STRONG-2020

QCD in strong magnetic background field

Lorenzo Maio

Università di Pisa and INFN sezione di Pisa

May 26^{th}, 2022

UNIVERSITÀ DI PISA

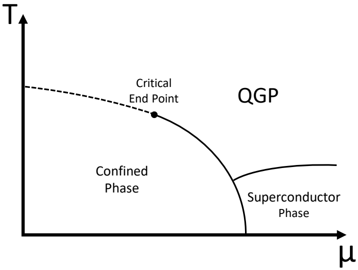
INFN PISA Istituto Nazionale di Fisica Nucleare Sezione di Pisa

Collaborators: Massimo D'Elia, Francesco Sanfilippo and Alfredo Stanzione

Contents

Lorenzo Maio

Introduction



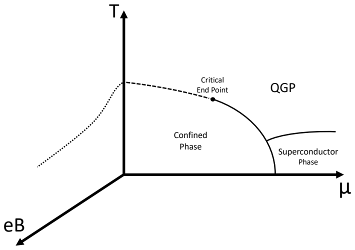
Quantum Chromodynamics (QCD) is the quantum field theory describing strong interactions. The fundamental degrees of freedom of the theory are quark and gluons, which carry color charge.

The strong interacting matter described by the theory can exist in different phases.

At ordinary temperatures and baryonic chemical potential it is in confined phase: quark and gluons cannot freely propagate.

At high temperatures and chemical potentials, the model undergoes a phase transitions to a deconfined phase called Quark Gluon Plasma (QGP).

The phase diagram in this plane is known to have a critical line and an end point.



When a magnetic field is present, it is able to interact with the matter fields which carry electric charge (i.e. quarks).

The vacuum properties are affected by the external field. Its effects are relevant even in the pure gauge sector of the theory, because of vacuum polarization effects.

The magnetic field, as well as the chemical potential, has effects on the thermodynamics of the theory. A drop in the critical temperature is the first signature of its presence.

Different predictions suggest that at extremely strong magnetic field intensities, the thermal crossover is expected to turn in a discontinuous phase transition.

Strong interacting matter can be found under such extreme conditions in the first stages of the Early Universe, in Magnetars and in heavy ion collisions.

Magnetic field on the lattice

Continuum Dirac Operator in the presence of Abelian and non-Abelian gauge fields:

$$\overline{\psi}^f(x) D^f_\mu(x) \psi^f(x) = \overline{\psi}^f(x) \left(\partial_\mu + igG^a_\mu(x) T^a + iq^f A_\mu(x) + m^f \right) \psi^f(x)$$

The discretization can be performed through the following substitution in the usual LQCD quark action

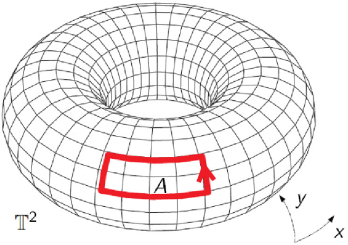
$$U_{i;\mu} \to u^f_{i;\mu} U_{i;\mu}$$

Where the Abelian phases $u^f_{i;\mu}$ are defined as follows

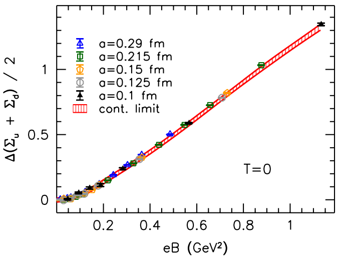
$$u^f_{i;y} = e^{ia^2 q_f B_z i_x}, \qquad u^f_{i;x}|_{i_x=L_x} = e^{-ia^2 q_f L_x B_z i_y}$$

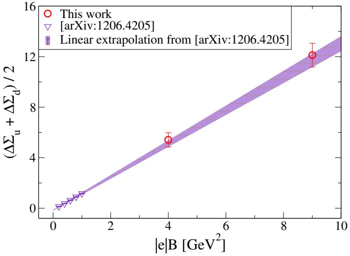
And are quantized in order to avoid ambiguities in the definition:

$$eB = \frac{6\pi b_z}{a^2 L_x L_y}, \quad \text{with} \quad b_z \in \mathbb{Z} \quad \text{and} \quad b_z < \frac{L_x L_y}{6}.$$



