IR Phase of Thermal QCD

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Outline:

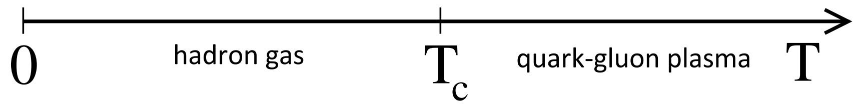
- ✓ Glue for thought
- ✓ IR phase of thermal QCD [A.A & I.H. 1906.08047, also 1502.07732]
- ✓ IR scale invariance: exact or asymptotic? [A.A. & I.H. 2103.05607]
  - IR effective dimensions [I.H. and R.M. 1807.03995, I.H. , P.M. & R.M. 2205.11520]
  - Anderson connection [A.A. & I.H. 2110.04833]
- ✓ Current picture

I. Glue for Thought

[1906.08047 , 1502.07732]

setting:

$T > 0$, $\mu_B = 0$ $\longrightarrow$ LHC, early universe



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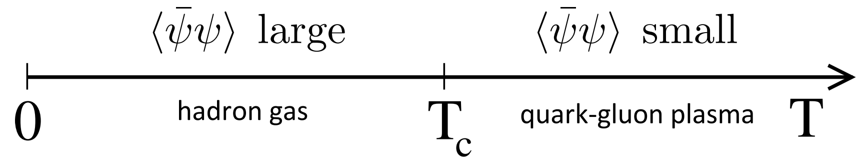
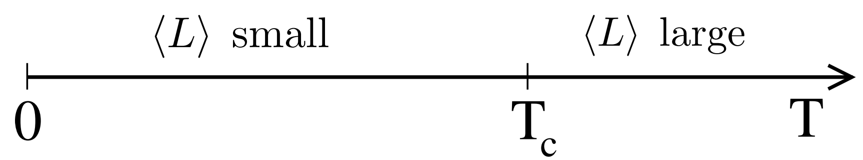


glue view:



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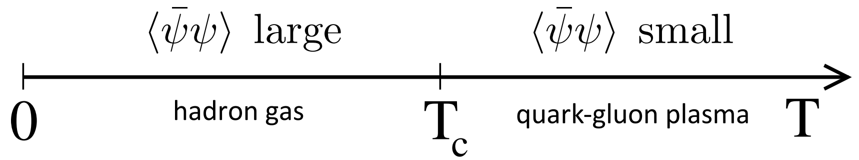

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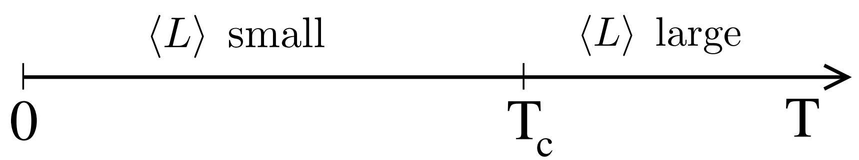

Quarks won the popularity contest [$T_c \approx 155$ MeV, crossover , Aoki et al, 2007]

But it would be great to have a robust glue-based phase criterion!

[For one, we could take it and scan all quark contents, masses, reps, temperatures etc.]

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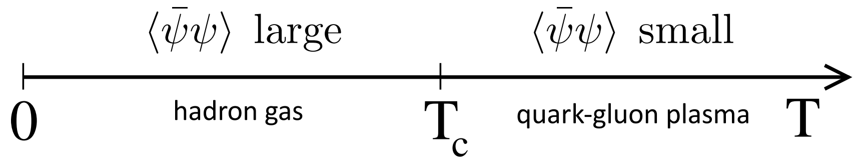

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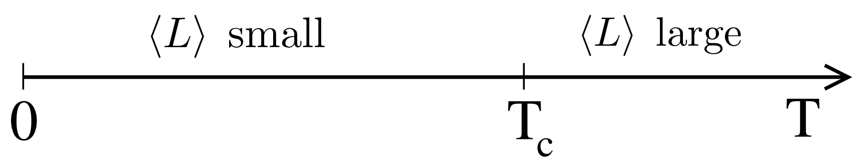

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But it would be nice to have a robust glue-based phase criterion!

[For one, we could take it and scan all quark contents, masses, reps, temperatures etc.]

Point 1: $\langle\bar\psi\psi\rangle$ is cleaner because it reflects deeper IR of glue

Point 2: both $\langle\bar\psi\psi\rangle$ and $\langle L\rangle$ are limited in terms of reflecting glue

Point 3: need glue probe that is sensitive to any scale by construction

object with explicit scale dependence

I. Glue for Thought…

Point 4: DIRAC SPECTRAL DENSITY FITS THE BILL What??? [quark-looking construct and all]

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- Dirac spectral density as glue operator

$$S = S_g(U) + \sum_{f=1}^{N_f} \bar{\psi}_f [\, D_s(U) + m_f \,] \psi_f$$

$$\text{arbitrary } \; D = D(U) \; : \; D\chi_k = \lambda_k \chi_k$$

$$\sigma(\lambda, U) \equiv \frac{1}{V} \sum_{0 \le \lambda_k(U) < \lambda} 1$$

gauge invariant
scale dependent
glue operator

$$\sigma(\lambda, m_f) \, = \, \langle\, \sigma(\lambda, U) \,\rangle_{m_f}$$

$$\rho(\lambda, m_f) \, = \, \frac{\partial}{\partial \lambda} \sigma(\lambda, m_f)$$

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$\longrightarrow$ one that resolves IR best $\longrightarrow$ spectral density of overlap operator

- Can such complex composite operator give us clues about glue?

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- Freedom to choose the best “glue operator of Dirac spectral density” σ=σ(D)

  $\longrightarrow$ one that resolves IR the best $\longrightarrow$ spectral density of overlap operator

- Can such monster of a composite operator tell us something useful about glue?

