Fractional Topological Charge

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to appear

For $SU(N)/Z(N)$ without quarks:

As volume $\to \infty$, conf.'s $\bar{c}$ top. chg. $1/N$

(Unsal, Poppitz, Anber...)

"Quantum": size ~ confinement scale. Not all ρ.

Dominate @ $T \lesssim T_{deconf}$ not @ high T

$\bar{c}$ quarks - much more complicated

e.g.: vacuum, cold, dense qks

Instantons @ large N

$$S_I \sim \frac{8\pi^2}{g^2} = \left(\frac{8\pi^2}{g^2 N}\right) N \Rightarrow e^{-S_I} \sim e^{-\# N} \quad \text{as} \quad N \to \infty$$

But $\frac{\chi}{\sigma^2} = \frac{1}{\sigma^2} \frac{\partial^2 F(\theta)}{\partial \theta^2} \sim 1$ as $N \to \infty$

Witten, Veneziano '78

<u>If</u> $Q_{top} \sim \frac{1}{N}$, $e^{-S_{frac}} \sim e^{-8\pi^2/(g^2 N)} \sim 1$

't Hooft '81, Sedlacek '82 ...

$Q_{top} \sim 1/N$ <u>only</u> $\bar{c}$ $\mathbb{Z}(N)$ twisted boundary conditions

True for us.

But vacuum $= \sum_{1/N} Q_{top}$.

Lattice

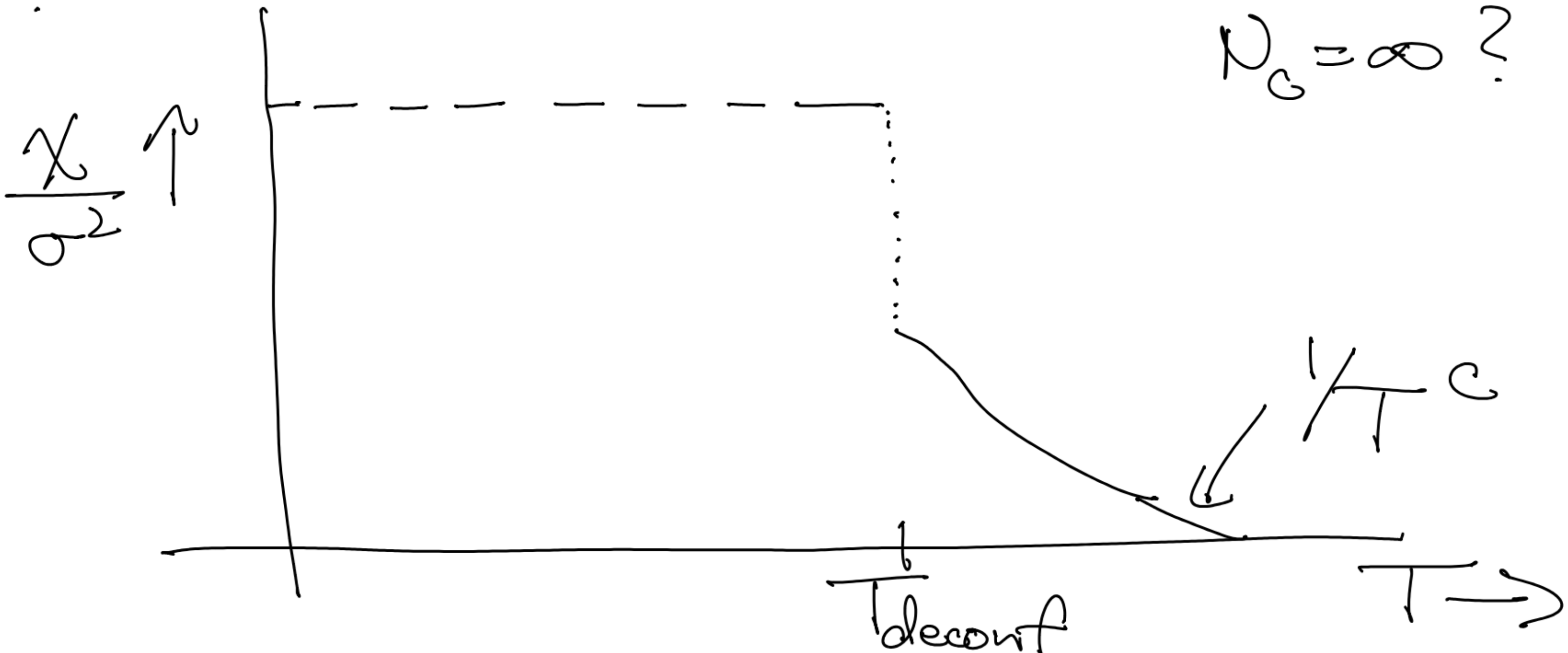
$$\chi \sim e^{-S_I} \sim e^{-8\pi^2/g^2(T)} \qquad \#/T^c \qquad T \to \infty$$

$$g^2 \sim \frac{8\pi^2}{c \ln T}$$

True for $N_c = 3$, $T \gtrsim 300$ MeV

Larger N_c?