Effects on the QCD vacuum





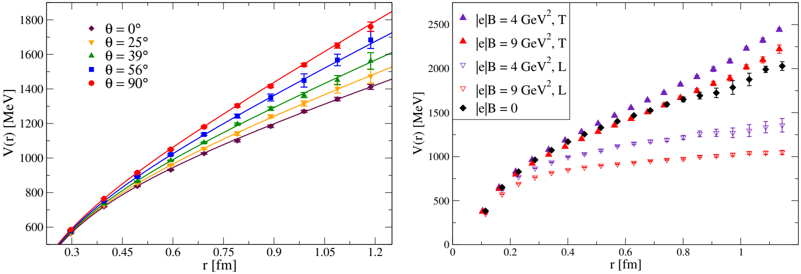
$$\Delta\Sigma(B,T)_f = \frac{2m_f}{M_\pi^2 F^2}\left[\overline{\psi}\psi(B,T)_f - \overline{\psi}\psi(0,0)_f\right], \qquad \overline{\psi}\psi(B,T)_f = \frac{T}{V}\frac{\partial \log Z(B,T)}{\partial m_f}$$

The presence of a magnetic field enhances the chiral symmetry brake in QCD. It can be seen in the growth of the chiral condensate.

This effect is named *“chiral catalysis”*. It can be predicted making use of the Lowest Landau Levels approximation and the Banks-Casher relation.

G. S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S. D. Katz and A. Schafer, Phys. Rev. D **86**, 071502 (2012) [arXiv:1206.4205 [hep-lat]].

M. D’Elia, LM, F. Sanfilippo and A. Stanzione, Phys. Rev. D **104** (2021) no.11, 114512 [arXiv:2109.07456 [hep-lat]].



The QCD potential between a $Q\overline{Q}$ pair placed at distance r can be parameterized in first approximation as

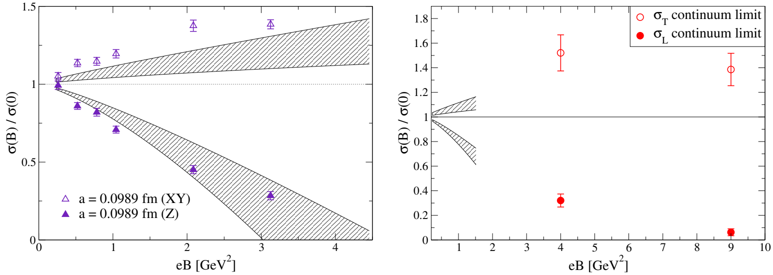
$$V(r) = V_0 + \frac{\alpha}{r} + \sigma r$$

We can study this observable via quark-antiquark correlation function. In the infinite quark mass limit, Wilson loops in the rt-plane have the suitable quantum numbers.

$$aV(r) = \lim_{n_t \to \infty} \log \left(\frac{\langle W(r, an_t) \rangle}{\langle W(r, an_t + 1)) \rangle} \right)$$

C. Bonati, M. D'Elia, M. Mariti, M. Mesiti, F. Negro, A. Rucci and F. Sanfilippo, Phys. Rev. D **94** (2016) no.9, 094007 [arXiv:1607.08160 [hep-lat]].

M. D'Elia, LM, F. Sanfilippo and A. Stanzione, Phys. Rev. D **104** (2021) no.11, 114512 [arXiv:2109.07456 [hep-lat]].



The coefficient (σ) of the linear term in the Cornell ansatz is named *"string tension"* and it represents the confining potential strength. In the $\sigma \to 0$ limit, the Cornell potential is equal to the Coulomb potential.

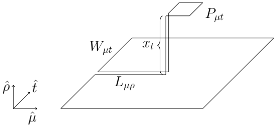
The magnetic background field affects the string tension, causing an enhancement for $Q\bar{Q}$ separations on the transverse plane, and a drop in the longitudinal direction.

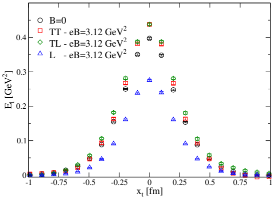
Anisotropic deconfinement could arise for even stronger magnetic field intensities.