[hep-th/9811212 & other]

Example: set $D = Ds$, $N_f = 2$, $m_1 = m_2$ chiral expansion of $\rho(\lambda, m)$: terms $+\, c \, \ln(m/\lambda)$

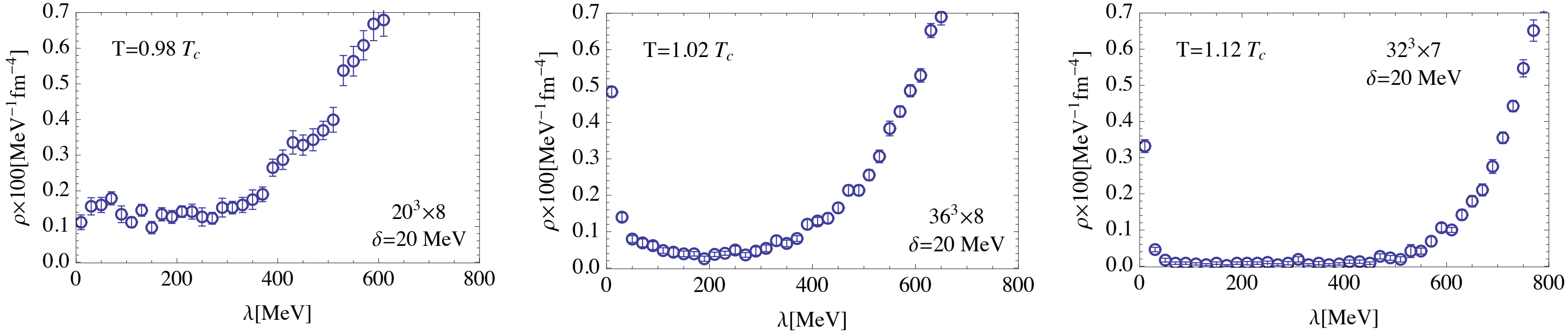
Is the existence of this logarithmic IR peak telling us something about IR glue?

Most certainly! We just don’t know what. BUT

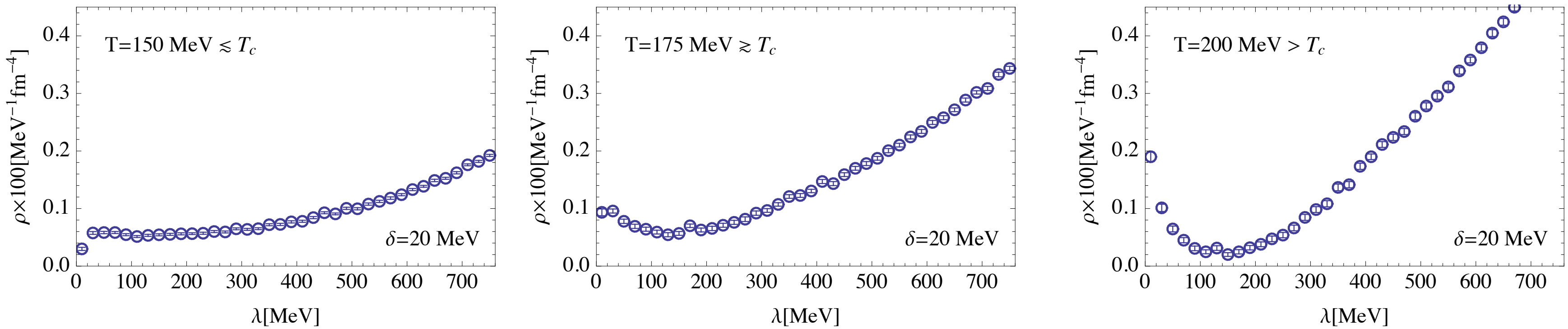
I. Glue for Thought… BUT…

- Peak in IR overlap spectrum upon crossing T_c (pure glue) [Edwards, Heller, Narayanan, Kiskis, 1999]

- Our version of it [AA & IH, 1502.07732]

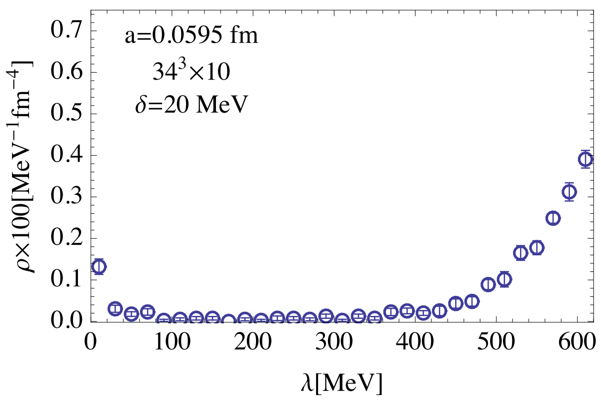


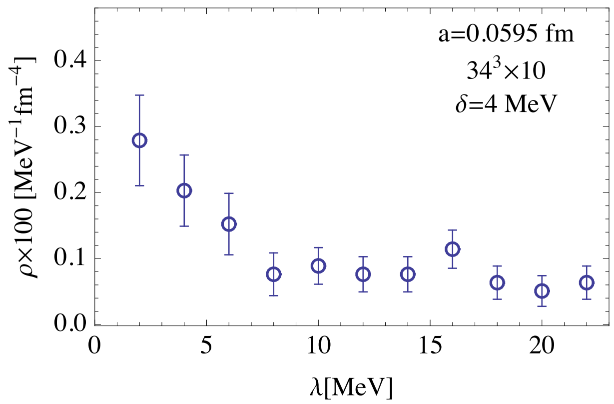
- knee-jerk reaction was: quenched chiral condensate may diverge in high-temperature pure glue
- knee-jerk reaction should be: what on earth is glue doing to produce this? [1502.07732]
- didn't know but went on with it, e.g., around chiral crossover we got this:

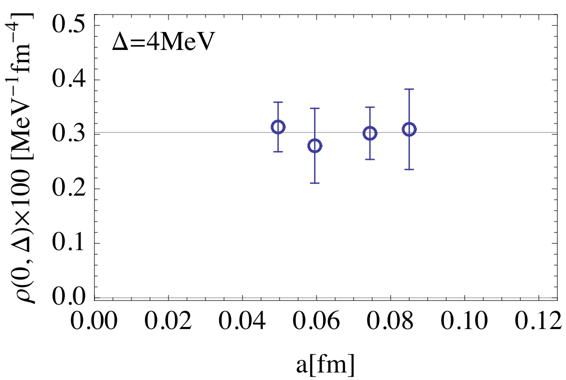


I. Glue for Thought… BUT…

Key for taking this IR structure seriously: Persists into continuum limit [AA & IH, 1502.07732]