$N_c = \infty$?



$$\chi = \frac{\partial^2}{\partial \theta^2} F(\theta)$$

$CP(N-1)$, 1+1 dim's

z^i, $i = 1 \dots N$, $z^i \to \omega z^i$

$$\mathcal{L} = \frac{1}{g^2} \sum |D_\mu z^i|^2$$

$D_\mu \to \partial_\mu - i A_\mu$

$z^i \to U^i_j z^j \Rightarrow SU(N)$

<u>but</u> if $U = e^{2\pi i j/N} \mathbb{1}$, $z^i \to e^{2\pi i j/N} z^i$

$e^{2\pi i j/N} \to \sim \omega \Rightarrow$ same

Sym. $= SU(N)/Z(N)$

A_μ not dynamical, $Q_{top} = \frac{1}{2\pi} \int d^2x\, \varepsilon^{\mu\nu} \partial_\mu A_\nu$

Soluble as $N \to \infty$, $m \sim e^{-\#/g^2 N}$

Witten '79 d'Adda, Luscher, Di Vecchia

Classical Instantons

Soln's of $\quad D_\mu^2 z = 0 \qquad A_\mu = -\frac{i}{2}\left(\bar{z}\partial_\mu z - \partial_\mu \bar{z}\, z\right)$ known.

Self-dual, $D_\mu z = i\,\varepsilon_{\mu\nu} D_\nu z$

$$z^i \sim (x_1 - i x_2)\, u^i \Big/ \sqrt{x_1^2 + x_2^2 + \rho^2}$$

Scale inv. $\Rightarrow$ scale size $\rho : 0 \to \infty$

$S_I \sim N \Rightarrow$ small, $\sim e^{-N}$ as $N \to \infty$

(N.B.: moduli space not simple except for $N = 2 \Rightarrow O(3)$)

But direct calc. shows as $N \to \infty$, $\chi_{top} \sim 1$, not e^{-N} ?

Quantum Inst.

As $N \to \infty$, $$S_{eff} = N \,\mathrm{tr} \ln \left(-D_\mu^2 + i\lambda\right) - i \int \frac{\lambda}{g^2}$$

Eqs of motion:

$$\mathrm{tr}\, D_\mu \frac{1}{-D_\mu^2 + m^2} = 0 \qquad A_\mu$$

$$N \,\mathrm{tr} \frac{1}{-D_\mu^2 + m^2} = \frac{i}{g^2} \qquad i\lambda = m^2(x)$$

Vacuum: $A_\mu = 0$, $m^2 = e^{-\#/g^2 N}$ = confinement scale

λ = constraint field, $\sim \lambda(|z|^2 - 1)$

Quantum Inst.'s

If $A_\varphi \sim \frac{Q}{r}$, $r \to \infty$, $Q_{top} = Q$.

Very hard finding soln. – but presumably $S_{eff}(Q) > S_{eff}(vac)$

$\Rightarrow$ exponentially supp.'d.

But – size of Q.I. fixed, $\sim 1/m^{-1}$

Scale invariance broken dyn.'y by $m \neq 0$

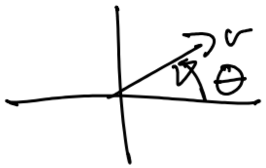
N.B.: possible that $S_{eff}(Q) - S_{eff}(vac) = N * 0 + O(N^0)$

Witten '78

If so, not obvious

Fractional Q.I.'s

Consider $\quad z^i(r,\theta) = e^{i\theta/N} f^i(r) \Rightarrow z(2\pi) = e^{2\pi i/N} z(0)$



Multi-valued, but consistent $\bar{c}$ $Z(N)$ sym.