C. Bonati, M. D'Elia, M. Mariti, M. Mesiti, F. Negro, A. Rucci and F. Sanfilippo, Phys. Rev. D **94** (2016) no.9, 094007 [arXiv:1607.08160 [hep-lat]].

M. D'Elia, LM, F. Sanfilippo and A. Stanzione, Phys. Rev. D **104** (2021) no.11, 114512 [arXiv:2109.07456 [hep-lat]].

Effects on the QCD vacuum





The QCD flux tube allows us to study the field strength tensor in the presence of two color sources

$$\rho^{\mu t}_{conn}(x_t) = \frac{\langle Tr(W(an_\mu, an_t)LP^{\mu t}(x_t)L^\dagger)\rangle}{\langle Tr(W(an_\mu, an_t))\rangle} - \frac{\langle Tr(W(an_\mu, an_t))Tr(P^{\mu t}(x_t))\rangle}{3\,\langle Tr(W(an_\mu, an_t))\rangle}$$

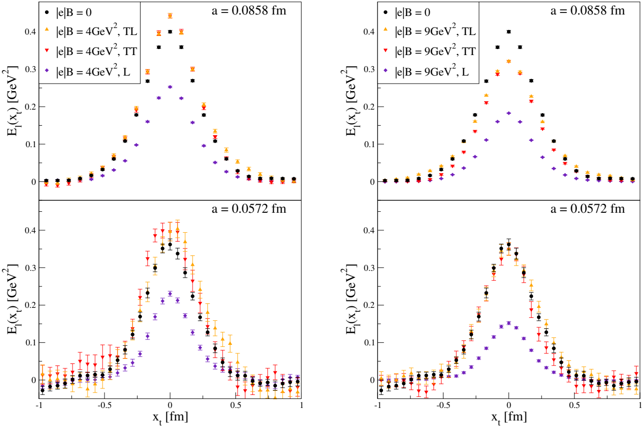
In the continuum limit it can be shown that it reduces to

$$\rho^{\mu t}_{conn} \propto \frac{\left\langle Tr\left[iW(d)LF^{\mu t}(x_t)L^\dagger\right]\right\rangle}{\langle Tr\, W\rangle} = E_l(d, x_t)$$

Where E_l is the longitudinal component of the chromoelectric field between a $Q\bar{Q}$ pair with separation d.

C. Bonati, S. Calì, M. D'Elia, M. Mesiti, F. Negro, A. Rucci and F. Sanfilippo, Phys. Rev. D **98** (2018) no.5, 054501 [arXiv:1807.01673 [hep-lat]].

Effects on the QCD vacuum



The asymmetry which is present in the static potential, is clearly visible also in the flux tube profile.

As expected from the string tension results, the confining chromoelectric field doesn't annihilate, not even in the strongest explored magnetic field.

M. D'Elia, LM, F. Sanfilippo and A. Stanzione, Phys. Rev. D **104** (2021) no.11, 114512 [arXiv:2109.07456 [hep-lat]].

Simulations at finite temperature

Exploiting the equivalence between the path integral formulation of the QFTs and the statistical mechanics, we can perform simulations at finite temperature.

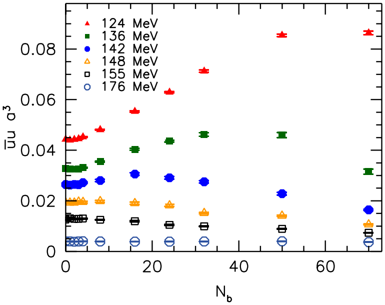
It can be done through the imposition of periodic boundary conditions on the Euclidean time direction of the lattice.

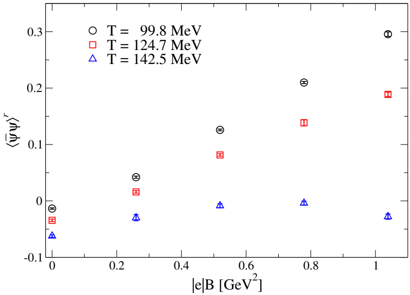
Then the temperature is linked to the physical extent of the time axis according to

$$T(\beta, m_l, m_s, N_t) = \frac{1}{a(\beta, m_l, m_s)N_t} \qquad (\text{with} \quad K_B = 1).$$

So, different temperatures can be probed both changing the lattice spacing a (suitably tuning the parameters) and changing the lattice sites number in the temporal axis N_t.