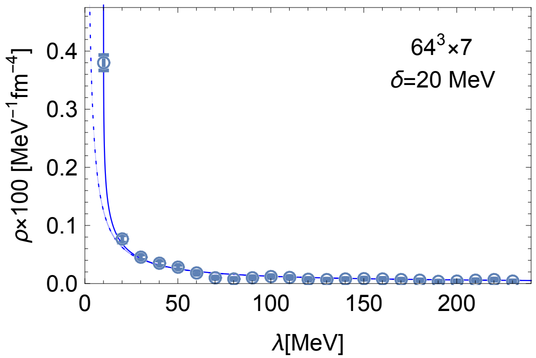


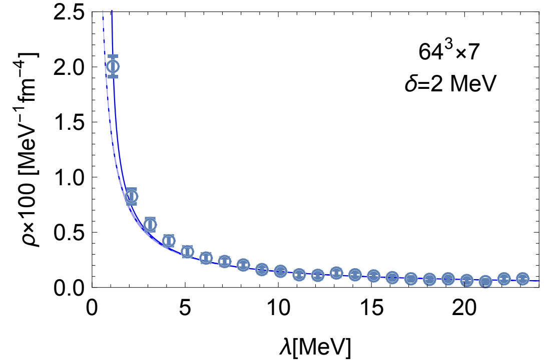


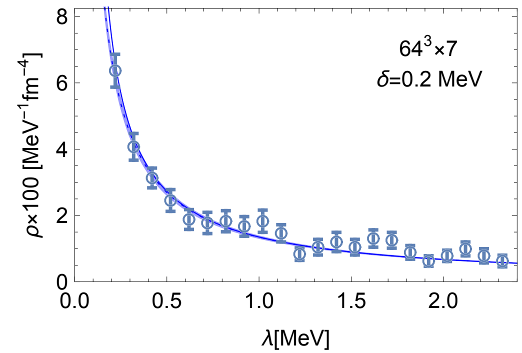


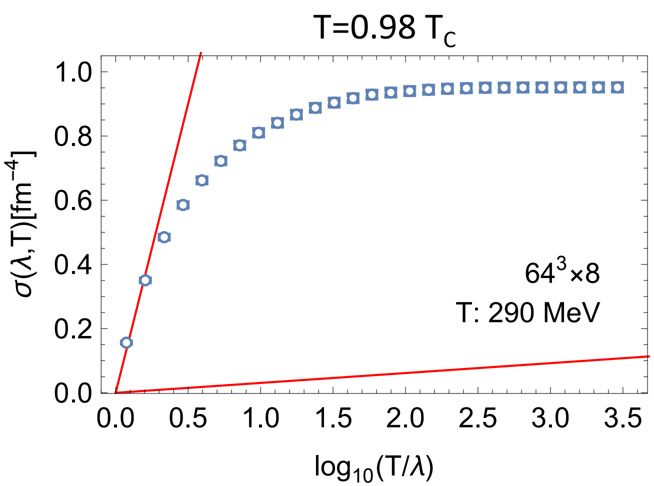
II. IR Phase of Thermal QCD: Point 1 [AA & IH 1906.08047]

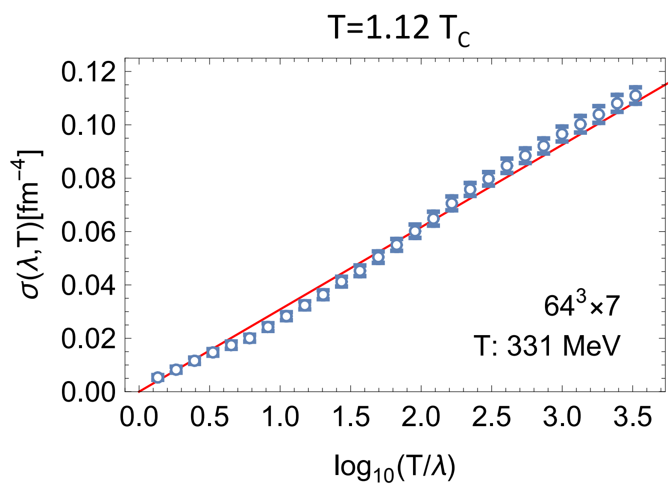
Surprise: fits to ρ(λ) ∝ 1/λ [pure-glue QCD]







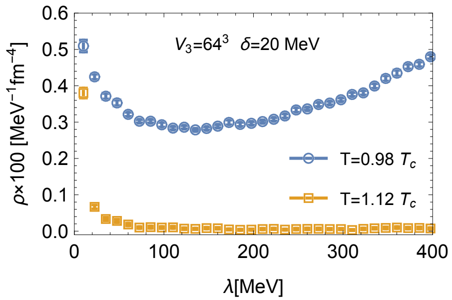


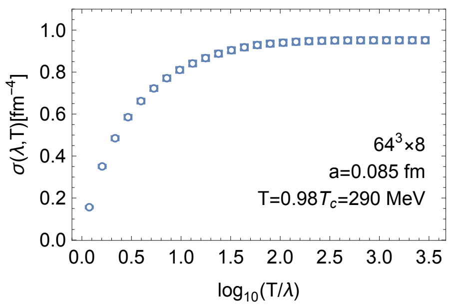


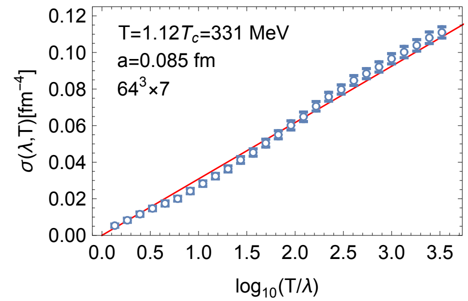
<u>Conjencture</u>: IR scale invariant density ⟶ IR scale invariance of glue restored

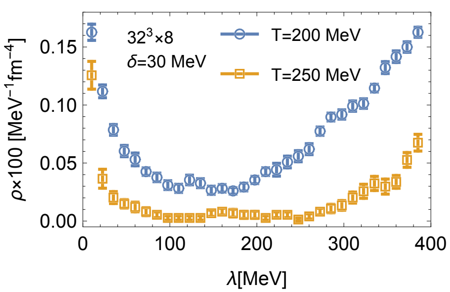
II. IR Phase of Thermal QCD: Point 1 [AA & IH 1906.08047] ...

