Topology and the Dirac spectrum in the high temperature phase of QCD

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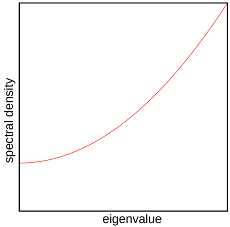
The Dirac spectrum across the transition

Standard lore

Below T_c

- Chiral symmetry broken
- Order parameter: $\rho(0) \neq 0$
  (spectral density at zero)



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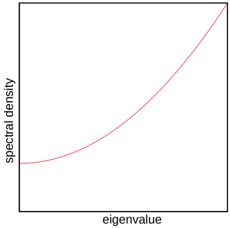
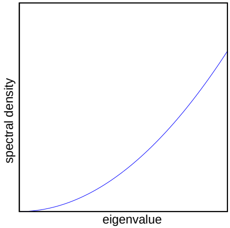


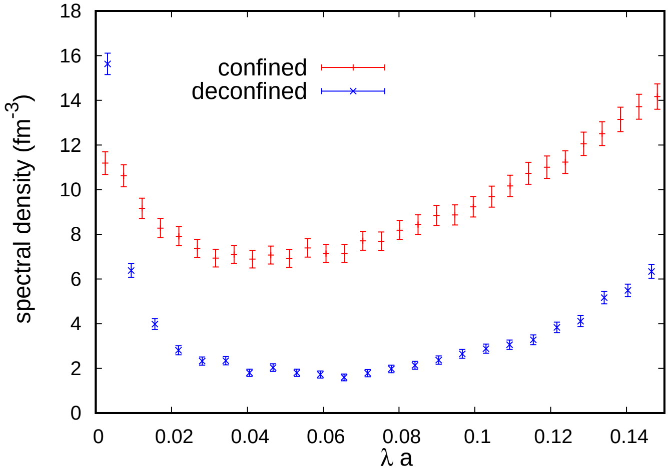
Above T_c

- Chiral symmetry restored
- Order parameter $\rho(0) = 0$



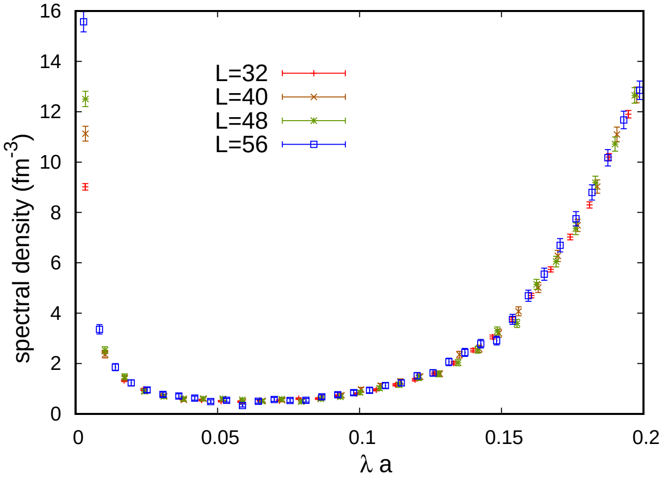
The Dirac spectral density across the transition

Lattice (quenched, $N_t = 8$, $T = T_c$)



