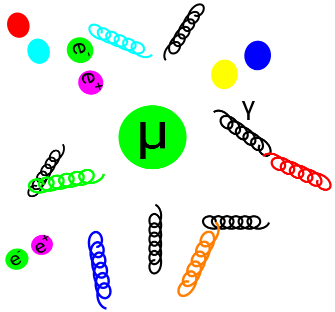


Outline

3. 
White Paper
Standard Model
17.5
18
18.5
19
19.5
20
20.5
21
21.5

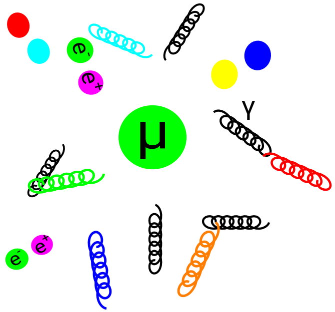

1.5 σ
BMW
lattice QCD
Standard Model
Experimental
Average
4.2 σ
White Paper
Standard Model
17.5
18
18.5
19
19.5
20
20.5
21
21.5
$a_\mu \times 10^9 - 1165900$

Theory: Standard Model



Sum over all known physics:

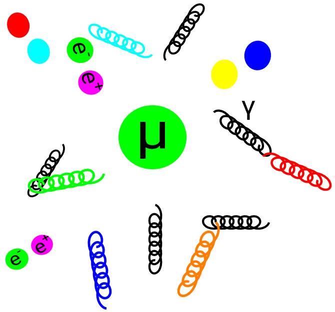
Theory: Standard Model



Sum over all known physics:

1. quantum electrodynamics (QED): photons, leptons

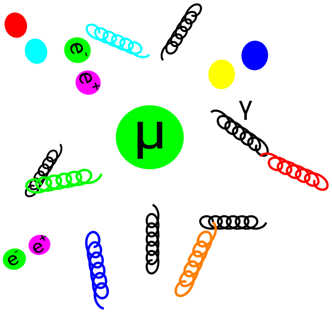
Theory: Standard Model



Sum over all known physics:

1. quantum electrodynamics (QED): photons, leptons
2. electroweak (EW): W, Z bosons, neutrinos, Higgs

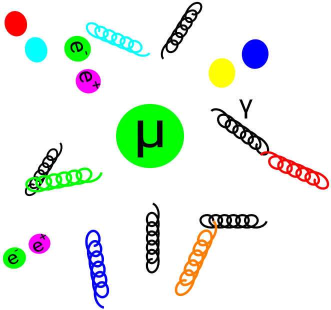
Theory: Standard Model



Sum over all known physics:

1. quantum electrodynamics (QED): photons, leptons
2. electroweak (EW): W, Z bosons, neutrinos, Higgs
3. strong (QCD): quarks and gluons

Theory: Standard Model

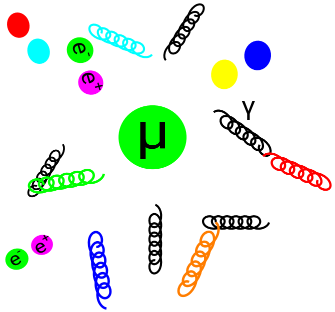


Sum over all known physics:

1. quantum electrodynamics (QED): photons, leptons
2. electroweak (EW): W, Z bosons, neutrinos, Higgs
3. strong (QCD): quarks and gluons

- [2006.04822] White Paper of Muon g-2 Theory Initiative

Theory: Standard Model



Sum over all known physics:

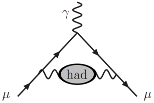
1. quantum electrodynamics (QED): photons, leptons
2. electroweak (EW): W, Z bosons, neutrinos, Higgs
3. strong (QCD): quarks and gluons

- [2006.04822] White Paper of Muon g-2 Theory Initiative

	$a_\mu \times 10^{-10}$
QED	11658471.9(0.0)
electroweak	15.4(0.1)
strong	693.7(4.3)
total	11659181.0(4.3)

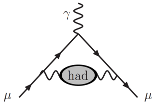
Hadronic contributions

- LO hadron vacuum polarization (LO-HVP, $(\frac{\alpha}{\pi})^2$)

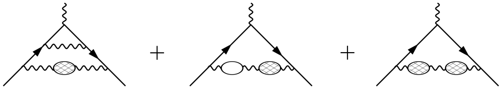


Hadronic contributions

- LO hadron vacuum polarization (LO-HVP, $(\frac{\alpha}{\pi})^2$)

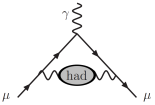


- NLO hadron vacuum polarization (NLO-HVP, $(\frac{\alpha}{\pi})^3$)

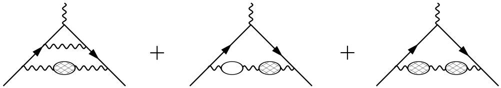


Hadronic contributions

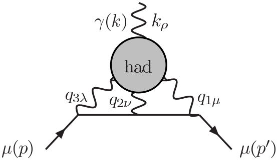
- LO hadron vacuum polarization (LO-HVP, $(\frac{\alpha}{\pi})^2$)