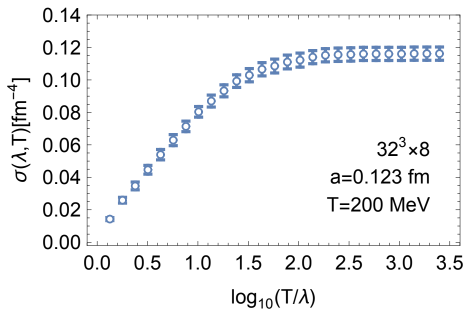
- Real-world QCD (N_f=2+1, BMW configs)

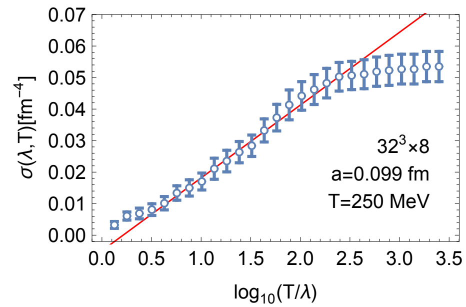




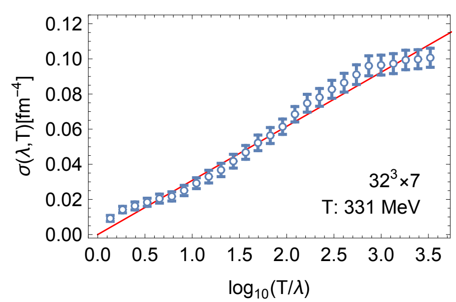








pure-glue lattice of comparable volume

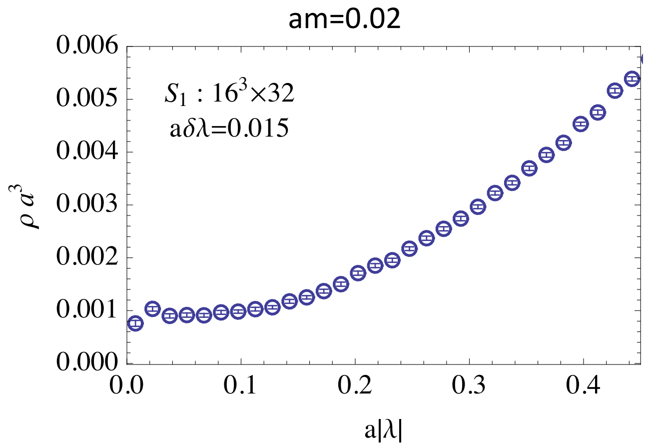


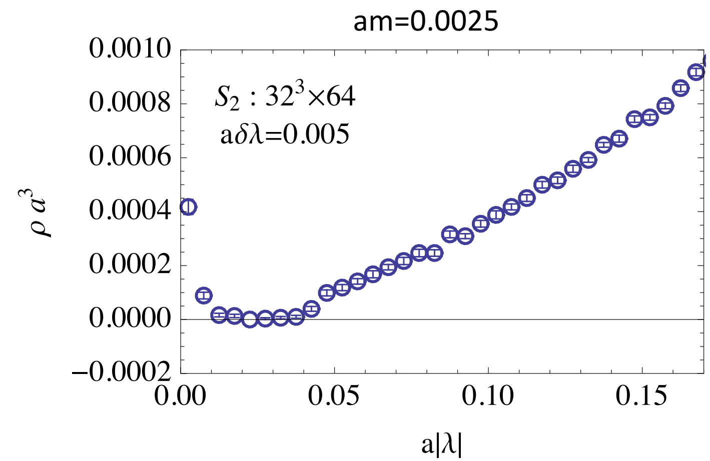
Proposition: QCD has a phase with scale-invariant glue - IR phase

$$200\,\mathrm{MeV} \;<\; T_{\mathrm{IR}} \;<\; 250\,\mathrm{MeV}$$

II. IR Phase of Thermal QCD: Point 2 [AA & IH 1906.08047]

N_f=12, T=0   Ensembles: A. Hasenfratz et al, 1207.7162   staggered with nHYP, β_F=2.8, β_A/β_F=-0.25





A.A, I.H. 1405.2968, 1411.1777

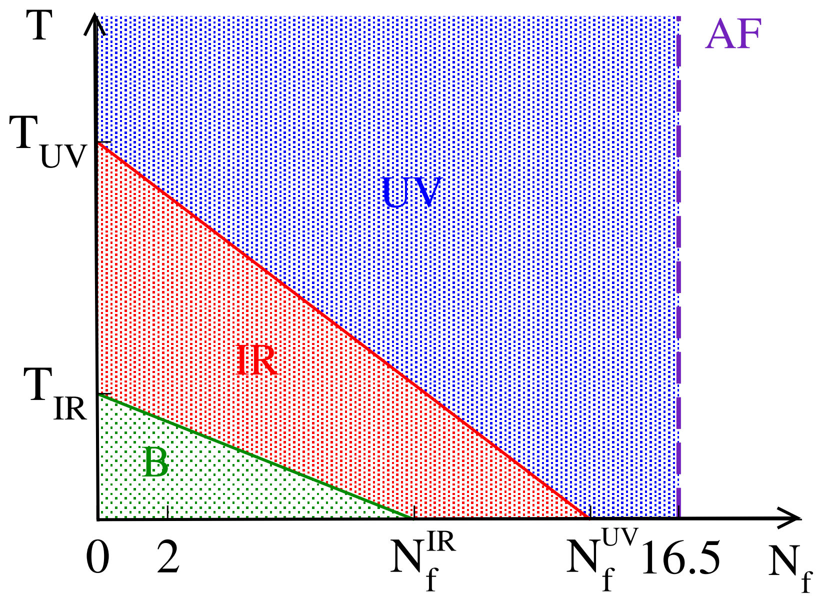
- Light quarks generate bimodality without temperature!
- Since near conformal window, it is expected $\rho(\lambda) \propto \lambda^p$ but then p<0!