Volume dependence of the spectral density

Lattice $N_t = 8$, $T = 1.045\,T_c$



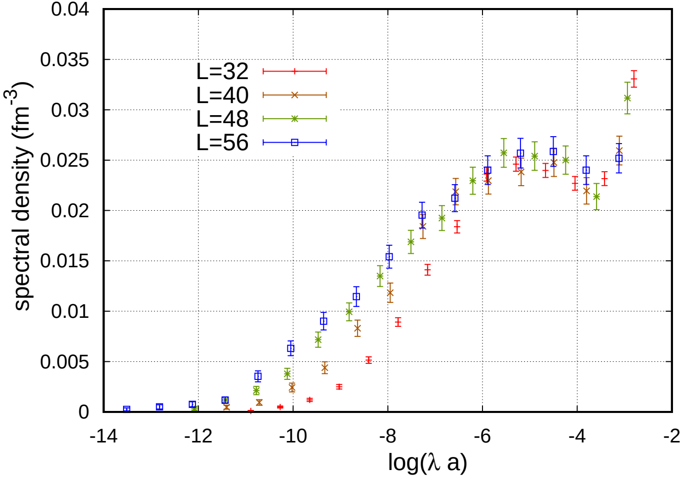
Spectral peak at $\lambda = 0$

- Spectral peak first noticed
  Edwards et al. 2000
- Spectral peak is not a quenched artifact
  Alexandru & Horvath 2015 Kaczmarek, Mazur, Sharma 2021
- Spectral density at $\lambda = 0$ nonzero
  $\Rightarrow$ chiral symmetry not restored
- Possible consequences for $U(1)_A$ breaking
- Strong volume dependence of the spectral density peak

Zooming in on the spectral peak

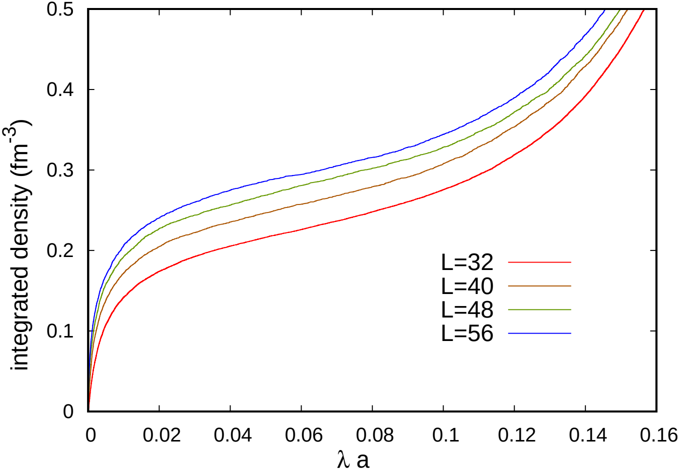
The distribution of $\log(\lambda)$

If $\rho(\lambda) \propto \lambda^{\alpha}$, then $\tilde{\rho}(y) \propto e^{(\alpha+1)y}$ $(y = \log\lambda)$



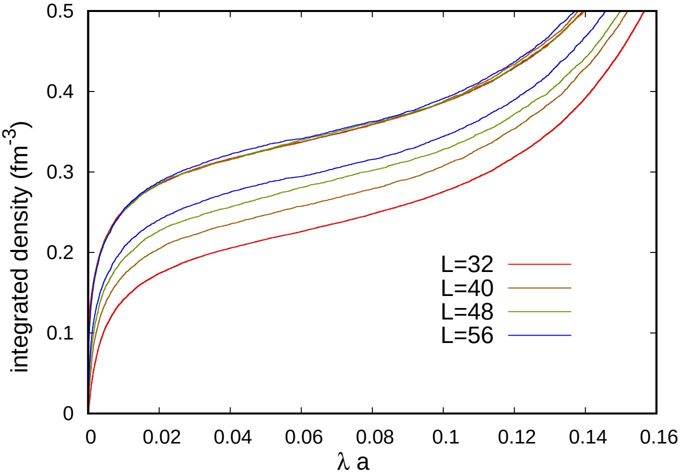
Integrated spectral density

Lattice $N_t = 8$, $T = 1.045\,T_c$



Integrated spectral density including zero modes

Lattice $N_t = 8$, $T = 1.045\,T_c$



Possible explanation of volume dependence

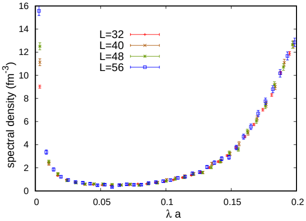
- Above T_c dilute instanton gas
  (weakly interacting lumps of charge ± 1)
- Density of topological objects $\propto V^0$
- Corresponding would be zero modes will mix to become:
  - Exact topological zero modes; density $\propto V^{-1/2}$
  - Non-chiral near zero modes; density increases with V
- Spectral peak $\equiv$ mixing zero modes (ZMZ)?