$A_\mu \sim \frac{1}{N} h(r)$. $\quad$ Eqs of motion: $\quad h \to 0 \quad r \to 0$

$h \to \text{const} \quad r \gg \frac{1}{m}$

Ansatz constructed so $\quad Q = \frac{1}{N}$.

<u>Not</u> self-dual: natural mass scale $\sim 1/m$

Frac. Q.I.'s

With $A_\mu^{frac} \sim \frac{1}{N}$, expand in A_μ @ large N.

As A_μ^{frac} small, take $i\lambda(x) = m^2$,

$$S_{eff} = S_{eff}^{vac} + N \int A_\mu^{frac} \underbrace{D_\mu \frac{1}{-D^2+m^2}}_{=0} + \underbrace{N \iint A_\mu^{fr} \Delta_{\mu\nu} A_\mu^{fr}}_{1/N} + \ldots$$

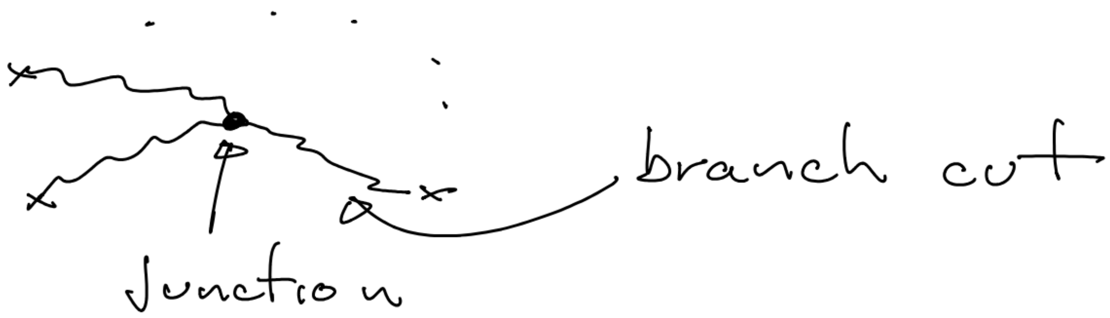
Not easy solving

But action $\sim 1/N$ not ~ 1.

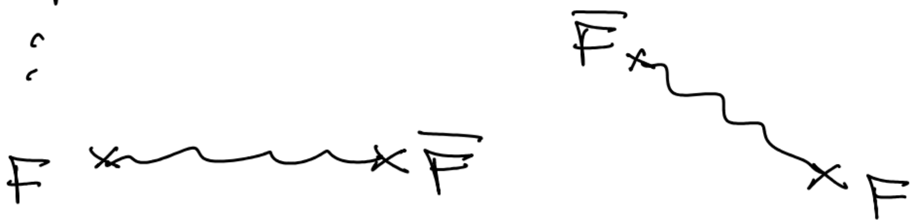
$\Rightarrow \chi(\theta) \sim 1$ dominated by interactions btwn frac. Q.I.'s

Frac. Q.I.'s

Instanton: (I)



Is vacuum superposition of I & $\bar{I}$'s or of frac.'y chg. F's:



Unsal 2007.03880

SU(N) gauge, 3+1, usual

$A_0 = 0$ gauge, coupling $g = 1$

$$A_i^{\mathcal{U}} = i\, \mathcal{U}^\dagger D_i\, \mathcal{U} \qquad D_i \longrightarrow \partial_\mu - i A_\mu$$

Consider $x : 0 \to 1$

$$A_i(\bar{x}, x) = (1 - x)\, A_i(\bar{x}) + x\, A_i^{\mathcal{U}}(\bar{x})$$

If $\Omega(\bar{x} \to \infty) = 1$, usual instanton

$$Q_{top} = \frac{1}{8\pi^2} \int \mathrm{tr}\, F\tilde{F} = \frac{1}{24\pi^2} \int d^3x\, \mathrm{tr}\,(\mathcal{U}^\dagger \partial\, \mathcal{U})^3$$

$= \underline{\text{integer}}$

$Z(N)$ vacua

But: $U \to e^{2\pi i j/N}\mathbb{1}$ as $\bar{x}\to\infty$ if <u>NO</u> dynamical gks

$$= e^{\frac{2\pi i j}{N}\begin{pmatrix}\mathbb{1}_{N-1} & \\ & -(N-1)\end{pmatrix}\varphi} \qquad U(1) = e^{2\pi i j/N}$$