<u>Proposition</u>: Conformal window has a strongly coupled part with p < 0

$$N_f^c \equiv N_f^{\rm IR} < N_f < N_f^{\rm UV} \leq 16.5$$

II. IR Phase of Thermal QCD: Point 2 [AA & IH 1906.08047] …

- Near-massless SU(3) theories with fundamental quarks



IR phase of QCD is in the same contiguous phase as strongly-coupled part of conf window

- Scale invariant glue is expected in the conformal window!

- Conjecture: IR phase of thermal QCD may feature full IR scale invariance

II. IR Phase of Thermal QCD: [AA & IH 1906.08047] ...

PHASE STRUCTURE OF QCD AND pgQCD IN TERMS OF IR SCALE INVARIANCE

HADRON GAS	NEAR-PERFECT FLUID	WEAKLY-COUPLED QGP
IR-broken	IR-symmetric	IR-trivial

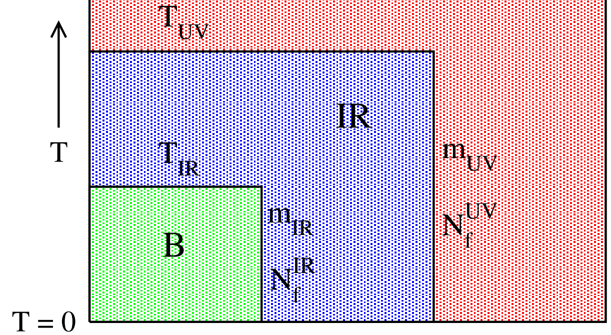
≈150   200-250   > 1000

STRONGLY COUPLED	STRONGLY COUPLED	PERTURBATIVE	
p = 0	p ≳ -1	p > 0	P = leading IR power in $\rho(\lambda)$

For details see this Wuhan talk:

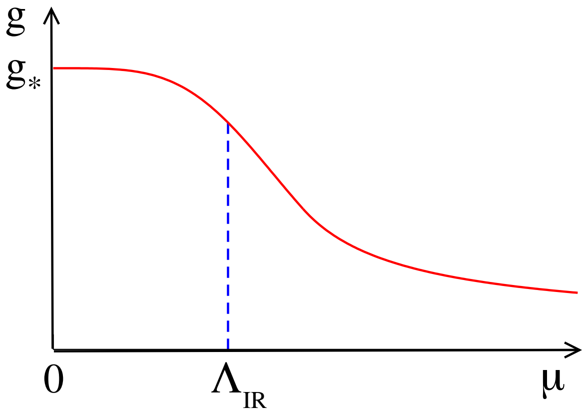
https://indico.cern.ch/event/764552/contributions/3420459/attachments/1865996/3068382/WuHan_jun_2019_infra.pdf

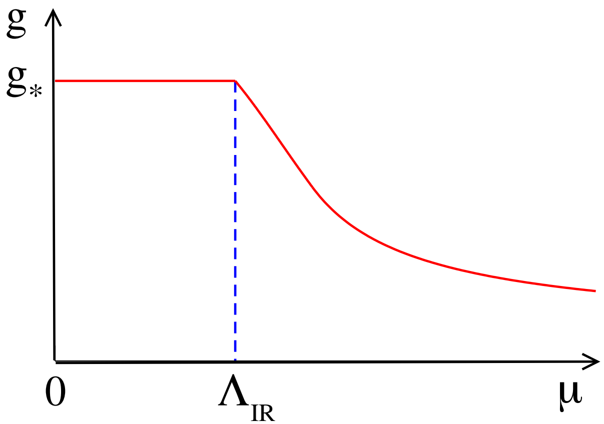
SU(3) theories with fundamental quarks



Λ$_{IR}$ = onset of scale invariance

III. Asymptotic or Exact?

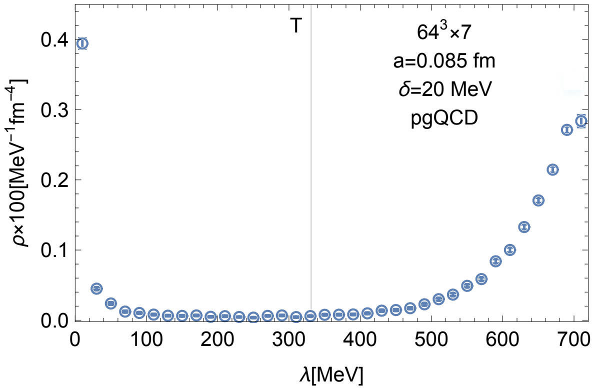




g(μ) non-analytic at Λ_{IR}

Q: How does the non-analyticity arise?

Proposed answer: [AA & IH 1906.08047]



bimodality → IR glue fluctuates independently of bulk

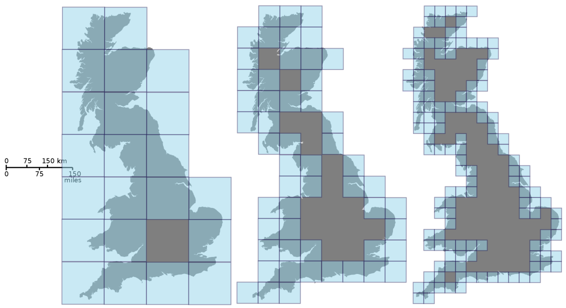
Makes non-analyticity natural (mismatch)!