Compare distribution of number of modes in the ZMZ

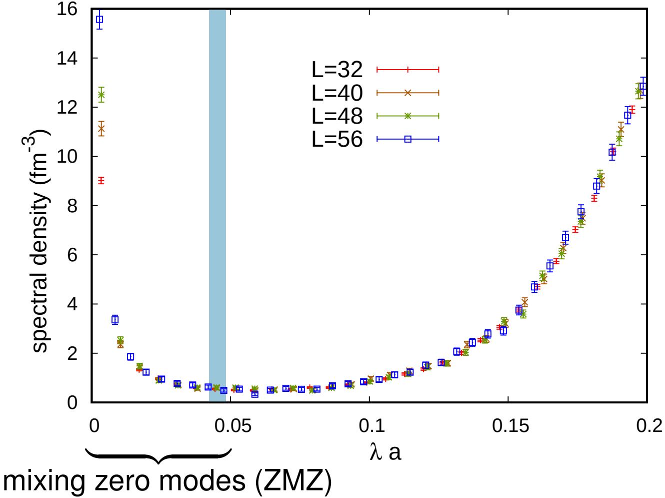
with the one expected from a non-interacting instanton gas.

How to count modes in the ZMZ?

- Assume that $|\lambda| < \lambda_{\text{ZMZ}}$ are the topology-related ev.-s
- Total number (incl. 0-modes): $n_+ + n_-$
  (number of instntons plus antiinstantons)
- Can all be counted if we know λ_{ZMZ}
- Assume ideal gas: $\Rightarrow \chi V = \langle Q^2 \rangle = \langle n_+ + n_- \rangle \Rightarrow \lambda_{\text{ZMZ}}$
- $\chi \implies \lambda_{\text{ZMZ}}$

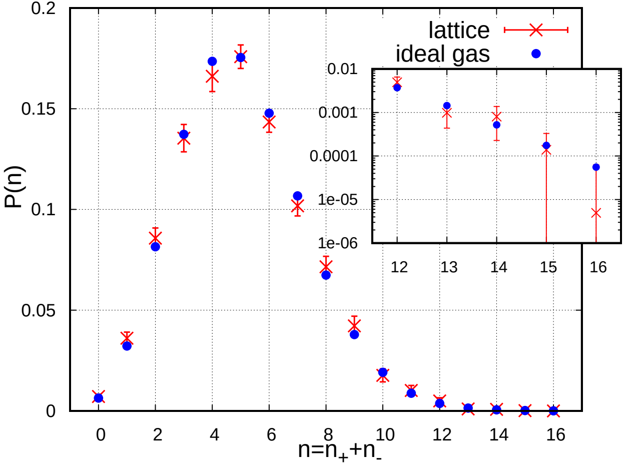


The zero mode zone obtained from $\langle Q^2 \rangle = \langle n_+ + n_- \rangle$



The distribution of $n_+ + n_-$

lattice: quenched, $N_t = 8$, $T = 1.04\,T_c$ vs. ideal gas

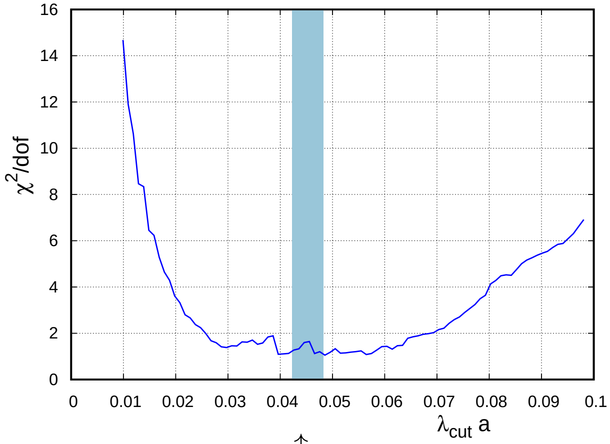


Is λ_{zmz} a special point in the spectrum?

- Distribution of number of $|\lambda| < \lambda_{\text{zmz}}$ eigenvalues Poisson
- Is this only true for λ_{zmz} obtained from $\langle Q^2 \rangle$?