- NLO hadron vacuum polarization (NLO-HVP, $(\frac{\alpha}{\pi})^3$)



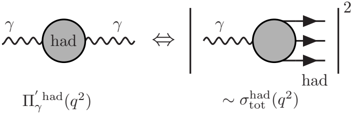
- Hadronic light-by-light (HLbL, $(\frac{\alpha}{\pi})^3$)



- pheno $a_\mu^{\text{HLbL}} = 9.2(1.9)$
  [Colangelo, Hoferichter, Kubis, Stoffer et al '15–'20]
- lattice a_μ^{HLbL}=7.9(3.1)(1.8) or 10.7(1.5)
  [RBC/UKQCD '19 and Mainz '21]

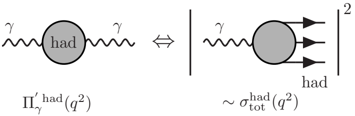
HVP from R-ratio

- Optical theorem



HVP from R-ratio

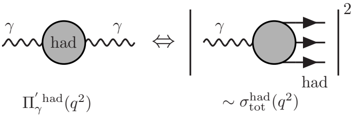
- Optical theorem



Use $e^+e^- \to$ had data of CMD, SND, BES,
KLOE, BABAR, ...
systematics limited

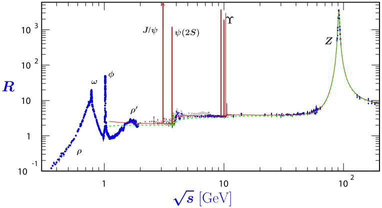
HVP from R-ratio

- Optical theorem



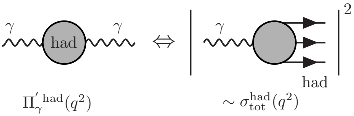
Use $e^+e^- \to$ had data of CMD, SND, BES, KLOE, BABAR, ...
systematics limited

$$a_\mu^{\text{LO-HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int \frac{ds}{s^2} K_\mu(s) R(s)$$

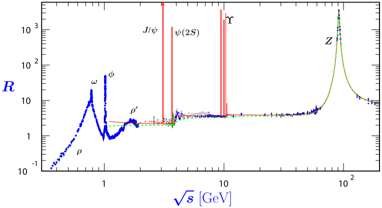


HVP from R-ratio

- Optical theorem



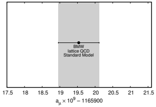
Use $e^+e^- \to$ had data of CMD, SND, BES, KLOE, BABAR, ...
systematics limited

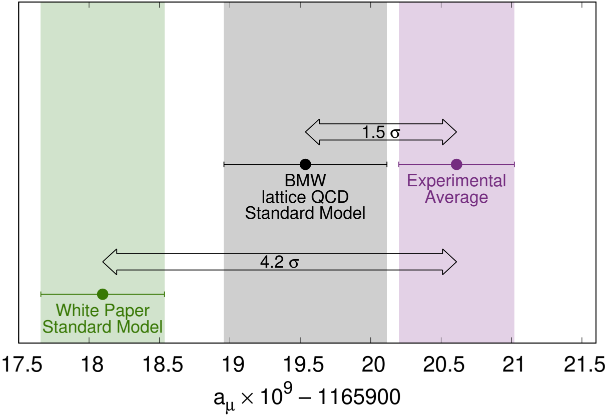


$$a_\mu^{\text{LO-HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int \frac{ds}{s^2} K_\mu(s) R(s)$$

LO	[Jegerlehner '18]	688.1(4.1)	0.60%
LO	[Davier et al '19]	693.9(4.0)	0.58%
LO	[Keshavarzi et al '19]	692.78(2.42)	0.35%
LO	[Hoferichter et al '19]	692.3(3.3)	0.48%
NLO	[Kurz et al '14]	-9.87(0.09)	
NNLO	[Kurz et al '14]	1.24(0.01)	

Outline

4. 
BMW
lattice QCD
Standard Model
17.5
18
18.5
19
19.5
20
20.5
21
21.5
$a_\mu \times 10^9 - 1165900$


1.5 σ
BMW
lattice QCD
Standard Model
Experimental
Average
4.2 σ
White Paper
Standard Model
17.5
18
18.5
19
19.5
20
20.5
21
21.5
$a_\mu \times 10^9 - 1165900$

$a_\mu^{\text{LO-HVP}}$ from lattice QCD Nature 593 (2021) 7857, 51

- Compute electromagnetic current-current correlator

$a_\mu^{\text{LO-HVP}}$ from lattice QCD Nature 593 (2021) 7857, 51

- Compute electromagnetic current-current correlator

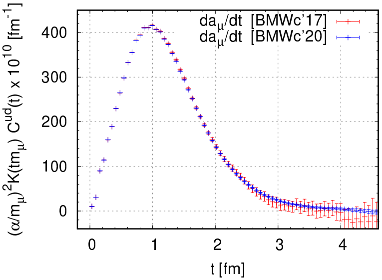
$$C(t) = \langle J_\mu(t) J_\nu(0) \rangle$$