High temperature effects



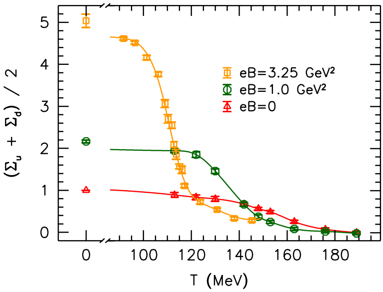


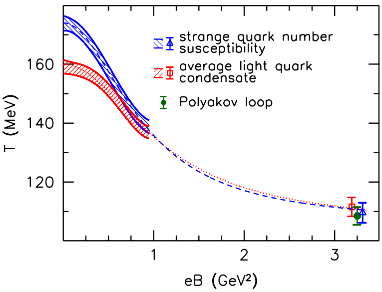
The magnetic catalysis is known to revert when the temperature approaches the pseudo critical region in which the crossover transition to the QGP phase takes place.

The stronger the magnetic field is, the more relevant the *"reverse chiral catalysis"* is. This effect combined with the chiral catalysis observed at low temperatures, results in a "strengthening" of the transition with the magnetic field.

G. S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S. D. Katz, S. Krieg, A. Schafer and K. K. Szabo, JHEP **1202**, 044 (2012) [arXiv:1111.4956 [hep-lat]].

C. Bonati, M. D'Elia, M. Mariti, M. Mesiti, F. Negro, A. Rucci and F. Sanfilippo Phys. Rev. D **94**, 094007 (2016) [arXiv:1607.08160 [hep-lat]].



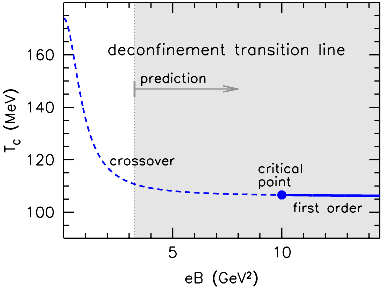


In this picture the strengthening of the transition is even clearer and another effect is highlighted: a strong background magnetic field causes a drop in the (pseudo) critical temperature.

The critical temperatures looking at different order parameters are closer, meaning that the crossover transition is slowly going to become a real and discontinuous phase transition.

G. Endrodi, JHEP **1507**, 173 (2015) [arXiv:1504.08280 [hep-lat]].

High temperature effects

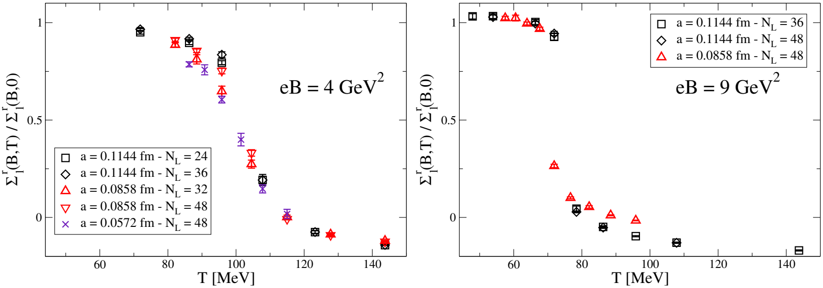


The same author provided a speculative proposal for the position of the critical point in the QCD phase diagram in the (eB, T) plane.

The existence of a first order transition in a strong field regime is deduced from a model of anisotropic pure gauge theory, which is expected to approximate the $eB \gg \Lambda_{QCD}^2$ limit of QCD.

The localization of the critical point is based on an extrapolation to zero of the width of the susceptibility peak at the transition. No direct observations were performed.

G. Endrodi, JHEP **1507**, 173 (2015) [arXiv:1504.08280 [hep-lat]].



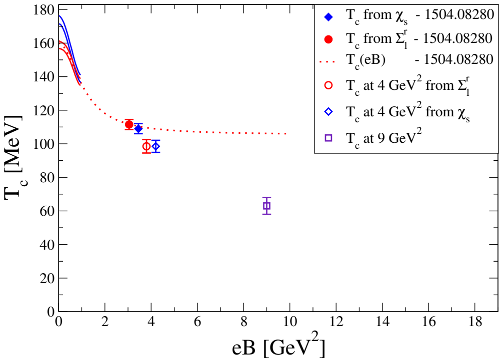
We performed simulations at higher values of the magnetic field, the same of the $T = 0$ section, bringing up to $eB = 9$ GeV^2 the limit of the direct observations.