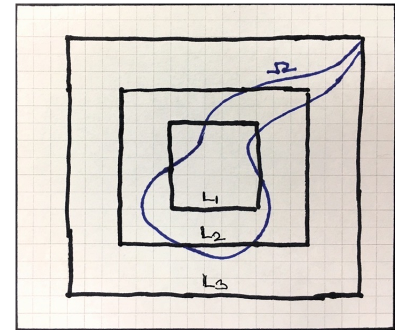
$$\Lambda_{\rm IR} < \lambda_{min} < T$$

Not very convincing: needs more

III. Asymptotic or Exact?...

Study <u>IR measure-based dimension</u>: global features, L $\longrightarrow \infty$



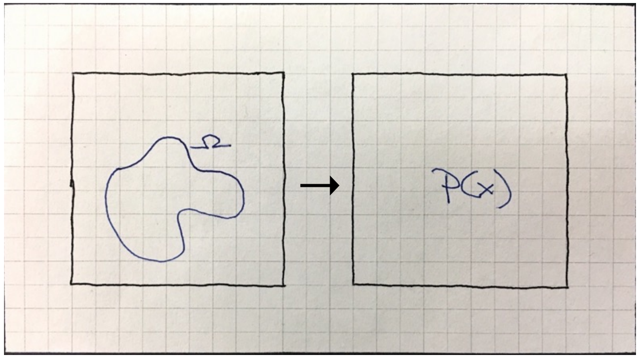


$$V(\Omega, L) \propto L^{d_{\mathrm{IR}}(\Omega)}$$

Rarely discussed but useful!

III. Asymptotic or Exact?...

But how to proceed in field theory?



$$P(x) \quad \Longrightarrow \quad \Omega_{\rm eff}$$

How???

[And we are on the lattice to begin with…]

Since measure has to be assigned to $\Omega_{\rm eff}$ for dimension à la Minkowski/Hausdorff:

1) count how many points $\mathcal{N} = \mathcal{N}[P] = \mathcal{N}(p_1, p_2, \ldots, p_N)$ are effectively selected by *P* (must be additive counting in order to obtain measure)

2) select $\Omega_{\rm eff}$ as $\mathcal{N}$ most probable points on the lattice

3) proceed as with Box

Consistent realization of the above program leads to unique effective dimension

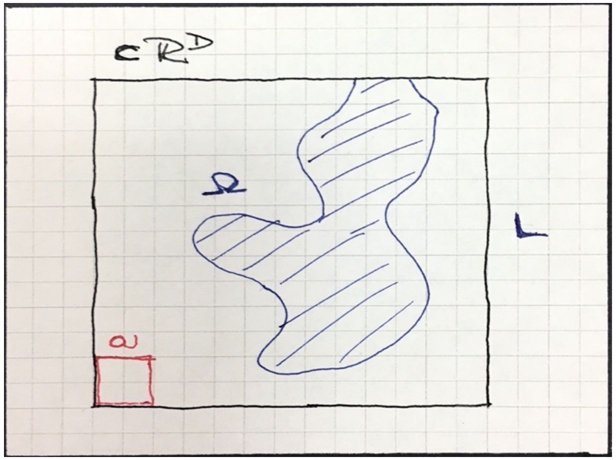
$$\mathcal{N}_\star[P] = \sum_{i=1}^{N} \mathfrak{n}_\star(Np_i) \quad , \quad \mathfrak{n}_\star(c) = \min\{c, 1\}$$

I.H. and R. M., 1807.03995
I.H. , P. M. and R.M. 2205.11520

$$\text{Box:} \quad N \longrightarrow N_+ \qquad \text{Effective:} \quad N \longrightarrow \mathcal{N}_\star[P]$$

Defines measure-based dimension for probabilistic sets (effective subsets).

III. Asymptotic or Exact?...



characterize both fine (UV) and global (IR) features

$$N \propto (L/a)^D$$

$$\text{UV:} \quad N_+(a,L) \propto a^{-d_{\rm UV}(L)} \;, \quad a \to 0$$

$$\text{IR:} \quad N_+(a,L) \propto L^{d_{\rm IR}(a)} \;, \quad L \to \infty$$

Effective Dimensions of Dirac eigenmodes:

$$\text{UV:} \quad \langle \mathcal{N}_\star \rangle_{a,L,\lambda} \propto a^{-d_{\rm UV}(L,\lambda)} \quad , \quad a \to 0$$

$$\text{IR:} \quad \langle \mathcal{N}_\star \rangle_{a,L,\lambda} \propto L^{d_{\rm IR}(a,\lambda)} \quad , \quad L \to \infty$$

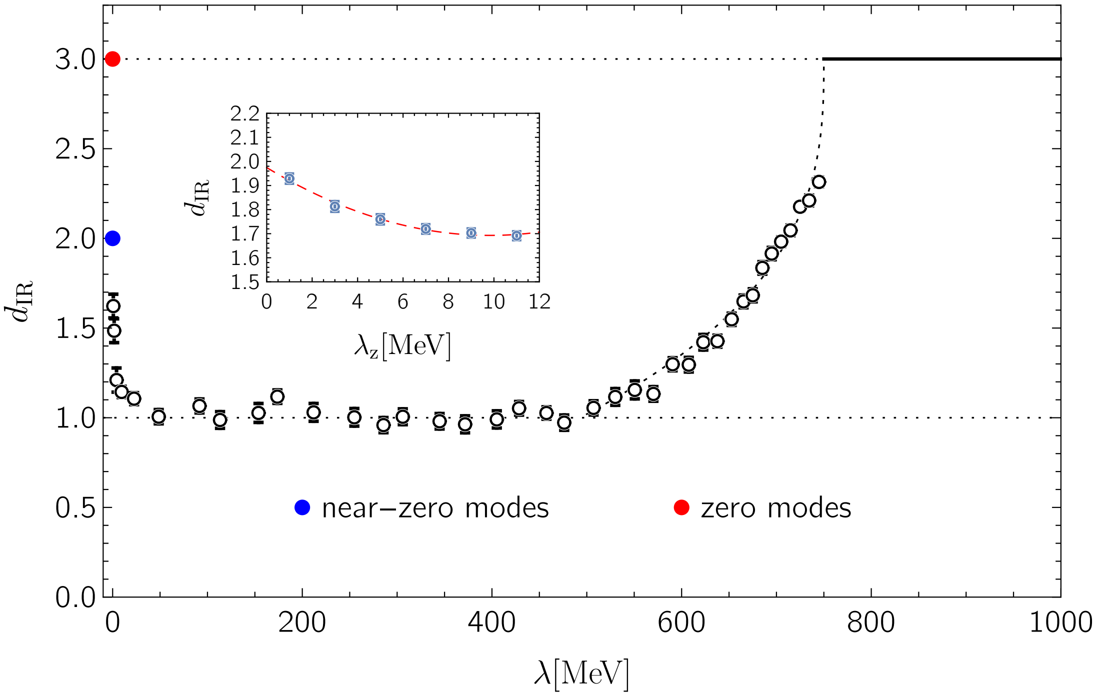
Finite T: $N = \left(\frac{L}{a}\right)^3 \frac{1}{Ta}$ $\qquad P=(p_1,p_2,\ldots,p_N) \;, \quad p_i=\psi_\lambda^+\psi_\lambda(x_i)$