Need U as fnc. of $r=\sqrt{x^2+y^2}$ & φ

$$Y_{ij} = \frac{\sigma_i \hat{x}}{2} + \frac{1}{N} - \frac{1}{2} \quad i,j=1,2 \quad ; \quad Y_{ij} = \frac{\delta^{ij}}{N} \quad i,j=3\ldots,N$$

$$U(\bar{x},\varphi) = e^{iY\theta(r,\varphi)} \qquad \begin{aligned}\theta &= 0 \ @\ r=0,\ \varphi=0\\ &= 2\pi\varphi \ @\ r=\infty\end{aligned}$$

Top. Chg

$$F \Rightarrow U^{+} F U + \frac{\partial}{\partial x} A^{u} = U^{+} (F - Da) U$$

Variation in x like "time", $a = \frac{d\Omega}{dx} \Omega^{-1}$

$a \sim A_0$

$$Q_{top} = \frac{1}{8\pi^2} \int tr (F - Da)^2 \, d^4x = \frac{1}{4\pi^2} \int_{x=\infty} d^3x \int dx \; tr(aF)$$

Need magnetic chg. With

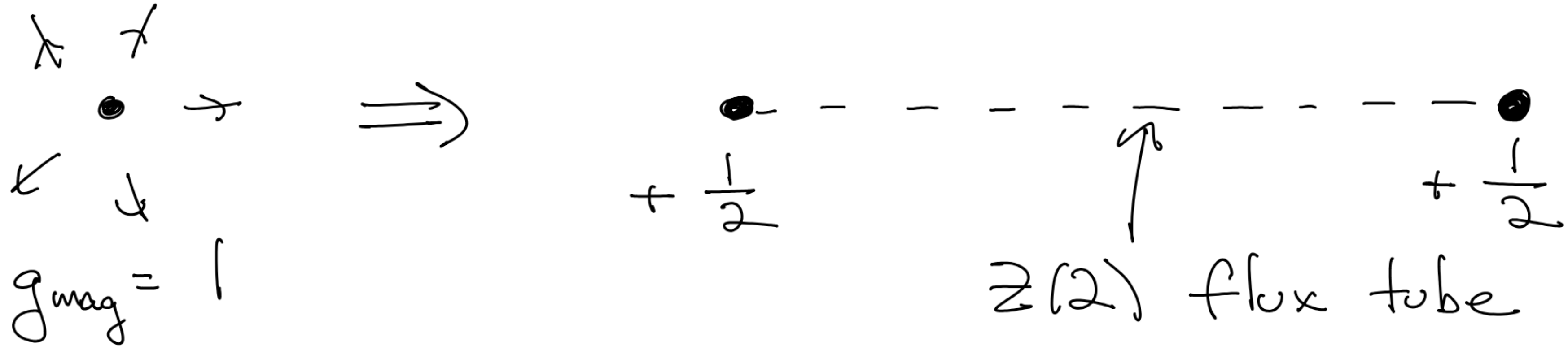
$$F_{ij} \sim -\frac{i}{2} \sigma \cdot \hat{x} \; \varepsilon_{ijk} \frac{\hat{x}^k}{r^2}$$

Q_{top} = integer

So?

"Split" $\mathbb{Z}_2$ monopole

Take $SU(2)$:



Without quarks, $Z(2)$ flux tube invisible.

Each end has $Q_{top} = +\frac{1}{2}$. To observe each half, need $Z(2)$ twisted boundary conditions.

$Z(N)$ dyon

At temp, $T > T_{deconf}$. <u>No</u> qks, adopt gauge $\bar{c}$ $A_0 \neq 0$

$$A_0^\infty = \frac{2\pi T}{N} k \; ; \quad k = k_1 = \begin{pmatrix} \mathbb{1}_{N-1} & \\ & -N+1 \end{pmatrix}, \quad k_2 = \begin{pmatrix} -N+1 & \\ & \mathbb{1}_{N-1} \end{pmatrix}$$

$$\mathbb{L} = e^{i \int A_0 dt} = e^{\frac{2\pi i}{N} k}$$

k = nontrivial holonomy, minimum of holonomous potential

$$A_0 = \frac{2\pi T}{N} q \; , \quad V_{1\text{-loop}}(q) = |q|^2 (1 - |q|)^2$$

$q \to q \bmod 1$

$Z(N)$ monopole

$$A^{\pm}_{\phi} \underset{r\to\infty}{=} \frac{1}{N\,2r}\, m\, \frac{(\pm 1 - \cos\theta)}{\sin\theta} = \frac{1}{N} * \text{Dirac monopole}$$

$$m \sim k_1, k_2$$

$$e^{i\oint A^+}\, e^{-i\oint A^-} = e^{\frac{2\pi i}{N} m}$$

$$r\to\infty: \quad A_0(r) = \frac{2\pi T}{N} k - \frac{1}{2Nr} m + \ldots$$

Assume reg. soln. $\forall\, r$, static soln.