$a_\mu^{\text{LO-HVP}}$ from lattice QCD Nature 593 (2021) 7857, 51

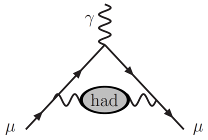
- Compute electromagnetic current-current correlator

$$C(t) = \langle J_\mu(t) J_\nu(0) \rangle$$

$$a_\mu^{\text{LO-HVP}} = \alpha^2 \int_0^\infty dt\, K(t)\, C(t)$$



$K(t)$ describes the leptonic part of diagram



Simulation setup

Simulation setup

- 6 lattice spacings: 0.13 fm – 0.064 fm $\longrightarrow$ controlled continuum limit

Simulation setup

- 6 lattice spacings: 0.13 fm – 0.064 fm $\longrightarrow$ controlled continuum limit
- Box size: $L \sim 6$ fm

Simulation setup

- 6 lattice spacings: 0.13 fm – 0.064 fm $\longrightarrow$ controlled continuum limit
- Box size: $L \sim 6$ fm

  $L \sim 11$ fm at one lattice spacing $\longrightarrow$ FV effects

Simulation setup

- 6 lattice spacings: 0.13 fm – 0.064 fm $\longrightarrow$ controlled continuum limit
- Box size: $L \sim 6$ fm

  $L \sim 11$ fm at one lattice spacing $\longrightarrow$ FV effects

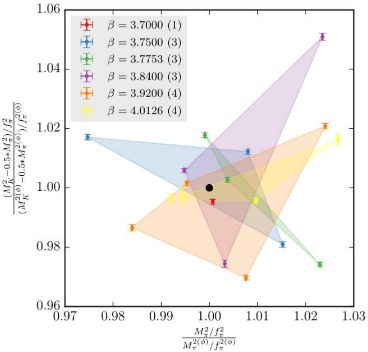
1 fm $= 10^{-15}$ m $\sim$ size of proton

Simulation setup

- 6 lattice spacings: 0.13 fm – 0.064 fm $\longrightarrow$ controlled continuum limit
- Box size: $L \sim 6$ fm

  $L \sim 11$ fm at one lattice spacing $\longrightarrow$ FV effects

  1 fm $= 10^{-15}$ m $\sim$ size of proton
- Quark masses bracketing their physical values

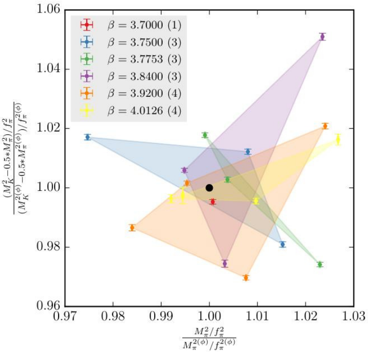


Simulation setup

- 6 lattice spacings: 0.13 fm – 0.064 fm $\longrightarrow$ controlled continuum limit
- Box size: $L \sim 6$ fm

  $L \sim 11$ fm at one lattice spacing $\longrightarrow$ FV effects

  1 fm $= 10^{-15}$ m $\sim$ size of proton
- Quark masses bracketing their physical values



β	a[fm]	$L \times T$	#conf
3.7000	0.1315	48×64	904
3.7500	0.1191	56×96	2072
3.7753	0.1116	56×84	1907
3.8400	0.0952	64×96	3139
3.9200	0.0787	80×128	4296
4.0126	0.0640	96×144	6980

Topology: did we hit the brick-wall?

Depending on the action: topology is frozen for a<0.05 fm

Topology: did we hit the brick-wall?

Depending on the action: topology is frozen for a$<$0.05 fm
$\Longrightarrow$ open boundary condition (CLS lattice group)

Topology: did we hit the brick-wall?

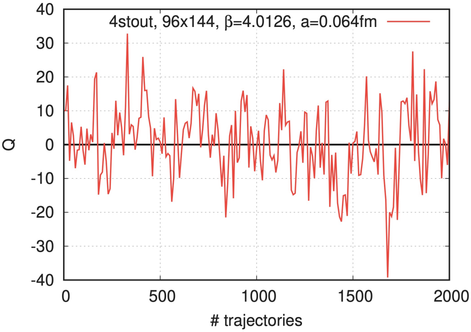
Depending on the action: topology is frozen for a$<$0.05 fm
$\Longrightarrow$ open boundary condition (CLS lattice group)
Take the smallest lattice spacing of us of 0.064 fm

Topology: did we hit the brick-wall?

Depending on the action: topology is frozen for a<0.05 fm
$\Longrightarrow$ open boundary condition (CLS lattice group)
Take the smallest lattice spacing of us of 0.064 fm
Topological charge at a Wilson-flow time of $\sqrt{8t} \approx 0.6$ fm

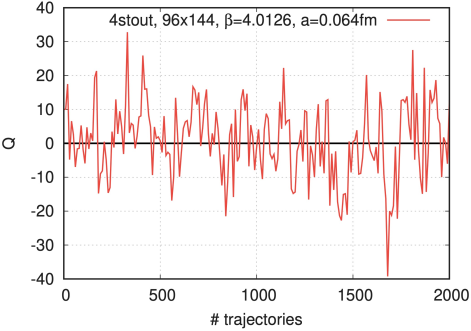
Topology: did we hit the brick-wall?