The critical temperature for the transition is still dropping down to lower values as the magnetic field is increased.

The step in the chiral condensate across the transition looks steeper as the field grows, signaling that the phase transition could actually be a first order.

M. D'Elia, LM, F. Sanfilippo and A. Stanzione, Phys. Rev. D **105** (2022) no.3, 034511 [arXiv:2111.11237 [hep-lat]].

High temperature effects



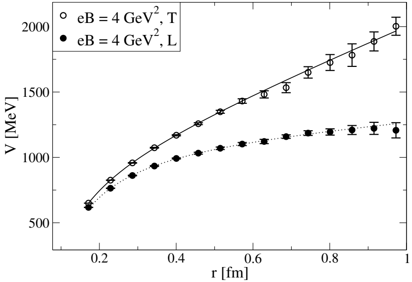
The drop in T_c that we observe is much higher than the predicted one.

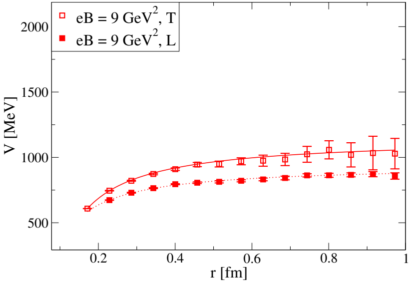
An extrapolation on these data to $T_c = 0$ would return the deconfining magnetic field intensity $eB \simeq 18$ GeV^2.

There is no a ~ 4 GeV energy scale in QCD which would explain such a value for a deconfining magnetic field.

M. D'Elia, LM, F. Sanfilippo and A. Stanzione, Phys. Rev. D **105** (2022) no.3, 034511 [arXiv:2111.11237 [hep-lat]].

High temperature effects



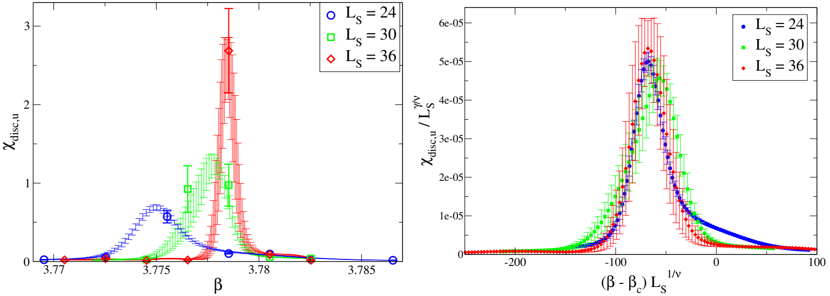


Looking again at the static potential, we found that it is compatible with a Coulomb description across the transition temperature.

An asymmetry is still present, but both in the longitudinal and transverse directions, this picture is compatible with deconfinement at $eB = 9\ \text{GeV}^2$.

M. D'Elia, LM, F. Sanfilippo and A. Stanzione, Phys. Rev. D **105** (2022) no.3, 034511 [arXiv:2111.11237 [hep-lat]].

High temperature effects



Because of the fixed scale prescription, a study of the nature of the transition at $eB = 9$ GeV^2 on the same constant physics line was not affordable with the available computational resources.

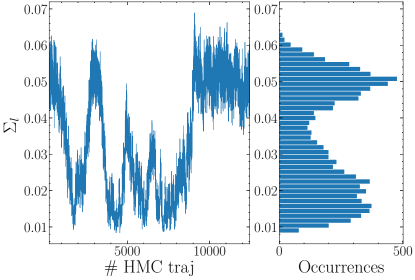
Since the presence of a first order transition is stable under small variations of the parameters, we performed a fine tuning in order to cross the critical line.