$Z(N)$ dyon

$$Q_{top} = \frac{1}{4\pi^2}\int \partial_i \, \mathrm{tr}\,(A_0 B_i) = \frac{1}{N^2}\, m \cdot k$$

$m = k = k_1$: $\quad Q = \frac{N-1}{N}$ $\qquad$ $m = k_1$, $k = k_2$ $\quad Q = -\frac{1}{N}$

Same as 't Hooft $\bar{c}$ $Z(N)$ twisted b.c.'s

$\neq$ Kraan, van Baal, Lee, Lu '98

There: <u>maximum</u> of holonomous pot. Here: minimum

" : integral magnetic chg. " : fractional

$Z(N)$ dyon

$T > T_d$: magnetic chg. confined. $\Rightarrow$

$Z(N)$ dyon $\bar{c}$ frac. top. chg. confined

$T < T_d$: $Z(N)$ dyons propagate freely

Geometry of configurations: dyons twisted in space-time - ?

Presumably - size $\sim$ confinement scale (as for $CP(N-1)$)

Femto-slabs (tori...)

Unsal ..., 2007.03880 Poppitz ... 2111.10423

Femto-slab: one spatial dimension $r_s \ll \Lambda_{QCD}^{-1} \Rightarrow g^2(r_s) \ll 1$

Reduces to 2+1 dim's (Polyakov '77)

Monopole-instantons $\bar{c}$ $Q_{top} = \frac{1}{N}$ <u>ubiquitous</u>

Size $\sim r_s$. What happens as $r_s \sim \Lambda_{QCD}^{-1}$?

Their size gets stuck; $\approx$ fixed size

(If $\sim \Lambda_{QCD}^{-1}$, geometry not simple: <u>not</u> necessarily point like)

Lattice - pure glue

To measure $Q_{top} \sim \frac{1}{N}$, use χ-sym quark prop. (as external probe only) in adjoint (not fond.) representation

Edwards, Heller, Narayanan lat/9806011

For $Q_{top} = \frac{1}{N}$, $\bar{c}$ adjoint prop. see one zero mode

From eigenvector, estimate size: if $r \sim \Lambda^{-1}_{QCD}$, big!

Are objects $\bar{c}$ $Q \sim 1/N$ dilute, or densely packed?

(Fodor +... 0905.3586: SU(3) $\bar{c}$ sextet rep.

No frac. Q, but then $Q_{top} \sim \frac{1}{5}$, not $\frac{1}{3}$)

With dynamical qks?

Frac. top. chg. $\Rightarrow$ $Z(N)$ mag. chg.

Dynamical qks see $Z(N)$ vortices $\Rightarrow$ confine $Z(N)$ mag. chg.

$\mu = 0$, $T \neq 0$: $T_{ch} \sim 150$ MeV $\quad T_{deconf} \sim 300$ (pure glue)

$T < T_{ch}$: $\chi \sim$ frac. inst.'s + massive qks

$T_{ch} < T < T_{deconf}$: $\chi \sim$ frac. inst.'s + massless qks

$T_{deconf} \lesssim T$: $\chi \sim$ instantons

$T \neq 0$ vs $\mu \neq 0$

Lattice: instantons dominate for > 300 MeV

Instantons suppressed by Debye mass

$$m_D^2 = g^2 \left(\frac{1}{3} \underbrace{\left(N_c + \frac{N_f}{2} \right)}_{\text{big}} T^2 + \underbrace{2 N_f \left(\frac{\mu_{qk}}{2\pi} \right)^2}_{\text{small}} \right)$$

Insti's don't dominate until very large $\mu_{qk} > 2$ GeV

RDP + Rennecke 1910.14052

$\Rightarrow$ for μ_{qk}: 300 MeV $\to$ 2 GeV,

fractional instantons dominate?

Frac. insti's? Need <u>very</u> fine lattices $\bar{c}$ <u>pure</u> gauge

Start $\bar{c}$ SU(2), in vacuum, then near T_{deconf}

Karthik, Sharma, Narayanan

With dyn. quarks?

$T = 0$, $\mu \neq 0$: analyze in quarkyonic regime?