Depending on the action: topology is frozen for a<0.05 fm
$\Longrightarrow$ open boundary condition (CLS lattice group)
Take the smallest lattice spacing of us of 0.064 fm
Topological charge at a Wilson-flow time of $\sqrt{8t} \approx 0.6$ fm



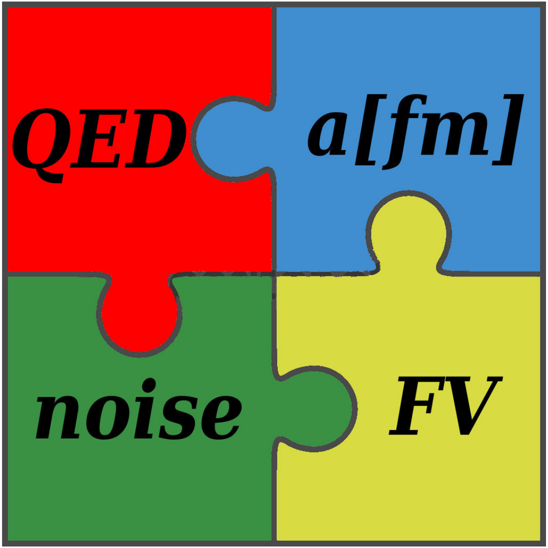
Topology: did we hit the brick-wall?

Depending on the action: topology is frozen for a<0.05 fm
$\Longrightarrow$ open boundary condition (CLS lattice group)
Take the smallest lattice spacing of us of 0.064 fm
Topological charge at a Wilson-flow time of $\sqrt{8t} \approx 0.6$ fm



The integrated autocorrelation time of Q is 19(2) trajectories.

New challenges



Scale determination

Lattice spacing a enters into a_μ determination:

Scale determination

Lattice spacing a enters into a_μ determination:

- physical value of m_μ
- physical values of m_π, m_K

Scale determination

Lattice spacing a enters into a_μ determination:

- physical value of m_μ
- physical values of m_π, m_K

$\longrightarrow$ $\Delta_{\text{scale}} a_\mu \sim 2 \cdot \Delta(\text{scale})$

Scale determination

Lattice spacing a enters into a_μ determination:

- physical value of m_μ
- physical values of m_π, m_K

$\longrightarrow$ $\Delta_{\text{scale}} a_\mu \sim 2 \cdot \Delta(\text{scale})$

1. For final results: M_Ω scale setting $\longrightarrow$ $a = (aM_\Omega)^{\text{lat}}/M_\Omega^{\text{exp}}$
   - Experimentally well known: 1672.45(29) MeV [PDG 2018]
   - Moderate m_q dependence
   - Can be precisely determined on the lattice

Scale determination

Lattice spacing a enters into a_μ determination:

- physical value of m_μ
- physical values of m_π, m_K

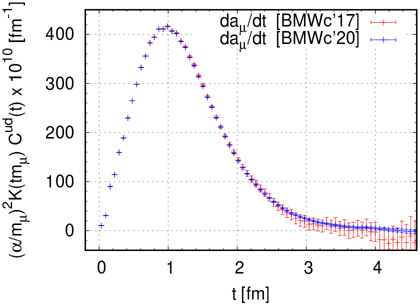
$\longrightarrow$ $\Delta_{\text{scale}} a_\mu \sim 2 \cdot \Delta(\text{scale})$

1. For final results: M_Ω scale setting $\longrightarrow$ $a = (aM_\Omega)^{\text{lat}} / M_\Omega^{\text{exp}}$
   - Experimentally well known: 1672.45(29) MeV [PDG 2018]
   - Moderate m_q dependence
   - Can be precisely determined on the lattice
2. For separation of isospin breaking effects: w_0 scale setting
   - Moderate m_q dependence
   - Can be precisely determined on the lattice
   - No experimental value
     $\longrightarrow$ Determine value of w_0 from $M_\Omega \cdot w_0$

$$w_0 = 0.17236(29)(63)[70] \text{ fm}$$

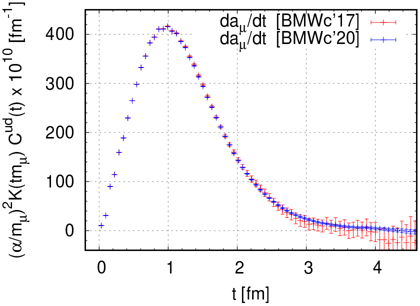
Noise reduction

- noise/signal in $C(t) = \langle J(t)J(0)\rangle$ grows for large distances



Noise reduction

- noise/signal in $C(t) = \langle J(t)J(0)\rangle$ grows for large distances

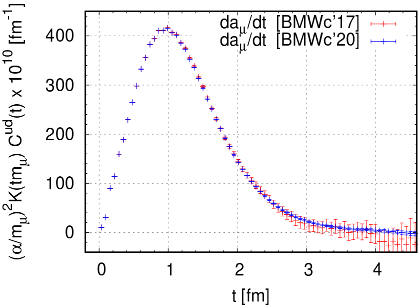


- Low Mode Averaging: use exact (all2all) quark propagator in IR and stochastic in UV
- decrease noise by replacing $C(t)$ by upper/lower bounds above t_c