λ_{zmz} is indeed special

Fit distribution of the number of $|\lambda| < \lambda_{cut}$ eigenvalues with Poisson



↑

λ_{zmz} obtained from $\langle Q^2 \rangle = \langle n_+ + n_- \rangle$

Conclusions

- Above T_c lowest part of the Dirac spectrum
  $\Longrightarrow$ mixing would be zero modes (ZMZ)

- Quenched: consistent with ideal instanton gas

- Spectral peak at zero generic feature,
  not a quenched artifact

- $\chi_{\text{top}} \neq 0 \quad \Longrightarrow \quad$ nonzero spectral density in ZMZ

- Chiral symmetry restoration is not trivial
  depends on the scales m_{q} and λ_{ZMZ}

Questions, speculations

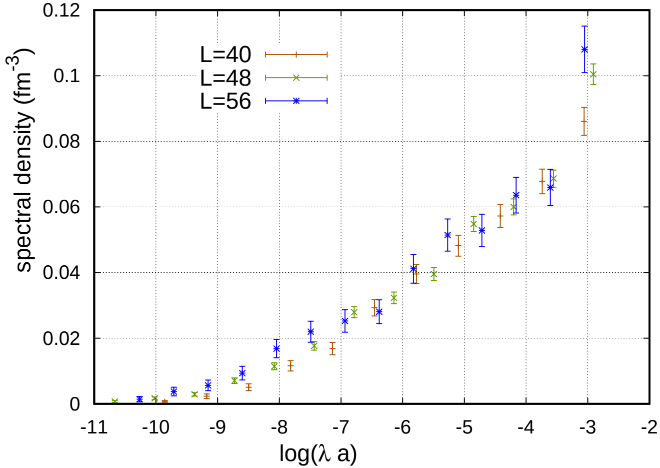
- Are there instanton interactions in the presence of dynamical quarks?
- Finite density: most dangerous eigenvalues in the ZMZ
- What is the connection bewteen
  - deconfinement
  - topological chatrge fluctuations
  - chiral symmetry restoration
  - localization

  ?

Backup slides

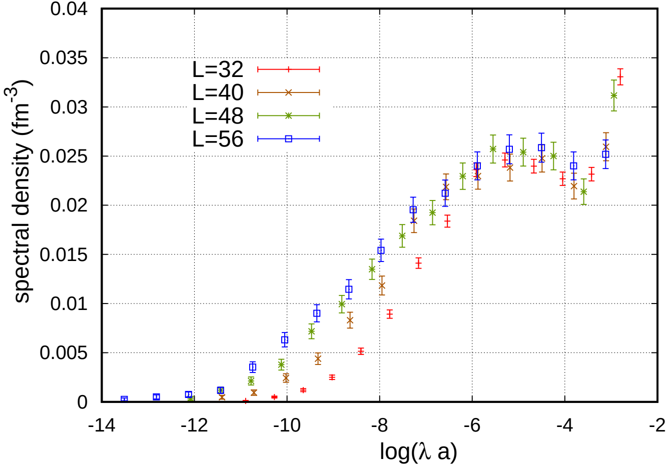
Spectral density at $T = T_c$

(density of $log\lambda$)



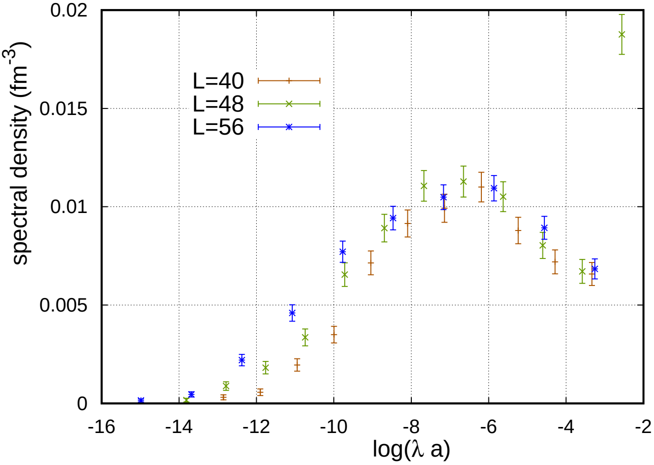
Spectral density at $T = 1.045\,T_c$

(density of $log\lambda$)



Spectral density at $T = 1.11\,T_c$

(density of $log\lambda$)



Spectral density at at different temperatures above T_c

(density of $log\lambda$)