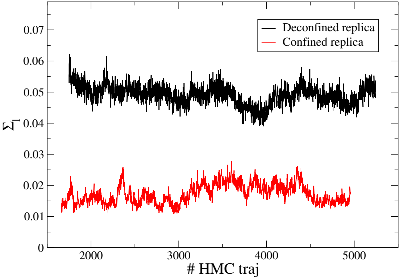
Using as starting point a simulation performed at a temperature and magnetic field close to the transition, we slightly changed β until the transition occurred.

Using this prescription, a finite size scaling was now affordable without lack in reliability, and it clearly shows a first order transition.

M. D'Elia, LM, F. Sanfilippo and A. Stanzione, Phys. Rev. D **105** (2022) no.3, 034511 [arXiv:2111.11237 [hep-lat]].

High temperature effects





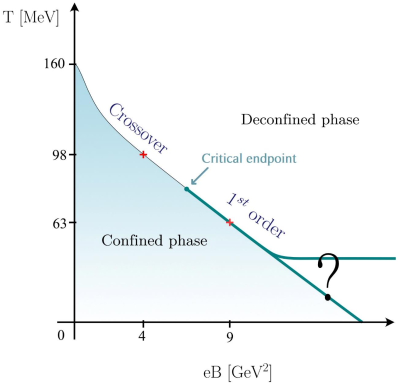
Other smoking guns of a first order transition are found on these samples.

A simulation running on the pseudo critical value of β shows indeed a bi-stable history of the chiral condensate.

At a larger volume the tunneling probability becomes small and simulations with the same parameters but different starting points show parallel histories.

M. D'Elia, LM, F. Sanfilippo and A. Stanzione, Phys. Rev. D **105** (2022) no.3, 034511 [arXiv:2111.11237 [hep-lat]].

Phase diagram proposal



We propose a new hint for the phase diagram of the QCD in the (eB, T) plane, based upon our results.

Since the nature of the transition appears to be a crossover at $eB = 4$ GeV^2 and a first order transition at $eB = 9$ GeV^2, the critical endpoint should be located somewhere in that interval.

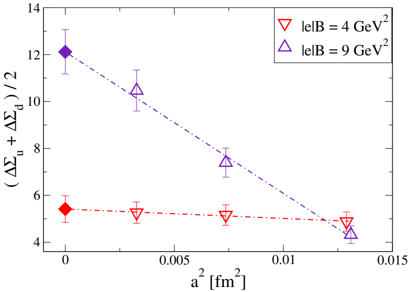
M. D'Elia, LM, F. Sanfilippo and A. Stanzione, Phys. Rev. D **105** (2022) no.3, 034511 [arXiv:2111.11237 [hep-lat]].

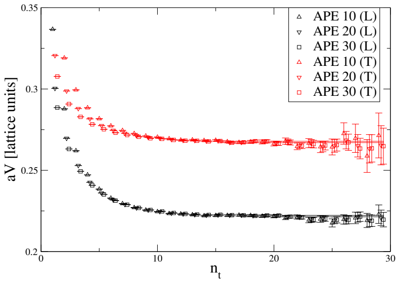
Summary

- Chiral condensate keeps growing linearly with the magnetic field up to $eB = 9$ GeV^2;
- Confinement properties are heavily affected by strong magnetic fields at vanishing temperature;
- The string tension is not vanishing in the magnetic field direction for the $|e|B = 4$ GeV^2 case, and it shows a $\sim 1.7\sigma$ tension with 0 in the $|e|B = 9$ GeV^2 case;
- The drop in T_c for $|e|B = 9$ GeV^2 is larger than expected;
- We found an analytical crossover transition at $eB = 4$ GeV^2, and a first order transition at $eB = 9$ GeV^2;
- A critical endpoint is expected to lie between this two magnetic fields at a temperature in the range $T_{CEP} \in (63, 98)$ MeV.

Thanks for your attention!

email: lorenzo.maio@phd.unipi.it


|e|B = 4 GeV^2
|e|B = 9 GeV^2
(ΔΣ_u + ΔΣ_d) / 2
a^2 [fm^2]
14
12
10
8
6
4
0
0.005
0.01
0.015


APE 10 (L)
APE 20 (L)
APE 30 (L)
APE 10 (T)
APE 20 (T)
APE 30 (T)
aV [lattice units]
0.35
0.3
0.25
0.2
0
10
20
30
n_t

Backup slides