$$0 \le C(t) \le C(t_c) e^{-E_{2\pi}(t-t_c)}$$

Noise reduction

- noise/signal in $C(t) = \langle J(t)J(0)\rangle$ grows for large distances



- Low Mode Averaging: use exact (all2all) quark propagator in IR and stochastic in UV
- decrease noise by replacing $C(t)$ by upper/lower bounds above t_c

$$0 \le C(t) \le C(t_c) e^{-E_{2\pi}(t-t_c)}$$

$\longrightarrow$ few permil level accuracy on each ensemble

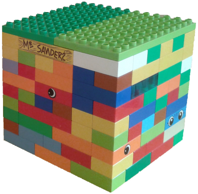
Finite-size effects

- Typical lattice runs use $L < 6\,\mathrm{fm}$, earlier model estimates gave $O(2)\%$ FV effect.

Finite-size effects

- Typical lattice runs use $L < 6\,\text{fm}$, earlier model estimates gave $O(2)\%$ FV effect.

$L_{\text{ref}} = 6.272\,\text{fm}$

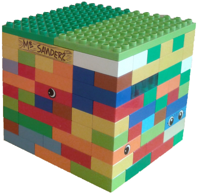


$L_{\text{big}} = 10.752\,\text{fm}$

Finite-size effects

- Typical lattice runs use $L < 6\,\mathrm{fm}$, earlier model estimates gave $O(2)\%$ FV effect.

$L_{\mathrm{ref}} = 6.272\,\mathrm{fm}$

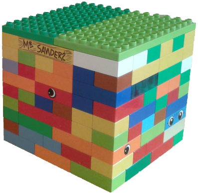


$L_{\mathrm{big}} = 10.752\,\mathrm{fm}$

1. $a_\mu(\mathrm{big}) - a_\mu(\mathrm{ref})$
   - perform numerical simulations in $L_{\mathrm{big}} = 10.752\,\mathrm{fm}$
   - perform analytical computations to check models

Finite-size effects

- Typical lattice runs use $L < 6\,\mathrm{fm}$, earlier model estimates gave $O(2)\%$ FV effect.

$L_{\mathrm{ref}} = 6.272\,\mathrm{fm}$   $L_{\mathrm{big}} = 10.752\,\mathrm{fm}$

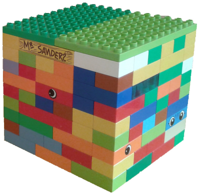
1. $a_\mu(\mathrm{big}) - a_\mu(\mathrm{ref})$
   - perform numerical simulations in $L_{\mathrm{big}} = 10.752\,\mathrm{fm}$
   - perform analytical computations to check models

lattice	NLO XPT	NNLO XPT	MLLGS	HP	RHO
$18.1(2.0)_{\mathrm{stat}}(1.4)_{\mathrm{cont}}$	11.6	15.7	17.8	16.7	15.2

Finite-size effects

- Typical lattice runs use $L < 6\,\mathrm{fm}$, earlier model estimates gave $O(2)\%$ FV effect.

$L_{\mathrm{ref}} = 6.272\,\mathrm{fm}$



$L_{\mathrm{big}} = 10.752\,\mathrm{fm}$

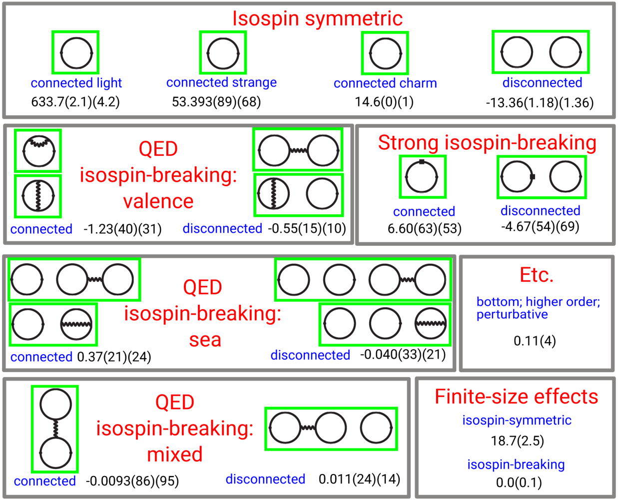
1. $a_\mu(\mathrm{big}) - a_\mu(\mathrm{ref})$
    - perform numerical simulations in $L_{\mathrm{big}} = 10.752\,\mathrm{fm}$
    - perform analytical computations to check models

lattice	NLO XPT	NNLO XPT	MLLGS	HP	RHO
$18.1(2.0)_{\mathrm{stat}}(1.4)_{\mathrm{cont}}$	11.6	15.7	17.8	16.7	15.2