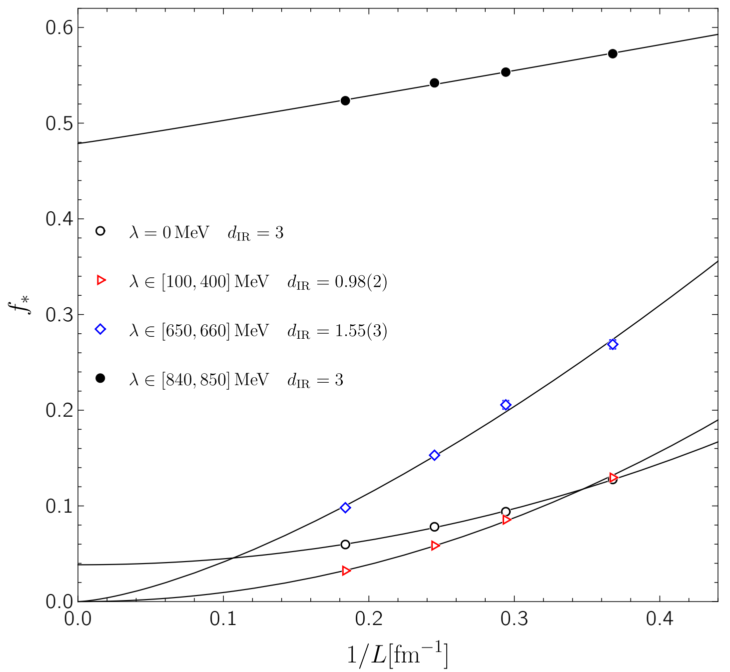
We are interested in IR dimension here.

III. Asymptotic or Exact?...

pure-glue QCD, *a*=0.085 fm, Wilson action, overlap Dirac, T=1.12T_{IR}, L_{max} = 5.5 fm

$$f_\star = \frac{\mathcal{N}_\star}{N} \propto \left(\frac{1}{L}\right)^{3-d_{\rm IR}}$$



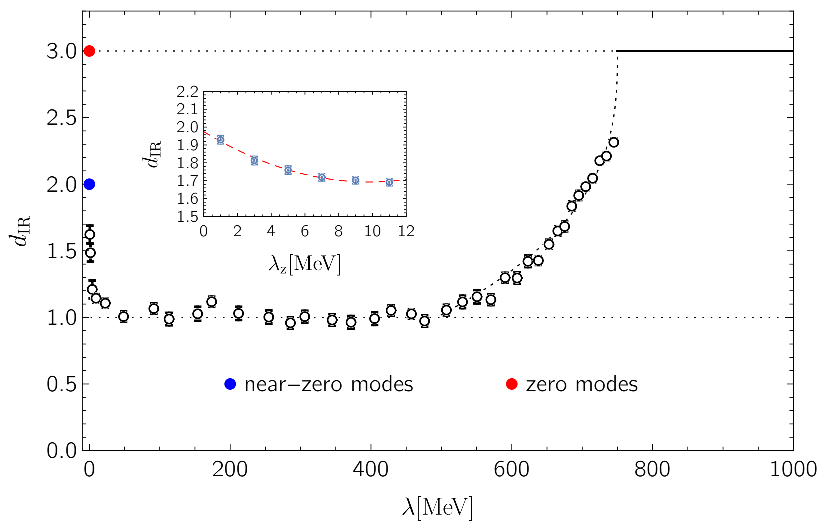


$\lambda_{\rm IR} = 0$ $\lambda_A \approx 750\,{\rm MeV}$

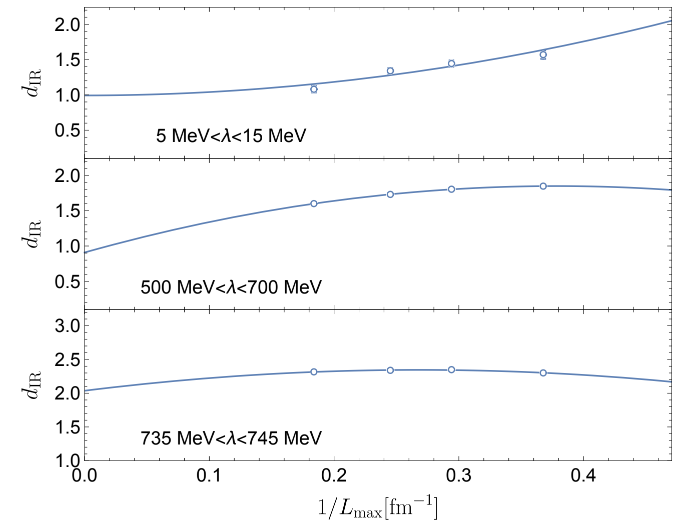
What is λ_A? This dimension break coincides with Anderson-like mobility edge!