2. $a_\mu(\infty) - a_\mu(\mathrm{big})$
    - use models for remnant finite-size effect of “big” $\sim 0.1\%$

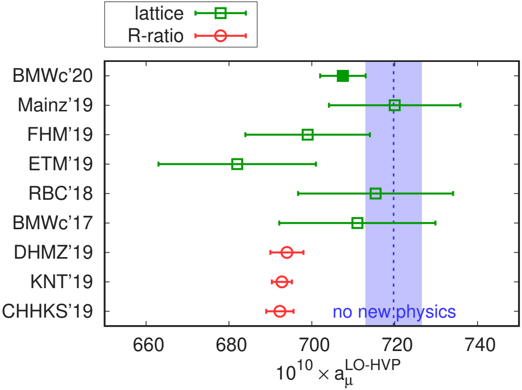
Isospin breaking effects

- Include leading order IB effects: $\mathcal{O}(e^2)$, $\mathcal{O}(\delta m)$

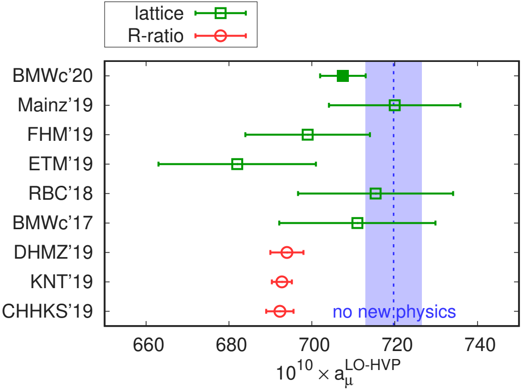


$$10^{10}\times a_\mu^{\text{LO-HVP}} = 707.5(2.3)_{\text{stat}}(5.0)_{\text{sys}}[5.5]_{\text{tot}}$$

Final result

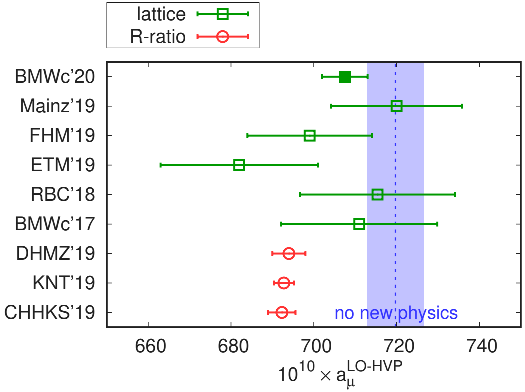


Final result



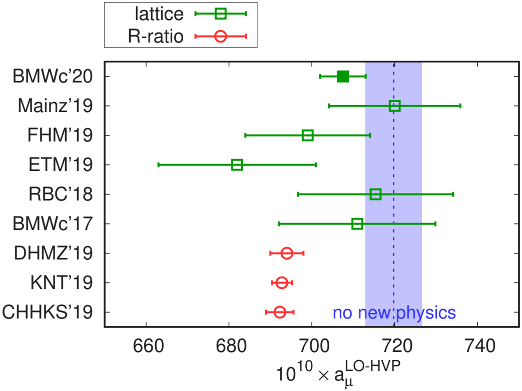
- $a_\mu^{\text{LO-HVP}} = 707.5(2.3)(5.0)[5.5]$ with 0.8% accuracy

Final result



- $a_\mu^{\text{LO-HVP}} = 707.5(2.3)(5.0)[5.5]$ with 0.8% accuracy
- consistent with new FNAL experiment

Final result

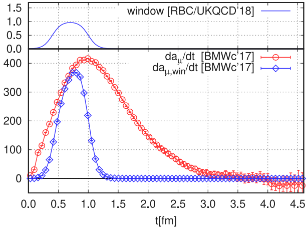


- $a_\mu^{\text{LO-HVP}} = 707.5(2.3)(5.0)[5.5]$ with 0.8% accuracy
- consistent with new FNAL experiment
- 2.0σ larger than [DHMZ'19], 2.5σ than [KNT'19]

Window observable

- Restrict correlator to window between $t_1 = 0.4\,\text{fm}$ and $t_2 = 1.0\,\text{fm}$

[RBC/UKQCD'18]

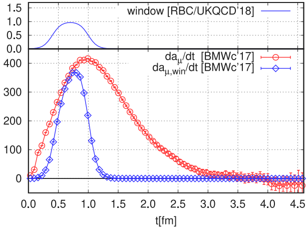


(144×96^3, $a \sim 0.064$ fm, $M_\pi \sim 135$ MeV)

Window observable

- Restrict correlator to window between $t_1 = 0.4\,\text{fm}$ and $t_2 = 1.0\,\text{fm}$

[RBC/UKQCD'18]



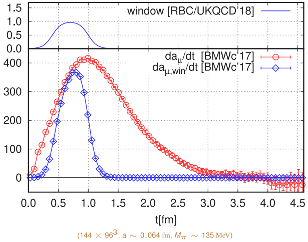
$(144 \times 96^3,\ a \sim 0.064\ \text{fm},\ M_\pi \sim 135\ \text{MeV})$

- Less challenging than full a_μ

Window observable

- Restrict correlator to window between $t_1 = 0.4\,\mathrm{fm}$ and $t_2 = 1.0\,\mathrm{fm}$

[RBC/UKQCD'18]

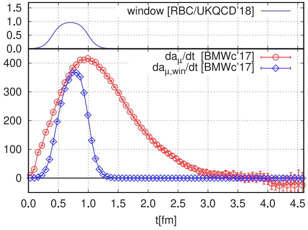


- Less challenging than full a_μ
  - signal/noise
  - finite size effects
  - lattice artefacts (short & long)

Window observable

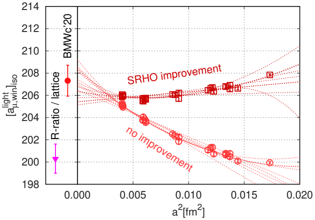
- Restrict correlator to window between $t_1 = 0.4\,\mathrm{fm}$ and $t_2 = 1.0\,\mathrm{fm}$

[RBC/UKQCD'18]



(144 × 96³, $a \sim 0.064$ fm, $M_\pi \sim 135$ MeV)

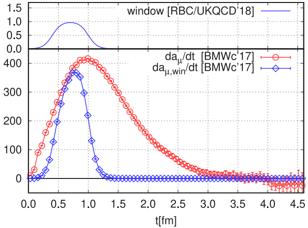
- Less challenging than full a_μ
  - signal/noise
  - finite size effects
  - lattice artefacts (short & long)



Window observable

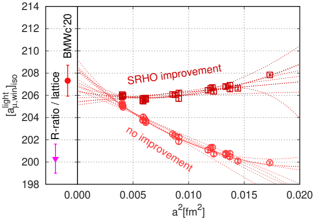
- Restrict correlator to window between $t_1 = 0.4\,\mathrm{fm}$ and $t_2 = 1.0\,\mathrm{fm}$

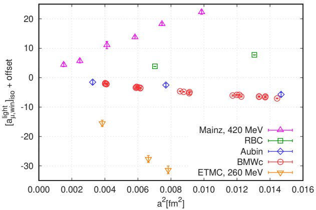
[RBC/UKQCD'18]



(144 × 96^3, $a \sim 0.064$ fm, $M_\pi \sim 135$ MeV)

- Less challenging than full a_μ
  - signal/noise
  - finite size effects
  - lattice artefacts (short & long)

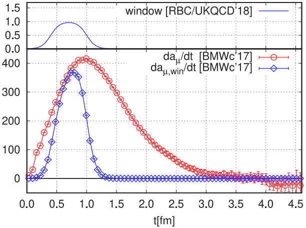




Window observable

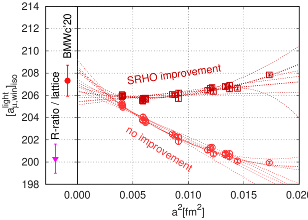
- Restrict correlator to window between $t_1 = 0.4\,\text{fm}$ and $t_2 = 1.0\,\text{fm}$

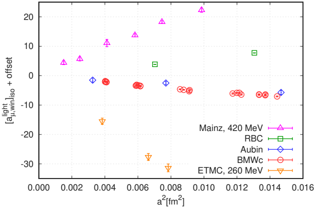
[RBC/UKQCD'18]

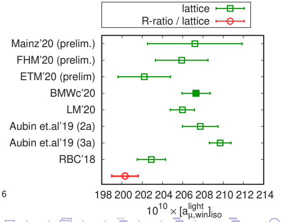


$(144 \times 96^3,\ a \sim 0.064\ \text{fm},\ M_\pi \sim 135\ \text{MeV})$

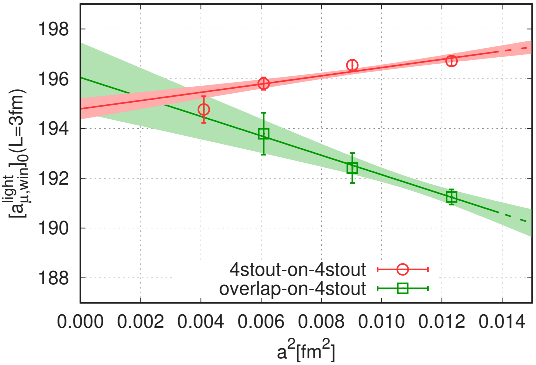
- Less challenging than full a_μ
  - signal/noise
  - finite size effects
  - lattice artefacts (short & long)