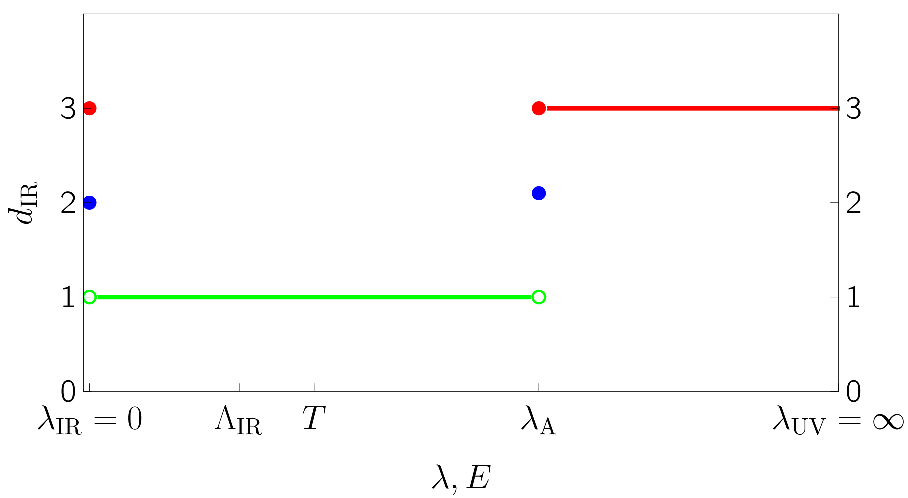
determined via methods used in localization [e.g. Giordano, Kovacs, Pitler, 1312.1179 & their other works]

III. Asymptotic or Exact?...



Volume trends toward thermodynamic limit

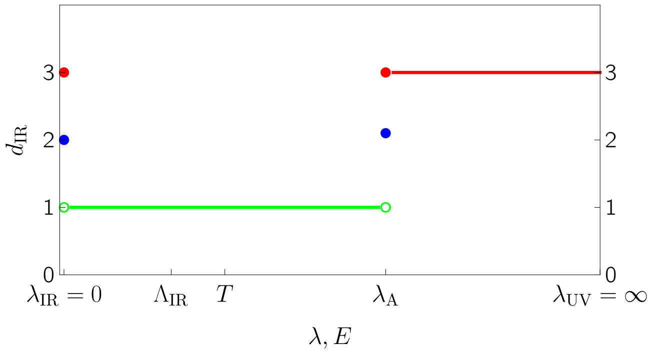




- Unusual dimension structure
- Non-analytic
- No finite Dirac scales below Λ_{IR} detected, consistently with IR scale invariance restoration.

$$d_{IR}(0^+) \equiv \lim_{L\to\infty} \lim_{\lambda\to 0^+} d_{IR}(\lambda, L)$$

Ramifications & Current Picture



I. Anderson viewpoint:

λ_{IR} appears to pass as a new "mobility edge"

[criticality and long-range IR physics] Andrei talk

II. Topological viewpoint 1:

Studied measure-based dimension but got near-integers: IR phase may have topological origin

[At T_{IR} thermal agitation destroys low QCD scales (→IR scale invariance) but not topology .]

III. Scale invariance viewpoint 1:

Crossing T_{IR} (external parameter) internal Dirac singularities λ_{IR} and λ_A appear.

[Propose: reflects non-analytic running at Λ_{IR}]

IV. Scale invariance viewpoint 2: Why does λ_A exist? Does the logic of IR phase need it?

Internal "phase transition" at λ_A (non-analyticity) makes Dirac "states above and below unrelated.

[This feature facilitates the conjectured decoupling of scale-invariant IR and makes non-analyticity of running at Λ_{IR} concrete: crossing λ_A in the RG process makes the break.]

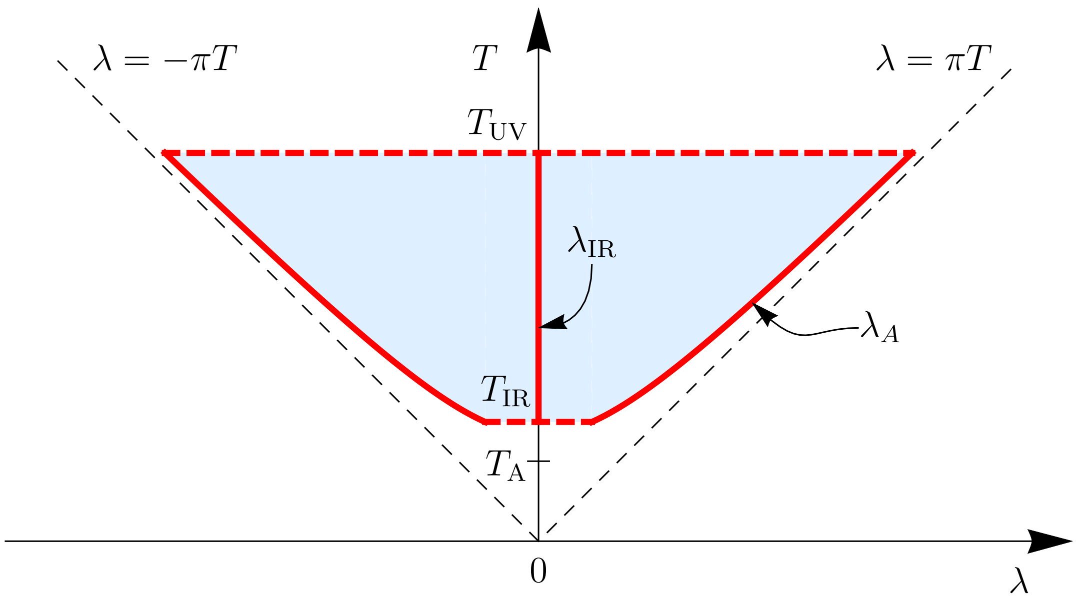
V. Topological viewpoint 2:

How to explain this dimensional structure in terms of known topology?

[Explaining d_{IR}=2 an interesting problem.]

For works that may add content to this see recent Cardinali, D'Elia, Pasqui and Vig, Kovacs

Ramifications & Current Picture...



Dirac spectral phase diagram