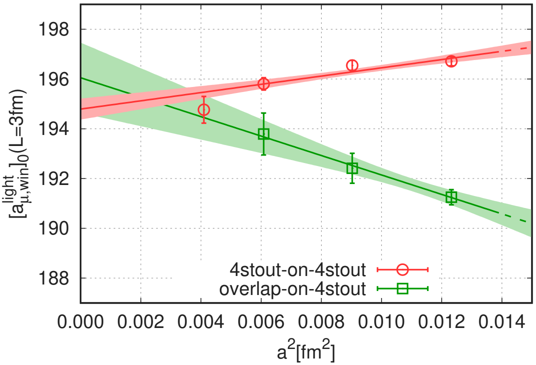




Crosscheck – overlap

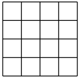


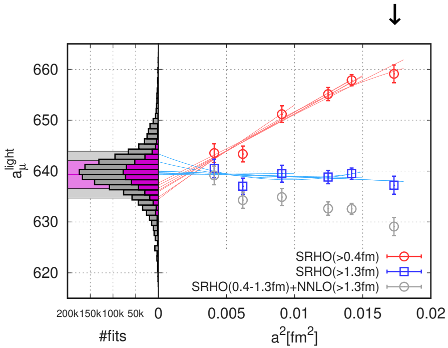
Crosscheck – overlap



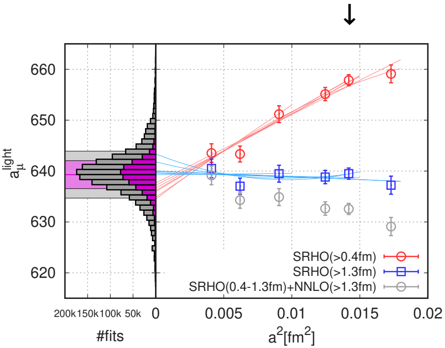
- compute $a_{\mu,\text{win}}$ with overlap valence
- local current instead of conserved $\longrightarrow$ had to compute Z_V
- cont. limit in $L = 3$ fm box consistent with staggered valence

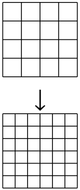
Continuum limit



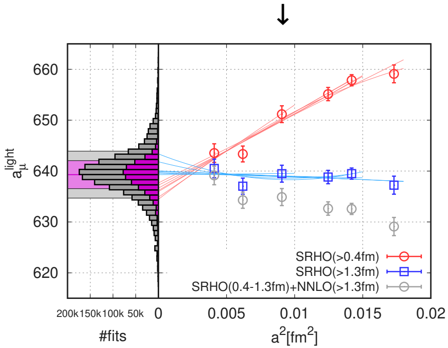


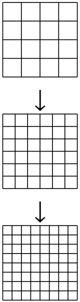
Continuum limit



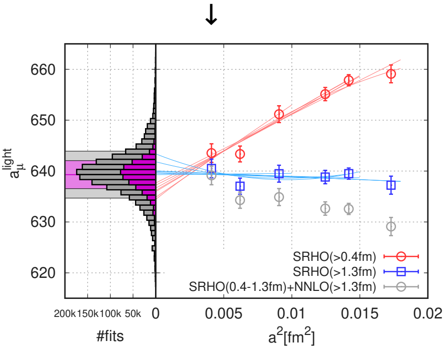


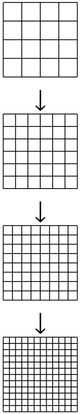
Continuum limit



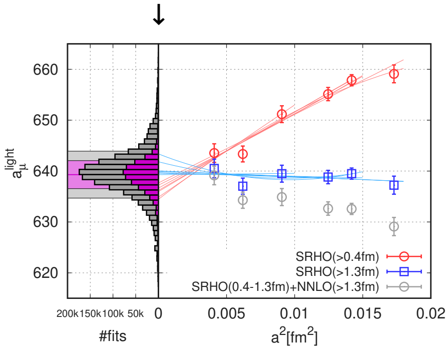


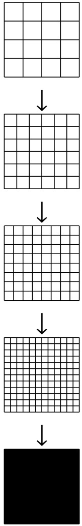
Continuum limit





Continuum limit

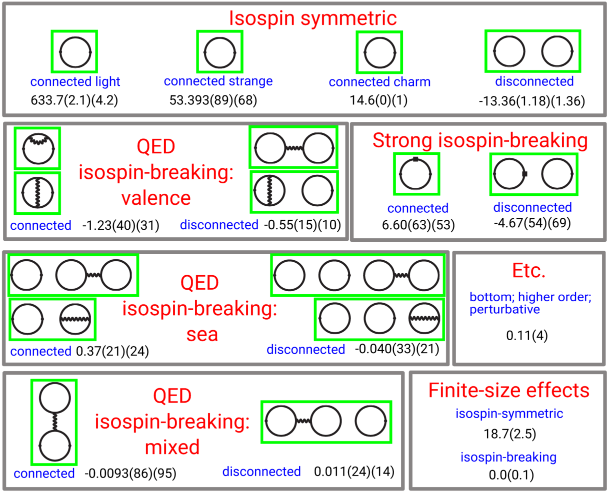




Outline

5. Summary

Final result



$$10^{10} \times a_\mu^{\text{LO-HVP}} = 707.5(2.3)_{\text{stat}}(5.0)_{\text{sys}}[5.5]_{\text{tot}}$$

Tensions: take-home message